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Exchange Rates, Information, and Crises

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By

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Abstract

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In this dissertation, I theoretically investigate how the actions of central banks affect the information and beliefs of rational agents. I focus primarily on my models' equilibrium predictions during crisis episodes, including situations in which agents' actions are strategic complements. The first two parts explore the implications of central bank transparency during foreign exchange interventions and develop dynamic models in which investors are heterogeneously informed about both interventions and fundamentals. In the first part, the benchmark two-period model presents the main result that transparency can often exacerbate any misalignment between the exchange rate and fundamentals. This is a consequence of two distinct effects of transparency. First, transparency reveals some information about fundamentals to investors (the truth-telling effect). Second, transparency increases the precision of the exchange rate as a signal of those fundamentals that remain unknown (the signal-precision effect). If a central bank announcement reveals little information about fundamentals, then this second effect dominates and transparency magnifies exchange rate misalignment. In effect, partial information revelation is worse than no information revelation. An important implication of this result is that a policy of ambiguity can increase the effectiveness of intervention to support a declining currency during times of crisis. In the second part, the benchmark model is extended to an infinite horizon and also expanded into a Bayesian signalling game. In both cases, I demonstrate that the principal results do not change.

In the third part, I examine a global coordination game in which the information of the agents is manipulated before it reaches them. I assume that the regime is imperfectly informed about the underlying state of fundamentals and must trade off the cost of biasing the signals of agents upwards with the benefit of being more likely to defeat the agents' attack. The main conclusion is that the effect of information manipulation depends on the extent to which the regime is better informed about the outcome of the game. In the limit as both the regime and the agents' information becomes arbitrarily precise, the effectiveness of information manipulation depends on whether the regime learns about fundamentals faster than the agents. If agents learn faster than the regime, then information manipulation is ineffective and incurs costs for the regime without diminishing the size of coordinated attacks. In these cases, the regime prefers to take no action but cannot credibly commit to do so.

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Contents

Introduction

This dissertation comprises three separate sections, each of which theoretically examines how the actions of central banks affect the information and beliefs of market participants and what the implications of those effects are. I derive several results about central bank transparency during foreign exchange interventions and information manipulation by the incumbent regime in coordination games. Throughout this discussion, I focus primarily on these results' implications during crisis episodes.

In the first two parts of the dissertation, I develop dynamic models of foreign exchange intervention in which investors are heterogeneously informed about both interventions and fundamentals. I examine the effects of credible and truthful public announcements about the size and timing of interventions as opposed to deliberate attempts to be secretive and create uncertainty about those interventions. The fist part presents the benchmark two-period model, which posits that foreign exchange interventions contain information about part of exchange rate fundamentals. In that setup, I put forward the main result that transparency can often exacerbate any misalignment between the exchange rate and fundamentals. This is a consequence of two distinct effects of transparency. First, transparency reveals some information about fundamentals to investors (the truth-telling effect), which reduces misalignment. Second, transparency increases the precision of the exchange rate as a signal of those fundamentals that remain unknown (the signal-precision effect), which compels rational Bayesian investors to weigh that public signal more heavily in their expectations and thus magnifies misalignment. The key conclusion is that if a central bank announcement reveals little information about fundamentals, then the signal-precision effect dominates and transparency magnifies currency mispricing. In effect, partial information revelation is worse than no information revelation.

An important implication of this result is that a policy of ambiguity will often increase the effectiveness of central bank intervention during periods of crisis and large capital outflows. In these episodes, asymmetric information, pro-cyclical liquidity provision, and psychology often lead to excessive sales of risky assets. My model predicts that it is precisely in situations like these, when risky countries' currencies are undervalued and it is difficult to credibly reveal information about fundamentals, that transparent interventions to support a currency are less effective than more opaque and secretive interventions. This prediction matches both central banks' observed behavior during these turbulent episodes and their justifications for more secretive intervention policies.

To examine the robustness of the results, in part two I both extend the benchmark model to an infinite horizon and expand into a Bayesian signalling game. In the signalling game, the central bank has a clearly defined objective function that investors are aware of. Given a set of assumptions for the model's primitives, I prove the existence of a partially-separating Bayesian equilibrium that preserves the intuition and analysis from the benchmark model. In the infinite-horizon extension, I consider environments in which higher-order expectations both are and are not part of the equilibrium exchange rate. If investors have common knowledge of the past, then higher-order expectations disappear and it is possible to analytically characterize the equilibrium. The results in this setting match the benchmark model's predictions. If investors do not have common knowledge of the past, then higher-order expectations remain and the equilibrium must be approximated. In this setting, unobserved transitory shocks have persistent effects on investors' beliefs. I show that these persistent effects can be magnified by transparency in the same way as in the benchmark model.

The final part of this dissertation explores the equilibrium implications of information manipulation on the part of the incumbent regime in global coordination games. This assumption extends more traditional global games in which the private information of coordinating agents is accurate and unbiased, and it captures the reality that in many cases the regime can take actions to alter or hide this information. I adopt the setup of Edmond (2008a), in which an incumbent regime can take a costly hidden action that biases the private signals of the game's coordinating agents, and then extend this setup so that the regime is imperfectly informed about the underlying state. This implies that the regime does not know the outcome of the coordination game ex-ante, so that it must trade off the cost of biasing the signals of agents upwards with the benefit of being more likely to defeat the agents' attack. Given this setup, I show that the effect of information manipulation depends on the extent to which the regime is better informed about the outcome of the game. If the precision of agents' information about the underlying state grows faster than the precision of the regime's information, then in the limit information manipulation is ineffective and incurs costs for the regime without diminishing the size of coordinated attacks. In these cases, the regime prefers to take no action but cannot credibly commit to do so. I also show that information manipulation is sometimes ineffective even if the precision of the regime's information grows faster than the precision of the agents' information.

Part I
Exchange Rate Manipulation and Constructive Ambiguity: The Meaning of Transparency

Over the past decade, a growing body of evidence has demonstrated that all but a few countries exert some control over the value of their exchange rates. According to Calvo and Reinhart (2002), this "fear of floating" is common not only among countries that openly admit it, but also among those that claim not to let currency prices affect policy. Just as central banks broadly agree about the desire to control their exchange rates, they broadly disagree about the policies that should accompany these interventions, especially with regard to transparency. In this chapter, I develop dynamic models of foreign exchange intervention that address these questions.

I focus on the issue of central bank transparency, specifically on the implications of credible and truthful public announcements about the size and timing of foreign exchange interventions as opposed to deliberate attempts to be secretive and create uncertainty about those interventions. While there are other important aspects of central bank intervention policy, the question of transparency is among both the most important and the most disputed. Indeed, there is extensive evidence that central banks from around the world hold opposing views about the implications of predictability versus unpredictability, and that they implement different policies for different reasons (Bank for International Settlements 2005, Canales-Kriljenko 2003, Chiu 2003).

Two examples from the financial crisis highlight this lack of policy consensus. Both Mexico and Russia faced intense capital outflows and speculative pressure as the price of risky assets throughout the world declined in the months after the collapse of Lehman Brothers in September 2008.[1] The Bank of Mexico has a longtime commitment to transparent foreign exchange intervention, but at the height of this crisis in early February 2009, the Bank became convinced that transparency was hurting its efforts to stabilize the peso and abruptly switched to a secretive and purposely ambiguous policy. In that month alone, the Bank spent nearly two billion dollars of its reserves in unannounced interventions.[2] In this same period, the Bank of Russia fought a protracted battle with the markets over the falling ruble. Its well-publicized attempts to initially guide the currency to an orderly and predictable depreciation eventually gave way to a looser, more ambiguous policy in which the target band for the ruble was substantially widened and made more flexible.[3] Ultimately, the Bank

[1] Between August 2008 and March 2009, both the Mexican peso and the Russian ruble lost more than one third of their values against the US dollar before eventually stabilizing at slightly higher levels.

[2] Although these interventions were intentionally kept secret, the Bank of Mexico did reveal their size afterwards. For a discussion of the Bank's normally transparent policy, see Sidaoui (2005).

[3] In the second half of 2008, the Bank of Russia widened the target band for the ruble to 16.9% (top to

of Russia's extensive interventions contributed to a loss of more than 200 billion dollars in foreign exchange reserves (nearly 40% of the Bank's total reserves) in a period of only six months. In both of these cases, policymakers appear to have been uncertain about the best way to complement their interventions and to help effectively stabilize and defend their currencies. In this era of enormous foreign exchange reserves and large-scale interventions, a better understanding of the implications of these different policies is important.

The main prediction of my analysis is that central bank transparency can in fact magnify any existing misalignment between the exchange rate and fundamentals. This follows because a transparent intervention policy improves the precision of the exchange rate as a signal of fundamentals (the signal-precision effect of transparency), and thus compels rational Bayesian investors to weigh that public signal more heavily in their expectations. Although transparency reveals some information about fundamentals (the truth-telling effect of transparency) and thus also diminishes the signal value of the exchange rate, this extra information can be outweighed by the increased precision provided by a public announcement. It is precisely in these cases, when central bank announcements do not credibly reveal sufficient information about fundamentals, that exchange rate misalignment worsens.[4] Figure 0.1 plots the relationship between exchange rate misalignment and information revelation. As shown, transparency magnifies misalignment for low levels of information revelation but there exists a threshold at which transparency starts to reduce this misalignment. In effect, partial transparency is worse than no transparency, while full transparency is best.

This conclusion has many implications. Arguably the most important is that a policy of ambiguity will often increase the effectiveness of central bank intervention during periods of crisis and large capital outflows. In these episodes, asymmetric information, pro-cyclical liquidity provision, and psychology often lead to excessive sales of risky assets, as shown by Brunnermeier and Pedersen (2009) and Shleifer and Vishny (1997). My model predicts that it is precisely in situations like these, when risky countries' currencies are undervalued and it is difficult to credibly reveal information about fundamentals, that transparent interventions to support a currency are less effective than more opaque and secretive interventions. In the case of Mexico and Russia, the model argues that both countries would have likely benefited from more secrecy and ambiguity—as they eventually chose—to go along with their extensive foreign exchange interventions.

I build on a simple model of a cashless economy in which investors are heterogeneously informed about both central bank interventions and fundamentals. The first model I present, the benchmark two-period model, posits that foreign exchange interventions contain information about part of exchange rate fundamentals. In the style of Grossman and Stiglitz

bottom) via a series of small adjustments. It then widened the band further to 28.9% in a little over one week in January 2009. Two examples of some of the press coverage surrounding this episode are the articles "The Flight from the Rouble" and "Down in the Dumps" from *The Economist*, November 20, 2008 and February 5, 2009, respectively.

[4]In all of the models I present in this and the next chapter, the concept of exchange rate misalignment is equivalent to the concept of exchange rate informativeness from market microstructure theory. Specifically, more exchange rate misalignment is the same as a less informative exchange rate.

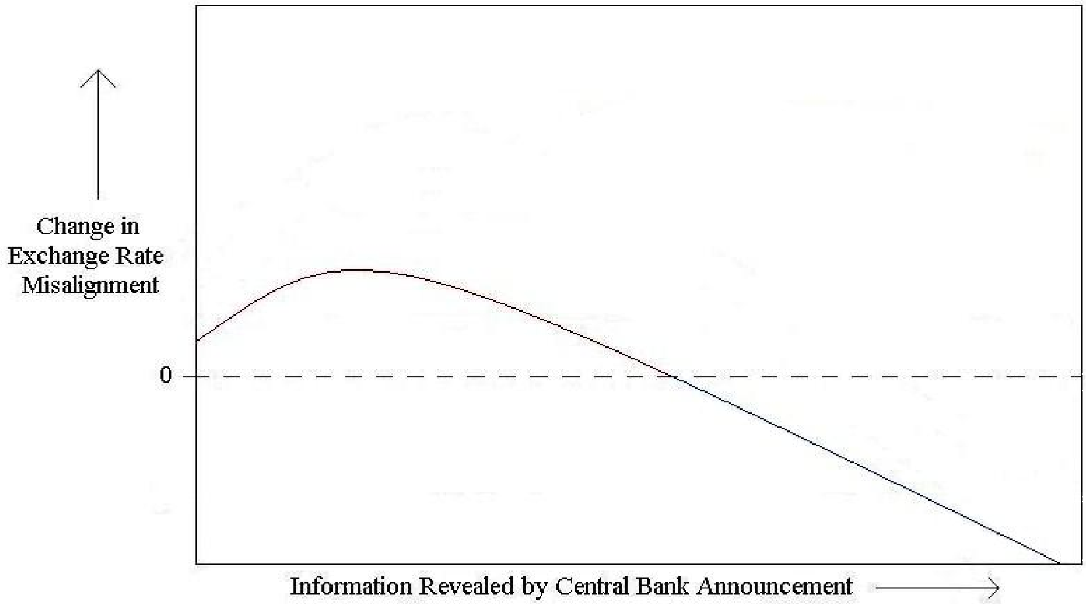

Figure 0.1: The relationship between exchange rate misalignment and information revelation.

(1976), information about all future fundamentals is embedded in the current exchange rate so that, by observing the price of currency, investors learn about these fundamentals and update their beliefs. This learning is imperfect, however, as noise traders push the exchange rate away from its fundamental value. Since the price of foreign currency is a publicly observable signal, any time that the exchange rate differs from its fundamental value average beliefs about fundamentals will differ from the true value of fundamentals. Within this framework, I demonstrate that transparency worsens exchange rate misalignment whenever interventions reveal little information about fundamentals.

The second model that I present extends the benchmark model to include foreign exchange interventions that respond to movements in the exchange rate. This is an important consideration, because a central bank will often take into account more than just its knowledge of fundamentals when choosing how extensively to intervene in the foreign exchange market. Indeed, an intervening bank is usually also concerned with the value of its currency and possible presence of misalignment. If this is the case, the central bank's intervention will be a function of both fundamentals and exchange rate misalignment.

Throughout this chapter, I consider the implications of a policy of publicly and truthfully announcing the size of interventions versus a policy of secrecy. One advantage of focusing on these two policies is that they have a clear economic interpretation in terms of the information sets of investors, making rigorous theoretical analysis easier. In practice, however, a central bank wishing to be transparent will often announce not only the size of a current intervention, but also the size of past interventions, the size and timing of interventions planned for the future, and the likely stance of other policies in the future.[5] These considerations have a

[5]Dominguez and Panthaki (2007) and Gnabo, Laurent, and Lecourt (2009) provide empirical evidence

natural interpretation in my models. In particular, all of the results about central bank transparency are statements about the extent of information that is revealed to investors, and the conclusion is that the more information that is credibly communicated through a public announcement, the less likely it is that transparency will exacerbate exchange rate misalignment (as shown in Figure 0.1).

A truthful central bank announcement affects investors' beliefs in two different ways in my models. First, and more apparently, any parameters the central bank reveals to investors eliminate the role of the exchange rate as a signal of those parameters. This is the *truth-telling effect* of transparency. Second, and less apparently, any parameters the central bank reveals to investors increase the precision of the exchange rate as a signal of other, still-unknown parameters, and hence increase the weight that investors place on the exchange rate signal when forming their beliefs about those unknown parameters. This is the *signal-precision effect* of transparency. These two effects push in opposite directions. The truth-telling effect directly raises expectations of parameters for which average beliefs are too low. This tends to reduce misalignment and appreciate an exchange rate that, because of sales by noise traders, is undervalued relative to fundamentals. Conversely, the signal-precision effect indirectly lowers expectations of parameters for which average beliefs are too low and tends to increase misalignment and further depreciate an already undervalued exchange rate. A large signal-precision effect explains why misalignment increases in the left side of Figure 0.1 while a large truth-telling effect explains why misalignment decreases in the right side of the figure. The main results of this chapter characterize the conditions for which one effect dominates the other.

There are several important conditions that imply that transparency will magnify exchange rate misalignment. The most essential of these is that a central bank announcement reveals only partial information about fundamentals (as shown in Figure 0.1), a condition that limits the size of the truth-telling effect of transparency relative to the signal-precision effect. If foreign exchange interventions instead contain extensive information about future policies and fundamentals, then a transparent intervention becomes an important and credible source of information, a point emphasized by Dominguez and Frankel (1993a), Mussa (1981), and the whole literature about the signalling hypothesis.[6] My models are consistent with this observation since they predict that transparency reduces exchange rate misalignment and increases the effectiveness of interventions (if the central bank's goal is to reduce misalignment) in these cases. One of this chapter's contributions, however, is to build on this logic of the signalling hypothesis by exploring the interaction between partial information revelation and currency mispricing and showing that transparency can in fact exacerbate exchange rate misalignment if interventions are not sufficiently informative about future fundamentals and policies.

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that many kinds of central bank statements related to foreign exchange interventions affect the exchange rate.

[6] Sarno and Taylor (2001) and Vitale (2007) both provide excellent surveys of the signalling-hypothesis literature (and the intervention literature, more broadly), while Kaminsky and Lewis (1996) empirically examine the relationship between interventions and future fundamentals.

The mechanism I describe in this first chapter matches well with the justification that central banks often provide for their ambiguous policies. In particular, survey evidence from Bank for International Settlements (2005) and Chiu (2003) indicates that central banks worry that unsuccessful transparent interventions might undermine both a bank's credibility and the market's confidence in its currency. Central banks are concerned that highly visible and extensive interventions coupled with continued undesirable movements in the exchange rate will intensify doubts about a bank's ability to achieve its goals. Indeed, a transparent failure of this nature publicly reveals the market's true sentiment about exchange rate fundamentals and magnifies pessimism among market participants with different beliefs. This chapter gives these intuitive but vague ideas a precise meaning within a clearly specified economic model.

1 Related Literature

My models assume that domestic and foreign assets are imperfect substitutes, which ensures that foreign exchange interventions alter the currency risk premium and have a permanent effect on the exchange rate. There remains, however, a considerable amount of both theoretical and empirical uncertainty about the relative impact of interventions that leave interest rates and the money supply unchanged. Indeed, as described by Edison (1993), some of the earliest literature on this topic concluded that interventions only affect the exchange rate by enhancing the credibility of future policy. I emphasize that this and the next chapter's main results do not require that interventions have a persistent impact on the exchange rate. In fact, even if I assume that interventions have no predictable effect on the exchange rate at any horizon, the results remain intact as long as interventions have an effect on the volatility of the exchange rate.[7]

Recently, a growing empirical literature has shown that foreign exchange interventions do have an immediate and statistically significant impact on exchange rates regardless of whether or not a central bank publicly announces the size and timing of its interventions. This literature includes Chaboud and Humpage (2005), Dominguez and Frankel (1993b), Dominguez and Panthaki (2007), Fatum and Hutchison (2003), Ghosh (1992), Ito (2002), Kearns and Rigobon (2005), and Payne and Vitale (2003), among others. No consensus has been reached, however, about how much of this impact is due to direct, portfolio-balance effects versus indirect, signalling effects, and how persistent these effects are.

Much recent research has emphasized the interaction between market expectations and central bank interventions. On the theoretical side, both Bhattacharya and Weller (1997) and Vitale (1999) incorporate ideas from the literature on microstructure and order flow in asset pricing and develop models in which interventions have large effects on market expectations. The market participants in their models observe order flow and rationally infer what an intervention reveals about fundamentals so that an intervention that has only a temporary effect on order-flow can still have a lasting impact on the exchange rate by

[7] Beine, Lahaye, Laurent, Neely, and Palm (2007) provide recent evidence that interventions increase exchange rate volatility, while Vitale (2007) presents a survey of some of the past literature on this topic.

affecting the foreign exchange market's information. These models do not examine the interaction between transparency, central bank information revelation, and exchange rate misalignment as I do, but they still find that public announcements are often neither desirable nor credible if central banks' objectives are not consistent with exchange rate fundamentals. On the empirical side, Dominguez and Panthaki (2007) show that both falsely reported interventions and unrequited interventions—interventions that the market expects but do not materialize—have statistically significant effects on the exchange rate. This observation strongly suggests that interventions influence the beliefs of currency traders in important ways.

There is a vast and insightful literature on managed exchange rates. Its focus is primarily on fixed currency pegs, in which no movement in the exchange rate is allowed, and target zones, in which the exchange rate is allowed to float freely only within some specified range. Among the most notable contributions are those of Flood and Garber (1984), Hellwig, Mukherji, and Tsyvinski (2006), Jeanne and Rose (2002), Krugman (1991), Morris and Shin (1998), and Obstfeld (1996). In general, the fixed exchange rate literature focuses on the causes and consequences of speculative attacks and currency crises, while the target zone literature focuses on the effects of policy on expectations of the future and hence on the value of today's exchange rate.[8] I consider foreign exchange interventions as part of a managed floating exchange rate, although the logic behind my results applies to fixed currency pegs and target zones as well.[9]

The structure of my models shares much in common with other models of imperfect information in asset-pricing and crises. Indeed, the benchmark two-period model operates in an environment that is similar to the asset-pricing and crisis hybrid model of Angeletos and Werning (2006). It is no surprise, then, that my model replicates one of their main insights—the positive relationship between the precision of agents' private signals of fundamentals and the precision of the exchange rate as an endogenous public signal of those fundamentals. Angeletos and Werning (2006) examine this relationship's implications for the equilibrium outcome in global coordination games but do not consider the possibility of price manipulation as I do. Given the similarity between the two models, an extension of this chapter's main results about transparency and currency mispricing to a global-games setting is likely to be a promising direction for future research.

The idea that transparency might have counterintuitive implications and lead to bad outcomes is also explored by Angeletos and Pavan (2007), Cornand and Heinemann (2004), and Morris and Shin (2002). These papers consider environments in which high levels of coordination among agents can be socially suboptimal and examine how public information facilitates this coordination and can lead to undesirable effects. These environments are static and highly stylized so that actions, information, and payoffs may be interpreted to represent many different things. My model avoids any analysis of total welfare and is instead a positive

[8] Krugman and Rotemberg (1992) link these two and analyze speculative attacks against target zones.

[9] In the case of a fixed currency peg, interest rates are an important price signal that can be manipulated by central banks.

exercise in the interaction of asset-price manipulation and central bank transparency.

Bannier and Heinemann (2005) examine the effects of central bank transparency in the context of currency crises and global games. Their main conclusion is that transparency helps prevent a crisis when prior beliefs about fundamentals are pessimistic since transparency causes agents to place greater weight on their private information when forming expectations. While this chapter's results do share some of this same logic, my emphasis is primarily on the interaction between partial information revelation and deviations of asset prices from their fundamental values.

Chamley (2003) develops a model in which speculators learn about fundamentals by observing the exchange rate move within a target band. He examines how speculators' ability to coordinate an attack against this band is affected by the information present in the exchange rate, and concludes that any central bank policy that reduces exchange rate volatility facilitates such coordination. Once again, my results do share some of this same logic, but my emphasis is on partial information revelation and asset mispricing rather than coordination.

Bond and Goldstein (2011) present a model in which the government intervenes to help firms with weak fundamentals. They investigate how different intervention and transparency policies affect price misalignment and find that price-based trading rules usually worsen this misalignment. While the authors do discuss the potentially deleterious effects of transparency, their emphasis is primarily on the benefits of government actions that rely on private rather than public information.

The chapter is organized as follows. Section 2 presents the benchmark two-period model and the main results about central bank transparency. Section 3 extends the benchmark model to analyze how these results change once interventions can depend on both exchange rate misalignment and fundamentals. Section 4 concludes. The proofs for all of the results are provided in the last section.

2 Benchmark Two-Period Model

There are two periods, $t \in \{1, 2\}$, and two countries, home and foreign. I shall refer to the home country's currency as the dollar and the foreign country's currency as the peso. There is only one good and its price in each country is linked by the law of one price, so that $e_t + p_t^* = p_t$ in each period t, where p_t is the log of the price of the good in the home country, p_t^* is the log of the price of the good in the foreign country, and e_t is the log of the nominal exchange rate, which is defined as the dollar price of one peso.

Three assets are traded in this economy: a nominal one-period bond issued by the domestic central bank with return i_1, a nominal one-period bond issued by the foreign central bank with return i_1^*, and a risk-free technology with real return r. The payoffs of all assets are realized in period two. I assume that the domestic central bank credibly commits to a constant domestic price level in all periods so that the interest rate on dollar bonds i_1 is equal to r. Without loss of generality, this constant price level is normalized so that $p_1 = p_2 = 0$, which implies that the log-linearized real return on foreign bonds is equal to $-p_2^* - e_1 + i_1^* = e_2 - e_1 + i_1^*$. In the foreign country, the interest rate in period one is given by $i_1^* = \mu + r$, where $\mu \in \mathbb{R}$. All investors observe i_1, i_1^*, and e_1 publicly in period one.

In this benchmark model, the exchange rate in period two is exogenously given by

$$e_2 = f + \kappa, \tag{2.1}$$

where $f \in \mathbb{R}$ represents exchange rate fundamentals in period two and $\kappa \sim \mathrm{N}(0, \sigma_\kappa^2)$ is a shock to the exchange rate in period two.[10] The infinite-horizon extension of this model presented in Section 7 gives a more precise meaning to the parameters f and κ. In that model, exchange rate fundamentals are equal to the time-discounted sum of spreads between foreign and domestic interest rates plus the time-discounted sum of risk premia, with the discount factor determined by the structure of the foreign central bank's interest rate rule.[11] The shock to the exchange rate is then the sum of the innovations in the stochastic processes for the foreign central bank's interest rates and purchases of peso bonds.

The economy is populated by a continuum of investors indexed by $i \in [0, 1]$. Each investor is endowed with real wealth $w_i \in \mathbb{R}_{++}$ at the beginning of period one and has negative exponential utility (CARA) over her consumption in period two. Because the log-linearized excess return of peso bonds is equal to $e_2 - e_1 + i_1^* - i_1 = e_2 - e_1 + \mu$, the maximization problem solved by each investor i is given by

$$\max_{b_i \in \mathbb{R}} -E_{i1} \exp\{-\gamma c_{i2}\}, \qquad \text{subject to} \quad c_{i2} = (1 + i_1)w_i + (e_2 - e_1 + \mu)b_i, \tag{2.2}$$

where b_i is the dollar amount of investor i's purchases of peso bonds in period one, c_{i2} is the quantity of the economy's only good consumed by investor i in period two, $\gamma > 0$ is the coefficient of absolute risk aversion, and $E_{i1}[\cdot]$ denotes the conditional expectation with respect to the information set of investor i in period one. In addition to the investors, the economy is also populated by a mass of noise traders that purchases ξ dollars worth of peso bonds in period one, where $\xi \sim \mathrm{N}(0, \sigma_\xi^2)$. The net supply of peso bonds is equal to zero.

The foreign central bank complements its interest rate policy in period one with a foreign exchange intervention in which it purchases $\nu \in \mathbb{R}$ dollars worth of peso bonds. This intervention affects the exchange rate in period one since it changes the total demand for peso bonds in that period. In period two, the relationship between the exchange rate and

[10] An alternative interpretation of κ is that it represents the part of fundamentals in period two that cannot be predicted or known in period one. This does not change any of the model's predictions.

[11] In a standard dynamic monetary model, fundamentals are equal to the time-discounted sum of future values of the foreign money supply (relative to the domestic, constant money supply), with the discount factor determined by the semi-elasticity of money demand with respect to the interest rate.

the central bank's intervention is more complex. I assume that exchange rate fundamentals in period two are given by

$$f = \theta_f f_0 + \theta_\nu f_\nu, \tag{2.3}$$

where $f_0 \in \mathbb{R}$ represents the part of fundamentals that is unrelated to the foreign central bank's intervention, $f_\nu \in \mathbb{R}$ represents the part of fundamentals that is related to the bank's intervention, and $\theta_f, \theta_\nu > 0$ are constants. The constant θ_ν measures the extent of the relationship between fundamentals and the central bank's intervention, with an increase (decrease) in θ_ν corresponding to a greater (lesser) connection between fundamentals and intervention. To keep this two-period model simple, I assume that the bank's intervention is equal to the part of fundamentals related to that intervention:

$$\nu = f_\nu. \tag{2.4}$$

Equation (2.4) implies that all of the foreign central bank's intervention in period one conveys information about fundamentals, but it is important to emphasize that the model's predictions do not change if this is generalized so that there is a noise term as part of the intervention.[12] The form of equations (2.1), (2.3), and (2.4) are common knowledge among all investors.

The relationship between exchange rate fundamentals in period two and the foreign central bank's intervention in period one as described by equations (2.3) and (2.4) merits some discussion. The most narrow interpretation of the constant θ_ν from these equations is that it measures only the time-discounted effect of persistent interventions on future risk premia (a determinant of fundamentals), and that interventions are unrelated to all other determinants of the exchange rate. This implies that interventions only have direct, portfolio-balance effects on the exchange rate and are useful as signals about only future intervention policy. In the infinite-horizon extension of this model presented in Section 7, I consider precisely this kind of setup.

The constant θ_ν captures more than just the direct effect of persistent central bank interventions, however. In particular, a higher value may also represent a partial correlation between other exchange rate fundamentals in period two and the bank's intervention in period one. For example, a large foreign exchange intervention may be a highly credible signal of the central bank's future macroeconomic policies (which affect exchange rate fundamentals), as emphasized by Dominguez and Frankel (1993a) and Mussa (1981). Even if an intervention is not a clear signal of future policies, it is still likely that the bank's choice of intervention is influenced by its beliefs about fundamentals and its future policy intentions. In this case, the intervention is still a source of information about future fundamentals as in the setups of Bhattacharya and Weller (1997) and Vitale (1999).

Because θ_ν measures the extent of the relationship between fundamentals and interven-

[12] Vitale (1999) starts from a central bank loss function and derives an optimal intervention rule that consists of one part that is a linear function of fundamentals as in equation (2.4) and another part that is a linear function of the bank's target value for the exchange rate. Because the bank's target is both uncorrelated with fundamentals and unknown to investors, this part of the intervention is like a noise term.

tion, it also measures the extent of information revelation about fundamentals when the foreign central bank publicly and credibly announces the value of ν. In particular, the more information about fundamentals that is contained in the bank's intervention, the more information about fundamentals that is revealed by publicizing that intervention. The central result I present from this two-period model states that information revelation must be large (θ_ν must be large) if transparency is to reduce exchange rate misalignment. This is because the truth-telling effect of transparency is increasing in the extent of information revelation, so that this effect is larger than the signal-precision effect once the information about fundamentals that is revealed by the central bank's intervention is sufficiently extensive.

In this benchmark model, I assume that investors have uninformative priors for f_0 and ν.[13] Each investor i receives private signals $x_i = f_0 + \epsilon_i$ and $y_i = \nu + \eta_i$ in period one, where $\epsilon_i \sim \mathrm{N}(0, \sigma_\epsilon^2)$, $\eta_i \sim \mathrm{N}(0, \sigma_\eta^2)$, ϵ_i and η_i are independent, and all noise terms are independent across investors. In equilibrium, investors rationally combine their private signals with the information about both f_0 and ν that is present in the exchange rate in period one.

Let $\mathcal{F}$ denote the information set consisting of all common public information together with f_0 and ν. The aggregate demand for peso bonds by the investors is equal to the average demand of the investors and is denoted by $B = E\left[b_i \mid \mathcal{F}\right]$.[14] It follows that the total demand for peso bonds in period one is equal to $B+\xi+\nu$. Let $\overline{E}_1[\cdot] = E\left[E_{i1}[\cdot] \mid \mathcal{F}\right]$ denote the average expectation of investors in period one, and let $\mathrm{Var}_{i1}[\cdot]$ denote the conditional variance with respect to the information set of investor i in period one and $\overline{\mathrm{Var}}_1[\cdot] = E\left[\mathrm{Var}_{i1}[\cdot] \mid \mathcal{F}\right]$ the average conditional variance of investors in period one. Finally, let $\sigma_1^2 = \overline{\mathrm{Var}}_1[e_2]$ denote the average conditional variance of the exchange rate in period two.

Definition 2.1. An equilibrium of this economy is a linear function for the exchange rate in period one e_1, such that: (i) the demand for peso bonds by each investor b_i solves the maximization problem (2.2), where investor i's information set consists of all common public information together with x_i, y_i, e_1, and, if the foreign central bank announces its intervention, ν as well; (ii) the peso bond market clears: $B + \xi + \nu = 0$; (iii) the exchange rate is a function of the demand for peso bonds by noise traders ξ, the foreign central bank's intervention ν, the interest rate parameter μ, and the fundamentals parameter f.

In this definition of equilibrium, the foreign central bank's transparency policy does not convey any information about the parameters of the model, an assumption that is essential in order to keep the analysis in this model tractable. I relax this assumption in Section 6 and investigate how signalling affects the equilibrium predictions of this model. All proofs from this section are in Section 5.

[13] An alternative but equivalent assumption is that investors' priors for f_0 and ν are uniform over $\mathbb{R}$.

[14] This notation is commonly written $B = \int_0^1 b_i \, di$, with the understanding that this integral is equal to the average across investors. As detailed by Judd (1985), however, the law of large numbers often does not hold for a continuum of random variables. I avoid this technical issue by explicitly defining continuums of this kind as the expected value of an individual investor's demand conditional on observing the parameters of the model.

Theorem 2.2. *The equilibrium exchange rate in period one is given by*

$$e_1 = \mu + f + \gamma\sigma_1^2\nu + \lambda\xi, \tag{2.5}$$

where λ *and* σ_1^2 *are such that*

$$\lambda = \frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma_1^2, \tag{2.6}$$

$$\sigma_1^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}. \tag{2.7}$$

The parameter λ in the expression for the equilibrium exchange rate from Theorem 2.2 is always positive and measures the magnitude of currency mispricing for any demand by noise traders ξ. An increase in λ corresponds to an increase in exchange rate misalignment, holding other terms constant.[15] A number of important properties of the equilibrium exchange rate stand out. First, the effects of noise traders on the exchange rate extend beyond the standard demand channel since $\lambda > \gamma\sigma_1^2$. In models with rational expectations and heterogeneously informed investors such as this, the equilibrium exchange rate is a publicly observable signal of both the part of future exchange rate fundamentals unrelated to the intervention f_0 and the part that is related to the intervention f_ν (or equivalently, the exchange rate is a signal of the central bank's intervention ν). Noise traders drive the exchange rate away from its fundamental value by altering the total demand for peso bonds, which then biases the average expectations of investors about both f_0 and f_ν. The difference between λ and $\gamma\sigma_1^2$ captures this extra effect and is exactly equal to the bias in investors' expectations.

A sketch of the proof of Theorem 2.2 illustrates this point. Market clearing implies that the exchange rate in period one is of the form $e_1 = \mu + \overline{E}_1[f] + \gamma\sigma_1^2(\nu + \xi)$. Solving for the equilibrium requires evaluating the average expectation $\overline{E}_1[f]$ and determining how much weight it places on the noise term ξ. This weight makes up the bias of investors' average expectations of fundamentals f. Evaluating this expectation is accomplished using standard Bayesian formulas. In particular, these formulas imply that for each investor i,

$$E_{i1}[f] = \theta_f x_i + \theta_\nu y_i + \frac{\mathrm{Cov}_i[f, e_1]}{\mathrm{Var}_i[e_1]}(e_1 - E_i[e_1]), \tag{2.8}$$

where $E_i[\cdot], \mathrm{Var}_i[\cdot]$, and $\mathrm{Cov}_i[\cdot,\cdot]$ denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of μ and the private signals x_i and y_i (no observation of e_1 in this information set). The exchange rate in period one is of the form $e_1 = \mu + f + \gamma\sigma_1^2\nu + \lambda\xi = \mu + \theta_f f_0 + (\theta_\nu + \gamma\sigma_1^2)\nu + \lambda\xi$, so it follows that $e_1 - E_i[e_1] = f - (\theta_f x_i + \theta_\nu y_i) + \gamma\sigma_1^2(\nu - y_i) + \lambda\xi$ and hence that $e_1 - \overline{E}_1[e_1] = \lambda\xi$. This last

[15] Note that λ also measures how informative the equilibrium exchange rate is, with a higher value of λ corresponding to a less informative exchange rate.

equality implies that

$$\overline{E}_1[f] = f + \frac{\mathrm{Cov}_i[f, e_1]}{\mathrm{Var}_i[e_1]}\lambda\xi, \tag{2.9}$$

so that the bias of investors' average expectations is equal to the last term in equation (2.9). This term reflects the fact that the exchange rate in period one contains information about f (since $\mathrm{Cov}_i[f, e_1]$ is nonzero) and thus its value contributes to equilibrium expectations.

For most parameterizations of this model, λ is increasing in both the variance of investors' private signals about future fundamentals σ_ϵ and the extent of the relation between exchange rate fundamentals and the central bank's intervention θ_ν. An increase in the unpredictability of noise traders σ_ξ can either increase or decrease the value of λ, although the magnitude of this response tends to be significantly smaller than the response to an increase in σ_ϵ or θ_ν.

The fact that λ is decreasing in the precision of investors' private signals about f_0 implies that the precision of the exchange rate as a public signal of f_0 is increasing in this precision. This follows because the term λ multiplies ξ in equation (2.6), so a decrease in λ implies a decrease in the variance of the exchange rate assuming that σ_ξ remains unchanged. Intuitively, this increase in precision is a consequence of investors with better private information trading more aggressively and moving the value of the exchange rate closer to its fundamental value, a property examined by Angeletos and Werning (2006) in a model of asset-pricing with heterogeneous private information similar to this one.

The effect of an increase in the variance of investors' private signals about the foreign central bank's intervention in period one σ_η are the most interesting. As I shall prove in Theorem 2.4 below, if the parameter λ is greater than the corresponding parameter when the central bank makes a public announcement about ν (denoted by $\tilde{\lambda}$), then this must be the case for all $\sigma_\eta > 0$. In other words, if the bias in investors' average expectations is larger (smaller) with transparency than without transparency, then this bias must be larger (smaller) regardless of the precision of investors' signals about central bank interventions. I also find that λ is increasing in σ_η whenever $\lambda > \tilde{\lambda}$ and decreasing in σ_η whenever $\lambda < \tilde{\lambda}$. This implies that decreases in the precision of investors' signals about interventions always magnify the difference in exchange rate misalignment with and without transparency.

In order to examine the effects of transparency on the price of the peso, it is necessary to solve for the equilibrium exchange rate when the central bank credibly and publicly announces the value of ν in period one. Let $\tilde{e}_1$ denote the exchange rate in period one if the central bank truthfully announces the value of ν to the investors.

Theorem 2.3. *If the foreign central bank credibly and publicly announces the value of ν in period one, then the equilibrium exchange rate is given by*

$$\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2\nu + \tilde{\lambda}\xi, \tag{2.10}$$

where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are such that

$$\tilde{\lambda} = \frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}_1^2, \tag{2.11}$$

$$\tilde{\sigma}_1^2 = \theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2 - \frac{\theta_f^4\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}. \tag{2.12}$$

In contrast to the system of equations from Theorem 2.2, this system of equations is simple enough to solve analytically. In the equilibrium with transparency, the effects of noise traders on the exchange rate again extend beyond the standard demand channel and bias investors' average expectations of fundamentals. As in the equilibrium with no transparency, the difference between $\tilde{\lambda}$ and $\gamma\tilde{\sigma}_1^2$ captures this extra effect and is equal to the bias of investors' expectations. Furthermore, $\tilde{\lambda}$ is again positive and measures the magnitude of exchange rate misalignment with fundamentals, so it follows that any time $\tilde{\lambda} > \lambda$ transparency magnifies this misalignment. The final step is to compare the values of the parameters λ and $\tilde{\lambda}$ and examine when this inequality holds.

