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1 Improved geophysical monitoring of carbon sequestration through parameter linkage to reservoir
2 modeling.

3
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7
8 **ABSTRACT**

9 Predictive reservoir modeling, even if present in the form of only basic hydrogeological model
10 assumptions, is expected to accompany the majority of carbon capture and sequestration
11 monitoring activities. It thus represents a source of prior information about the migration of
12 injected fluids that can benefit geophysical survey planning and ensuing monitoring.
13 Constraining the imaging of geophysical monitoring data with reservoir modeling is preferable
14 over standalone geophysical imaging because of additional complementary hydrogeological
15 information. However, fully coupled hydrogeophysical data inversion for flow-modeling
16 parameters that control saturation predictions is an involved process. Within the context of three-
17 dimensional electromagnetic (EM) inversion of data from borehole-to-surface layouts, we
18 employ a "poor people's" alternative. The approach constrains geophysical inversion parameters
19 through saturation predictions. The coupling is realized through spatially variable lower and
20 upper parameter bounds that scale with gas saturation magnitudes, the latter provided by
21 reservoir modeling. Enhancement of three-dimensional time-lapse plume EM imaging is
22 demonstrated for simulated sequestration into a depleted gas reservoir.

23
24 **1. Introduction**

25 Regional-scale deployment of geologic carbon sequestration (GCS) requires reliable stewardship
26 in form of failsafe monitoring in order to reach industrial maturity (e.g., Arts et al., 2008; Shi et
27 al., 2008; Ringrose, 2020). Geophysical remote-sensing methods offer the volume coverage
28 needed for long-term monitoring of regional-scale GCS activities (Gasperikova and Hoversten,
29 2006; Michael et al., 2010; Jenkins et al., 2015; Davis et al., 2019; Gasperikova et al., 2022;
30 Tveit and Mannseth, 2022). Among geophysical rock properties, electrical properties have
31 shown to correlate with saturation levels of stored carbon dioxide (CO₂) in a porous system (Kim
32 et al., 2010; Alemu et al., 2011). Owing to their sensitivity to fluid-induced electrical property
33 changes and large exploration depths, geophysical controlled-source electromagnetic (CSEM)
34 methods (e.g., Streich et al., 2010; Wirianto et al., 2010; Vilamajó et al., 2013) and concomitant
35 CSEM-data inversion methods (e.g., Ayani et al., 2020; Grana et al., 2021; Tveit et al., 2015;
36 Tveit et al., 2020) have become viable techniques for GCS monitoring.

37 Typical reservoir depths exceed 800 m in order to meet the pressure and temperature
38 requirements needed to store CO₂ as a supercritical fluid (van der Meer, 1993). Large reservoir
39 depths render borehole-to-surface electromagnetic (BSEM) surveying as suitable because the
40 proximity of transmitters to target zones retains its sensitivities (He et al., 2005; Marsala et al.,
41 2011; Gasperikova et al., 2022). BSEM is defined by a borehole-deployed transmitter and a
42 receiver spread on the surface. Despite its advantage of strategically placing instruments near

43 zones of interest, a major challenge to deep reservoir monitoring is a typically thin nature of
44 fluid-confining formations (Marsala et al., 2014). There is growing consensus that vertical
45 electric dipole (VED) antennas deployed as sources can alleviate this issue because their
46 dominating EM-wave mode maximizes sensitivities to thin target structures (Wirianto et al.,
47 2010; Girard et al., 2011; Vilamajó et al., 2013; Grayver et al., 2014; Schaller et al., 2014).

48 While technical feasibility can be regarded as established, we believe that economic feasibility is
49 an equally important but more overlooked aspect. This contribution simulates a low-cost
50 scenario by making the BSEM survey design deliberately sparse in terms of the total
51 instrumentation. While we exploit the resolution advantages of VED sources, the BSEM setup of
52 our imaging demonstrations implies a volume coverage that is relatively small compared to the
53 regional target scale. To offset the deficiencies due to a limited instrumentation, we propose the
54 incorporation of model constraints provided by reservoir modeling.

55 Reservoir modeling (e.g., Hosseini et al., 2012) can be regarded as some subsurface equivalent to
56 weather prediction because fluid flow and transport forecasting improves as more monitoring
57 data becomes available over time. Reservoir models are expected to be a standard part of GCS
58 site management. Doughty and Oldenburg (2020) use the term *operational* reservoir model for a
59 hydrogeologic model that is expected to be initially crude at the pre-injection stage due to a
60 limited data base. Operational reservoir modeling then involves the assimilation of the
61 monitoring data stream for an ongoing model improvement. A model that is deemed sufficiently
62 mature thus provides a free source of prior complementary information to help optimize
63 geophysical plume-mapping. To offset weak model resolution due to limited instrumentation, we
64 hence propose to employ flow predictions in order to construct inversion constraints in the form
65 of target-adapted model parameter bounds.

66 Spatially variable (lower and upper) model parameter bounds facilitate the incorporation of prior
67 information as inversion constraints (e.g., Abubakar et al., 2008; Sosa et al., 2013; Aghamiry et
68 al., 2019; Ogarko et al., 2021), because parameter ranges can be scaled with uncertainties.
69 Recent studies (Commer et al., 2022) in a crosswell EM context have employed spatially
70 variable scaling of bound widths under a different premise. For poorly resolved interwell
71 regions, it was shown that bound intervals that are enlarged with respect to a global default can
72 benefit the solution-finding process. Principally, this is achieved through diminishing the chance
73 for a premature convergence towards local solution minima. The approach is related to the
74 concept of inversion constraints with a spatially varying degree of enforcement in order to
75 balance out varying sensitivities inherent in a given survey geometry (Yi et al., 2003). In a
76 similar manner, to locally enhance model fidelity, we augment lower and upper parameter
77 bounds in zones where operational reservoir modeling predicts significant gas saturation
78 changes.

79 For brevity, we will use the abbreviation ORM for operational reservoir model in the following.
80 Section 2 outlines the hydrogeophysical aspects underlying the construction of spatially variable
81 parameter bounds from an ORM. These bounds are the input to the constrained inversion of
82 synthetic BSEM data simulated for a large-scale reservoir model. The inversion approach
83 presented in Section 3 and results in Section 4 demonstrate that expanded parameter bounds have
84 the potential to offset weak model resolution in thin target zones that undergo CO₂-induced
85 changes in electrical conductivity.

86

87 2. Hydrogeophysical reservoir modeling

88 Our EM-imaging experiments are based on a reservoir model that is representative of typical
89 large-scale depleted natural gas reservoirs in the Sacramento River Delta region of California.
90 Reservoirs of this kind are prospective candidates for long-term storage. Extensive CO₂ plume
91 evolution simulations by Doughty and Oldenburg (2020) assessed long-term forecasting
92 uncertainties associated with pressure and saturation evolution. Their simulated scenario
93 involves 8 Mt/year of CO₂ injection over 20 years.

94 Doughty and Oldenburg (2020) used two types of reservoir flow models: actual and operational
95 (Fig. 1). The actual model (Fig. 1a) features a complex heterogeneous 3D permeability
96 representation of a regional-scale geologic model of a depleted gas field. Stochastic modeling
97 added small-scale spatial model heterogeneity to ensure a more realistic geological complexity.
98 A high-permeability sandstone layer located between depths of 1.4 and 1.8 km forms the actual
99 storage reservoir, its thickness averaging roughly 400 m. The storage reservoir is overlain by a
100 very low permeability shale (not shown in Fig. 1) that serves as cap rock for the reservoir.

101 Starting from an initially crude vertically-layered representation, a series of progressively more
102 complex reservoir models was derived. Their complexity reflected incorporation of the gradually
103 increasing informational content due to the accumulation and history-matching of periodically
104 sampled pressure, saturation, and gas composition data, simulated for a set of 14 observation
105 wells. These wells are distributed over approximately $7 \times 13 \text{ km}^2$ as indicated by black circles in
106 the horizontal sections of Fig. 1. Fig. 1b shows the estimated permeability model that would be
107 realistically available to the site operator after 5 years of injection and history-matching of the
108 corresponding monitoring data.

109 The actual model represents the (unknown) true state. Synthetic-data generation is based on this
110 model. The ORM represents the best available approximation to the true state at the time 5 years
111 after start of injection (at time zero). Geophysical inversion constraints will be constructed from
112 this model.

