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Author
Siddiqui, Afzal S.

Publication Date
2000-10-01

LBNL-47645

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Afzal S. Siddiqui

Environmental Energy Technologies Division
Ernest Orlando Lawrence Berkeley National Laboratory
Berkeley, California 94720

December 2000

Presented at the Institute for Operations Research and the Management Sciences (INFORMS) Annual Meeting in San Antonio, TX, November 2000

The work described in this study was funded by the Assistant Secretary of Energy Efficiency and Renewable Energy, Office of Power Technologies of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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Afzal S. Siddiqui
Environmental Energy Technologies Division
Ernest Orlando Lawrence Berkeley National Laboratory
Berkeley, California 94720
email: ASSiddiqui@lbl.gov

Abstract

In competitive electricity markets, the vertically integrated utilities that were responsible for ensuring system reliability in their own service territories, or groups of territories, often cease to exist. Typically, the burden falls to an independent system operator (ISO) to ensure that enough ancillary services (AS) are available for safe, stable, and reliable operation of the grid, typically defined, in part, as compliance with officially approved engineering specifications for minimum levels of AS. In order to characterize the behavior of market participants (generators, retailers, and an ISO) in a competitive electricity market with reliability requirements, we model a spot market for electricity and futures markets for both electricity and AS. By assuming that each participant seeks to maximize its expected utility of wealth and that all markets clear, we solve for the optimal quantities of electricity and AS traded in each market by all participants, as well as the corresponding market-clearing prices. We show that futures prices for both electricity and AS depend on expectations of the spot price, statistical aspects of system demand, and production cost parameters. More important, our model captures the fact that electricity and AS are substitute products for the generators, implying that anticipated changes in the spot market will affect the equilibrium futures positions of *both* electricity and AS. We apply our model to the California electricity and AS markets to test its viability.

Keywords: Ancillary services, competitive electricity markets, pricing futures contracts.

1. Introduction

Once thought of as a "natural monopoly" industry, i.e., one in which cost declines with output, thereby rendering competition infeasible, the electric power sector is now undergoing policy and regulatory changes intended to foster competition. In the U. S., the electric power industry has predominantly been vertically integrated with its various functions conglomerated under the auspices of an investor-owned utility company that holds an exclusive franchise to provide services to a certain geographic area. According to [1], the four main electricity supply functions provided by a utility are:

- **generation**: conversion of primary energy to electricity.
- **transmission**: transportation of electricity along meshed high-voltage wires to substations.
- **distribution**: transportation of electricity along low-voltage wires to customer meters.
- **retailing**: arrangements for billing and demand management.

California was among the first U. S. states to deregulate its electric power sector. Similar to electricity market reforms in other regions of the world, the changes in California's industry included *unbundling* the various services previously offered by its three major incumbent investor-owned electricity utilities. Now, instead of allowing these utilities to control all aspects of electricity supply, California state legislators passed Assembly Bill (AB) 1890 which separates the industry into:

1. a competitive part, consisting of the generation and retail functions, and
2. a regulated monopoly structure that retains control over the transmission and distribution systems.

Two non-profit corporations were created: the California Independent System Operator (CAISO) and the California Power Exchange (CalPX). The former provides system control services to all electricity suppliers, while the latter operates forward competitive energy markets from which distribution companies must buy electricity for their retained retail customers for a transitional period, and into which generators sell. In addition, the CAISO manages a real-time imbalance energy market, which is essentially a spot market for wholesale electricity. In order to balance energy and safeguard the reliability of the grid, reserve generating capacity, known as ancillary services (AS), is required under North American Electric Reliability Council (NERC) and Western System Coordinating Council (WSCC) rules. AS are procured by the CAISO in competitive day- and hour-ahead markets.

Alternatively, each scheduling coordinator (SC) can choose to supply its own AS, and the Federal Energy Regulatory Commission (FERC) has approved a block forward AS market operated by CalPX. Nonetheless, to date, virtually all AS have been procured in the CAISO's first in the world open competitive AS markets. That is, the CAISO accepts generator offers and buys AS on behalf of almost all loads. The AS procured in this way are:

- **regulation service**: generation resources that are available and running, and can be used to maintain real-time energy balance.
- **spinning reserves**: generation resources that are running and synchronous with additional capacity available.
- **non-spinning reserves**: generation resources that are available quickly but not running.
- **replacement reserves**: generation resources that are capable of starting up and running for a sustained period.

In addition, *reactive power support* and *black-start generation capability* are AS that are procured through annual contracts. For a more complete description of the restructured California electricity industry, see either [2] or [3].

Regardless of the form of deregulation, it has been documented that introduction of competition in the electricity generation sector leads to some improvements in social welfare, as wholesale prices tend to be lower and labor productivity increases (see [4]). Along with greater economic efficiency in the generation sector, however, deregulation has also introduced new problems into an industry that was once insulated from the forces of the free market. Some of these issues include:

- **market power**: evidence exists that both the British and California wholesale electricity markets have at least the conditions that reward through higher prices the withholding of generation capacity from the market (see [5], [6], and [7]).
- **price volatility**: although some price volatility is to be expected in any competitive commodity market due to supply and demand fluctuations, seasonality, and lack of storability, there has been evidence that some price volatility may have been caused or exacerbated by firms exercising market power (see either [2] or [8]).
- **management of system reliability**: the vertically integrated utilities operated in a strictly hierarchical manner with a central controller responsible for marshalling all the resources necessary to operate the system reliably. Particularly, AS were self-provided by utilities based on engineering specifications *suggested* by regional reliability councils. In restructured markets, their procurement is typically the duty of an independent system operator (ISO) which is responsible for the transmission grid. Consequently, the AS markets must offer a "fair" price to generators in order for them to provide sufficient amounts of AS to the ISO.

While the first two issues have received considerable attention in the literature, the third one has not been addressed via an integrated model that acknowledges the fundamental link between the electricity and AS markets, viz., that electricity and AS are substitute products, thereby implying that the physical and financial characteristics of one market will have consequences for the other (see [9] for a thorough discussion). Indeed, the problem of managing system reliability in a competitive environment may be exacerbated if there is no market-based methodology for pricing AS. For example, in the California AS markets, numerous problems existed due to market design that did not account for the dependencies between the AS and electricity markets. Consequently, these markets experienced bid insufficiency, price spikes, price reversals, and in general, did not provide the CAISO with an effective method for procuring AS.

It is with this issue in mind that a spot market for electricity and futures markets for both electricity and AS in a perfectly competitive framework are modeled here. Our objective in solving the model is to assess all spot and futures prices and to determine what factors will affect the trading decisions of the various agents in the markets. We believe that this will enable market participants to have a clearer understanding of the relationship between the futures and spot prices. As a result, they will be able to make rational risk management decisions, thereby leading to better functioning AS markets. The structure of this paper is as follows:

- **Section 2** introduces the model of electricity production and markets.
- **Section 3** solves for the equilibrium prices and quantities traded in each market and discusses the intuition behind the resulting price structure.
- **Section 4** compares the results from Section 3 to California market data.
- **Section 5** summarizes the main results and gives direction for future research in this area.

2. Electricity Production and Markets

Since our objective is to assess equilibrium spot and futures prices for electricity and futures prices for AS, we require a model of these markets and of the transactions conducted there. We assume perfectly competitive[1] spot and futures markets for electricity and a futures market for one type of AS (as opposed to the four that actually exist in California). We analyze production decisions for only a single future time period because the non-storability of electricity creates markets that are independent over time. For simplicity, we assume that there is no uncertainty in real-time. Underlying this assumption is the fact that power companies are able to forecast demand in the immediate future, i.e., the next hour, with precision. Here, we also abstract from transmission constraints

[1]The degree to which the California electricity markets are competitive is open to debate. Our concern, however, is more with pricing once market mechanisms are fully in place.

by supposing that electricity can be transmitted costlessly. Of course, in reality transmission bottlenecks play a significant role in determining the pattern of electricity generation and pricing. However, our focus is on the short term strategies of market agents that will determine equilibrium prices rather than on congestion pricing. Although market agents are assumed to face no uncertainty while making decisions in the real-time spot market, this assumption is invalid if applied to the futures market. This supposition, together with risk aversion on part of market agents, implies that there will be considerable demand for futures contracts as market agents try to hedge their spot market positions.[2] In order to formalize this concept of risk averse agents, we assume that the objective of each market agent i is to maximize its expected utility of profit function, which is of the form $E_\omega[U(\pi_i(\omega))] \equiv E[\pi_i(\omega)] - \frac{A_i}{2} Var(\pi_i(\omega))$. Here, ω is a random variable that depicts the state of the world, which is unknown to the market agent when making futures market decisions (but is realized when making spot market decisions). Naturally, agent i's profit $\pi_i(\omega)$ depends on the state of the world. $A_i > 0$ is a risk aversion parameter that can differ across agent types.

Within this framework, we have three distinct types of agents who have various interests in the markets:

- $n_1 \in \mathcal{Z}_+$ **identical flexible generators**: generator p_i has $\alpha_{p_i} > 0$ megawatts (MW) of production capacity available for any given period. It can use this capacity either to generate electricity and sell it into the electricity spot or futures markets or to reserve the capacity and sell it into the AS futures markets. For selling the output from $X^S_{p_i}(\omega)$ MW of capacity into the electricity spot market, generator p_i receives the endogenously-determined electricity spot price $P^S_X(\omega)$. The futures market decisions are as follows: by selling the output from $X^F_{p_i}(\omega)$ MW of capacity into the electricity futures market, generator p_i receives the endogenously-determined electricity futures price $P^F_X(\omega)$. If it sells $Y^F_{p_i}(\omega)$ MW of capacity into the AS futures market, the generator receives the endogenously-determined per MW AS futures price $P^F_Y(\omega)$. This payment has two components: one compensates the provision of AS reserves, and the other compensates the actual generation from this reserved capacity.

- $n_2 \in \mathcal{Z}_+$ **identical inflexible generators**: generator p_j has $\alpha_{p_j} > 0$ megawatts (MW) of production capacity available for any given period. It can use this capacity only to generate electricity and sell it into the electricity spot or futures markets. For selling the output from $X^S_{p_j}(\omega)$ MW of capacity into the electricity spot market, generator p_j receives the endogenously-determined electricity spot price $P^S_X(\omega)$. In the futures market, it can sell the output from $X^F_{p_j}(\omega)$ MW of capacity into the electricity futures market. For this, generator p_j receives the endogenously-determined electricity futures price $P^F_X(\omega)$.

[2] If we assume that market agents are risk neutral, then there is little incentive for them to use futures contracts.

- $m \in \mathcal{Z}_+$ **identical retailers**: retailer r_k purchases electricity from the spot and futures markets and sells it to customers in its exclusive franchise area at a fixed unit price of $P_{r_k} \geq 0$. The total retail demand for electricity in its area, $X_{r_k}(\omega)$, is unknown at the time of the decision to purchase futures and must be satisfied, i.e., the retailer faces an obligation to serve a totally inelastic demand in the short run. This setup reflects the fact that in California, most end-use consumers do not yet see volatile spot prices; rather, they are guaranteed fixed per unit prices. The retailer, however, has to take the risk of purchasing from a volatile market. This would seem to imply that retailers would like to purchase futures contracts to lock in their purchase prices. Hence, retailer r_k's purchases in the spot and futures markets ($X^S_{r_k}(\omega)$ and $X^F_{r_k}(\omega)$, respectively) are used to meet its retail demand.

- **an ISO**: the ISO procures enough AS from the futures markets to comply with the minimum levels required for reliability, $Y_I(\omega)$. Usually in California, this implies that the amount of AS procured by the ISO is approximately a fixed percentage of overall electricity demand. The ISO, thus, acquires enough AS from the futures market ($Y^F_I(\omega)$) to meet its requirements.

As we shall show in Section 3, all agents act out of self-interest in order to maximize their respective expected utilities of wealth. Their interaction in the markets then determines the equilibrium prices for electricity and AS.

3. Market Trading

In this section, we solve the optimization problems of the four types of market agents using the market equilibrium approach presented in [10], where spot and futures markets for only electricity are considered. Since we have two types of markets, i.e., spot and futures, there are two time stages to the agents' problems. In actuality, agents first take positions in the futures markets based on their expectations of spot market conditions. Then, in real-time, as demand conditions are realized, market agents make spot market transactions (facing no uncertainty). We, however, begin by evaluating the agents' real-time, i.e., spot market, decisions taking futures positions and prices as fixed. Once we determine the spot market price and positions, we then step back in time to determine the optimal futures positions and equilibrium futures prices.

3.1. Spot Market

At this stage, ω has been realized, so there is no uncertainty. Furthermore, all of the futures positions ($X^F_{p_i}(\omega)$, $X^F_{p_j}(\omega)$, $X^F_{r_k}(\omega)$, $Y^F_{p_i}(\omega)$, and $Y^F_I(\omega)$) have been taken, and futures prices ($P^F_X(\omega)$ and $P^F_Y(\omega)$) determined. They are, thus, treated as fixed. We, therefore, solve for the spot market positions and price as each market agent i attempts to maximize its profits $\pi_i(\omega)$.