Theorem 2.4. *There exists a unique threshold $\hat{\theta}_\nu > 0$ such that $\tilde{\lambda} > \lambda$ if and only if $\theta_\nu < \hat{\theta}_\nu$. This threshold is given by $\hat{\theta}_\nu = \tilde{\lambda} - \gamma\tilde{\sigma}_1^2$, and satisfies*

$$\lim_{\sigma_\xi \to 0} \hat{\theta}_\nu = \infty, \qquad \lim_{\sigma_\xi \to \infty} \hat{\theta}_\nu = 0,$$

$$\lim_{\sigma_\kappa \to 0} \hat{\theta}_\nu = \frac{\gamma^2\theta_f^2\sigma_\epsilon^2}{1 + \gamma^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2}, \qquad \lim_{\sigma_\kappa \to \infty} \hat{\theta}_\nu = 0,$$

$$\lim_{\theta_f \to 0} \hat{\theta}_\nu = 0, \qquad \lim_{\theta_f \to \infty} \hat{\theta}_\nu = \frac{1}{\gamma\sigma_\xi^2},$$

$$\lim_{\gamma \to 0} \hat{\theta}_\nu = 0, \qquad \lim_{\gamma \to \infty} \hat{\theta}_\nu = 0.$$

Corollary 2.5. *If $\theta_\nu < \hat{\theta}_\nu$, then there exists a threshold $\hat{\xi} \in \mathbb{R}$ such that $\tilde{e}_1 < e_1$ if and only if $\xi < \hat{\xi}$.*

Theorem 2.4 and Corollary 2.5 together present the main results of all of the models and extensions presented in this chapter. The theorem states that exchange rate misalignment is magnified by transparency ($\tilde{\lambda} > \lambda$) whenever the information content of the central bank's intervention is sufficiently limited ($\theta_\nu < \hat{\theta}_\nu$). Because transparency also affects the peso risk premium (usually by lowering it), this magnification must be significant enough to outweigh this change in the risk premium if an announcement is to depreciate the peso. The corollary describes precisely when this magnification is significant enough. It states that transparency depreciates the exchange rate relative to ambiguity whenever the peso is sufficiently undervalued relative to fundamentals. Theorem 2.4 and Corollary 2.5 together imply that exchange rate undervaluation together with transparency can in fact magnify currency mispricing and reduce the effectiveness of foreign exchange interventions intended to move the exchange rate closer to its fundamental value.

This result has implications for policy during times of crisis. In these episodes, asymmetric information, pro-cyclical liquidity provision, and psychology often lead to excessive sales of risky assets, as shown by Brunnermeier and Pedersen (2009) and Shleifer and Vishny (1997). This translates to a negative value of ξ in this benchmark model, so that if an intervention does not contain much information about future policies and fundamentals ($\theta_\nu < \hat{\theta}_\nu$), Corollary 2.5 implies that a public announcement about that intervention often depreciates the exchange rate. In this case, the central bank can achieve a higher exchange rate if it does not publicly announce the size of its intervention.

Theorem 2.4 implies that it is only if the information revealed by a public announcement of the foreign central bank's intervention is sufficiently incomplete ($\theta_\nu < \hat{\theta}$) that exchange rate misalignment may be magnified by transparency. Recall the two distinct effects of transparency: the *truth-telling effect*, which reduces currency mispricing, and the *signal-precision effect*, which magnifies currency mispricing. The truth-telling effect refers to the fact that any parameters the central bank reveals to investors eliminate the role of the exchange rate as a signal of those parameters. The signal-precision effect refers to the fact that any parameters the bank reveals to investors also increase the precision of the exchange rate as a signal of other, still-unknown parameters. Theorem 2.4 states that it is precisely when information revelation is incomplete that the truth-telling effect of transparency is small relative to the signal-precision effect of transparency. If information revelation is complete ($\theta_\nu > \hat{\theta}_\nu$), on the other hand, Theorem 2.4 implies that the truth-telling effect will exceed the signal-precision effect and transparency will lessen exchange rate misalignment. While this analysis ignores the effect that transparency has on the conditional variance of the exchange rate in period two (which is part of the peso bond risk premium $\gamma\sigma_1^2$), it captures the essence of how transparency affects the equilibrium outcome of the model.[16]

The behavior of λ relative to $\tilde{\lambda}$ is shown graphically in Figures 2.1, 2.2, 2.3, and 2.4. The baseline parameterization shown in Figure 2.1 is chosen to match the baseline parameterization of the richer dynamic model of Section 7 (shown in Figure 7.1 of that section). Figure 2.2 presents this same parameterization except that the variance of investors' private signals about the central bank's intervention σ_η is smaller. This has the effect of bringing λ and $\tilde{\lambda}$ closer together without changing the threshold $\hat{\theta}_\nu$ (the point where the two lines intersect). Figure 2.3 presents the same parameterization as in Figure 2.2 except that now the unpredictability of noise traders σ_ξ is smaller. This has the effect of increasing both λ and $\tilde{\lambda}$ and increasing the threshold $\hat{\theta}_\nu$. Finally, Figure 2.4 presents the same parameterization as in Figure 2.3 except that now a smaller part of fundamentals is unrelated to the central bank's intervention (θ_f is smaller). This has the effect of decreasing both λ and $\tilde{\lambda}$ and decreasing the threshold $\hat{\theta}_\nu$.

A more detailed discussion of the truth-telling and signal-precision effects of transparency is warranted. If the foreign central bank credibly and truthfully announces the value of its

[16] If $\tilde{\lambda} > \lambda$, then transparency sometimes increases this conditional variance by increasing the noise in e_1.

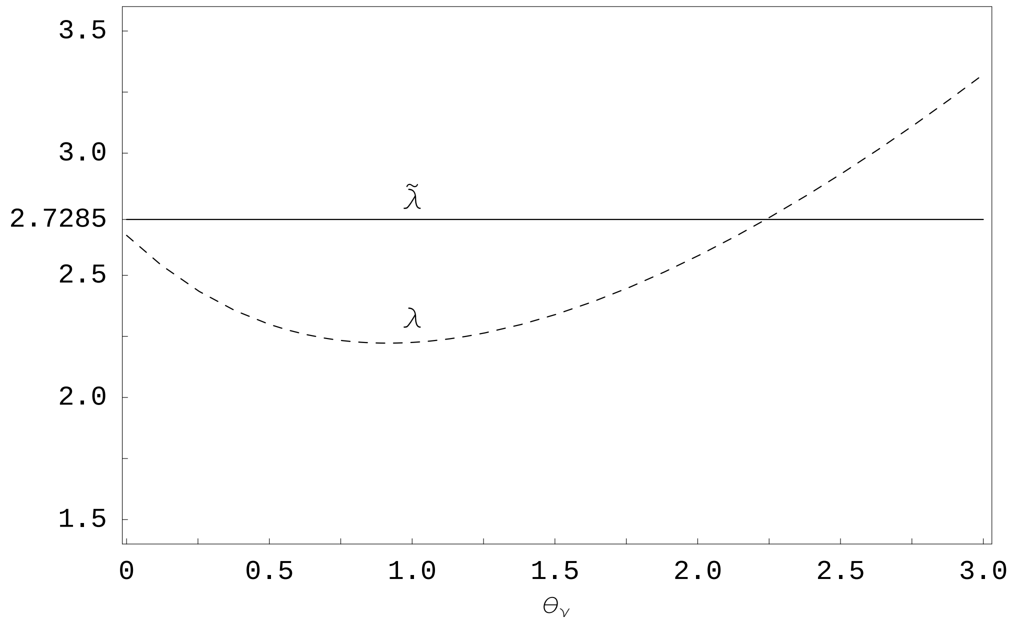

Figure 2.1: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the level of information revelation θ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.35$, $\sigma_\xi = 0.12$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 2$)

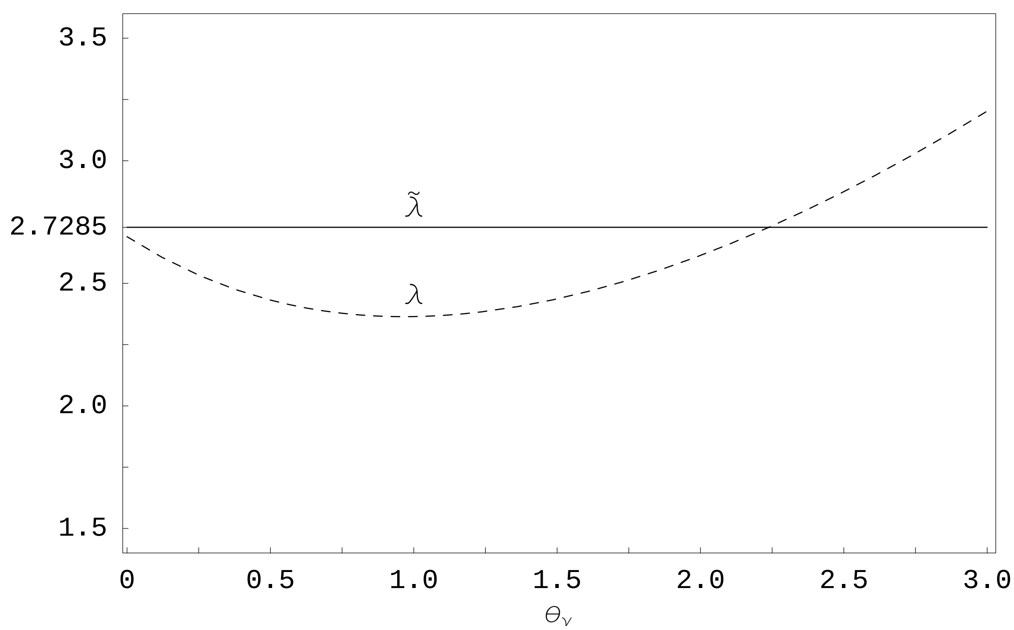

Figure 2.2: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the level of information revelation θ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.12$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 2$)

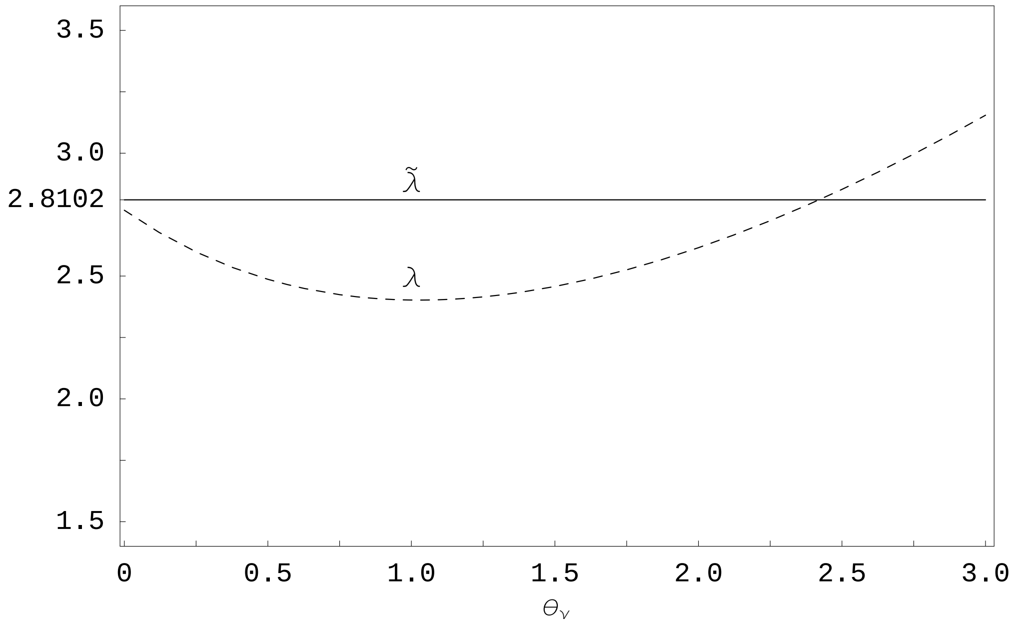

Figure 2.3: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the level of information revelation θ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.10$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 2$)

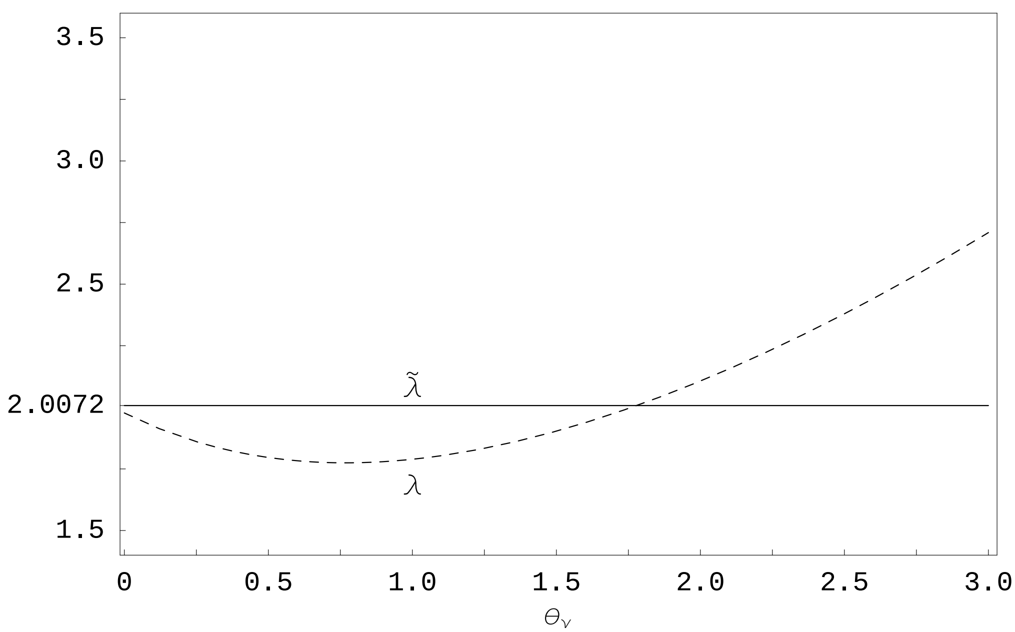

Figure 2.4: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the level of information revelation θ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.10$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 1.6$)

intervention ν in period one, then investors all perfectly learn both this value and the value of the part of fundamentals correlated with the intervention f_ν. This implies that they no longer form expectations of f_ν as part of their expectations of the fundamental value of the peso, so that $\mathrm{Cov}_i[f, e_1]$ becomes smaller and the multiplier on the noise traders' demand ξ in the average expectation $\overline{E}_1[f]$ decreases, as shown by equation (2.9). If the demand of noise traders is negative ($\xi < 0$), then the exchange rate is undervalued and investors' expectations of fundamentals f are biased downwards. In this case, learning f_ν eliminates some of this bias and causes investors' expectations to increase and approach the true value of f. This is the truth-telling effect of transparency.

In addition to revealing f_ν to investors, a foreign central bank announcement increases the precision of the exchange rate in period one as a signal of the part of fundamentals that is not related to this intervention f_0. This means that $\mathrm{Var}_i[e_1]$ also becomes smaller, which increases the multiplier on the noise traders' demand ξ in the average expectation $\overline{E}_1[f]$ (again by equation (2.9)). This is simply a consequence of rational Bayesian investors placing a greater weight on a more precise signal when forming their beliefs about these fundamentals, and the implication is that some of the bias of investors' expectations of f (specifically, the bias of expectations of f_0) actually is magnified after a central bank announcement. If the demand of noise traders is negative (and hence this bias is negative), then learning the value of ν causes investors' expectations to decrease further away from the true value of f. This is the signal-precision effect of transparency.

Consider two special cases. First, in the limit as $\theta_\nu \to 0$, the foreign central bank's intervention in period one neither directly affects nor conveys any information about exchange rate fundamentals in period two. This intervention introduces only noise into the exchange rate in period one. In this case, learning the value of ν tells investors nothing about fundamentals f and eliminates none of the bias of investors' expectations of f, but it does increase the precision of e_1 as a signal of f. This means that there is no truth-telling effect and only a signal-precision effect of transparency. Theorem 2.4 confirms that this is indeed the case, since the threshold $\hat{\theta}_\nu$ is always positive and hence $\theta_\nu < \hat{\theta}_\nu$ and $\tilde{\lambda} > \lambda$ once θ_ν is sufficiently close to zero.

Second, in the limit as $\theta_f \to 0$, the foreign central bank's intervention in period one fully reveals all future exchange rate fundamentals (since f_ν becomes all of fundamentals). Much of the early literature about the signalling hypothesis, such as Dominguez and Frankel (1993a) and Mussa (1981), posits an environment similar to this special case when arguing that transparency is desirable and can effectively reduce exchange rate misalignment. Theorem 2.4 demonstrates that this benchmark model is consistent with these authors' analysis, since $\hat{\theta}_\nu \to 0$ as $\theta_f \to 0$ and θ_ν is positive by assumption. It is important to emphasize, however, that as the information about future fundamentals that is embedded in the central bank's intervention declines, the benefits of transparency become more tenuous.

An important implication of Theorem 2.4 is that whether or not transparency magnifies exchange rate misalignment does not depend on the variance of investors' private signals about central bank interventions σ_η (this is shown in Figure 2.2). This follows because the threshold $\hat{\theta}_\nu$ is only a function of the exchange rate parameters $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$, which do not depend on σ_η since they correspond to a central bank policy of transparency (and hence $\sigma_\eta = 0$). As mentioned earlier, an important consequence of this is that changes in the precision of investors' private signals of ν cannot swing the balance between the truth-telling and signal-precision effects of transparency. More precisely, if $\lambda > \tilde{\lambda}$ or $\lambda < \tilde{\lambda}$, then this relationship must hold for all $\sigma_\eta > 0$.

In fact, I find that increases in σ_η tend to magnify the difference between the parameters λ and $\tilde{\lambda}$. Because $\tilde{\lambda}$ does not change as σ_η increases, this implies that λ is increasing in σ_η whenever $\lambda > \tilde{\lambda}$ (and hence $\theta_\nu > \hat{\theta}_\nu$) and decreasing in σ_η whenever $\lambda < \tilde{\lambda}$ (and hence $\theta_\nu < \hat{\theta}_\nu$). These properties are shown in Figure 2.5. This result is significant because it implies that all of this model's predictions about transparency and exchange rate misalignment apply even when the foreign central bank reduces rather than eliminates the variance of investors' private signals about interventions. In reality, rather than choosing between full transparency and full ambiguity, central banks choose from a set of different policies that are distinguished by their overall effect on the level of transparency.

This benchmark model formalizes the intuitive but vague justifications that central banks often provide for their ambiguous policies. Theorem 2.4 shows that banks are right to worry that unsuccessful transparent interventions might undermine the market's confidence in their currencies, since transparency makes it easier for investors with different beliefs to learn each others' information and hence for pessimism to intensify and spread. In other words, if investors observe a depreciated currency together with an extensive intervention, then they conclude that fundamentals are worse than they previously thought. This reasoning implies that both Mexico and Russia would have likely benefited from more ambiguous intervention policies during the financial crisis, and it provides an explanation for why Mexico and Russia eventually made such a policy switch.

The model provides two key insights that guide this intuition of the central banks. First, it is only if the information that banks reveal to the public is sufficiently partial that transparency can magnify exchange rate misalignment. If central banks can credibly reveal enough information about fundamentals, then transparency is usually stabilizing and will tend to reduce currency misalignment. This highlights the importance of a central bank's ability to reassure markets by making credible public announcements about current and future policies. Second, if transparency does magnify exchange rate misalignment, then ambiguity appreciates only an undervalued currency. This observation highlights the importance of the information advantage of central banks. In a world with rational expectations, it is only if a currency is undervalued that ambiguity can increase the effectiveness of an intervention designed to appreciate that currency. If this model is interpreted literally, then it is natural to assume that the foreign central bank has more information about fundamentals than the investors since fundamentals are entirely determined by the bank's policies. In a more realistic and complete model of exchange rate determination, however, there are many other components of exchange rate fundamentals that central banks are not necessarily more informed about.

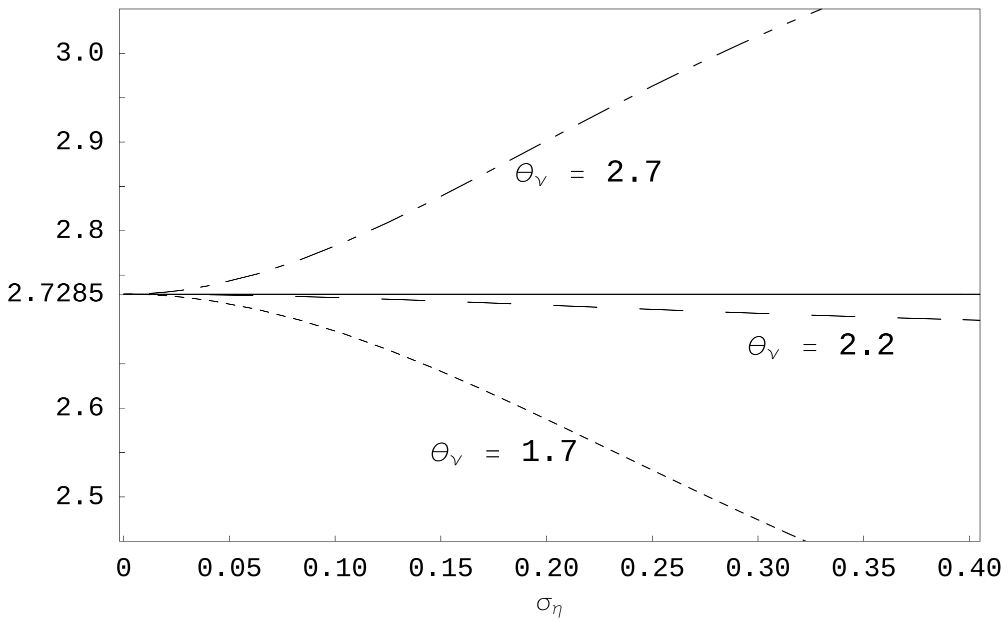

Figure 2.5: The value of λ as private uncertainty about interventions σ_η increases. ($\sigma_\epsilon = 0.35$, $\sigma_\xi = 0.12$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 2$)

Finally, I should emphasize that this model does not imply that an ambiguous intervention policy is always better than a transparent intervention policy. In fact, a transparent intervention policy is often better even if the conditions of Theorem 2.4 hold and $\tilde{\lambda} > \lambda$. This is because central bank policy is an important determinant of currency risk premia and transparency can be an effective way to reduce these risk premia. The purpose of my analysis is to examine and emphasize a mechanism by which transparency can in fact exacerbate exchange rate misalignment, rather than to capture all of the factors that affect exchange rates. While this mechanism is likely to be very important during times of great uncertainty about policy and fundamentals, it is unlikely to be as important during more normal times.

3 Intervention and Exchange Rate Misalignment

Throughout the benchmark model, I assume that the foreign central bank's intervention is only a function of some part of exchange rate fundamentals. This assumption simplifies the analysis and is sufficient to present the main results of this chapter and to develop the underlying logic and intuition. In reality, however, a central bank will often take into account more than just its knowledge about fundamentals when choosing how extensively to intervene in the foreign exchange market. An intervening bank is usually also concerned with the value of the exchange rate and the possible presence of misalignment.

There are several reasons why a central bank might intervene in response to movements in the exchange rate. One possibility is that the bank targets some specific value for the exchange rate, as in the intervention models of Bhattacharya and Weller (1997) and Vitale (1999). Another possibility is that the bank's objective is to resist exchange rate misalignment, as in the last section of Chamley (2003). A third possibility is that the central bank wishes to disseminate aggregated private information about exchange rate disturbances generated by noise traders, as in the setup of Popper and Montgomery (2001). Finally, a fourth possibility is that the bank learns about fundamentals from movements in the exchange rate and intervenes based on this learning, as in Bond and Goldstein (2011) and Goldstein, Ozdenoren, and Yuan (2011). Regardless of the underlying motivation, however, the implication is always that intervention is a function of both exchange rate fundamentals and exchange rate misalignment.[17]

This section extends the benchmark model of Section 2 to include foreign exchange interventions that respond to movements in the exchange rate. I focus primarily on the implications of central bank transparency, with the goal of investigating the robustness of the previous section's results about public announcements and exchange rate misalignment. In this setup, the foreign central bank's intervention ν is both a function of part of exchange rate fundamentals, f_ν, and the noise traders' demand for peso bonds in period one, ξ. As in the previous section, the exchange rate in period two is given by $e_2 = f + \kappa$ (equation 2.1) and exchange rate fundamentals are separated into two parts so that $f = \theta_f f_0 + \theta_\nu f_\nu$ (equation 2.3). In this extended model, however, the foreign central bank's intervention in period one is given by

$$\nu = a_\nu f_\nu + a_\xi \xi, \tag{3.1}$$

where the constants a_ν and a_ξ are such that $a_\nu > 0$ and $-1 < a_\xi < 0$. The assumption that $a_\nu > 0$ captures the reality that a central bank's choice of foreign exchange intervention is generally positively correlated with some part of exchange rate fundamentals. As described in Section 2, this positive correlation can be the consequence of either interventions that are affected by information about fundamentals (Bhattacharya and Weller 1997, Vitale 1999), interventions that are credible signals about future monetary policy (Mussa 1981), or interventions that permanently alter currency risk premia. The assumption that $-1 < a_\xi < 0$ reflects a focus on interventions that reduce exchange rate misalignment. This is not an important restriction, and the model is easily extended to consider the possibility that $a_\xi > 0$. The form of equation (3.1) is common knowledge among all investors.

As in Section 2, the goal is to examine how a credible and truthful public announcement about ν affects exchange rate misalignment. Because ν is no longer simply equal to f_ν in

[17]This assumes that a central bank is not restricted to placing only market orders that cannot depend on the exchange rate, as in the setup of Vitale (1999). Indeed, in order for a foreign exchange intervention to be a function of misalignment, a bank must be able to observe the exchange rate before choosing the size of its intervention.

this section's setup, it is necessary to clarify what private signals the investors observe. In particular, I assume that each investor i receives private signals $x_i = f_0 + \epsilon_i$ and $y_i = f_\nu + \eta_i$ in period one, where $\epsilon_i \sim \mathrm{N}(0, \sigma_\epsilon^2)$, $\eta_i \sim \mathrm{N}(0, \sigma_\eta^2)$, ϵ_i and η_i are independent, and all noise terms are independent across investors. This is equivalent to the benchmark model because in that model ν is equal to f_ν and investors observe private signals about ν (which they know are signals about f_ν, as well). As a consequence, if $a_\nu = 1$, then this section's setup becomes identical to the benchmark setup in the limit as $a_\xi \to 0$, as I demonstrate below.

The definition of an equilibrium exchange rate in this setup is the same as definition 7.1 from Section 2. I also use the same notation, so that e_1 denotes the exchange rate in period one in the absence of a central bank announcement about ν and $\tilde{e}_1$ denotes the exchange rate in period one if there is such an announcement. In addition, this section adopts the previous section's assumptions about investors' preferences and about dollar and peso bonds (so that the log-linearized excess return of peso bonds is equal to $e_2 - e_1 + \mu$). All proofs from this section are in Section 5.

Theorem 3.1. *The equilibrium exchange rate in period one is given by*

$$e_1 = \mu + f + \gamma\sigma_1^2\nu + \lambda\xi, \tag{3.2}$$

where λ and σ_1^2 are given by the solution to

$$\lambda = \frac{\theta_f^2(a_\xi\gamma\sigma_1^2 + \lambda)\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma_1^2)(a_\xi\gamma\sigma_1^2 + \lambda)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2} + \gamma\sigma_1^2, \tag{3.3}$$

$$\sigma_1^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma_1^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2}. \tag{3.4}$$

A simple comparison of equations (2.6) and (2.7) from Theorem 2.2 with equations (3.3) and (3.4) from Theorem 3.1 shows that if $a_\nu = 1$, then in the limit as $a_\xi \to 0$ the equilibrium exchange rate in this setup converges to the equilibrium exchange rate in the benchmark setup. The parameter λ in the equilibrium exchange rate equation (3.2) is always positive and measures the magnitude of exchange rate misalignment for any demand by noise traders ξ, taking the foreign central bank's intervention ν as given. Because this intervention is a function of both f_ν and ξ (recall from equation 3.1 that $\nu = a_\nu f_\nu + a_\xi\xi$, with $a_\nu > 0$ and $-1 < a_\xi < 0$), the total misalignment of the exchange rate is in fact equal to $(\lambda + a_\xi\gamma\sigma_1^2)\xi < \lambda\xi$. I focus primarily on the parameter λ rather than $\lambda + a_\xi\gamma\sigma_1^2$, however, because λ captures the extent of misalignment that exists absent the direct effect of intervention. Indeed, it is obvious that a more extensive intervention more effectively reduces exchange rate misalignment. The more challenging and interesting question is how to maximize the effectiveness of this intervention, holding its size constant. This is answered by examining the parameter λ.

Misalignment-dependent interventions have both direct and indirect effects on exchange rate misalignment. The direct effect refers to the fact that any purchase or sale of peso bonds alters the risk premium and reduces the overall misalignment from $\lambda\xi$ to $(\lambda + a_\xi \gamma\sigma_1^2)\xi$. The indirect effect refers to the fact that by directly altering misalignment, any purchase or sale of peso bonds also alters the precision of the exchange rate as a signal of fundamentals and hence affects the misalignment that arises from investors' biased expectations. Consider the market clearing condition, which yields an equilibrium exchange rate of the form $e_1 = \mu + \overline{E}_1[f] + \gamma\sigma_1^2(\nu + \xi)$. By equation (3.1), this implies that $e_1 = \mu + \overline{E}_1[f] + \gamma\sigma_1^2 a_\nu f_\nu + \gamma\sigma_1^2(1 + a_\xi)\xi$. To solve for the misalignment arising from investors' biased expectations, it is necessary to evaluate the average expectation $\overline{E}_1[f]$ and determine how much weight it places on the noise term ξ. This weight tends to decrease as the quantity $\gamma\sigma_1^2(1 + a_\xi)$ decreases, making investors' expectations less biased. In the limit as $a_\xi \to -1$, the central bank eliminates all of both the bias in investors' expectations and the misalignment in the exchange rate.[18]

Interestingly, however, investors do not learn about fundamentals perfectly in the limit as $a_\xi \to -1$. This is a consequence of the foreign central bank's intervention being a function of the two unknown quantities f_ν and ξ. Indeed, even though $e_1 \to \mu + f + \gamma\sigma_1^2 a_\nu f_\nu$, so that the exchange rate is no longer affected by the noise traders' demand for peso bonds, investors still cannot perfectly learn f by observing e_1 since they do not know the value of f_ν. Perhaps surprisingly, this incomplete learning result remains true even if the bank announces its intervention. The next step is to solve for the equilibrium exchange rate in period one if the foreign central bank truthfully announces the value of ν to the investors.

Theorem 3.2. *If the foreign central bank credibly and publicly announces the value of ν in period one, then the equilibrium exchange rate in period one is given by*

$$\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2\nu + \tilde{\lambda}\xi, \tag{3.5}$$

where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by the solution to

$$\tilde{\lambda} = \frac{\tilde{\lambda} a_\nu^2 \theta_f^2 \sigma_\epsilon^2 \sigma_\eta^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2} + \gamma\tilde{\sigma}_1^2, \tag{3.6}$$

$$\tilde{\sigma}_1^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{a_\nu^2\theta_f^2(\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2)\sigma_\epsilon^2\sigma_\eta^2 + \left(a_\xi\theta_f^2\sigma_\epsilon^2 + \theta_\nu(a_\xi\theta_\nu - a_\nu\tilde{\lambda})\sigma_\eta^2\right)^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2}. \tag{3.7}$$

In this equilibrium exchange rate with transparency, the effects of noise traders on the exchange rate again extend beyond the standard demand channel and bias investors' average expectations about fundamentals. As always, the difference between $\tilde{\lambda}$ and $\gamma\tilde{\sigma}_1^2$ measures the extent of this bias. Equations (3.6) and (3.7) show that if $a_\nu = 1$, then in the limit as $a_\xi \to 0$ the equilibrium exchange rate of Theorem 3.2 converges to the equilibrium exchange rate

[18] Note that the elimination of all misalignment implies that $\lambda + a_\xi\gamma\sigma_1^2 \to 0$ and hence that λ converges to $\gamma\sigma_1^2 > 0$ as $a_\xi \to -1$.

of Theorem 2.3. Furthermore, these equilibrium equations indicate that $\tilde{\lambda}$ actually grows larger than $\gamma\tilde{\sigma}_1^2$ as $a_\xi \to -1$. This occurs because the foreign central bank's intervention ν is a function of ξ and hence investors base their expectations about fundamentals on this disturbance (because they base their expectations on ν) once the intervention is revealed to them. The final step is to compare the values of the misalignment parameters λ and $\tilde{\lambda}$.

Theorem 3.3. *There exist thresholds $\hat{\theta}_\nu > 0$, $\widehat{a\sigma} > 0$, $\hat{\sigma}_\eta > 0$, and $\hat{a}_\nu > 0$ such that:*

(i) if $\theta_\nu < \hat{\theta}_\nu$ and $a_\nu \sigma_\eta > \widehat{a\sigma}$, then $\tilde{\lambda} > \lambda$.

(ii) if $\sigma_\eta < \hat{\sigma}_\eta$, then $\tilde{\lambda} < \lambda$.

(iii) if $a_\nu < \hat{a}_\nu$, then $\tilde{\lambda} < \lambda$.

Theorem 3.3 extends the results from Theorem 2.4 in the previous section and presents this section's main results about central bank transparency and exchange rate misalignment. The first part of the theorem states that exchange rate misalignment is magnified by transparency whenever the information content of the central bank's intervention is sufficiently limited ($\theta_\nu < \hat{\theta}_\nu$), and both the investors' private information about the part of fundamentals related to the bank's intervention is sufficiently imprecise and the bank's intervention is sufficiently related to fundamentals ($a_\nu \sigma_\eta > \widehat{a\sigma}$). This result establishes that central bank announcements that reveal little information about fundamentals are likely to exacerbate misalignment, even when those announcements reveal some direct information about that misalignment. Clearly, this conclusion is consistent with the discussion about the potentially undesirable effects of transparency in Section 2.

The second and third parts of the theorem highlight the ways in which the effects of transparency change once the foreign central bank's intervention contains information about the demand of noise traders. In particular, the last two parts of the theorem state that exchange rate misalignment is reduced by transparency whenever either the investors' private information about the part of fundamentals related to the bank's intervention is sufficiently precise ($\sigma_\eta < \hat{\sigma}_\eta$) or the bank's intervention is sufficiently unrelated to fundamentals ($a_\nu < \hat{a}_\nu$). As a consequence, the last parts of Theorem 3.3 imply that a central bank announcement can reduce misalignment even if θ_ν is small and that announcement reveals little direct information about exchange rate fundamentals. Unlike the first part of the theorem, this result contrasts sharply with the previous section's discussion.

The contrasting results of Theorem 3.3 are a direct consequence of the foreign central bank's two-part intervention rule $\nu = a_\nu f_\nu + a_\xi \xi$. Given this rule, an announcement about ν reveals information about both f_ν and ξ without revealing the exact value of either one. As long as this announcement does not reveal precise information about either fundamentals or noise traders' demand for peso bonds, then it is the case that the signal-precision effect of transparency dominates the truth-telling effect as described in the previous section.[19] It

[19]Recall that exchange rate fundamentals are given by $f = \theta_f f_0 + \theta_\nu f_\nu$, so information about f_ν is also information about fundamentals.

is important to emphasize that an announcement that reveals precise information about ξ also reveals precise information about f. The exchange rate in period one is given by $\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2\nu + \tilde{\lambda}\xi$, so it follows that if investors learn both ν and ξ from the central bank's announcement, then they can effectively filter all of the noise out of the exchange rate and learn the value of f perfectly. In this case, the truth-telling effect of transparency dominates the signal-precision effect and transparency reduces exchange rate misalignment.

There are three different ways in which an announcement about ν can reveal precise information about f or ξ. One possibility is that fundamentals f are approximately equal to f_ν (because θ_ν is large relative to θ_f), so that information about f_ν is information about nearly all of f and a bank announcement can reduce exchange rate misalignment. This corresponds to the first part of Theorem 3.3, and the logic is the same as it was in Section 2. A second possibility, unique to this section's setup, is that the bank's intervention barely depends on fundamentals and is instead almost entirely a function of ξ. This corresponds to a scenario in which $a_\nu \to 0$ (the third part of Theorem 3.3), and the implication is that an announcement about ν becomes equivalent to an announcement about ξ. A third possibility, also unique to this section's setup, is that investors have very precise private information about f_ν and hence an announcement about ν again becomes equivalent to an announcement about ξ. This corresponds to a scenario in which $\sigma_\eta \to 0$ as in the second part of the theorem.

4 Conclusion

In this chapter, I have theoretically examined the implications of central bank transparency during foreign exchange interventions. The central feature of all my models is that investors are heterogeneously informed about both interventions and fundamentals. Information about future fundamentals is embedded in the current exchange rate so that investors learn about these fundamentals when they observe the price of foreign currency.

This chapter has identified and emphasized two distinct effects of transparency. The first is the truth-telling effect, which corresponds to the fact that any parameters the central bank reveals to investors eliminate the role of the exchange rate as a signal of those parameters. The second is the signal-precision effect, which corresponds to the fact that any parameters the central bank reveals to investors increase the precision of the exchange rate as a signal of other, still-unknown parameters. The truth-telling effect directly raises expectations of parameters for which average beliefs are too low, while the signal-precision effect indirectly lowers expectations of parameters for which average beliefs are too low. I find that the truth-telling effect grows relative to the signal-precision effect as the extent of information about fundamentals that is revealed by a transparent intervention policy increases.

The key implication of my analysis is that central bank transparency can in fact magnify any existing misalignment between the exchange rate and fundamentals. This occurs if a central bank can credibly reveal only partial information about fundamentals to market participants, so that the signal-precision effect of transparency is larger than the truth-telling effect of transparency. In effect, partial information revelation is worse than no information revelation, while full information revelation is best. This result implies that a policy of ambiguity will often increase the effectiveness of central bank intervention during periods of crisis and large capital outflows. In these episodes, asymmetric information, pro-cyclical liquidity provision, and psychology often lead to excessive sales of risky assets, causing risky countries' currencies to be undervalued and making it difficult to credibly reveal information about fundamentals. This prediction and the intuition behind it match well with the justification that central banks often provide for their ambiguous intervention policies.