113

114 2.1. Geophysical monitoring objective: time-lapse gas saturation changes

115 Our monitoring objective employs the 5-year ORM (Fig. 1b) for making gas-saturation
116 predictions for up to 12 years. Note that the 5-year timeframe refers to model maturity. In a real-
117 world case, this maturity would be attained by history-matching the monitoring data collected up
118 to 5 years. Predicting the reservoir state up to 12 years hence translates to matching the 5-year
119 monitoring history and forecasting the 7 following years. Fig. 2 (left column) compares the 12-
120 year forecast of gas saturations between the actual model (a) and (b) the ORM. Striking is the
121 more heterogeneous distribution of zones of altered gas saturations at 12 years in the actual
122 model (compare subplots of left plot column). While the match is poor on a fine-scale, the ORM
123 appears to reproduce regions of heightened saturations on a gross scale.

124 Gas saturation includes CH₄ (methane) and CO₂. Isolated pockets of gas as visible in Fig. 2a
125 (left) have different origins. First, residual free-phase CH₄ is initially trapped in some attic
126 regions forming static accumulations. Second, over the course of injection, degassed CH₄ can
127 form due to addition of CO₂. Third, injection-induced changes in pressure and composition can
128 create isolated free-phase CO₂ out of dissolved CO₂ as the latter is present more widely.

129 Our inversion objective is based on earlier assessments about the storage potential of the
 130 Sacramento River Delta region (Oldenburg et al., 2001). The region hosts many geological
 131 structures with gas storage capacity. Reinjecting gas into a depleted reservoir associates the
 132 question of permanence with properly forecasting where buoyant rise would make the gas
 133 accumulate over time. Therefore, we choose the attic region underneath the caprock (indicated as
 134 Northern attic in Fig. 1 and Fig. 2) as focus area as it is representative for locally closed high-
 135 permeability structures that are crucial as long-term repositories. To predict saturation changes
 136 due to fluid transfer and substitution processes, we choose the time span between 8 and 12 years.
 137 Fig. 2a (right column) exhibits that the particular permeability makeup of the actual model leads
 138 to significant saturation changes within this time interval. Properly forecasting these changes in
 139 the reservoir cap region can aid the reduction of reservoir performance uncertainty.

140 The ORM's prediction of the time-lapse saturation change between 8 and 12 years (Fig. 2b, right
 141 column) appears similar to the absolute saturations: there exists agreement on a gross scale;
 142 however, saturation changes in the Northern attic region appear with an error in the predicted
 143 elevation. Our objective thus also involves the question to what degree deviations from the true
 144 case would corrupt geophysical-data inversions that use prior information derived from the
 145 ORM.

146

147 *2.2. Petrophysical transformation of the reservoir model*

148 Petrophysical transformation functions are the key linking elements between two physical
 149 systems: the reservoir flow model and the geophysical model for EM data simulation. The
 150 petrophysical relationship derived in the following establishes a connection between rock
 151 properties that control fluid transport and electrical properties.

152 Synthetic geophysical EM data creation is based on the actual permeability model (Fig. 1a) and
 153 its corresponding hydrogeological flow state during CO₂ injection. In a first step, the function

$$\rho_f = \frac{\rho_{18c}}{1 + \alpha(T - 18)} \quad (1)$$

154 calculates pore fluid electrical resistivity ρ_f (with unit Ωm) from temperature T and content c of
 155 total dissolved solids (TDS), the latter affecting the reference resistivity at 18 degrees Celsius,

$$\rho_{18c} = \frac{3549}{c^{0.924}}. \quad (2)$$

156 Eq. (1) uses the constant $\alpha = 0.025$ which together with the constants in Eq. (2) reflects the
 157 temperature dependence of electrical resistivity as detailed by Hayashi (2004). These constants
 158 are chosen here as representative for the bulk resistivity ρ_b of the Sacramento River Delta region
 159 expressed by means of Archie's law,

$$\rho_b = a\phi^{-m}\rho_f, \quad (3)$$

160 where $a = 1$, ϕ is porosity, and the cementation exponent m varies between 1.1 for clay layers
 161 and 2 for sand layers. The bulk resistivity value of each model grid cell results from Eq. (3) and
 162 the cell's gas saturation value S_g ,

$$\rho = \frac{1}{\sigma} = \frac{\rho_b}{(1-S_g)^n}, \quad (4)$$

163 using a constant exponent of $n = 2$. Our imaging method operates on the electrical conductivity
164 $\sigma = \frac{1}{\rho}$ (with unit S/m). Eq. (4) represents the final petrophysical transformation function linking
165 the (geophysical) σ -model to the reservoir flow model.

166

167 2.3. BSEM survey layout

168 Eqs. (1)-(4) represent the hydrogeophysical linkage between the modeling of the reservoir states
169 of interest (between 8 and 12 years) and the corresponding geophysical property evolution. The
170 outcome of Eq. (4) produces the model of electrical rock resistivity used to calculate synthetic
171 data of a BSEM layout shown in Fig. 2. Recall that these data calculations are based on the ρ -
172 distribution originating from the actual reservoir model. Synthetic data are given as electric fields
173 that would be excited by borehole source antennas and measured via the surface electrode array.
174 The employed controlled-source EM forward-modeling algorithm approximates Maxwell's
175 equations on a finite-difference grid representing the geophysical modeling domain.
176 Computational and algorithmic details can be found in the work of Commer and Newman
177 (2008).

178 CSEM sources are deployed in monitoring wells U1 and U2 (indicated in Fig. 1 and Fig. 2). The
179 sources are vertical electric dipole antennas with a length of 50 m, centered at a depth of 1850 m.
180 We simulate data for both sources at four frequencies, 0.25, 0.5, 1, and 2 Hz. There are 13 north-
181 south oriented receiver profiles spaced 250 m apart (along the Easting direction), with a receiver
182 separation of 125 m (along the Northing direction). Two perpendicular electric-field
183 components, E_x (parallel to Easting) and E_y (parallel to Northing) are recorded at 45 receiver
184 stations per profile, thus amounting to a total of 585 stations and 1170 complex electric field data
185 points per source frequency.

186 Magnitudes of artificial noise imposed on the synthetic data are based upon 0.5 % of the data
187 amplitude and an additional noise floor of 10^{-12} V/m (normalized to unit dipole moment).
188 Theoretical estimates for sensor noise-floor limits were reported between 10^{-14} V/m (Streich *et al.*
189 *al.*, 2010) to 10^{-13} V/m (Wirianto *et al.*, 2010).

190

191 2.4. Simulating operational model uncertainty

192 Uncertainty in the understanding of reservoir processes is prevalent owing to monitoring data
193 that is always insufficient in view of the reservoir system's size and complexity. Uncertainties
194 associated with the history-matching process and the ensuing incompleteness of a given state-of-
195 the-art reservoir model have been discussed at length (e.g., Subbey *et al.*, 2004; Ma, 2011). Our
196 scope touches on synthetic uncertainty, namely, does the ORM's incompleteness with respect to
197 the actual model properly reflect the degree of incompleteness one would face in a corresponding
198 real-world case?

199 Table 1 of Doughty and Oldenburg (2020) details reservoir properties of the actual model and
200 the initial (at 0 years) ORM. Without going too much into detail, we reiterate here that the
201 difference in permeability distribution, three-dimensionally heterogeneous versus layered (Fig.
202 1), already precludes the ORM from achieving a perfect data fit. Moreover, the ORM uses
203 simpler relative permeability and capillary pressure functions. While both are based on the van
204 Genuchten (1980) formulation, the actual model includes hysteresis and all function parameters

205 depend on permeability. In contrast, the ORM is non-hysteretic and only the parameter capillary
206 pressure strength depends on permeability via Leverett scaling.

207 The following initial reservoir conditions match their actuals by the 5-year stage of the ORM:
208 Initial (hydrostatic) pressure, temperature (using a geothermal gradient), initial CH₄ (methane)
209 distribution, and vertical salinity profile as derived from regional estimates by Kang and Jackson
210 (2016). Note that CH₄ is initially absent and only becomes an added feature in the 5-year ORM.
211 One could argue that a more realistic simulation of an incomplete ORM would involve more
212 deviating initial conditions. However, assuming the existence of prior knowledge due to the
213 site's depletion history, realistic deviations would not significantly affect the degree of error that
214 is already present in the saturation predictions (Fig. 2). In other words, fine-scale improvements
215 in the saturation prediction will not affect the geophysical plume imaging as the latter only
216 requires a gross-scale saturation profile. More of these aspects are also discussed further below.