Applying the notation and assumptions of Section 2, we can express the profit-maximization problem of **flexible generator** p_i:

$$\pi^*_{p_i}(X^S_{p_i}(\omega)) = \max_{X^S_{p_i}(\omega)} \{P^S_X(\omega)X^S_{p_i}(\omega) + P^F_X(\omega)X^F_{p_i}(\omega) + P^F_Y(\omega)Y^F_{p_i}(\omega) - \frac{\theta}{2\alpha_{p_i}}(X^S_{p_i}(\omega) + X^F_{p_i}(\omega) + f(\omega)Y^F_{p_i}(\omega))^2\} \quad (1)$$

where $\pi^*_{p_i}(\cdot)$ is the maximized profit level, $\theta > 0$ is the per MW input (e.g., fuel) costs, and $0 \leq f(\omega) \leq 1$ denotes the fraction of AS capacity sold that is called upon to generate.[3] Fuel cost is incurred only for actual electricity generation, i.e., to produce electricity sold as energy, and to operate any AS capacity that is specifically required by the ISO to generate. Furthermore, the cost term exhibits the quadratic form, which implies increasing marginal costs of generation. Intuitively, this models the fact that as demand increases, less efficient sources of generation are brought on line. For the purposes of this model, we assume that continuous quadratic functions reasonably approximate generation costs, even though actual generation costs may be discontinuous. Analogously, the profit-maximization problem of **inflexible generator** p_j is:

$$\pi^*_{p_j}(X^S_{p_j}(\omega)) = \max_{X^S_{p_j}(\omega)} \{P^S_X(\omega)X^S_{p_j}(\omega) + P^F_X(\omega)X^F_{p_j}(\omega) - \frac{\theta}{2\alpha_{p_j}}(X^S_{p_j}(\omega) + X^F_{p_j}(\omega))^2\} \quad (2)$$

The profit-maximization problem of **retailer** r_k is as follows:

$$\pi^*_{r_k}(X^S_{r_k}(\omega)) = \max_{X^S_{r_k}(\omega)} \{P_{r_k}X_{r_k}(\omega)(1+\gamma f(\omega)) - P^S_X(\omega)X^S_{r_k}(\omega) - P^F_X(\omega)X^F_{r_k}(\omega)\}$$
$$\text{subject to} \quad X^S_{r_k}(\omega) + X^F_{r_k}(\omega) \geq X_{r_k}(\omega)(1+\gamma f(\omega)) \quad (3)$$

where $\pi^*_{r_k}$ is the maximized profit level, $0 \leq \gamma \leq 1$ is the total AS requirement expressed as the fraction of overall electricity load, and $X_{r_k}(\omega)$ is the realized total electricity demand in the franchise area of retailer r_k. Note that the $1 + \gamma f(\omega)$ term simply accounts for the fact that retailer r_k receives an extra $\gamma f(\omega)X_{r_k}$ MW of electricity due to the ISO's purchases for reliability reasons. While it doesn't actually transact in the AS market, retailer r_k is billed for these reliability measures that are required in its own region. In turn, it passes along these costs to its end-use consumers. Similarly, the **ISO**'s optimization problem can be written as follows:

$$\pi^*_I(Y^F_I(\omega)) = \max_{Y^F_I(\omega)} \{P^S_X(\omega)\gamma f(\omega)X'_R(\omega) - P^F_I(\omega)Y^F_I(\omega)\}$$
$$\text{subject to} \quad Y^F_I(\omega) \geq Y_I(\omega) \equiv \gamma X'_R(\omega) \quad (4)$$

[3] We assume that generators have sufficient capacity to meet system demand.

where π_I^* is the maximized profit level, $Y_I^F(\omega)$ is the amount of AS purchased by the ISO on behalf of the retailers from the futures market, and $Y_I(\omega)$ is its total purchase requirement. Note that the total amount of AS required equals some fixed fraction $0 \leq \gamma \leq 1$ of the overall electricity load $X_R'(\omega) \equiv \sum_{k=1}^m X_{r_k}(\omega)$.

Flexible generator p_i's real-time decision is to select the quantity of electricity to sell into the spot market that maximizes its profits. The **first-order necessary condition** is:

$$\begin{aligned} \tfrac{\partial \pi_{p_i}^*(X_{p_i}^S(\omega))}{\partial X_{p_i}^S(\omega)} &= 0 \\ \Rightarrow P_X^S(\omega) - \tfrac{\theta}{\alpha_{p_i}}(X_{p_i}^{S*}(\omega) + X_{p_i}^{F*}(\omega) + f(\omega)Y_{p_i}^{F*}(\omega)) &= 0 \\ \Rightarrow X_{p_i}^{S*}(\omega) = \tfrac{\alpha_{p_i}}{\theta}P_X^S(\omega) - X_{p_i}^{F*}(\omega) - f(\omega)Y_{p_i}^{F*}(\omega) & \end{aligned} \quad (5)$$

The **second-order sufficiency condition** is also satisfied:

$$\frac{\partial^2 \pi_{p_i}^*(X_{p_i}^S(\omega))}{\partial X_{p_i}^{S\,2}(\omega)} = -\frac{\theta}{\alpha_{p_i}} < 0. \quad (6)$$

Hence, because generator p_i's problem is guaranteed to have a global maximum, the profit-maximizing quantity of electricity that generator p_i sells into the spot market can be determined.

Inflexible generator p_j has a similar real-time decision problem with the following **first-order necessary condition**:

$$\begin{aligned} \tfrac{\partial \pi_{p_j}^*(X_{p_j}^S(\omega))}{\partial X_{p_j}^S(\omega)} &= 0 \\ \Rightarrow P_X^S(\omega) - \tfrac{\theta}{\alpha_{p_j}}(X_{p_j}^{S*}(\omega) + X_{p_j}^{F*}(\omega)) &= 0 \\ \Rightarrow X_{p_j}^{S*}(\omega) = \tfrac{\alpha_{p_j}}{\theta}P_X^S(\omega) - X_{p_j}^{F*}(\omega) & \end{aligned} \quad (7)$$

The **second-order sufficiency condition** is also satisfied:

$$\frac{\partial^2 \pi_{p_j}^*(X_{p_j}^S(\omega))}{\partial X_{p_j}^{S\,2}(\omega)} = -\frac{\theta}{\alpha_{p_i}} < 0. \quad (8)$$

On the other hand, retailer r_k has little choice in selecting $X_{r_k}^S(\omega)$ since it *must* meet the net demand in its franchise area, $X_{r_k}(\omega)(1 + \gamma f(\omega)) - X_{r_k}^F(\omega)$. This implies that $X_{r_k}^{S*}(\omega) = X_{r_k}(\omega)(1 + \gamma f(\omega)) - X_{r_k}^{F*}(\omega)$. Since the ISO has no real-time decision, we discuss its role in Section 3.2.

We can now use Equations 5 and 7 together with the retailers' and ISO's purchase requirements to evaluate the equilibrium prices that ensure all markets clear. The **market-clearing conditions** can, thus, be expressed as follows:

$$\sum_{i=1}^{n_1} X_{p_i}^{S*}(\omega) + \sum_{j=1}^{n_2} X_{p_j}^{S*}(\omega) + \sum_{i=1}^{n_1} f(\omega)Y_{p_i}^{F*}(\omega) = \sum_{k=1}^{m} X_{r_k}^{S*}(\omega) \quad (9)$$

$$\sum_{i=1}^{n_1} X_{p_i}^{F*}(\omega) + \sum_{j=1}^{n_2} X_{p_j}^{F*}(\omega) = \sum_{k=1}^{m} X_{r_k}^{F*}(\omega) \tag{10}$$

$$\sum_{i=1}^{n_1} Y_{p_i}^{F*}(\omega) = Y_I^{F*}(\omega) \tag{11}$$

Equation 9 states that in order for an equilibrium to occur in the electricity spot market, the total sales by the generators *plus* the AS called upon to generate equal the total spot market purchases by the retailers (including the AS reserves required to generate). Equations 10 and 11 simply state that total supply equals total demand in the futures markets for electricity and AS, respectively.

Substituting the retailers' purchase requirements as well as Equations 5 and 7 into Equation 9, we obtain the **equilibrium spot market price for electricity**:

$$P_X^{S*}(\omega) = \frac{\theta}{\alpha} X_R'(\omega)(1 + \gamma f(\omega)) \equiv \frac{\theta}{\alpha} X_R(\omega) \tag{12}$$

where $\alpha \equiv \sum_{i=1}^{n_1} \alpha_{p_i} + \sum_{j=1}^{n_2} \alpha_{p_j}$. The details of this derivation are left for Appendix A. Intuitively, the electricity spot price is simply the pro-rated cost of meeting the overall electricity retail demand plus AS reserves required to generate.

At this point, the generators' equilibrium spot market sales of electricity can be explicitly evaluated by substituting Equation 12 into Equations 5 and 7. Doing so, we obtain

$$X_{p_i}^{S*}(\omega) = \frac{\alpha_{p_i}}{\alpha} X_R(\omega) - X_{p_i}^{F*}(\omega) - f(\omega) Y_{p_i}^{F*}(\omega) \tag{13}$$

and

$$X_{p_j}^{S*}(\omega) = \frac{\alpha_{p_j}}{\alpha} X_R(\omega) - X_{p_j}^{F*}(\omega) \tag{14}$$

Since it was assumed that all flexible generators are identical and both electricity and AS requirements are fixed at $X_R'(\omega)$ and $\gamma X_R'(\omega)$, respectively, in equilibrium each flexible generator will sell its pro-rated share of the overall requirements into the spot market less its futures commitments. For inflexible generators, this reasoning also holds.

3.2. Futures Markets

Stepping back in time to when positions in the futures markets are taken, we can determine the optimal quantities traded by each type of agent in both the electricity and AS markets. In solving for these quantities, we also obtain the equilibrium futures prices for both electricity and AS. Unlike decisions made in real-time, futures markets positions are taken in face of uncertainty about the state of the world, expressed by the random variable ω.

At this stage, we can express the profit of **flexible generator** p_i as follows:

$$\begin{aligned}\pi_{p_i}(\omega) = \quad & P_X^{S*}(\omega)X_{p_i}^{S*}(\omega) + P_X^F(\omega)X_{p_i}^F(\omega) + P_Y^F(\omega)Y_{p_i}^F(\omega) \\ & -\frac{\theta}{2\alpha_{p_i}}(X_{p_i}^{S*}(\omega) + X_{p_i}^F(\omega) + f(\omega)Y_{p_i}^F(\omega))^2 \end{aligned} \qquad (15)$$

By setting $X_{p_i}^F(\omega) = 0$ and $Y_{p_i}^F(\omega) = 0$, we can define the *unhedged* profit level:

$$\rho_{p_i}^*(\omega) = P_X^{S*}(\omega)X_{p_i}^{S*}(\omega) - \frac{\theta}{2\alpha_{p_i}}X_{p_i}^{S*^2}(\omega) \qquad (16)$$

Substituting in Equation 13 with $X_{p_i}^{F*}(\omega) = 0$ and $Y_{p_i}^{F*}(\omega) = 0$, we obtain:

$$\begin{aligned} \rho_{p_i}^*(\omega) &= \frac{\theta}{\alpha}X_R(\omega)\frac{\alpha_{p_i}}{\alpha}X_R(\omega) - \frac{\theta}{2\alpha_{p_i}}(\frac{\alpha_{p_i}}{\alpha}X_R(\omega))^2 \\ \Rightarrow \quad \rho_{p_i}^*(\omega) &= \frac{\alpha_{p_i}\theta}{2\alpha^2}X_R^2(\omega) \end{aligned} \qquad (17)$$

By using Equations 12 and 17 together with Equation 13 as usual, we obtain:

$$\pi_{p_i}(\omega) = \rho_{p_i}^*(\omega) + X_{p_i}^F(\omega)(P_X^F(\omega) - P_X^{S*}(\omega)) + Y_{p_i}^F(\omega)(P_Y^F(\omega) - f(\omega)P_X^{S*}(\omega))$$

Flexible generator p_i's objective now is to select $X_{p_i}^F(\omega)$ and $Y_{p_i}^F(\omega)$ in order to maximize $E_\omega[U(\pi_{p_i}(\omega))] \equiv E[\pi_{p_i}(\omega)] - \frac{A_P}{2}Var(\pi_{p_i}(\omega))$, where $A_P > 0$ and is common to all generators (both flexible and inflexible). Hence, flexible generator p_i's optimization problem can be expressed as:

$$\begin{aligned} & \max_{X_{p_i}^F(\omega), Y_{p_i}^F(\omega)} \{E[\rho_{p_i}^*(\omega)] + X_{p_i}^F(\omega)(P_X^F(\omega) - E[P_X^{S*}(\omega)]) \\ & +Y_{p_i}^F(\omega)(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)]) - \frac{A_P}{2}Var(\rho_{p_i}^*(\omega) + X_{p_i}^F(\omega)(P_X^F(\omega) - P_X^{S*}(\omega)) \\ & +Y_{p_i}^F(\omega)(P_Y^F(\omega) - f(\omega)P_X^{S*}(\omega)))\} \\ \Rightarrow \quad & \max_{X_{p_i}^F(\omega), Y_{p_i}^F(\omega)} \{E[\rho_{p_i}^*(\omega)] + X_{p_i}^F(\omega)(P_X^F(\omega) - E[P_X^{S*}(\omega)]) \\ & +Y_{p_i}^F(\omega)(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)]) - \frac{A_P}{2}[Var(\rho_{p_i}^*(\omega)) + X_{p_i}^{F^2}(\omega)Var(P_X^{S*}(\omega)) \\ & +Y_{p_i}^{F^2}(\omega)Var(f(\omega)P_X^{S*}(\omega)) - 2X_{p_i}^F(\omega)Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega)) \\ & -2Y_{p_i}^F(\omega)Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega)) \\ & +2X_{p_i}^F(\omega)Y_{p_i}^F(\omega)Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))]\} \end{aligned} \qquad (18)$$