Beyond foreign exchange intervention, this chapter considers general price manipulation and highlights a mechanism by which transparency can undermine the intended effect of that manipulation. While public information and transparency are normally desirable, I find that if they do not credibly communicate information about fundamentals and future policies, then the signal-precision effect of transparency may lead to undesirable outcomes. Given the ubiquity of price manipulation in today's economic environment, these subtler effects of transparency deserve further analysis and consideration.

5 Proofs

Proof of Theorem 2.2 Suppose that the exchange rate in period two is normally distributed conditional on investor i's information set. Then, the investors' problem (2.2) is a standard CARA-normal maximization problem, and the demand for peso bonds by investor i is given by

$$b_i = \frac{E_{i1}[e_2] - e_1 + \mu}{\gamma \operatorname{Var}_{i1}[e_2]}. \tag{5.1}$$

Suppose also that $\operatorname{Var}_{i1}[e_2]$ is equal for all $i \in [0,1]$ and hence that $\overline{\operatorname{Var}}_1[e_2] = \operatorname{Var}_{i1}[e_2]$. It follows that $\sigma_1^2 = \operatorname{Var}_{i1}[e_2]$ and that the aggregate investor demand for peso bonds in period one is given by

$$B = \frac{\overline{E}_1[e_2] - e_1 + \mu}{\gamma \sigma_1^2}, \tag{5.2}$$

which, together with the market clearing condition in the peso bond market, implies that

$$e_1 = \overline{E}_1[e_2] + \mu + \gamma \sigma_1^2 (\nu + \xi). \tag{5.3}$$

The exchange rate in period two is given by $e_2 = \theta_f f_0 + \theta_\nu f_\nu + \kappa$, so that $E_{i1}[e_2] = \theta_f E_{i1}[f_0] + \theta_\nu E_{i1}[\nu]$ (recall that $f_\nu = \nu$ by equation (2.4)). I am interested in the rational expectations equilibrium of this economy, so investors must take into account the fact that the value of the exchange rate in period one is a signal of both f_0 and ν. In other words, the exchange rate e_1 is part of investors' information sets in period one.

Let $E_i[\cdot], \text{Var}_i[\cdot]$, and $\text{Cov}_i[\cdot,\cdot]$ denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of μ and the private signals x_i and y_i. In equilibrium, the exchange rate in period one is of the form

$$e_1 = \mu + f + \gamma\sigma_1^2\nu + \lambda\xi = \mu + \theta_f f_0 + (\theta_\nu + \gamma\sigma_1^2)\nu + \lambda\xi, \tag{5.4}$$

so that $\text{Cov}_i[f_0, e_1] = \theta_f\sigma_\epsilon^2$ and $\text{Cov}_i[\nu, e_1] = (\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2$. The goal is to solve for the undetermined coefficients λ and σ_1^2 in equation (5.4). Standard Bayesian inference implies that the exchange rate in period two is normally distributed conditional on investor i's information set (this justifies the assumption of conditional normality) and that

$$\begin{aligned} E_{i1}[f_0] &= E_i[f_0] + \frac{\text{Cov}_i[f_0, e_1]}{\text{Var}_i[e_1]}(e_1 - E_i[e_1]) \\ &= x_i + \frac{\theta_f\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \mu - \theta_f x_i - (\theta_\nu + \gamma\sigma_1^2)y_i\right), \end{aligned}$$

and

$$\begin{aligned} E_{i1}[\nu] &= E_i[\nu] + \frac{\text{Cov}_i[\nu, e_1]}{\text{Var}_i[e_1]}(e_1 - E_i[e_1]) \\ &= y_i + \frac{(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \mu - \theta_f x_i - (\theta_\nu + \gamma\sigma_1^2)y_i\right). \end{aligned}$$

It follows, then, that

$$\overline{E}_1[f_0] = f_0 + \frac{\lambda\theta_f\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\xi, \tag{5.5}$$

and

$$\overline{E}_1[\nu] = \nu + \frac{\lambda(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\xi. \tag{5.6}$$

Substituting equations (5.5) and (5.6) into equation (5.3) above yields

$$\begin{aligned} e_1 &= \mu + \theta_f f_0 + (\theta_\nu + \gamma\sigma_1^2)\nu + \left(\frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma_1^2 \right) \xi \\ &= \mu + f + \gamma\sigma_1^2\nu + \left(\frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma_1^2 \right) \xi. \end{aligned} \tag{5.7}$$

The next step is to solve for σ_1^2, the conditional variance of the exchange rate in period two. Because $e_2 = \theta_f f_0 + \theta_\nu \nu + \kappa$, this conditional variance is given by $\sigma_1^2 = \theta_f^2 \overline{\mathrm{Var}}_1[f_0] + \theta_\nu^2 \overline{\mathrm{Var}}_1[\nu] + \sigma_\kappa^2 + 2\theta_f\theta_\nu \overline{\mathrm{Cov}}_1[f_0, \nu]$. As before, standard Bayesian inference implies that

$$\begin{aligned} \overline{\mathrm{Var}}_1[f_0] &= \mathrm{Var}_i[f_0] - \frac{\mathrm{Cov}_i[f_0, e_1]^2}{\mathrm{Var}_i[e_1]} = \sigma_\epsilon^2 - \frac{\theta_f^2\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}, \\ \overline{\mathrm{Var}}_1[\nu] &= \mathrm{Var}_i[\nu] - \frac{\mathrm{Cov}_i[\nu, e_1]^2}{\mathrm{Var}_i[e_1]} = \sigma_\eta^2 - \frac{(\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}, \end{aligned}$$

and that

$$\overline{\mathrm{Cov}}_1[f_0, \nu] = \mathrm{Cov}_i[f_0, \nu] - \frac{\mathrm{Cov}_i[f_0, e_1]\,\mathrm{Cov}_i[\nu, e_1]}{\mathrm{Var}_i[e_1]} = -\frac{\theta_f(\theta_\nu + \gamma\sigma_1^2)\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}.$$

It follows, then, that

$$\sigma_1^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}. \tag{5.8}$$

Note that this justifies the assumption that the conditional variance is equal for all investors i. The proof of existence is complete once we equate the undetermined coefficients from equation (5.4) above with the implied expressions from equations (5.7) and (5.8).

The system of equations that determines λ and σ_1^2 jointly is nonlinear and of too high an order to solve analytically. All of the numerical solutions to this system I have computed indicate that there exists a unique real solution (together with four complex solutions). Even if multiple real solutions do exist for some set of parameters, all of the important results about transparency described in Section 2 are true for all possible real solutions. □

Proof of Theorem 2.3 This proof follows the proof of Theorem 2.2 very closely. If I again assume that the exchange rate in period two is normally distributed conditional on investor i's information set, then it can be shown in a similar manner to before that market clearing in the peso bond market implies that $e_1 = \overline{E}_1[e_2] + \mu + \gamma\tilde{\sigma}_1^2(\nu + \xi)$. In equilibrium, this exchange rate is of the form $e_1 = \mu + f + \gamma\tilde{\sigma}_1^2\nu + \tilde{\lambda}\xi$, so that standard Bayesian inference both justifies the assumption of conditional normality and yields aggregate expectations about f_0 that are similar to those when ν remained unknown:

$$\overline{E}_1[f_0] = f_0 + \frac{\tilde{\lambda}\theta_f \sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}\xi. \tag{5.9}$$

Substituting this equation into the expression for the exchange rate in period one yields

$$\begin{aligned}\tilde{e}_1 &= \mu + \theta_f f_0 + (\theta_\nu + \gamma\tilde{\sigma}_1^2)\nu + \left(\frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}_1^2\right)\xi \\ &= \mu + f + \gamma\tilde{\sigma}_1^2\nu + \left(\frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}_1^2\right)\xi. \end{aligned} \tag{5.10}$$

The conditional variance of the exchange rate in period two, $\tilde{\sigma}_1^2$, is also determined in a manner similar to the previous proof. In particular, standard Bayesian inference implies that

$$\overline{\mathrm{Var}}_1[f_0] = \sigma_\epsilon^2 - \frac{\theta_f^2\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}.$$

The computation is simpler in this case because ν is known with certainty and hence both $\overline{\mathrm{Var}}_1[\nu]$ and $\overline{\mathrm{Cov}}_1[f_0, \nu]$ are equal to zero. It follows, then, that

$$\tilde{\sigma}_1^2 = \theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2 - \frac{\theta_f^2\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}, \tag{5.11}$$

which shows that the conditional variance is equal for all investors i, and together with equation (5.10) completes the proof of existence. In this simpler case, the system of equations (2.11) and (2.12) can be solved analytically. There exists only one real solution to this system and this unique real solution corresponds to the unique equilibrium exchange rate $\tilde{e}_1$. $\square$

Proof of Theorem 2.4 I first show that $\lambda > \tilde{\lambda}$ whenever $\lambda < \theta_\nu + \gamma\sigma_1^2$ and $\lambda < \tilde{\lambda}$ whenever $\lambda > \theta_\nu + \gamma\sigma_1^2$, and then show that $\tilde{\lambda} - \gamma\tilde{\sigma}_1^2 > \theta_\nu$ whenever $\lambda - \gamma\sigma_1^2 > \theta_\nu$ and $\tilde{\lambda} - \gamma\tilde{\sigma}_1^2 < \theta_\nu$ whenever $\lambda - \gamma\sigma_1^2 < \theta_\nu$. Together, these two facts imply that $\lambda > \tilde{\lambda}$ whenever $\theta_\nu > \tilde{\lambda} - \gamma\tilde{\sigma}_1^2$ and $\lambda < \tilde{\lambda}$ whenever $\theta_\nu < \tilde{\lambda} - \gamma\tilde{\sigma}_1^2$.

According to equation (2.7) from Theorem 2.2,

$$\begin{aligned}
\sigma_1^2 &= \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\theta_f^4\sigma_\epsilon^4 + 2\theta_f^2\theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\epsilon^2\sigma_\eta^2 + \theta_\nu^2(\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} \\
&= \sigma_\kappa^2 + \frac{\left((\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2\right)\theta_f^2\sigma_\epsilon^2 + \left(\theta_f^2\sigma_\epsilon^2 + \lambda^2\sigma_\xi^2\right)\theta_\nu^2\sigma_\eta^2 - 2\theta_f^2\theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} \\
&= \sigma_\kappa^2 + \frac{\theta_f^2\gamma^2\sigma_1^4\sigma_\epsilon^2\sigma_\eta^2 + \lambda^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2 + \lambda^2\theta_\nu^2\sigma_\eta^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}, \qquad (5.12)
\end{aligned}$$

so that by equation (2.6) also

$$\lambda = \frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma_\kappa^2 + \frac{\gamma(\gamma\sigma_1^2)^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^2 + \gamma\lambda^2\sigma_\xi^2\theta_f^2\sigma_\epsilon^2 + \gamma\lambda^2\sigma_\xi^2\theta_\nu^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}. \qquad (5.13)$$

Similarly, equation (2.12) from Theorem 2.3 implies that $\tilde\sigma_1^2 = \sigma_\kappa^2 + \frac{\tilde\lambda^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + \tilde\lambda^2\sigma_\xi^2}$, so that by equation (2.11) also

$$\tilde\lambda = \frac{\tilde\lambda\theta_f^2\sigma_\epsilon^2 + \tilde\lambda^2\gamma\theta_f^2\sigma_\epsilon^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + \tilde\lambda^2\sigma_\xi^2} + \gamma\sigma_\kappa^2. \qquad (5.14)$$

Equations (5.13) and (5.14) imply that

$$\lambda^2\sigma_\xi^2(\lambda - \gamma\theta_f^2\sigma_\epsilon^2 - \gamma\sigma_\kappa^2) = \gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\eta^2\left(\theta_f^2\gamma^2\sigma_1^4\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2 - \lambda\sigma_1^2(\theta_\nu+\gamma\sigma_1^2)\right),$$

and

$$\tilde\lambda^2\sigma_\xi^2(\tilde\lambda - \gamma\theta_f^2\sigma_\epsilon^2 - \gamma\sigma_\kappa^2) = \gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2. \qquad (5.15)$$

Let

$$\Delta = \theta_f^2\gamma^2\sigma_1^4\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2 - \lambda\sigma_1^2(\theta_\nu+\gamma\sigma_1^2), \qquad (5.16)$$

so that

$$\lambda^2\sigma_\xi^2(\lambda - \gamma\theta_f^2\sigma_\epsilon^2 - \gamma\sigma_\kappa^2) = \gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\eta^2\Delta \qquad (5.17)$$

and also

$$\lambda = \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2} + \frac{\gamma\sigma_\eta^2\Delta}{\lambda^2\sigma_\xi^2}. \qquad (5.18)$$

It follows that λ is increasing in Δ with $\lambda = \tilde\lambda$ if and only if $\Delta = 0$ or $\sigma_\eta = 0$. Equation (5.18) also implies that $\lambda > \tilde\lambda$ whenever $\Delta > 0$ and $\sigma_\eta > 0$, and $\lambda < \tilde\lambda$ whenever $\Delta < 0$ and $\sigma_\eta > 0$. The bulk of this proof amounts to showing that $\Delta > 0$ whenever $\theta_\nu > \lambda - \gamma\sigma_1^2$ and that $\Delta < 0$ whenever $\theta_\nu < \lambda - \gamma\sigma_1^2$.

Before proving these inequalities, note that equation (2.6) implies that

$$\lambda - \gamma\sigma_1^2 = \frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2},$$

so that

$$(\lambda - \gamma\sigma_1^2)\left(\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2\right) = \lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2. \tag{5.19}$$

Some algebra then yields

$$\begin{aligned}\lambda^2\sigma_\xi^2(\lambda - \gamma\sigma_1^2) &= \gamma\sigma_1^2\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)\left(\lambda\theta_\nu - (\lambda - \gamma\sigma_1^2)(\theta_\nu + \gamma\sigma_1^2)\right)\sigma_\eta^2 \\ &= \gamma\sigma_1^2\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_1^2(\theta_\nu + \gamma\sigma_1^2)(\theta_\nu + \gamma\sigma_1^2 - \lambda)\sigma_\eta^2,\end{aligned}$$

so that

$$\lambda^2\sigma_\xi^2\theta_\nu = \frac{\gamma\sigma_1^2\theta_\nu}{\lambda - \gamma\sigma_1^2}\theta_f^2\sigma_\epsilon^2 + \frac{\gamma\sigma_1^2(\theta_\nu + \gamma\sigma_1^2)\theta_\nu}{\lambda - \gamma\sigma_1^2}(\theta_\nu + \gamma\sigma_1^2 - \lambda)\sigma_\eta^2. \tag{5.20}$$

Equation (5.20) is crucial to the proof of Theorem 2.4. It implies that $\lambda^2\sigma_\xi^2\theta_\nu > \gamma\sigma_1^2\theta_f^2\sigma_\epsilon^2$ and $\frac{\lambda - \gamma\sigma_1^2}{\gamma\sigma_1^2} > \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ whenever $\lambda < \theta_\nu + \gamma\sigma_1^2$, and also that $\lambda^2\sigma_\xi^2\theta_\nu < \gamma\sigma_1^2\theta_f^2\sigma_\epsilon^2$ and $\frac{\lambda - \gamma\sigma_1^2}{\gamma\sigma_1^2} < \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ whenever $\lambda > \theta_\nu + \gamma\sigma_1^2$.

Suppose that $\theta_\nu > \lambda - \gamma\sigma_1^2$, so that $\lambda < \theta_\nu + \gamma\sigma_1^2$. As I just showed in equation (5.20), this implies that both $\lambda^2\sigma_\xi^2\theta_\nu > \gamma\sigma_1^2\theta_f^2\sigma_\epsilon^2$ and $\frac{\lambda - \gamma\sigma_1^2}{\gamma\sigma_1^2} > \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$. It follows that

$$\begin{aligned}\gamma^2\sigma_1^4\theta_f^2\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma_1^2)^2 &> (\gamma\sigma_1^2)^2\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_1^2\theta_\nu\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma_1^2)^2 \\ &= \gamma\sigma_1^2(\theta_\nu + \gamma\sigma_1^2)\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma_1^2)^2 \\ &= (\theta_\nu + \gamma\sigma_1^2)^2\left(\gamma\sigma_\kappa^2 + \gamma\theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma_1^2}{\theta_\nu + \gamma\sigma_1^2}\right).\end{aligned} \tag{5.21}$$

Suppose now that $\Delta \leq 0$. It follows by equation (5.18), then, that $\lambda \leq \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ and hence

$$\gamma\sigma_1^2 = \lambda - (\lambda - \gamma\sigma_1^2) \leq \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2} - (\lambda - \gamma\sigma_1^2). \tag{5.22}$$

Because $\frac{\lambda - \gamma\sigma_1^2}{\gamma\sigma_1^2} > \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ in this case, inequality (5.22) implies that

$$\gamma\sigma_1^2 < \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{(\lambda - \gamma\sigma_1^2)\gamma\sigma_\kappa^2}{\gamma\sigma_1^2} - (\lambda - \gamma\sigma_1^2) = \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\lambda - \gamma\sigma_1^2}{\gamma\sigma_1^2}(\gamma\sigma_\kappa^2 - \gamma\sigma_1^2),$$

which then implies that

$$\gamma\sigma_1^2\left(1+\frac{\lambda-\gamma\sigma_1^2}{\gamma\sigma_1^2}\right)<\gamma\sigma_\kappa^2\left(1+\frac{\lambda-\gamma\sigma_1^2}{\gamma\sigma_1^2}\right)+\gamma\theta_f^2\sigma_\epsilon^2. \tag{5.23}$$

Inequality (5.23) yields

$$\gamma\sigma_1^2<\gamma\sigma_k^2+\gamma\theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma_1^2}{\lambda},$$

from which it follows that

$$\begin{aligned}\lambda(\theta_\nu+\gamma\sigma_1^2)\gamma\sigma_1^2&<\lambda(\theta_\nu+\gamma\sigma_1^2)\gamma\sigma_k^2+(\theta_\nu+\gamma\sigma_1^2)\gamma\theta_f^2\sigma_\epsilon^2\gamma\sigma_1^2\\&<(\theta_\nu+\gamma\sigma_1^2)^2\left(\gamma\sigma_\kappa^2+\gamma\theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma_1^2}{\theta_\nu+\gamma\sigma_1^2}\right).\end{aligned} \tag{5.24}$$

Of course, inequality (5.24) together with inequality (5.21) from above implies that

$$\lambda(\theta_\nu+\gamma\sigma_1^2)\gamma\sigma_1^2<\gamma^2\sigma_1^4\theta_f^2\sigma_\epsilon^2+\lambda^2\theta_\nu^2\sigma_\xi^2+\sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2,$$

which, because $\Delta=\theta_f^2\gamma^2\sigma_1^4\sigma_\epsilon^2+\lambda^2\theta_\nu^2\sigma_\xi^2+\sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2-\lambda\sigma_1^2(\theta_\nu+\gamma\sigma_1^2)$ by equation (5.16), contradicts the assumption that $\Delta\le 0$ and proves that $\Delta>0$ whenever $\lambda<\theta_\nu+\gamma\sigma_1^2$.

Suppose that $\theta_\nu<\lambda-\gamma\sigma_1^2$, so that $\lambda>\theta_\nu+\gamma\sigma_1^2$. As shown above in equation (5.20), this implies that both $\lambda^2\sigma_\xi^2\theta_\nu<\gamma\sigma_1^2\theta_f^2\sigma_\epsilon^2$ and $\frac{\lambda-\gamma\sigma_1^2}{\gamma\sigma_1^2}<\frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$. It follows that

$$\begin{aligned}\gamma^2\sigma_1^4\theta_f^2\sigma_\epsilon^2+\lambda^2\theta_\nu^2\sigma_\xi^2+\sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2&<(\gamma\sigma_1^2)^2\theta_f^2\sigma_\epsilon^2+\gamma\sigma_1^2\theta_\nu\theta_f^2\sigma_\epsilon^2+\sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2\\&=\gamma\sigma_1^2(\theta_\nu+\gamma\sigma_1^2)\theta_f^2\sigma_\epsilon^2+\sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2\\&=(\theta_\nu+\gamma\sigma_1^2)^2\left(\gamma\sigma_\kappa^2+\gamma\theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma_1^2}{\theta_\nu+\gamma\sigma_1^2}\right).\end{aligned} \tag{5.25}$$

Suppose now that $\Delta\ge 0$. It follows by equation (5.18), then, that $\lambda\ge\gamma\theta_f^2\sigma_\epsilon^2+\gamma\sigma_\kappa^2+\frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ and hence

$$\gamma\sigma_1^2=\lambda-(\lambda-\gamma\sigma_1^2)\ge\gamma\theta_f^2\sigma_\epsilon^2+\gamma\sigma_\kappa^2+\frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}-(\lambda-\gamma\sigma_1^2). \tag{5.26}$$

Because $\frac{\lambda-\gamma\sigma_1^2}{\gamma\sigma_1^2}<\frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ in this case, inequality (5.26) implies that

$$\gamma\sigma_1^2>\gamma\theta_f^2\sigma_\epsilon^2+\gamma\sigma_\kappa^2+\frac{(\lambda-\gamma\sigma_1^2)\gamma\sigma_\kappa^2}{\gamma\sigma_1^2}-(\lambda-\gamma\sigma_1^2)=\gamma\theta_f^2\sigma_\epsilon^2+\gamma\sigma_\kappa^2+\frac{\lambda-\gamma\sigma_1^2}{\gamma\sigma_1^2}(\gamma\sigma_\kappa^2-\gamma\sigma_1^2),$$

which then implies that

$$\gamma\sigma_1^2\left(1+\frac{\lambda-\gamma\sigma_1^2}{\gamma\sigma_1^2}\right) > \gamma\sigma_\kappa^2\left(1+\frac{\lambda-\gamma\sigma_1^2}{\gamma\sigma_1^2}\right)+\gamma\theta_f^2\sigma_\epsilon^2. \tag{5.27}$$

Inequality (5.27) yields

$$\gamma\sigma_1^2 > \gamma\sigma_k^2+\gamma\theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma_1^2}{\lambda},$$

from which it follows that

$$\begin{aligned}\lambda(\theta_\nu+\gamma\sigma_1^2)\gamma\sigma_1^2 &> \lambda(\theta_\nu+\gamma\sigma_1^2)\gamma\sigma_k^2+(\theta_\nu+\gamma\sigma_1^2)\gamma\theta_f^2\sigma_\epsilon^2\gamma\sigma_1^2\\ &> (\theta_\nu+\gamma\sigma_1^2)^2\left(\gamma\sigma_\kappa^2+\gamma\theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma_1^2}{\theta_\nu+\gamma\sigma_1^2}\right). \end{aligned}\tag{5.28}$$

Of course, inequality (5.28) together with inequality (5.25) from above implies that

$$\lambda(\theta_\nu+\gamma\sigma_1^2)\gamma\sigma_1^2 > \gamma^2\sigma_1^4\theta_f^2\sigma_\epsilon^2+\lambda^2\theta_\nu^2\sigma_\xi^2+\sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2,$$

which, because $\Delta = \theta_f^2\gamma^2\sigma_1^4\sigma_\epsilon^2+\lambda^2\theta_\nu^2\sigma_\xi^2+\sigma_\kappa^2(\theta_\nu+\gamma\sigma_1^2)^2-\lambda\sigma_1^2(\theta_\nu+\gamma\sigma_1^2)$, contradicts the assumption that $\Delta \geq 0$ and proves that $\Delta < 0$ whenever $\lambda > \theta_\nu+\gamma\sigma_1^2$. These two inequalities also imply that $\Delta = 0$ if and only if $\lambda = \theta_\nu+\gamma\sigma_1^2$, which by equation (5.18) and continuity implies that if $\lambda > \tilde{\lambda}$ ($\lambda < \tilde{\lambda}$) for some $\sigma_\eta > 0$, then $\lambda > \tilde{\lambda}$ ($\lambda < \tilde{\lambda}$) for all $\sigma_\eta > 0$.[20]

The final step of the proof is to show that $\tilde{\lambda}-\gamma\tilde{\sigma}_1^2 > \theta_\nu$ whenever $\lambda-\gamma\sigma_1^2 > \theta_\nu$ and $\tilde{\lambda}-\gamma\tilde{\sigma}_1^2 < \theta_\nu$ whenever $\lambda-\gamma\sigma_1^2 < \theta_\nu$. Suppose that $\lambda-\gamma\sigma_1^2 > \theta_\nu \geq \tilde{\lambda}-\gamma\tilde{\sigma}_1^2$. As was just proved, this implies that $\lambda-\gamma\sigma_1^2 > \theta_\nu$ for all $\sigma_\eta > 0$. Since $\lambda = \tilde{\lambda}$ and $\sigma_1^2 = \tilde{\sigma}_1^2$ if $\sigma_\eta = 0$, it follows by continuity that $\lambda-\gamma\sigma_1^2 = \theta_\nu = \tilde{\lambda}-\gamma\tilde{\sigma}_1^2$ if $\sigma_\eta = 0$. But, I just proved that this implies that $\lambda-\gamma\sigma_1^2 = \theta_\nu$ for all $\sigma_\eta > 0$ as well, so there is a contradiction and it must be that $\tilde{\lambda}-\gamma\tilde{\sigma}_1^2 > \theta_\nu$. A similar argument proves that $\tilde{\lambda}-\gamma\tilde{\sigma}_1^2 < \theta_\nu$ whenever $\lambda-\gamma\sigma_1^2 < \theta_\nu$ as well. □

Proof of Corollary 2.5 Recall that $e_1 = \mu+f+\gamma\sigma_1^2\nu+\lambda\xi$ and that a similar expression describes $\tilde{e}_1$, with $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ replacing λ and σ_1^2, respectively. It is immediate, then, that $\tilde{e}_1 - e_1$ is strictly increasing in ξ whenever $\tilde{\lambda} > \lambda$ and that for ξ large enough, this quantity is greater than zero regardless of the value of ν. This implies the existence of a unique threshold $\hat{\xi} \in \mathbb{R}$ such that $\tilde{e}_1 < e_1$ if and only if $\xi < \hat{\xi}$. This threshold is decreasing (increasing) in ν whenever $\sigma_1^2 > \tilde{\sigma}_1^2$ ($\sigma_1^2 < \tilde{\sigma}_1^2$). □

Proof of Theorem 3.1 Suppose that the exchange rate in period two is normally distributed conditional on investor i's information set. In a manner similar to the proofs

[20]This requires that also $\frac{\partial\lambda}{\partial\sigma_\eta^2} = \frac{\partial\sigma_1^2}{\partial\sigma_\eta^2} = 0$ whenever $\lambda = \theta_\nu+\gamma\sigma_1^2$ (and hence $\Delta = 0$), which is not difficult to show.

of Theorems 2.2 and 2.3, it can be shown that market clearing in the peso bond market implies that $e_1 = \overline{E}_1[e_2] + \mu + \gamma\sigma_1^2(\nu + \xi)$. The equilibrium exchange rate is of the form $e_1 = \mu + f + \gamma\sigma_1^2\nu + \lambda\xi$, which by equation (3.1) implies that $e_1 = \mu + f + \gamma\sigma_1^2(a_\nu f_\nu + a_\xi\xi) + \lambda\xi$. It follows by standard Bayesian inference that the exchange rate in period two is normally distributed conditional on investor i's information set (this justifies the assumption of conditional normality) and that

$$\begin{pmatrix} E_{i1}[f_0] \\ E_{i1}[f_\nu] \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} \theta_f\sigma_\epsilon^2 \\ (\theta_\nu + a_\nu\gamma\sigma_1^2)\sigma_\eta^2 \end{pmatrix} \frac{e_1 - E_i[e_1]}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2},$$

and hence also that

$$\begin{pmatrix} \overline{E}_1[f_0] \\ \overline{E}_1[f_\nu] \end{pmatrix} = \begin{pmatrix} f_0 \\ f_\nu \end{pmatrix} + \begin{pmatrix} \theta_f\sigma_\epsilon^2 \\ (\theta_\nu + a_\nu\gamma\sigma_1^2)\sigma_\eta^2 \end{pmatrix} \frac{(a_\xi\gamma\sigma_1^2 + \lambda)\xi}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2}.$$

Substituting this last equation into the expression for the exchange rate in period one (recall that $\overline{E}_1[e_2] = \overline{E}_1[f]$) yields

$$e_1 = \mu + f + \gamma\sigma_1^2\nu + \left(\frac{\theta_f^2(a_\xi\gamma\sigma_1^2 + \lambda)\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma_1^2)(a_\xi\gamma\sigma_1^2 + \lambda)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2} + \gamma\sigma_1^2 \right) \xi. \tag{5.29}$$

The next step is to solve for σ_1^2, the conditional variance of the exchange rate in period two. Because $e_2 = \theta_f f_0 + \theta_\nu f_\nu + \kappa$, this conditional variance is given by $\sigma_1^2 = \theta_f^2\overline{\mathrm{Var}}_1[f_0] + \theta_\nu^2\overline{\mathrm{Var}}_1[f_\nu] + \sigma_\kappa^2 + 2\theta_f\theta_\nu\overline{\mathrm{Cov}}_1[f_0, f_\nu]$, just as in the earlier theorems' proofs. Bayesian inference implies that

$$\begin{aligned} \begin{pmatrix} \overline{\mathrm{Var}}_1[f_0] & \overline{\mathrm{Cov}}_1[f_0, f_\nu] \\ \overline{\mathrm{Cov}}_1[f_0, f_\nu] & \overline{\mathrm{Var}}_1[f_\nu] \end{pmatrix} &= \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \\ &- \frac{1}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2} \begin{pmatrix} \theta_f\sigma_\epsilon^2 \\ (\theta_\nu + a_\nu\gamma\sigma_1^2)\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \theta_f\sigma_\epsilon^2 & (\theta_\nu + a_\nu\gamma\sigma_1^2)\sigma_\eta^2 \end{pmatrix}, \end{aligned}$$

so that

$$\begin{aligned} \overline{\mathrm{Var}}_1[f_0] &= \sigma_\epsilon^2 - \frac{\theta_f^2\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2}, \\ \overline{\mathrm{Var}}_1[f_\nu] &= \sigma_\eta^2 - \frac{(\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2}, \end{aligned}$$

and

$$\overline{\mathrm{Cov}}_1[f_0, f_\nu] = \frac{-\theta_f(\theta_\nu + a_\nu\gamma\sigma_1^2)\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2}.$$

It follows that

$$\sigma_1^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma_1^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2}. \tag{5.30}$$

Note that this justifies the assumption that the conditional variance is equal for all investors i. The proof of existence is complete once we equate the undetermined coefficients λ and σ_1^2 with the implied expressions from equations (5.29) and (5.30). □

Proof of Theorem 3.2 Suppose that the exchange rate in period two is normally distributed conditional on investor i's information set. In a manner similar to the proofs of Theorems 2.2, 2.3, and 3.1, it can be shown that market clearing in the peso bond market implies that $\tilde{e}_1 = \overline{E}_1[e_2] + \mu + \gamma\tilde{\sigma}_1^2(\nu + \xi)$. The equilibrium exchange rate is of the form $\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2\nu + \tilde{\lambda}\xi$, which implies that $\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2(a_\nu f_\nu + a_\xi\xi) + \tilde{\lambda}\xi$. It follows by standard Bayesian inference that the exchange rate in period two is normally distributed conditional on investor i's information set (this justifies the assumption of conditional normality) and that

$$\begin{aligned}\begin{pmatrix} E_{i1}[f_0] \\ E_{i1}[f_\nu] \end{pmatrix} = &\begin{pmatrix} x_i \\ y_i \end{pmatrix} \\ &+ \begin{pmatrix} 0 & \theta_f\sigma_\epsilon^2 \\ a_\nu\sigma_\eta^2 & \pi_\nu\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2 & a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2 \\ a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2 & \theta_f^2\sigma_\epsilon^2 + \pi_\nu^2\sigma_\eta^2 + \pi_\xi^2\sigma_\xi^2 \end{pmatrix}^{-1} \begin{pmatrix} \nu - a_\nu y_i \\ \tilde{e}_1 - E_i[\tilde{e}_1] \end{pmatrix},\end{aligned}$$

where $\pi_\nu = \theta_\nu + a_\nu\gamma\tilde{\sigma}_1^2$ and $\pi_\xi = a_\xi\gamma\tilde{\sigma}_1^2 + \tilde{\lambda}$. Let $\Psi = \theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2$. Averaging this last expression across all investors then yields

$$\begin{aligned}\begin{pmatrix} \overline{E}_1[f_0] \\ \overline{E}_1[f_\nu] \end{pmatrix} &= \begin{pmatrix} f_0 \\ f_\nu \end{pmatrix} + \frac{1}{\Psi}\begin{pmatrix} 0 & \theta_f\sigma_\epsilon^2 \\ a_\nu\sigma_\eta^2 & \pi_\nu\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \theta_f^2\sigma_\epsilon^2 + \pi_\nu^2\sigma_\eta^2 + \pi_\xi^2\sigma_\xi^2 & -a_\nu\pi_\nu\sigma_\eta^2 - a_\xi\pi_\xi\sigma_\xi^2 \\ -a_\nu\pi_\nu\sigma_\eta^2 - a_\xi\pi_\xi\sigma_\xi^2 & a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2 \end{pmatrix} \begin{pmatrix} a_\xi\xi \\ \pi_\xi\xi \end{pmatrix} \\ &= \begin{pmatrix} f_0 \\ f_\nu \end{pmatrix} + \frac{1}{\Psi}\begin{pmatrix} -\theta_f\sigma_\epsilon^2(a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2) & \theta_f\sigma_\epsilon^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2) \\ a_\nu\theta_f^2\sigma_\epsilon^2\sigma_\eta^2 + \pi_\xi(a_\nu\pi_\xi - a_\xi\pi_\nu)\sigma_\eta^2\sigma_\xi^2 & a_\xi(a_\xi\pi_\nu - a_\nu\pi_\xi)\sigma_\eta^2\sigma_\xi^2 \end{pmatrix} \begin{pmatrix} a_\xi\xi \\ \pi_\xi\xi \end{pmatrix} \\ &= \begin{pmatrix} f_0 \\ f_\nu \end{pmatrix} + \frac{1}{\Psi}\begin{pmatrix} a_\nu\theta_f(a_\nu\tilde{\lambda} - a_\xi\theta_\nu)\sigma_\epsilon^2\sigma_\eta^2\xi \\ a_\nu a_\xi\theta_f^2\sigma_\epsilon^2\sigma_\eta^2\xi \end{pmatrix}.\end{aligned}$$

Finally, substituting the last equation into the expression for the exchange rate in period one implies that

$$\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2\nu + \left(\frac{\tilde{\lambda} a_\nu^2 \theta_f^2 \sigma_\epsilon^2 \sigma_\eta^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2} + \gamma\tilde{\sigma}_1^2 \right)\xi. \tag{5.31}$$

The next step is to solve for $\tilde{\sigma}_1^2$, the conditional variance of the exchange rate in period two. As in the proof of Theorem 3.1, this conditional variance is given by $\tilde{\sigma}_1^2 = \theta_f^2\overline{\mathrm{Var}}_1[f_0] + \theta_\nu^2\overline{\mathrm{Var}}_1[f_\nu] + \sigma_\kappa^2 + 2\theta_f\theta_\nu\overline{\mathrm{Cov}}_1[f_0, f_\nu]$. Bayesian inference implies that

$$\begin{aligned}
\begin{pmatrix} \overline{\mathrm{Var}}_1[f_0] & \overline{\mathrm{Cov}}_1[f_0, f_\nu] \\ \overline{\mathrm{Cov}}_1[f_0, f_\nu] & \overline{\mathrm{Var}}_1[f_\nu] \end{pmatrix} &= \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \\
&- \begin{pmatrix} 0 & \theta_f\sigma_\epsilon^2 \\ a_\nu\sigma_\eta^2 & \pi_\nu\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2 & a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2 \\ a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2 & \theta_f^2\sigma_\epsilon^2 + \pi_\nu^2\sigma_\eta^2 + \pi_\xi^2\sigma_\xi^2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & a_\nu\sigma_\eta^2 \\ \theta_f\sigma_\epsilon^2 & \pi_\nu\sigma_\eta^2 \end{pmatrix},
\end{aligned}$$

so that

$$\begin{aligned}
\begin{pmatrix} \overline{\mathrm{Var}}_1[f_0] & \overline{\mathrm{Cov}}_1[f_0, f_\nu] \\ \overline{\mathrm{Cov}}_1[f_0, f_\nu] & \overline{\mathrm{Var}}_1[f_\nu] \end{pmatrix} &= \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \\
&- \frac{1}{\Psi} \begin{pmatrix} -\theta_f\sigma_\epsilon^2(a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2) & \theta_f\sigma_\epsilon^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2) \\ a_\nu\theta_f^2\sigma_\epsilon^2\sigma_\eta^2 + \pi_\xi(a_\nu\pi_\xi - a_\xi\pi_\nu)\sigma_\eta^2\sigma_\xi^2 & a_\xi(a_\xi\pi_\nu - a_\nu\pi_\xi)\sigma_\eta^2\sigma_\xi^2 \end{pmatrix} \begin{pmatrix} 0 & a_\nu\sigma_\eta^2 \\ \theta_f\sigma_\epsilon^2 & \pi_\nu\sigma_\eta^2 \end{pmatrix}.
\end{aligned}$$

It follows that

$$\begin{aligned}
\overline{\mathrm{Var}}_1[f_0] &= \sigma_\epsilon^2 - \frac{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^4}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2}, \\
\overline{\mathrm{Var}}_1[f_\nu] &= \sigma_\eta^2 - \frac{a_\nu^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^4 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^4\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2},
\end{aligned}$$

and

$$\overline{\mathrm{Cov}}_1[f_0, f_\nu] = \frac{a_\xi\theta_f(a_\nu\tilde{\lambda} - a_\xi\theta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2},$$

and hence also that

$$\tilde{\sigma}_1^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{a_\nu^2\theta_f^2(\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2)\sigma_\epsilon^2\sigma_\eta^2 + \left(a_\xi\theta_f^2\sigma_\epsilon^2 + \theta_\nu(a_\xi\theta_\nu - a_\nu\tilde{\lambda})\sigma_\eta^2\right)^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2}. \tag{5.32}$$

Note that this justifies the assumption that the conditional variance is equal for all investors i. The proof of existence is complete once we equate the undetermined coefficients $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$

with the implied expressions from equations (5.31) and (5.32). $\square$

Proof of Theorem 3.3 Suppose that $\theta_\nu = 0$. According to equations (3.3) and (3.4), in this case λ and σ_1^2 are given by

$$\lambda = \frac{\theta_f^2(a_\xi\gamma\sigma_1^2 + \lambda)\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2} + \gamma\sigma_1^2, \tag{5.33}$$

$$\sigma_1^2 = \sigma_\kappa^2 + \frac{\theta_f^2\sigma_\epsilon^2\left((a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2\right)}{\theta_f^2\sigma_\epsilon^2 + (a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma_1^2 + \lambda)^2\sigma_\xi^2}, \tag{5.34}$$

and according to equations (3.6) and (3.7), in this case $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by

$$\tilde{\lambda} = \frac{\tilde{\lambda}a_\nu^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + a_\nu^2\tilde{\lambda}^2\sigma_\eta^2\sigma_\xi^2} + \gamma\tilde{\sigma}_1^2, \tag{5.35}$$

$$\tilde{\sigma}_1^2 = \sigma_\kappa^2 + \frac{a_\nu^2\theta_f^2\tilde{\lambda}^2\sigma_\epsilon^2\sigma_\eta^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + a_\nu^2\tilde{\lambda}^2\sigma_\eta^2\sigma_\xi^2}. \tag{5.36}$$

Consider now the limit as $a_\nu\sigma_\eta \to \infty$. As long as λ does not diverge to infinity, equations (5.33) and (5.34) imply that

$$\lim_{a_\nu\sigma_\eta\to\infty} \lambda = \lim_{a_\nu\sigma_\eta\to\infty} \gamma\sigma_1^2 = \gamma\sigma_\kappa^2 + \gamma\theta_f^2\sigma_\epsilon^2.$$

Furthermore, it is not difficult to show that the equilibrium equations imply that λ cannot diverge to infinity. In a similar manner, as long as $\tilde{\lambda}$ does not diverge to infinity, equations (5.35) and (5.36) imply that

$$\lim_{a_\nu\sigma_\eta\to\infty} \tilde{\lambda} = \lim_{a_\nu\sigma_\eta\to\infty} \frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}_1^2, \tag{5.37}$$

$$\lim_{a_\nu\sigma_\eta\to\infty} \tilde{\sigma}_1^2 = \lim_{a_\nu\sigma_\eta\to\infty} \sigma_\kappa^2 + \frac{\theta_f^2\tilde{\lambda}^2\sigma_\epsilon^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}. \tag{5.38}$$

It is straightforward to show that the equilibrium conditions imply that $\tilde{\lambda}$ cannot diverge infinity, as well. Note that the equilibrium conditions given by equations (5.37) and (5.38) are identical to the equilibrium conditions given by equations (2.11) and (2.12) from Theorem 2.3 in Section 2, so that the parameters $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ in this model converge to the same value as the simpler model's parameters in the limit as $\theta_\nu \to 0$ and $a_\nu\sigma_\eta \to \infty$. Equations (5.37) and (5.38) together imply that

$$\lim_{a_\nu \sigma_\eta \to \infty} \tilde{\lambda}^3 \sigma_\xi^2 = \lim_{a_\nu \sigma_\eta \to \infty} \gamma \sigma_\kappa^2 (\theta_f^2 \sigma_\epsilon^2 + \tilde{\lambda}^2 \sigma_\xi^2) + \gamma \theta_f^2 \tilde{\lambda}^2 \sigma_\epsilon^2 \sigma_\xi^2,$$

and hence that

$$\lim_{a_\nu \sigma_\eta \to \infty} \tilde{\lambda} = \lim_{a_\nu \sigma_\eta \to \infty} \gamma \sigma_\kappa^2 \left(1 + \frac{\theta_f^2 \sigma_\epsilon^2}{\tilde{\lambda}^2 \sigma_\xi^2} \right) + \gamma \theta_f^2 \sigma_\epsilon^2 > \gamma \sigma_\kappa^2 + \gamma \theta_f^2 \sigma_\epsilon^2 = \lim_{a_\nu \sigma_\eta \to \infty} \lambda.$$

It follows by continuity, then, that there exist thresholds $\hat{\theta}_\nu > 0$ and $\widehat{a\sigma} > 0$ such that if $\theta_\nu < \hat{\theta}_\nu$ and $a_\nu \sigma_\eta > \widehat{a\sigma}$, then $\tilde{\lambda} > \lambda$.