217

218 **3. Constrained inversion of BSEM data**

219 The total number of estimated parameters that make up the geophysical inversion domain
220 amounts to 1,538,974 active finite-difference grid cells, where each cell hosts an electrical-
221 conductivity parameter. The large discrepancy between the sizes of the parameter and data
222 spaces causes solution ambiguity rendering the inverse problem ill-posed. Bound constraints aim
223 at reducing the ambiguity by narrowing the solution space in zones where physical property
224 ranges are known prior to inversion. The common approach reduces bound intervals according to
225 the prior information's certainty (e.g., Kim and Kim, 2011). Our approach differs by widening
226 bound intervals in zones where the ORM suggests significant time-lapse property changes. In the
227 following, we describe the method of mapping the degree of gas saturation changes to its
228 function parameters controlling spatially variable bound widths. The general concept of bound
229 constraints was introduced earlier together with our employed inversion algorithm (Commer and
230 Newman, 2008). The algorithm employs a non-linear conjugate gradient (NLCG) scheme which
231 uses a line-search based upon quadratic interpolation, safeguarded with backtracking.
232 Logarithmically transformed model parameters define the search space, which facilitates bound
233 constraints as logarithmic functions are amenable to inequality constraints.

234 *3.1. Spatially variable lower and upper parameter bounds for enhanced model resolution*

235 Lower and upper parameter bounds are also known as inequality constraints (e.g., Kim et al.,
236 1999) and are abbreviated by a and b in the following. Inequality refers to the evolution of a
237 model parameter m during an inversion process, so that m is bounded by $a < m < b$. In order to
238 impose a positivity constraint on electrical conductivity, the bounds usually occur in conjunction
239 with logarithmic or hyperbolic types of parameter transformation functions. Generalized
240 functions were formulated by Kim and Kim (2011),

$$x = \frac{1}{n} \log \left(\frac{m-a}{b-m} \right) = \frac{2}{n} \tanh^{-1} \left(\frac{2m-b-a}{b-a} \right); \quad a < m < b, \quad (5)$$

241 where x is the transformed parameter and n is a positive integer constant. Our studies employ a
242 version realized by $n = 1$. Other choices for n are discussed by Kim and Kim (2011).
243 Transforming x back into the original model parameter space then leads to (Commer and
244 Newman, 2008)

$$m = \frac{a+b \exp(x)}{1+\exp(x)}; \quad -\infty < x < \infty. \quad (6)$$

245 Constant bounds may be most suitable for inversion applications where no prior information is
 246 available, that is, the bound interval $[a, b]$ applies to every cell parameter m_i ($i = 1, \dots, M$) of the
 247 finite-difference mesh representing the inversion domain. In the presence of location-dependent
 248 prior information, $[a, b]$ can become spatially variable so that each cell m_i is subjected to an
 249 individual bounding interval $[a_i, b_i]$.

250

251

252 3.2. Designing bounds from gas-saturation predictions

253 Gas saturation predictions provide the basis for constructing lower and upper conductivity
 254 parameter bounds in order to achieve a local resolution-enhancing effect. The general approach
 255 is to widen a preset default bound interval $[a, b] = [10^{-3}, 1.5]$ S/m within regions of interest, so
 256 that $[a, b]$ becomes $[a \cdot f_a, b \cdot f_b]$ with positive factors f_a and f_b . The choice of the default
 257 bounds are based on estimates of extremal conductivity values that result from reservoir
 258 modeling over the whole 12-year period of interest. The focus regions are linked to injection-
 259 induced gas saturation changes predicted from the ORM (Fig. 2b). With the focus being on
 260 delineating gas saturation changes between year 8 (quantified by S_g^{08y}) and year 12 (S_g^{12y}), the
 261 zones of interest are defined by their absolute percentage saturation change

$$|\Delta S_g^{\%}| = \left| \frac{S_g^{12y} - S_g^{08y}}{S_g^{08y}} \right| \cdot 100 \quad (7)$$

262 exceeding a given threshold $\Delta S_g^{thr\%}$. This threshold is set to $\Delta S_g^{thr\%} = 5\%$ which is the
 263 minimum gas saturation change in percent that has to occur in order to augment the default
 264 interval $[a, b]$. In practice, the widening is achieved through two factors, $f_a = \frac{1}{f_{lb}(x,y,z)}$ and $f_b =$
 265 $f_{ub}(x, y, z)$, applied to $[a, b]$ so that the interval becomes spatially variable, $[a, b] \rightarrow$
 266 $\left[\frac{a}{f_{lb}}, b \cdot f_{ub} \right]$.

267 Dividing by the lower-bound factor $f_{lb} = f_{lb}(\Delta S_g^{\%})$ decreases the lower bound a . In our
 268 particular case, f_{lb} varies linearly between 1 and 100, i.e. $f_{lb}(\Delta S_g^{\%} = 0) = 1$ and
 269 $f_{lb}(\Delta S_g^{\%} \geq 100) = 100$. Hence, with higher saturation changes, the lower bound decreases up to
 270 a minimum of $a/100$. Enlarging the upper bound is achieved via multiplication of b with the
 271 factor f_{ub} , which varies linearly between 1 and 10, i.e. $f_{ub}(\Delta S_g^{\%} = 0) = 1$ and $f_{ub}(\Delta S_g^{\%} \geq$
 272 $100) = 10$. The default bound interval thus increases to an extremum of $[a_{min}, b_{max}] =$
 273 $[10^{-5}, 15]$ S/m.

274 Setting the expansion factor for the lower bounds an order of magnitude larger than for the upper
 275 bounds reflects the fact that positive gas saturation changes and corresponding conductivity
 276 decreases dominate over the 12-year simulation period. Undocumented trial inversions with
 277 larger expansion factors, for example $f_{lb}(\Delta S_g^{\%} \geq 100) = 1000$ and $f_{ub}(\Delta S_g^{\%} \geq 100) = 100$,
 278 revealed no significant differences in the inversion outcome. Fig. 3 illustrates both bound
 279 extremes, the lower-bound (left) range $[a_{min}, a_{max}]$ and the upper-bound (right) range

280 $[b_{min}, b_{max}]$, for one vertical cross section. The selected plane is parallel to the Northing axes
281 and cuts through the Easting coordinate at $E=240$ m.

282 Bounds construction starts by feeding the saturation S_g^{08y} and S_g^{12y} , calculated from the ORM, to
283 Eq. (7). The output are sets of lower and upper bounds with augmented intervals for zones where
284 the 5-%-threshold criterion is met. Upon input of the inversion's starting model, an individual
285 bound pair a_i, b_i is then assigned to each cell parameter m_i of the inversion domain.

286

287 3.3. Gradient weighting and parameter masking

288 In addition to parameter bounds, we employ two other approaches that essentially impose
289 constraints on the NLCG inversion process, namely gradient weighting and parameter masking.
290 However, we abstain from a study that would systematically investigate the interplay of all three
291 constraining methods as this would go beyond our intended scope. More systematic studies of
292 combinations of different constraints were carried out by Portniaguine and Zhdanov (1999) and
293 Boulanger and Chouteau (2001) in the context of gravity data inversion. Here, we use both
294 gradient weighting and parameter masking in a rather empirical manner owing to their
295 individually demonstrated benefit.

296 Gradient weighting as used in this work is a method of assigning weighting factors to each
297 component of the gradient vector \mathbf{g} of the NLCG scheme. The gradient vector \mathbf{g} contains
298 derivative information and is at the base of optimizing the direction of the line-search in model
299 space. Methodological details and successful application of gradient weighting within the same
300 NLCG scheme were demonstrated earlier (Commer et al., 2016). Principally, high sensitivities of
301 cell parameters m_i pertaining to regions near surface spreads or instrumented wells are damped
302 through assigning weighting factors $w_i < 1$. The corresponding weighted gradient component
303 $w_i g_i$ then becomes down-weighted with respect to cells in unweighted model regions. Similar to
304 the bound constraints, gradient weighting basically boosts the NLCG line-search in areas of low
305 sensitivity, causing weak resolution, at the expense of high-sensitivity zones. For BSEM, the
306 desired effect is to nudge model updates away from the vicinity of transmitters and receivers,
307 where sensitivities are high, towards less resolved center regions.

