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Prices, capacities and service quality in a congestible Bertrand duopoly

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Abstract

We study the duopolistic interaction between congestible facilities that supply perfect substitutes. Firms are assumed to make sequential decisions on capacities and prices. Since the outcomes directly affect consumers’ time cost of accessing or using a facility, the capacity sharing rule is endogenous. We study this two-stage game for different firm objectives and compare the duopoly outcomes with those under monopoly and at the social optimum. Our findings include the following. First, for profit maximizing firms both capacity provision and service quality, defined as the inverse of time costs of using the facility, are distorted under duopoly: they are below the socially optimal levels. This contrasts with the monopoly outcome, where pricing and capacity provision are such that the monopolist does provide the socially optimal level of service quality. Second, duopoly prices are lower than monopoly prices, but higher than in the social optimum. Hence, while price competition between duopolists yields benefits for consumer, capacity competition is harmful. Third, price-capacity competition implies that higher capacity costs may lead to higher profits for both facilities. Finally, if firms also care about output, this mainly affects pricing behavior; strategic interaction in capacities are much less affected. If duopolists attach a higher weight to output and a correspondingly lower weight to profits, this leads to a deterioration of the quality of service.

Keywords: congestion, price-capacity games, imperfect competition

JEL code: L13, L86

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1. Introduction

Many facilities, like seaports, airports, internet access providers, and roads, are prone to congestion. When the volume of simultaneous users increases and capacity is constant, the time cost of using these facilities increases. More generally, the quality of the service provided by a facility may decrease when it gets crowded. Facility management can respond to quality deterioration by changing prices, but also by adapting the capacity of the facility. This paper asks how capacity and price decisions are made for congestible facilities in an oligopolistic market structure, and compares the oligopoly result to the monopoly outcome and the socially optimal outcome.

More specifically, we study the duopolistic interaction between congestion-prone facilities that supply perfect substitutes in the framework of a sequential game. The facilities first decide simultaneously on capacities; next, they simultaneously choose prices, given capacity decisions. Prices and capacities jointly determine consumers’ time cost of accessing or using a particular facility. The quality of service, defined as the inverse of time costs of using a facility, declines with crowding. We analyze the two-stage game for two different objective functions of the congestible facilities. First, we assume profit maximizing facilities. Second, however, we also look at the case where the facilities care about output as well as profits\(^1\). For profit maximizing facilities, the analysis shows that at the Nash equilibrium capacities and prices, service quality is distorted compared to the social optimum. For plausible parameter values, it is below the socially optimal quality. This contrasts to the monopoly solution, in which pricing and capacity choice does result in the socially optimal service quality. Prices are higher under monopoly than under duopoly. Hence, while price competition between duopolists yields benefits for consumer, capacity competition is harmful. If firms also care about output, we find that this mainly affects pricing behavior; strategic interaction in capacities is much less affected. If duopolists attach a higher weight to output and a correspondingly lower weight to profits, this generally leads to a deterioration of the quality of service.

\(^1\) Recent work on airport pricing and capacity choices by, e.g., Starkie (2001) and Zhang and Zhang (2003), suggests that airports are likely to care about output as well as profits. For example, they may get part of their revenues out of concessions, implying a strong interest in output as such.
The analysis of this paper is relevant to a number of situations. Competition between airports in metropolitan areas (e.g. San Francisco Airport and Oakland Airport in the San Francisco Bay Area) is one example. The airports are congestible, so that service quality declines with the number of passengers and plane movements. If airport management maximizes profits\(^2\), then price decisions and capacity choices will interact with service quality (congestion). A second example relates to competition between ports that serve the same hinterland (e.g. the ports of Long Beach and of Los Angeles in Southern California, or the ports of Antwerp and Rotterdam in Western Europe). Here too, port capacities and port charges can be chosen by the port authorities to maximize profits. Competition between internet service providers is another example, although our maintained no entry assumption is less straightforward in this case. The quality of internet service can be measured as a weighted average of (mainly) download speed, upload speed and mail processing speed; the capacity (computing power, disk space and network capacity) that is required to keep quality constant is approximately a linear function of the number of simultaneous users.\(^3\) The relation between congestion and the need for internet access pricing is studied in, e.g., Mason (2000), who uses simpler representations of congestion than we do and does not consider the connection between capacity and price decisions, quality of service, and market power.

Our analysis of price and capacity decisions in a homogenous goods duopoly as a sequential game in capacities and prices naturally builds upon earlier literature. First, Braid (1986) and Van Dender (2004) study duopoly pricing decisions of congested facilities, but they do not consider capacity adjustments. Second, de Palma and Leruth (1989) do study a two-stage game in capacities and prices; however, they focus on a discrete demand representation (users either consume one or zero units of the good), which does not allow discussing the role of specific model parameters in much detail.\(^4\) Third, Baake and Mitusch (2004) recently developed a model that is quite similar to ours, but mainly focus on the comparison between Cournot and Bertrand models in the pricing

\(^2\) At present, many airports do not act as profit-maximizers, as they are constrained by regulation and by long run contracts with (dominant) carriers. In a fully deregulated environment, however, market power deriving from airport congestion is more likely to accrue to airports than to airlines.

\(^3\) Personal communication with Francis Depuydt, Team Manager Integrated Service Platforms, Belgacom.

\(^4\) In their model, the Nash equilibrium in capacities will occur where capacities are restricted up to the point of zero consumer surplus.
stage of the game. The present paper provides a more detailed analysis of Bertrand pricing policies, it pays more attention to the distortion of service quality in the duopoly case, and it contains a detailed numerical illustration of price, capacity and service quality levels under different market structures. Moreover, we focus on different objective functions. Fourth, our analysis is closely related to an emerging literature on tax competition between countries or regional authorities that operate congestible road networks. For example, De Borger et al. (2004a, 2004b) discuss how a welfare-maximizing government decides on tolls and capacities when the network is used by domestic users and by transit users, when two countries compete for toll revenue from transit users. In the current paper we do not focus on tolling by governments but instead concentrate on the interaction of private congested facilities. Moreover, we use a slightly simpler model structure, yielding much more transparent theoretical results.