Suppose that $\sigma_\eta = 0$. According to equations (3.3) and (3.4), in this case λ and σ_1^2 are given by

$$\lambda = \frac{\theta_f^2 (a_\xi \gamma \sigma_1^2 + \lambda) \sigma_\epsilon^2}{\theta_f^2 \sigma_\epsilon^2 + (a_\xi \gamma \sigma_1^2 + \lambda)^2 \sigma_\xi^2} + \gamma \sigma_1^2,$$

$$\sigma_1^2 = \sigma_\kappa^2 + \frac{\theta_f^2 (a_\xi \gamma \sigma_1^2 + \lambda)^2 \sigma_\epsilon^2 \sigma_\xi^2}{\theta_f^2 \sigma_\epsilon^2 + (a_\xi \gamma \sigma_1^2 + \lambda)^2 \sigma_\xi^2},$$

and according to equations (3.6) and (3.7), in this case $\tilde{\lambda} = \gamma \tilde{\sigma}_1^2 = \gamma \sigma_\kappa^2$. Because $\lambda > \gamma \sigma_\kappa^2 = \tilde{\lambda}$, it follows by continuity that there exists a threshold $\hat{\sigma}_\eta > 0$ such that if $\sigma_\eta < \hat{\sigma}_\eta$, then $\lambda > \tilde{\lambda}$.

In a similar manner, suppose that $a_\nu = 0$ (but now also $\sigma_\eta > 0$) and note that equations (3.3) and (3.4) imply that in this case λ and σ_1^2 are given by

$$\lambda = \frac{\theta_f^2 (a_\xi \gamma \sigma_1^2 + \lambda) \sigma_\epsilon^2 + \theta_\nu^2 (a_\xi \gamma \sigma_1^2 + \lambda) \sigma_\eta^2}{\theta_f^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2 + (a_\xi \gamma \sigma_1^2 + \lambda)^2 \sigma_\xi^2} + \gamma \sigma_1^2,$$

$$\sigma_1^2 = \sigma_\kappa^2 + \frac{(\theta_f^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2)(a_\xi \gamma \sigma_1^2 + \lambda)^2 \sigma_\xi^2}{\theta_f^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2 + (a_\xi \gamma \sigma_1^2 + \lambda)^2 \sigma_\xi^2}.$$

As in the previous case, equations (3.6) and (3.7) also imply that in this case $\tilde{\lambda} = \gamma \tilde{\sigma}_1^2 = \gamma \sigma_\kappa^2$. Because $\lambda > \gamma \sigma_\kappa^2 = \tilde{\lambda}$, it follows by continuity that there exists a threshold $\hat{a}_\nu$ such that if $a_\nu < \hat{a}_\nu$, then $\lambda > \tilde{\lambda}$. □

Part II

Extensions to the Benchmark Intervention Model

In the all of the models of the previous chapter, I assume that either the central bank's chosen policy of transparency is independent of the underlying state of the economy or investors are naive and unable to infer anything from this choice of policy. While these assumptions simplify the analysis, they are not realistic. Indeed, there is both theoretical (Angeletos, Hellwig, and Pavan 2006, Mussa 1981) and empirical (Bank for International Settlements 2005, Chiu 2003) evidence that transparency policy is an important signal to investors. To explore this question, in the first section of this chapter I expand the two-period benchmark model into a Bayesian signalling game in which the central bank has a clearly defined objective function and investors are not naive. Given a set of assumptions for the model's primitives, I prove the existence of a partially-separating Bayesian equilibrium that preserves the intuition and analysis from the benchmark model.

In the second section of this chapter, I extend the two-period benchmark model to an infinite horizon. This exercise examines the robustness of the results in a more complete setting in which exchange rate fundamentals are equal to the sum of time-discounted interest rate spreads and risk premia (which are affected by foreign exchange interventions). The first of these infinite-horizon models assumes that investors have common knowledge of the past. This causes higher-order expectations to disappear and keeps the analysis relatively tractable, so that even though a full analytic solution is not possible, an analytic characterization of the equilibrium conditions can be obtained. In this richer setup, I describe some cases in which transparency magnifies exchange rate misalignment and provide exact numerical values for all of the model's endogenous parameters. The results match the benchmark model's predictions. The second infinite-horizon model assumes that investors have imperfect common knowledge of the past. This causes higher-order expectations to be part of the steady-state equilibrium as in similarly structured dynamic macroeconomic models with information heterogeneity such as Bacchetta and van Wincoop (2006), Lorenzoni (2009), and Nimark (2010a).[21] Without common knowledge of the past, transitory noise trades permanently affect investors' expectations of fundamentals and lead to persistent exchange rate misalignment. In this setting, I show that this persistent misalignment can also be magnified by transparency

[21] There is a class of models in which the equilibrium is fully revealing even though agents are heterogeneously informed about fundamentals. The most famous example of this is given by Townsend (1983). As shown by Kasa (2000), Pearlman and Sargent (2005), and Sargent (1991), the agents in Townsend's model can actually infer the information of others so that higher-order expectations are not part of equilibrium. The investors in my dynamic model, in contrast, cannot infer other agents' information and higher-order expectations do not disappear.

The extension of the benchmark model to an infinite horizon with perpetually disparately informed traders adopts assumptions that are similar to those in the asset-pricing models of Allen, Morris, and Shin (2006), Bacchetta and van Wincoop (2008), Kasa, Walker, and Whiteman (2007), and Nimark (2010b). Each of these papers shows that persistent gaps between prices and fundamentals are common in such an environment, as is the case in my model. These papers emphasize this gap and offer a compelling explanation for several important empirical puzzles in finance, but they do not examine price manipulation as I do.

This chapter is organized as follows. Section 6 expands the benchmark two-period model into a Bayesian signalling game. Section 7 extends the benchmark two-period model to an infinite horizon, with Section 7.1 considering the case in which investors have common knowledge of the past and Section 7.2 considering the case in which investors are perpetually disparately informed. The proofs for all of the results are provided in the last section.

6 Policy as a Signal of Fundamentals

All of the results I have presented so far assume that either central bank interventions are independent of the underlying state of the economy or that investors are naive and unable to infer anything from the bank's chosen policy of transparency. While this keeps the analysis tractable, it is an unrealistic assumption as there is plenty of evidence that central banks' decisions whether or not to announce the size of their interventions are careful, highly strategic decisions. Rational investors are aware of this strategic element, and they use a bank's chosen level of transparency to better infer the underlying state of the economy. In other words, central banks and investors play a Bayesian signalling game.

In this section, I relax this assumption and investigate how the benchmark model's predictions are affected. I consider a Bayesian signalling game between the foreign central bank and the investors in which the central bank has a clear objective that investors are not naive about. With the example of a central bank defending a falling exchange rate in mind, I first examine a game in which the bank's objective is to appreciate the peso exchange rate. I then reverse this objective and consider a game in which the central bank's goal is to depreciate the peso exchange rate. It is important to note, however, that all of this analysis is easily extended to a game in which the bank targets a publicly known value of the exchange rate. Indeed, if the central bank targets a specific value of the exchange rate, then the game that is played involves either the central bank increasing the exchange rate—if the exchange rate is below the target—or the central bank decreasing the exchange rate—if the exchange rate is above the target. In either case, investors observe the value of the exchange rate relative to the target and are aware of the central bank's desire to achieve either appreciation or depreciation.

This section's most important contribution is to construct a partially-separating Bayesian equilibrium in which the foreign central bank's goal is to appreciate the peso and it announces its intervention whenever the exchange rate is sufficiently overvalued. This equilibrium demonstrates that the previous results about central bank ambiguity reducing exchange rate misalignment are consistent with an environment in which policy choice is a signal to investors. Furthermore, the existence of a non-pooling equilibrium proves that self-fulfilling beliefs about the meaning of central bank transparency need not dwarf the effects I describe in the previous sections. In fact, self-fulfilling pooling equilibria often exist only together with highly unintuitive and implausible out-of-equilibrium beliefs.[22]

This section's Bayesian signalling games between the foreign central bank and investors take place in the two-period setup of Section 2. I assume that the central bank knows the value of exchange rate fundamentals f and that it chooses between two possible actions: either adopt a policy of transparency and announce the size of its intervention in period one, or adopt a policy of ambiguity and do not announce anything. Implicitly, then, I assume that the central bank cannot credibly reveal the value of all parts of fundamentals f to investors. This is justified by the fact that the bank's objective is either to increase or decrease the exchange rate and hence no unverifiable announcement about f could possibly be credible.[23] In reality, many announcements about future policies that affect fundamentals inherently lack credibility, especially promises to engage in large-scale interventions or to alter monetary policy in ways that might significantly disrupt the domestic economy.

Signalling games of this kind together with a model that features asset-pricing under imperfect information presents many technical difficulties. Most significantly, investors' beliefs about f_0 and ν are generally not normally distributed, a fact that makes it very difficult to characterize the investors' aggregate demand for peso bonds and the equilibrium exchange rate. Indeed, investors' utility functions are exponential, so if their beliefs about fundamentals are not normally distributed (which requires a normally distributed exchange rate in period one) then their demand is impossible to characterize analytically. If the demand of investors cannot be characterized, then the exchange rate in period one also cannot be characterized and it becomes very difficult to prove even simple equilibrium properties. Worse still, these technical difficulties do not go away even if exponential utility is replaced by mean-variance utility.[24]

In both the game in which the foreign central bank's objective is to appreciate the peso exchange rate and the game in which its objective is to depreciate the peso exchange rate, I prove the existence of a partially-separating Bayesian equilibrium given a set of assumptions for the model's primitives. One key to constructing these equilibria is that absent any investor interpretation of transparency policy, the foreign central bank prefers one policy over another for some combination of fundamentals. This ensures that regardless

[22]The intuitive criterion of Cho and Kreps (1987) does not restrict the set of pooling equilibria in this game since the value of the central bank's policy is purely determined by the investors' interpretation of that policy. In other words, neither transparency nor ambiguity is ever strictly dominated.

[23]Vitale (1999) also concludes that central bank announcements are not credible if the bank's goals are inconsistent with exchange rate fundamentals.

[24]Although it may be possible to analytically characterize the investors' demand for peso bonds with mean-variance utility, to characterize an equilibrium of this game one must also find a fixed point between investors' beliefs about fundamentals and the exchange rate. Since investors' beliefs are not normally distributed (beliefs are truncated in any partially-separating equilibrium), this is impossible to do analytically.

of which policy is interpreted as an undesirable signal of currency misalignment (if the bank's goal is appreciation, then signalling overvaluation is undesirable, and if the bank's goal is depreciation, then signalling undervaluation is undesirable), the bank does not shun that policy in equilibrium. In this section's signalling games, a preference for one policy over another exists because the risk premium on peso bonds varies depending on both the conditional variance of the exchange rate in period two and the extent of central bank intervention (this alters the available supply of peso bonds). As long as different transparency policies imply different conditional variances, the central bank will never strictly prefer one policy over the other. More succinctly, if the exchange rate in period one is approximately given by

$$e_1 = \mu + \overline{E}_1[f] + \gamma\sigma_1^2(\nu + \xi), \tag{6.1}$$

then as long as the difference between $\overline{E}_1[f]$ with and without transparency is finite and $\sigma_1^2 \neq \tilde{\sigma}_1^2$, there will always be a nonempty set of fundamentals for which the central bank chooses each policy.

This also implies that self-fulfilling pooling equilibria often require highly unintuitive out-of-equilibrium beliefs. Although large shifts in the exchange rate should be expected if central bank policy ever signals to investors that fundamentals are much different than what is implied by the value of the exchange rate, the preceding argument shows that for some range of fundamentals these shifts are less important than changes in the risk premium. Of course, this requires that the risk premium actually changes with the central bank's transparency policy.

In the first partially-separating equilibrium I construct, the bank makes an announcement only if the exchange rate is sufficiently overvalued in period one. The construction of this equilibrium is aided by a technical assumption that ensures that less uncertainty about the exchange rate in period two reduces the risk premium on peso bonds and raises the peso exchange rate. Specifically, I assume that there is a fixed supply of peso bonds equal to $S > 0$ dollars and that the bank's intervention ν is always less than this supply. This changes the risk premium term in equation (6.1) above to $(\theta_\nu + \gamma\sigma_1^2)(\nu - S)$ and ensures that this term is always negative.

Assumption 6.1. *There is a positive net supply of peso bonds denoted by $S > 0$. The central bank's intervention ν is bounded, so that $|\nu| \leq \bar{\nu} < S$, and investors' common prior for ν is uniform over the interval $[-\bar{\nu}, \bar{\nu}]$.*

Besides aiding with the technical details of Theorem 6.3 below, Assumption 6.1 better reflects the reality of a country for which transparency often reduces both the uncertainty and the risk premium of its assets. Indeed, a more realistic version of the benchmark model applied to risky assets certainly must assume that interventions are bounded and risk premia are always positive ($S > \nu$) and increasing in uncertainty.

Before presenting the formal definition of a Bayesian equilibrium of this game, it is necessary to introduce some additional notation. Let $\tau \in \{T, N\}$ denote the foreign central bank's choice of transparency policy, with $\tau = T$ corresponding to an announcement about the bank's intervention and $\tau = N$ corresponding to no announcement. In this section's first signalling game, the central bank chooses its transparency policy so that

$$\tau(\nu, f) = \arg\max_{\tau \in \{T,N\}} e_1(\tau). \tag{6.2}$$

Note that the bank's intervention ν is assumed to be exogenous, as in the benchmark model of Section 2.

Definition 6.2. A Bayesian equilibrium of this economy is a strategy for the foreign central bank and a function for the exchange rate in period one e_1, such that (i) the demand for peso bonds by each investor b_i solves the maximization problem (2.2), where investor i's information set consists of all common public information together with x_i, y_i, e_1, and, if the central bank announces its intervention policy, ν as well; (ii) the foreign central bank chooses its transparency policy according to (6.2); (iii) the peso bond market clears: $B + \xi + \nu = S$; (iv) the exchange rate is a function of the central bank's transparency policy, the demand for peso bonds by noise traders ξ, the supply of peso bonds S, the foreign central bank's intervention ν, the interest rate parameter μ, and the fundamentals parameter f.

All expectations and variances in this game are functions of the bank's policy choice. In order to emphasize this point, the conditional expectations and variances with respect to the information set of investor i in period one are denoted by $E_{i1}(\tau)[\cdot]$ and and $\mathrm{Var}_{i1}(\tau)[\cdot]$, respectively.

Theorem 6.3. *Suppose that $\theta_\nu < \hat{\theta}_\nu$ and that the foreign central bank's objective is given by (6.2). There exist bounds $\hat{S}, \hat{\nu}, \hat{\sigma}_\xi > 0$ such that if $S \geq \hat{S}$, $\bar{\nu} \geq \hat{\nu}$, and $\sigma_\xi \leq \hat{\sigma}_\xi$, then there exists a partially-separating Bayesian equilibrium in which the central bank announces the size of its intervention if and only if $\xi \geq \hat{\xi}(\nu)$. In this equilibrium, the threshold function $\hat{\xi}(\nu)$ is positive and decreasing in ν.*

The proof of Theorem 6.3 is in Section 8. The theorem states that there exists a partially-separating equilibrium in which the foreign central bank chooses a transparent policy if the exchange rate is sufficiently overvalued relative to fundamentals. Although rational investors infer that this policy choice is a sign of an overvalued currency and adjust their beliefs accordingly, the central bank still prefers to be transparent because it reduces the unpredictability of the exchange rate in period two and therefore lowers the risk premium on peso bonds (and thus raises the peso exchange rate). This is an important result because it demonstrates that the benchmark model's predictions about central bank ambiguity reducing exchange rate misalignment are not overturned once signalling is introduced into the model.

Theorem 6.3 requires that the demand of noise traders be highly predictable (low value of σ_ξ). This ensures that investors' beliefs about fundamentals are approximately a linear function of those fundamentals, despite the fact that beliefs about ξ are truncated above or below depending upon the central bank's choice of policy. Without approximate linearity, it is impossible to analytically characterize the equilibrium exchange rate, as mentioned earlier. If the exchange rate cannot be characterized in this way, even by approximation, then it is impossible to compare the value of the exchange rate under different transparency policies.

To better understand the role of this assumption about σ_ξ, consider a simplified version of this game. Forget about the two parts of fundamentals f as given by equation (2.3), and suppose instead that each investor i observes both $f_i = f + \epsilon_i$ and the exchange rate in period one. In this example, a central bank announcement reveals to investors that $\xi \geq \hat{\xi} > 0$. Suppose that $\tilde{e}_1 = f + \tilde{\lambda}(\xi - \hat{\xi})$. This means that the distribution of f conditional on the information of investor i is truncated normal with mean $f_i + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}\left(\tilde{e}_1 - f_i + \tilde{\lambda}\hat{\xi}\right)$, variance $\frac{\sigma_\epsilon^2\tilde{\lambda}^2\sigma_\xi^2}{\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}$, and truncation $f < \tilde{e}_1$. By l'Hôpital's rule, this implies that in the aggregate

$$\lim_{\sigma_\xi \to 0} \overline{E}_1(T)\exp\{-f\} = \lim_{\sigma_\xi \to 0} \exp\{-\tilde{e}_1\} = \lim_{\sigma_\xi \to 0} \exp\{-f - \tilde{\lambda}(\xi - \hat{\xi})\}. \tag{6.3}$$

If $e_2 = f + \kappa$ and investors care only about e_2, it follows that $\lim_{\sigma_\xi \to 0} \tilde{e}_1 = \lim_{\sigma_\xi \to 0} f + \tilde{\lambda}(\xi - \hat{\xi})$ and hence that the exchange rate in period one is indeed normally distributed in the limit.

On the other hand, if there is no central bank announcement then investors learn that $\xi < \hat{\xi}$. Let $e_1 = f + \lambda\xi$. This means that the distribution of f conditional on the information of investor i is truncated normal with mean $f_i + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \lambda^2\sigma_\xi^2}(e_1 - f_i)$, variance $\frac{\sigma_\epsilon^2\lambda^2\sigma_\xi^2}{\sigma_\epsilon^2 + \lambda^2\sigma_\xi^2}$, and truncation $f > e_1 - \lambda\hat{\xi}$. Average expectations are simpler this time, with

$$\lim_{\sigma_\xi \to 0} \overline{E}_1(N)\exp\{-f\} = \lim_{\sigma_\xi \to 0} \exp\left\{-f - \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}(e_1 - f)\right\} = \lim_{\sigma_\xi \to 0} \exp\{-f - \lambda\xi\}. \tag{6.4}$$

This follows because the truncation communicates nothing about f in the limit since the conditional mean of f is on average greater than the truncation. Once again, this implies that indeed $\lim_{\sigma_\xi \to 0} e_1 = \lim_{\sigma_\xi \to 0} f + \lambda\xi$, confirming the initial guess.

Although this example is simpler than the full setup of this section, it does capture the role of the assumption $\sigma_\xi \leq \hat{\sigma}_\xi$ in the proof of Theorem 6.3. One implication is that for σ_ξ small enough, the difference between e_1 and $\tilde{e}_1$ is approximately given by

$$e_1 - \tilde{e_1} = \xi(\lambda - \tilde{\lambda}) + \tilde{\lambda}\hat{\xi}. \tag{6.5}$$

This relationship shows that if $\tilde{\lambda} > \lambda$ in this setting, then it is not possible to construct an equilibrium in which the central bank only makes an announcement if $\xi < \hat{\xi}$. According to equation (6.5), regardless of the value of $\hat{\xi}$ (or if $\hat{\xi}$ multiplies λ instead of $\tilde{\lambda}$), if ξ is sufficiently negative, then $e_1 > \tilde{e}_1$ and the central bank prefers an ambiguous intervention policy. This is an important observation, because together with the existence of self-fulfilling pooling equilibria, another concern is that investors' interpretation of central bank policy may dictate whether transparency signals an overvalued or undervalued currency in equilibrium. This example shows that this is generally not possible.

The next step is to reverse the foreign central bank's objective and to examine a signalling game in which the bank wishes to depreciate the peso exchange rate. In the notation of the model, the foreign central bank chooses its transparency policy so that

$$\tau(\nu, f) = \arg\min_{\tau \in \{T,N\}} e_1(\tau). \tag{6.6}$$

This game reflects not a crisis environment in which the goal is to stem rapid capital outflows, but instead a calmer environment in which the goal is to maintain a competitively devalued currency. In this depreciation game, I construct a partially-separating equilibrium that is very similar to the equilibrium described in Theorem 6.3. This construction relies on many of the same technical assumptions as before, including Assumption 6.1, which ensures that less uncertainty about the exchange rate in period two reduces the risk premium on peso bonds and raises the peso exchange rate. An equilibrium of this game is defined as in Definition 6.2 above, except that in part (ii) the bank now chooses its transparency policy according to (6.6).

Theorem 6.4. *Suppose that $\theta_\nu > \hat{\theta}_\nu$ and that the foreign central bank's objective is given by (6.6). There exist bounds $\hat{S}, \hat{\nu}, \hat{\sigma}_\xi > 0$ such that if $S \geq \hat{S}$, $\bar{\nu} \geq \hat{\nu}$, and $\sigma_\xi \leq \hat{\sigma}_\xi$, then there exists a partially-separating Bayesian equilibrium in which the central bank announces the size of its intervention if and only if $\xi \geq \hat{\xi}(\nu)$. In this equilibrium, the threshold function $\hat{\xi}(\nu)$ is positive and decreasing in ν.*

The proof of Theorem 6.4 is in Section 8. As in the previous theorem, this theorem states that there exists an equilibrium in which the foreign central bank chooses a transparent policy if the exchange rate is sufficiently overvalued relative to fundamentals. Unlike the previous equilibrium, the central bank in this equilibrium benefits from transparency because this signals to investors that the exchange rate is overvalued (which compels rational investors to sell peso bonds and lower the exchange rate). Transparency, however, also lowers the risk premium on peso bonds and thus raises the peso exchange rate, creating an incentive for the central bank to make no announcement once the peso is sufficiently undervalued. These two contrasting effects make it possible to construct a partially-separating equilibrium.

The most notable difference between the equilibrium requirements of Theorems 6.3 and 6.4 is that the latter theorem insists that $\theta_\nu > \hat{\theta}_\nu$. By Theorem 2.4, this assumption implies that $\lambda > \tilde{\lambda}$. While this assumption serves primarily to aid with the technical details of the theorem, it also has an interesting and plausible economic interpretation. Recall from the discussion of the benchmark model in Section 2 that θ_ν measures the extent of information about fundamentals that the foreign central bank can credibly reveal to the investors. If θ_ν

is large, then a central bank announcement reveals much relevant information to the public. It can be argued that a bank seeking to depreciate its currency has more resources available and is more likely to achieve that goal than a bank seeking to appreciate its currency. In terms of the model, this implies a larger value of θ_ν since any statements by the central bank must be taken seriously. Despite this motivation, an extension of both this and the previous game to consider the opposite case in which θ_ν is either smaller or larger than $\hat{\theta}_\nu$ might be a promising direction for future research.

7 Infinite-Horizon Model

Time is discrete and indexed by t and there are two countries. As in Section 2, I shall refer to the home country's currency as the dollar and the foreign country's currency as the peso. There is only one good for consumption and its price in each country is linked by the law of one price, so that $e_t + p_t^* = p_t$ for all $t \in \mathbb{N}$. As before, the exchange rate is defined as the dollar price of a peso, and its log in period t is given by e_t.

In this infinite-horizon extension, three assets are traded in each period: a nominal one-period bond issued by the domestic central bank with return i_t, a nominal one-period bond issued by the foreign central bank with return i_t^*, and a risk-free technology with real return r in each period. As in the two-period model, I assume that the domestic central bank credibly commits to a constant domestic price level in all periods so that the interest rate on dollar bonds i_t is equal to r for all $t \geq 1$. This price level is normalized so that $p_t = 0$, which implies that the log-linearized real return on foreign bonds in period t is equal to $-p_{t+1}^* - e_t + i_t^* = e_{t+1} - e_t + i_t^*$.

The foreign central bank's interest rate policy is more complicated in this setup. In particular, I assume that the foreign central bank follows a Wicksellian interest rate rule in which the price target is equal to zero.[25] This policy is subject to uncertainty, however, so that investors face risk when investing in peso bonds. Specifically, in each period t, the interest rate on peso bonds is given by $i_t^* = ap_t^* + f_t + r$, where f_t follows an autoregressive process of order one (AR(1)) and $a > 0$ is a constant that measures the response of interest rate policy to deviations from the price target. The stochastic process for interest rate deviations is given by $f_t = \rho_f f_{t-1} + \zeta_t$, where $0 < \rho_f < 1$ is a constant and ζ_t is i.i.d. normal, with mean zero and variance σ_ζ^2. The stochastic process for f_t is common knowledge among all investors, as is the value of f_t in period t since all current and past interest rates are publicly observable.

The economy is populated by overlapping generations of investors such that, in each

[25] Woodford (2003) provides a detailed discussion of the implications of Wicksellian, price-targeting interest rate rules in cashless economies such as this one.

period t, a new generation of investors is born while the old generation of investors dies.[26] Each newly born investor in period t chooses her portfolio and then, in period $t+1$, liquidates her positions and consumes all of her realized wealth before dying. As in the previous section, investors are indexed by $i \in [0,1]$ and each investor i born in period t solves the maximization problem

$$\max_{b_{it}\in\mathbb{R}} -E_{it}\exp\{-\gamma c_{it+1}\}, \qquad \text{subject to} \quad c_{it+1} = (1+i_t)w_{it} + (e_{t+1} - e_t + i_t^* - i_t)b_{it}, \quad (7.1)$$

where $w_{it} \in \mathbb{R}_{++}$ is investor i's endowment of real wealth at birth, $e_{t+1} - e_t + i_t^* - i_t$ is the log-linearized excess return of peso bonds in period t, b_{it} is the dollar amount of investor i's purchases of peso bonds in period t, c_{it+1} is the quantity of the economy's only good consumed by investor i in period $t+1$, $\gamma > 0$ is the coefficient of absolute risk aversion, and $E_{it}[\cdot]$ denotes the conditional expectation with respect to the information set of investor i in period t. The net supply of peso bonds is constant and equal to zero. In each period t, a mass of noise traders purchases ξ_t dollars worth of peso bonds, where ξ_t is i.i.d. normal, with mean zero and variance σ_ξ^2.[27] Noise traders liquidate all their assets from the previous period before making any purchases.

As in the two-period model, the foreign central bank complements its interest rate policy by performing foreign exchange interventions in each period. I assume specifically that the central bank purchases $\nu_t \in \mathbb{R}$ dollars worth of peso bonds in each period t and that these interventions follow an AR(1) process, so that $\nu_t = \rho_\nu \nu_{t-1} + \delta_t$, where $0 < \rho_\nu < 1$ is a constant and δ_t is i.i.d. normal, with mean zero and variance σ_δ^2. The stochastic process for ν_t is common knowledge among all investors.

This assumption implies that foreign exchange interventions affect exchange rate fundamentals only through their direct effects in this infinite-horizon model. Since the empirical evidence about these direct effects is inconclusive (especially over longer time horizons), I emphasize that this assumption is made only for expositional convenience and that it can be easily relaxed so that interventions also convey information about other exchange rate fundamentals. Indeed, none of this section's qualitative results changes if I assume that interventions are correlated with future interest rates.[28]

In this infinite-horizon model, I assume that in each period t each investor i receives the private signals $x_{it} = f_{t+1} + \epsilon_{it}$ and $y_{it} = \nu_t + \eta_{it}$, where $\epsilon_{it} \sim \mathrm{N}(0, \sigma_\epsilon^2)$, $\eta_{it} \sim \mathrm{N}(0, \sigma_\eta^2)$, ϵ_{it} and η_{it} are both i.i.d. and independent of each other, and all noise terms are independent across investors. Following Bacchetta and van Wincoop (2006), I also assume that the generation

[26] An alternative assumption is that investors live forever and have log preferences, with the risk-free interest rate then determined by the investors' patience. The difficulty with such a setup is that the model becomes intractable once higher-order expectations become part of the equilibrium as in Section 7.2.

[27] The assumption that noise traders' demand is i.i.d. is made for analytical convenience. The principal results do not change if the model is extended so that shocks to this demand persist over time.

[28] Suppose, for example, that the interest rate parameter f_{t+1} is split so that $f_{t+1} = f_{t+1}^0 + \theta_\nu f_{t+1}^\nu$ where $\nu_t = f_{t+1}^\nu$ and $\theta_\nu > 0$. In this case, all predictions remain the same except that increases in θ_ν have the same effect as increases in ρ_ν.

of investors that is born in period t inherits all of the private information from the generation that dies in period t. More precisely, I assume that in each period t, each newly born investor i inherits all of the private information of investor i from the generation born in period $t-1$.

I shall consider two different specifications for the investors' information. In the first, investors perfectly learn about past values of ν_t which causes higher-order expectations to collapse into more simple average beliefs.[29] The exchange rate can be characterized analytically in this setup, and the equilibrium is similar to the equilibrium from the two-period model in Section 2. It is not surprising, then, that most of the previous conclusions about transparency and exchange rate misalignment continue to be valid. In the second specification, investors do not learn about past values of ν_t so that higher-order expectations remain part of the equilibrium exchange rate. This, however, makes an analytic solution intractable as discussed by Bacchetta and van Wincoop (2006) and Lorenzoni (2009). As a consequence, I solve numerically for an approximate steady-state solution using results from Nimark (2010a). Before specifying the details of investors' information sets, it is useful to first solve for the equilibrium exchange rate without any assumptions about these information sets.

In this infinite-horizon setup, I adopt notation similar to that from the benchmark model in the previous section. For all $t \in \mathbb{N}$, let $\mathcal{F}_t$ denote the information set consisting of all common public information in period t together with ν_s and e_s for all $1 \leq s \leq t$ and f_s for all $1 \leq s \leq t+1$.[30] The aggregate demand for peso bonds by the investors in period t is equal to the average demand of the investors in period t and is denoted by $B_t = E\left[b_{it} \mid \mathcal{F}_t\right]$. It follows that the total demand for peso bonds in period t is equal to $B_t + \nu_t + \xi_t$.

Let $\overline{E}_t[\cdot] = E\left[E_{it}[\cdot] \mid \mathcal{F}_t\right]$ denote the average expectation of investors in period t, and let $\mathrm{Var}_{it}[\cdot]$ denote the conditional variance with respect to the information set of investor i in period t and $\overline{\mathrm{Var}}_t[\cdot] = E\left[\mathrm{Var}_{it}[\cdot] \mid \mathcal{F}_t\right]$ the average conditional variance of investors in period t. I denote higher-order expectations in this environment by $\overline{E}_t^0[\cdot] = \cdot$, $\overline{E}_t^1[\cdot] = \overline{E}_t[\cdot]$, and, in general, $\overline{E}_t^n[\cdot] = \overline{E}_t \overline{E}_{t+1} \cdots \overline{E}_{t+n-1}[\cdot]$. The information set of investor i in period t is denoted by $\mathcal{G}_{it}$. Finally, let $\mathcal{G}_{i0} = \varnothing$, $\sigma_t^2 = \overline{\mathrm{Var}}_t[e_{t+1}]$, and $\alpha = \frac{1}{1+a}$.

Definition 7.1. A steady-state equilibrium of this economy is a stochastic process for the exchange rate $\{e_t : t \in \mathbb{N}\}$, such that for all t: (i) the demand for peso bonds by each investor i solves the maximization problem (7.1), where investor i's information set $\mathcal{G}_{it}$ consists of all common public information in period t together with x_{it}, y_{it}, $\mathcal{G}_{it-1}$, and, if the foreign central bank announces its intervention in period t, ν_t as well; (ii) the peso bond market clears: $B_t + \xi_t + \nu_t = 0$; (iii) the exchange rate is a linear function of the demand for peso bonds by noise traders $\{\xi_s : 1 \leq s \leq t\}$, the foreign central bank's interventions $\{\nu_s : 1 \leq s \leq t\}$,

[29] Investors already learn about current and past values of f_t because interest rates are publicly observable.

[30] In this setup, investors observe signals of f_{t+1} in period t, so that in some sense (if the probability space and the corresponding filtration were explicitly defined) this interest rate parameter is measurable with respect to time t.

and the interest rate parameters $\{f_s : 1 \leq s \leq t+1\}$; (iv) the exchange rate is in a steady state: there exists $\sigma^2 > 0$ such that $\sigma_t^2 = \sigma^2$ in all periods $t \in \mathbb{N}$.

Lemma 7.2. *Suppose that the conditional variance* $\mathrm{Var}_{it}[e_{t+1}]$ *is equal for all investors* $i \in [0,1]$ *in all periods* t *and that* e_{t+1} *is normally distributed conditional on the information set of investor* i *in period* t*. Then, a steady-state equilibrium exchange rate satisfies*

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [f_{t+n}] + \gamma\sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [\nu_{t+n}] + \alpha\gamma\sigma^2 \xi_t. \tag{7.2}$$

Proof. If e_{t+1} is normally distributed conditional on the information set of investor i in period t, then problem (7.1) is a standard CARA-normal maximization and the demand for peso bonds by investor i in period t is given by

$$b_{it} = \frac{E_{it}[e_{t+1}] - e_t + i_t^* - i_t}{\gamma \, \mathrm{Var}_{it}[e_{t+1}]}. \tag{7.3}$$

If the conditional variance $\mathrm{Var}_{it}[e_{t+1}]$ is equal for all investors $i \in [0,1]$, then $\mathrm{Var}_{it}[e_{t+1}] = \overline{\mathrm{Var}}_t[e_{t+1}] = \sigma_t^2$ and hence

$$B_t = \frac{\overline{E}_t[e_{t+1}] - e_t + i_t^* - i_t}{\gamma\sigma_t^2}. \tag{7.4}$$

Recall that in each period t, the total demand for peso bonds is equal to $B_t + \nu_t + \xi_t$ while the domestic and foreign interest rates are equal to r and $-ae_t + f_t + r$, respectively. In a steady-state equilibrium, $\sigma_t^2 = \sigma^2$ for all t, so that

$$B_t = \frac{\overline{E}_t[e_{t+1}] - (1+a)e_t + f_t}{\gamma\sigma^2}, \tag{7.5}$$

and then, by market clearing,

$$e_t = \alpha \overline{E}_t[e_{t+1}] + \alpha f_t + \alpha\gamma\sigma^2(\nu_t + \xi_t). \tag{7.6}$$

The noise traders' demand is i.i.d. over time, so it follows that $\overline{E}_t[\xi_{t+n}] = 0$ for all $n \geq 1$. Forward iteration of equation (7.6), then, yields

$$e_t = \alpha^2 \overline{E}_t \overline{E}_{t+1}[e_{t+2}] + \alpha^2 \overline{E}_t[f_{t+1}] + \alpha f_t + \alpha^2\gamma\sigma^2 \overline{E}_t[\nu_{t+1}] + \alpha\gamma\sigma^2\nu_t + \alpha\gamma\sigma^2\xi_t \tag{7.7}$$

$$= \alpha^3 \overline{E}_t^3[e_{t+3}] + \sum_{n=0}^{2} \alpha^{n+1} \overline{E}_t^n [f_{t+n}] + \gamma\sigma^2 \sum_{n=0}^{2} \alpha^{n+1} \overline{E}_t[\nu_{t+n}] + \alpha\gamma\sigma^2\xi_t. \tag{7.8}$$

Finally, as demonstrated above, repeated forward iteration implies that the equilibrium exchange rate in period t must satisfy

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [f_{t+n}] + \gamma\sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [\nu_{t+n}] + \alpha\gamma\sigma^2 \xi_t, \tag{7.9}$$

which completes the proof. $\square$

In order to keep the analysis tractable in this infinite-horizon model, I focus only on steady-state equilibria in which the foreign central bank either announces the size of its intervention ν_t in each period t or never announces its intervention. In reality, however, central banks switch between these two policies so that the true steady-state equilibrium is somewhere in between these two extremes. If investors have common knowledge of the past, then the implication of this is only that the true steady-state variances and risk premia with and without transparency are much closer together (depending on assumptions about the probability of switching from one transparency regime to another). This implies that the truth-telling and signal-precision effects are even more important determinants of the effects of transparency on exchange rate misalignment.