308 Parameter masking as employed here is a straightforward way of disabling an arbitrary subset of
309 cell parameters of a rectangular inversion domain. Geological horizons derived from seismic
310 data usually provide the prior information needed to define parameter masks (e.g., Hoversten et
311 al., 2021). We employ the upper horizon shown in Fig. 4 in order to deactivate all mesh cell
312 parameters that are above it. The underlying assumption is a no-leakage scenario, that is, our
313 imaging objective does not consider the case of buoyancy-driven gas flow into the caprock. The
314 present inversion study assumes that the background geology above the delineating seismic
315 horizon would be known from preliminary baseline (pre-injection) data inversions and prior
316 information.

317

318 3.3. Sequential inversion scheme

319 Delineating property changes between the two observation times (8 years and 12 years after the
320 start of injection) involves a three-step inversion procedure. Three inversion runs pertain to the
321 refinement of the baseline (pre-injection) model and, with respect to this baseline, the delineation

322 of fluid-induced anomalous resistivity alterations after 8 and 12 years. Synthetic time-lapse data
323 for these inversions are obtained by using the actual reservoir model and the BSEM
324 configuration for three times, referred to as Year00, Year08, and Year12. For these times, Eqs.
325 (1)-(4) convert the corresponding flow properties (porosity, saturation, and content of TDS) to
326 electrical conductivity (σ).

327 The first step of the inversion sequence refines the 3D baseline model. Fig. 4 shows slices along
328 the Northing direction for the initial σ -distribution resulting from the transform of the pre-
329 injection state calculated for the actual (left) and ORM (right). The latter one is the starting
330 model for the first NLCG inversion sweep which refines the pre-injection state (at year 0) below
331 the upper seismic horizon (annotated in Fig. 4, cell parameters are fixed above). The output
332 $\sigma(\text{Year00})$ serves as the starting model for the second inversion sweep, estimating the σ -
333 distribution after 8 years of injection. In the third sweep, the estimated model $\sigma(\text{Year08})$ then
334 becomes the initial guess for the delineation of the plume after 12 years of injection, the output
335 referred to as $\sigma(\text{Year12})$.

336 We perform the three-step inversion sequence twice in a comparative way. The first sequence
337 uses constant lower and upper parameter bounds $[a, b] = [10^{-5}, 15]$ S/m; the second uses
338 variable bounds (Fig. 3), $[a, b] = \left[\frac{a}{f_{lb}}, b \cdot f_{ub}\right]$, that widen according to the factors f_{lb} and f_{ub} as
339 described above. Variable bounds are active only for the estimation of the Year08 and Year12
340 states, that is, both inversion sequences employ constant bounds for the Year00-inversion. Note
341 that the constant-bound sequence uses the extremal interval $[a_{min}, b_{max}]$ occurring in the
342 variable-bound inversion sequence. The initial reasoning for a globally wide interval was to let
343 the inversion practically be unbounded in order to avoid any bias due to potentially too tight
344 bounds. However, other undocumented trial inversion experiments with the narrower default
345 bound interval $[a, b] = [10^{-3}, 1.5]$ S/m yielded no significant difference in the final model
346 outcomes.

347 Computing times amounted to a total of 126 hours for the inversion sequence with constant
348 bounds versus 147 total hours for the variable-bounds sequence. All inversions employed 250
349 cores of a cluster architecture with Intel Cascade Lake processor compute nodes (40 cores per
350 node) connected with a Mellanox EDR infiniband fabric.

351

352

353 4. Results and Discussion

354 The final plume image output is represented in Fig. 5 as differences in σ between the estimated
355 Year08 and Year12 models. We compare the differences $\Delta\sigma = \sigma(\text{Year12}) - \sigma(\text{Year08})$ in
356 percent for two realizations of the three-step sequential inversion outlined above. The first uses
357 constant bounds, the second uses spatially variable bounds. Fig. 5a shows the true distribution
358 $\Delta\sigma^{true}$ which is calculated from the petrophysical transform of the actual reservoir model state at
359 Year08 and Year12. Fig. 5b and Fig. 5c compare the estimated counterparts, these are,
360 respectively, the differences $\Delta\sigma^{con}$ (constant bounds) and $\Delta\sigma^{var}$ (variable bounds). Negative
361 changes greatly dominate in the actual outcome $\Delta\sigma^{true}$ and are visualized here since our
362 inversion objective focuses on the time-lapse accumulation of CO_2 which causes negative time-
363 lapse changes.

364 Most notable between the two estimates $\Delta\sigma^{con}$ and $\Delta\sigma^{var}$ is a lack of delineation of any
365 property changes in zones of interest away from the injection well for the image $\Delta\sigma^{con}$. In
366 contrast, $\Delta\sigma^{var}$ exhibits the fluid-induced property alterations across the regional scale. The
367 variable-bound method thus yields indicators for negative time-lapse changes associated with
368 CO₂ accumulation and migration. Deviations from the true case manifest in an overestimated
369 volume undergoing fluid-induced conductivity alterations in the Northern attic region.
370 Concurrently, conductivity decreases in the region around the wells U1 and U2 remain
371 underestimated in terms of their magnitudes.

372 A certain degree of image blurring can be expected from CSEM inversions of low-frequency
373 data, because model resolution is limited due to corresponding large wavelengths of the
374 generated EM fields. However, despite the low resolution and the limited coverage using two
375 transmitter wells, the final image of $\Delta\sigma^{var}$ is sufficient to support predictions of CO₂-
376 breakthrough in the Northern attic region (Fig. 2b) with first-order accuracy. For this zone, Fig. 2
377 revealed a discrepancy in the elevation of predicted saturation changes (between Year08 and
378 Year12). This discrepancy appears only marginally in the image $\Delta\sigma^{var}$.

379 The large image improvement provided by the modified bounds indicates an optimization of the
380 model-updating line-search procedure. In essence, the method translates to steering and scaling
381 the conjugate search direction of our NLCG inversion scheme such that it more closely
382 resembles the Newton direction. The latter is characterized by faster convergence and less
383 potential for a premature end of the line-search in a local minimum, yet is computationally very
384 expensive. This point of view also hints at a resemblance with the method of regularized
385 focusing inversion (Portniaguine and Zhdanov, 1999), where stabilizing functionals convey prior
386 information with the goal of image focusing.

387

388 5. Conclusions

389 Spatially variable lower and upper parameter bounds provide a straightforward way of
390 incorporating prior model information in an inversion workflow. Our example inversion studies
391 demonstrated that such prior model information can be obtained through rough spatial estimates
392 of anticipated fluid-induced property alterations. Regions of property alterations that qualify as
393 anomalous with respect to the property range of the underlying baseline geology are target zones
394 of interest in time-lapse imaging problems. Prior-model information of such target zones
395 translates to enlarged bound intervals. Our inversion study demonstrated that widened bound
396 intervals can selectively enhance model resolution in target zones. Spatially balanced bounds can
397 thus greatly improve the imaging process through their potential of steering the NLCG search
398 direction away from local solution minima.

399 Since reservoir modeling will likely be a minimum required tool for GCS site management, it
400 represents an inexpensive source of auxiliary information for constraining geophysical plume
401 imaging problems. One may thus call the presented method a "poor people's approach" to
402 hydrogeophysical data analysis, because inexpensive also holds for the relative simplicity in
403 incorporating flow information as parameter bounds. Approximate bound intervals make the
404 method somewhat forgiving towards petrophysical uncertainties while still constraining
405 geophysical parameter estimation to the physical processes of the flow system. In contrast, more
406 rigorous and involved fully-coupled hydrogeophysical joint inversion approaches are generally
407 susceptible towards erroneous petrophysical relationships (e.g., Sun and Li, 2017).

408 In light of a need for cost-effective monitoring, our synthetic experiments consider limitations
409 likely present at future GCS sites. A relatively sparse volume coverage results from employing
410 two existing monitoring wells for source deployment. However, fluid-induced conductivity
411 alterations could still be mapped, suggesting that the variable-bounds concept can help offset
412 model resolution loss due to limited survey coverage.