Finally, the present sequential capacity-price game can be contrasted to the literature evolving from the seminal paper by Kreps and Scheinkman (1983). They show that, with an L-shaped marginal cost function and with an efficient capacity-sharing rule, the two-stage capacity-price game yields the same result as a one-stage Cournot game in quantities. Later papers, e.g. by Maggi (1996), Dastidar (1995, 1997), and Boccard and Wauthy (2000, 2004), find that this result does not hold when marginal costs increase before capacity is reached (or when capacity can be exceeded at a higher marginal cost) or when different sharing rules are used. In all these papers, however, the cost function represents costs incurred by the firm, whereas we consider an upward sloping cost function incurred by users of the firms’ facilities. The marginal costs incurred by the firm are constant, so that, if there were no congestion, our model produces the standard Bertrand paradox (the competitive outcome is obtained). Note that the introduction of an upward sloping user cost function in combination with the consumer equilibrium constraint leads to ‘endogenous sharing’, as the distribution of output over the facilities is determined within the model, rather than through an externally defined sharing rule (which is required in the homogenous goods case without congestion in order to determine the distribution of market demand over firms). Not surprisingly, in this context the two-stage capacity-price game does not reduce to a one-stage Cournot game.
The structure of this paper is as follows. Section two contains the theoretical analysis. First, the structure of the model and the reduced form demand system are laid out. Then the second stage (price competition) and the first stage (capacity competition) of the duopoly game are analyzed. The duopoly solution is compared to the monopoly outcome and to the social welfare optimum. Finally, the role of different firm objectives is analyzed. Section three then uses a numerical example to clarify the properties of the model and to illustrate the role of various parameters. Section four concludes.

2. Analytical model

2.1. Model set-up

There are two facilities, $A$ and $B$, providing identical services. Aggregate willingness to pay is described by a downward sloping linear inverse demand function

$$G = \alpha - \beta q = \alpha - \beta \left( q_A + q_B \right)$$

where $q_i$ ($i = A, B$) is the number of simultaneous users of facility $i$. Consumers pay a price $p_A$ to use facility $A$ and $p_B$ to use facility $B$. In addition, they incur a time cost, which depends on their marginal time cost and on congestion, which is defined as the ratio between the number of (simultaneous) users $q_i$, $i = \{A, B\}$ and a facility’s capacity $K_i$, $i = \{A, B\}$. Congestion can be interpreted literally, as an increase in time costs, or it can be taken to reflect quality of service; this declines with the extent of crowding of the facility. As in de Palma and Leruth (1989), we denote the inverse of capacity by $R_i$, so that the time cost at each facility is $q_i R_i$, $i = \{A, B\}$. The marginal cost of capacity, $c_i$, $i = \{A, B\}$, is assumed to be constant.

---

5 We assume linear demand to simplify some of the derivations and to keep the theoretical results transparent. Note that the linear demand specification given by (1) can be explicitly derived under specific assumptions on the form of the utility function and the distribution of the value of the service to consumers, see e.g. Brueckner (2004).

6 Using inverse capacity facilitates many of the derivations below.
Throughout, we assume an interior solution, in which case consumer equilibrium requires that generalized prices (the sum of prices and time costs) at both locations are equal to the marginal willingness to pay.\(^7\) The game is solved by backward induction in sections 2.3 and 2.4; before doing so, we discuss the reduced form demand system in section 2.2.

### 2.2 Reduced form demands

The structural form of the demand system implies that marginal willingness-to-pay equals generalized prices at both facilities, where generalized prices are the sum of prices and time costs. It can be written as:

\[
G[q_A + q_B] = p_A + q_A R_A
\]

\[
G[q_A + q_B] = p_B + q_B R_B
\]

where \(G(\cdot)\) is given by (1) above. System (2) implicitly defines the reduced form demand functions that express demand at each facility as a function of prices and capacities at both facilities. Using superscript \(r\) for the reduced form demand functions, they can be written in general as:

\[
q_A = q_A^r(p_A, p_B, R_A, R_B)
\]

\[
q_B = q_B^r(p_A, p_B, R_A, R_B)
\]

To derive the impact of price and capacity changes on demand, we differentiate system (2) and write the result in matrix notation:

\[
\begin{bmatrix}
-\partial - R_A & -\partial \\
-\partial & -\partial - R_B
\end{bmatrix}
\begin{bmatrix}
dq_A \\
dq_B
\end{bmatrix} =
\begin{bmatrix}
dp_A + q_A dR_A \\
dp_B + q_A dR_B
\end{bmatrix}
\]

Applying Cramer’s rule we immediately derive the following effects:

\[
\frac{dq_A}{dp_A} \cdot \frac{q_A^r}{p_A} = -\frac{\partial - R_B}{|d|} < 0
\]

\(^7\) The assumption of interiority is reasonable when there is a homogenous value of time, but when consumer types differ by their willingness to pay for service quality, it can be shown that the pricing equilibrium will involve at least partial separation of types across facilities, even when price discrimination is allowed.
\[
\frac{dq_A}{dp} = \frac{q_A'}{p_A} = \frac{\partial}{|A|} > 0
\]  \hspace{1cm} (6)

\[
\frac{dq_A}{dR} = \frac{q_A'}{R} = \frac{q_A \left(-\frac{\partial}{|A|} - R_b\right)} > 0
\]  \hspace{1cm} (7)

\[
\frac{dq_A}{dR} = \frac{q_A'}{R} = \frac{\partial q_B}{|A|} > 0
\]  \hspace{1cm} (8)

where

\[
|A| = \det \left( \begin{bmatrix}
R_A & R_B \\
\frac{\partial}{|A|} & R_b
\end{bmatrix} \right) > 0
\]  \hspace{1cm} (9)

is the determinant of the matrix on the left-hand side of (4).