If investors do not have common knowledge of the past, then the true steady-state equilibria are more difficult to characterize. In particular, the fact that investors learn ν_t forever once the foreign central bank makes an announcement implies that they will never again be perpetually disparately informed about interventions, even if higher-order expectations remain in equilibrium. This makes the equilibrium without transparency more similar to the equilibrium if investors have common knowledge of the past, although the importance of this past observation diminishes the longer the foreign central bank goes without making another announcement.

7.1 Common Knowledge of the Past

Suppose that in each period $t > 1$, the value of the previous period's intervention ν_{t-1} becomes common knowledge among all investors. This assumption implies that the higher-order expectations from equation (7.2) collapse into more simple average expectations.

In the next section, I relax the assumption about public revelation of ν_{t-1} and also assume that the interest rate on peso bonds depends on a factor that is not perfectly observed. This creates an environment where higher-order expectations are an important part of the equilibrium steady state regardless of whether or not the foreign central bank announces the value of its intervention ν_t. In this case, the transitory demand of noise traders has persistent effects on the exchange rate. I demonstrate that transparency can magnify the persistent effect of this noise, in addition to magnifying its immediate effect as in this and the previous section's models.

This section's assumptions about the investors' information yield an equilibrium exchange rate that is similar to the two-period model analyzed in Section 2. In doing so, this section provides an interpretation of the exchange rate fundamentals from that benchmark model, with those fundamentals now equal to the time-discounted sum of spreads between foreign and domestic interest rates plus the time-discounted sum of risk premia. The discount factor is determined by the parameter $\alpha = \frac{1}{1+a}$, which measures the sensitivity of the foreign central bank's interest rate rule to deviations from the price target.

To better see this connection, recall that exchange rate fundamentals in the benchmark model are given by $f = \theta_f f_0 + \theta_\nu f_\nu$ (this is equation (2.3)), where f_0 represents the part of fundamentals that is unrelated to the foreign central bank's intervention and f_ν represents the part of fundamentals that is related to this intervention. The bank's interventions are independent of interest rates and other disturbances in this infinite-horizon setup, so $\theta_f f_0$ is replaced by the time-discounted sum of spreads between foreign and domestic interest rates (the first term in equation (7.2) from Lemma 7.2) and $\theta_\nu f_\nu$ is replaced by the time-discounted sum of risk premia (the second term in equation (7.2) from Lemma 7.2). As I show below, the extent of the relationship between the central bank's intervention and the time-discounted sum of risk premia in this setup is highly dependent on the persistence of interventions ρ_ν. Not surprisingly, then, this setup reproduces many of the two-period setup's predictions with ρ_ν replacing the parameter θ_ν.

I present the equilibrium exchange rate with no central bank announcement about ν_t before presenting the equilibrium exchange rate with a central bank announcement. These two cases are then compared, and the implications of transparency are stated and discussed. As always, I assume that an announcement by the foreign central bank is truthful and credible. All proofs from this section are in Section 8.

Theorem 7.3. *If the value of ν_{t-1} becomes common knowledge among all investors in period t, then the steady-state equilibrium exchange rate is given by*

$$e_t = (\alpha - \rho_f \beta_f) f_t + (\psi_f + \beta_f) f_{t+1} - \rho_\nu \beta_\nu \nu_{t-1} + (\psi_\nu + \beta_\nu)\nu_t + \lambda \xi_t, \tag{7.10}$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$, $\psi_\nu = \frac{\alpha\gamma\sigma^2}{1-\alpha\rho_\nu}$ *and* $\lambda, \beta_f, \beta_\nu$, *and* σ^2 *are such that*

$$\lambda = \frac{\lambda\psi_f(\psi_f+\beta_f)(\sigma_\eta^2+\sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + \lambda\alpha\rho_\nu\psi_\nu(\psi_\nu+\beta_\nu)(\sigma_\epsilon^2+\sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2}{\Psi} + \alpha\gamma\sigma^2, \tag{7.11}$$

$$\beta_f = \frac{\alpha\rho_\nu\psi_\nu(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\delta^2 - \psi_f\left((\sigma_\eta^2+\sigma_\delta^2)\lambda^2\sigma_\xi^2 + (\psi_\nu+\beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right)\sigma_\epsilon^2}{\Psi}, \tag{7.12}$$

$$\beta_\nu = \frac{\psi_f(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\delta^2 - \alpha\rho_\nu\psi_\nu\left((\sigma_\epsilon^2+\sigma_\zeta^2)\lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right)\sigma_\eta^2}{\Psi}, \tag{7.13}$$

$$\begin{aligned}\sigma^2 = {} & \frac{\psi_f^2}{\alpha^2}\sigma_\epsilon^2 + \rho_\nu^2\psi_\nu^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\zeta^2 + (\psi_\nu+\beta_\nu)^2\sigma_\delta^2 \\ & - \frac{\psi_f^2\sigma_\epsilon^4}{\alpha^2\Psi}\left[(\sigma_\eta^2+\sigma_\delta^2)\left(\lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\zeta^2\right) + (\psi_\nu+\beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right] \\ & - \frac{\rho_\nu^2\psi_\nu^2\sigma_\eta^4}{\Psi}\left[(\sigma_\epsilon^2+\sigma_\zeta^2)\left(\lambda^2\sigma_\xi^2 + (\psi_\nu+\beta_\nu)^2\sigma_\delta^2\right) + (\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right] \\ & - \frac{2\rho_\nu\psi_f\psi_\nu}{\alpha\Psi}(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\sigma_\delta^2,\end{aligned} \tag{7.14}$$

with $\Psi = (\psi_f+\beta_f)^2(\sigma_\eta^2+\sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + (\psi_\nu+\beta_\nu)^2(\sigma_\epsilon^2+\sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2 + (\sigma_\epsilon^2+\sigma_\zeta^2)(\sigma_\eta^2+\sigma_\delta^2)\lambda^2\sigma_\xi^2$.

If a real-valued solution to the system of equations given by Theorem 7.3 exists, then there exist two real solutions distinguished by the value of the steady-state variance σ^2. A thorough discussion of the viability of these multiple equilibria is beyond the scope of this chapter, but in general, the high-variance equilibrium is not stable in the sense that any perceived deviation of the variance from this steady-state value generates an even larger actual deviation from that steady state.[31] With this instability in mind, I follow Bacchetta and van Wincoop (2006) and focus primarily on the low-variance steady-state equilibrium exchange rate. I emphasize that all of the results I present in Theorem 7.5 below apply also to the high-variance equilibria with and without transparency.

Equation (7.10) from Theorem 7.3 implies that the exchange rate in period $t+1$ is given by

$$\begin{aligned} e_{t+1} &= (\alpha - \rho_f\beta_f)f_{t+1} + (\psi_f+\beta_f)f_{t+2} - \rho_\nu\beta_\nu\nu_t + (\psi_\nu+\beta_\nu)\nu_{t+1} + \lambda\xi_{t+1} \\ &= \frac{\psi_f}{\alpha}f_{t+1} + \psi_\nu\rho_\nu\nu_t + \lambda\xi_{t+1} + (\psi_f+\beta_f)\zeta_{t+2} + (\psi_\nu+\beta_\nu)\delta_{t+1}. \end{aligned} \tag{7.15}$$

In the benchmark two-period model, the exchange rate in period two is given by $e_2 = f + \kappa$, with $f = \theta_f f_0 + \theta_\nu\nu$, so there are clearly similarities between that setup and this infinite-horizon setup. In particular, equation (7.15) shows that this model's expression for e_{t+1} is

[31] Consider any positive $\sigma_0^2 \neq \sigma^2$. One implication of this instability is that if investors observe past variances of the exchange rate and choose σ_t^2 in each period t as a weighted average of these past, observed variances, then σ_t^2 will never converge to the high-variance equilibrium value of σ^2.

the same as that model's expression for e_2, with θ_f replaced by $\frac{\psi_f}{\alpha}$, f_0 replaced by f_{t+1}, θ_ν replaced by $\rho_\nu\psi_\nu$, ν replaced by ν_t, and κ replaced by $\lambda\xi_{t+1} + (\psi_f + \beta_f)\zeta_{t+2} + (\psi_\nu + \beta_\nu)\delta_{t+1}$. As mentioned earlier, this model's transparency results are much like those from Section 2, with θ_ν now replaced by $\rho_\nu\psi_\nu$.

Before presenting these results, it is first necessary to characterize the steady-state equilibrium exchange rate when the foreign central bank makes a credible and truthful announcement of its intervention in period t. As in the benchmark model, let $\tilde{e}_t$ denote the exchange rate in period t if the central bank announces the value of ν_t to the investors in period t.

Theorem 7.4. *If the foreign central bank credibly and publicly announces the value of ν_t in period t, then the steady-state equilibrium exchange rate is given by*

$$\tilde{e}_t = (\alpha - \rho_f\tilde{\beta}_f)f_t + (\psi_f + \tilde{\beta}_f)f_{t+1} + \psi_\nu\nu_t + \tilde{\lambda}\xi_t, \tag{7.16}$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$, $\psi_\nu = \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}$, and $\tilde{\lambda}, \tilde{\beta}_f$, and $\tilde{\sigma}^2$ are such that

$$\tilde{\lambda} = \frac{\tilde{\lambda}\psi_f(\psi_f + \tilde{\beta}_f)\sigma_\epsilon^2\sigma_\zeta^2}{(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2} + \alpha\gamma\tilde{\sigma}^2, \tag{7.17}$$

$$\tilde{\beta}_f = -\frac{\psi_f\sigma_\epsilon^2\tilde{\lambda}^2\sigma_\xi^2}{(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}, \tag{7.18}$$

$$\tilde{\sigma}^2 = \frac{\psi_f^2}{\alpha^2}\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2 + \psi_\nu^2\sigma_\delta^2 - \frac{\psi_f^2\sigma_\epsilon^4\left(\tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2\right)}{\alpha^2(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + \alpha^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}. \tag{7.19}$$

In this infinite-horizon model, investors know both the values of f_t and ν_{t-1} (and also ν_t in the case of transparency) and the stochastic processes for these variables. This implies that investors have common priors about the values of f_{t+1} and ν_t, a fact that shows up in Theorems 7.3 and 7.4 in the form of the parameters β_f, β_ν, and $\tilde{\beta}_f$. While these extra parameters complicate the equilibrium exchange rate expressions, the parameters λ and $\tilde{\lambda}$ still measure the extent of exchange rate misalignment as a result of noise traders' demand while the differences $\lambda - \alpha\gamma\sigma^2$ and $\tilde{\lambda} - \alpha\gamma\tilde{\sigma}^2$ still measure the bias of investors' expectations of fundamentals as a result of this demand.

Theorem 7.5. *The parameters λ and $\tilde{\lambda}$ satisfy*

$$\lim_{\sigma_\epsilon\to\infty}\lambda > \lim_{\sigma_\epsilon\to\infty}\tilde{\lambda} = 0, \qquad \lim_{\sigma_\xi\to 0}\lambda < \lim_{\sigma_\xi\to 0}\tilde{\lambda} = \infty,$$
$$\lim_{\sigma_\zeta\to 0}\lambda = \lim_{\sigma_\zeta\to 0}\tilde{\lambda} = 0, \qquad \lim_{\sigma_\delta\to 0}\lambda = \lim_{\sigma_\delta\to 0}\tilde{\lambda} > 0.$$

The limits of both λ and $\tilde{\lambda}$ as either σ_ξ, σ_ζ, or σ_δ increases to infinity are undefined since the systems of equations that define the steady-state equilibria cease to have real solutions in those limits. Theorem 7.5 establishes several comparative statics for the parameters λ and $\tilde{\lambda}$, many of which reproduce results from the benchmark model (see Theorem 2.4).

As shown by equation (7.15) above, the product $\rho_\nu \psi_\nu$ in this model replaces the parameter θ_ν from the two-period model of Section 2. This product is equal to the time-discounted sum of future risk premia, and the term ρ_ν measures the persistence of the foreign central bank's interventions and hence the extent to which an intervention in period t affects peso bond risk premia in future periods (more persistence implies more effect). Since future risk premia are part of exchange rate fundamentals, a higher value of ρ_ν implies that the central bank's intervention in period t has a larger effect on those fundamentals. In other words, the truth-telling effect of transparency is increasing in ρ_ν in this infinite-horizon model.

Figures 7.1 - 7.4 show that the parameter λ tends to be less than $\tilde{\lambda}$ for smaller values of ρ_ν and greater than $\tilde{\lambda}$ for larger values of ρ_ν. These figures are similar to the parameterizations of the benchmark two-period model given by Figures 2.1 - 2.4, and they show that λ is again increasing relative to $\tilde{\lambda}$ as the extent of information revealed by a central bank announcement increases. Although this section's parameterizations all generate standard deviations for changes in the exchange rate that are roughly consistent with what is observed in quarterly data, the spirit of these numerical exercises is to illustrate the mechanism by which exchange rate misalignment can be magnified rather than to create a quantitatively precise simulation. Indeed, all of the models that I discuss are highly stylized and intended to explore and characterize the interaction between the truth-telling and signal-precision effects of transparency rather than to produce a precise model of exchange rate determination.

The first, baseline parameterization, depicted in Figure 7.1, features a choice of parameters that yields an unconditional standard deviation of ten percent for changes in the exchange rate (this is roughly consistent with the data). The second parameterization, depicted by Figure 7.2, presents this same parameterization except the variance of investors' private signals about the central bank's intervention σ_η is smaller. This has the effect of bringing λ and $\tilde{\lambda}$ closer together. The third parameterization, depicted in Figure 7.3, presents the same parameterization as in Figure 7.2 except that now the unpredictability of noise traders σ_ξ is smaller. This has the effect of increasing both λ and $\tilde{\lambda}$. Finally, Figure 7.4 presents the same parameterization as in Figure 7.3 except that now the persistence of innovations in the interest rate on peso bonds is smaller (ρ_f is smaller). This has the effect of decreasing both λ and $\tilde{\lambda}$.

The behavior of λ and $\tilde{\lambda}$ in these figures is very similar to the behavior shown graphically in the benchmark model. Indeed, the main conclusion to draw from this infinite-horizon model with common knowledge of the past is that the results largely reproduce the results from the two-period model. This is important because it shows that the previous discussion about truth-telling and signal-precision effects of transparency and its implications for central bank intervention policy are perfectly consistent with a richer infinite-horizon setup.

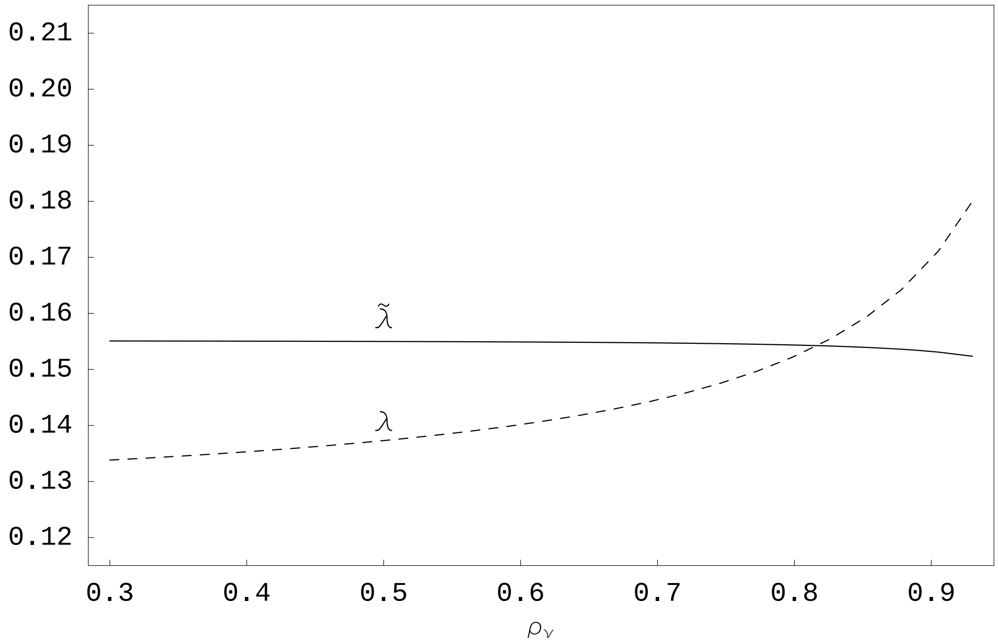

Figure 7.1: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.35$, $\sigma_\xi = 0.12$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$)

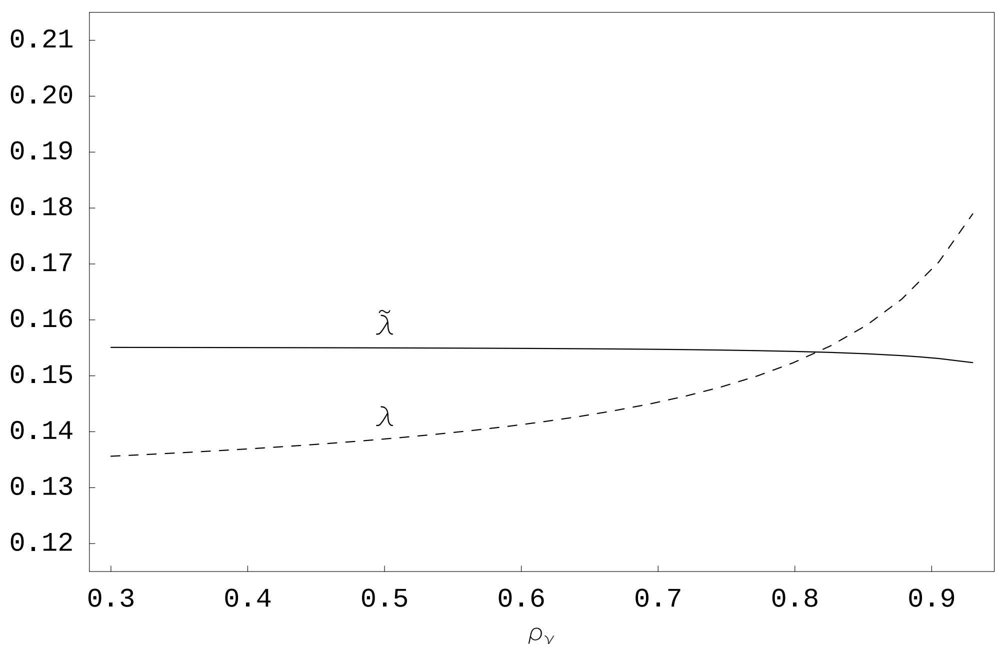

Figure 7.2: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.12$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$)

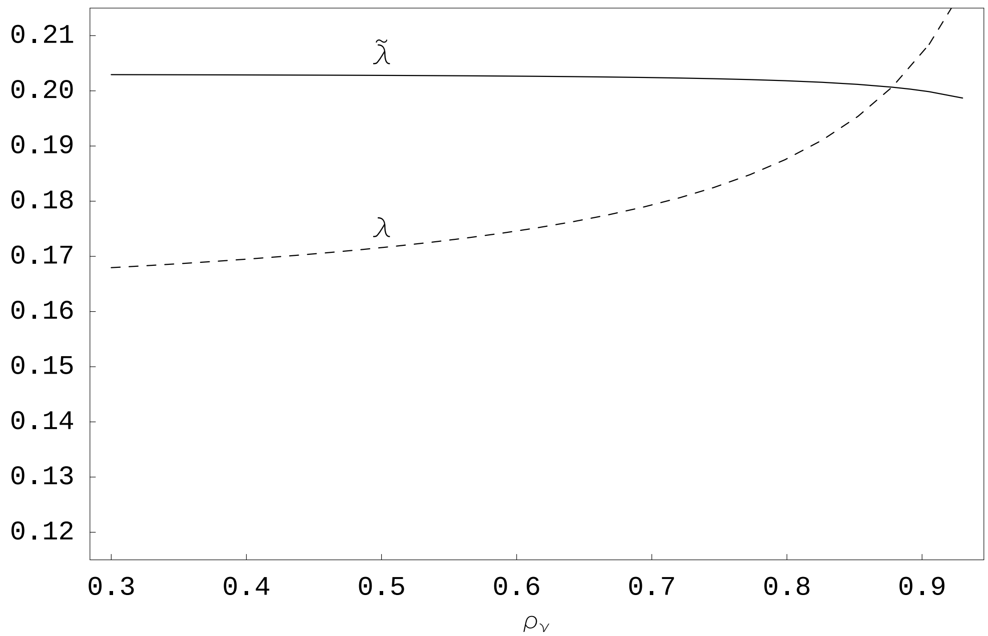

Figure 7.3: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.1$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$)

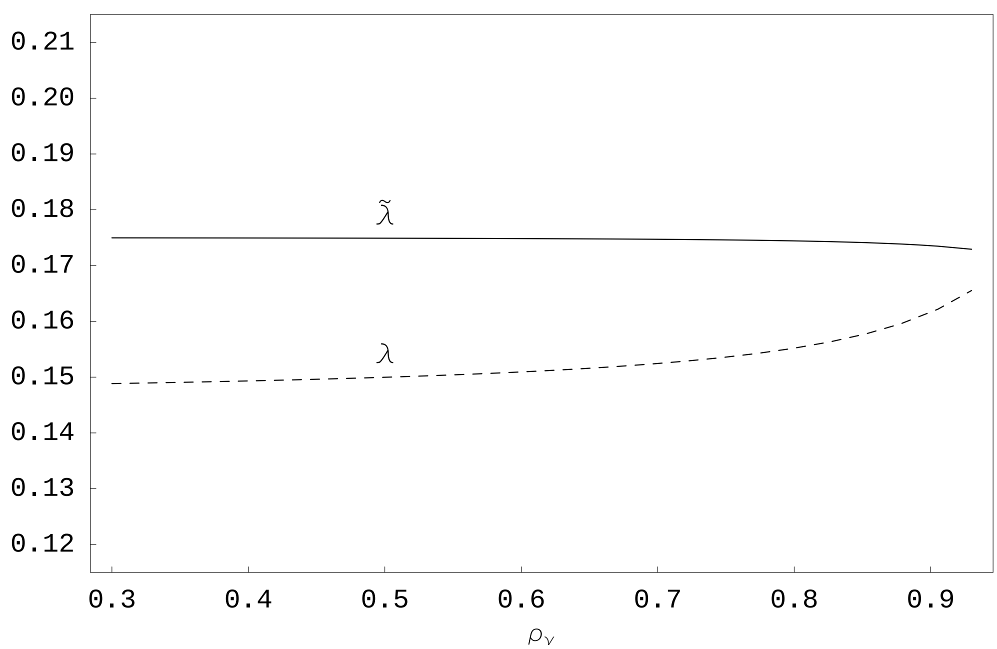

Figure 7.4: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.1$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.55$)

7.2 Imperfect Common Knowledge of the Past

Suppose that the value of ν_{t-1} does not become common knowledge among all investors in period t. Suppose also that the interest rate on peso bonds in period t is now given by $i_t^* = ap_t^* + f_t + \chi_t + r$, where χ_t is i.i.d. normal with mean zero and variance σ_χ^2. Since investors only observe i_t^* and p_t^* in each period t, these assumptions imply that investors have imperfect common knowledge about the value of f_t and, if the central bank does not announce the size of its intervention, also about the value of ν_t. It follows that higher-order expectations are always part of the equilibrium exchange rate.

There have been a number of dynamic macroeconomic models that feature higher-order expectations, including the early models of Townsend (1983) and Singleton (1987), and more recently, the models of Bacchetta and van Wincoop (2006) and Lorenzoni (2009). With the exception of Townsend (1983), all of these setups cannot be solved directly and must instead be approximated. This is usually accomplished by assuming that the past exogenously becomes common knowledge with some lag, a technique that keeps the state space in these models finite and makes it possible to solve for the steady-state equilibrium using standard methods. There is, however, another technique for solving these models as described by Nimark (2010a). Rather than assuming that the past becomes common knowledge, Nimark (2010a) shows that the steady-state equilibrium of a model in which agents are perpetually disparately informed can be approximated arbitrarily well by exogenously bounding the order of agents' expectations. As this bound grows to infinity, the approximate equilibrium converges to the true equilibrium.

In this section, I use this technique to consider the equilibrium of this infinite-horizon model when investors do not have common knowledge of the past. In models with higher-order expectations such as this one, it is typical for transitory shocks to have permanent effects on the beliefs of agents, as shown by Allen, Morris, and Shin (2006), Bacchetta and van Wincoop (2008), Lorenzoni (2009), and Nimark (2010b). Although these permanent effects diminish over time, they still introduce substantial excess volatility and disconnect between prices and fundamentals. The goal of this extension is to examine how the persistent effects of transitory changes in noise traders' demand for peso bonds compare with and without foreign central bank transparency. Consistent with all the other results in this and the previous chapter, I find that central bank transparency often worsens the exchange rate misalignment caused by transitory shocks to noise traders' demand in the past. In these cases, persistent deviations of the exchange rate from its fundamental value are magnified by transparency.

Before presenting this section's results, it is necessary to introduce some notation. Let $\bar{i}_t = i_t^* - ap_t^* - r$, and note that in each period t, investors observe the common public signal $\bar{i}_t = f_t + \chi_t$ but are unable to infer the value of f_t because of the unobserved disturbance χ_t. Furthermore, in order to maintain symmetry and simplify the solution, suppose now that each investor i observes the private signal $x_{it} = f_t + \epsilon_{it}$ rather than the private signal $x_{it} = f_{t+1} + \epsilon_{it}$ in each period t.[32] Strictly speaking, the definition of a steady-state equilibrium exchange

[32]This assumption is without loss of generality since $\rho_f x_{it} = \rho_f f_t + \rho_f \epsilon_{it} = f_{t+1} - \zeta_{t+1} + \rho_f \epsilon_{it}$, and hence

rate 7.1 must now be appended to include the disturbances χ_s for all $1 \leq s \leq t$ and to restrict the equilibrium to be a function of f_s only for all $1 \leq s \leq t$ (rather than for all $1 \leq s \leq t+1$). For the sake of brevity, I only mention these technical details rather than restating the full definition of equlibrium.

The equilibrium exchange rate in this setup is expressed as a function of higher-order expectations at time t only, so let $\overline{E}(0)_t[\cdot] = \cdot$, $\overline{E}(1)_t[\cdot] = \overline{E}_t[\cdot]$, and in general, $\overline{E}(j)_t[\cdot] = \overline{E}_t\overline{E}_t \cdots \overline{E}_t[\cdot]$ with the expectation repeated j times. For all $0 \leq j \leq k$, let

$$q_{jt} = \left(\overline{E}(j)_t[f_t] \quad \overline{E}(j)_t[\nu_t]\right)', \tag{7.20}$$

and for all $t \in \mathrm{N}$, let

$$Q_t(k) = \left(q'_{0t} \quad q'_{1t} \quad \cdots \quad q'_{kt}\right)', \tag{7.21}$$

$$w_t = \left(\sigma_\zeta^{-1}\zeta_t \quad \sigma_\delta^{-1}\delta_t \quad \sigma_\chi^{-1}\chi_t \quad \sigma_\xi^{-1}\xi_t\right)'. \tag{7.22}$$

Let $h_1 = (\,1\ 0\ 0\ \cdots)'$ and $h_2 = (\,0\ 1\ 0\ 0\ \cdots)'$, and let the matrix H be given by

$$H = \begin{pmatrix} \mathbf{0}_{2k+2\times 2} & \begin{matrix} I_{2k} \\ \mathbf{0}_{2\times 2k} \end{matrix} \end{pmatrix}, \tag{7.23}$$

where I_{2k} is equal to the identity matrix of dimension $2k$. This matrix evaluates the average expectation of a vector and then annihilates the highest-order expectation, so that

$$HQ_t(k) = \left(q'_{1t} \quad q'_{2t} \quad \cdots \quad q'_{kt} \quad 0 \quad 0\right)' = \left(\overline{E}_t[q'_{0t}] \quad \overline{E}_t[q'_{1t}] \quad \cdots \quad \overline{E}_t[q'_{k-1t}] \quad 0 \quad 0\right)'. \tag{7.24}$$

All proofs from this section are in Section 8.

Theorem 7.6. *If the interest rate on peso bonds is given by $i^*_t = ap^*_t + f_t + \chi_t + r$ in each period t and the value of ν_{t-1} does not become common knowledge among all investors in period t, then the steady-state equilibrium exchange rate is approximately given by the system of equations*

$$e_t = AQ_t(k) + \alpha\gamma\sigma^2\xi_t, \tag{7.25}$$

$$Q_t(k) = MQ_{t-1}(k) + Nw_t, \tag{7.26}$$

where the vector A satisfies

$$A = \sum_{n=0}^{\infty} \alpha^{n+1}(h'_1 + \gamma\sigma^2 h'_2)(MH)^n. \tag{7.27}$$

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a private signal of f_t is also a private signal of f_{t+1}.

As the order of truncation k grows to infinity, the solution to this system of equations converges to the true steady-state equilibrium exchange rate.

If the foreign central bank announces the value of ν_t in period t, then investors continue to have imperfect common knowledge about f_t while commonly learning the value of ν_t. In order to characterize the equilibrium exchange rate in this case, it is necessary again to introduce more notation. For all $0 \leq j \leq k$, let $\tilde{q}_{jt} = \overline{E}(j)_t[f_t]$, and for all $t \in \mathrm{N}$, let

$$\tilde{Q}_t(k) = \begin{pmatrix} \tilde{q}_{0t} & \tilde{q}_{1t} & \cdots & \tilde{q}_{kt} \end{pmatrix}', \tag{7.28}$$

$$\tilde{H} = \begin{pmatrix} \mathbf{0}_{k+1\times 1} & \begin{matrix} I_k \\ \mathbf{0}_{1\times k} \end{matrix} \end{pmatrix}, \tag{7.29}$$

$$\tilde{w}_t = \begin{pmatrix} \sigma_\zeta^{-1}\zeta_t & \sigma_\chi^{-1}\chi_t & \sigma_\xi^{-1}\xi_t \end{pmatrix}'. \tag{7.30}$$

Theorem 7.7. *If the interest rate on peso bonds is given by $i_t^* = ap_t^* + f_t + \chi_t + r$ in each period t and the foreign central bank credibly and publicly announces the value of ν_t in each period t, then the steady-state equilibrium exchange rate is approximately given by the system of equations*

$$\tilde{e}_t = \tilde{A}\tilde{Q}_t(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t, \tag{7.31}$$

$$\tilde{Q}_t(k) = \tilde{M}\tilde{Q}_{t-1}(k) + \tilde{N}\tilde{w}_t, \tag{7.32}$$

where the vector $\tilde{A}$ satisfies

$$\tilde{A} = \sum_{n=0}^{\infty} \alpha^{n+1} h_1'(\tilde{M}\tilde{H})^n. \tag{7.33}$$

As the order of truncation k grows to infinity, the solution to this system of equations converges to the true steady-state equilibrium exchange rate.

The matrices M and N and the steady-state variance σ^2 from Theorem 7.6 as well as the matrices $\tilde{M}$ and $\tilde{N}$ and the steady-state variance $\tilde{\sigma}^2$ from Theorem 7.7 must all be approximated numerically. They are determined by the solution to two systems of matrix equations as detailed in Section 8. As in Section 7.1, there are two solutions to both systems of equations, one corresponding to a high-variance steady state and the second corresponding to a low-variance steady state. Numerical approximations indicate that the high-variance steady state is unstable in the sense described earlier.

In Figure 7.5, I plot the response of the steady-state equilibrium exchange rates with and without transparency to a negative shock to the noise traders' demand for peso bonds in period t_0. This shock is normalized so that the exchange rate with transparency $\tilde{e}_t$ decreases five percent in period t_0. The persistent effect of this transitory shock is plotted over time.

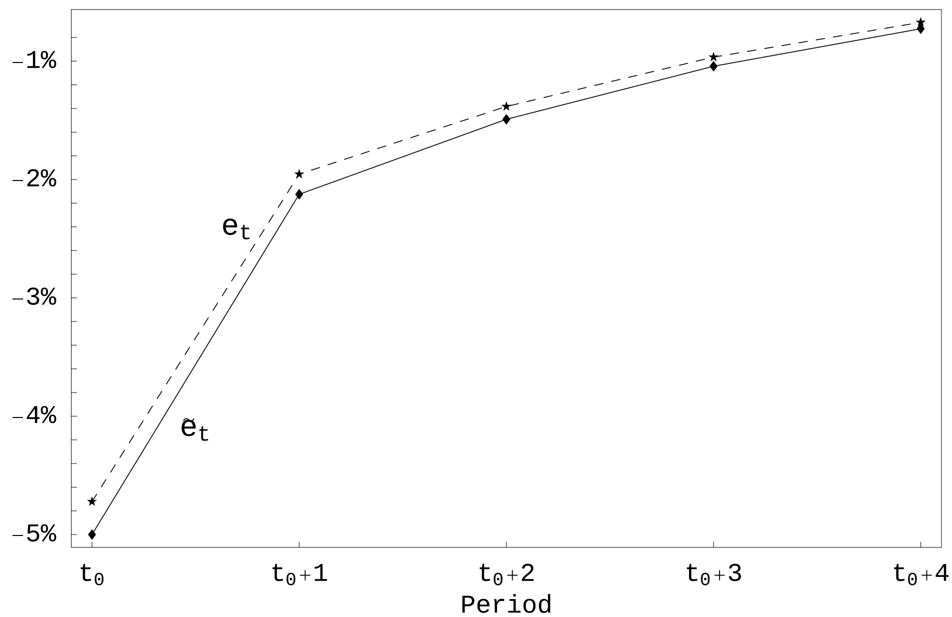

Figure 7.5: The response of the exchange rate with and without transparency to a shock to the noise traders' demand for peso bonds ξ_t in period t_0. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.35$, $\sigma_\xi = 0.1$, $\sigma_\zeta = 0.03$, $\sigma_\delta = 0.07$, $\sigma_\chi = 0.005$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$, $\rho_\nu = 0.1$, $k = 50$)

The parameterization shown in Figure 7.5 is similar to the baseline parameterization shown in Figure 7.1 from the previous section. The main difference is that the variance terms σ_ξ and σ_ζ in this section's figure are slightly smaller in order to compensate for the extra noise term χ_t and to keep the unconditional variance of changes in the exchange rate close to ten percent (which is roughly consistent with the data). In the parameterization shown in the figure, higher-order expectations are truncated at $k = 50$. I find that the results do not change if this is increased even further.

The message of Figure 7.5 is similar to the message of Section 7.1: transparency magnifies exchange rate misalignment for low values of ρ_ν, even if that misalignment arises from shocks to noise traders' demand for peso bonds in the past. In particular, this result is a generalization of the previous sections' result that $\tilde{\lambda} > \lambda$ since the equilibrium exchange rate in period t is now a function of $\xi_{t-1}, \xi_{t-2}, \ldots$ as well as ξ_t, and the multipliers on all of these noise terms are larger if the foreign central bank is transparent. More precisely, the exchange rate in period t is now of the form $e_t = \lambda\xi_t + \lambda_1\xi_{t-1} + \lambda_2\xi_{t-2} + \cdots$ (with a corresponding expression for $\tilde{e}_t$), and for low values of ρ_ν my numerical approximations demonstrate that $\tilde{\lambda} > \lambda$, $\tilde{\lambda}_1 > \lambda_1$, $\tilde{\lambda}_2 > \lambda_2$, and so on. One implication of this result is that periods of sustained exchange rate misalignment are likely to imply large differences between mispricing with and without transparency as the larger multipliers with either policy start to add up.

The policy implication of this setup with higher-order expectations is similar to the implication in all previous sections. If central bank announcements reveal sufficiently partial information about exchange rate fundamentals, then the truth-telling effect is likely to be smaller than the signal-precision effect and transparency is likely to exacerbate exchange rate misalignment. This section shows that this applies also to misalignment between the exchange rate and fundamentals in the future, since both the immediate and persistent effects of temporary disturbances are magnified in a similar manner.

8 Proofs

Proof of Theorem 6.3 Suppose that the foreign central bank announces its intervention if and only if $\xi \geq \hat{\xi}(\nu)$, where $\hat{\xi}(\nu)$ is positive, bounded, and decreasing in ν. It is important to emphasize that investors only know the exact value of $\hat{\xi}$ if they learn ν via a central bank announcement, otherwise they are only aware of the equilibrium relationship between these variables.

Suppose that $\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu))$, where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by the solution to equations (2.11) and (2.12) from Theorem 2.3. Because investors observe that the foreign central bank has announced the value of ν, they all learn that $\xi \geq \hat{\xi}(\nu)$, which is equivalent to learning that $\tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}\hat{\xi}(\nu) - \theta_f f_0 \geq \tilde{\lambda}\hat{\xi}(\nu)$ (recall that $f = \theta_f f_0 + \theta_\nu\nu$ by equations (2.3) and (2.4)). Bayesian inference implies that for each investor i, the distribution of $\theta_f f_0$ conditional on investor i's information set is truncated normal, with mean $\theta_f x_i + \frac{\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2}\left(\tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}\hat{\xi}(\nu) - \theta_f x_i\right)$, variance $\frac{\tilde{\lambda}^2\sigma_\xi^2\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2}$, and truncation $\theta_f f_0 \leq \tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S)$.

