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Event-related potentials of single-digit addition, subtraction, and multiplication

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Abstract

This study compared the event-related potentials elicited by single-digit addition, subtraction, and multiplication problems. With a delayed verification paradigm, 18 Chinese undergraduates were first asked to solve the arithmetic problems that were presented visually for 200 ms and, after 1.5 s, to judge whether a presented solution was correct or not. Results showed that, compared to addition and subtraction, multiplication elicited a greater N300 at the left frontal electrodes peaking around 320 ms (in the interval between 275 and 334 ms after the onset of the arithmetic problem). To control for the confounding effects of task difficulty and solution size, comparisons were further made between “large” addition problems (with sums between 11 and 17) and “small” multiplication problems (with products between 6 and 24). Similar results were obtained (i.e., a significant difference between addition and multiplication in the N300 component between 296 and 444 ms). Source analyses demonstrated that a single dipole in the left anterior brain areas could have contributed to the topographies of the difference waveforms (“multiplication–addition”, “multiplication–subtraction”, and “‘small’ multiplication–‘large’ addition”). These results are interpreted in terms of the greater reliance on phonological processing for the retrieval of multiplication facts than for the retrieval of addition and subtraction facts.

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Keywords: Arithmetic facts; Representation; Cognitive arithmetic; Numerical cognition; Event-related potentials

1. Introduction

Single-digit addition, subtraction, and multiplication are fundamental operations of arithmetic. Children acquire the facts of single-digit arithmetic mainly through two types of strategies: procedural strategies and rote verbal memory (e.g., Dehaene & Cohen, 1997; Roussel, Fayol, & Barrouillet, 2002; Zhou & Dong, 2003). Procedural strategies, such as counting, transformation (e.g., $6 + 7 = 6 + 6 + 1$, $9 + 7 = 9 + 1 + 6$), and repeated addition, typically involve quantity manipulation along the mental number line. With the rote memory strategy, people repeatedly recite arithmetic facts so that the facts could be stored

in memory as a type of modularized phonological associations between a digit pair and their answer.

School children are usually taught to use procedural strategies for simple addition and subtraction, but to use rote memory strategy to memorize multiplication facts (e.g., Dehaene & Cohen, 1997; Roussel et al., 2002; Zhou & Dong, 2003). These differential strategies during the acquisition of arithmetic facts may play an important role in shaping their mental representations (e.g., Siegler & Shipley, 1995; Siegler & Shrager, 1984). It is possible that mental representations of multiplication facts have greater reliance on verbal memory than those for addition and subtraction facts. This hypothesis has been supported by several neuropsychological studies. Researchers have found that patients with lesions in the left perisylvian language region and those with low verbal fluency had more difficulty in single-digit multiplication than in addition and subtraction (e.g., Cohen,

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Dehaene, Chochon, Lehéricy, & Naccache, 2000; Dehaene & Cohen, 1997; Delazer & Benke, 1997; Lemer, Dehaene, Spelke, & Cohen, 2003; Pesenti, Seron, & van der Linden, 1994; van Harskamp, Rudge, & Cipolotti, 2002, 2005).

In contrast, neuroimaging studies with PET and fMRI have yielded inconsistent results. Consistent with the lesion studies, two fMRI studies have found that, compared to subtraction or numerical–magnitude comparison, multiplication elicited more activation in the left perisylvian language regions (either the left angular gyrus or the left inferior frontal gyrus) (e.g., Lee, 2000; Rickard et al., 2000). Several other neuroimaging studies, however, failed to replicate these results (Chochon, Cohen, Van de Moortele, & Dehaene, 1999; Dehaene et al., 1996; Hayashi, Ishii, Kitagaki, & Kazui, 2000; Kawashima et al., 2004; Kazui, Kitagaki, & Mori, 2000). One possible reason for these inconsistent findings is that PET and fMRI have poor temporal resolution and are not always sensitive enough to detect potential differences in a small time window among arithmetic operations. After all, a typical single-digit arithmetic problem involves three stages: converting stimulus into appropriate internal codes, retrieving or calculating the answer, and reporting the answer (Campbell & Epp, 2005). Only the second stage is expected to differ across the arithmetic operations in terms of the involvement of verbal processing. By averaging signals from all three stages, PET and fMRI studies have a greatly reduced sensitivity to differences across arithmetic operations.

No studies have used the event-related potential (ERP) technique to systematically compare the neural basis of single-digit addition, subtraction, and multiplication. With its high temporal resolution and moderate capability for spatial localization, the ERP technique is a useful tool to examine the time-locked difference in neural bases of the three arithmetic operations. Using the ERP technique, the present study tested our hypothesis that single-digit multiplication has greater reliance on phonological processing than does single-digit addition and subtraction. With source analysis, we further expected that differences among arithmetic operations can be localized at either of the following two areas: the left inferior frontal gyrus, especially the Broca's area, which has the function of phonological processing (e.g., Burton, 2001; Poldrack et al., 1999), or the left angular gyrus, which has been linked to number processing (Dehaene, Piazza, Pinel, & Cohen, 2003).

Comparisons of neural bases for different single-digit arithmetic operations are often subject to the confounding effect of task difficulty. Previous research has clearly documented that different arithmetic operations have different levels of difficulty. In general, multiplication has been found to be more difficult than other arithmetic operations (e.g., addition) or other number tasks (e.g., numerical–magnitude comparison) (e.g., Chochon et al., 1999; Kawashima et al., 2004; Zhou & Dong, 2003). To control for the potential confounding effect of task difficulty, we further compared “large” single-digit addition problems (with sums between 11 and 17) and “small” single-digit multiplication problems (with products between 6 and 24). Previous research has shown that these problems were matched in problem difficulty for Chinese subjects (Zhou & Dong, 2003).