To avoid confusion in interpreting the sign of the last two expressions, recall that
\(R\) indicates the inverse of capacity. The signs then correspond to intuition: ceteris
paribus, a higher price at a particular facility reduces demand at that facility and increases
demand at the other; more capacity at a facility (i.e., conditional on demand, better
service quality) increases demand at that facility and reduces demand at the other.

2.3 Stage two: Nash equilibrium in prices

We take the point of view of facility \(A\). Its objective is to maximize profits:

\[
\max_{p_A} \pi_A = p_A q_A - \frac{c_A}{R_A}
\]

where demand is given by (3). The first-order condition can be developed as:

\[
q'_A(\cdot) + p_A \frac{q_A'}{p_A} = 0
\]  \hspace{1cm} (10)

\[
q'_A(\cdot) - p_A \frac{\partial + R_b}{\left( R_A R_b + \frac{\partial (R_A + R_B)}{R_A + R_B} \right)} = 0
\]

It follows that:

\[
p_A = q'_A(\cdot) R_A + q'_A(\cdot) \frac{\partial R_b}{\partial + R_b}
\]  \hspace{1cm} (11)
A similar expression holds for facility B. Note that expression (11) is conceptually identical to the ones obtained in Braid (1986) and Van Dender (2004). The optimal price, conditional on capacities at both facilities, consists of two components. The first one implies that each facility charges the marginal congestion cost at its facility, i.e. consumers pay for the marginal reduction in quality of service that their presence at the facility imposes on other (simultaneously present) users. The second component is a positive markup over marginal external congestion cost; it increases when demand becomes less elastic and when the competing facility is more congestible. Note that, in the Bertrand setting, congestion costs are the only source of market power: with \( \alpha = 0 \), prices are equal to marginal production costs (normalized to zero); otherwise said, in the absence of congestion costs, the textbook Bertrand paradox is obtained.

The pricing rules for A and B are implicit representations of the price reaction functions (superscript R) \( p_A^R = p_A^R(p_B, R_A, R_B) \) and \( p_B^R = p_B^R(p_A, R_A, R_B) \), conditional on capacities. To find the slope of the price reaction function for A, write the price rule in implicit form as follows:

\[
\cdot (p_A, p_B, R_A, R_B) = p_A - q_A(p_A, p_B, R_A, R_B) \left[ R_A + \frac{\partial R_B}{\partial p_B} \right] = 0, \quad (12)
\]

where the dependence of demand on capacities and prices, see (3), has been made explicit. Then use the implicit function theorem to find, after simple algebra:

\[
\cdot p_A^R = \frac{-\cdot R_B}{\cdot p_B} = \frac{\partial}{\partial p_B} > 0 \quad (13)
\]

\[
\cdot p_B^R = \frac{-\cdot R_A}{\cdot p_A} = \frac{\partial}{\partial p_A} = 0 \quad (14)
\]

\[
\cdot p_A^R = \frac{-\cdot R_B}{\cdot p_B} = \frac{\partial}{\partial p_B} > 0 \quad (15)
\]
Analogous results hold for $B$. The price reaction functions, conditional on capacity, are linear in the price of the competing facility, and they have an intercept between zero and one, guaranteeing a unique interior Nash equilibrium in prices, for given capacities. As is clear from (15), the reaction of prices to capacities at the competitor’s facility is not linear. As could be expected, the expression implies that a marginal capacity decrease at $B$ (i.e. a marginal increase in $R_B$) leads to a higher price at $A$.

Remarkably, equation (14) shows that along the reaction function, a facility’s price does not respond to a change in its capacity determined at the previous stage of the game. Intuitively, there are two opposing effects from a marginal capacity increase. The first one is that, holding demand in $A$ constant, an increase in capacity in $A$ reduces the time cost in $A$, so reducing the optimal price. The second effect is that more capacity at $A$ increases demand in $A$, and this increases both the time cost and the markup, raising the price. Given the specific model structure used (linear demands and congestion cost functions), one easily shows that these two effects cancel out. Of course, in more general models (e.g. with nonlinear congestion functions), the two effects will have opposite signs but their absolute size need not be identical.

The Nash-equilibrium prices, for given capacities, are denoted $p_A^{NE}(R_A, R_B)$, $p_B^{NE}(R_A, R_B)$, respectively. Formally, they are determined by the intersection of the reaction functions:

$$
p_A^{NE}(R_A, R_B) = p_A^R(p_B^{NE}, R_A, R_B)
$$

$$
p_B^{NE}(R_A, R_B) = p_B^R(p_A^{NE}, R_A, R_B)
$$

The sign of the effect of a marginal capacity increase at $A$ and at $B$ on these prices is easily determined by differentiating system (16). We find, using (13)-(15) and the analogous effects for the reaction function in $B$:

$$\frac{p_A^{NE}}{p_A^{NE}} \cdot \frac{p_B^{NE}}{p_B^{NE}} \cdot \frac{p_A^R}{p_A^R} \cdot \frac{p_B^R}{p_B^R} \cdot R_A > 0
$$

(17)
By (13) and its equivalent for \( B \), the denominator of (17) is positive and smaller than one. By (13) and (15), the numerator is positive. Consequently, a marginal capacity decrease at \( A \) raises the Nash-equilibrium price at \( A \). Similarly, a marginal capacity decrease at \( B \) raises the Nash-equilibrium price at \( A \) as well, see (18). In other words, we can say that a more congestible system (i.e. a lower combined capacity of both facilities) is characterized by higher Nash-equilibrium prices.