The difference between the truncation and the mean of $\theta_f f_0$ is equal to

$$\frac{\tilde{\lambda}^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}\left(\tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}\hat{\xi}(\nu) - \theta_f x_i\right) - \tilde{\lambda}\hat{\xi}(\nu).$$

Because $\hat{\xi}(\nu)$ is positive for all $\nu \in [-\bar{\nu}, \bar{\nu}]$ and $\tilde{\lambda}\sigma_\xi \to 0$ as $\sigma_\xi \to 0$ (this is not hard to prove), it follows that this difference does not converge to a nonnegative value as $\sigma_\xi \to 0$. In this case, Lemma 8.1 implies (it is not difficult to show that the conditions of the lemma are satisfied) that

$$\lim_{\sigma_\xi\to 0} E_{i1}(T)\left[e^{-\theta_f f_0}\right] = \lim_{\sigma_\xi\to 0} e^{-\left(\tilde{e}_1-\mu-\theta_\nu\nu-\gamma\tilde{\sigma}_1^2(\nu-S)\right)+\frac{1}{2}\left(\frac{\tilde{\lambda}^2\sigma_\xi^2\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2}\right)}. \tag{8.1}$$

Utility is exponential, so equation (8.1) implies that each investor i's demand for peso bonds in period one satisfies

$$\lim_{\sigma_\xi\to 0} b_{i1} = \lim_{\sigma_\xi\to 0} \frac{(\tilde{e}_1 - \mu - \theta_\nu \nu - \gamma\tilde{\sigma}_1^2(\nu - S)) + \theta_\nu \nu - \tilde{e}_1 + \mu}{\gamma \operatorname{Var}_{i1}(T)[e_2]},$$

so that by dominated convergence

$$\begin{aligned}\lim_{\sigma_\xi\to 0} B_1 &= \lim_{\sigma_\xi\to 0} \frac{(\tilde{e}_1 - \mu - \theta_\nu \nu - \gamma\tilde{\sigma}_1^2(\nu - S)) + \theta_\nu \nu - \tilde{e}_1 + \mu}{\gamma\tilde{\sigma}_1^2} \\ &= \lim_{\sigma_\xi\to 0} \frac{\theta_f f_0 + \tilde{\lambda}(\xi - \hat{\xi}(\nu)) + \theta_\nu \nu - \tilde{e}_1 + \mu}{\gamma\tilde{\sigma}_1^2}.\end{aligned}$$

This last equality together with the market clearing condition for the peso bond market implies that

$$\begin{aligned}\lim_{\sigma_\xi\to 0} \tilde{e}_1 &= \lim_{\sigma_\xi\to 0} \mu + \theta_f f_0 + \theta_\nu \nu + \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu)) \\ &= \lim_{\sigma_\xi\to 0} \mu + f + \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu)),\end{aligned} \tag{8.2}$$

where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by the solution to equations (2.11) and (2.12) from Theorem 2.3. Of course, if $\tilde{e}_1 \to \mu + f + \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu))$ as $\sigma_\xi \to 0$, then all of the above statements are true in the limit and it follows that the limit relationship (8.2) indeed holds.

Suppose that $e_1 = \mu + f + \gamma\sigma_1^2(\nu - S) + \lambda\xi$, where λ and σ_1^2 are given by the solution to equations (2.6) and (2.7) from Theorem 2.2. Because investors observe that the foreign central bank has not announced the value of ν, they all learn that $\xi < \hat{\xi}(\nu)$ without learning the exact value of ν. This is equivalent to learning that $e_1 - \mu - f - \gamma\sigma_1^2(\nu - S) < \lambda\hat{\xi}(\nu)$. Bayesian inference implies that for each investor i, the distribution of f conditional on investor i's information set is truncated normal, with mean

$$\theta_f x_i + \theta_\nu y_i + \frac{\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\big(e_1 - \mu - \theta_f x_i - \theta_\nu y_i - \gamma\sigma_1^2(y_i - S)\big)$$

(recall again that $f = \theta_f f_0 + \theta_\nu \nu$), variance

$$\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 - \frac{\big(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2\big)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2},$$

and truncations $f > e_1 - \mu - \gamma\sigma_1^2(\nu - S) - \lambda\hat{\xi}(\nu)$ and $\theta_f f_0 - \theta_\nu\bar{\nu} \le f \le \theta_f f_0 + \theta_\nu\bar{\nu}$.

Conditional on knowing the value of ν, the difference between the first truncation and the mean of f is equal to

$$-\lambda\hat{\xi}(\nu) + \frac{\lambda^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \mu - \theta_f x_i - \theta_\nu \nu - \gamma\sigma_1^2(\nu - S)\right).$$

Because $\hat{\xi}(\nu)$ is positive for all $\nu \in [-\bar{\nu}, \bar{\nu}]$ and $\lambda\sigma_\xi \to 0$ as $\sigma_\xi \to 0$, it follows that the expectation of this difference does not converge to a positive value as $\sigma_\xi \to 0$. In this case, Lemma 8.1 implies (as before, it is not difficult to show that the conditions of the lemma are satisfied) that

$$\lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} E_{i1}(N)\left[e^{-f}\right] = \lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} \frac{e^{-\theta_f x_i - \theta_\nu y_i - \frac{\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \mu - \theta_f x_i - \theta_\nu y_i - \gamma\sigma_1^2(y_i - S)\right)}}{e^{-\frac{1}{2}\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\right)}}. \tag{8.3}$$

Utility is exponential, so equation (8.3) implies that each investor i's demand for peso bonds in period one satisfies

$$\lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} b_{i1} =$$

$$\lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} \frac{\theta_f x_i + \theta_\nu y_i + \frac{\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \mu - \theta_f x_i - \theta_\nu y_i - \gamma\sigma_1^2(y_i - S)\right) - e_1 + \mu}{\gamma \operatorname{Var}_{i1}(N)[e_2]},$$

so that by dominated convergence

$$\lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} B_1 = \lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} \frac{f + \frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\xi - e_1 + \mu}{\gamma\sigma_1^2}.$$

This last equality together with the market clearing condition for the peso bond market implies that

$$\lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} e_1 = \lim_{\sigma_\xi \to 0}\lim_{\bar{\nu}\to\infty} \mu + f + \gamma\sigma_1^2(\nu_1 - S) + \lambda\xi, \tag{8.4}$$

where λ and σ_1^2 are given by the solution to equations (2.6) and (2.7) from Theorem 2.2. Of course, if $e_1 \to \mu + f + \gamma\sigma_1^2(\nu - S) + \lambda\xi$ as $\sigma_\xi \to 0$ and $\bar{\nu} \to \infty$, then all of the above statements are true in the limit and it follows that the limit (8.4) indeed holds.

I have shown that if the foreign central bank announces its intervention if and only if $\xi \geq \hat{\xi}(\nu)$, where $\hat{\xi}(\nu)$ is positive and decreasing in ν, then as $\sigma_\xi \to 0$ and $\bar{\nu} \to \infty$, if there is a central bank announcement, the exchange rate in period one is arbitrarily close to $\mu + f + \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu))$, where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by the solution to equations (2.11) and (2.12), and if there is no central bank announcement, the exchange rate in period one is arbitrarily close to $\mu + f + \gamma\sigma_1^2(\nu - S) + \lambda\xi$, where λ and σ_1^2 are given by the solution to equations (2.6) and (2.7). The assumption that $\theta_\nu < \hat{\theta}_\nu$ guarantees that $\tilde{\lambda} > \lambda$. Furthermore, equations (2.11) and (2.12) imply that $\tilde{\sigma}_1^2 \to \sigma_\kappa^2$ as $\sigma_\xi \to 0$ (see Theorem 2.4), while equations (2.6) and (2.7) imply that $\lim_{\sigma_\xi \to 0} \sigma_1^2 > \sigma_\kappa^2$. It follows that as $\bar{\nu} \to \infty$ and $\sigma_\xi \to 0$, the difference $e_1 - \tilde{e}_1$ is arbitrarily close to

$$\gamma(\nu - S)(\sigma_1^2 - \sigma_\kappa^2) + \lambda\xi + \tilde{\lambda}(\hat{\xi}(\nu) - \xi).$$

As long as $S > \bar{\nu}$, then for each $\nu \in [-\bar{\nu}, \bar{\nu}]$, there exists $\hat{\xi}(\nu)$ such that $e_1 - \tilde{e}_1 = 0$ whenever $\xi = \hat{\xi}(\nu)$, $e_1 - \tilde{e}_1 < 0$ whenever $\xi > \hat{\xi}(\nu)$, and $e_1 - \tilde{e}_1 > 0$ whenever $\xi < \hat{\xi}(\nu)$ and such that $\hat{\xi}(\nu)$ is always strictly positive and decreasing in ν. □

Proof of Theorem 6.4 Suppose that the foreign central bank announces its intervention if and only if $\xi \geq \hat{\xi}(\nu)$, where $\hat{\xi}(\nu)$ is positive, bounded, and decreasing in ν. As in the previous proof, it is important to emphasize that investors only know the exact value of $\hat{\xi}$ if they learn ν via a central bank announcement, otherwise they are only aware of the equilibrium relationship between these variables.

Suppose that $\tilde{e}_1 = \mu + f + \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu))$, where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by the solution to equations (2.11) and (2.12) from Theorem 2.3. Because investors observe that the foreign central bank has announced the value of ν, they all learn that $\xi \geq \hat{\xi}(\nu)$, which is equivalent to learning that $\tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}\hat{\xi}(\nu) - \theta_f f_0 \geq \tilde{\lambda}\hat{\xi}(\nu)$ (recall that $f = \theta_f f_0 + \theta_\nu \nu$ by equations (2.3) and (2.4)). Bayesian inference implies that for each investor i, the distribution of $\theta_f f_0$ conditional on investor i's information set is truncated normal, with mean $\theta_f x_i + \frac{\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}\left(\tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}\hat{\xi}(\nu) - \theta_f x_i\right)$, variance $\frac{\tilde{\lambda}^2\sigma_\xi^2\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}$, and truncation $\theta_f f_0 \leq \tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S)$.

The difference between the truncation and the mean of $\theta_f f_0$ is equal to

$$\frac{\tilde{\lambda}^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}\left(\tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}\hat{\xi}(\nu) - \theta_f x_i\right) - \tilde{\lambda}\hat{\xi}(\nu).$$

Because $\hat{\xi}(\nu)$ is positive for all $\nu \in [-\bar{\nu}, \bar{\nu}]$ and $\tilde{\lambda}\sigma_\xi \to 0$ as $\sigma_\xi \to 0$ (this is not hard to prove), it follows that this difference does not converge to a nonnegative value as $\sigma_\xi \to 0$. In this case, Lemma 8.1 implies (it is not difficult to show that the conditions of the lemma are satisfied) that

$$\lim_{\sigma_\xi \to 0} E_{i1}(T)\left[e^{-\theta_f f_0}\right] = \lim_{\sigma_\xi \to 0} e^{-\left(\tilde{e}_1 - \mu - \theta_\nu\nu - \gamma\tilde{\sigma}_1^2(\nu - S)\right) + \frac{1}{2}\left(\frac{\tilde{\lambda}^2\sigma_\xi^2\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}\right)}. \tag{8.5}$$

Utility is exponential, so equation (8.5) implies that each investor i's demand for peso bonds in period one satisfies

$$\lim_{\sigma_\xi \to 0} b_{i1} = \lim_{\sigma_\xi \to 0} \frac{(\tilde{e}_1 - \mu - \theta_\nu \nu - \gamma \tilde{\sigma}_1^2(\nu - S)) + \theta_\nu \nu - \tilde{e}_1 + \mu}{\gamma \operatorname{Var}_{i1}(T)[e_2]},$$

so that by dominated convergence

$$\begin{aligned}\lim_{\sigma_\xi \to 0} B_1 &= \lim_{\sigma_\xi \to 0} \frac{(\tilde{e}_1 - \mu - \theta_\nu \nu - \gamma \tilde{\sigma}_1^2(\nu - S)) + \theta_\nu \nu - \tilde{e}_1 + \mu}{\gamma \tilde{\sigma}_1^2} \\ &= \lim_{\sigma_\xi \to 0} \frac{\theta_f f_0 + \tilde{\lambda}(\xi - \hat{\xi}(\nu)) + \theta_\nu \nu - \tilde{e}_1 + \mu}{\gamma \tilde{\sigma}_1^2}.\end{aligned}$$

This last equality together with the market clearing condition for the peso bond market implies that

$$\begin{aligned}\lim_{\sigma_\xi \to 0} \tilde{e}_1 &= \lim_{\sigma_\xi \to 0} \mu + \theta_f f_0 + \theta_\nu \nu + \gamma \tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu)) \\ &= \lim_{\sigma_\xi \to 0} \mu + f + \gamma \tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu)),\end{aligned} \tag{8.6}$$

where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by the solution to equations (2.11) and (2.12) from Theorem 2.3. Of course, if $\tilde{e}_1 \to \mu + f + \gamma \tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu))$ as $\sigma_\xi \to 0$, then all of the above statements are true in the limit and it follows that the limit relationship (8.6) indeed holds.

Suppose that $e_1 = \mu + f + \gamma \sigma_1^2(\nu - S) + \lambda \xi$, where λ and σ_1^2 are given by the solution to equations (2.6) and (2.7) from Theorem 2.2. Because investors observe that the foreign central bank has not announced the value of ν, they all learn that $\xi < \hat{\xi}(\nu)$ without learning the exact value of ν. This is equivalent to learning that $e_1 - \mu - f - \gamma \sigma_1^2(\nu - S) < \lambda \hat{\xi}(\nu)$. Bayesian inference implies that for each investor i, the distribution of f conditional on investor i's information set is truncated normal, with mean

$$\theta_f x_i + \theta_\nu y_i + \frac{\theta_f^2 \sigma_\epsilon^2 + \theta_\nu(\theta_\nu + \gamma \sigma_1^2)\sigma_\eta^2}{\theta_f^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\eta^2 + \lambda^2 \sigma_\xi^2} \big(e_1 - \mu - \theta_f x_i - \theta_\nu y_i - \gamma \sigma_1^2(y_i - S)\big)$$

(recall again that $f = \theta_f f_0 + \theta_\nu \nu$), variance

$$\theta_f^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2 - \frac{\big(\theta_f^2 \sigma_\epsilon^2 + \theta_\nu(\theta_\nu + \gamma \sigma_1^2)\sigma_\eta^2\big)^2}{\theta_f^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\eta^2 + \lambda^2 \sigma_\xi^2},$$

and truncations $f > e_1 - \mu - \gamma \sigma_1^2(\nu - S) - \lambda \hat{\xi}(\nu)$ and $\theta_f f_0 - \theta_\nu \bar{\nu} \le f \le \theta_f f_0 + \theta_\nu \bar{\nu}$.

Conditional on knowing the value of ν, the difference between the first truncation and the mean of f is equal to

$$-\lambda\hat{\xi}(\nu) + \frac{\lambda^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma_1^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \mu - \theta_f x_i - \theta_\nu \nu - \gamma\sigma_1^2(\nu - S)\right).$$

Because $\hat{\xi}(\nu)$ is positive for all $\nu \in [-\bar{\nu}, \bar{\nu}]$ and $\lambda\sigma_\xi \to 0$ as $\sigma_\xi \to 0$, it follows that the expectation of this difference does not converge to a positive value as $\sigma_\xi \to 0$. In this case, Lemma 8.1 implies (as before, it is not difficult to show that the conditions of the lemma are satisfied) that

$$\lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty} E_{i1}(N)\left[e^{-f}\right] = \lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty}\frac{e^{-\theta_f x_i - \theta_\nu y_i - \frac{\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2+(\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2+\lambda^2\sigma_\xi^2}\left(e_1-\mu-\theta_f x_i-\theta_\nu y_i-\gamma\sigma_1^2(y_i-S)\right)}}{e^{-\frac{1}{2}\left(\theta_f^2\sigma_\epsilon^2+\theta_\nu^2\sigma_\eta^2-\frac{\left(\theta_f^2\sigma_\epsilon^2+\theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2+(\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2+\lambda^2\sigma_\xi^2}\right)}}. \tag{8.7}$$

Utility is exponential, so equation (8.7) implies that each investor i's demand for peso bonds in period one satisfies

$$\lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty} b_{i1} =$$

$$\lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty}\frac{\theta_f x_i + \theta_\nu y_i + \frac{\theta_f^2\sigma_\epsilon^2+\theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2+(\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2+\lambda^2\sigma_\xi^2}\left(e_1 - \mu - \theta_f x_i - \theta_\nu y_i - \gamma\sigma_1^2(y_i - S)\right) - e_1 + \mu}{\gamma\,\mathrm{Var}_{i1}(N)[e_2]},$$

so that by dominated convergence

$$\lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty} B_1 = \lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty}\frac{f + \frac{\lambda\theta_f^2\sigma_\epsilon^2+\lambda\theta_\nu(\theta_\nu+\gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2+(\theta_\nu+\gamma\sigma_1^2)^2\sigma_\eta^2+\lambda^2\sigma_\xi^2}\xi - e_1 + \mu}{\gamma\sigma_1^2}.$$

This last equality together with the market clearing condition for the peso bond market implies that

$$\lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty} e_1 = \lim_{\sigma_\xi\to 0}\lim_{\bar{\nu}\to\infty} \mu + f + \gamma\sigma_1^2(\nu_1 - S) + \lambda\xi, \tag{8.8}$$

where λ and σ_1^2 are given by the solution to equations (2.6) and (2.7) from Theorem 2.2. Of course, if $e_1 \to \mu + f + \gamma\sigma_1^2(\nu - S) + \lambda\xi$ as $\sigma_\xi \to 0$ and $\bar{\nu} \to \infty$, then all of the above statements are true in the limit and it follows that the limit (8.8) indeed holds.

I have shown that if the foreign central bank announces its intervention if and only if $\xi \geq \hat{\xi}(\nu)$, where $\hat{\xi}(\nu)$ is positive and decreasing in ν, then as $\sigma_\xi \to 0$ and $\bar{\nu} \to \infty$, if there is a central bank announcement, the exchange rate in period one is arbitrarily close to $\mu + f + \gamma\tilde{\sigma}_1^2(\nu - S) + \tilde{\lambda}(\xi - \hat{\xi}(\nu))$, where $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by the solution to equations (2.11) and (2.12), and if there is no central bank announcement, the exchange rate in period one is arbitrarily close to $\mu + f + \gamma\sigma_1^2(\nu - S) + \lambda\xi$, where λ and σ_1^2 are given by the solution to equations (2.6) and (2.7). The assumption that $\theta_\nu > \hat{\theta}_\nu$ guarantees that $\lambda > \tilde{\lambda}$. Furthermore, equations (2.11) and (2.12) imply that $\tilde{\sigma}_1^2 \to \sigma_\kappa^2$ as $\sigma_\xi \to 0$ (see Theorem 2.4), while equations (2.6) and (2.7) imply that $\lim_{\sigma_\xi \to 0} \sigma_1^2 > \sigma_\kappa^2$. It follows that as $\bar{\nu} \to \infty$ and $\sigma_\xi \to 0$, the difference $e_1 - \tilde{e}_1$ is arbitrarily close to

$$\gamma(\nu - S)(\sigma_1^2 - \sigma_\kappa^2) + \lambda\xi + \tilde{\lambda}(\hat{\xi}(\nu) - \xi).$$

As long as $S > \bar{\nu}$, then for each $\nu \in [-\bar{\nu}, \bar{\nu}]$, there exists $\hat{\xi}(\nu)$ such that $e_1 - \tilde{e}_1 = 0$ whenever $\xi = \hat{\xi}(\nu)$, $e_1 - \tilde{e}_1 < 0$ whenever $\xi < \hat{\xi}(\nu)$, and $e_1 - \tilde{e}_1 > 0$ whenever $\xi > \hat{\xi}(\nu)$ and such that $\hat{\xi}(\nu)$ is always strictly positive and decreasing in ν. $\square$

Lemma 8.1. *Let $x \sim \mathrm{N}(\mu(z), \sigma^2(z))$ with $z > 0$, and suppose that $\lim_{z\to 0} \sigma^2(z) = 0$ and that $\lim_{z\to 0} \mu(z)$ and $\lim_{z\to 0} \hat{x}(z)$ exist or are equal to plus or minus infinity. Also, suppose that all functions are continuously differentiable and*

$$\frac{\sigma'(z)}{\frac{d}{dz}\left(\frac{\hat{x}(z)-\mu(z)}{\sigma(z)}\right)} \to 0 \tag{8.9}$$

as $z \to 0$. Then

$$\lim_{z\to 0} E\left[e^x \mid x < \hat{x}(z)\right] = \lim_{z\to 0} e^{\mu(z)+\frac{1}{2}\sigma^2(z)} \tag{8.10}$$

whenever $\lim_{z\to 0} \hat{x}(z) - \mu(z) \geq 0$, and

$$\lim_{z\to 0} E\left[e^x \mid x < \hat{x}(z)\right] = \lim_{z\to 0} e^{\hat{x}(z)+\frac{1}{2}\sigma^2(z)} \tag{8.11}$$

whenever $\lim_{z\to 0} \hat{x}(z) - \mu(z) < 0$.

Proof. Let $x \sim \mathrm{N}(\mu(z), \sigma^2(z))$ with $z > 0$, and suppose that $\lim_{z\to 0} \sigma^2(z) = 0$ and that $\lim_{z\to 0} \mu(z)$ and $\lim_{z\to 0} \hat{x}(z)$ exist or are equal to plus or minus infinity. For all $z > 0$,

$$E\left[e^x \mid x < \hat{x}(z)\right] \Phi\left(\frac{\hat{x}(z) - \mu(z)}{\sigma(z)}\right) = e^{\mu(z)+\frac{1}{2}\sigma^2(z)} \Phi\left(\frac{\hat{x}(z) - \mu(z) - \sigma^2(z)}{\sigma(z)}\right),$$

and hence

$$\lim_{z\to 0} E\left[e^x \mid x < \hat{x}(z)\right] \Phi\left(\frac{\hat{x}(z) - \mu(z)}{\sigma(z)}\right) = \lim_{z\to 0} e^{\mu(z)+\frac{1}{2}\sigma^2(z)} \Phi\left(\frac{\hat{x}(z) - \mu(z) - \sigma^2(z)}{\sigma(z)}\right). \tag{8.12}$$

If $\lim_{z\to 0}\hat{x}(z)-\mu(z)\geq 0$, then it is immediate by equation (8.12) that

$$\lim_{z\to 0} E\left[e^x \mid x<\hat{x}(z)\right] = \lim_{z\to 0} e^{\mu(z)+\frac{1}{2}\sigma^2(z)}.$$

The limit relationship is more complicated if $\lim_{z\to 0}\hat{x}(z)-\mu(z)<0$, however, since this implies that $\Phi\left(\frac{\hat{x}(z)-\mu(z)}{\sigma(z)}\right)\to 0$ and $\Phi\left(\frac{\hat{x}(z)-\mu(z)-\sigma^2(z)}{\sigma(z)}\right)\to 0$ as $z\to 0$. In this case, by l'Hôpital's rule and by assumption,

$$\begin{aligned}
\lim_{z\to 0}\frac{\Phi\left(\frac{\hat{x}(z)-\mu(z)-\sigma^2(z)}{\sigma(z)}\right)}{\Phi\left(\frac{\hat{x}(z)-\mu(z)}{\sigma(z)}\right)} &= \lim_{z\to 0}\frac{\phi\left(\frac{\hat{x}(z)-\mu(z)-\sigma^2(z)}{\sigma(z)}\right)}{\phi\left(\frac{\hat{x}(z)-\mu(z)}{\sigma(z)}\right)} \\
&= \lim_{z\to 0}\exp\left\{-\frac{(\hat{x}(z)-\mu(z)-\sigma^2(z))^2}{2\sigma^2(z)}+\frac{(\hat{x}(z)-\mu(z))^2}{2\sigma^2(z)}\right\} \\
&= \lim_{z\to 0} e^{\hat{x}(z)-\mu(z)}.
\end{aligned}$$

It follows that $\lim_{z\to 0} E\left[e^x \mid x<\hat{x}(z)\right] = \lim_{z\to 0} e^{\hat{x}(z)+\frac{1}{2}\sigma^2(z)}$ in this case. □

Proof of Theorem 7.3 Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t and that the conditional variance $\mathrm{Var}_{it}[e_{t+1}]$ is equal for all investors i (it must be equal in all periods t by definition). Lemma 7.2 then implies that the equilibrium exchange rate in period t must satisfy

$$e_t = \alpha f_t + \sum_{n=1}^{\infty}\alpha^{n+1}\overline{E}_t^n[f_{t+n}] + \gamma\sigma^2\sum_{n=0}^{\infty}\alpha^{n+1}\overline{E}_t^n[\nu_{t+n}] + \alpha\gamma\sigma^2\xi_t. \tag{8.13}$$

The exchange rate in period t is of the form

$$e_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu \nu_t + \lambda\xi_t + \beta_f\zeta_{t+1} + \beta_\nu\delta_t, \tag{8.14}$$

so the goal is to solve for the coefficients $\psi_f, \psi_\nu, \lambda, \beta_f$, and β_ν, which requires solving for the steady-state variance σ^2 as well.

The next step, then, is to solve for the average expectations $\overline{E}_t^n[f_{t+n}]$ and $\overline{E}_t^n[\nu_{t+n}]$. This requires first solving for the individual expectations $E_{it}[f_{t+1}]$ and $E_{it}[\nu_{t+1}]$, with the latter equal to $\rho_\nu E_{it}[\nu_t]$ since investors in period t have private signals of ν_t only. These expectations are more difficult to compute now that investors have prior distributions.

Let $E_{it}^0[\cdot]$, $\mathrm{Var}_{it}^0[\cdot]$, and $\mathrm{Cov}_{it}^0[\cdot]$ denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of f_t and the private signals x_{it} and y_{it}. If the form of the exchange rate in equation (8.14) is taken as given, then Bayesian inference implies both that the exchange rate in period $t+1$ is conditionally normally distributed (this justifies the assumption of conditional normality) and that

$$\begin{pmatrix} E_{it}[f_{t+1}] \\ E_{it}[\nu_t] \end{pmatrix} = \begin{pmatrix} x_{it} \\ y_{it} \end{pmatrix} + \begin{pmatrix} \sigma_\epsilon^2 & 0 & \psi_f \sigma_\epsilon^2 \\ 0 & \sigma_\eta^2 & \psi_\nu \sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\epsilon^2 + \sigma_\zeta^2 & 0 & \pi_f \\ 0 & \sigma_\eta^2 + \sigma_\delta^2 & \pi_\nu \\ \pi_f & \pi_\nu & \mathrm{Var}_{it}^0[e_t] \end{pmatrix}^{-1} \begin{pmatrix} \rho_f f_t - x_{it} \\ \rho_\nu \nu_{t-1} - y_{it} \\ e_t - E_{it}^0[e_t] \end{pmatrix},$$

where $\pi_f = \psi_f \sigma_\epsilon^2 - \beta_f \sigma_\zeta^2$ and $\pi_\nu = \psi_\nu \sigma_\eta^2 - \beta_\nu \sigma_\delta^2$. The inverse of the variance matrix in the above expression is equal to

$$\frac{1}{\Psi} \begin{pmatrix} (\sigma_\eta^2 + \sigma_\delta^2)\,\mathrm{Var}_{it}^0[e_t] - \pi_\nu^2 & \pi_f \pi_\nu & -(\sigma_\eta^2 + \sigma_\delta^2)\pi_f \\ \pi_f \pi_\nu & (\sigma_\epsilon^2 + \sigma_\zeta^2)\,\mathrm{Var}_{it}^0[e_t] - \pi_f^2 & -(\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu \\ -(\sigma_\eta^2 + \sigma_\delta^2)\pi_f & -(\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu & (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) \end{pmatrix}, \tag{8.15}$$

where

$$\begin{aligned} \Psi &= (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2)\,\mathrm{Var}_{it}^0[e_t] - (\sigma_\eta^2 + \sigma_\delta^2)\pi_f^2 - (\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu^2 \\ &= (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2 + (\psi_f^2 + 2\psi_f\beta_f + \beta_f^2)(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 \\ &\qquad + (\psi_\nu^2 + 2\psi_\nu\beta_\nu + \beta_\nu^2)(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2 \\ &= (\psi_f + \beta_f)^2(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2. \end{aligned} \tag{8.16}$$

Note that $\overline{E}_t[x_{it}] = f_{t+1}$, $\overline{E}_t[y_{it}] = \nu_t$, and $\overline{E}_t[e_t - E_{it}^0[e_t]] = \lambda\xi_t + \beta_f\zeta_{t+1} + \beta_\nu\delta_t$, since $E\left[x_{it} \mid \mathcal{F}_t\right] = f_{t+1}$ and $E\left[y_{it} \mid \mathcal{F}_t\right] = \nu_t$ for all $i \in [0,1]$ and all $t \in \mathbb{N}$. Let

$$\Delta_f = \psi_f(\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) - (\sigma_\eta^2 + \sigma_\delta^2)\pi_f = (\psi_f + \beta_f)(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\zeta^2,$$

and

$$\Delta_\nu = \psi_\nu(\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) - (\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu = (\psi_\nu + \beta_\nu)(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\delta^2.$$

Because $\mathrm{Var}_{it}^0[e_t] = \psi_f^2\sigma_\epsilon^2 + \psi_\nu^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2 + \beta_f^2\sigma_\zeta^2 + \beta_\nu^2\sigma_\delta^2$, it follows that

$$\begin{aligned} \overline{E}_t[f_{t+1}] &= f_{t+1} + \lambda\Delta_f\sigma_\epsilon^2\xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu\psi_f - \pi_f\pi_\nu + \beta_\nu\Delta_f\right)\delta_t \\ &\qquad + \frac{\sigma_\epsilon^2}{\Psi}\left(\pi_\nu^2 + (\sigma_\eta^2 + \sigma_\delta^2)\pi_f\psi_f - (\sigma_\eta^2 + \sigma_\delta^2)\,\mathrm{Var}_{it}^0[e_t] + \beta_f\Delta_f\right)\zeta_{t+1} \\ &= f_{t+1} + \lambda\Delta_f\sigma_\epsilon^2\xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\psi_f + \beta_f)\sigma_\zeta^2\pi_\nu + \beta_\nu\Delta_f\right)\delta_t \\ &\qquad - \frac{\sigma_\epsilon^2}{\Psi}\left((\sigma_\eta^2 + \sigma_\delta^2)(\lambda^2\sigma_\xi^2 + \beta_f^2\sigma_\zeta^2 + \psi_f\beta_f\sigma_\zeta^2) + (\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 - \beta_f\Delta_f\right)\zeta_{t+1}, \end{aligned}$$

so that

$$\overline{E}_t[f_{t+1}] = f_{t+1} + \lambda(\psi_f+\beta_f)(\sigma_\eta^2+\sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2\xi_t + \frac{(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\delta_t - \sigma_\epsilon^2\left((\sigma_\eta^2+\sigma_\delta^2)\lambda^2\sigma_\xi^2+(\psi_\nu+\beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right)\zeta_{t+1}}{\Psi}. \tag{8.17}$$

Similarly, it follows that

$$\begin{aligned}\overline{E}_t[\nu_t] &= \nu_t + \lambda\Delta_\nu\sigma_\eta^2\xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\sigma_\eta^2+\sigma_\delta^2)\pi_f\psi_\nu - \pi_f\pi_\nu + \beta_f\Delta_\nu\right)\zeta_{t+1} \\ &\quad + \frac{\sigma_\eta^2}{\Psi}\left(\pi_f^2 + (\sigma_\epsilon^2+\sigma_\zeta^2)\pi_\nu\psi_\nu - (\sigma_\epsilon^2+\sigma_\zeta^2)\operatorname{Var}_{it}^0[e_t] + \beta_\nu\Delta_\nu\right)\delta_t \\ &= \nu_t + \lambda\Delta_\nu\sigma_\eta^2\xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\psi_\nu+\beta_\nu)\sigma_\delta^2\pi_f + \beta_f\Delta_\nu\right)\zeta_{t+1} \\ &\quad - \frac{\sigma_\eta^2}{\Psi}\left((\sigma_\epsilon^2+\sigma_\zeta^2)(\lambda^2\sigma_\xi^2+\beta_\nu^2\sigma_\delta^2+\psi_\nu\beta_\nu\sigma_\delta^2) + (\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2 - \beta_\nu\Delta_\nu\right)\delta_t,\end{aligned}$$

so that

$$\overline{E}_t[\nu_t] = \nu_t + \lambda(\psi_\nu+\beta_\nu)(\sigma_\epsilon^2+\sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2\xi_t + \frac{(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\delta^2\zeta_{t+1} - \sigma_\eta^2\left((\sigma_\epsilon^2+\sigma_\zeta^2)\lambda^2\sigma_\xi^2+(\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right)\delta_t}{\Psi}. \tag{8.18}$$

Equations (8.17) and (8.18) state that both $\overline{E}_t[f_{t+1}]$ and $\overline{E}_t[\nu_t]$ are not functions of past noise trades or disturbances, so that higher-order beliefs collapse. More precisely, higher-order expectations are such that $\overline{E}_t^n[f_{t+n}] = \rho_f^{n-1}\overline{E}_t[f_{t+1}]$ and $\overline{E}_t^n[\nu_{t+n}] = \rho_\nu^n\overline{E}_t[\nu_t]$ for all $n>1$. This important observation implies that the expression from equation (8.13) simplifies to

$$\begin{aligned} e_t &= \alpha f_t + \sum_{n=1}^{\infty}\alpha^{n+1}\rho_f^{n-1}\overline{E}_t[f_{t+1}] + \alpha\gamma\sigma^2\nu_t + \gamma\sigma^2\sum_{n=1}^{\infty}\alpha^{n+1}\rho_\nu^n\overline{E}_t[\nu_t] + \alpha\gamma\sigma^2\xi_t \\ &= \alpha f_t + \frac{\alpha^2}{1-\alpha\rho_f}\overline{E}_t[f_{t+1}] + \alpha\gamma\sigma^2\nu_t + \gamma\sigma^2\frac{\alpha^2\rho_\nu}{1-\alpha\rho_\nu}\overline{E}_t[\nu_t] + \alpha\gamma\sigma^2\xi_t. \end{aligned} \tag{8.19}$$

Substituting equations (8.17) and (8.18) into equation (8.19) yields

$$e_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu\nu_t + \lambda\xi_t + \beta_f\zeta_{t+1} + \beta_\nu\delta_t, \tag{8.20}$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$ and $\psi_\nu = \frac{\alpha\gamma\sigma^2}{1-\alpha\rho_\nu}$, and λ, β_f, and β_ν are given by the solution to equations (7.11), (7.12), and (7.13).

The final step is to solve for σ^2, the steady-state variance of the exchange rate, which is accomplished by first solving for $\overline{\text{Var}}_t[f_{t+1}], \overline{\text{Var}}_t[\nu_t]$, and $\overline{\text{Cov}}_t[f_{t+1}, \nu_t]$. Bayesian inference implies that

$$\begin{aligned}
\begin{pmatrix} \overline{\text{Var}}_t[f_{t+1}] & \overline{\text{Cov}}_t[f_{t+1}, \nu_t] \\ \overline{\text{Cov}}_t[f_{t+1}, \nu_t] & \overline{\text{Var}}_t[\nu_t] \end{pmatrix} &= \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \\
&- \begin{pmatrix} \sigma_\epsilon^2 & 0 & \psi_f \sigma_\epsilon^2 \\ 0 & \sigma_\eta^2 & \psi_\nu \sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\epsilon^2 + \sigma_\zeta^2 & 0 & \pi_f \\ 0 & \sigma_\eta^2 + \sigma_\delta^2 & \pi_\nu \\ \pi_f & \pi_\nu & \text{Var}_{it}^0[e_t] \end{pmatrix}^{-1} \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \\ \psi_f \sigma_\epsilon^2 & \psi_\nu \sigma_\eta^2 \end{pmatrix},
\end{aligned}$$

where $\pi_f = \psi_f \sigma_\epsilon^2 - \beta_f \sigma_\zeta^2$ and $\pi_\nu = \psi_\nu \sigma_\eta^2 - \beta_\nu \sigma_\delta^2$ as before. It follows by equation (8.15) that

$$\begin{aligned}
\overline{\text{Var}}_t[f_{t+1}] &= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4}{\Psi} \left((\sigma_\eta^2 + \sigma_\delta^2) \text{Var}_{it}^0[e_t] - \pi_\nu^2 - 2\psi_f (\sigma_\eta^2 + \sigma_\delta^2) \pi_f + \psi_f^2 (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) \right) \\
&= \sigma_\epsilon^2 - \\
&\quad \frac{\sigma_\epsilon^4}{\Psi} \left[(\sigma_\eta^2 + \sigma_\delta^2) \left(\psi_f^2 \sigma_\epsilon^2 + \psi_\nu^2 \sigma_\eta^2 + \lambda^2 \sigma_\xi^2 + \beta_f^2 \sigma_\zeta^2 + \beta_\nu^2 \sigma_\delta^2 - 2\psi_f \pi_f + \psi_f^2 (\sigma_\epsilon^2 + \sigma_\zeta^2) \right) - \pi_\nu^2 \right] \\
&= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4}{\Psi} \left[(\sigma_\eta^2 + \sigma_\delta^2) \left(\psi_\nu^2 \sigma_\eta^2 + \lambda^2 \sigma_\xi^2 + (\psi_f + \beta_f)^2 \sigma_\zeta^2 + \beta_\nu^2 \sigma_\delta^2 \right) - (\psi_\nu \sigma_\eta^2 - \beta_\nu \sigma_\delta^2)^2 \right] \\
&= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4}{\Psi} \left[(\sigma_\eta^2 + \sigma_\delta^2) \left(\lambda^2 \sigma_\xi^2 + (\psi_f + \beta_f)^2 \sigma_\zeta^2 \right) + (\psi_\nu + \beta_\nu)^2 \sigma_\eta^2 \sigma_\delta^2 \right],
\end{aligned}$$

that

$$\begin{aligned}
\overline{\text{Var}}_t[\nu_t] &= \sigma_\eta^2 - \frac{\sigma_\eta^4}{\Psi} \left((\sigma_\epsilon^2 + \sigma_\zeta^2) \text{Var}_{it}^0[e_t] - \pi_f^2 - 2\psi_\nu (\sigma_\epsilon^2 + \sigma_\zeta^2) \pi_\nu + \psi_\nu^2 (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) \right) \\
&= \sigma_\eta^2 - \\
&\quad \frac{\sigma_\eta^4}{\Psi} \left[(\sigma_\epsilon^2 + \sigma_\zeta^2) \left(\psi_f^2 \sigma_\epsilon^2 + \psi_\nu^2 \sigma_\eta^2 + \lambda^2 \sigma_\xi^2 + \beta_f^2 \sigma_\zeta^2 + \beta_\nu^2 \sigma_\delta^2 - 2\psi_\nu \pi_\nu + \psi_\nu^2 (\sigma_\eta^2 + \sigma_\delta^2) \right) - \pi_f^2 \right] \\
&= \sigma_\eta^2 - \frac{\sigma_\eta^4}{\Psi} \left[(\sigma_\epsilon^2 + \sigma_\zeta^2) \left(\psi_f^2 \sigma_\epsilon^2 + \lambda^2 \sigma_\xi^2 + \beta_f^2 \sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2 \sigma_\delta^2 \right) - (\psi_f \sigma_\epsilon^2 - \beta_f \sigma_\zeta^2)^2 \right] \\
&= \sigma_\eta^2 - \frac{\sigma_\eta^4}{\Psi} \left[(\sigma_\epsilon^2 + \sigma_\zeta^2) \left(\lambda^2 \sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2 \sigma_\delta^2 \right) + (\psi_f + \beta_f)^2 \sigma_\epsilon^2 \sigma_\zeta^2 \right],
\end{aligned}$$

and that

$$
\begin{aligned}
\overline{\mathrm{Cov}}_t[f_{t+1},\nu_t] &= -\frac{\sigma_\epsilon^2\sigma_\eta^2}{\Psi}\left(\pi_f\pi_\nu - \psi_f(\sigma_\epsilon^2+\sigma_\zeta^2)\pi_\nu - \psi_\nu(\sigma_\eta^2+\sigma_\delta^2)\pi_f + \psi_f\psi_\nu(\sigma_\epsilon^2+\sigma_\zeta^2)(\sigma_\eta^2+\sigma_\delta^2)\right) \\
&= -\frac{\sigma_\epsilon^2\sigma_\eta^2}{\Psi}\left(\psi_\nu(\sigma_\eta^2+\sigma_\delta^2)(\psi_f+\beta_f)\sigma_\zeta^2 - \pi_\nu(\psi_f+\beta_f)\sigma_\zeta^2\right) \\
&= -\frac{(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\sigma_\delta^2}{\Psi}.
\end{aligned}
$$

As before, Ψ is given by equation (8.16). Equation (8.20) implies that the steady-state variance is equal to

$$
\begin{aligned}
\sigma^2 = \frac{\psi_f^2}{\alpha^2}\overline{\mathrm{Var}}_t[f_{t+1}] + \rho_\nu^2\psi_\nu^2\overline{\mathrm{Var}}_t[\nu_t] + \frac{2\rho_\nu\psi_f\psi_\nu}{\alpha}\overline{\mathrm{Cov}}_t[f_{t+1},\nu_t] \\
+ \lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\zeta^2 + (\psi_\nu+\beta_\nu)^2\sigma_\delta^2,
\end{aligned}
$$

which justifies the assumption that the conditional variance is equal for all investors i. Equation (7.14) follows. □

Proof of Theorem 7.4 This proof follows the proof of Theorem 7.3 very closely. Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t and that the conditional variance $\mathrm{Var}_{it}[\tilde{e}_{t+1}]$ is equal for all investors i. Lemma 7.2 then implies that the equilibrium exchange rate in period t satisfies equation (8.13). The exchange rate in period t is again of the form $\tilde{e}_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu\nu_t + \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}$ and the goal remains to solve for the coefficients $\psi_f, \psi_\nu, \tilde{\lambda}$, and $\tilde{\beta}_f$ as well as the conditional variance $\tilde{\sigma}^2$.