413 While we deem the simulated BSEM survey layout as relatively economic, more involved
414 studies similar to the works of Eidsvik et al. (2008) and Trainor-Guitton et al. (2014) could
415 figure out an informative cost measure in form of a benefit-cost ratio. Such a measure would
416 essentially help the operator decide whether the BSEM data's added value in form of more
417 accurate plume predictions outweighs the survey costs. Eidsvik et al. (2008) used the decision-
418 theoretic concept of value of information (VOI) with rock physics and spatial statistics in order
419 to compare the value of seismic amplitude-versus-offset data against CSEM. In our case, this
420 comparison would be between well-based reservoir monitoring data against BSEM data.

421 Trainor-Guitton et al. (2014) presented a VOI methodology that quantifies the impact of
422 inaccuracies of multidimensional geophysical inversions on geothermal resource identification.
423 Translated to GCS contexts, a VOI assessment of constrained EM data inversions as presented
424 here might help analyze aspects of interest to the operator that go beyond cost, for example
425 leakage risk and environmental impact.

426 A final related comment concerns ORM uncertainty, as discussed in Section 2.4., as a major
427 factor influencing VOI of BSEM monitoring. Despite fairly significant deviations from the
428 actual model, the 5-year ORM produced a gross-scale match of the actual gas saturation that
429 turned out sufficient for the bound construction and ensuing delineation of anticipated target
430 zones. A follow-up study similar to Harp et al. (2019) would investigate at what point larger
431 deviations, caused by larger ORM parameter errors, would render the BSEM imaging outcome
432 too distorted. Harp et al. (2019) developed a metric for quantifying the degree to which ORM
433 parameters can deviate from their actuals without violating preset reservoir performance criteria.
434 We envision a similar kind of robustness measure for assessing when an ORM may become too
435 erroneous for properly constraining geophysical reservoir imaging.

436

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443 **REFERENCES**

444 Abubakar, A., Habashy, T.M., Druskin, V.L., Knizhnerman, L., Alumbaugh, D., 2008. 2.5D
445 forward and inverse modeling for interpreting low-frequency electromagnetic measurements.
446 *Geophysics* 73, F165-F177.

447 Aghamiry, H.S., Gholami, A., Operto, S., 2019. Implementing bound constraints and total-
448 variation regularization in extended full waveform inversion with the alternating direction
449 method of multiplier: Application to large contrast media. *Geophys. J. Int.* 218, 855-872.

450 Alemu, B.L., Aker, E., Soldal, M., Johnsen, Ø., Aagaard, P., 2011. Influence of CO₂ on rock
451 physics properties in typical reservoir rock: A CO₂ flooding experiment of brine saturated
452 sandstone in a CT-scanner. *Energy Procedia* 4, 4379-4386.

453 Arts, R.J., Chadwick, A., Eiken, O., Thibeau, S., Nooner, S., 2008. Ten years' experience of
454 monitoring CO₂ injection in the Utsira Sand at Sleipner, offshore Norway. *First break*, 26 (1),
455 65-72.

456 Ayani, M., Dario, G., Liu, M., 2020. Stochastic inversion method of time-lapse controlled source
457 electromagnetic data for CO₂ plume monitoring. *Int. J. Greenh. Gas Control* 100, 103098.

458 Boulanger, O., Chouteau, M., 2001. Constraints in 3D gravity inversion. *Geophysical*
459 *Prospecting* 49, 265-280.

460 Commer, M., Newman, G.A., 2008. New advances in three-dimensional controlled-source
461 electromagnetic inversion, *Geophys. J. Int.* 172, 513-535.

462 Commer, M., Doetsch J., Dafflon B., Wu Y., Daley T.M., Hubbard S.S., 2016. Time-lapse 3-D
463 electrical resistance tomography inversion for crosswell monitoring of dissolved and
464 supercritical CO₂ flow at two field sites: Escatawpa and Cranfield, Mississippi, USA. *Int. J.*
465 *Greenh. Gas Control*, 49, 297-311.

466 Commer, M., Alumbaugh, D.A., Hoversten, G.M., Um, E.S., Vasco, D.W., Wilt, M., Nichols, E.,
467 Marchesini, P., Macquet, M., 2022. Enhanced multi-dimensional inversion through target-
468 specific inversion parameter bounds with an application to crosswell EM for sequestration
469 monitoring, submitted to *Frontiers in Earth Science*.

470 Davis, T.L., Landrø, M., Wilson, M., 2019. *Geophysics and geosequestration*. Cambridge
471 University Press.

472 Doughty, C., Oldenburg, C.M., 2020. CO₂ plume evolution in a depleted natural gas reservoir:
473 Modeling of conformance uncertainty reduction over time. *Int. J. Greenh. Gas Control* 97,
474 103026.

475 Eidsvik, J., Bhattacharjya, D., Mukerji, T., 2008. Value of information of seismic amplitude and
476 CSEM resistivity. *Geophysics* 73. R59-R69.

477 Gasperikova, E., Hoversten, G.M., 2006. A feasibility study of non-seismic geophysical methods
478 for monitoring geologic CO₂ sequestration. *The Leading Edge*, 25 (10), 1282–1288.

479 Gasperikova, E., Appriou, D., Bonneville, A., Feng, Z., Huang, L., Gao, K., Yang, X., Daley, T.,
480 2022. Sensitivity of geophysical techniques for monitoring secondary CO₂ storage plumes, *Int. J.*
481 *Greenh. Gas Control* 114, 103585.

482 Grana, D., Liu, M., Ayani, M., 2021. Prediction of CO₂ saturation spatial distribution using
483 geostatistical inversion of time-lapse geophysical Data. *IEEE Trans. Geosci. Rem. Sens.* 59,
484 3846-3856.

485 Grayver, A.V., Streich, R., Ritter, O., 2014. 3D inversion and resolution analysis of land-based
486 CSEM data from the Ketzin CO₂ storage formation. *Geophysics* 79 (2), E101-E114.

487 Girard, J.-F., Coppo, N., Rohmer, J., Bourgeois, B., Naudet, V., Schmidt-Hattenberger, C., 2011.
488 Time-lapse CSEM monitoring of the Ketzin (Germany) CO₂ injection using 2xMAM
489 configuration. *Energy Procedia* 4, 3322-3329.

490 Harp, D.R., Oldenburg, C.M., Pawar, R., 2019. A metric for evaluating conformance robustness
491 during geologic CO₂ sequestration operations. *Int. J. Greenh. Gas Control*. 85, 100-108.

492 Hayashi, M., 2004. Temperature-electrical conductivity relation of water for environmental
493 monitoring and geophysical data inversion. *Environmental monitoring and assessment* 96. 119
494 -128.

495 He, Z., Liu, X., Qiu, W., Huang, Z., 2005. Mapping reservoir boundary by borehole-surface
496 TFEM: two case studies. *Lead. Edge*, 24 (9), 898-900.

497 Hosseini, S.A., Lashgari, H., Choi, J.W., Nicot, J.-P., Lu, J., Hovorka, S.D., 2013. Static and
498 dynamic reservoir modeling for geological CO₂ sequestration at Cranfield, Mississippi, U.S.A.
499 *Int. J. Greenh. Gas. Contr.* 18, 449-462.

500 Hoversten, G.M., Mackie, R.L., Hua, Y., 2021. Reexamination of controlled-source
501 electromagnetic inversion at the Lona prospect, Orphan Basin Canada. *Geophysics* 86 (3), E157–
502 E170.

503 Jenkins, C., Chadwick, A., Hovorka, S.D., 2015. The state of the art in monitoring and
504 verification - ten years on. *Int. J. Greenh. Gas Contr.*, 40, 312–349.

505 Kang, M., Jackson, R.B., 2016. Salinity of deep groundwater in California: Water quantity,
506 quality, and protection. *Proc. Nat. Acad. Sci.* 113 (28), 7768-7773.

507 Kim, H.J., Song, Y., Lee, K.H., 1999. Inequality constraint in least-squares inversion of
508 geophysical data. *Earth Planets Space* 51, 255-259.

509 Kim, J., Xue, Z., Matsuoka, T., 2010. Experimental study of CO₂ monitoring and saturation with
510 combined P-wave velocity and resistivity, *CPS/SPE Int. Oil & Gas Conf. and Exhib.*, SPE
511 130284.

512 Kim, H.J., Kim Y.H., 2011. A unified transformation function for lower and upper bounding
513 constraints on model parameters in electrical and electromagnetic inversion, *Journal of*
514 *Geophysics and Engineering* 8, 21–26.