2. Methods

2.1. Participants

Because we used a delayed verification task (see details in the next section), two separate samples were needed: one for the ERP experiment, and the other for the behavioral experiment. Thirty-six undergraduate subjects were recruited from Beijing Normal University for this study. Half of them were randomly (stratified by gender) assigned to the ERP experiment and the other half to the behavioral experiment. The average age of the subjects was 21.3 years, ranging from 17.8 to 27.2 years. All participants were right-handed and had normal or corrected-to-normal eyesight. They had not participated in any experiments similar to the present one (i.e., involving simple arithmetic tasks of addition, subtraction, and multiplication) during the past half a year. Subjects gave written informed consent before the experiment.

2.2. Materials

Single-digit addition, subtraction, and multiplication problems were used in this study. Because repeated-operand or “tie” problems (e.g., $3 + 3$, $3 - 3$, and 3×3) and those with 0 and 1 as an operand (e.g., $1 + 5$, $0 + 5$, 1×5 , 0×5) are rule-based problems (e.g., LeFevre et al., 1996), subjects typically use special strategies to solve these problems. Therefore, they were not used in this study. Subtraction problems were further limited to the large-operand-first problems (e.g., $3 - 2$) to avoid negative solutions. Due to these constraints and the need to have a balanced set of problems across operations, there were only 28 problems we could use for each operation. Consequently, to allow for enough trials for the ERP recording, we had to present each problem four times, twice followed by the correct solution, and twice followed by an incorrect solution. In the end, there were 112 trials for each type of operation. It should be pointed out that the incorrect solutions were not randomly generated. They had to come from the single-digit arithmetic table of the same operation type and had to have the same number of digits (either one or two digits) as the correct solution would have.

The addition and multiplication problems were further divided into “small” and “large” problems in order to control for the confounding effect of task difficulty. Based on Zhou and Dong's (2003) study, “large” addition problems included those with sums from 11 to 17 and “small” multiplication problems included those with products ranging from 6 to 24.

2.3. Procedure

Subjects were seated 105 cm away from the computer screen in a dimly lit, sound-attenuated room. All stimuli were presented visually in white against black background at the center of the screen. For each trial, a fixation sign “◇” (3 cm × 4 cm in size) was first presented for 200 ms, followed by a blank screen for 500 ms. Then two single-digit operands (each 2 cm × 3 cm in size), separated by a 4.5 cm blank space, were presented for 200 ms, followed by a blank screen for 1300 ms. During the presentation of two operands and the blank screen, subjects would perform addition, subtraction, or multiplication as instructed on the two operands. After the blank screen, a solution was presented. Subjects responded by pressing a key to indicate whether the presented solution was the same as their solution. Half of the subjects responded “Yes” with their right hand and “No” with their left hand, and the other half in the opposite way. After the response, the presented solution disappeared from the screen and 2000 ms later the next trial began. Instructions emphasized both speed and accuracy. Subjects showed high accuracy in their responses (error rate < 2%) and fast RT (436, 427, and 432 ms for addition, subtraction, and multiplication conditions, respectively, from the onset of the solution to response).

We used a delayed verification task rather than a production task or a standard verification task in the ERP experiment for several reasons. First, in verbal-production tasks, the tongue movement and muscle activities might add artifacts to the EEG recording. For this reason alone, many other similar studies have used delayed verification tasks (e.g., Galfano, Mazza, Angrilli, & Umiltà, 2004; Niedeggen & Rosler, 1999; Szucs & Csepe, 2005). Second, multiplication problems generally involve larger numbers as their answers (ranging from 6 to 72) than do other operations such as addition (5–17) and subtraction (1–7).

Consequently, verbal production might result in more verbal processing for multiplication than for addition and subtraction. The verification paradigm helps to minimize the confounding effect of verbal production. Finally, the standard verification task (i.e., operands and proposed solution are presented simultaneously, e.g., “ $2 + 3 = 8$ ”) has been found to involve mental processes that are different from those for verbal production of answers (e.g., Campbell & Tarling, 1996; Lemaire & Fayol, 1995). That is because the proposed solution in the standard verification tasks would allow subjects to make a plausibility judgment, whereas the production paradigm and the delayed verification paradigm would require either direct retrieval of problem solutions or calculation.

Previous research has shown strong evidence of interference among arithmetic operations when subjects had to switch among them (e.g., Campbell & Oliphant, 1992). To reduce such interference, problems were presented in separate blocks. Each type of operations had two blocks (about 5 min each). Problems were randomly presented within a block, with the constraint that consecutive problems did not have a common operand or the same solution. At the beginning of each block, the arithmetic operation to be performed was cued on the screen. Subjects had a 2-min rest between blocks.

Before the formal test, there were practice trials with problems with 0 and 1 as one of the operands (e.g., 0×2 , 1×2). During the practice stage, subjects were instructed to avoid eye-blinks during a trial (from the beginning of fixation through pressing the key). Subjects were given feedback if they made eye blinks, made too many response errors, took too long to respond, or had obvious head movement.

2.4. Electroencephalography (EEG) recording and analysis

Scalp voltages were recorded by a NeuroSCAN system, using a 64-channel Quick-cap with silver chloride electrodes (Neurosoft, Inc., Sterling, USA). Linked-ears served as reference (see Section 4 for a comment on potential problems with this procedure), and the middle of the forehead served as ground. Two channels were placed at the outer canthi of both eyes to record the horizontal electrooculogram (HEOG), and another two channels above and below the left eye for vertical electrooculogram (VEOG). The sampling rate was 1000 Hz. The impedance of all electrodes was kept below 5 k Ω . Immediately following scalp electrical recording, electrode positions and physical landmarks were digitized using Polhemus Fastrak digitizer and 3D SpaceDx software contained within the NeuroScan software package.