### 2.4 Stage one: Nash equilibrium in capacities

The first order condition for profit maximization in stage 1 is:

\[
\cdot \frac{p_{NE}^A}{R_A} q'_A(\cdot) + p_A \frac{dq_A'}{dR_A} + \frac{c_A}{R_A^2} = 0
\]

where

\[
\frac{dq_A'}{dR_A} = \frac{q'_A}{R_A} \frac{p_A}{<0} + \frac{q'_A \cdot p_{NE}^A}{R_A} \frac{R_A}{<0} + \frac{q'_A \cdot p_{NE}^B}{R_A} \frac{R_A}{<0}
\]

is the total effect of a capacity change in \( A \) on demand. It consists of the direct effect, holding prices constant, and indirect effects through Nash equilibrium price adjustments at the pricing stage of the game. The signs of the partial derivatives of the reduced form demand system and of the Nash-equilibrium prices – indicated beneath the expressions – were defined in (5), (6), (7), and in (17) and (18). It follows that the sign of (20) is ambiguous. As long as the first term on the right hand side (i.e., the direct effect of capacity on reduced-form demand) dominates the indirect effects through price reactions of capacity changes, it will be negative. If this is the case, marginally increasing \( R_A \) – marginally decreasing capacity at \( A \) – reduces demand at \( A \).

Note that, combining (19) and (20) and using the first order condition for optimal pricing behavior in \( A \) (see (10)), condition (19) for optimal capacity choice can be formulated equivalently as follows:
This suggests that duopolistic firms will supply higher capacity to the extent that capacity directly raises their demand (first term on the left-hand side) and that it does not too strongly reduce their demand via price adjustments by the competitor (second term): higher capacity in \( A \) reduces the Nash equilibrium price of the competitor \( B \), which in turn reduces demand in \( A \).

Equation (19) implicitly defines the reaction function in capacity for facility \( A \). Writing it in implicit form as

\[
0 = \frac{p_A}{R_A} \frac{q_A'}{q_A} + p_A \frac{dq_A'}{dR_A} - \frac{c_A}{R_A^2}
\]

and applying the implicit function theorem yields the following expression for the slope of the capacity reaction function:

\[
\frac{\partial R_A}{\partial R_B} = -M = \frac{1}{R_A} \left[ \frac{p_A}{R_A} \frac{dq_A'}{dR_B} + q_A \frac{2}{R_A} \frac{p_A}{R_B} \frac{dq_A'}{dR_A} + \frac{c_A}{R_A} \frac{d^2 q_A'}{dR_A dR_B} \right]
\]

where \( M \) is negative by the second order condition for profit maximization in capacity.

In general, the sign of (22) is ambiguous. In view of our earlier results (see, e.g., (17), (18) and (20)) the first term between the square brackets is plausibly positive; similarly, the third is plausibly negative. Little can be said about the second derivative terms a priori. For plausible parameter values and in a fully symmetric case, the slope can be shown to be negative (see also the numerical illustration). This implies that, the (inverse) capacity of \( A \) declines with that of \( B \). Intuitively, two opposite forces are at play. First, more capacity in \( B \) provides \( A \) an incentive to defend its market share by responding with a capacity increase as well. Second, higher capacity in \( B \) reduces Nash equilibrium prices at both facilities. Firm \( A \) then has an incentive to reduce capacity in order to increase congestion and, as a consequence, its price. The downward sloping capacity reaction functions suggest that the second effect dominates the first.

Finally, note that, although demand and cost functions were assumed to be linear, the capacity reaction function is not. This is not surprising because (see (15)) price
reactions to marginal capacity changes at the competitive location were shown not to be linear.

We further explore the nature of price and capacity reactions, and of the Nash equilibrium, in Section 3. First, however, we compare the duopoly solution to the monopoly outcome and to the social welfare maximum, and we briefly explore the role of objectives other than profit maximizing behavior.

2.5 Comparing duopoly, monopoly and the social optimum

Congestion (or service quality) and capacity are interdependent through technology. How this interdependence affects prices and capacity choices (or service quality choices) depends on the market structure. The comparison of different market structures provides further insight into the role of the oligopolistic interaction on which this paper focuses. Here, we derive price and capacity rules for a monopolist and for a social welfare-maximizer. Note that in these cases, it does not matter whether the problem is analyzed as a simultaneous or sequential choice of capacities and prices, as choices are made by a single agent, leaving no scope for strategic interaction. We opt for the simpler simultaneous approach.

Assume first that both facilities are operated by a single profit-maximizer. Profits are given by:

$$\sum_{i=A,B} p_i q_i'(p_A, p_B, R_A, R_B) - \sum_{i=A,B} \frac{c_i}{R_i}$$

and maximized with respect to the two prices and capacity levels. The first-order conditions can be written as:

$$\begin{align*}
p_A \cdot q_A' + q_A'(\cdot) + p_B \cdot q_B' = 0; \quad p_A \cdot q_A' + q_A'(\cdot) + p_B \cdot q_B' = 0 \\
p_A \cdot q_A' \cdot p_A + p_B \cdot q_B' \cdot p_B + \frac{c_B}{(R_B)^2} = 0; \quad p_A \cdot q_A' \cdot p_A + p_B \cdot q_B' \cdot p_B + \frac{c_B}{(R_B)^2} = 0
\end{align*}$$

These equations can be manipulated, using the reduced-form derivatives derived before (see (5)-(8)), to yield:

$$p_i = (q_A + q_B) \frac{\partial}{\partial t} + q_i. R_i. \{A, B\}$$
\[
\frac{1}{R_i} = \left( \frac{1}{c_i} \right)^{1/2} q_i, \quad i \in \{A, B\}
\]  

(26)

Interpretation is obvious. According to (25), the price at each facility is the sum of the marginal congestion cost at that location and a term relating to the elasticity of demand. Comparing to (11), it follows that the elasticity-related markup is higher than in the duopoly case. According to (26), capacity – the inverse of \( R_i \) – is inversely related to the marginal cost of capacity, it is increasing in the marginal value of time, and it is proportional to demand at the facility. Because the monopolist fully controls all instruments, the monopolist’s choice of capacity does not directly take account of effects on the equilibrium price. This contrasts to the duopoly case, where capacity choices do affect the Nash equilibrium price through strategic interactions.