515 Ma, Y.Z., 2011. Uncertainty analysis in reservoir characterization and management: How much
516 should we know about what we don't know?, in: Ma, Y.Z., La Pointe, P.R. (Eds.), *Uncertainty*
517 *analysis and reservoir modeling*. AAPG Memoir 96, pp. 1–15.

518 Marsala, A.F., Al-Buali, M.H., Ali, Z.A., Ma, S.M., He, Z., Biyan, T., Zhao, G., He, T., 2011.
519 First pilot of borehole to surface electromagnetic in Saudi Arabia: A new technology to enhance
520 reservoir mapping and monitoring, *73rd Annual Int. Conf. and Exhibition, EAGE, Extended*
521 *Abstracts*, I005.

522 Marsala, A.F., Zhdanov, M.S, Endo, M., Black, N., 2014. 3D inversion of borehole to surface
523 electromagnetic data in a multiple reservoirs survey, *84th Annual Int. Meeting, SEG, Denver,*
524 *USA, Expanded Abstracts*, 2600-2604.

525 Michael, K., Golab, A., Shulakova, V., Ennis-King, J., Allinson, G., Sharma, S., and Aiken, T.,
526 2010. Geological storage of CO₂ in saline aquifers - A review of the experience from existing
527 storage operations. *Int. J. Greenhouse Gas Control*, 4, 659-667.

528 Ogarko, V., Giraud, J., Martin, R., and Jessell, M., 2021. Disjoint interval bound constraints
529 using the alternating direction method of multipliers for geologically constrained inversion:
530 Application to gravity data. *Geophysics* 86, G1-G11.

531 Oldenburg, C.M., Pruess, K., Benson, S.M., 2001. Process modeling of CO₂ injection into
532 natural gas reservoirs for carbon sequestration and enhanced gas recovery. *Energy and Fuels* 15,
533 293-298.

534 Portniaguine, O., Zhdanov, M.S., 1999. Focusing geophysical inversion images. *Geophysics* 64
535 (3), 874-887.

536 Ringrose, P., 2020. How to store CO₂ underground: insights from early-mover CCS projects,
537 Springer, Berlin/Heidelberg.

538 Schaller, A., Hunziker, J., Streich, R., Drijkoningen, G., 2014. Sensitivity of the near-surface
539 vertical electric field in land controlled- source electromagnetic monitoring, 84th Annual Int.
540 Meeting, SEG, Denver, USA, Expanded Abstracts, 838–843.

541 Shi, J.-Q., Durucan, S., Fujioka, M., 2008. A reservoir simulation study of CO₂ injection and N₂
542 flooding at the Ishikari coalfield CO₂ storage pilot project, Japan. *Int. J. Greenh. Gas Control*, 2
543 (1), 47-57.

544 Sosa, A., Velasco, A.A., Velazquez, L., Argaez, M., and Romero, R., 2013. Constrained
545 optimization framework for joint inversion of geophysical data sets. *Geophys. J. Int.* 195, 1745-
546 1762.

547 Streich, R., Becken, M., Ritter, O., 2010. Imaging of CO₂ storage sites, geothermal reservoirs,
548 and gas shales using controlled-source magnetotellurics: Modeling studies. *Chemie der Erde* 70
549 (S3), 63–75.

550 Subbeya, S., Christie, M., Sambridge, M., 2004. Prediction under uncertainty in reservoir
551 modeling. *J. Petrol. Sci. Eng.* 44 (1-2), 143-153.

552 Sun, J., Li, Y., 2017. Joint inversion of multiple geophysical and petrophysical data using
553 generalized fuzzy clustering algorithms. *Geophys. J. Int.* 208, 1201-1216.

554 Trainor-Guitton, W., Hoversten, G.M., Ramirez, A., Roberts, J., Juliusson, E., Key, K., Mellors,
555 R., 2014. The value of spatial information for determining well placement: A geothermal
556 example. *Geophysics* 79 (5), W27-W41.

557 Tveit, S., Bakr, S.A., Lien, M., Mannseth, T., 2015. Ensemble-based Bayesian inversion of
558 CSEM data for subsurface structure identification. *Geophys. J. Int.*, 201 (3), 1849-1867.

559 Tveit, S., Mannseth, T., Park, J., Sauvin, G., Agersborg, R., 2020. Combining CSEM or gravity
560 inversion with seismic AVO inversion, with application to monitoring of large-scale CO₂
561 injection. *Computational Geosciences* 24, 1201-1220.

562 Tveit, S., Mannseth, T., 2022. Monitoring of large-scale CO₂ injection using CSEM,
563 gravimetric, and seismic AVO data, in: Huang, L. (Ed.), *Geophysical Monitoring for Geologic*
564 *Carbon Storage*. American Geophysical Union, pp. 273-301.

565 Van der Meer, L.G.H., 1993. The conditions limiting CO₂ storage in aquifers. *Energy*
566 *Conversion and Management* 34 (9), 959-966.

567 van Genuchten, M.Th., 1980. A closed-form equation for predicting the hydraulic conductivity
568 of unsaturated soils. *Soil Sci. Soc. Am. J.* 44 (5), 892–898.

569 Vilamajó, E., Queralt, P., Ledo, J., Marcuello, A., 2013. Feasibility of monitoring the Hontomín
570 (Burgos, Spain) CO₂ storage site using a deep EM source. *Surveys in Geophysics* 34, 441–461.

571 Wirianto, M., Mulder, W.A., Slob, E.C., 2010. A feasibility study of land CSEM reservoir
572 monitoring in a complex 3-D model. *Geophysical Journal International* 181, 741–755.

573 Yi, M.-J., Kim, J.H., Chung, S.-H., 2003. Enhancing the resolving power of least-squares
574 inversion with active constraint balancing. *Geophysics* 68, 931–941.

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Figures

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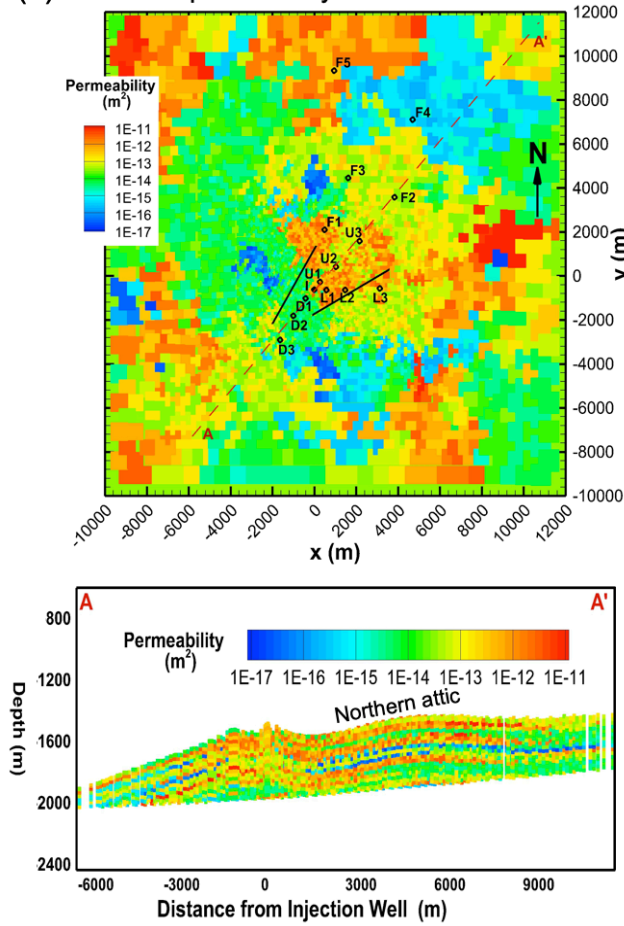
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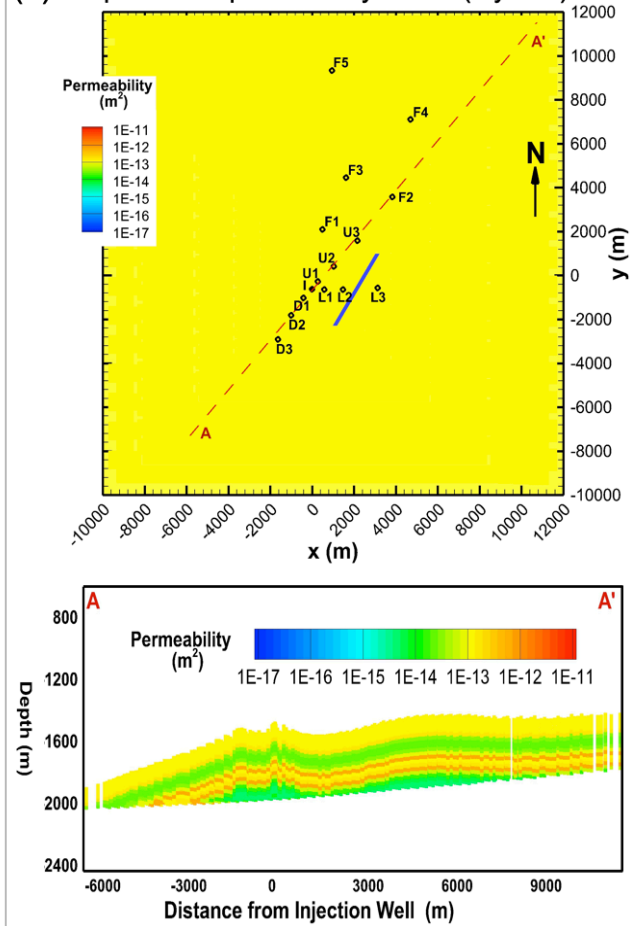
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(a) Actual permeability model