Offline, trials were rejected for incorrect responses, movement artifacts, or amplifier saturation. A DC correction was applied, and ocular artifacts were then corrected with NeuroScan EDIT (Version 4.3). The trigger threshold for ocular artifacts was set to 10%. The minimum number of sweeps that were required to construct an averaged VEOG artifact was 20. The duration of the average artifacts was 400 ms. After the correction of ocular artifacts, the continuous EEG data were segmented into epochs from 200 ms prestimulus (i.e., 200 ms before the onset of digit pairs) until 1500 ms poststimulus. The 200 ms prestimulus served as the baseline. EEG were detrended and baseline-corrected. Epochs exceeding the range of -100 to $100 \mu\text{V}$ at any channel except HEOG and VEOG were rejected as artifacts. The remaining trials were averaged for each operation separately for each subject. The percentage of valid trials used for averaging were $90 \pm 13\%$ for addition, $92 \pm 10\%$ for subtraction, and $93 \pm 11\%$ for multiplication. The averaged waveform was filtered with a lowpass of 30 Hz (zero-phase, 12 dB/octave). The grand average was obtained by averaging across the subjects' averages separately for each arithmetic operation (i.e., addition, subtraction, and multiplication).

2.5. Statistical analysis

The event-related potentials to be analyzed were time-locked to the onset of digit pairs. Sample-by-sample nonparametric statistics were performed on these data by using the Friedman test to isolate the location and time window among experimental conditions. The *p*-values for all samples over 64 electrodes were visualized with a software programmed in-house. A difference among experimental conditions in a scalp region was considered significant when it appeared for 30 consecutive samples (a duration of 30 ms) simultaneously on at least five electrodes with a significance level of .05. This criterion for significance was similar to that used in Pinel, Dehaene, Riviere, and LeBihan's (2001)

study on number-comparison task. We selected groups of adjacent electrodes over which the operation effect reached the highest level of significance. The voltages averaged across those electrodes and associated time windows were then entered into a repeated measures ANOVA with multiple pairwise comparisons (with Bonferroni adjustment). Scalp topographies were visualized with EEGLAB (<http://scen.ucsd.edu/eeglab/>).

Similar analyses were conducted to compare “large” addition and “small” multiplication problems. For these analyses, we used the sample-by-sample nonparametric statistics with the Wilcoxon test. Because there were only two conditions, post hoc contrasts were not needed.

For both the event-related potential experiment and the behavioral experiment, the correct trials with greater than 3 s in reaction time were discarded. Reaction times for the correct trials were further trimmed for each subject by excluding trials with RT that were longer than three standard deviations above that individual's mean RT. Reaction times and error rates from each experiment were subjected to a repeated measures ANOVA.

2.6. Source analysis

The software Brain Electrical Source Analysis (BESA, Version 5.1.4) (Scherg & Ebersole, 1993) was used to construct source models for the operation effects. The default four-shell ellipsoidal head model was used to model intracranial generator(s) as a single dipole source. The averaged digitized electrode locations across subjects were imported as electrode configurations. The grand mean difference waveforms among operations were analyzed. The interval of waveform for source analysis was defined on the basis of the global field power (GFP) (Lehman & Skrandies, 1986; Schlereth, Baumgartner, Magerl, Stoeter, & Treede, 2003). Goodness-of-fit was estimated in terms of residual variance (RV), i.e., the percentage of variance in the interval that could not be explained by the model.

Two types of source models were tested. First, we tested source models that placed a single dipole in a region of interest and examined how well the data fit the models. Specifically, in two separate models for each type of scalp topography of the difference waveforms, a single dipole for the onset phase of the GFP peak was placed at one of the two brain regions that might be involved in the phonological processing of numbers: the left inferior frontal gyrus and the left angular gyrus. Summarizing the neuroimaging and neuropsychological studies on number processing (Chochon et al., 1999; Dehaene, Spelke, Stanescu, Pinel, & Tsivkin, 1999; Lee, 2000; Simon, Cohen, Mangin, Bihan, & Dehaene, 2002; Stanescu-Cosson et al., 2000), Dehaene et al. (2003) concluded that the angular gyrus was the language system for number processing. The average Talairach coordinates of activation were -41 , -66 , and 36 . In addition, relative to other tasks (i.e., letter naming, eye fixation, perceptual motor control task, number-magnitude comparison, or digit reading), single-digit multiplication activated the left inferior frontal gyrus (Talairach coordinates: -36 , 27 , 3 ; -54 , 16 , 24 ; -40 , 13 , 32) and the adjacent insula (Talairach coordinates: -30 , 18 , 2) (Chochon et al., 1999; Kawashima et al., 2004; Richard et al., 2000; Zago et al., 2001). We averaged these Talairach coordinates to define another potential neural source for phonological processing in multiplication. The averaged coordinates were -40 , 19 , and 15 , which is adjacent to the averaged coordinates (-43 , 18 , 17) for phonological tasks (e.g., rhyme generation, phoneme monitoring, and phonetic discrimination) as reported in Poldrack et al.'s (1999) summary of 18 studies. A single dipole was placed at locations in the Talairach space -40 , 19 , and 15 (inferior frontal gyrus) or -41 , -66 , and 36 (angular gyrus). The BESA then freely adjusted its orientation to match the scalp topography of the difference waveforms.

Second, to supplement the model-fitting analyses, we constructed a second type of models that allowed one dipole to fit the data without limiting the location. We could then examine how close the final fitted location is to the original region of interest (e.g., the left inferior frontal gyrus).

2.7. Behavioral experiment

The behavioral experiment was conducted in order to examine the relative difficulty of the different arithmetic operations. (The ERP experiment provided reaction times and error rates only on the verification stage, thus not relevant to the task difficulty involved in the generation of the solution.) The materials and

procedure were the same as those used for the ERP experiment with the exception that subjects were asked to orally report the solutions. A voice-activated relay was used as switch that controlled a software clock to record the reaction times. Subjects' responses were evaluated by the experimenter to calculate the error rates.