Bayesian inference again implies that the exchange rate in period $t+1$ is conditionally normally distributed, so the initial assumption is justified. As in the previous proof, $\overline{E}_t[x_{it}] = f_{t+1}$ and $\overline{E}_t[e_t - E_{it}^0[e_t]] = \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}$. Furthermore, the average expectation of ν_t is equal to ν_t itself since the intervention is common knowledge, and so it follows that the average expectation of f_{t+1} is given by

$$
\begin{aligned}
\overline{E}_t[f_{t+1}] &= f_{t+1} + \begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2+\sigma_\zeta^2 & \psi_f\sigma_\epsilon^2 - \tilde{\beta}_f\sigma_\zeta^2 \\ \psi_f\sigma_\epsilon^2 - \tilde{\beta}_f\sigma_\zeta^2 & \psi_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2 + \tilde{\beta}_f^2\sigma_\zeta^2\end{pmatrix}^{-1}\begin{pmatrix}-\zeta_{t+1} \\ \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}\end{pmatrix} \\
&= f_{t+1} + \frac{1}{D}\begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\psi_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2 + \tilde{\beta}_f^2\sigma_\zeta^2 & \tilde{\beta}_f\sigma_\zeta^2 - \psi_f\sigma_\epsilon^2 \\ \tilde{\beta}_f\sigma_\zeta^2 - \psi_f\sigma_\epsilon^2 & \sigma_\epsilon^2+\sigma_\zeta^2\end{pmatrix}\begin{pmatrix}-\zeta_{t+1} \\ \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}\end{pmatrix},
\end{aligned}
$$

where $D = (\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2$. It follows that

$$\begin{aligned}
\overline{E}_t[f_{t+1}] &= f_{t+1} + \frac{1}{D}\left(\left(\tilde{\lambda}^2\sigma_\xi^2 + \tilde{\beta}_f(\psi_f + \tilde{\beta}_f)\sigma_\zeta^2\right)\sigma_\epsilon^2 \quad (\tilde{\beta}_f + \psi_f)\sigma_\epsilon^2\sigma_\zeta^2\right)\begin{pmatrix} -\zeta_{t+1} \\ \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}\end{pmatrix} \\
&= f_{t+1} + \frac{\tilde{\lambda}(\tilde{\beta}_f + \psi_f)\sigma_\epsilon^2\sigma_\zeta^2\xi_t - \tilde{\lambda}^2\sigma_\xi^2\sigma_\epsilon^2\zeta_{t+1}}{(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}. \qquad (8.21)
\end{aligned}$$

Equation (8.21) states that $\overline{E}_t[f_{t+1}]$ is not a function of past noise trades or disturbances, so it follows that higher-order beliefs again collapse in this case. Furthermore, investors have no information about future values of ν_t besides knowledge of the current value of ν_t and the stochastic process that governs its motion. This implies that $\overline{E}_t^n[f_{t+n}] = \rho_f^{n-1}\overline{E}_t[f_{t+1}]$ and $\overline{E}_t^n[\nu_{t+n}] = \rho_\nu^n\nu_t$ for all $n > 1$, so that equation (8.13) simplifies to

$$\tilde{e}_t = \alpha f_t + \frac{\alpha^2}{1-\alpha\rho_f}\overline{E}_t[f_{t+1}] + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t \qquad (8.22)$$

Substituting equation (8.21) into equation (8.22) yields

$$\tilde{e}_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu\nu_t + \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}, \qquad (8.23)$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$ and $\psi_\nu = \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}$, and $\tilde{\lambda}$ and $\tilde{\beta}_f$ are given by the solution to equations (7.17) and (7.18).

The final step of the proof is to solve for the steady-state variance of the exchange rate, $\tilde{\sigma}^2$. If investors know the value of ν_t in period t, then standard Bayesian inference implies that

$$\begin{aligned}
\overline{\mathrm{Var}}_t[f_{t+1}] &= \sigma_\epsilon^2 - \begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2 + \sigma_\zeta^2 & \psi_f\sigma_\epsilon^2 - \tilde{\beta}_f\sigma_\zeta^2 \\ \psi_f\sigma_\epsilon^2 - \tilde{\beta}_f\sigma_\zeta^2 & \psi_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2 + \tilde{\beta}_f^2\sigma_\zeta^2\end{pmatrix}^{-1}\begin{pmatrix}\sigma_\epsilon^2 \\ \psi_f\sigma_\epsilon^2\end{pmatrix} \\
&= \sigma_\epsilon^2 - \frac{1}{D}\begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\psi_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2 + \tilde{\beta}_f^2\sigma_\zeta^2 & \tilde{\beta}_f\sigma_\zeta^2 - \psi_f\sigma_\epsilon^2 \\ \tilde{\beta}_f\sigma_\zeta^2 - \psi_f\sigma_\epsilon^2 & \sigma_\epsilon^2 + \sigma_\zeta^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2 \\ \psi_f\sigma_\epsilon^2\end{pmatrix} \\
&= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^2}{D}\begin{pmatrix}\tilde{\lambda}^2\sigma_\xi^2 + \tilde{\beta}_f(\psi_f + \tilde{\beta}_f)\sigma_\zeta^2 & (\psi_f + \tilde{\beta}_f)\sigma_\zeta^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2 \\ \psi_f\sigma_\epsilon^2\end{pmatrix} \\
&= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4\left(\tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2\right)}{(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}.
\end{aligned}$$

Equation (8.23) implies that the steady-state variance is equal to

$$\tilde{\sigma}^2 = \frac{\psi_f^2}{\alpha^2}\overline{\mathrm{Var}}_t[f_{t+1}] + \tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2 + \psi_\nu^2\sigma_\delta^2,$$

which justifies the assumption that the conditional variance is equal for all investors i. Equation (7.19) follows. $\square$

Proof of Theorem 7.5 Let $\tilde{\Psi} = (\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2$, and recall that

$$\frac{\Psi}{\sigma_\eta^2 + \sigma_\delta^2} = (\psi_f + \beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\frac{\sigma_\eta^2\sigma_\delta^2}{\sigma_\eta^2 + \sigma_\delta^2} + (\sigma_\epsilon^2 + \sigma_\zeta^2)\lambda^2\sigma_\xi^2. \tag{8.24}$$

According to equations (7.14) and (7.19),

$$\begin{aligned}\sigma^2 = {} & \frac{\psi_f^2\sigma_\epsilon^2\sigma_\zeta^2\left((\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right)}{\alpha^2\Psi} \\ & + \frac{\rho_\nu^2\psi_\nu^2\sigma_\eta^2\sigma_\delta^2\left((\sigma_\epsilon^2 + \sigma_\zeta^2)\lambda^2\sigma_\xi^2 + (\psi_f + \beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right)}{\Psi} \\ & + \lambda^2\sigma_\xi^2 + (\psi_f + \beta_f)^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2\sigma_\delta^2 - \frac{2\rho_\nu\psi_f\psi_\nu}{\alpha\Psi}(\psi_f + \beta_f)(\psi_\nu + \beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\sigma_\delta^2\end{aligned} \tag{8.25}$$

and

$$\tilde{\sigma}^2 = \frac{\psi_f^2\sigma_\epsilon^2\sigma_\zeta^2\tilde{\lambda}^2\sigma_\xi^2}{\alpha^2\tilde{\Psi}} + \tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2 + \psi_\nu^2\sigma_\delta^2. \tag{8.26}$$

Throughout this proof, I assume that the parameters of the model are such that there exist real solutions λ and $\tilde{\lambda}$ to the systems of equations given by Theorems 7.3 and 7.4. If this is not the case, then these limits are undefined.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\xi \to 0$ and suppose that $\tilde{\lambda}$ does not diverge to infinity. In this case, $\tilde{\lambda}^2\sigma_\xi^2 \to 0$ so that by equations (7.17) and (7.18) it follows that $\tilde{\beta}_f \to 0$ and $\lim_{\sigma_\xi\to 0}\tilde{\lambda} = \lim_{\sigma_\xi\to 0}\tilde{\lambda} + \alpha\gamma\tilde{\sigma}^2$. Of course, the limit of $\tilde{\lambda}$ and $\tilde{\lambda} + \alpha\gamma\tilde{\sigma}^2$ can only be equal if either $\tilde{\lambda} \to 0$ or $\tilde{\lambda} \to \infty$. Equation (8.26) implies that $\tilde{\sigma}^2 \geq \psi_f^2\sigma_\zeta^2 > 0$ in the limit, so it must be that $\tilde{\lambda} \to \infty$ as $\sigma_\xi \to 0$. On the other hand, if λ does not diverge to infinity as $\sigma_\xi \to 0$, then equations (8.24), (7.11), and (8.25) imply that

$$\lim_{\sigma_\xi\to 0}\lambda = \lim_{\sigma_\xi\to 0}\frac{\lambda\psi_f(\psi_f + \beta_f)(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2\sigma_\delta^2 + \lambda\alpha\rho_\nu\psi_\nu(\psi_\nu + \beta_\nu)(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2}{(\psi_f + \beta_f)^2(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2} + \alpha\gamma\sigma^2.$$

As long as $\sigma_\eta > 0$, it follows that λ converges to a finite limit.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\epsilon \to \infty$. If $\tilde{\lambda}$ converges to a finite limit in this case, then equation (7.18) implies that $\tilde{\beta}_f \to -\psi_f$ so that $\lim_{\sigma_\epsilon\to\infty}\tilde{\lambda} = \lim_{\sigma_\epsilon\to\infty}\alpha\gamma\tilde{\sigma}^2$. Equation (8.26) implies that

$$\lim_{\sigma_\epsilon\to\infty}\tilde{\sigma}^2 = \lim_{\sigma_\epsilon\to\infty}\tilde{\lambda}^2\sigma_\xi^2 + \psi_\nu^2\sigma_\delta^2 = \lim_{\sigma_\epsilon\to\infty}\alpha^2\gamma^2\tilde{\sigma}^4\sigma_\xi^2 + \frac{\alpha^2\gamma^2\tilde{\sigma}^4}{(1 - \alpha\rho_\nu)^2}\sigma_\delta^2.$$

The only real solution to the equation $\tilde{\sigma}^2 = \alpha^2\gamma^2\tilde{\sigma}^4\sigma_\xi^2 + \frac{\alpha^2\gamma^2\tilde{\sigma}^4}{(1-\alpha\rho_\nu)^2}\sigma_\delta^2$ is $\tilde{\sigma}^2 = 0$, so it follows that both $\tilde{\sigma}^2 \to 0$ and $\tilde{\lambda} \to 0$ as $\sigma_\epsilon \to \infty$. According to equation (8.24),

$$\lim_{\sigma_\epsilon\to\infty} \frac{\Psi}{\sigma_\epsilon^2} = \lim_{\sigma_\epsilon\to\infty} (\psi_f + \beta_f)^2(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 + (\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2,$$

so that, much like in the case of $\tilde{\beta}_f$, equation (7.12) implies that $\beta_f \to -\psi_f$ as $\sigma_\epsilon \to \infty$. These properties imply that

$$\lim_{\sigma_\epsilon\to\infty} \lambda = \lim_{\sigma_\epsilon\to\infty} \frac{\lambda\alpha\rho_\nu\psi_\nu(\psi_\nu + \beta_\nu)\sigma_\eta^2\sigma_\delta^2}{(\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 + (\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2} + \alpha\gamma\sigma^2. \tag{8.27}$$

The key equation is equation (7.13), which implies that

$$\lim_{\sigma_\epsilon\to\infty} \beta_\nu = \lim_{\sigma_\epsilon\to\infty} \frac{-\alpha\rho_\nu\psi_\nu\sigma_\eta^2\lambda^2\sigma_\xi^2}{(\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 + (\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2} + \alpha\gamma\sigma^2,$$

so that $\psi_\nu + \beta_\nu$ does not converge to zero since $\alpha\rho_\nu < 1$. All that remains is to show that σ^2 and hence ψ_ν does not converge to zero as $\sigma_\epsilon \to \infty$. This follows by equation (8.25), which implies that

$$\lim_{\sigma_\epsilon\to\infty} \sigma^2 = \lim_{\sigma_\epsilon\to\infty} \frac{\psi_f^2}{\alpha^2}\sigma_\zeta^2 + \rho_\nu^2\psi_\nu^2\sigma_\eta^2\sigma_\delta^2 + \lambda^2\sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2\sigma_\delta^2. \tag{8.28}$$

The solution to this equation in the limit must be greater than zero since it contains the constant term $\frac{\psi_f^2}{\alpha^2}\sigma_\zeta^2 > 0$. It follows by equation (8.27) that λ converges to a constant greater than zero as $\sigma_\epsilon \to \infty$.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\zeta \to 0$. As in the case of $\sigma_\epsilon \to \infty$, equation (7.18) implies that $\tilde{\beta}_f \to -\psi_f$ in this case and hence by equation (8.26) it follows that $\tilde{\sigma}^2 \to 0$ and $\tilde{\lambda} \to 0$. It is not difficult to show that a limit equation identical to equation (8.27) obtains for this case where $\sigma_\zeta \to 0$, and that a similar equation to equation (8.28) also obtains. The key difference, however, is that if $\sigma_\zeta \to 0$, equation (8.28) changes so that

$$\lim_{\sigma_\zeta\to 0} \sigma^2 = \lim_{\sigma_\zeta\to 0} \rho_\nu^2\psi_\nu^2\sigma_\eta^2\sigma_\delta^2 + \lambda^2\sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2\sigma_\delta^2,$$

and hence both σ^2 and ψ_ν converge to zero in the limit. It follows by equation (8.27) that $\lambda \to 0$ as $\sigma_\zeta \to 0$.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\delta \to 0$. Equation (8.24) implies that

$$\lim_{\sigma_\delta\to 0} \Psi = \lim_{\sigma_\delta\to 0} (\psi_f + \beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\sigma_\eta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\lambda^2\sigma_\xi^2,$$

and hence equations (7.11) and (8.25) imply that

$$\lim_{\sigma_\delta \to 0} \lambda = \lim_{\sigma_\delta \to 0} \frac{\lambda \psi_f (\psi_f + \beta_f) \sigma_\epsilon^2 \sigma_\zeta^2}{(\psi_f + \beta_f)^2 \sigma_\epsilon^2 \sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2) \lambda^2 \sigma_\xi^2},$$

and

$$\lim_{\sigma_\delta \to 0} \sigma^2 = \lim_{\sigma_\delta \to 0} \frac{\psi_f^2 \sigma_\epsilon^2 \sigma_\zeta^2 \lambda^2 \sigma_\xi^2}{\alpha^2 (\psi_f + \beta_f)^2 \sigma_\epsilon^2 \sigma_\zeta^2 + \alpha^2 (\sigma_\epsilon^2 + \sigma_\zeta^2) \lambda^2 \sigma_\xi^2} + \lambda^2 \sigma_\xi^2 + (\psi_f + \beta_f)^2 \sigma_\zeta^2.$$

Equation (7.12) also implies that

$$\lim_{\sigma_\delta \to 0} \tilde{\beta}_f = \lim_{\sigma_\delta \to 0} - \frac{\psi_f \sigma_\epsilon^2 \lambda^2 \sigma_\xi^2}{(\psi_f + \beta_f)^2 \sigma_\epsilon^2 \sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2) \lambda^2 \sigma_\xi^2}.$$

Meanwhile, equations (7.17), (7.18), and (8.26) imply that an identical set of equations jointly determine the value of $\tilde{\lambda}$ as $\sigma_\delta \to 0$, so it follows that $\lim_{\sigma_\delta \to 0} \lambda = \lim_{\sigma_\delta \to 0} \tilde{\lambda}$. □

Proof of Theorem 7.6 Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t. Suppose also that the conditional variance $\mathrm{Var}_{it}[e_{t+1}]$ is equal for all investors i. Lemma 7.2 then implies that the equilibrium exchange rate in period t must satisfy

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [f_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t. \tag{8.29}$$

The exchange rate in period t is of the form

$$e_t = A Q_t(k) + \alpha \gamma \sigma^2 \xi_t, \tag{8.30}$$
$$Q_t(k) = M Q_{t-1}(k) + N w_t, \tag{8.31}$$

where $k > 0$ is the level at which higher-order expectations are truncated in the model. The goal is to solve for the equilibrium conditions that characterize the matrices M and N, the vector A, and the steady-state variance σ^2.

The definitions of the higher-order expectations vector $Q_t(k)$ and the matrices M and N imply that $\overline{E}_t^n [f_{t+n}] = h_1' (MH)^n Q_t(k)$ and $\overline{E}_t^n [\nu_{t+n}] = h_2' (MH)^n Q_t(k)$ for all $n \geq 1$. Equation (8.29) then implies that

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} (h_1' + \gamma \sigma^2 h_2') (MH)^n Q_t(k) + \alpha \gamma \sigma^2 \xi_t,$$

so it follows by equation (8.30) that the vector A must satisfy

$$A = \sum_{n=0}^{\infty} \alpha^{n+1}(h_1' + \gamma\sigma^2 h_2')(MH)^n.$$

Note that this equation matches equation (7.27) exactly, so that all that remains of this proof is to characterize the state transition matrices M and N and the steady-state variance σ^2.

Recall that $\bar{i}_t = i_t^* - ap_t^* - r = f_t + \chi_t$. In each period t, each investor i observes

$$z_{it} = \begin{pmatrix} x_{it} \\ y_{it} \\ \bar{i}_t \\ e_t \end{pmatrix} = DQ_t(k) + R \begin{pmatrix} \sigma_\epsilon^{-1}\epsilon_{it} \\ \sigma_\eta^{-1}\eta_{it} \\ \sigma_\zeta^{-1}\zeta_t \\ \sigma_\delta^{-1}\delta_t \\ \sigma_\chi^{-1}\chi_t \\ \sigma_\xi^{-1}\xi_t \end{pmatrix},$$

where

$$D = \begin{pmatrix} 1 & 0 & \\ 0 & 1 & \mathbf{0}_{3\times 2k} \\ 1 & 0 & \\ & & A \end{pmatrix},$$

and $R = \begin{pmatrix} R_1 & R_2 \end{pmatrix}$, with

$$R_1 = \begin{pmatrix} \sigma_\epsilon & 0 \\ 0 & \sigma_\eta \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} & 0 & 0 \\ \mathbf{0}_{4\times 2} & 0 & 0 \\ & \sigma_\chi & 0 \\ & 0 & \alpha\gamma\sigma^2\sigma_\xi \end{pmatrix}.$$

If the state vector of higher-order expectations evolves according to equation (8.31), then Bayesian updating implies both that the exchange rate in period $t+1$ is conditionally normally distributed (this justifies the assumption of conditional normality) and that

$$E_{it}[Q_t(k)] = ME_{it-1}[Q_{t-1}(k)] + K\left(z_{it} - DME_{it-1}[Q_{t-1}(k)]\right),$$

where K is the Kalman gain matrix. Averaging this equation over all investors yields

$$\begin{aligned} \overline{E}_t[Q_t(k)] &= M\overline{E}_{t-1}[Q_{t-1}(k)] + K\left(DMQ_{t-1}(k) + (DN + R_2)w_t - DM\overline{E}_{t-1}[Q_{t-1}(k)]\right) \\ &= (M - KDM)\overline{E}_{t-1}[Q_{t-1}(k)] + KDMQ_{t-1}(k) + K(DN + R_2)w_t. \end{aligned} \tag{8.32}$$

Equation (8.32) implies that

$$Q_t(k) = \begin{pmatrix} q_{0t} \\ \overline{E}_t[Q_t(k-1)] \end{pmatrix} = M \begin{pmatrix} q_{0t-1} \\ \overline{E}_{t-1}[Q_{t-1}(k-1)] \end{pmatrix} + Nw_t = MQ_{t-1}(k) + Nw_t, \quad (8.33)$$

where

$$M = \begin{pmatrix} \begin{matrix} \rho_f & 0 \\ 0 & \rho_\nu \end{matrix} & \mathbf{0}_{2\times 2k} \\ \multicolumn{2}{c}{\mathbf{0}_{2k\times 2k+2}} \end{pmatrix} + \begin{pmatrix} \multicolumn{2}{c}{\mathbf{0}_{2\times 2k+2}} \\ \mathbf{0}_{2k\times 2} & [M-KDM]_- \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{2\times 2k+2} \\ [KDM]_- \end{pmatrix}, \quad (8.34)$$

$$N = \begin{pmatrix} \begin{matrix} \sigma_\zeta & 0 \\ 0 & \sigma_\delta \end{matrix} & \mathbf{0}_{2\times 2} \\ [K(DN+R_2)]_- & \end{pmatrix}, \quad (8.35)$$

and $[M-KDM]_-$ is the matrix $M-KDM$ with the last two rows and columns removed and $[KDM]_-$ and $[K(DN+R_2)]_-$ are, respectively, the matrices KDM and $K(DN+R_2)$ with the last two rows removed. The Kalman gain matrix K is given by

$$K = (PD' + NR_2')(DPD' + RR')^{-1}, \quad (8.36)$$

where P satisfies the matrix Riccati equation

$$P = M\left(P - (PD' + NR_2')(DPD' + RR')^{-1}(PD' + NR_2')'\right)M' + NN'. \quad (8.37)$$

The next step is to solve for the steady-state variance of the exchange rate σ^2. In order to do this, it is necessary to compute the variance-covariance matrix

$$\hat{P} = \mathrm{Var}_{it}\begin{bmatrix} Q_{t+1}(k) \\ \xi_{t+1} \end{bmatrix} = \overline{\mathrm{Var}}_t\begin{bmatrix} Q_{t+1}(k) \\ \xi_{t+1} \end{bmatrix},$$

which depends on the steady-state dynamics of a system slightly more general than the system from equation (8.31). Note that

$$\begin{pmatrix} Q_t(k) \\ \xi_t \end{pmatrix} = \begin{pmatrix} M & \mathbf{0}_{2k+2\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times 2k+3}} \end{pmatrix} \begin{pmatrix} Q_{t-1}(k) \\ \xi_{t-1} \end{pmatrix} + \begin{pmatrix} N_1 & & N_2 \\ 0 \quad 0 & 0 & \sigma_\xi \end{pmatrix} \begin{pmatrix} \sigma_\zeta^{-1}\zeta_t \\ \sigma_\delta^{-1}\delta_t \\ \sigma_\chi^{-1}\chi_t \\ \sigma_\xi^{-1}\xi_t \end{pmatrix},$$

where N_1 and N_2 consist, respectively, of the first two columns and the last two columns of the matrix N from equation (8.35) above, and that

$$z_{it} = \begin{pmatrix} x_{it} \\ y_{it} \\ \bar{i}_t \\ e_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & \\ 0 & 1 & \mathbf{0}_{3\times 2k+1} \\ 1 & 0 & \\ & A & \alpha\gamma\sigma^2 \end{pmatrix} \begin{pmatrix} Q_t(k) \\ \xi_t \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{it} \\ \eta_{it} \\ \chi_t \end{pmatrix}.$$

This system of equations both justifies the assumption that the conditional variance is equal for all investors i and implies that the matrix $\hat{P}$ is given by the solution to the Riccati equation

$$\hat{P} = \hat{M}\left(\hat{P} - (\hat{P}\hat{D}' + \hat{N}\hat{R}_2')(\hat{D}\hat{P}\hat{D}' + \hat{R}\hat{R}')^{-1}(\hat{P}\hat{D}' + \hat{N}\hat{R}_2')'\right)\hat{M}' + \hat{N}\hat{N}', \tag{8.38}$$

where

$$\hat{M} = \begin{pmatrix} M & \mathbf{0}_{2k+2\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times 2k+3}} \end{pmatrix}, \qquad \hat{N} = \begin{pmatrix} N_1 & & & N_2 \\ 0 & 0 & 0 & \sigma_\xi \end{pmatrix}, \qquad \hat{D} = \begin{pmatrix} 1 & 0 & \\ 0 & 1 & \mathbf{0}_{3\times 2k+1} \\ 1 & 0 & \\ & A & \alpha\gamma\sigma^2 \end{pmatrix}$$

$$\hat{R} = \begin{pmatrix} \hat{R}_1 & \hat{R}_2 \end{pmatrix}, \qquad \hat{R}_1 = \begin{pmatrix} \sigma_\epsilon & 0 \\ 0 & \sigma_\eta \\ \multicolumn{2}{c}{\mathbf{0}_{2\times 2}} \end{pmatrix}, \qquad \hat{R}_2 = \begin{pmatrix} \multicolumn{4}{c}{\mathbf{0}_{2\times 4}} \\ 0 & 0 & \sigma_\chi & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Because $e_{t+1} = AQ_{t+1}(k) + \alpha\gamma\sigma^2\xi_{t+1}$, it follows that

$$\sigma^2 = \begin{pmatrix} A & \alpha\gamma\sigma^2 \end{pmatrix} \hat{P} \begin{pmatrix} A & \alpha\gamma\sigma^2 \end{pmatrix}'. \tag{8.39}$$

I conclude that the matrices M and N and the steady-state variance σ^2 from the approximate equilibrium of Theorem 7.6 are given by the joint solution to equations (8.34), (8.35), (8.36), (8.37), (8.38), and (8.39). The fact that this approximation converges to the true steady-state equilibrium of this model is shown by Nimark (2010a). □

Proof of Theorem 7.7 Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t. Suppose also that the conditional variance $\mathrm{Var}_{it}[\tilde{e}_{t+1}]$ is equal for all investors i. Lemma 7.2 then implies that the equilibrium exchange rate in period t must satisfy

$$\tilde{e}_t = \sum_{n=0}^{\infty} \alpha^{n+1}\overline{E}_t^n[f_{t+n}] + \gamma\tilde{\sigma}^2 \sum_{n=0}^{\infty} \alpha^{n+1}\overline{E}_t^n[\nu_{t+n}] + \alpha\gamma\tilde{\sigma}^2\xi_t. \tag{8.40}$$

The exchange rate in period t is of the form

$$\tilde{e}_t = \tilde{A}\tilde{Q}_t(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t, \tag{8.41}$$

$$Q_t(k) = \tilde{M}\tilde{Q}_{t-1}(k) + \tilde{N}\tilde{w}_t, \tag{8.42}$$

where $k > 0$ is the level at which higher-order expectations are truncated in the model. The goal is to solve for the equilibrium conditions that characterize the matrices $\tilde{M}$ and $\tilde{N}$, the vector $\tilde{A}$, and the steady-state variance $\tilde{\sigma}^2$.

As in Theorem 7.6, the investors do not publicly observe the value of f_t in each period t, and so higher-order expectations of this interest rate parameter are part of the equilibrium exchange rate. However, unlike in Theorem 7.6, the investors do publicly observe ν_t and hence there are no higher-order expectations of current or future interventions. It follows that $\overline{E}_t^n[f_{t+n}] = h_1'(\tilde{M}\tilde{H})^n\tilde{Q}_t(k)$ for all $n \geq 1$ as before, while now $\overline{E}_t^n[\nu_{t+n}] = \rho_\nu^n\nu_t$ for all $n \geq 1$. Equation (8.40) then implies that

$$\tilde{e}_t = \sum_{n=0}^{\infty}\alpha^{n+1}h_1'(\tilde{M}\tilde{H})^n\tilde{Q}_t(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t,$$

so it follows by equation (8.41) that the vector $\tilde{A}$ must satisfy

$$\tilde{A} = \sum_{n=0}^{\infty}\alpha^{n+1}h_1'(\tilde{M}\tilde{H})^n.$$

Note that this equation matches equation (7.33) exactly, so that all that remains of this proof is to characterize the state transition matrices $\tilde{M}$ and $\tilde{N}$ and the steady-state variance $\tilde{\sigma}^2$.

Let $\tilde{\tilde{e}}_t = \tilde{e}_t - \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t$. If the foreign central bank announces the value of ν_t publicly, the relevant observations for each investor i in each period t are given by

$$\tilde{z}_{it} = \begin{pmatrix} x_{it} \\ \bar{i}_t \\ \tilde{\tilde{e}}_t \end{pmatrix} = D\tilde{Q}_t(k) + R\begin{pmatrix} \sigma_\epsilon^{-1}\epsilon_{it} \\ \sigma_\zeta^{-1}\zeta_t \\ \sigma_\chi^{-1}\chi_t \\ \sigma_\xi^{-1}\xi_t \end{pmatrix},$$

where

$$D = \begin{pmatrix} 1 & \mathbf{0}_{2\times k} \\ 1 & \\ & \tilde{A} \end{pmatrix},$$

and $R = \begin{pmatrix} R_1 & R_2 \end{pmatrix}$, with

$$R_1 = \begin{pmatrix} \sigma_\epsilon \\ 0 \\ 0 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_\chi & 0 \\ 0 & 0 & \alpha\gamma\tilde{\sigma}^2\sigma_\xi \end{pmatrix}.$$

If the state vector of higher-order expectations evolves according to equation (8.42), then Bayesian updating implies both that the exchange rate in period $t+1$ is conditionally normally distributed (this justifies the assumption of conditional normality) and that

$$E_{it}[\tilde{Q}_t(k)] = \tilde{M}E_{it-1}[\tilde{Q}_{t-1}(k)] + K\left(\tilde{z}_{it} - D\tilde{M}E_{it-1}[\tilde{Q}_{t-1}(k)]\right),$$

where K is the Kalman gain matrix. Averaging this equation over all investors yields

$$\begin{aligned} \overline{E}_t[\tilde{Q}_t(k)] &= \tilde{M}\overline{E}_{t-1}[\tilde{Q}_{t-1}(k)] + K\left(D\tilde{M}\tilde{Q}_{t-1}(k) + (D\tilde{N} + R_2)\tilde{w}_t - D\tilde{M}\overline{E}_{t-1}[\tilde{Q}_{t-1}(k)]\right) \\ &= (\tilde{M} - KD\tilde{M})\overline{E}_{t-1}[\tilde{Q}_{t-1}(k)] + KD\tilde{M}\tilde{Q}_{t-1}(k) + K(D\tilde{N} + R_2)\tilde{w}_t. \end{aligned} \tag{8.43}$$

Equation (8.43) implies that

$$\tilde{\Pi}_t(k) = \begin{pmatrix} \tilde{q}_{0t} \\ \overline{E}_t[\tilde{Q}_t(k-1)] \end{pmatrix} = \tilde{M}\begin{pmatrix} \tilde{q}_{0t-1} \\ \overline{E}_{t-1}[\tilde{Q}_{t-1}(k-1)] \end{pmatrix} + \tilde{N}\tilde{w}_t = \tilde{M}\tilde{Q}_{t-1}(k) + \tilde{N}\tilde{w}_t, \tag{8.44}$$

where

$$\tilde{M} = \begin{pmatrix} \rho_f & \mathbf{0}_{1\times k} \\ \multicolumn{2}{c}{\mathbf{0}_{k\times k+1}} \end{pmatrix} + \begin{pmatrix} \multicolumn{2}{c}{\mathbf{0}_{1\times k+1}} \\ \mathbf{0}_{k\times 1} & [\tilde{M} - KD\tilde{M}]_- \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{1\times k+1} \\ [KD\tilde{M}]_- \end{pmatrix}, \tag{8.45}$$

$$\tilde{N} = \begin{pmatrix} \sigma_\zeta \quad 0 \quad\quad 0 \\ [K(D\tilde{N} + R_2)]_- \end{pmatrix}, \tag{8.46}$$

and $[\tilde{M} - KD\tilde{M}]_-$ is the matrix $\tilde{M} - KD\tilde{M}$ with the last row and column removed and $[KD\tilde{M}]_-$ and $[K(D\tilde{N} + R_2)]_-$ are, respectively, the matrices $KD\tilde{M}$ and $K(D\tilde{N} + R_2)$ with the last row removed. The Kalman gain matrix K is given by

$$K = (PD' + \tilde{N}R_2')(DPD' + RR')^{-1}, \tag{8.47}$$

where P satisfies the matrix Riccati equation

$$P = \tilde{M}\left(P - (PD' + \tilde{N}R_2')(DPD' + RR')^{-1}(PD' + \tilde{N}R_2')'\right)\tilde{M}' + \tilde{N}\tilde{N}'. \tag{8.48}$$

As in the proof of Theorem 7.6, the final step is to solve for the steady-state variance of the exchange rate $\tilde{\sigma}^2$. In order to do this, it is necessary to compute the variance-covariance matrix

$$\hat{P} = \mathrm{Var}_{it} \begin{bmatrix} \tilde{Q}_{t+1}(k) \\ \xi_{t+1} \end{bmatrix} = \overline{\mathrm{Var}}_t \begin{bmatrix} \tilde{Q}_{t+1}(k) \\ \xi_{t+1} \end{bmatrix},$$

which depends on the steady-state dynamics of a system slightly more general than the system from equation (8.42). Note that

$$\begin{pmatrix} \tilde{Q}_t(k) \\ \tilde{\xi}_t \end{pmatrix} = \begin{pmatrix} \tilde{M} & \mathbf{0}_{k+1\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times k+2}} \end{pmatrix} \begin{pmatrix} \tilde{Q}_{t-1}(k) \\ \tilde{\xi}_{t-1} \end{pmatrix} + \begin{pmatrix} \multicolumn{2}{c}{\tilde{N}_1} & \tilde{N}_2 \\ 0 & 0 & \sigma_\xi \end{pmatrix} \begin{pmatrix} \sigma_\zeta^{-1}\zeta_t \\ \sigma_\chi^{-1}\chi_t \\ \sigma_\xi^{-1}\xi_t \end{pmatrix},$$

where $\tilde{N}_1$ and $\tilde{N}_2$ consist, respectively, of the first two columns and the last column of the matrix $\tilde{N}$ from equation (8.46) above, and that

$$\tilde{z}_{it} = \begin{pmatrix} x_{it} \\ \bar{i}_t \\ \tilde{\bar{e}}_t \end{pmatrix} = \begin{pmatrix} 1 & \\ 1 & \mathbf{0}_{2\times k+1} \\ A & \alpha\gamma\tilde{\sigma}^2 \end{pmatrix} \begin{pmatrix} \tilde{Q}_t(k) \\ \xi_t \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{it} \\ \chi_t \end{pmatrix}.$$

This system of equations both justifies the assumption that the conditional variance is equal for all investors i and implies that the matrix $\hat{P}$ is given by the solution to the Riccati equation

$$\hat{P} = \hat{M}\left(\hat{P} - (\hat{P}\hat{D}' + \hat{N}\hat{R}_2')(\hat{D}\hat{P}\hat{D}' + \hat{R}\hat{R}')^{-1}(\hat{P}\hat{D}' + \hat{N}\hat{R}_2')'\right)\hat{M}' + \hat{N}\hat{N}', \tag{8.49}$$

where

$$\hat{M} = \begin{pmatrix} \tilde{M} & \mathbf{0}_{k+1\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times k+2}} \end{pmatrix}, \qquad \hat{N} = \begin{pmatrix} \multicolumn{2}{c}{\tilde{N}_1} & \tilde{N}_2 \\ 0 & 0 & \sigma_\xi \end{pmatrix}, \qquad \hat{D} = \begin{pmatrix} 1 & \\ 1 & \mathbf{0}_{2\times k+1} \\ \tilde{A} & \alpha\gamma\tilde{\sigma}^2 \end{pmatrix}$$

$$\hat{R} = \begin{pmatrix} \hat{R}_1 & \hat{R}_2 \end{pmatrix}, \qquad \hat{R}_1 = \begin{pmatrix} \sigma_\epsilon \\ 0 \\ 0 \end{pmatrix}, \qquad \hat{R}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_\chi & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Because $\tilde{e}_{t+1} = \tilde{A}\tilde{\Pi}_{t+1}(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_{t+1}$, it follows that

$$\tilde{\sigma}^2 = \begin{pmatrix} \tilde{A} & \alpha\gamma\tilde{\sigma}^2 \end{pmatrix} \hat{P} \begin{pmatrix} \tilde{A} & \alpha\gamma\tilde{\sigma}^2 \end{pmatrix}' + \left(\frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\right)^2 \sigma_\delta^2. \tag{8.50}$$

I conclude that the matrices $\tilde{M}$ and $\tilde{N}$ and the steady-state variance $\tilde{\sigma}^2$ from the approximate equilibrium of Theorem 7.7 are given by the joint solution to equations (8.45), (8.46), (8.47), (8.48), (8.49), and (8.50). The fact that this approximation converges to the true steady-state equilibrium of this model is shown by Nimark (2010a). $\square$

Part III

Information Manipulation in Global Coordination Games

A wide variety of economic outcomes are affected by the ability of agents to coordinate their actions. In all of these cases, the information available to the agents influences both their actions and their capacity for coordinating those actions. This information is usually assumed to be truthful and unbiased, but if players in these games can filter or alter the information that reaches the agents, then this assumption is more tenuous. The effect of information manipulation is especially important in coordination settings, since the optimal action of an individual depends not only on her ability to remove the bias from her information, but also on the ability of others to remove the bias from their information. This interaction between information manipulation and the heterogeneity of beliefs can potentially have significant effects on the equilibrium outcome of coordination games.