The **first-order necessary conditions** are:

$$\frac{\partial E_\omega[U(\pi_{p_i}(X^F_{p_i}(\omega),Y^F_{p_i}(\omega)))]}{\partial X^F_{p_i}(\omega)} = 0$$

$$\Rightarrow P^F_X(\omega) - E[P^{S*}_X(\omega)] - A_P X^F_{p_i}(\omega)Var(P^{S*}_X(\omega)) + A_P Cov(\rho^*_{p_i}(\omega), P^{S*}_X(\omega))$$
$$-A_P Y^F_{p_i}(\omega)Cov(P^{S*}_X(\omega), f(\omega)P^{S*}_X(\omega)) = 0$$
$$\Rightarrow A_P X^F_{p_i}(\omega)Var(P^{S*}_X(\omega)) = P^F_X(\omega) - E[P^{S*}_X(\omega)] + A_P Cov(\rho^*_{p_i}(\omega), P^{S*}_X(\omega))$$
$$-A_P Y^F_{p_i}(\omega)Cov(P^{S*}_X(\omega), f(\omega)P^{S*}_X(\omega))$$
$$\Rightarrow X^{F*}_{p_i}(\omega) = \frac{P^F_X(\omega)-E[P^{S*}_X(\omega)]}{A_P Var(P^{S*}_X(\omega))} + \frac{Cov(\rho^*_{p_i}(\omega),P^{S*}_X(\omega))}{Var(P^{S*}_X(\omega))} - \frac{Y^{F*}_{p_i}(\omega)Cov(P^{S*}_X(\omega),f(\omega)P^{S*}_X(\omega))}{Var(P^{S*}_X(\omega))} \quad (19)$$

and

$$\frac{\partial E_\omega[U(\pi_{p_i}(X^F_{p_i}(\omega),Y^F_{p_i}(\omega)))]}{\partial Y^F_{p_i}(\omega)} = 0$$

$$\Rightarrow P^F_Y(\omega) - E[f(\omega)P^{S*}_X(\omega)] - A_P Y^F_{p_i}(\omega)Var(f(\omega)P^{S*}_X(\omega))$$
$$+A_P Cov(\rho^*_{p_i}(\omega), f(\omega)P^{S*}_X(\omega))$$
$$-A_P X^F_{p_i}(\omega)Cov(P^{S*}_X(\omega), f(\omega)P^{S*}_X(\omega)) = 0$$
$$\Rightarrow A_P Y^F_{p_i}(\omega)Var(f(\omega)P^{S*}_X(\omega)) = P^F_Y(\omega) - E[f(\omega)P^{S*}_X(\omega)]$$
$$+A_P Cov(\rho^*_{p_i}(\omega), f(\omega)P^{S*}_X(\omega)) - A_P X^F_{p_i}(\omega)Cov(P^{S*}_X(\omega), f(\omega)P^{S*}_X(\omega))$$
$$\Rightarrow Y^{F*}_{p_i}(\omega) = \frac{P^F_Y(\omega)-E[f(\omega)P^{S*}_X(\omega)]}{A_P Var(f(\omega)P^{S*}_X(\omega))} + \frac{Cov(\rho^*_{p_i}(\omega),f(\omega)P^{S*}_X(\omega))}{Var(f(\omega)P^{S*}_X(\omega))}$$
$$-\frac{X^{F*}_{p_i}(\omega)Cov(P^{S*}_X(\omega),f(\omega)P^{S*}_X(\omega))}{Var(f(\omega)P^{S*}_X(\omega))} \quad (20)$$

Equation 19 states that generator p_i increases its sales in the futures market for electricity either

- in response to the bias in the futures price compared to the spot price, or
- to minimize its variance of profits, resulting from positive covariance between its unhedged profits and the spot price of electricity.

Equation 20 has a similar structure. However, these expressions differ from those presented in [10] due to the presence of a third term that reflects the fact that electricity and AS are substitutes.

In order for the **second-order sufficiency conditions** to be satisfied, we make the assumption that $f(\omega)$ is independent of $P^{S*}_X(\omega)$ (and therefore, of $X'_R(\omega)$). Intuitively, there is no reason to believe that the *fraction* of reserves called upon to generate in real-time is affected by the real-time load. Proceeding with the analysis, we see that the hessian matrix, $H_{\pi_{p_i}(X^{F*}_{p_i}(\omega),Y^{F*}_{p_i}(\omega))}$, is negative definite, i.e., the determinants of the principal minors are nonzero and alternate in sign with the first ones being negative:

$$H_{\pi_{p_i}(X^{F*}_{p_i}(\omega),Y^{F*}_{p_i}(\omega))}$$
$$= \begin{bmatrix} -A_P Var(P^{S*}_X(\omega)) & -A_P Cov(P^{S*}_X(\omega), f(\omega)P^{S*}_X(\omega)) \\ -A_P Cov(P^{S*}_X(\omega), f(\omega)P^{S*}_X(\omega)) & -A_P Var(f(\omega)P^{S*}_X(\omega)) \end{bmatrix} \quad (21)$$

$$\Rightarrow \det(-A_P Var(P_X^{S*}(\omega))) = -A_P Var(P_X^{S*}(\omega)) < 0 \tag{22}$$

$$\det(-A_P Var(f(\omega)P_X^{S*}(\omega))) = -A_P Var(f(\omega)P_X^{S*}(\omega)) < 0 \tag{23}$$

and

$$\begin{aligned}
\det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) = A_P^2 Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega)) \\
-A_P^2 Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega)) \\
\Rightarrow \det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) = A_P^2[Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega)) \\
-Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))] \\
\Rightarrow \det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) = A_P^2[Var(P_X^{S*}(\omega))[Var(f(\omega))Var(P_X^{S*}(\omega)) \\
+(E[f(\omega)])^2 Var(P_X^{S*}(\omega)) + (E[P_X^{S*}(\omega)])^2 Var(f(\omega))] - (E[f(\omega)]E[P_X^{S*^2}(\omega)] \\
-E[f(\omega)](E[P_X^{S*}(\omega)])^2)^2] \\
\Rightarrow \det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) = A_P^2[Var(f(\omega))(Var(P_X^{S*}(\omega)))^2 \\
+(E[f(\omega)])^2(Var(P_X^{S*}(\omega)))^2 + (E[P_X^{S*}(\omega)])^2 Var(f(\omega))Var(P_X^{S*}(\omega)) \\
-(E[f(\omega)])^2(Var(P_X^{S*}(\omega)))^2] \\
\Rightarrow \det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) = A_P^2[Var(f(\omega))(Var(P_X^{S*}(\omega)))^2 \\
+(E[P_X^{S*}(\omega)])^2 Var(f(\omega))Var(P_X^{S*}(\omega))] \\
\Rightarrow \det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) = A_P^2 Var(f(\omega))Var(P_X^{S*}(\omega))[Var(P_X^{S*}(\omega)) \\
+(E[P_X^{S*}(\omega)])^2] \\
\Rightarrow \det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) = A_P^2 Var(f(\omega))Var(P_X^{S*}(\omega))E[P_X^{S*2}(\omega)] \\
\Rightarrow \det(H_{\pi_{p_i}(X_{p_i}^{F*}(\omega),Y_{p_i}^{F*}(\omega))}) > 0
\end{aligned} \tag{24}$$

Hence, there is a global maximum to flexible generator p_i's problem.[4]

In order to isolate the $X_{p_i}^{F*}(\omega)$ and $Y_{p_i}^{F*}(\omega)$ terms, we solve Equations 19 and 20 simultaneously. Leaving the details for Appendix B, we merely state the results here:

$$\begin{aligned}
X_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega),X_R(\omega),\theta,\alpha)}[\frac{(P_X^F(\omega)-E[P_X^{S*}(\omega)])Var(f(\omega)X_R(\omega))}{A_P} \\
+Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))Var(f(\omega)X_R(\omega)) - \frac{(P_Y^F(\omega)-E[f(\omega)P_X^{S*}(\omega)])Cov(X_R(\omega),f(\omega)X_R(\omega))}{A_P} \\
-Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))]
\end{aligned} \tag{25}$$

and

$$Y_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega),X_R(\omega),\theta,\alpha)}[\frac{(P_Y^F(\omega)-E[f(\omega)P_X^{S*}(\omega)])Var(X_R(\omega))}{A_P}$$

[4] We make use of two facts concerning expressions with independent random variables A and B:

1. $Cov(A, AB) = E[B]Var(A)$.
2. $Var(AB) = Var(A)Var(B) + (E[B])^2 Var(A) + (E[A])^2 Var(B)$.

$$+Cov(\rho^*_{p_i}(\omega), f(\omega)P^{S*}_X(\omega))Var(X_R(\omega)) - \frac{(P^F_X(\omega)-E[P^{S*}_X(\omega)])Cov(X_R(\omega),f(\omega)X_R(\omega))}{A_P}$$
$$-Cov(\rho^*_{p_i}(\omega), P^{S*}_X(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))] \quad (26)$$

where

$$Z(f(\omega), X_R(\omega), \theta, \alpha) \equiv \frac{\theta^2}{\alpha^2}\left[Var(X_R(\omega))Var(f(\omega)X_R(\omega)) - Cov^2(X_R(\omega), f(\omega)X_R(\omega))\right]$$

(which also equals $\frac{\theta^2}{\alpha^2}Var(f(\omega))Var(X_R(\omega))E[X^2_R(\omega)]$).

Similarly, the profit of **inflexible generator** p_j is:

$$\pi_{p_j}(\omega) = \quad P^{S*}_X(\omega)X^{S*}_{p_j}(\omega) + P^F_X(\omega)X^F_{p_j}(\omega) - \frac{\theta}{2\alpha_{p_j}}(X^{S*}_{p_j}(\omega) + X^F_{p_j}(\omega))^2 \quad (27)$$

Again, by setting $X^F_{p_j}(\omega) = 0$, we can define the *unhedged* profit level:

$$\rho^*_{p_j}(\omega) = P^{S*}_X(\omega)X^{S*}_{p_j}(\omega) - \frac{\theta}{2\alpha_{p_j}}X^{S*^2}_{p_j}(\omega) \quad (28)$$

By using Equation 14 with $X^{F*}_{p_j}(\omega) = 0$, we obtain:

$$\rho^*_{p_j}(\omega) = \frac{\theta}{\alpha}X_R(\omega)\frac{\alpha_{p_j}}{\alpha}X_R(\omega) - \frac{\theta}{2\alpha_{p_j}}(\frac{\alpha_{p_j}}{\alpha}X_R(\omega))^2$$
$$\Rightarrow \quad \rho^*_{p_j}(\omega) = \frac{\alpha_{p_j}\theta}{2\alpha^2}X^2_R(\omega) \quad (29)$$

By using Equations 12 and 29 together with Equation 14 as usual, we obtain:

$$\pi_{p_j}(\omega) = \rho^*_{p_j}(\omega) + X^F_{p_j}(\omega)(P^F_X(\omega) - P^{S*}_X(\omega))$$

Now, the objective of inflexible generator p_j is to select $X^F_{p_j}(\omega)$ in order to maximize $E_\omega[U(\pi_{p_j}(\omega))] \equiv E[\pi_{p_j}(\omega)] - \frac{A_P}{2}Var(\pi_{p_j}(\omega))$. Hence, inflexible generator p_j's optimization problem is:

$$\max_{X^F_{p_j}(\omega)}\{E[\rho^*_{p_j}(\omega)] + X^F_{p_j}(\omega)(P^F_X(\omega) - E[P^{S*}_X(\omega)])$$
$$-\frac{A_P}{2}Var(\rho^*_{p_j}(\omega) + X^F_{p_j}(\omega)(P^F_X(\omega) - P^{S*}_X(\omega)))\}$$
$$\Rightarrow \quad \max_{X^F_{p_j}(\omega)}\{E[\rho^*_{p_j}(\omega)] + X^F_{p_j}(\omega)(P^F_X(\omega) - E[P^{S*}_X(\omega)])$$
$$-\frac{A_P}{2}[Var(\rho^*_{p_j}(\omega)) + X^{F^2}_{p_j}(\omega)Var(P^{S*}_X(\omega))$$
$$-2X^F_{p_j}(\omega)Cov(\rho^*_{p_j}(\omega), P^{S*}_X(\omega))]\} \quad (30)$$

The **first-order necessary condition** is:

$$\frac{\partial E_\omega[U(\pi_{p_j}(X_{p_j}^F(\omega)))]}{\partial X_{p_j}^F(\omega)} = 0$$

$$\Rightarrow P_X^F(\omega) - E[P_X^{S*}(\omega)] - A_P X_{p_j}^F(\omega) Var(P_X^{S*}(\omega)) + A_P Cov(\rho_{p_j}^*(\omega), P_X^{S*}(\omega)) = 0$$

$$\Rightarrow A_P X_{p_j}^F(\omega) Var(P_X^{S*}(\omega)) = P_X^F(\omega) - E[P_X^{S*}(\omega)] + A_P Cov(\rho_{p_j}^*(\omega), P_X^{S*}(\omega))$$

$$\Rightarrow X_{p_j}^{F*}(\omega) = \frac{P_X^F(\omega) - E[P_X^{S*}(\omega)]}{A_P Var(P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_j}^*(\omega), P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} \quad (31)$$

Equation 31 is similar to Equation 19 without the presence of a third term that reflects the substitution of generation capacity between electricity and AS. Indeed, these inflexible generators do not have the capability to ramp up production during a short time span, and thus, are unable to offer capacity for AS in the futures market.