3. Results

3.1. Comparisons among addition, subtraction, and multiplication

Fig. 1 shows the mean reaction times and error rates from the behavioral experiment. One-factor repeated measures ANOVA with arithmetic operation as the within-subject factor showed a significant main effect of operation on reaction times, $F(2, 34) = 46.79$, $MSe = 361.93$, $p < .001$. Pairwise comparisons with Bonferroni adjustment at .05 level showed that RT was longer for multiplication than for addition and subtraction problems and longer for addition than for subtraction problems. Analyses of the error rates showed similar results, with a significant main effect of operation, $F(1, 17) = 16.15$, $MSe = 8.38$, $p < .001$. Pairwise comparisons showed more errors for multiplication than for addition and subtraction problems.

The raw waveforms elicited by single-digit addition, subtraction, and multiplication are shown in Fig. 2. Based on sample-by-sample analysis, we selected one group of electrodes mainly over the left frontal regions, including F3, F5, F7, FC3, FC5, FT7, AF3, and C5. The time window for significant effects was from 275 through 334 ms. One-factor repeated measures ANOVA with operation as the within-subject factor was conducted. The mean amplitudes differed significantly across operations, $F(2, 34) = 9.81$, $MSe = 4.70$, $p < .001$. Pairwise comparisons with Bonferroni adjustment showed that multiplication problems elicited more negative potentials than either addition or subtraction problems at the .001 significance level (see Fig. 2). No differences in the mean amplitudes were found between addition and subtraction problems. The operation effect seemed to occur at the N300 potentials. The significant difference was in its peak amplitude, $F(2, 34) = 9.31$, $MSe = 6.56$, $p < .005$, not its peak latency (in the interval of 275–334 ms). Pairwise comparisons revealed that multiplication had greater N300 than either addition or subtraction at the .01 level. Peak amplitudes did not differ between addition and subtraction (Fig. 3).

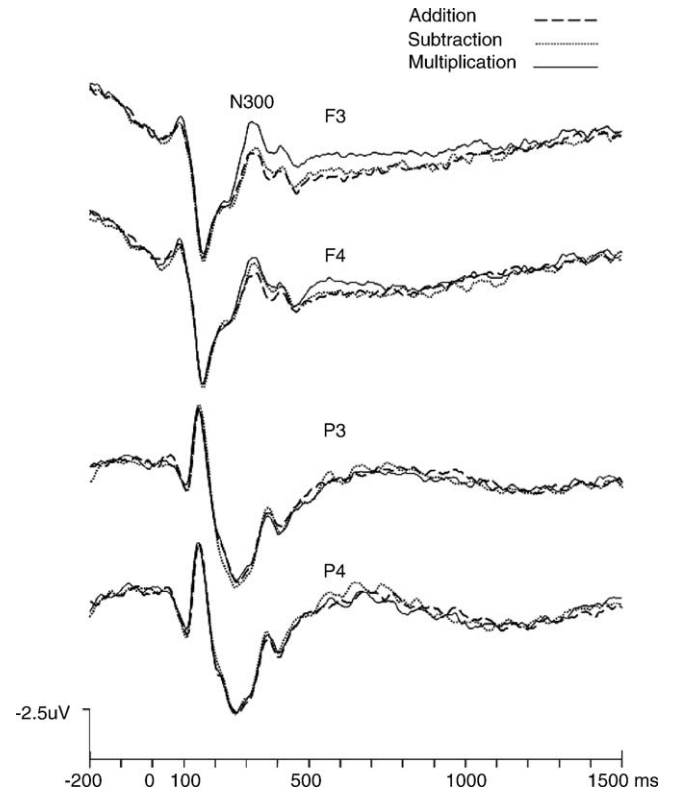


Fig. 2. The grand mean event-related potentials elicited by single-digit addition, subtraction, and multiplication over representative electrodes F3, F4, P3, and P4. Typical differences among operations located at the electrodes over the left and anterior parts of the scalp.

To investigate the locations of neural sources underlying differences between arithmetic operations, the brain electrical source analysis (BESA) software was used to derive a single-dipole model based on the mean difference waveforms. Modeling was conducted separately for two contrasts of the arithmetic operations: “multiplication–addition” and “multiplication–subtraction.” For the contrast between multiplication and addition, the onset phase of the GFP peak was from 263 to 323 ms, and the first spatial component of the principal component analysis (PCA) accounted for 97.0% of the variance in the topography of that interval. When the dipole was set at the left inferior frontal gyrus (Talairach coordinates: $-40, 19, 15$),

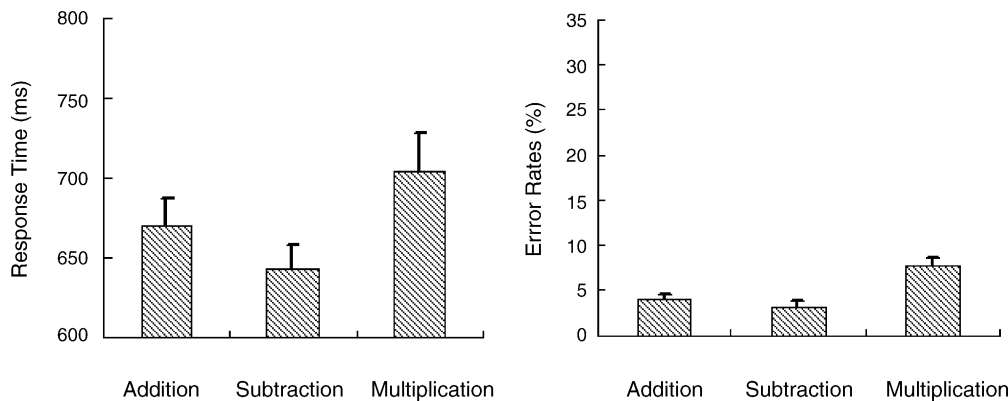


Fig. 1. Response times (ms) and error rates (%) when solving single-digit arithmetic problems: the behavioral experiment.