Next, assume the facilities are operated by a welfare-maximizing government. It maximizes the difference between total net surplus and total social costs:

\[
\sum_{i=A,B} \left( \int_0^q G[u] du \right) - \sum_{i=A,B} \left( (G - p_i) q_i + \frac{c_i}{R_i} \right)
\]

(27)

where, as before, demands are given by (3) and \( G \) is defined in (2). This last expression implies

\[ G - p_i = R_i q_i \]

Using this information, the first order conditions can be written as:

\[
(G - 2 \cdot R_A q_A) \cdot \frac{q_A'}{R_A} + (G - 2 \cdot R_B q_B) \cdot \frac{q_B'}{R_B} = 0; \quad (G - 2 \cdot R_A q_A) \cdot \frac{q_A'}{P_A} + (G - 2 \cdot R_B q_B) \cdot \frac{q_B'}{P_B} = 0
\]

(28)

\[
(G - 2 \cdot R_A q_A) \cdot \frac{q_A'}{R_A} + (G - 2 \cdot R_B q_B) \cdot \frac{q_B'}{R_B} - q_A^2 + \frac{c_A}{(R_A)^2} = 0
\]

\[
(G - 2 \cdot R_A q_A) \cdot \frac{q_A'}{R_A} + (G - 2 \cdot R_B q_B) \cdot \frac{q_B'}{R_B} - q_B^2 + \frac{c_B}{(R_B)^2} = 0
\]

Again using (2), we have \( G - 2 \cdot R_A q_i = p_i - R_i q_i, \quad i = A, B \). Substitution then immediately implies the following price and capacity rules.

\[ p_i = q_i \cdot R_i, \quad i \in \{A, B\} \]

(29)
Social welfare maximization internalizes the externality: the price equals the marginal external congestion cost. The capacity rule is identical to that of the monopoly case (but as it holds at a different price and a different level of demand, the optimal capacity level will be different). Interestingly, since there are constant returns to scale in the provision of capacity, one easily shows that optimal pricing and optimal provision of capacity lead to exact cost recovery in this case. To see this, note that by (29) we have that total revenues equal
\[ \sum_i p_i q_i = \sum_i q_i^2 \cdot R_i \] using (30), total expenditures can be written
\[ \sum_i \frac{c_i}{R_i} = \sum_i q_i^2 \cdot R_i , \] establishing equality of revenues and expenditures. Self-financing facilities imply that the social welfare maximum can be implemented without distortionary taxes, viz. by a combination of congestion tolls and competitive pricing at each facility. The competitive price equals the marginal private production cost at each facility (which we normalized to zero).

It is clear that marginal congestion costs are a component of the price in all market structures, but that the markup differs between market structures. Expressions (11), (25) and (29) imply that the markup is zero in the social welfare maximum, it is positive at the duopoly outcome and, conditional on demand and capacity levels, it is highest in the monopoly case. Comparing in more detail the prices, capacity levels and quality of service under the three market structures leads to a number of observations. First, while capacity levels differ between the monopoly outcome and the social welfare maximum, the quality levels (as measured by time costs of using a facility, \( R_i q_i \)) will be identical. To see this, note that the optimal capacity rules (26) and (30) imply that, both under monopoly and at the social welfare optimum, the time cost equals \( R_i q_i = \left( \frac{c_i}{R_i} \right)^{1/2} \). Hence, a monopolist has no incentive to distort quality, as all benefits of providing it accrue to the firm itself. This observation is consistent with Spence (1975)\(^8\), who clarifies

\[ \frac{1}{R_i} = \left( \frac{1}{c_i} \right)^{1/2}, \quad i \in \{A, B\} \]

\(^8\) Spence (1975) shows that quality at the monopolists’ output level is below (above) the socially optimal level when the partial derivative of willingness to pay with respect to output and to quality is negative.
that the result is contingent on the additive structure of the generalized price. Second, together with (25) and (29) equal quality of service levels immediately imply monopoly prices that necessarily exceed prices at the social optimum. Moreover, with linear demands, duopoly prices will not only structurally but also numerically be between those under monopoly and at the social optimum. Indeed, (10) implies that a duopolist will operate where the price elasticity of reduced-form demand equals minus one, whereas (24) suggests the monopolist operates at an elasticity exceeding one in absolute value.

Third, whereas a monopolist does not distort quality compared to the social optimum, a duopolist does. In the duopoly case capacity choices are affected by strategic interactions (both at the pricing and capacity stages of the game, see (11) and (19), and quality of service levels will generally differ from their socially optimal values. The intuition is that a capacity increase reduces the generalized cost and, therefore, boosts demand at both facilities. This implies that the benefits of a capacity increase at one location partially accrue to the other. This externality is fully internalized in both the social optimum and the monopoly case, but it is not under duopoly. To get more intuition on this matter, note that (21) and (24) imply that the first order conditions for optimal capacity under monopoly and duopoly can, after simple rearrangement, be written as, respectively:

\[
\begin{align*}
 p_A \cdot \frac{q_A^f}{R_A} + p_B \cdot \frac{q_B^f}{R_A} &= -\frac{c_A}{R_A^2} \tag{31} \\
p_A \left( \frac{q_A^f}{R_A} + \frac{q_B^f \cdot P_B^{NE}}{P_A \cdot R_A} \right) &= -\frac{c_A}{R_A^2} \tag{32}
\end{align*}
\]

The monopolist takes into account the effect of raising capacity in \(A\) on demand in \(B\); moreover, he controls both the prices in \(A\) and \(B\). This allows him to fully internalize the effect of capacity at facility \(A\). The duopolist operating facility \(A\), however, can only take account of the indirect impact of raising capacity on its own demand via price adjustments by the competitor. For plausible parameter values, one expects each facility (positive). In our linear and additive specification of demand this derivative is zero, so that the monopolist supplies optimal quality.
owner to react to the ‘leakage of benefits’ of its capacity to its competitor by providing
less capacity. The numerical analysis in the next section confirms this suggestion. We
find that the duopoly leads to less capacity and lower quality of service, than under either
monopoly or maximal social welfare.