That the **second-order sufficiency condition** is satisfied can be readily verified:

$$\frac{\partial^2 E_\omega[U(\pi_{p_j}(X_{p_j}^F(\omega)))]}{\partial X_{p_j}^{F^2}(\omega)} = -A_P Var(P_X^{S*}(\omega)) < 0. \quad (32)$$

We can similarly set up an expression for the profit earned by **retailer** r_k by substituting the binding constraint from Equation 3 into the expression for $\pi_{r_k}(\omega)$:

$$\begin{aligned} \pi_{r_k}(\omega) = \quad & P_{r_k} X_{r_k}(\omega)(1+\gamma f(\omega)) - P_X^F(\omega) X_{r_k}^F(\omega) - P_X^{S*}(\omega) X_{r_k}^{S*}(\omega) \\ \Rightarrow \pi_{r_k}(\omega) = \quad & P_{r_k} X_{r_k}(\omega)(1+\gamma f(\omega)) - P_X^F(\omega) X_{r_k}^F(\omega) - P_X^{S*}(\omega)(X_{r_k}(\omega)(1+\gamma f(\omega)) \\ & -X_{r_k}^F(\omega)) \\ \Rightarrow \pi_{r_k}(\omega) = \quad & (P_{r_k} - P_X^{S*}(\omega)) X_{r_k}(\omega)(1+\gamma f(\omega)) + (P_X^{S*}(\omega) - P_X^F(\omega)) X_{r_k}^F(\omega) \quad (33) \end{aligned}$$

By letting $\rho_{r_k}^*(\omega) \equiv (P_{r_k} - P_X^{S*}(\omega)) X_{r_k}(\omega)(1+\gamma f(\omega))$ be the *unhedged* profit level for retailer r_k, Equation 33 then becomes:

$$\pi_{r_k}(\omega) = \rho_{r_k}^*(\omega) + (P_X^{S*}(\omega) - P_X^F(\omega)) X_{r_k}^F(\omega) \quad (34)$$

Retailer r_k's objective is then to select $X_{r_k}^F(\omega)$ in order to maximize $E_\omega[U(\pi_{r_k}(\omega))] \equiv E[\pi_{r_k}(\omega)] - \frac{A_R}{2} Var(\pi_{r_k}(\omega))$, where $A_R > 0$ and is common to all retailers:

$$\begin{aligned} \max_{X_{r_k}^F(\omega)} \{ & E[\rho_{r_k}^*(\omega)] + X_{r_k}^F(\omega)(E[P_X^{S*}(\omega)] - P_X^F(\omega)) - \frac{A_R}{2}[Var(\rho_{r_k}^*(\omega)) \\ & + X_{r_k}^{F^2}(\omega) Var(P_X^{S*}(\omega)) + 2 X_{r_k}^F(\omega) Cov(\rho_{r_k}^*(\omega), P_X^{S*}(\omega))]\} \quad (35) \end{aligned}$$

The **first-order necessary condition** is:

$$\frac{\partial E_\omega[U(\pi_{r_k}(X_{r_k}^F(\omega)))]}{\partial X_{r_k}^F(\omega)} = 0$$

$$\Rightarrow E[P_X^{S*}(\omega)] - P_X^F(\omega) - A_R X_{r_k}^F(\omega) Var(P_X^{S*}(\omega)) - A_R Cov(\rho_{r_k}^*(\omega), P_X^{S*}(\omega)) = 0$$

$$\Rightarrow X_{r_k}^{F*}(\omega) = \frac{E[P_X^{S*}(\omega)] - P_X^F(\omega)}{A_R Var(P_X^{S*}(\omega))} - \frac{Cov(\rho_{r_k}^*(\omega), P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} \quad (36)$$

Similar to Equation 31, Equation 36 states that retailer r_k's futures purchases increase in response to the bias in the spot price over the futures price. Its futures purchases are reduced (increased) if there exists positive (negative) covariance between its unhedged profits and the electricity spot price.

The **second-order sufficiency condition** is also satisfied:

$$\frac{\partial^2 E_\omega[U(\pi_{r_k}(X^F_{r_k}(\omega)))]}{\partial X^{F2}_{r_k}(\omega)} = -A_R Var(P^{S*}_X(\omega)) < 0. \tag{37}$$

By inspecting Equation 4, we see that the **ISO** simply has to purchase enough AS futures to satisfy the reserve requirements. As such, the ISO has no demand for risk-reduction, and thus:

$$Y^{F*}_I(\omega) = \gamma E[X'_R(\omega)] \tag{38}$$

In order to understand completely the demand for hedging, however, it is useful to evaluate the expressions for covariance between unhedged profits and the spot price. From Equations 18, 30, and 35, we see that futures trading is able to reduce risk for the market agents as long as these covariances are non-zero. Using the expressions for the unhedged profits, spot price and positions, we arrive at:

Lemma 1

$$Cov(\rho^*_{p_i}(\omega), P^{S*}_X(\omega)) = \frac{\alpha_{p_i}\theta^2}{2\alpha^3} Cov(X^2_R(\omega), X_R(\omega))$$

Proof: Using Equation 12 together with Equation 17, we obtain:

$$Cov(\rho^*_{p_i}(\omega), P^{S*}_X(\omega)) \quad = Cov(\frac{\alpha_{p_i}\theta}{2\alpha^2} X^2_R(\omega), \frac{\theta}{\alpha} X_R(\omega)) \tag{39}$$

The result is now immediate. ■

Lemma 2

$$Cov(\rho^*_{p_i}(\omega), f(\omega) P^{S*}_X(\omega)) = \frac{\alpha_{p_i}\theta^2}{2\alpha^3} Cov(X^2_R(\omega), f(\omega) X_R(\omega))$$

Proof: This follows from Lemma 1. ■

Lemma 3

$$\begin{aligned} Cov(\rho^*_{r_k}(\omega), P^{S*}_X(\omega)) = \quad & \frac{\theta P_{r_k}}{\alpha} Cov(X_{r_k}(\omega)(1+\gamma f(\omega)), X_R(\omega)) \\ & -\frac{\theta^2}{\alpha^2} Cov(X_{r_k}(\omega)(1+\gamma f(\omega)) X_R(\omega), X_R(\omega)) \end{aligned}$$

Proof:

$$\begin{aligned}
\rho^*_{r_k}(\omega) = & \quad (P_{r_k} - P_X^{S*}(\omega))X_{r_k}(\omega)(1+\gamma f(\omega)) \\
\Rightarrow \rho^*_{r_k}(\omega) = & \quad (P_{r_k} - \frac{\theta}{\alpha}X_R(\omega))X_{r_k}(\omega)(1+\gamma f(\omega)) \\
\Rightarrow Cov(\rho^*_{r_k}(\omega), P_X^{S*}(\omega)) = & \quad Cov(P_{r_k}X_{r_k}(\omega)(1+\gamma f(\omega)) \\
& \quad -\frac{\theta}{\alpha}X_R(\omega)X_{r_k}(\omega)(1+\gamma f(\omega)), \frac{\theta}{\alpha}X_R(\omega))
\end{aligned}$$

The result then follows. ■

Now, we can substitute these expressions for the covariances into the expressions for the optimal futures positions:

$$\begin{aligned}
X_{p_i}^{F*}(\omega) \quad = \quad & \frac{1}{Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{(P_X^F(\omega) - E[P_X^{S*}(\omega)])Var(f(\omega)X_R(\omega))}{A_P} \\
& + \frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega), X_R(\omega))Var(f(\omega)X_R(\omega))}{2\alpha^3} \\
& - \frac{(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])Cov(X_R(\omega), f(\omega)X_R(\omega))}{A_P} \\
& - \frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))}{2\alpha^3}]
\end{aligned} \tag{40}$$

$$\begin{aligned}
Y_{p_i}^{F*}(\omega) \quad = \quad & \frac{1}{Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])Var(X_R(\omega))}{A_P} \\
& + \frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))Var(X_R(\omega))}{2\alpha^3} \\
& - \frac{(P_X^F(\omega) - E[P_X^{S*}(\omega)])Cov(X_R(\omega), f(\omega)X_R(\omega))}{A_P} \\
& - \frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega), X_R(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))}{2\alpha^3}]
\end{aligned} \tag{41}$$

$$X_{p_j}^{F*}(\omega) \quad = \quad \frac{P_X^F(\omega) - E[P_X^{S*}(\omega)]}{A_P Var(P_X^{S*}(\omega))} + \frac{\alpha_{p_j}Cov(X_R^2(\omega), X_R(\omega))}{2\alpha Var(X_R(\omega))} \tag{42}$$

$$\begin{aligned}
X_{r_k}^{F*}(\omega) \quad = \quad & \frac{E[P_X^{S*}(\omega)] - P_X^F(\omega)}{A_R Var(P_X^{S*}(\omega))} - \frac{\alpha P_{r_k} Cov(X_{r_k}(\omega)(1+\gamma f(\omega)), X_R(\omega))}{\theta Var(X_R(\omega))} \\
& + \frac{Cov(X_R(\omega)X_{r_k}(\omega)(1+\gamma f(\omega)), X_R(\omega))}{Var(X_R(\omega))}
\end{aligned} \tag{43}$$

Summarizing, the flexible generators' demand for hedging increases in response to the bias in the futures price relative to the spot price and the desire for minimizing the variance of profits. Conversely, they reduce futures sales of one product if the substitute product is proportionately more lucrative. Inflexible generators respond only to the bias in the futures price relative to the spot price and the desire for minimizing the variance of profits. The retailers increase electricity futures purchases in response to a bias in the spot price relative to the futures price. Furthermore, they optimally reduce their futures purchases if retail revenues covary positively with the spot price. Finally, they increase futures purchases if their local demand is highest when overall system demand (and, therefore, spot prices) are greatest.

3.3. Equilibrium Futures Prices

We can now use the market-clearing conditions (Equations 9, 10, and 11) together with the optimal futures positions (Equations 40, 41, 42, 43, and 38) to determine equilibrium futures prices for both electricity and AS. We leave the details of the derivation for Appendix B and focus here on discussing the intuitive properties of the expressions. Solving, we obtain:

$$\begin{aligned} P_X^{F*}(\omega) &= E[P_X^{S*}(\omega)] + \frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta} \\ &\quad - \frac{\theta}{\alpha\eta}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] Var(X_R(\omega)) \end{aligned} \tag{44}$$

and

$$\begin{aligned} P_Y^{F*}(\omega) &= E[f(\omega)P_X^{S*}(\omega)] + \frac{E[f(\omega)]\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta} \\ &\quad - \frac{\theta}{\alpha\eta}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] E[f(\omega)]Var(X_R(\omega)) \\ &\quad + \frac{\theta\gamma E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha(1+\gamma E[f(\omega)])} \end{aligned} \tag{45}$$

Here, $\eta \equiv \frac{n_1}{A_P} + \frac{n_2}{A_P} + \frac{m}{A_R}$ reflects the number of firms trading in the electricity markets and their degree of risk-aversion; $\eta' \equiv \frac{n_1}{A_P}$ reflects the number of flexible generators and their degree of risk-aversion; and $\beta_{r_k} \equiv \frac{Cov(X_{r_k}(\omega)(1+\gamma f(\omega)), X_R(\omega))}{Var(X_R(\omega))}$ is the degree to which demand (plus AS requirements) in retailer r_k's franchise area is correlated with overall system demand.

Equations 44 and 45 are similar in structure to the expression for the equilibrium futures price for electricity derived in [10] except that here the futures price for electricity depends also on the average proportion of reserves required to generate (as compared to the overall load). Given the fundamental link between these two products, this result is to be expected. Specifically, the futures price for electricity is equal to the expected spot price for electricity plus a term related to the skewness of overall system demand and one related to the variance of overall system demand. If either the number of market agents approaches infinity or any type of market agents is risk-neutral, then the electricity futures price approaches the expected spot price. Under the assumptions we made, however, this is not a possibility. Hence, we have the implication that the electricity futures price is increasing in the skewness of system demand and decreasing in the variance of system demand. The former is straightforward to verify, but the latter is not so clear. For this, we need to know the relationship between the beta-weighted average of the retail price and the expected spot price.