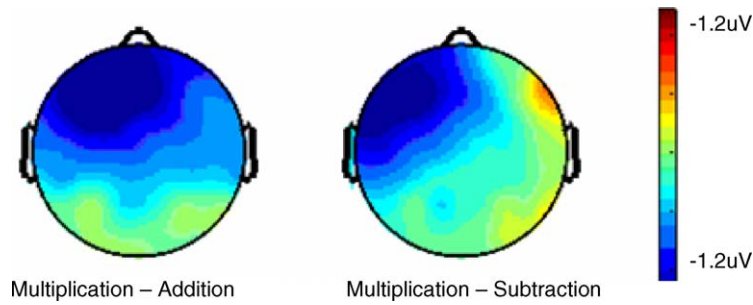


Fig. 3. Topographies of mean difference potentials among single-digit addition, subtraction and multiplication in the interval 275–334 ms. The left of the picture corresponds to the left scalp.

the residual variance (RV) was 12.4% (see Fig. 4), and when it was set at the angular gyrus (Talairach coordinates: $-41, -66, 36$), the RV was 34.4%. When the dipole was allowed to freely adjust its location and orientation, the Talairach coordinates were $-21, 5, 19$, and the RV was 8.5%.

For the contrast between multiplication and subtraction, the onset phase of the GFP peak was from 261 to 301 ms, and the first spatial component accounted for 98.2% of the variance. When the dipole was set at the left inferior frontal gyrus (Talairach coordinates: $-40, 19, 15$), the RV was 8.5% (see Fig. 4), and

when it was set at the angular gyrus (Talairach coordinates: $-41, -66, 36$), the RV was 42.7%. When the dipole was allowed to freely adjust its location and orientation, the Talairach coordinates were $-24, 25, 13$, and the RV was 6.0%.

3.2. Comparisons between “large” addition and “small” multiplication problems

Data from the behavioral experiment showed that the mean reaction times were 711 (± 28) ms for “large” addition problems and 702 (± 26) ms for “small” multiplication problems. Corresponding error rates were 4.6% ($\pm 1.1\%$) and 3.7% ($\pm .5\%$). Repeated measures ANOVA of these data found no significant differences either in reaction times or error rates, which is consistent with previous findings based on a similar sample (Zhou & Dong, 2003).

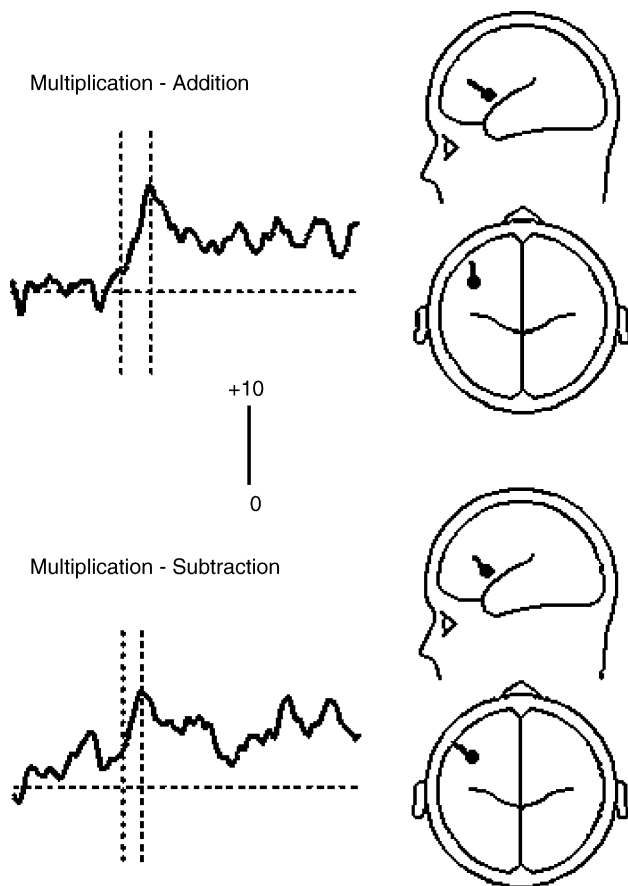


Fig. 4. Source waveforms (1–800 ms) and models for the difference potentials for “multiplication–addition” and “multiplication–subtraction”. The corresponding intervals for dipole fitting are 263–323 and 261–301 ms. The single dipole position is in left inferior frontal gyrus (Talairach coordinates: $-40, 19, 15$).

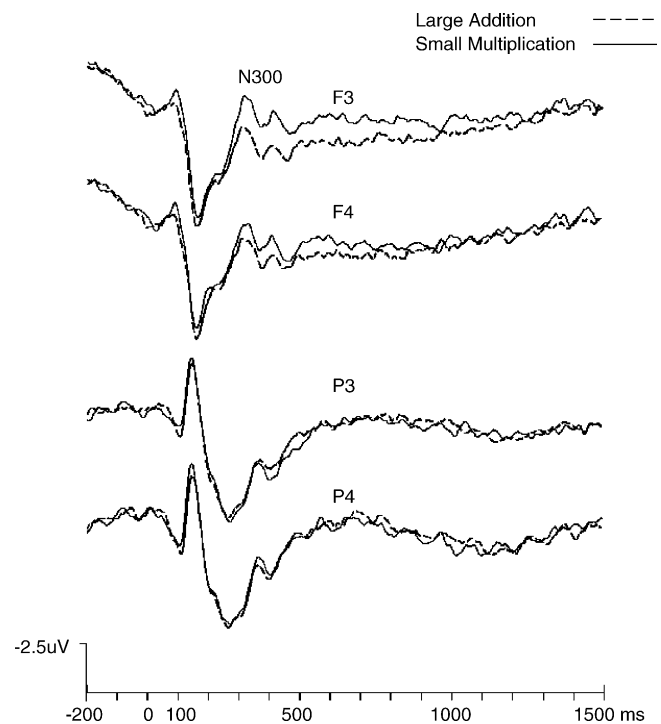


Fig. 5. The grand mean event-related potentials elicited by single-digit “large” addition and “small” multiplication over representative electrodes F3, F4, P3, and P4. Typical differences between operations located at the electrodes over the left and anterior parts of the scalp.