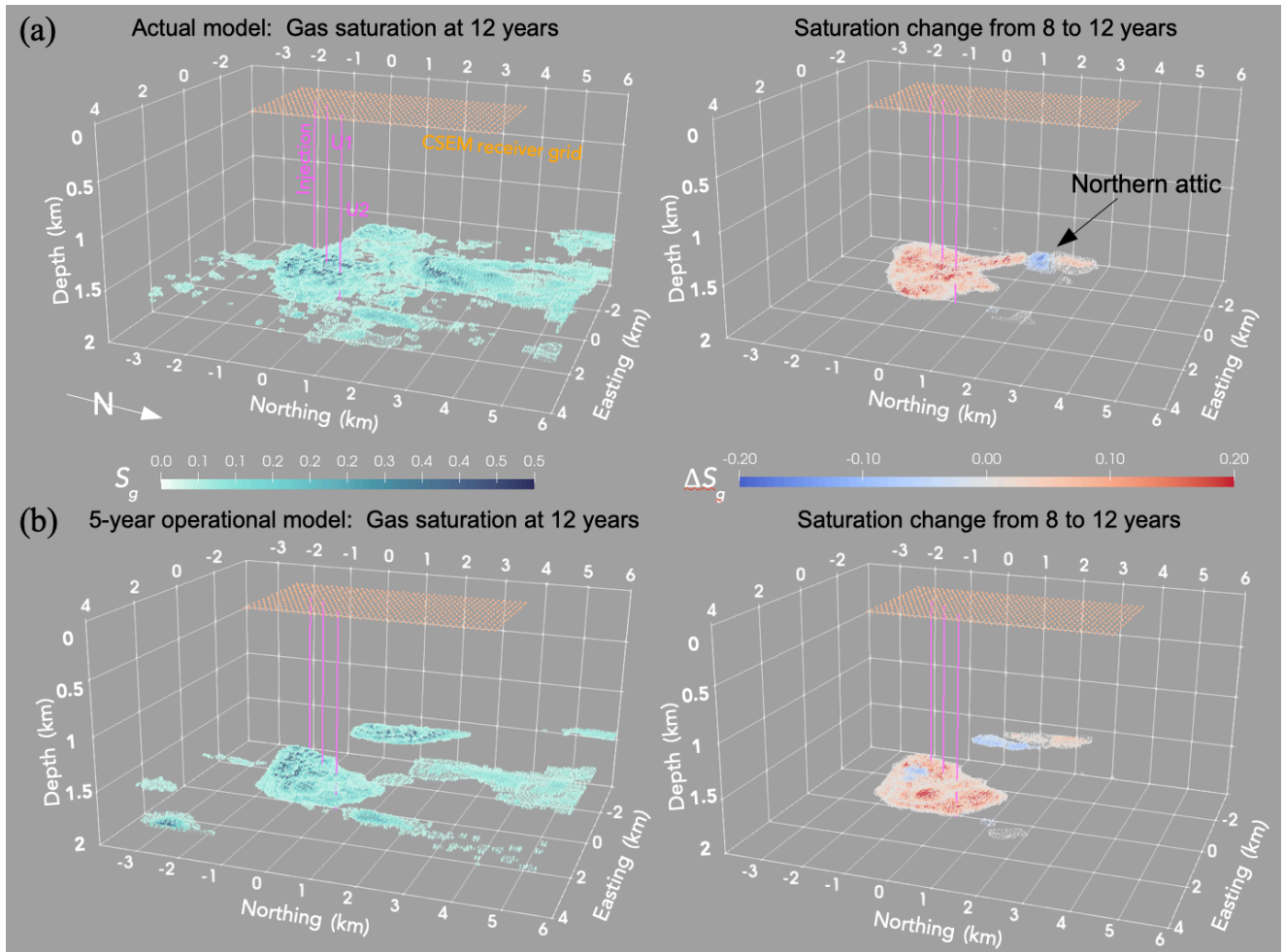
(b) Operational permeability model (5 years)



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593 **Fig. 1:** Actual (a) and operational (b) permeability model. Vertical slices (lower subplots) are
594 along the diagonal (A-A', dashed) line and show the stratified reservoir heterogeneity with high-
595 permeability attic zones. The actual model features vertical faults as black line segments (one
596 blue line in the simplified model). Vertical exaggeration is two times. Note that the (x-y) plan
597 views are not horizontal as they are a projection of the top layer in the reservoir model.

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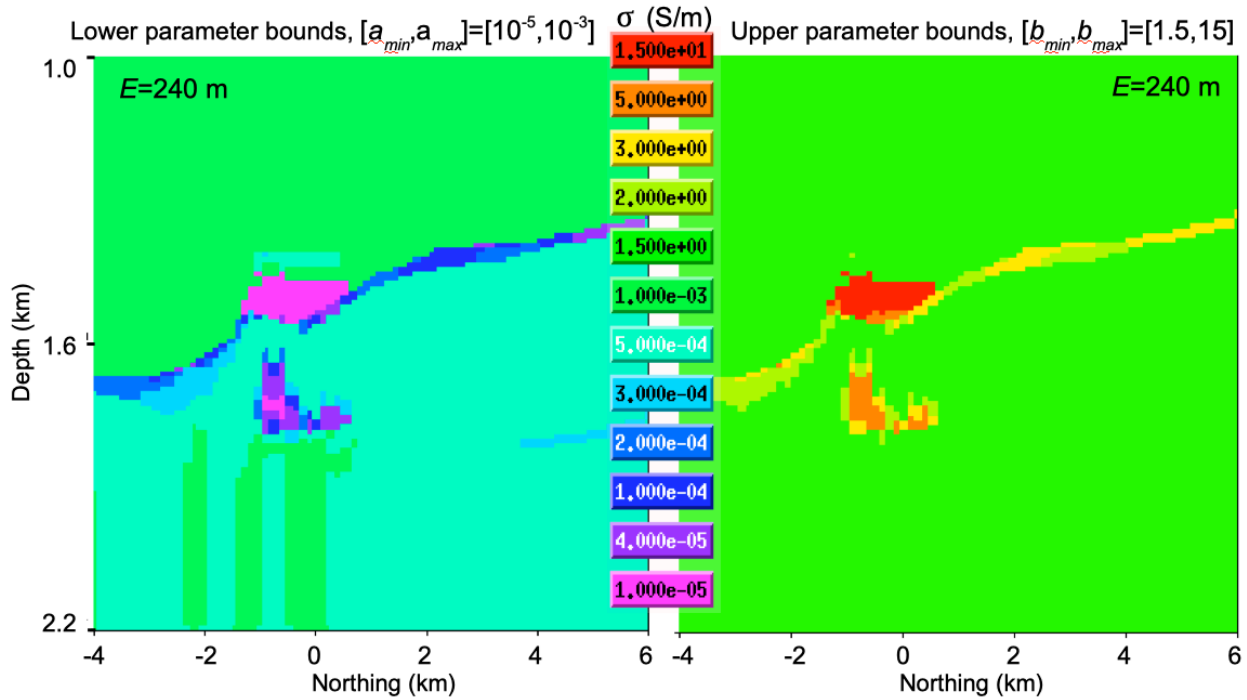