Intuitively, the futures price increases with system demand skewness because high skewness (implying extreme positive demand realizations) spurs retailers to purchase electricity from the futures market. In contrast, generators are attracted to sell electricity in the spot market. This combination of greater demand and reduced supply in the futures electricity market then naturally leads to a higher futures price of electricity. The degree to which the futures price of electricity increases due to an increase in system demand skewness varies positively with the risk-aversion of market participants. On the other hand, the futures price of electricity decreases with the variance of system demand because transacting in the futures market eliminates the risk from demand uncertainty. To show this, we need the following:

Lemma 4

$$\frac{1}{m}\sum_{k=1}^{m} Cov(P_{r_k}X_{r_k}(\omega)(1+\gamma f(\omega)), P_X^{S*}(\omega)) \geq 0$$

Proof:

$$\begin{aligned}
& \frac{1}{m}\sum_{k=1}^{m} Cov(P_{r_k}X_{r_k}(\omega)(1+\gamma f(\omega)), P_X^{S*}(\omega)) \\
= \; & \frac{1}{m}\sum_{k=1}^{m} Cov(P_{r_k}X_{r_k}(\omega)(1+\gamma f(\omega)), \frac{\theta}{\alpha}X_R(\omega)) \\
= \; & \frac{1}{m}[Cov(P_{r_1}X_{r_1}(\omega)(1+\gamma f(\omega)), \frac{\theta}{\alpha}X_R(\omega)) + \cdots \\
& + Cov(P_{r_m}X_{r_m}(\omega)(1+\gamma f(\omega)), \frac{\theta}{\alpha}X_R(\omega))] \\
= \; & \frac{1}{m}[\frac{P_{r_1}\theta}{\alpha}Cov(X_{r_1}(\omega)(1+\gamma f(\omega)), X_R(\omega)) + \cdots \\
& + \frac{P_{r_m}\theta}{\alpha}Cov(X_{r_m}(\omega)(1+\gamma f(\omega)), X_R(\omega))] \\
& (P_{min} \equiv \min_{k=1,\ldots,m} P_{r_k} \geq 0)
\end{aligned}$$

$$
\begin{aligned}
&\geq \frac{1}{m}[\frac{P_{min}\theta}{\alpha}Cov(X_{r_1}(\omega)(1+\gamma f(\omega)), X_R(\omega)) + \cdots \\
&\quad + \frac{P_{min}\theta}{\alpha}Cov(X_{r_m}(\omega)(1+\gamma f(\omega)), X_R(\omega))] \\
&= \frac{P_{min}\theta}{m\alpha}[Cov(X_{r_1}(\omega)(1+\gamma f(\omega)), X_R(\omega)) + \cdots + Cov(X_{r_m}(\omega)(1+\gamma f(\omega)), X_R(\omega))] \\
&= \frac{P_{min}\theta}{m\alpha}[Cov((X_{r_1}(\omega) + \cdots + X_{r_m}(\omega))(1+\gamma f(\omega)), X_R(\omega))] \\
&= \frac{P_{min}\theta}{m\alpha}Cov(X_R'(\omega)(1+\gamma f(\omega)), X_R(\omega)) \\
&= \frac{P_{min}\theta}{m\alpha}Var(X_R(\omega)) \geq 0
\end{aligned}
$$

■

Lemma 4 states that on average retailers' revenues covary non-negatively with the electricity spot price. Consequently, retailers are interested in selling electricity in the futures markets in order to remove this exposure. Therefore, in order for the electricity futures market to clear, a decrease in the electricity futures price is required, which results in an offsetting increase in the quantity of futures demanded. From Equation 44, we can see that the electricity futures price will decrease with the variance of system demand (which, by Lemma 4, is related to the covariance between retail revenue and the spot price) the greater the beta-weighted retail price is than the average electricity spot price (plus an adder that represents payments for called AS generation). Since risk-averse retailers will not enter the industry unless $\sum_{k=1}^{m} P_{r_k}\beta_{r_k} > E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}$, we can conclude that the electricity futures price is indeed decreasing in the variance of system demand.

Since its structure is similar to that of Equation 44, Equation 45 retains many of the aforementioned intuitive properties. Rewriting it as $P_Y^{F*}(\omega) = E[f(\omega)]P_X^{F*}(\omega) + \frac{\theta\gamma}{\eta'\alpha(1+\gamma E[f(\omega)])}E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]$, we see that the per MW AS futures price has two terms, the first of which compensates the generator (at the futures electricity price) for missing out on profiting from the average fraction of reserves that are actually called upon to generate. This reflects the fact that by providing reserves in the futures market, a flexible generator incurs opportunity costs from foregone electricity futures sales. On average, a fraction $E[f(\omega)]$ of the reserves sold into the AS futures market will be called upon to generate. Hence, the capacity payment of $E[f(\omega)]P_X^{F*}(\omega)$ covers the flexible generator's opportunity costs, i.e., revenues it would have received from the other lucrative endeavor. Conversely, the second term compensates the generator for electricity actually called upon to generate. Towards that end, the flexible generator is paid (on average) the spot market electricity price (but scaled down to reflect the fact that the spot market price is already scaled up relative to the case with no AS) when it's called upon to generate from its AS reserves. Furthermore, the $Var(f(\omega))E[X_R^2(\omega)]$ term scales the average spot price to account for the uncertainty facing the generator. Finally, it is interesting to note that the AS futures price decreases in the variance of system demand in proportion to the beta-weighted average of the retail price. The retailers' average revenue term enters the expression because more power is sold when the electricity spot price is high (by Lemma 4). From the above explanation of Equation 44, retailers are spurred by their positive exposure to the electricity spot price to sell electricity futures, thereby lowering the electricity futures price. This is precisely what motivates generators to consider selling AS futures instead of electricity futures. Since they are substitute products, if the price of one commodity decreases, then *ceteris paribus*, the other one seems relatively more lucrative. The resulting increase in supply of AS futures, thus, lowers the equilibrium futures price.

3.4. Optimal Futures Positions

We now describe the futures positions taken by the various market participants. We leave the derivations for Appendix C, and instead discuss which factors may give certain agents an advantage in trading in the futures markets (as opposed to the spot market). The optimal futures positions are:

$$\begin{aligned} X_{p_i}^{F*}(\omega) &= \frac{\alpha_{p_i}}{\alpha}E[X_R(\omega)] + \left[\frac{1}{2\eta A_P} + \frac{\alpha_{p_i}}{2\alpha}\right]\frac{Skew(X_R(\omega))}{Var(X_R(\omega))} \\ &\quad - \frac{\alpha}{\eta\theta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] \\ &\quad - \frac{\gamma E[f(\omega)]E[X_R'(\omega)]}{n_1} \end{aligned} \tag{46}$$

$$Y_{p_i}^{F*}(\omega) = \frac{\gamma E[X_R'(\omega)]}{n_1} \tag{47}$$

$$\begin{aligned} X_{p_j}^{F*}(\omega) &= \frac{\alpha_{p_j}}{\alpha}E[X_R(\omega)] + \left[\frac{1}{2\eta A_P} + \frac{\alpha_{p_j}}{2\alpha}\right]\frac{Skew(X_R(\omega))}{Var(X_R(\omega))} \\ &\quad - \frac{\alpha}{\eta\theta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] \end{aligned} \tag{48}$$

$$\begin{aligned} X_{r_k}^{F*}(\omega) &= E[X_{r_k}(\omega)(1+\gamma f(\omega))] + \frac{\alpha}{\theta\eta A_R}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] \\ &\quad + \frac{Coskew(X_{r_k}(\omega)(1+\gamma f(\omega)), X_R(\omega))}{Var(X_R(\omega))} - \frac{Skew(X_R(\omega))}{2\eta A_R Var(X_R(\omega))} \\ &\quad - \frac{\alpha}{\theta}\beta_{r_k}\left[P_{r_k} - E[P_X^{S*}(\omega)]\right] \end{aligned} \tag{49}$$

$$Y_I^{F*}(\omega) = \gamma E[X_R'(\omega)] \tag{50}$$

Equations 46 and 48 indicate that generators' optimal electricity futures positions deviate from their pro-rated share of forecasted system load by two (three, in the case of flexible generators) terms:

- the term related to the skewness of system load reflects the fact that positively skewed demand induces retailers to purchase electricity from the futures market, which then increases $P_X^{F*}(\omega)$ and attracts generators to sell forward (with higher capacity generators increasing futures sales more).

- the term related to the difference in the beta-weighted retail price and the average electricity spot price reflects the effect of the downward bias in the futures price due to the retailers' desire to sell forward in order to hedge their retail revenues.

- the third term in Equation 46 reflects the fact that electricity and AS are substitute products, and the effect of selling more AS futures is obviously to reduce the amount of electricity futures that can be sold by any flexible generator.

From Equation 49, we see that retailers' futures purchases (or sales, if this term is negative) differ from their forecasted local loads plus AS requirements by four terms:

- they increase futures purchases the greater the difference in the beta-weighted retail price and the average electricity spot price since this term reflects the downward bias in the electricity futures price due to hedging of retail revenue exposure.

- they also increase futures purchases in the coskewness of local demand with system demand, reflecting the fact high load realizations impose large purchase costs on retailers since the electricity spot price is also high at this time.

- the term related to the skewness of demand reflects the fact that high demand skewness increases the electricity futures price, and thus, spurs retailers to reduce futures purchases.

- the final term reflects the fact that firms with higher betas or retail rates (in relation to the average electricity spot price) reduce their futures purchases the most since they have less opportunity to engage in risk reduction practices.

Finally, Equations 47 and 50 simply indicate that each flexible generator satisfies its pro-rated share of the forecasted AS requirement, and the ISO purchases enough AS to satisfy the reserve requirements.

4. Empirical Analysis

Using California market data, we can empirically test the hypotheses developed in Section 3 regarding the relationship between spot and futures prices. Before proceeding with the analysis, however, some speculation is required as to which California markets are closest to the ideal markets in our perfectly competitive model. For the AS, the spinning reserve day-ahead market is used, and for electricity, the ISO ex-post supplemental (imbalance) energy market and PX day-ahead constrained market are used for spot and futures markets, respectively. Hourly data for one year (1 June 1999 to 31 May 2000) obtained from the CAISO and CalPX are analyzed. Furthermore, we obtain the fraction of spinning reserves called upon to generate for each hour by dividing the quantity produced in the ISO ex-post spin energy market by the amount sold into the spinning reserve day-ahead market. [5]

In order to test Equation 44, we construct the following linear regression model:

$$P^F_{X_i} = \mu_0 + \mu_1 Skew(P^S_{X_i}) + \mu_2 Var(P^S_{X_i}) \tag{51}$$

We then simply regress the average daily PX day-ahead prices on the daily skewness and variance of the ISO imbalance energy price (we use the imbalance energy price instead of system demand because PX day-ahead constrained quantities were unavailable). If our hypothesis regarding futures pricing is correct, then we would expect μ_0 to be equal to the average imbalance energy (spot) price for the particular zone, $\mu_1 > 0$, and $\mu_2 < 0$. To test Equation 45, we use a similar regression model:

$$P^F_{X_i} = \nu_0 + \nu_1 E[f_i] Skew(P^S_{X_i}) + \nu_2 E[f_i] Var(P^S_{X_i}) \tag{52}$$

Here, we regress the average daily spinning reserve day-ahead prices on the daily skewness and variance of the ISO imbalance energy price times the average fraction of reserves called upon to generate for day i. Note that we don't include the third term from Equation 45, which compensates the flexible generator for called reserves. This reflects the fact that in California, the spinning reserve price is only a capacity payment. By contrast, our Equation 45 includes the energy payment term. We would, thus, expect ν_0 to be equal to the average ISO imbalance energy price times the average fraction of reserves called upon to generate, $\nu_1 > 0$, and $\nu_2 < 0$.

In Tables 1 and 2, results of the ordinary least-squares (OLS) electricity futures regression are presented for the two largest zones in California (the standard errors appear in parentheses). While the intercepts correspond closely to the average electricity spot prices in these zones (see Table 5), the coefficient estimates on both skewness and variance of spot price have different signs than predicted. Given the problems with market power and price volatility experienced in the California electricity markets and the fact

[5] We cap this ratio at 1 because some there were some observations greater than 1. This may be because the quantity produced in the ISO ex-post spin energy market comes from reserves obtained through imports and the spinning reserve hour-ahead market in addition to the day-ahead market.