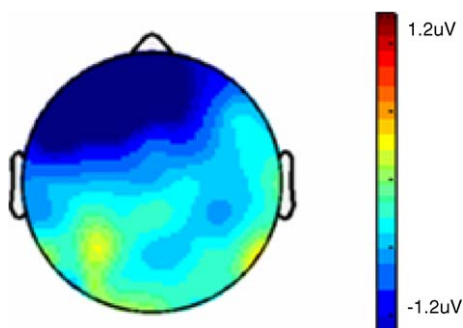


Fig. 6. Topography of mean difference potentials for the contrast between “small” multiplication and “large” addition in the interval 296–444 ms. The left of the picture corresponds to the left scalp.

The raw waveforms elicited by the difficulty-matched arithmetic problems are shown in Fig. 5. Based on sample-by-sample analysis with the Wilcoxon test, significant differences between “large” addition and “small” multiplication problems were found at the electrodes over the left frontal regions, including F1, F3, F5, F7, AF3, AF7, FP1, FC5, and FC3. The time window for significant effects was from 296 through 444 ms. During this interval, the peak amplitudes were $-3 \mu\text{V}$ for “large” addition and $1.1 \mu\text{V}$ for “small” multiplication problems, which differed significantly from each other, $F(1, 17) = 13.19$, $MSe = 17.34$, $p < .005$ (Fig. 6).

The averaged difference waveforms between “small” multiplication and “large” addition problems were submitted to the BESA for source analyses. The onset phase of the GFP peak was from 275 to 412 ms, and the first spatial component accounted for 94.0% of the variance. When the dipole was set in the left inferior frontal gyrus (Talairach coordinates: $-40, 19, 15$), the RV was 15.8% (see Fig. 7). When the dipole was set in the angular gyrus (Talairach coordinates: $-41, -66, 36$), the RV was 48.4%. When the dipole was allowed to freely match the scalp topography, the Talairach coordinates were $-32, 21$, and 26 , and the RV was 14.9%.

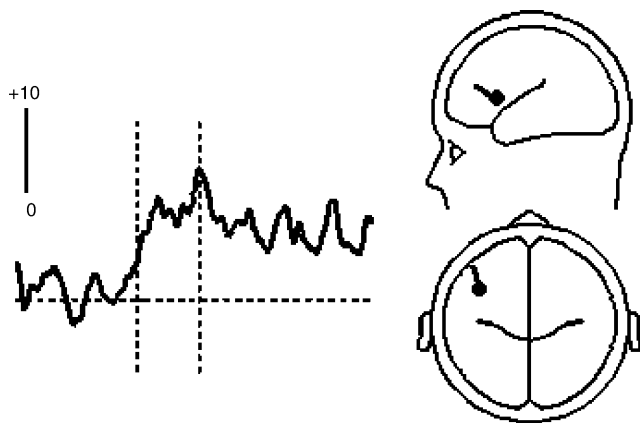


Fig. 7. Source waveform (1–800 ms) and model for difference potentials for the contrast between “small” multiplication and “large” addition. The interval for dipole fitting is 275–412 ms. The single dipole position is in left inferior frontal gyrus (Talairach coordinates: $-40, 19, 15$).

4. Discussion

The present study compared the event-related potentials elicited by single-digit addition, subtraction, and multiplication problems. Results showed that, compared to addition and subtraction, multiplication elicited more negative potentials on the electrodes over the scalp of left frontal lobe between 275 and 334 ms. The significant operation effects were in the N300 component peaking around 320 ms. Source analyses suggested that a single-dipole model at the left anterior brain region could have accounted for the topography of the difference waveforms (“multiplication–addition” and “multiplication–subtraction”). To control for the effect of task difficulty, same analyses were conducted with “large” addition and “small” multiplication that matched in problem difficulty and solution size. Similar results were obtained with a significant N300 effect between 296 and 444 ms. These findings were consistent with our hypothesis that the retrieval of multiplication facts has a greater reliance on phonological processing (in the left anterior brain region including the Broca’s area) than the retrieval of addition and subtraction facts. In the following sections, we discuss specifically the operation effects and their origin.

4.1. Operation effects

Based on the behavioral experiment, our subjects (Chinese college undergraduates) typically solved addition, subtraction, and multiplication within 800 ms. Within that period, they had to go through three stages of cognitive processes: encoding two visually presented digits, retrieving the answer (either through direct retrieval from the memory or through calculation), and reporting the solution to activate voice-control switch. Given that only the second stage differs across the three arithmetic operations, it is likely that subjects retrieved answers (with different strategies for different operations) in the interval of 200–400 ms, which created the N300 effect.

Of the source models we tested, the model with a single dipole placed at the angular gyrus did not fit the data, with RVs ranging from 34 through 48%. Results of the other two models (a single dipole placed at the left inferior frontal gyrus or allowed to freely adjust its location) appear to converge. They had similar RV, ranging from 8 to 16% for the left-inferior-frontal-gyrus (LIFG) model and 6–15% for the free-fitting model. These models located the source in the left anterior brain region, although the free-fitting models locate a more medial source (e.g., insula) than the LIFG model. It is worth noting that, when the task difficulty was controlled, the source location from the free-fitting model was especially close to that from the LIFG model.

Based on these data, we can draw a tentative conclusion that the left anterior brain region (likely to involve Broca’s area) may be more involved in multiplication than in subtraction and addition. In other words, this conclusion suggests that single-digit multiplication seems to involve phonological processing more than do single-digit addition and subtraction. As discussed in Section 1, this notion that multiplication facts are represented as phonological codes is consistent with previous neuropsychological findings (e.g., Delazer & Benke, 1997; Lemer et al., 2003;

van Harskamp et al., 2002) and an fMRI study (Richard et al., 2000).