2.6 Alternative assumptions on firms’ objectives

As many congestible facilities (airport, ports, roads) are publicly owned or are
strongly regulated, it is reasonable to at least briefly consider objectives other than pure
profit maximization. For example, as referred to in the introduction, Starkie (2001) and
Zhang and Zhang (2003) argue that output is a relevant partial objective for many airports
that generate revenues out of concessions. Moreover, recent experiences in Europe also
suggest that the social role of airports encompasses more than profit, but that generating
activities in itself is a valid objective, for example, for reasons of employment
opportunities. This section therefore briefly explores the equilibria that result when
facilities’ objectives consist of a weighted sum of output and profit. When no weight is
given to profits, the facilities are output maximizers. When no weight is given to output,
they are profit-maximizers, and the analysis of the previous sections is obtained. Again,
we look at alternative ownership arrangements: duopoly refers to separate
ownership of the facilities, monopoly implies joint ownership.⁹

Duopoly: separate ownership

Suppose each facility is interested both in generating output (e.g. because of
lobbying by concessionary activities at an airport) and in profits. Assume that output and
profits receive an exogenous weight, normalize the output weight to one, and denote the

⁹ There is a potential semantic issue here, as duopoly and monopoly are usually understood to imply both a
particular ownership structure and the profit maximization objective. Strictly speaking, when profit
maximization is replaced by a different objective, one could argue that the duopoly and monopoly labels
are no longer appropriate. We stick to this terminology, however, even under conditions of output
maximizing behavior.
profit weight by \( \propto > 0 \). In stage 2 of the game, prices are set; the owner of facility \( A \) maximises:

\[
q_A + \propto \left( p_A q_A - \frac{c_A}{R_A} \right)
\]

subject to the consumer equilibrium constraints; i.e., demand in is given by the reduced-form demands derived before. The first-order conditions lead to the following pricing rule, conditional on capacities:

\[
p_A = q_A R_A + q_A \frac{\partial R_B}{\partial q_A} \frac{1}{\propto}
\]

Compared to the case of profit-maximizing duopolists, see (11), the price rule is amended by the extra term \(-1/\propto\). When this term is zero (i.e. as \( \propto \) approaches infinity), profits completely outweigh output in the objective, and (11) is obtained. When \( \propto \) becomes very small, output maximization becomes the main objective, and the last term in (34) dominates, implying a subsidy (i.e., prices become negative). For smaller \( \propto \), strategic interactions become relatively less important: output-maximization is obtained by subsidies (and a complete disregard for congestion costs), whatever the other facility does. In general, the strategic capacity setting decisions pertain to the profit-maximizing part of the objective function (note the separable nature of (34) and the exogeneity of \( \propto \)), so that the structure of the first stage of the game (capacity choices) is strongly similar to the profit-maximizing duopoly case.

**Monopoly: joint ownership**

Now consider joint ownership of both facilities; it maximizes:

\[
\sum_{i=A,B} q_i + \propto \left( \sum_{i=A,B} p_i q_i - \frac{c_i}{R_i} \right)
\]

subject to reduced-form demands, i.e., satisfying the consumer equilibrium constraints. The corresponding price and capacity rules are:

\[\text{[Equation]}\]

10 Using profits leads to the same results as using an exogenously defined allowable deficit.
The capacity provision rule is the same as for a profit maximizing monopolist (see (24)) and a social welfare maximizer, see (30). Not surprisingly, the price rule again reduces to that of a profit maximizing monopolist when \( \frac{1}{\infty} \). Interestingly, for \( \propto = \frac{1}{q} \) the welfare maximizing rule is obtained, see (29) above. Intuitively, \( \propto = \frac{1}{q} \) indicates that output is not the only objective (and becomes less important as output is high and \( q \) is large), because supply is ‘costly’.

It will be interesting to assess the effect of varying the relative weight of profit and output objectives on the separate and joint ownership results. Specifically, the effect on service quality needs to be assessed. These issues are explored in the numerical example.

3. Numerical analysis

This section explores the capacity-price game using a simple numerical example. The parameters of the example are given in Table 1; they were arbitrarily chosen. While the solution procedure of the numerical model slightly differs from the approach used in theoretical analysis\(^{11}\), the structure of the model is identical. We discuss a central\(^{11}\)

\(^{11}\) The numerical model does not explicitly use the reduced-form demands, but it finds the Nash equilibrium by simultaneously solving a system of equations that consists of (a) the consumer equilibrium conditions, i.e. the structural form of the demand system, (b) the congestion functions, (c) the first-order conditions of the price game for both facilities, and (d) the first-order conditions of the capacity game (conditional on the price rules) for both facilities. The numerical solution is subjected to the following check to confirm that the solution is a Nash equilibrium:

\[
\pi_i\left(R^*, R^*, p^*\left(R^*, R^*\right), p_j^*\left(R^*, R^*\right)\right) = \\
\pi_i\left(R_i, R_j^*, p_i^*\left(R_i, R_j^*\right), p_j^*\left(R_j, R_i^*\right)\right), i, j \in \{A, B\}, i \neq j, R_i = 0.
\]
scenario and assess the effect of changing parameters; next we compare market structures and, finally, we consider the effects of different objective functions.

3.1 Nash equilibrium – central scenario

Figures 1 and 2 depict the price and the capacity reaction functions for the central scenario, using the parameters reported in Table 1. When capacities are given, the price reaction function is linear and positively sloped. This says that price responses are positive and constant. The function is drawn for the capacities of the Nash equilibrium; changing these capacities results in a parallel shift of the function.

The capacity reaction function is not linear and it is downward sloping. The negative slope implies that the optimal response to a capacity increase at the competing facility is to reduce capacity at the own facility. For “low” capacity levels at the competing facility, the response function is concave, and for “high” capacity levels it is convex. In fact, it turns out that the Nash equilibrium occurs at the inflection point (where the absolute value of the marginal capacity response takes its maximal value). That is, the Nash equilibrium occurs where the response to a capacity increase at the competitor’s facility would entail the largest capacity reduction at the own facility.
3.2 Varying parameters

In this section we assess how the solution, i.e., the Nash equilibrium, responds when the inverse demand function shifts, when its slope changes, when the value of time changes (shift of the congestion function), and when marginal costs of capacity change. More specifically, as illustrated by Table 1, we compare the following scenarios:
• Central: benchmark parameter values, randomly chosen;
• \( \alpha < 0 \): reduces the intercept of the inverse demand function;
• \( \odownarrow < 0 \): reduces the slope of the inverse demand function;
• \( \odot < 0 \): reduces the time cost;
• \( c < 0 \): reduces the marginal cost of capacity.