	Coefficients
$\hat{\mu_0}$	\$31.150/MW
	(0.908)
$\hat{\mu_1}$	-1.33×10^{-5}MW^2/\$^{2}
	(2×10^{-6})
$\hat{\mu_2}$	0.0049MW/\$
	(0.0005)
R^2	0.240
SER	16.081

Table 1: Regression of Electricity Futures Prices on Electricity Spot Prices for NP15 (366 observations)

	Coefficients
$\hat{\mu_0}$	\$30.503/MW
	(0.759)
$\hat{\mu_1}$	-2.78×10^{-6}MW^2/\$^{2}
	(1.05×10^{-6})
$\hat{\mu_2}$	0.0016MW/\$
	(0.0003)
R^2	0.175
SER	13.903

Table 2: Regression of Electricity Futures Prices on Electricity Spot Prices for SP15 (366 observations)

that our model assumes perfectly competitive markets, these results are not particularly surprising. In defense of the model, $\hat{\mu_1}$ for both zones is negative by a small amount (although it is statistically significant). Moreover, by looking at Equation 44, a positive value for $\hat{\mu_2}$ implies that beta-weighted retail prices in California may be less than the average electricity spot price (plus an adder for generation from AS futures). Evidence of this stems from the fact that once the San Diego Gas and Electric Company began passing on market prices to consumers, the average retail bill almost doubled (from \$55 per month to \$90 per month) in comparison to that under the regulated retail rate (see [11]). In this case, a positive value for $\hat{\mu_2}$ would not be unexpected.

The OLS AS futures regression (see Tables 3 and 4) produces similar results. Once again, the intercepts are of the predicted magnitude, approximately equaling the average electricity spot price times the average fraction of AS futures required to generate. However, the signs on the skewness and variance of spot price terms are different from what

	Coefficients
$\hat{\nu}_0$	\$3.993/MW
	(0.453)
$\hat{\nu}_1$	-2.95$\times10^{-5}$MW^2/\$^{2}
	(3$\times10^{-6}$)
$\hat{\nu}_2$	0.0115MW/\$
	(0.0010)
R^2	0.343
SER	8.457

Table 3: Regression of AS Futures Prices on Electricity Spot Prices for NP15 (366 observations)

	Coefficients
$\hat{\nu}_0$	\$5.132/MW
	(0.622)
$\hat{\nu}_1$	-1.98$\times10^{-5}$MW^2/\$^{2}
	(4$\times10^{-6}$)
$\hat{\nu}_2$	0.0076MW/\$
	(0.0012)
R^2	0.117
SER	11.565

Table 4: Regression of AS Futures Prices on Electricity Spot Prices for SP15 (366 observations)

was predicted by the model. Given that electricity and AS are substitute products, it's not surprising that deviations from competition in the electricity market also effect the AS market.

5. Conclusions

In this paper, we analyze a theoretical model for managing electricity reliability risk in a deregulated framework similar to that of California's. We posit that the performance of California's AS markets may benefit from the development of a comprehensive pricing methodology. Rather than use a "no-arbitrage" approach to pricing, we take the market equilibrium approach. By solving our integrated model, which accounts for the underlying physical and financial links between the AS and electricity markets, we obtain equilibrium

Zone	Mean ($/MW)	S. D. ($/MW)
NP15	36.537	40.226
SP15	34.302	46.717

Table 5: Electricity Spot Price Summary Statistics (8784 observations)

Zone	Mean	S. D.
NP15	0.088	0.180
SP15	0.105	0.269

Table 6: Summary Statistics for the Fraction of Day-Ahead Spinning Reserves Called Upon to Generate (8784 observations)

prices for both electricity and AS futures markets. These prices are biased measures of the electricity spot price and depend on statistical aspects of system demand. In addition, the extent of the AS futures price's dependence on the electricity market prices is formalized. We also obtain optimal positions taken in the competitive markets by the various agents.

Empirical analysis using recent California market data documents the departures from competition in the newly deregulated setting. Specifically, we confirm that end-use consumers' electricity bills have actually risen, rather than decreased, from their regulated levels. Furthermore, deviations from competition in the electricity markets have propagated to the AS markets as well.

Although our integrated model abstracts from reality by assuming perfectly competitive markets, no transmission constraints, and only one type of AS, we are, nevertheless, able to gain insight into the factors that affect AS futures prices. In particular, we obtain the result that the AS futures price has two components: a capacity payment (based on the electricity futures price) that compensates generators for opportunity costs, and an energy payment (based on the electricity spot price) that reimburses generators for electricity actually produced. We feel that these insights into market-based pricing of AS will aid market agents and the ISO to manage risk associated with trading of electricity reliability services more efficiently in a deregulated setting. Consequently, AS markets will function more smoothly as both market agents and the ISO alike begin to trust the price signals. For future research, it would be interesting to include departures from competition, trading of different kinds of AS, and other types of derivatives in the model.

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Appendix A: Solving for the Equilibrium Spot Market Price

Substituting Equation 5, Equation 7, and the retailers' purchase requirements into Equation 9, we obtain:

$$\sum_{i=1}^{n_1} \frac{\alpha_{p_i}}{\theta} P_X^S(\omega) - \sum_{i=1}^{n_1} X_{p_i}^{F*}(\omega) - \sum_{i=1}^{n_1} f(\omega) Y_{p_i}^{F*}(\omega) + \sum_{i=1}^{n_1} f(\omega) Y_{p_i}^{F*}(\omega)$$

$$+\sum_{j=1}^{n_2} \frac{\alpha_{p_j}}{\theta} P_X^S(\omega) - \sum_{j=1}^{n_2} X_{p_j}^{F*}(\omega) = \sum_{k=1}^{m} X_{r_k}(\omega)(1+\gamma f(\omega)) - \sum_{k=1}^{m} X_{r_k}^{F*}(\omega)$$

Making use of Equation 10, and letting $\alpha_1 \equiv \sum_{i=1}^{n_1} \alpha_{p_i}$, $\alpha_2 \equiv \sum_{j=1}^{n_2} \alpha_{p_j}$, $\alpha \equiv \alpha_1 + \alpha_2$, and $X_R(\omega) \equiv X'_R(\omega)(1+\gamma f(\omega))$:

$$\Rightarrow \frac{\alpha_1}{\theta} P_X^S(\omega) - \sum_{i=1}^{n_1} X_{p_i}^{F*}(\omega) + \frac{\alpha_2}{\theta} P_X^S(\omega) - \sum_{j=1}^{n_2} X_{p_j}^{F*}(\omega) = \sum_{k=1}^{m} X_{r_k}(\omega)(1+\gamma f(\omega)) - \sum_{k=1}^{m} X_{r_k}^{F*}(\omega)$$

$$\Rightarrow \frac{\alpha}{\theta} P_X^S(\omega) = \sum_{k=1}^{m} X_{r_k}(\omega) + \gamma f(\omega) X'_R(\omega)$$

$$\Rightarrow \frac{\alpha}{\theta} P_X^S(\omega) = X'_R(\omega) + \gamma f(\omega) X'_R(\omega)$$

This is equivalent to Equation 12.

Appendix B: Solving for Equilibrium Futures Prices

We first solve simultaneously for $X_{p_i}^{F*}(\omega)$ and $Y_{p_i}^{F*}(\omega)$ by inserting Equation 20 into Equation 19:

$$\begin{aligned}
& X_{p_i}^{F*}(\omega) = \frac{P_X^F(\omega) - E[P_X^{S*}(\omega)]}{A_P Var(P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} \\
& -\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))}[\frac{P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)]}{A_P Var(f(\omega)P_X^{S*}(\omega))} \\
& +\frac{Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)P_X^{S*}(\omega))} - \frac{X_{p_i}^{F*}(\omega)Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)P_X^{S*}(\omega))}] \\
\Rightarrow \quad & X_{p_i}^{F*}(\omega) = \frac{P_X^F(\omega) - E[P_X^{S*}(\omega)]}{A_P Var(P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} \\
& -\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])}{A_P Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega))} \\
& -\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega))} \\
& +X_{p_i}^{F*}(\omega)\frac{Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega))} \\
\Rightarrow \quad & X_{p_i}^{F*}(\omega)\left[\frac{Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega)) - Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega))}\right] \\
& = \frac{P_X^F(\omega) - E[P_X^{S*}(\omega)]}{A_P Var(P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} \\
& -\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])}{A_P Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega))}
\end{aligned}$$

$$-\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega))}$$

$$\Rightarrow \quad X_{p_i}^{F*}(\omega) = \frac{(P_X^F(\omega) - E[P_X^{S*}(\omega)])Var(f(\omega)P_X^{S*}(\omega))}{A_P[Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega)) - Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))]}$$
$$+\frac{Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega))}{[Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega)) - Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))]}$$
$$-\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])}{A_P[Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega)) - Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))]}$$
$$-\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{[Var(P_X^{S*}(\omega))Var(f(\omega)P_X^{S*}(\omega)) - Cov^2(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))]} \tag{53}$$

Then by employing the definition of $Z(f(\omega), X_R(\omega), \theta, \alpha)$ and using Equation 12, we arrive at Equation 25.

Next, by inserting Equation 25 into Equation 20, we obtain:

$$Y_{p_i}^{F*}(\omega) = \frac{P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)]}{A_P Var(f(\omega)P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)P_X^{S*}(\omega))}$$
$$-\frac{Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)P_X^{S*}(\omega))Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{(P_X^F(\omega) - E[P_X^{S*}(\omega)])Var(f(\omega)X_R(\omega))}{A_P}$$
$$+Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))Var(f(\omega)X_R(\omega))$$
$$-\frac{(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])Cov(X_R(\omega), f(\omega)X_R(\omega))}{A_P}$$
$$-Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))]$$
$$\Rightarrow \quad Y_{p_i}^{F*}(\omega) = \frac{P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)]}{A_P Var(f(\omega)P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)P_X^{S*}(\omega))}$$
$$-\frac{Cov(X_R(\omega), f(\omega)X_R(\omega))}{Var(f(\omega)X_R(\omega))Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{(P_X^F(\omega) - E[P_X^{S*}(\omega)])Var(f(\omega)X_R(\omega))}{A_P}$$
$$+Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))Var(f(\omega)X_R(\omega))$$
$$-\frac{(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])Cov(X_R(\omega), f(\omega)X_R(\omega))}{A_P}$$
$$-Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))]$$
$$\Rightarrow \quad Y_{p_i}^{F*}(\omega) = \frac{\alpha^2(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])}{\theta^2 A_P Var(f(\omega)X_R(\omega))} + \frac{\alpha^2 Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{\theta^2 Var(f(\omega)X_R(\omega))}$$
$$-\frac{Cov(X_R(\omega), f(\omega)X_R(\omega))(P_X^F(\omega) - E[P_X^{S*}(\omega)])}{A_P Z(f(\omega), X_R(\omega), \theta, \alpha)}$$
$$-\frac{Cov(X_R(\omega), f(\omega)X_R(\omega))Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))}{Z(f(\omega), X_R(\omega), \theta, \alpha)}$$

$$
\begin{aligned}
&+\frac{(P_Y^F(\omega)-E[f(\omega)P_X^{S*}(\omega)])Cov^2(X_R(\omega),f(\omega)X_R(\omega))}{A_P Var(f(\omega)X_R(\omega))Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&+\frac{Cov(\rho_{p_i}^*(\omega),f(\omega)P_X^{S*}(\omega))Cov^2(X_R(\omega),f(\omega)X_R(\omega))}{Var(f(\omega)X_R(\omega))Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
\Rightarrow\quad Y_{p_i}^{F*}(\omega) &= \frac{(P_Y^F(\omega)-E[f(\omega)P_X^{S*}(\omega)])}{A_P Var(f(\omega)X_R(\omega))}\left[\frac{\alpha^2}{\theta^2}+\frac{Cov^2(X_R(\omega),f(\omega)X_R(\omega))}{Z(f(\omega),X_R(\omega),\theta,\alpha)}\right] \\
&+\frac{Cov(\rho_{p_i}^*(\omega),f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)X_R(\omega))}\left[\frac{\alpha^2}{\theta^2}+\frac{Cov^2(X_R(\omega),f(\omega)X_R(\omega))}{Z(f(\omega),X_R(\omega),\theta,\alpha)}\right] \\
&-\frac{Cov(X_R(\omega),f(\omega)X_R(\omega))(P_X^F(\omega)-E[P_X^{S*}(\omega)])}{A_P Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&-\frac{Cov(X_R(\omega),f(\omega)X_R(\omega))Cov(\rho_{p_i}^*(\omega),P_X^{S*}(\omega))}{Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
\Rightarrow\quad Y_{p_i}^{F*}(\omega) &= \frac{(P_Y^F(\omega)-E[f(\omega)P_X^{S*}(\omega)])Var(X_R(\omega))}{A_P Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&+\frac{Cov(\rho_{p_i}^*(\omega),f(\omega)P_X^{S*}(\omega))Var(X_R(\omega))}{Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&-\frac{Cov(X_R(\omega),f(\omega)X_R(\omega))(P_X^F(\omega)-E[P_X^{S*}(\omega)])}{A_P Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&-\frac{Cov(X_R(\omega),f(\omega)X_R(\omega))Cov(\rho_{p_i}^*(\omega),P_X^{S*}(\omega))}{Z(f(\omega),X_R(\omega),\theta,\alpha)} \qquad (54)
\end{aligned}
$$

This is equivalent to Equation 26.