4.2. Problem difficulty and solution size

Problem difficulty and solution size are potential confounding factors in comparisons of cognitive arithmetic tasks. Normally, multiplication problems were more difficult and typically involved larger solution size than addition and subtraction. However, several findings from this and other studies seem to suggest that problem difficulty or solution size is not responsible for the greater N300 at the left frontal electrodes for multiplication than for addition and subtraction. First, there was also a significant difference in difficulty and solution size between addition and subtraction (i.e., subjects took longer to solve addition problems than subtraction problems). However, these two types of operations did not differ in the N300 component. Second, the topography of ERP for problem difficulty was different from that for the operation effect (Kong et al., 1999). Kong et al. (1999) reported that difficult problems (with carrying, e.g., $35 + 9$) elicited greater P2 at frontal regions than easy problems (without carrying, e.g., $35 + 2$). Finally, and perhaps most importantly, when we matched the difficulty and solution size of addition and multiplication problems, we still found a similar N300 effect.

4.3. Acquisition of arithmetic facts

To understand the arithmetic operation effects, it is necessary to examine the role experiences during the acquisition of arithmetic facts play in shaping mental representations of arithmetic facts (e.g., LeFevre & Liu, 1997; Siegler & Shipley, 1995; Siegler & Shrager, 1984). We should comment on the way multiplication table was learnt by Chinese children. Like children in some other cultures (e.g., Dehaene & Cohen, 1997; Roussel et al., 2002), Chinese children are explicitly taught to use the rote verbal strategy to learn the multiplication facts (Zhou & Dong, 2003). Chinese children start to memorize the multiplication table during the second semester of the first grade or the first semester of the second grade (as compared to the third grade in the U.S.). It takes about 4 months for students to learn all multiplication facts. To Chinese children's advantage, the multiplication table (also called multiplication rhyme) is organized as pithy mnemonic formulas with a reasonable rhyme. The equations for most multiplication facts only include the two operands and their product, without the word "times" or "multiply", for example, "liu qi si shi er" (literally, "six seven four-ten-two") for $6 \times 7 = 42$. Another advantage is that Chinese multiplication table includes only smaller-operand-first entries. Finally, the shorter pronunciation duration of Chinese digits (Chen & Stevenson, 1988; Stigler, Lee, & Stevenson, 1986) may also facilitate Chinese children's learning of the multiplication table.

4.4. Limitations of the present study

Several limitations of the current study need to be discussed. First, although it is out of necessity that we repeated the same

arithmetic problems (because of a small number of plausible and comparable problems), such repetitions may result in the use of different cognitive strategies in solving the arithmetic problems (e.g., calculation in the beginning and memorization for the repeated problems). Such potential differences in strategies are of concern in cognitive research. For the current study, however, we believe that the repetitions of simple arithmetic problems were not likely to have a major impact on our results for two reasons. First, previous studies of single-digit arithmetic problems showed that a small number of repetitions may have only a limited effect on cognitive processes. For example, the problem size effect (i.e., larger problems are more difficult than smaller problems) is typically stable even when the same single-digit multiplication problems were repeated three times (LeFevre & Liu, 1997). Second, if repetitions result in the situation that subjects just remember the answers (across all types of operations), instead of calculating those answers for addition and subtraction and reciting verbal codes for multiplication, the repeated trials would theoretically have reduced the differences between multiplication and addition and those between multiplication and subtraction. Therefore, studies like ours that use repetitions of problems would have under-estimated (not inflated) differences among operations.

A second limitation of our study was that we used linked-ears as reference. This error may have resulted in an irreparable distortion of the field distribution on the scalp (Miller, Lutzenberger, & Elbert, 1991). However, we think the effects of that problem were mitigated by the fact that we were comparing across three conditions that presumably would be similarly affected by the distortion due to linked-ears reference.

Finally, it should be noted that we only tested single-dipole models. Even though our models fit reasonably well with the data, other models involving multiple sources may also fit the data. Moreover, the precision of source localization is less than desirable, especially given the aforementioned limitation of linked-ears as reference in the current study.

4.5. Conclusions

The present study of Chinese adults showed that single-digit multiplication elicited a greater N300 component in the interval 275–334 ms than did single-digit addition and subtraction. The greater negative potentials appeared to have a single dipole at the left anterior brain areas, maybe at Broca's area. These results are consistent with the notion that the representation and the retrieval of multiplication facts have greater involvement of phonological processing than do the representation and the retrieval of addition and subtraction facts.

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References

- Burton, M. W. (2001). The role of inferior frontal cortex in phonological processing. *Cognitive Science*, 25, 695–709.