### Table 1  Duopoly solution: sensitivity to parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Central</th>
<th>( \alpha &lt; 0 )</th>
<th>( \odownarrow &lt; 0 )</th>
<th>( \odot &lt; 0 )</th>
<th>( c &lt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept inverse demand</td>
<td>( \alpha )</td>
<td>120</td>
<td>110</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Slope inverse demand</td>
<td>( \odownarrow )</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Marginal cost of capacity</td>
<td>( c_i )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Marginal value of time</td>
<td>( . )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>( q )</td>
<td>51.8</td>
<td>47.4</td>
<td>103.5</td>
<td>52.1</td>
<td>52.1</td>
</tr>
<tr>
<td>Price</td>
<td>( p_i )</td>
<td>10.8</td>
<td>10.0</td>
<td>10.8</td>
<td>10.3</td>
<td>10.3</td>
</tr>
<tr>
<td>Generalized price</td>
<td>( g_i )</td>
<td>16.5</td>
<td>15.3</td>
<td>16.5</td>
<td>15.8</td>
<td>15.8</td>
</tr>
<tr>
<td>Output per facility</td>
<td>( q_i )</td>
<td>25.9</td>
<td>23.7</td>
<td>51.8</td>
<td>26.1</td>
<td>26.1</td>
</tr>
<tr>
<td>Time cost</td>
<td></td>
<td>5.7</td>
<td>5.3</td>
<td>5.7</td>
<td>5.4</td>
<td>5.4</td>
</tr>
<tr>
<td>Profits</td>
<td>( \pi_i )</td>
<td>274.6</td>
<td>232.8</td>
<td>549.2</td>
<td>267.2</td>
<td>267.2</td>
</tr>
<tr>
<td>Inverse capacity</td>
<td>( R_i )</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Capacity</td>
<td>( K_i )</td>
<td>4.6</td>
<td>4.5</td>
<td>9.1</td>
<td>2.4</td>
<td>4.8</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td></td>
<td>2,680.3</td>
<td>2,242.1</td>
<td>5,361</td>
<td>2,716</td>
<td>2,716</td>
</tr>
<tr>
<td>Welfare (profits + surplus)</td>
<td></td>
<td>2,954.9</td>
<td>2,474.9</td>
<td>5,909.8</td>
<td>2,982.8</td>
<td>2,982.8</td>
</tr>
</tbody>
</table>

The results derived from Table 1 can be summarized as follows. First, as expected, reducing demand by shifting the inverse demand function inwards (\( \alpha < 0 \)) reduces prices, capacities, and profits. Second and remarkably, reducing the slope of the inverse demand function (\( \odownarrow < 0 \)) does not affect the equilibrium price. This implies that making demand more price sensitive does not affect the Nash equilibrium prices despite the presence of market power.\(^\text{12}\) The mechanism underlying this result is clear from the table. When the demand function becomes steeper (\( \odownarrow < 0 \)), facilities adjust expenditures on capacity upward so as to keep time costs equal at a higher output level: the linear congestion, demand and capacity cost functions imply that setting \( \odownarrow \) at half its initial value induces facilities to provide twice the initial capacity at twice the initial output;

---

\(^\text{12}\) The effect obviously also holds in the opposite direction. When the slope of inverse demand is halved, capacity, output and profits are doubled, but prices and time costs are constant. The result is not contingent on the level of the unit capacity costs, but it does require marginal capacity costs being constant.
time costs remain constant. Note that the result is contingent on the two-stage structure of
the game, allowing firms to adjust capacities: in a one-stage pricing game with constant
capacities the Bertrand price directly depends on the slope of the demand function (Van
Dender, 2004). Although the perfectly proportional adjustments in capacity and demand
are specific to the linear model structure, the intuition for the result does hold in more
general models: providing capacity contributes more to profit when demand is more
sensitive to reductions in time costs. Hence we can expect prices not to be very sensitive
to the demand response to a cost increase also in models with nonlinear congestion and
capacity cost functions. Finally note that, as we will see below (subsection 3.3), the
insensitivity of prices to the slope of the demand function also holds for other market
structures.

Third, given the linear model structure, a reduction of the value of time (\( \theta < 0 \))
has the same effect as the same proportional reduction in marginal capacity costs (\( c < 0 \)).
Reducing the value of time directly reduces the time cost of congestion. Reducing
capacity costs indirectly reduces the cost of congestion by raising capacity. Given the
model structure, expenditures on capacity are identical for both parameter changes. They
both intensify competition between facilities, and this leads to lower prices and higher
output than in the central scenario. Fourth, note that profits are lower after a capacity cost
reduction than in the central scenario, which seems counterintuitive\(^{13}\). The reason is that
the cost reduction in the provision of capacity reduces costs, but also intensifies
competition and reduces prices, indirectly reducing revenues. If the latter effect
dominates, as it does in the numerical example, lower capacity costs reduce profits. Of
course, this is not a general duopoly result. For example, in the numerical example
considered, an increase in capacity costs starting from relatively high initial capacity cost
levels does reduce profits. Moreover, the result obviously does not hold under the
assumption of monopoly or of surplus maximization, see below.

\(^{13}\) The result appears in a starker form when, starting from identical cost levels, the cost at one location is
increased. When, for example, the cost at location A rises, profits at A rise and those at B decline, and the
sum of profits rises.
3.3 Comparison of market structures

The analysis has focused on the interaction between congestion, pricing and capacity decisions in a duopoly. Clearly, the interaction also is present in other market structures. This section compares the monopoly solution and the social welfare maximum to the duopoly. It was argued before that the social welfare maximum is obtainable through a combination of pure competition and optimal congestion tolls. Table 2 summarizes solutions under the various assumptions on market structure. Three observations are noteworthy.