We now solve for $P_X^{F*}(\omega)$ by inserting Equations 40, 42, and 43 into Equation 10:

$$
\begin{aligned}
&\frac{n_1(P_X^F(\omega)-E[P_X^{S*}(\omega)])Var(f(\omega)X_R(\omega))}{A_P Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega),X_R(\omega))Var(f(\omega)X_R(\omega))}{2\alpha^3 Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&-\frac{n_1(P_Y^F(\omega)-E[f(\omega)P_X^{S*}(\omega)])Cov(X_R(\omega),f(\omega)X_R(\omega))}{A_P Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&-\frac{\alpha_1\theta^2 Cov(X_R^2(\omega),f(\omega)X_R(\omega))Cov(X_R(\omega),f(\omega)X_R(\omega))}{2\alpha^3 Z(f(\omega),X_R(\omega),\theta,\alpha)} \\
&+\frac{n_2(P_X^F(\omega)-E[P_X^{S*}(\omega)])}{A_P Var(P_X^{S*}(\omega))}+\frac{\alpha_2 Cov(X_R^2(\omega),X_R(\omega))}{2\alpha Var(X_R(\omega))} \\
&=\frac{m(E[P_X^{S*}(\omega)]-P_X^F(\omega))}{A_R Var(P_X^{S*}(\omega))}-\frac{\alpha\sum_{k=1}^m P_{r_k}Cov(X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{\theta Var(X_R(\omega))} \\
&+\frac{\sum_{k=1}^m Cov(X_{r_k}(\omega)(1+\gamma f(\omega))X_R(\omega),X_R(\omega))}{Var(X_R(\omega))}
\end{aligned}
$$

By letting $\eta \equiv n_1/A_P + n_2/A_P + m/A_R$, $\eta' \equiv n_1/A_P$, and $\beta_{r_k} \equiv \frac{Cov(X_{r_k}(\omega)(1+\gamma f(\omega)), X_R(\omega))}{Var(X_R(\omega))}$, and using the fact that $Cov(X_R^2(\omega), X_R(\omega)) \equiv Skew(X_R(\omega)) + 2E[X_R(\omega)]Var(X_R(\omega))$ and $P_X^{S*}(\omega) = \frac{\theta}{\alpha}X_R(\omega)$, we obtain:

$$(P_X^F(\omega) - E[P_X^{S*}(\omega)])[\frac{n_1 Var(f(\omega)X_R(\omega))}{A_P Z(f(\omega), X_R(\omega), \theta, \alpha)} + \frac{n_2\alpha^2}{A_P\theta^2 Var(X_R(\omega))}$$
$$+\frac{m\alpha^2}{A_R\theta^2 Var(X_R(\omega))}]$$
$$= \frac{Cov(X_R(\omega), f(\omega)X_R(\omega))}{Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])}{A_P}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))}{2\alpha^3}]$$
$$+Cov(X_R^2(\omega), X_R(\omega))\left[\frac{1}{Var(X_R(\omega))} - \frac{\alpha_2}{2\alpha Var(X_R(\omega))} - \frac{\alpha_1\theta^2 Var(f(\omega)X_R(\omega))}{2\alpha^3 Z(f(\omega), X_R(\omega), \theta, \alpha)}\right]$$
$$-\frac{\alpha\sum_{k=1}^m P_{r_k}\beta_{r_k}}{\theta}$$
$$\Rightarrow \frac{(P_X^F(\omega) - E[P_X^{S*}(\omega)])}{Z(f(\omega), X_R(\omega), \theta, \alpha)}\left[\frac{n_1 Var(f(\omega)X_R(\omega))}{A_P} + Var(f(\omega))E[X_R^2(\omega)]\left(\frac{n_2}{A_P} + \frac{m}{A_R}\right)\right]$$
$$= \frac{Cov(X_R(\omega), f(\omega)X_R(\omega))}{Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])}{A_P}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))}{2\alpha^3}]$$
$$+\frac{Cov(X_R^2(\omega), X_R(\omega))}{Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]}{\alpha^2} - \frac{\alpha_2\theta^2 Var(f(\omega))E[X_R^2(\omega)]}{2\alpha^3}$$
$$-\frac{\alpha_1\theta^2 Var(f(\omega)X_R(\omega))}{2\alpha^3}] - \frac{\alpha}{\theta}\sum_{k=1}^m P_{r_k}\beta_{r_k}$$
$$\Rightarrow (P_X^F(\omega) - E[P_X^{S*}(\omega)])[\frac{n_1}{A_P}(Var(f(\omega))Var(X_R(\omega)) + (E[f(\omega)])^2 Var(X_R(\omega))$$
$$+(E[X_R(\omega)])^2 Var(f(\omega))) + (\frac{n_2}{A_P} + \frac{m}{A_R})Var(f(\omega))E[X_R^2(\omega)]]$$
$$= Cov(X_R(\omega), f(\omega)X_R(\omega))[\frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])}{A_P}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))}{2\alpha^3}]$$
$$+Cov(X_R^2(\omega), X_R(\omega))[\frac{2\alpha\theta^2 Var(f(\omega))E[X_R^2(\omega)]}{2\alpha^3} - \frac{\alpha_2\theta^2 Var(f(\omega))E[X_R^2(\omega)]}{2\alpha^3}$$
$$-\frac{\alpha_1\theta^2(Var(f(\omega))Var(X_R(\omega)) + (E[f(\omega)])^2 Var(X_R(\omega)) + (E[X_R(\omega)])^2 Var(f(\omega)))}{2\alpha^3}]$$
$$-\frac{\alpha Z(f(\omega), X_R(\omega), \theta, \alpha)}{\theta}\sum_{k=1}^m P_{r_k}\beta_{r_k}$$

$$\Rightarrow \quad (P_X^F(\omega) - E[P_X^{S*}(\omega)])[\frac{n_1}{A_P}Var(f(\omega))E[X_R^2(\omega)] + \frac{n_1}{A_P}(E[f(\omega)])^2 Var(X_R(\omega))$$
$$+(\frac{n_2}{A_P} + \frac{m}{A_R})Var(f(\omega))E[X_R^2(\omega)]]$$
$$= \frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(X_R(\omega))}{A_P}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), X_R(\omega))(E[f(\omega)])^2 Var(X_R(\omega))}{2\alpha^3}]$$
$$+Cov(X_R^2(\omega), X_R(\omega))[\frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]}{2\alpha^3}(2\alpha - \alpha_1 - \alpha_2)$$
$$-\frac{\alpha_1\theta^2(E[f(\omega)])^2 Var(X_R(\omega)))}{2\alpha^3}]$$
$$-\frac{\alpha Z(f(\omega), X_R(\omega), \theta, \alpha)}{\theta}\sum_{k=1}^{m} P_{r_k}\beta_{r_k}$$
$$\Rightarrow \quad (P_X^F(\omega) - E[P_X^{S*}(\omega)])[\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega))]$$
$$= \frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(X_R(\omega))}{A_P}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), X_R(\omega))(E[f(\omega)])^2 Var(X_R(\omega))}{2\alpha^3}$$
$$+Cov(X_R^2(\omega), X_R(\omega))[\frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]}{2\alpha^2}$$
$$-\frac{\alpha_1\theta^2(E[f(\omega)])^2 Var(X_R(\omega)))}{2\alpha^3}]$$
$$-\frac{\alpha Z(f(\omega), X_R(\omega), \theta, \alpha)}{\theta}\sum_{k=1}^{m} P_{r_k}\beta_{r_k}$$
$$\Rightarrow \quad (P_X^F(\omega) - E[P_X^{S*}(\omega)])[\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega))]$$
$$= \frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(X_R(\omega))}{A_P}$$
$$+\frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]Skew(X_R(\omega))}{2\alpha^2}$$
$$+\frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]E[X_R(\omega)]Var(X_R(\omega))}{\alpha^2}$$
$$-\frac{\alpha Z(f(\omega), X_R(\omega), \theta, \alpha)}{\theta}\sum_{k=1}^{m} P_{r_k}\beta_{r_k}$$
$$\Rightarrow \quad (P_X^F(\omega) - E[P_X^{S*}(\omega)])[\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega))]$$
$$= \frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(X_R(\omega))}{A_P}$$
$$+\frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]Skew(X_R(\omega))}{2\alpha^2}$$

$$-\frac{\alpha Z(f(\omega),X_R(\omega),\theta,\alpha)}{\theta}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\right]$$

$$\Rightarrow \quad P_X^{F*}(\omega) = E[P_X^{S*}(\omega)] + \frac{n_1(P_Y^{F*}(\omega) - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(X_R(\omega))}{A_P(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))}$$
$$+\frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]Skew(X_R(\omega))}{2\alpha^2(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))}$$
$$-\frac{\alpha Z(f(\omega),X_R(\omega),\theta,\alpha)}{\theta(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\right] \quad (55)$$

We now arrive at a similar expression for $P_Y^{F*}(\omega)$ by inserting Equations 41 and 38 into Equation 11:

$$\frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])Var(X_R(\omega))}{A_P}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))Var(X_R(\omega))}{2\alpha^3}$$
$$-\frac{n_1(P_X^F(\omega) - E[P_X^{S*}(\omega)])Cov(X_R(\omega), f(\omega)X_R(\omega))}{A_P}$$
$$-\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), X_R(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))}{2\alpha^3}$$
$$= \gamma E[X_R'(\omega)]Z(f(\omega),X_R(\omega),\theta,\alpha)$$

$$\Rightarrow \quad \frac{n_1(P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)])Var(X_R(\omega))}{A_P} = \gamma E[X_R'(\omega)]Z(f(\omega),X_R(\omega),\theta,\alpha)$$
$$-\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))Var(X_R(\omega))}{2\alpha^3}$$
$$+\frac{n_1(P_X^F(\omega) - E[P_X^{S*}(\omega)])Cov(X_R(\omega), f(\omega)X_R(\omega))}{A_P}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), X_R(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))}{2\alpha^3}$$

$$\Rightarrow \quad P_Y^F(\omega) - E[f(\omega)P_X^{S*}(\omega)] = \frac{\gamma E[X_R'(\omega)]Z(f(\omega),X_R(\omega),\theta,\alpha)}{\eta' Var(X_R(\omega))}$$
$$+\frac{(P_X^F(\omega) - E[P_X^{S*}(\omega)])Var(X_R(\omega))E[f(\omega)]}{Var(X_R(\omega))}$$
$$+\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), X_R(\omega))Var(X_R(\omega))E[f(\omega)]}{2\alpha^3\eta' Var(X_R(\omega))}$$
$$-\frac{\alpha_1\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))Var(X_R(\omega))}{2\alpha^3\eta' Var(X_R(\omega))}$$

$$\Rightarrow \quad P_Y^{F*}(\omega) = E[f(\omega)P_X^{S*}(\omega)] + \frac{\gamma\theta^2 E[X_R'(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha^2}$$
$$+E[f(\omega)](P_X^{F*}(\omega) - E[P_X^{S*}(\omega)]) \quad (56)$$

By solving Equations 55 and 56 simultaneously, we arrive at Equations 44 and 45:

$$
\begin{aligned}
& P_X^{F*}(\omega) = E[P_X^{S*}(\omega)] + \frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]Skew(X_R(\omega))}{2\alpha^2(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))} \\
& -\frac{\alpha Z(f(\omega), X_R(\omega), \theta, \alpha)}{\theta(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\right] \\
& +\frac{n_1 E[f(\omega)]Var(X_R(\omega))}{A_P(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))} \\
& \left[\frac{\gamma\theta^2 E[X_R'(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha^2} + E[f(\omega)](P_X^{F*}(\omega) - E[P_X^{S*}(\omega)])\right] \\
\Rightarrow \quad & (P_X^{F*}(\omega) - E[P_X^{S*}(\omega)])\left[\frac{A_P\eta Var(f(\omega))E[X_R^2(\omega)]}{A_P(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))}\right] \\
& = \frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]Skew(X_R(\omega))}{2\alpha^2(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))} \\
& +\frac{n_1 E[f(\omega)]Var(X_R(\omega))\gamma\theta^2 E[X_R'(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha^2 A_P(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))} \\
& -\frac{\alpha Z(f(\omega), X_R(\omega), \theta, \alpha)}{\theta(\eta Var(f(\omega))E[X_R^2(\omega)] + \eta'(E[f(\omega)])^2 Var(X_R(\omega)))}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\right] \\
\Rightarrow \quad & (P_X^{F*}(\omega) - E[P_X^{S*}(\omega)])\eta Var(f(\omega))E[X_R^2(\omega)] \\
& = \frac{\theta^2 Var(f(\omega))E[X_R^2(\omega)]Skew(X_R(\omega))}{2\alpha^2} \\
& +\frac{n_1 E[f(\omega)]Var(X_R(\omega))\gamma\theta^2 E[X_R'(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha^2 A_P} \\
& -\frac{\theta}{\alpha}Var(f(\omega))Var(X_R(\omega))E[X_R^2(\omega)]\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\right] \\
\Rightarrow \quad & P_X^{F*}(\omega) - E[P_X^{S*}(\omega)] = \frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta} + \frac{E[f(\omega)]Var(X_R(\omega))\gamma\theta^2 E[X_R'(\omega)]}{\alpha^2\eta} \\
& -\frac{\theta}{\alpha\eta}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\right]Var(X_R(\omega)) \\
\Rightarrow \quad & P_X^{F*}(\omega) = E[P_X^{S*}(\omega)] + \frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta} \\
& -\frac{\theta}{\alpha\eta}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right]Var(X_R(\omega))
\end{aligned} \tag{57}
$$