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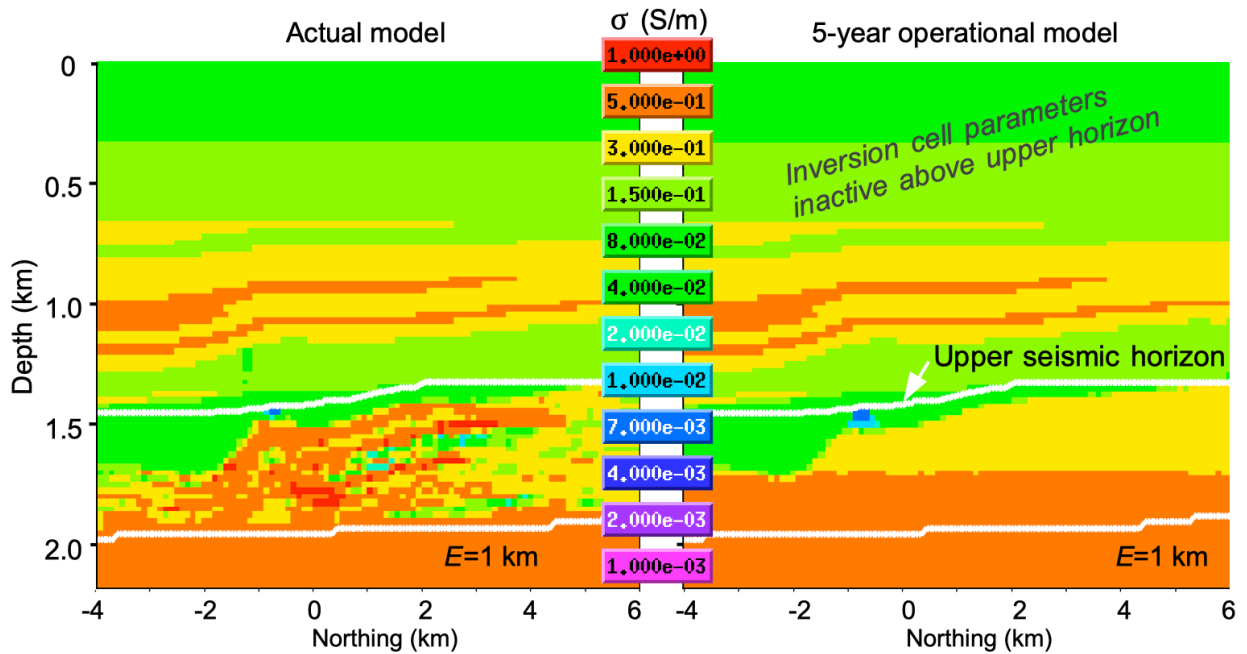
601 **Fig. 2:** Gas saturation predictions calculated from the actual (a) and 5-year operational (b)
 602 reservoir flow model. The right plot column shows gas saturation differences between
 603 predictions made at 8 and 12 years. Each monitoring well U1 and U2 hosts one borehole EM
 604 transmitter at a depth of 1850 m, sourcing the shown surface receiver grid. Receiver profiles
 605 extend from $y=-1500$ m to $y=4000$ m, with a station interval of $\Delta y=125$ m.

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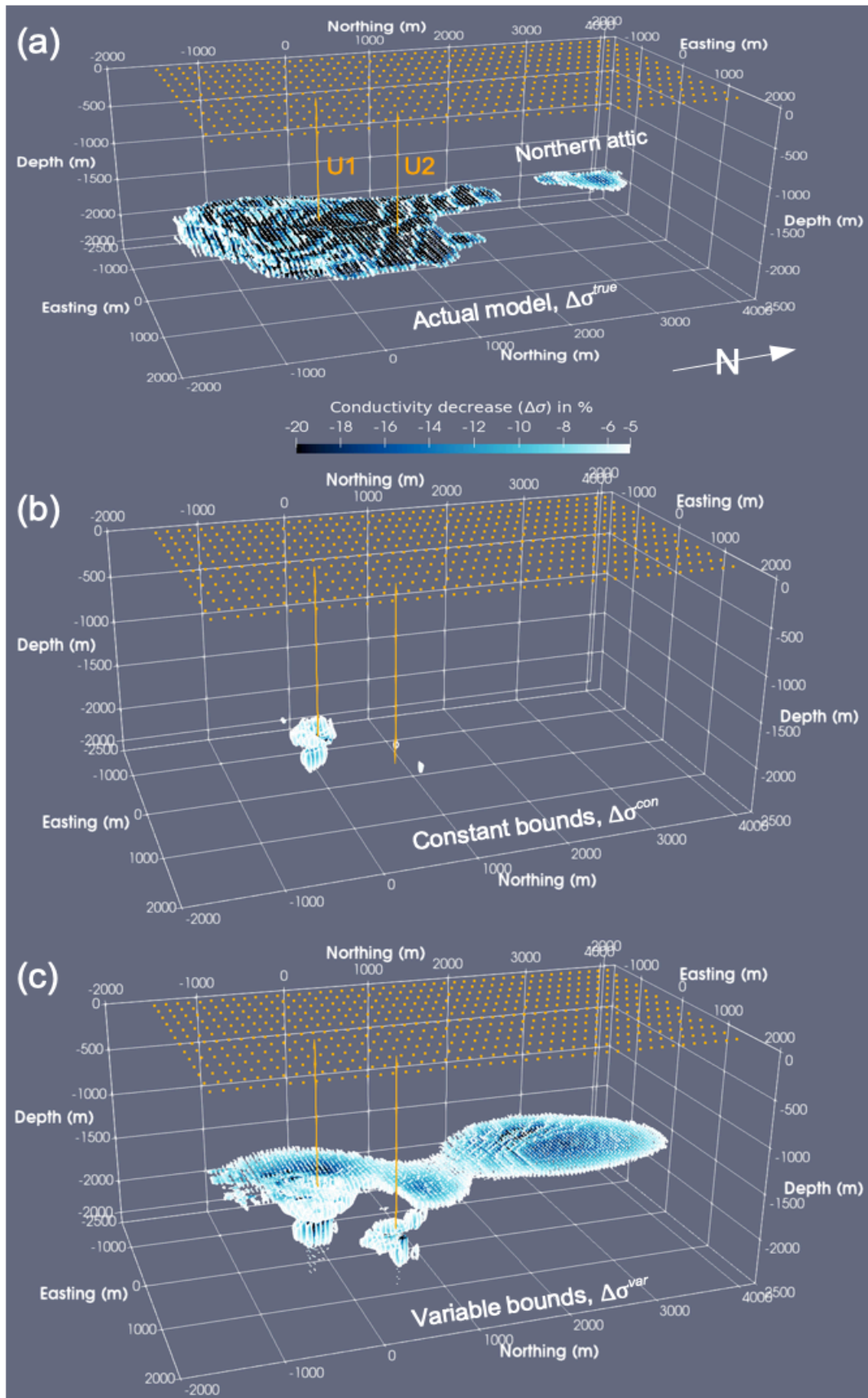
Fig. 3: Augmented lower and upper parameter bounds. The example cross sections are through the Easting coordinate $E=240$ m.



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Fig. 4: Petrophysical transform of the actual (left) and operational (right) flow model into the baseline (Year00) electrical conductivity. Conductivities derived from the operational model serve as the initial model guess for the inversion that refines the Year00 (pre-injection) model

616 below the upper seismic horizon (indicated by the white lines). The upper horizon delineates the
617 upper boundary of the inversion domain.



619 **Fig. 5:** Electrical conductivity change $\Delta\sigma = \sigma(\text{Year12}) - \sigma(\text{Year08})$ resulting from the
620 corresponding gas saturation changes between Year08 and Year12. (a) True difference $\Delta\sigma^{true}$
621 calculated from the actual model. (b) Estimated difference $\Delta\sigma^{con}$ resulting from the constant-
622 bound inversion sequence. (c) Estimated difference $\Delta\sigma^{var}$ resulting from the variable-bound
623 inversion sequence.

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