First, capacities in the monopoly outcome are higher than in the duopoly; the same holds for prices and profits. Service quality, as captured by the inverse of time costs, is higher in the monopoly case than under duopoly. Consistent with the theory, service quality is the same in the monopoly and the social welfare maximum. This confirms our earlier statement that, while the monopolist distorts output, service quality is optimal from the social point of view. In contrast, duopolists compete in prices as well as capacities, so they cannot capture as much surplus generated by high quality as a monopolist can: benefits partly accrue to the competitor. Therefore, duopolists will supply less of it than a monopolist or a social welfare-maximizer will. The consequence is lower capacities and higher time costs in the duopolistic equilibrium as compared to the monopoly or the social welfare maximum. Whereas price competition under duopoly benefits the consumer, capacity competition is detrimental to consumer welfare. Note from the numerical example that, despite the quality distortion under duopoly, consumer surplus and welfare are lower in the monopoly case than under duopoly due to the output distortion of monopolistic pricing.
Second, reducing the marginal value of time implies that capacity (or service quality) is valued less by consumers, so that less of it is provided under all market structures. Time costs fall with lower marginal values of time, but the reduction is mitigated by the reduction of capacity. Output increases with lower values of time. In the monopoly, prices do not depend on the marginal value of time, so that profits increase. In the duopoloy, less congestion means lower prices, and profits fall despite the increase of output.
Third, as previously illustrated for duopoly, prices are independent of the slope of the structural demand function. The consequence is that the price elasticity of the (structural) demand function is also independent of $\beta$ in each market structure. Given the linear demand function, it is clear that its absolute value is largest (and above one) in the monopoly outcome, smaller in the duopoly, and smallest in the welfare maximum. Prices, time costs and, therefore, generalized prices are also constant when the slope of the inverse demand function varies. The mechanism generating this result was explained before. The intuition is that providing capacity contributes more to profits or to social welfare when demand is more sensitive to reductions in time costs. This phenomenon is general; of course, the knife-edge result that price-elasticities remain perfectly constant within each market structure is particular to the linear structure of the model (i.e. to the additive structure of the generalized price and to the constant returns in the provision of capacity).

### 3.4 Alternative objective functions

We calculate the outcome of the model corresponding to Section 2.6, where a weighted sum of output and profits was maximized under joint and separate ownership of the facilities, using the parameters of the central scenario. The key results for various values of $\alpha$, the exogenous weight of profits, are summarized in Table 3.

**Table 3 Key Results for mixed objective**

<table>
<thead>
<tr>
<th>Exogenous weight of profits in the objective function ($\alpha$)</th>
<th>$\alpha \rightarrow #inf$</th>
<th>$\alpha = 1.5$</th>
<th>$\alpha = 1$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha \rightarrow 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Separate ownership</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>51.8</td>
<td>52.1</td>
<td>52.2</td>
<td>52.7</td>
<td>56.9</td>
</tr>
<tr>
<td>Price</td>
<td>10.8</td>
<td>10.2</td>
<td>9.9</td>
<td>8.9</td>
<td>0.05</td>
</tr>
<tr>
<td>Time cost</td>
<td>5.67</td>
<td>5.69</td>
<td>5.71</td>
<td>5.75</td>
<td>6.14</td>
</tr>
<tr>
<td>Capacity</td>
<td>4.56</td>
<td>4.57</td>
<td>4.57</td>
<td>4.58</td>
<td>4.63</td>
</tr>
<tr>
<td><strong>Single ownership</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>29.5</td>
<td>29.67</td>
<td>29.75</td>
<td>30.00</td>
<td>57.28</td>
</tr>
<tr>
<td>Price</td>
<td>60</td>
<td>59.67</td>
<td>59.50</td>
<td>59.00</td>
<td>4.4</td>
</tr>
<tr>
<td>Time cost</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Capacity</td>
<td>14.75</td>
<td>14.83</td>
<td>14.87</td>
<td>15.00</td>
<td>28.64</td>
</tr>
</tbody>
</table>
In the leftmost column, the profit weight approaches infinity and the same results are obtained as under pure duopoly and pure monopoly. When the relative weight of output increases, output increases and prices decrease due to a lower weight on profit. The difference between separate (duopoly) and single (monopoly) ownership lies in the quality of service. With single ownership, the quality of service is independent (and equal to the socially optimal level of quality) of the relative weights of profits and output. In the duopoly case, putting more weight on output (reducing \( \propto \)) leads to a deterioration of the quality of service. As explained in Section 2.6, in the case of monopoly, with a profit weight of \( \propto = 1/q \), the socially optimal solution is obtained. In this example, that profit weight is close to zero (compare Tables 2 and 3).

4. Concluding remarks

We studied duopolistic competition in capacities and in prices between congestible facilities within the framework of a sequential price-capacity game. Moreover, we compared the duopoly solution to the monopoly outcome and to the social welfare maximum. First, it was shown that the equilibrium level of service quality, as determined by the inverse ratio of demand and capacity, is lower in the duopoly than under either monopoly or the social optimum; given the additive structure of generalized prices in our model, quality levels are the same under monopoly and at the welfare optimum. Duopoly prices were shown to be below monopoly prices but above socially optimal prices. The implication is that, while price competition yields benefits for consumer, capacity competition between duopolists is harmful to consumers. Second, we showed that higher capacity costs may in fact lead to higher profits for both facilities, because the dampening effect on capacity provision implies higher prices. Third, the observation that providing extra capacity contributes more to profits (and to social welfare, for that matter) when demand is more sensitive to reductions in time costs implies, given the linear model structure, that prices were independent of the slope of the demand function under all market structures. Finally, we considered the effects of alternative objective functions. It was found that, when duopolists attach a higher weight
to output and a correspondingly lower weight to profits, this leads to a deterioration of the quality of service.

References