- Campbell, J. I. D., & Epp, L. J. (2005). Architectures for arithmetic. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 347–360). New York: Psychology Press.
- Campbell, J. I. D., & Oliphant, M. (1992). Representation and retrieval of arithmetic facts: A network-interference model and simulation. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 331–364). Amsterdam: Elsevier.
- Campbell, J. I. D., & Tarling, D. P. (1996). Retrieval processes in arithmetic production and verification. *Memory & Cognition*, 24, 156–172.
- Chen, C., & Stevenson, H. W. (1988). Cross-linguistic differences in digit span of preschool children. *Journal of Experimental Child Psychology*, 46, 150–158.
- Chochon, F., Cohen, L., Van de Moortele, P. F., & Dehaene, S. (1999). Differential contributions of the left and right inferior parietal lobules to number processing. *Journal of Cognitive Neuroscience*, 11, 617–630.
- Cohen, L., Dehaene, S., Chochon, F., Lehéricy, S., & Naccache, L. (2000). Language and calculation within the parietal lobe: A combined cognitive, anatomical and fMRI study. *Neuropsychologia*, 38, 1426–1440.
- Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*, 33, 219–250.
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20, 487–506.
- Dehaene, S., Spelke, E., Stanescu, R., Pinel, P., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*, 284, 970–974.
- Dehaene, S., Tzourio, N., Frak, V., Raynaud, L., Cohen, L., Mehler, J., et al. (1996). Cerebral activation during number multiplication and comparison: A PET study. *Neuropsychologia*, 34, 1097–1106.
- Delazer, M., & Benke, T. (1997). Arithmetic facts without meaning. *Cortex*, 33, 697–710.
- Galfano, G., Mazza, V., Angrilli, A., & Umiltà, C. (2004). Electrophysiological correlates of stimulus-driven multiplication facts retrieval. *Neuropsychologia*, 42, 1370–1382.
- Hayashi, N., Ishii, K., Kitagaki, H., & Kazui, H. (2000). Regional differences in cerebral blood flow during recitation of the multiplication table and actual calculation: A positron emission tomography study. *Journal of the Neurological Sciences*, 176, 102–108.
- Kawashima, R., Taira, M., Okita, K., Inoue, K., Tajima, N., Yoshida, H., et al. (2004). A functional MRI study of simple arithmetic—A comparison between children and adults. *Cognitive Brain Research*, 18, 225–231.
- Kazui, H., Kitagaki, H., & Mori, E. (2000). Cortical activation during retrieval of arithmetic facts and actual calculation: A functional magnetic resonance imaging study. *Psychiatry and Clinical Neurosciences*, 54, 479–485.
- Kong, J., Wang, Y., Shang, H., Wang, Y., Yang, X., & Zhuang, D. (1999). Brain potentials during mental arithmetic—effects of problem difficulty on event-related brain potentials. *Neuroscience Letters*, 260, 169–172.
- Lee, K. M. (2000). Cortical areas differentially involved in multiplication and subtraction: A functional magnetic resonance imaging study and correlation with a case of selective acalculia. *Annals of Neurology*, 48, 657–661.
- LeFevre, J., Bisanz, J., Daley, K. E., Buffone, L., Greenham, S. L., & Sadesky, G. S. (1996). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General*, 125, 284–306.
- LeFevre, J., & Liu, J. (1997). The role of experience in numerical skill: Multiplication performance in adults from Canada and China. *Mathematical Cognition*, 3, 31–62.
- Lehman, D., & Skrandies, W. (1986). Segmentation of EEG potential fields. *Electroencephalography Clinical Neurophysiology*, 38, 27–32.
- Lemaire, P., & Fayol, M. (1995). When plausibility judgments supersede fact retrieval: The example of the odd-even effect on product verification. *Memory & Cognition*, 23, 34–48.
- Lemer, C., Dehaene, S., Spelke, E., & Cohen, L. (2003). Approximate quantities and exact number words: Dissociable systems. *Neuropsychologia*, 41, 1942–1958.
- Miller, G. A., Lutzenberger, W., & Elbert, T. (1991). The linked-reference issue in EEG and ERP recording. *Journal of Psychophysiology*, 5, 273–276.
- Niedeggen, M., & Rosler, F. (1999). N400 effects reflect activation spread during retrieval of arithmetic facts. *Psychological Science*, 10, 271–276.
- Pesenti, M., Seron, X., & van der Linden, M. (1994). Selective impairment as evidence for mental organization of arithmetic facts: BB, a case of preserved subtraction. *Cortex*, 30, 661–671.
- Pinel, P., Dehaene, S., Riviere, D., & LeBihan, D. (2001). Modulation of parietal activation by semantic distance in a number comparison task. *NeuroImage*, 14, 1013–1026.
- Poldrack, R. A., Wagner, A. D., Prull, M. W., Desmond, J. E., Glover, G. H., & Gabrieli, J. D. (1999). Functional specialization for semantic and phonological processing in the left inferior prefrontal cortex. *NeuroImage*, 10, 15–35.
- Richard, T. C., Romero, S. G., Basso, G., Wharton, C., Flitman, S., & Grafman, J. (2000). The calculating brain: An fMRI study. *Neuropsychologia*, 38, 325–335.
- Roussel, J., Fayol, M., & Barrouillet, P. (2002). Procedural vs. direct retrieval strategies in arithmetic: A comparison between additive and multiplicative problem solving. *European Journal of Cognitive Psychology*, 14, 61–104.
- Scherg, M., & Ebersole, J. S. (1993). Models of brain sources. *Brain Topography*, 5, 419–423.
- Schlereth, T., Baumgartner, U., Magerl, W., Stoeter, P., & Treede, R. D. (2003). Left-hemisphere dominance in early nociceptive processing in the human parasylvian cortex. *NeuroImage*, 20, 441–454.
- Siegler, R. S., & Shipley, E. (1995). Variation, selection, and cognitive change. In G. Halford & T. Simon (Eds.), *Developing cognitive competence: New approaches to process modeling* (pp. 31–76). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
- Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229–293). Hillsdale, NJ: Lawrence Erlbaum Associates Inc.
- Simon, O., Cohen, L., Mangin, J. F., Bihan, D. L., & Dehaene, S. (2002). Topographical layout of hand, eye, calculation and language related areas in the human parietal lobe. *Neuron*, 33, 475–487.
- Stanescu-Cosson, R., Pinel, P., Van de Moortele, P. F., Le Bihan, D., Cohen, L., & Dehaene, S. (2000). Cerebral bases of calculation processes: Impact of number size on the cerebral circuits for exact and approximate calculation. *Brain*, 123, 2240–2255.
- Stigler, J. W., Lee, S., & Stevenson, H. W. (1986). Digit memory in Chinese and English: Evidence for a temporally limited store. *Cognition*, 23, 1–20.
- Szucs, D., & Csepe, V. (2005). The effect of numerical distance and stimulus probability on ERP components elicited by numerical incongruencies in mental addition. *Cognitive Brain Research*, 22, 289–300.
- van Harskamp, N. J., Rudge, P., & Cipolotti, L. (2002). Are multiplication facts implemented by the left supramarginal and angular gyri? *Neuropsychologia*, 40, 1786–1793.
- van Harskamp, N. J., Rudge, P., & Cipolotti, L. (2005). Does the left inferior parietal lobule contribute to multiplication facts? *Cortex*, 41, 742–752.
- Zhou, X., & Dong, Q. (2003). The representation of addition and multiplication. *Acta Psychologica Sinica*, 35, 345–351 (in Chinese).