This is identical to Equation 44. By substituting this into Equation 56, we obtain Equation 45:

$$
P_Y^{F*}(\omega) = E[f(\omega)P_X^{S*}(\omega)] + \frac{\gamma\theta^2 E[X_R'(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha^2}
$$

$$+\frac{E[f(\omega)]\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta}$$

$$-\frac{\theta}{\alpha\eta}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] E[f(\omega)]Var(X_R(\omega))$$

$$\Rightarrow \quad P_Y^{F*}(\omega) = E[f(\omega)P_X^{S*}(\omega)] + \frac{\gamma\theta E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha(1+\gamma E[f(\omega)])}$$

$$+\frac{E[f(\omega)]\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta}$$

$$-\frac{\theta}{\alpha\eta}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] E[f(\omega)]Var(X_R(\omega))$$

$$\Rightarrow \quad P_Y^{F*}(\omega) = E[f(\omega)P_X^{F*}(\omega)] + \frac{\gamma\theta E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha(1+\gamma E[f(\omega)])} \tag{58}$$

Appendix C: Solving for Equilibrium Futures Positions

Here, we derive the equilibrium futures positions. To obtain the expression for $X_{p_i}^{F*}(\omega)$, we substitute Equations 44 and 45 into Equation 40:

$$X_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{Var(f(\omega)X_R(\omega))\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta A_P}$$

$$-\frac{\theta}{\alpha\eta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] Var(f(\omega)X_R(\omega))Var(X_R(\omega))$$

$$+\frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega), X_R(\omega))Var(f(\omega)X_R(\omega))}{2\alpha^3}$$

$$-\frac{E[f(\omega)]\theta^2 Skew(X_R(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))}{2\alpha^2\eta A_P}$$

$$-\frac{\gamma\theta E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]Cov(X_R(\omega), f(\omega)X_R(\omega))}{\eta'\alpha(1+\gamma E[f(\omega)])A_P}$$

$$+\frac{\theta}{\alpha\eta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] (E[f(\omega)])^2(Var(X_R(\omega)))^2$$

$$-\frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega), f(\omega)X_R(\omega))Cov(X_R(\omega), f(\omega)X_R(\omega))}{2\alpha^3}]$$

$$\Rightarrow \quad X_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega), X_R(\omega), \theta, \alpha)}[\frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta A_P}[Var(f(\omega)X_R(\omega))$$

$$-(E[f(\omega)])^2 Var(X_R(\omega))] + \frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega), X_R(\omega))}{2\alpha^3}[Var(f(\omega)X_R(\omega))$$

$$
\begin{aligned}
&-(E[f(\omega)])^2Var(X_R(\omega))] + \frac{\theta}{\alpha\eta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] \\
&Var(X_R(\omega))[(E[f(\omega)])^2Var(X_R(\omega)) - Var(f(\omega)X_R(\omega))] \\
&-\frac{\gamma\theta^2 E[X_R'(\omega)]Var(f(\omega))E[X_R^2(\omega)]E[f(\omega)]Var(X_R(\omega))}{\eta'\alpha^2 A_P}] \\
\Rightarrow\quad & X_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega),X_R(\omega),\theta,\alpha)}[\frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta A_P}Var(f(\omega))E[X_R^2(\omega)] \\
&+\frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega),X_R(\omega))}{2\alpha^3}Var(f(\omega))E[X_R^2(\omega)] \\
&-\frac{\theta}{\alpha\eta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] Var(X_R(\omega))Var(f(\omega))E[X_R^2(\omega)] \\
&-\frac{\gamma\theta E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]E[f(\omega)]Var(X_R(\omega))}{\eta'\alpha A_P(1+\gamma E[f(\omega)])}] \\
\Rightarrow\quad & X_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega),X_R(\omega),\theta,\alpha)}[\frac{Skew(X_R(\omega))Z(f(\omega),X_R(\omega),\theta,\alpha)}{2\eta A_P Var(X_R(\omega))} \\
&+\frac{\alpha_{p_i}Skew(X_R(\omega))Z(f(\omega),X_R(\omega),\theta,\alpha)}{2\alpha Var(X_R(\omega))} + \frac{\alpha_{p_i}E[X_R(\omega)]Z(f(\omega),X_R(\omega),\theta,\alpha)}{\alpha} \\
&-\frac{\alpha}{\theta\eta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] Z(f(\omega),X_R(\omega),\theta,\alpha) \\
&-\frac{\gamma\alpha E[f(\omega)]E[P_X^{S*}(\omega)]Z(f(\omega),X_R(\omega),\theta,\alpha)}{n_1\theta(1+\gamma E[f(\omega)])}] \\
\Rightarrow\quad & X_{p_i}^{F*}(\omega) = \frac{\alpha_{p_i}}{\alpha}E[X_R(\omega)] + \left[\frac{1}{2\eta A_P} + \frac{\alpha_{p_i}}{2\alpha}\right]\frac{Skew(X_R(\omega))}{Var(X_R(\omega))} \\
&-\frac{\alpha}{\eta\theta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] - \frac{\gamma E[f(\omega)]E[X_R'(\omega)]}{n_1} \qquad (59)
\end{aligned}
$$

Similarly, by substituting Equations 44 and 45 into Equation 41, we obtain the expression for $Y_{p_i}^{F*}(\omega)$:

$$
\begin{aligned}
&Y_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega),X_R(\omega),\theta,\alpha)}[\frac{\theta^2 E[f(\omega)]Skew(X_R(\omega))Var(X_R(\omega))}{2\alpha^2\eta A_P} \\
&+\frac{\gamma\theta E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]Var(X_R(\omega))}{\eta'\alpha(1+\gamma E[f(\omega)])A_P} \\
&-\frac{\theta}{\alpha\eta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] E[f(\omega)](Var(X_R(\omega)))^2 \\
&+\frac{\alpha_{p_i}\theta^2 E[f(\omega)]Cov(X_R^2(\omega),X_R(\omega))Var(X_R(\omega))}{2\alpha^3} \\
&-\frac{\theta^2 Skew(X_R(\omega))E[f(\omega)]Var(X_R(\omega))}{2\alpha^2\eta A_P}
\end{aligned}
$$

$$+\frac{\theta E[f(\omega)]Var(X_R(\omega))}{\alpha\eta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right]Var(X_R(\omega))$$
$$-\frac{\alpha_{p_i}\theta^2 Cov(X_R^2(\omega),X_R(\omega))E[f(\omega)]Var(X_R(\omega))}{2\alpha^3}]$$
$$\Rightarrow \quad Y_{p_i}^{F*}(\omega) = \frac{1}{Z(f(\omega),X_R(\omega),\theta,\alpha)}\frac{\gamma\theta^2 E[X_R'(\omega)]Var(f(\omega))E[X_R^2(\omega)]Var(X_R(\omega))}{n_1\alpha^2}$$
$$\Rightarrow \quad Y_{p_i}^{F*}(\omega) = \frac{\gamma E[X_R'(\omega)]}{n_1} \tag{60}$$

Derivation of Equation 48 is similar to that of Equation 46, i. e., substitute Equation 44 into Equation 42:

$$X_{p_j}^{F*}(\omega) = \frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta A_P Var(P_X^{S*}(\omega))} + \frac{\alpha_{p_j} Cov(X_R^2(\omega),X_R(\omega))}{2\alpha Var(X_R(\omega))}$$
$$-\frac{\theta Var(X_R(\omega))}{\alpha\eta A_P Var(P_X^{S*}(\omega))}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right]$$
$$\Rightarrow \quad X_{p_j}^{F*}(\omega) = \frac{Skew(X_R(\omega))}{2\eta A_P Var(P_X^{S*}(\omega))} + \frac{\alpha_{p_j} Skew(X_R(\omega))}{2\alpha Var(X_R(\omega))} + \frac{\alpha_{p_j} E[X_R(\omega)]}{\alpha}$$
$$-\frac{\alpha Var(X_R(\omega))}{\theta\eta A_P Var(X_R(\omega))}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right]$$
$$\Rightarrow \quad X_{p_j}^{F*}(\omega) = \frac{\alpha_{p_j}}{\alpha}E[X_R(\omega)] + \left[\frac{1}{2\eta A_P} + \frac{\alpha_{p_j}}{2\alpha}\right]\frac{Skew(X_R(\omega))}{Var(X_R(\omega))}$$
$$-\frac{\alpha}{\eta\theta A_P}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] \tag{61}$$

Finally, by substituting Equation 44 into Equation 43 and making use of the fact that for any random variables A and B, $Cov(AB,B) = Coskew(A,B) + E[B]Cov(A,B) + E[A]Var(B)$, we obtain the expression for $X_{r_k}^{F*}(\omega)$:

$$X_{r_k}^{F*}(\omega) = -\frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta A_R Var(P_X^{S*}(\omega))} - \frac{\alpha P_{r_k}\beta_{r_k}}{\theta}$$
$$+\frac{\theta Var(X_R(\omega))}{\alpha\eta A_R Var(P_X^{S*}(\omega))}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right]$$
$$+\frac{Cov(X_R(\omega)X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{Var(X_R(\omega))}$$
$$\Rightarrow \quad X_{r_k}^{F*}(\omega) = -\frac{Skew(X_R(\omega))}{2\eta A_R Var(X_R(\omega))} - \frac{\alpha P_{r_k}\beta_{r_k}}{\theta}$$
$$+\frac{\alpha}{\theta\eta A_R}\left[\sum_{k=1}^{m} P_{r_k}\beta_{r_k} - E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right]$$

$$
\begin{aligned}
&+\frac{Cov(X_R(\omega)X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{Var(X_R(\omega))}\\
\Rightarrow\quad & X_{r_k}^{F*}(\omega)=\frac{\alpha}{\theta}\left[\frac{1}{\eta A_R}\sum_{k=1}^{m}P_{r_k}\beta_{r_k}-P_{r_k}\beta_{r_k}\right]-\frac{Skew(X_R(\omega))}{2\eta A_R Var(X_R(\omega))}\\
&-\frac{\alpha(1+2\gamma E[f(\omega)])}{\theta\eta A_R(1+\gamma E[f(\omega)])}E[P_X^{S*}(\omega)]+\frac{Cov(X_R(\omega)X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{Var(X_R(\omega))}\\
\Rightarrow\quad & X_{r_k}^{F*}(\omega)=\frac{\alpha}{\theta}\left[\frac{1}{\eta A_R}\sum_{k=1}^{m}P_{r_k}\beta_{r_k}-P_{r_k}\beta_{r_k}\right]-\frac{Skew(X_R(\omega))}{2\eta A_R Var(X_R(\omega))}\\
&-\frac{2E[X_R'(\omega)]Var(X_R(\omega))}{2\eta A_R Var(X_R(\omega))}+\frac{Cov(X_R(\omega)X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{Var(X_R(\omega))}\\
&-\frac{2\gamma E[f(\omega)]E[X_R'(\omega)]}{\eta A_R}\\
\Rightarrow\quad & X_{r_k}^{F*}(\omega)=\frac{\alpha}{\theta}\left[\frac{1}{\eta A_R}\sum_{k=1}^{m}P_{r_k}\beta_{r_k}-P_{r_k}\beta_{r_k}\right]-\frac{Skew(X_R(\omega))}{2\eta A_R Var(X_R(\omega))}\\
&-\frac{E[X_R'(\omega)]}{\eta A_R}+\frac{Coskew(X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{Var(X_R(\omega))}+E[X_{r_k}(\omega)(1+\gamma f(\omega))]\\
&+\frac{E[X_R(\omega)]Cov(X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{Var(X_R(\omega))}-\frac{2\alpha\gamma E[f(\omega)]}{\theta\eta A_R(1+\gamma E[f(\omega)])}E[P_X^{S*}(\omega)]\\
\Rightarrow\quad & X_{r_k}^{F*}(\omega)=E[X_{r_k}(\omega)(1+\gamma f(\omega))]-\frac{\alpha}{\theta}\beta_{r_k}\left[P_{r_k}-E[P_X^{S*}(\omega)]\right]\\
&+\frac{Coskew(X_{r_k}(\omega)(1+\gamma f(\omega)),X_R(\omega))}{Var(X_R(\omega))}-\frac{Skew(X_R(\omega))}{2\eta A_R Var(X_R(\omega))}\\
&+\frac{\alpha}{\theta\eta A_R}\left[\sum_{k=1}^{m}P_{r_k}\beta_{r_k}-E[P_X^{S*}(\omega)]\frac{(1+2\gamma E[f(\omega)])}{(1+\gamma E[f(\omega)])}\right] \qquad (62)
\end{aligned}
$$