This chapter investigates how information manipulation alters the equilibrium outcome of coordination games in which agents observe noisy private signals of the underlying state, i.e., global games. I consider a simple game in which a regime manipulates the private information of agents while the agents attempt to coordinate an attack against the regime. The success of this attack depends on both the attack's size and the underlying state, which affects the regime's ability to defend successfully. The main conclusion is that the effect of manipulation depends crucially on the regime learning about fundamentals faster than the agents do. In particular, I show that information manipulation incurs costs for the regime but has no effect on the agents' attack if the agents learn about the underlying state fastest. If instead the regime learns the state faster than the agents, then it is possible for information manipulation to weaken the agents' attack provided that this learning satisfies certain properties. In those cases in which the manipulation is ineffective, the regime prefers not to manipulate information but cannot credibly commit to do so.

Information manipulation in this setting is intended to capture the reality that the incumbent regime is often able to alter and filter the information that eventually reaches coordinating agents. Such manipulative actions are particularly apparent during both speculative attacks against fixed currency pegs and political revolutions against autocratic regimes. In the case of speculative attacks, the experience of the United Kingdom during the European ERM crisis of 1992 provides a nice example. Both Stephens (1996) and Thompson (1996) document that in the months prior to the pound's exit from the ERM, the British government often prevented or attempted to prevent the publication of news stories that could reveal either the divisions within the government itself or the tensions between the government and the German Bundesbank. During this episode, the pound was pegged to the Deutschmark along with several other European currencies, and the British government was hoping that the Germans would help support the peg by both lowering domestic interest rates and purchasing pounds sterling. The British authorities also sought to create the impression of stronger fundamentals by misleading the public about their ability and willingness to use foreign exchange reserves to defend the peg. Indeed, Stephens (1996) describes one instance in which British officials deliberately concealed some of their reserves in an attempt to surprise the market and convince it that both reserves and buyers of pound sterling were in great abundance. Manipulative actions such as those taken by the British government during the European ERM crisis of 1992 surely have some effect on the expectations of market participants, even if those participants are aware of the government's actions.

Much like central banks during currency crises, autocratic regimes almost always seek to control the information that reaches their subjects in an attempt to prevent or limit the intensity of revolutions. These manipulative actions include censoring newspaper and printed media content, banning open protests and dissident groups, censoring radio and television broadcasting, and most recently, monitoring and restricting the public's internet access. As emphasized by Edmond (2008b), the use of these authoritarian techniques to distort the public's information has been widespread and includes such different episodes as nineteenth century Ottoman Turkey and present day China. In the same way that the actions of central banks affect the expectations of market participants, the restrictive policies of autocratic regimes affect the beliefs of the general public, even if the public is rational and aware that its information is being manipulated. One of the goals of my analysis is to better understand how those beliefs are influenced and what the implications of that influence are.

The global games that I present in this chapter extend more standard global games so that the regime does not know the outcome of the game ex-ante. This is achieved by splitting the fundamentals of the game into two parts, one of which the regime is perfectly informed about (denoted by θ) and the other which the regime is unsure about (denoted by δ). Agents face uncertainty about both parts of fundamentals and receive private signals of these parameters, so that they have heterogeneous posterior distributions for fundamentals, as in the setup of Morris and Shin (1998).[33] Agents choose whether or not to join an attack against the regime based on their biased private signals, and agents' actions are strategic complements since the attack successfully ousts the regime only if enough agents join the attack. Of course, rational agents are aware of the potential for biased private signals and take this into account when forming their beliefs about the probability of a successful attack.

In this setting, I examine the implications of information manipulation by the regime. Following Edmond (2008a), I assume that the regime values successfully repelling the agents' attack and that it can take a costly hidden action that shifts the mean of the agents' private signals upwards. The maximum size of this bias is bounded above, so that the full effectiveness of this action is limited. Because the regime is uncertain about a part of fundamentals, the regime knows neither the exact size of the attack it will face nor its full ability to repel that attack. As a consequence, the regime is uncertain about the outcome of the coordina-

[33]The setup is somewhat related to Cheli and Della Posta (2007), who consider how biased private signals affect the outcome of a global coordination game. The key difference is that the authors primarily analyze the effects of unexpected bias rather than the equilibrium implications of information manipulation.

tion game when it chooses how much to bias agents' private signals. In equilibrium, then, the regime trades off the cost that information manipulation incurs with the benefit that it provides by reducing the likelihood of a successful attack by the agents.

The fact that both the regime and the agents face uncertainty in my setup is crucial. In particular, this uncertainty causes some weak regimes to bias the information of agents upwards since this action makes a successful attack by the agents less likely. Furthermore, because the effectiveness of information manipulation is bounded above, many regimes that wish to take an action that is large enough to practically eliminate the possibility of a defeat by the agents are unable to do so and instead only have the option to take an action that leaves a non-trivial probability of collapse. Rational agents are aware that weak regimes are biasing their private information and this compels them to discount higher signals more aggressively. As a consequence, the effectiveness of information manipulation depends on the extent to which the regime is better informed about fundamentals than the agents. This result extends the result of Edmond (2008a), who considers a game in which the regime faces no ex-ante uncertainty and concludes that information manipulation effectively reduces the intensity of the agents' attack. In my model, this maximum-effectiveness result only obtains in some cases.

There is a mass of receivers of information in my model (the agents), so the equilibrium effects of information manipulation are different from earlier analyses of strategic manipulation that considered a single receiver of information. In a one-to-one setup, Crawford and Sobel (1982) show that the single information receiver achieves higher expected utility as her preferences become more similar to the information sender's. Similarly, both Milgrom and Roberts (1982) and Matthews and Mirman (1983) consider an environment in which an established firm seeks to choose a price that maximizes current profits but also may deter another firm's entry as a competitor. These authors examine how and when limit pricing is able to effectively reduce those instances in which the second firm enters. While these authors' results do hint at the importance of more precise information about fundamentals, their focus is primarily on noisy signalling and asymmetric information.

This chapter contributes to a growing literature that examines the implications of uncertainty on the part of the regime in coordination games. Goldstein, Ozdenoren, and Yuan (2011) consider a setup in which the central bank does not know fundamentals perfectly and learns about them by observing the aggregate trading of currency speculators. They show that the bank can improve the effectiveness of its policy by committing to put a lower weight on information from the market. Kurlat (2009) develops a model in which the regime is uncertain about speculators' preferences and shows that there exist distributions of preferences that can justify any possible partial defense strategy by the regime. Bauer and Herz (2009) consider a setup in which the central bank faces uncertainty about the size of the speculators' attack against it and find that the strength of the defensive measures chosen by the bank are not monotonically increasing in fundamentals

The chapter is organized as follows. Section 9 presents the benchmark game in which agents play a global game with two sources of uncertainty but no information manipulation. Section 10 extends this game so that the regime can manipulate the information of the agents and presents the main results. Section 11 concludes. The proofs for all of the results are provided in the last section.

9 Benchmark Game

A game is played between a regime and a continuum of risk-neutral agents indexed by $i \in [0, 1]$. There are two periods in this game, and in period one each agent i chooses whether or not to attack the regime. The action $a_i = 1$ represents an attack and $a_i = 0$ represents no attack, and the total mass of attacking agents is denoted by $A \geq 0$.

The difference between this benchmark global coordination game and more standard setups is that fundamentals are separated into two distinct parts. In particular, the fundamentals in this game are parameterized by the sum

$$f = \theta + \delta, \tag{9.1}$$

where $\theta, \delta \in \mathbb{R}$. In period two, the regime survives the agents' attack if and only if $f \geq A$, so the sum $\theta + \delta$ measures how strong an attack must be in order for the regime to collapse. I separate fundamentals so that both the regime and the agents can be better informed about one part of fundamentals, as is the case in the setup of Section 10. The goal is then to investigate how the effectiveness of information manipulation by the regime depends on the degree to which the regime is better informed about fundamentals than the agents.

As is standard in coordination games, fundamentals are such that if $f < 0$, then the regime collapses in period two regardless of the size of the agents' attack, and if $f \geq 1$, then the regime survives in period two regardless of the size of the attack.[34] The agents' payoffs are realized at the end of period two and depend on the outcome of the attack. Specifically, the payoff of agent i is given by

$$u(a_i, A, f) = (1_{\{f<A\}} - r)a_i, \tag{9.2}$$

where $1 > r > 0$ is the cost of joining the attack against the regime. The actions of the agents in this setup are strategic complements since the probability of a successful attack increases as more agents join in.

In period one, each agent i receives a private signal about the fundamentals, $x_i = \theta + \delta + \epsilon_i$, where $\epsilon_i \sim \mathrm{N}(0, \sigma_\epsilon^2)$ and all noise terms are independent across agents. I assume that the common prior for θ is given by an improper uniform distribution over the real line, so that there is no public information about the value of this fundamental. These assumptions imply that each agent i's posterior beliefs about f are normally distributed with mean x_i and variance σ_ϵ^2. In equilibrium, each agent's period-one decision whether or not to attack

[34]This is because the total mass of attacking agents A is always greater than or equal to zero and less than or equal to one.

the regime is a function of her private signal only, so I write $a_i = a(x_i)$ for all $i \in [0, 1]$.

In this benchmark game, I assume that δ is normally distributed with mean zero and variance σ_δ^2. All agents share the same beliefs about δ and these beliefs are common knowledge. I also assume that the regime cannot take any actions in the first period, so that it only observes the game in period one and then collapses in period two if the agents' attack is sufficiently large ($\theta + \delta = f < A$). These assumptions imply that the regime's beliefs about δ are inconsequential, and that the setup is that of a standard global game. Let $\phi(\cdot)$ denote the density function of a standard normal random variable.

Definition 9.1. An equilibrium of this game is an agent's posterior density $\pi(f \,|\, x_i)$, an individual attack decision $a(x_i)$, and a mass of attackers $A(f)$ such that

$$\pi(f \,|\, x_i) = \int_{-\infty}^{\infty} \sigma_\delta^{-1}\sigma_\epsilon^{-1}\phi\left(\frac{x_i - f}{\sigma_\epsilon}\right)\phi\left(\frac{\theta - f}{\sigma_\delta}\right) d\theta, \tag{9.3}$$

$$a(x_i) = \arg\max_{a_i \in \{0,1\}} \left\{\int_{-\infty}^{\infty} u(a_i, A(f), f)\pi(f \,|\, x_i)\, df\right\}, \tag{9.4}$$

$$A(f) = \int_{-\infty}^{\infty} \sigma_\epsilon^{-1} a(x_i)\phi\left(\frac{x_i - \theta - \delta}{\sigma_\epsilon}\right) dx_i. \tag{9.5}$$

Note that equation (9.3) from this definition implies that the agents' posterior densities for fundamentals $f = \theta + \delta$ are given by

$$\pi(f \,|\, x_i) = \sigma_\epsilon^{-1}\phi\left(\frac{x_i - f}{\sigma_\epsilon}\right). \tag{9.6}$$

This is simply the density of a normal random variable with mean x_i, so it follows that agents' posterior beliefs about f are normally distributed. This is a consequence of agents having uninformative priors about θ, since that forces their observations of x_i to form all of their beliefs about the sum $\theta + \delta$. I emphasize that this assumption can be easily relaxed without changing the main results. Indeed, the key implication of assuming that agents have a common prior for θ is that equilibrium uniqueness is no longer guaranteed.

It is a well-known result that imperfect common knowledge about fundamentals in coordination games generates a unique equilibrium that is monotone in the agents' private signals. In this section's setup, this means that the agents' equilibrium strategies are monotonically decreasing functions of their private signals x_i: there exists a unique threshold x^* such that each agent i chooses to attack if and only if $x_i \leq x^*$. This uniqueness result was first proved by Carlsson and van Damme (1993), and has since been extended by Angeletos, Hellwig, and Pavan (2007), Dasgupta (2007), and Hellwig, Mukherji, and Tsyvinski (2006), among others. The fact that there exists a unique monotone equilibrium of this game when agents have heterogeneous information contrasts sharply with the equilibrium predictions when all information is instead public. Indeed, if all information about $\theta + \delta$ is shared by agents, then there exist multiple equilibria. More specifically, if $0 \leq \theta + \delta < 1$ and this value is common knowledge, then there exist two equilibria: either all agents attack the regime and this attack is successful, or no agents attack the regime and this attack is unsuccessful. In either case, the success or failure of the agents' attack against the regime is self-fulfilling.

More generally, a unique equilibrium in global games of this kind is guaranteed as long as the agents' private information about $\theta + \delta$ is sufficiently more informative than their public information. This result is discussed by Angeletos and Werning (2006) and Hellwig (2002). In terms of this benchmark game, the implication is that the unique equilibrium of this section still obtains if I add an informative common prior for θ, provided that that prior is sufficiently less informative than the agents' private signals x_i.

Theorem 9.2. *There exists a unique Bayesian equilibrium of this game in which there are thresholds x^* and $\theta^*(\delta)$ such that each agent i attacks the regime ($a(x_i) = 1$) if and only if $x_i \leq x^*$ and the regime is successfully overthrown ($\theta + \delta < A$) if and only if $\theta < \theta^*(\delta)$. These thresholds are uniquely determined by the solution to the equations*

$$\theta^*(\delta) = A(\theta^*(\delta) + \delta) = P(x_i \leq x^* \mid \theta^*(\delta)) = \Phi\left(\frac{x^* - \theta^*(\delta) - \delta}{\sigma_\epsilon}\right), \tag{9.7}$$

$$r = P(\theta \leq \theta^*(\delta) \mid x^*) = P(x^* - \theta^*(\delta) - \delta \leq \epsilon_i) = 1 - \Phi\left(\frac{x^* - \theta^*(\delta) - \delta}{\sigma_\epsilon}\right), \tag{9.8}$$

so that $x^* = 1 - r + \sigma_\epsilon \Phi^{-1}(1 - r)$ and $\theta^*(\delta) = 1 - r - \delta$.

Equation (9.6) shows that the agents' posterior distributions for fundamentals $f = \theta + \delta$ are heterogeneous, and as a consequence the proof of Theorem 9.2 proceeds by the usual iterated elimination of dominated strategies. Indeed, because of these heterogeneous posteriors, there is no practical difference between this setup and other, more standard global game setups. For this reason, I omit the proof of Theorem 9.2 for brevity and refer the reader to Hellwig (2002) and Morris and Shin (2003) for a proof of this uniqueness result.

I express the monotone equilibrium of this game in terms of a threshold for θ that is a function of δ only to facilitate the comparison of this game with other games in which θ represents all of fundamentals. Theorem 9.2 could easily be restated with a unique threshold for $f = \theta + \delta$ given by $f^* = 1 - r$. In this case, the regime is successfully overthrown if and only if $f = \theta + \delta < f^*$. An important consequence of the fact that $\theta^*(\delta) = 1 - r - \delta$ is that this threshold does not change as either σ_ϵ, the standard deviation of each agent's idiosyncratic noise term, or σ_δ, the standard deviation of δ, change. In other words, the range of fundamentals for which the regime survives the agents' attack against it does not depend on the precision of the agents' signals about those fundamentals. This result changes dramatically in Section 10.[35] Once the regime is able to manipulate the agents' information,

[35] Adding a public source of information about fundamentals also changes the implication that the precision of the agents' information is irrelevant. For a discussion of the effects of higher quality information in this setup, see Heinemann and Illing (2002), Metz (2002), and Bannier and Heinemann (2005).

the effect of an increase in precision can often be significant, especially in the limit as signals become arbitrarily precise. This result is examined in detail by Edmond (2008a).

Throughout my analysis, I shall focus primarily on the case in which δ is equal to zero. First, beliefs about δ are normally distributed around zero, so it is clearly most appropriate to think of this variable as being equal to zero rather than some other value, especially as $\sigma_\delta \to 0$. Second, this parameter is intended to capture the uncertainty that both agents and the regime have about fundamentals, even if these players turn out to be correct on average. Most global games assume that the regime knows the state of fundamentals perfectly, but this is clearly an unrealistic assumption. For example, the costly, extensive, and ultimately ineffective manipulative actions of the British government during the European ERM crisis of 1992 present strong evidence that this regime was imperfectly informed about the state of the economy and the intensity of the speculative pressure that was mounting against it. If the government was truly aware of its weak position, it surely would not have acted as aggressively as it did.[36] In addition to being unrealistic, there is reason to think that the assumption of perfect information on the part of the regime is more than just a harmless simplification in the case of information manipulation. Indeed, I show in the next section that the effectiveness of this manipulation depends crucially on the extent to which the regime is better informed than the agents. The next step is to extend the model and consider the implications of information manipulation by the regime.

10 Information Manipulation

Suppose that the regime can manipulate the information of the agents in period one. Specifically, suppose that the regime can take a costly hidden action $\nu \in [0, \bar{\nu}]$, where $0 < \bar{\nu} < \infty$. This action biases the private information of agents so that each agent i observes the private signal $x_i = \theta + \delta + \nu + \epsilon_i$ in period one. As before, the signals ϵ_i are normally distributed with mean zero and variance σ_ϵ^2 and are independent across agents.

The cost of this information manipulation is given by the convex function $C(\nu)$, where $C(0) = 0$, $C'(\nu) > 0$ for all $\nu > 0$, and $C''(\nu) \geq 0$ for all $\nu \in [0, \bar{\nu}]$. This assumption captures the fact that the regime is able to filter and alter the information that agents receive. Information manipulation creates the appearance of stronger fundamentals and hence weakens the agents' attack, but it also incurs some cost to the regime. Furthermore, this action is bounded above so that the maximum effect of this manipulation is limited. The game remains the same in period two, with the regime collapsing if and only if $f = \theta + \delta < A$ and agents receiving payoffs depending upon the attack's outcome ($1 - r > 0$ for attacking

[36] Additionally, both Stephens (1996) and Thompson (1996) look at internal communications and statements within the British government during the ERM crisis and find strong evidence of misinformation.

successfully, $-r < 0$ for attacking unsuccessfully, and 0 for not attacking).

The second important assumption involves the information of the agents and the regime. As I shall demonstrate below, the way in which this information differs between players has important implications for the effectiveness of information manipulation in equilibrium. I assume that the regime knows the value of θ perfectly but is unsure about the value of δ. Specifically, the regime believes that δ is normally distributed with mean zero and variance σ_r^2. The agents, on the other hand, believe that δ is normally distributed with mean zero and variance σ_a^2. This section's main exercise is to investigate how the ratio $\frac{\sigma_r}{\sigma_a}$ influences the effectiveness of information manipulation by the regime.

This setup is intended to capture a situation in which δ is equal to zero, but both the regime and the agents are unsure that this is indeed the case. It follows that the parameter σ_r measures the amount of uncertainty that the regime faces about fundamentals. The agents, however, face even more uncertainty about fundamentals because they also do not know the value of θ perfectly (they observe private signals $x_i = \theta + \delta + \nu + \epsilon_i$). In the results below, I investigate how the impact of manipulation varies with the relative informativeness of the agents and the regime. I show that this ratio determines the effectiveness of manipulation in the limit as both the agents' and the regime's information become infinitely precise.

The fact that the regime faces some uncertainty about fundamentals and hence also about the effectiveness of its hidden actions is an important part of this section's setup. If the regime instead knows the value of δ perfectly, then it will never incur the costs of manipulation without the benefit of succeeding against the agents' attack. In other words, the regime will only choose $\nu > 0$ if it is certain that this action will effectively thwart the agents' attack against the regime. As a consequence, any time the regime manipulates information, it is able to successfully ensure its survival. This is the setup analyzed by Edmond (2008a), who shows that this assumption implies that the agents' attack disappears in the limit as agents' private signals of fundamentals become infinitely precise. I show below that this same limit result obtains in this section's setup as well, but only in the special case in which the regime learns about δ faster than the agents learn about $\theta + \delta$.

If the agents instead learn about θ faster than the regime learns about δ, then the regime's option to manipulate information incurs costs on the regime without diminishing the intensity of the agents' attack in period two. This follows because the regime must trade off the costs of information manipulation with its uncertain benefits. Furthermore, because the effectiveness of this manipulation is bounded ($\nu \leq \bar{\nu}$), the regime is often unable to err on the side of caution and manipulate to such an extent that it is almost certain it will survive the attack in period two. Instead, the regime must choose between taking a costly and highly uncertain action and doing nothing.

The regime has a loss of one if it collapses in period two and its hidden action $\nu \in [0, \bar{\nu}]$ incurs a cost of $C(\nu)$. It follows, then, that the regime wishes to minimize the loss function

$$L(\theta + \delta, A, \nu) = C(\nu) + 1_{\{\theta+\delta<A\}}. \tag{10.1}$$

For a given choice of $\nu \geq 0$, let

$$\theta + \hat{\delta}(\theta, \nu) = \Phi\left(\frac{x^* - \theta - \nu - \hat{\delta}(\theta, \nu)}{\sigma_\epsilon}\right), \tag{10.2}$$

so that

$$\hat{\delta}(\theta, \nu) = x^* - \theta - \nu - \sigma_\epsilon \Phi^{-1}\left(\theta + \hat{\delta}(\theta, \nu)\right). \tag{10.3}$$

Because $A(\theta, \nu, \delta) = \Phi\left(\frac{x^* - \theta - \nu - \delta}{\sigma_\epsilon}\right)$, the regime collapses in period two with $\theta < A(\theta, \nu, \delta)$ if and only if $\delta < \hat{\delta}(\theta, \nu)$. This fact implies that the regime's expected loss is simply equal to the cost of its hidden manipulation plus the probability that $\delta < \hat{\delta}(\theta, \nu)$.

Definition 10.1. An equilibrium of this game is an agent's posterior density $\pi(f \mid x_i)$, an individual attack decision $a(x_i)$, a mass of attackers $A(\theta, \nu, \delta)$, and regime information manipulation $\nu(\theta)$ such that

$$\pi(f \mid x_i) = \frac{\int_{-\infty}^{\infty} \sigma_a^{-1} \sigma_\epsilon^{-1} \phi\left(\frac{x_i - f - \nu(\theta)}{\sigma_\epsilon}\right) \phi\left(\frac{\theta - f}{\sigma_a}\right) d\theta}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_a^{-1} \sigma_\epsilon^{-1} \phi\left(\frac{x_i - f - \nu(\theta)}{\sigma_\epsilon}\right) \phi\left(\frac{\theta - f}{\sigma_a}\right) d\theta df}, \tag{10.4}$$

$$a(x_i) = \underset{a_i \in \{0,1\}}{\arg\max} \left\{ \int_{-\infty}^{\infty} u(a_i, A(\theta, \nu, \delta), f) \pi(f \mid x_i) \, df \right\}, \tag{10.5}$$

$$A(\theta, \nu, \delta) = \int_{-\infty}^{\infty} \sigma_\epsilon^{-1} a(x_i) \phi\left(\frac{x_i - \theta - \nu(\theta) - \delta}{\sigma_\epsilon}\right) dx_i, \tag{10.6}$$

$$\nu(\theta) = \underset{\nu \in [0, \bar{\nu}]}{\arg\min} \left\{ C(\nu) + \int_{-\infty}^{\hat{\delta}(\theta)} \sigma_r^{-1} \phi\left(\frac{\delta}{\sigma_r}\right) d\delta \right\}. \tag{10.7}$$

This equilibrium definition extends Definition 9.1 from the standard global game with no information manipulation. The crucial difference between the two is that the regime in this game manipulates the agents' information in a way that minimizes its expected loss given its information about θ and δ. Rational Bayesian agents are aware of this information manipulation and attempt to filter it out when forming their expectations about $f = \theta + \delta$. This is evident from equation (10.4).

Unfortunately, the extra complexity that information manipulation together with uncertainty about δ adds makes it difficult to prove the existence of a unique equilibrium. As a consequence, I focus primarily on *monotone equilibria* in this setup. A monotone equilibrium of this game is given by two thresholds as in Theorem 9.2. The first threshold, $\theta^*(\delta)$, is such that the regime collapses in period two if and only if $\theta < \theta^*(\delta)$. The second threshold, x^*, is such that an agent with private signal x_i chooses to join the attack against the regime in period one if and only if $x_i \leq x^*$. Because the attack against the regime is successful in period two if and only if $\theta < A(\theta, \nu, \delta)$, it follows that the threshold $\theta^*(\delta)$ is given by the solution to the regime's indifference condition

$$\theta^*(\delta) = x^* - \delta - \nu(\theta^*(\delta)) - \sigma_\epsilon \Phi^{-1}(\theta^*(\delta) + \delta). \tag{10.8}$$

Similarly, as long as the agents' posterior probability of the regime collapsing is decreasing in their private signals x_i, the threshold x^* is given by the solution to the agents' indifference condition

$$P(\theta < \theta^*(\delta) \mid x^*, \nu(\cdot)) = r. \tag{10.9}$$

I show below that a unique monotone equilibrium exists in the limit as the uncertainty in this game disappears. As in the previous section, I focus on the case in which δ is equal to zero. Let $\boldsymbol{\sigma} = \begin{pmatrix} \sigma_a & \sigma_r & \sigma_\epsilon \end{pmatrix}'$ be a vector of noise terms.

Theorem 10.2. *Suppose that $\boldsymbol{\sigma} \to \mathbf{0}$. Suppose also that $C(\bar{\nu}) < \frac{1}{2}$ and $1 - r + \bar{\nu} < 1$. If $\frac{\sigma_\epsilon}{\sigma_a^2} \to 0$ and $\frac{1}{\sigma_a}\phi\left(\frac{\sigma_r\sqrt{\ln -\sigma_r}}{\sigma_a}\right) \to 0$, then in the limit there exists a unique monotone equilibrium with $\theta^*(0) \to 1 - r$.*

The proof of Theorem 10.2 is in Section 12. The second condition of the theorem, that $\frac{1}{\sigma_a}\phi\left(\frac{\sigma_r\sqrt{\ln -\sigma_r}}{\sigma_a}\right) \to 0$, implies that also $\frac{\sigma_a}{\sigma_r} \to 0$. Together with $\frac{\sigma_\epsilon}{\sigma_a^2} \to 0$, this implies that $\frac{\sigma_\epsilon}{\sigma_r} \to 0$ as well and so Theorem 10.2 states that if agents learn about fundamentals $\theta + \delta$ faster than the regime learns about δ, then information manipulation is ineffective. In particular, if δ is equal to zero in this case, then the threshold for θ at which the regime is defeated, $\theta^*(0)$, converges to $1 - r$ in the limit. This is the same value for the threshold as in the benchmark game with no information manipulation of Section 9. The implication is that fundamentals determine the regime's ability to survive the agents' attack in period two in the same way as if there were no hidden actions to bias the agents' private information.

In this game's unique monotone equilibrium (in the limit), a regime with $\theta = \theta^*(0)$ chooses $\nu = \bar{\nu}$ and manipulates the agents' information maximally. This action is effective as it reduces the number of agents with private signals x_i less than x^* so that the regime faces an attack that is smaller than if it were to take no action. However, the agents are aware that the regime may be manipulating information and they discount their private signals in a Bayesian manner. If the agents' information is sufficiently better than the regime's, then the agents perfectly discount their private signals and filter out all of the regime's manipulation. In equilibrium, then, the agents expect the regime to bias their signals and the regime validates these expectations in order to give itself a chance to survive by reducing the size of an otherwise enormous attack against it. The end result is that the regime takes a costly action with no net benefit. If it could do so credibly, the regime would prefer to commit to not manipulate the agents' information in these instances.

The proof of Theorem 10.2 is accomplished in two separate steps. First, I characterize the regime's choice of manipulation ν in the limit as both the regime and the agents' information about fundamentals become arbitrarily precise, assuming that $\bar{\nu} \leq x^* \leq 1$. In this case, the regime's choice of manipulation is a function of its type θ and varies across three different regions for this type parameter. The first region consists of those regimes that prefer not to bias the agents' information because of either very strong or very weak fundamentals. More precisely, I show that there exist thresholds $\theta_l < \theta_h$ such that all regimes with $\theta \leq \theta_l$ (weak fundamentals) or $\theta \geq \theta_h$ (strong fundamentals) choose $\nu(\theta) = 0$. The second region consists of those regimes with intermediate fundamentals whose optimal choice of ν is nonzero and is not affected by the upper-bound constraint $\nu \leq \bar{\nu}$. This means that there exists a threshold θ_m (satisfying $\theta_l < \theta_m < \theta_h$) such that all regimes with $\theta_m \leq \theta < \theta_h$ choose $0 < \nu(\theta) < \bar{\nu}$. In this range of fundamentals, regimes are able to manipulate the agents' information in a way that ensures that they survive the agents' attack for most values of δ, including the important case in which it is equal to zero. The third region consists of those regimes with intermediate fundamentals whose optimal choice of ν is nonzero but is constrained by the upper bound $\bar{\nu}$. In particular, I show that all regimes with $\theta_l < \theta < \theta_m$ choose $\nu(\theta) = \bar{\nu}$. An important consequence of the assumption that $C(\bar{\nu}) < \frac{1}{2}$ is that this region is nonempty, provided that the noise parameters σ_ϵ and σ_a are sufficiently small. This follows because a regime of type $\theta = x^* - \bar{\nu}$ that chooses $\nu(\theta) = \bar{\nu}$ survives the agents' attack in the limit if and only if $\delta > 0$, which occurs with probability one half. As long as $C(\bar{\nu}) < \frac{1}{2}$, the regime prefers to incur the costs of maximum manipulation rather than to survive the agents' attack with probability zero in the limit.

The second part of the proof of Theorem 10.2 involves solving for the agents' beliefs about the probability of a successful attack against the regime. Recall that the agents' attack threshold x^* is given by the solution to equation (10.9), and consider the behavior of the conditional probability on the left-hand side of this equation as $\boldsymbol{\sigma} \to \mathbf{0}$. An agent who observes the private signal $x^* = \theta + \nu(\theta) + \delta + \epsilon_i$ rationally weighs the possibility of θ being in each of the manipulation regions described in the previous paragraph. This is where the assumption that both $\frac{\sigma_a}{\sigma_r} \to 0$ and $\frac{\sigma_\epsilon}{\sigma_r} \to 0$ is crucial. Because θ_h converges to x^* and both θ_l and θ_m converge to $x^* - \bar{\nu}$ as $\boldsymbol{\sigma} \to \mathbf{0}$, it is necessary for agents to learn about fundamentals faster than the regime in order for an agent who observes x^* to conclude that $\theta \in (\theta_l, \theta_m)$. Once it is established that θ is in this intermediate range, it is very easy to filter out the regime's information manipulation because $\nu(\theta) = \bar{\nu}$ for all θ. As a consequence, the equilibrium conditions given by equations (10.8) and (10.9) become, in the limit, essentially the same as the conditions given by equations (9.7) and (9.8) from Theorem 9.2 in the previous section, except that x^* now filters out the regime's manipulation and hence converges to $1 - r + \bar{\nu}$ rather than $1 - r$.[37]

[37] Recall that in the coordination game with no information manipulation of Section 9, $x^* \to 1 - r$ as $\sigma_\epsilon \to 0$ while $\theta^*(0) = 1 - r$ does not change.

Theorem 10.3. *Suppose that $\boldsymbol{\sigma} \to \mathbf{0}$ and that $C(\bar{\nu}) < \frac{1}{2}$.*

(i) If $\dfrac{\sigma_\epsilon}{\sigma_a} \to \infty$, $\dfrac{1}{\sigma_\epsilon}\phi\left(\dfrac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_\epsilon}\right) \to \infty$, and $\lim\limits_{\boldsymbol{\sigma}\to\mathbf{0}} \dfrac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a} > \Phi^{-1}(1-r)$, then in the limit there exists a monotone equilibrium with $\theta^*(0) \to 0$ and there exist no monotone equilibria with $\theta^*(0) > 0$.

(ii) If $\dfrac{\sigma_\epsilon}{\sigma_a} \to \infty$ and $\lim\limits_{\boldsymbol{\sigma}\to\mathbf{0}} \dfrac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a} < \Phi^{-1}(1-r)$, then in the limit there exist no monotone equilibria with $\theta^*(0) < 1-\bar{\nu}$.

The proof of Theorem 10.3 is in Section 12. In the first part of the theorem, the assumption that $\frac{1}{\sigma_\epsilon}\phi\left(\frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_\epsilon}\right) \to \infty$ requires that $\frac{\sigma_r}{\sigma_\epsilon}$ converge to a limit less than one. Consequently, this part of the theorem states that if the regime learns about δ faster than the agents learn about $\theta+\delta$, then information manipulation is effective provided that the regime does not learn about δ too much faster than the agents. This result corresponds to the situation described by Edmond (2008a). In this case, both the regime and the agents learn that δ is equal to zero much faster than the agents learn about the value of θ. This implies that in the limit the agents are unable to filter out the regime's hidden actions so that the regime manipulates effectively and $\theta^*(0)$ converges to a limit strictly less than $1-r$. One of this chapter's main contributions is to show that this occurs only if the agents do not learn about fundamentals much faster than the regime.

The second part of Theorem 10.3 states that faster learning by the regime is not enough to ensure that information manipulation has the intended effect. In particular, if the regime learns about δ too much faster than the agents, then information manipulation can actually backfire and intensify the agents' attacks against the regime. This occurs because the regime learns that δ is equal to zero very quickly, and hence those regimes that choose to manipulate only bias the agents' signals up to a level that is barely above x^*. The regime believes that this action is sufficient to thwart the agents' attack against it because it correctly understands that a value of δ that is much below zero is nearly impossible. The problem is that the agents do not share this confidence about the low probability of observing values of δ much below zero. As a consequence, the agents believe that the regime's actions are insufficient and will not greatly reduce the possibility of a successful attack. If this effect is strong enough, the regime's actions can actually undermine the agents' confidence in a manipulated signal and cause attacks to intensify. In the limit, this makes any value of $x^* \leq 1$ impossible to reconcile with a monotone equilibrium.

Theorem 10.3 is proved in a similar manner to Theorem 10.2. As in the first theorem, I show that whenever $\bar{\nu} \leq x^* \leq 1$ the regime's choice of manipulation varies across three different regions for the fundamentals parameter θ as given by the thresholds $\theta_l < \theta_m < \theta_h$. However, unlike in the first theorem, the regime now learns that δ is equal to zero faster than the agents learn about $\theta+\delta$. As a consequence, an agent who observes the private signal x^*

no longer concludes that $\theta_l < \theta < \theta_m$ and $\nu(\theta) = \bar{\nu}$, but instead that $\theta_m \leq \theta < \theta_h$. I show that this implies that the conditional probability from equation (10.9) satisfies

$$\lim_{\boldsymbol{\sigma} \to \mathbf{0}} P(\theta < \theta^*(\delta) \mid x^*, \nu(\cdot)) = \lim_{\boldsymbol{\sigma} \to \mathbf{0}} \Phi\left(\frac{-\sigma_r\sqrt{-\ln \sigma_r}}{\sigma_a}\right). \tag{10.10}$$

The first part of Theorem 10.3 assumes that $\frac{\sigma_r\sqrt{-\ln \sigma_r}}{\sigma_a}$ converges to a limit that is strictly greater than $\Phi^{-1}(1-r)$, so it follows that the conditional probability on the left-hand side of equation (10.10) converges to a limit that is strictly less than r. The second part of the theorem assumes instead that $\frac{\sigma_r\sqrt{-\ln \sigma_r}}{\sigma_a}$ converges to a limit that is strictly less than $\Phi^{-1}(1-r)$, so that the conditional probability converges to a limit that is strictly greater than r. In both cases, the implication is that the equilibrium condition from equation (10.9) cannot be satisfied for any $\bar{\nu} \leq x^* \leq 1$.

The next step in the theorem's proof is to consider attack thresholds satisfying $0 \leq x^* < \bar{\nu}$. In general, a value of x^* that is in this range significantly complicates the calculation of both the regime's optimal choice of information manipulation and the agents' posterior probabilities of mounting a successful attack against the regime. Part of the difficulty is that if $x^* < \bar{\nu}$, then there exist regimes of type $\theta \leq 0$ with $\theta + \bar{\nu} > x^*$. These regimes often prefer manipulating the agents' information to not manipulating it, but in the limit as $\sigma_\epsilon \to 0$ they do not choose ν so that $\theta + \nu > x^*$ since they can only survive if δ is greater than zero and hence any manipulation above x^* is useless.[38] As a result, an agent who observes x^* must conclude that this observation potentially came from a regime with $\theta \leq 0$ that is unlikely to survive the agents' attack against it. This increases the conditional probability on the left-hand side of the equilibrium equation (10.9), with the property that this probabilty converges to one as $x^* \to 0$. Consequently, if $\lim_{\boldsymbol{\sigma} \to \mathbf{0}} \frac{\sigma_r\sqrt{-\ln \sigma_r}}{\sigma_a} > \Phi^{-1}(1-r)$, then there exists a threshold $0 \leq x^* < \bar{\nu}$ such that the equilibrium condition is satisfied in the limit. Because all regimes with $\theta > 0$ choose ν so that $\theta + \nu(\theta) \geq x^*$, it follows that $\theta^*(0) \to 0$ in this case.

If instead $\lim_{\boldsymbol{\sigma} \to \mathbf{0}} \frac{\sigma_r\sqrt{-\ln \sigma_r}}{\sigma_a} < \Phi^{-1}(1-r)$ as is assumed in the second part of Theorem 10.3, then the implication is more surprising. In particular, the higher conditional probability implied by a value of x^* that is less than $\bar{\nu}$ has the effect of further increasing the value of the left-hand side of equation (10.9) and thus further pushing the game out of equilibrium. The conclusion is that it is impossible to construct a monotone equilibrium in which $x^* \leq 1$.

In general, this section's setup is difficult to analyze if $x^* > 1$. This follows because as $\sigma_r \to 0$ and the regime learns that δ is equal to zero, all regimes with $\theta \geq 1$ will always prefer to not manipulate the agents' information. Indeed, manipulation is costly and hence once the regime learns that fundamentals are such that $\theta + \delta \geq 1$, the regime knows that it will never collapse in period two and hence prefers not to incur this cost. However, regimes with $\theta < 1$ may still prefer to manipulate, especially if x^* is not far above one. In this case, it is possible that the agents' private signals about $\theta + \delta$ will be non-monotonic enough so that

[38] Equivalently, these regimes all have $\hat{\delta}(\theta, \nu) \geq 0$.

the posterior probability $P(\delta \leq \hat{\delta}(\theta, \nu(\theta)) \mid x_i, \nu(\cdot))$ is actually increasing for some values of x_i. If this happens, then a monotonic equilibrium of this game may not exist. I avoid this technical issue by assuming that $1 - r + \bar{\nu} < 1$ in Theorem 10.2. The following lemma formalizes these ideas. Like the two previous theorems, its proof is in Section 12.

Lemma 10.4. *Suppose that $\boldsymbol{\sigma} \to \mathbf{0}$. If the regime manipulates the agents' information, then in the limit there exist no monotone equilibria with $x^* > 1$.*

In summary, there are two key implications of Theorems 10.2 and 10.3. First, the regime's ability to manipulate the private information of the agents is only beneficial if the agents do not learn about fundamentals significantly faster than the regime. Otherwise, the agents are able to effectively filter this manipulation out and the regime only incurs costs on itself through this action. The second implication is that it is still possible that information manipulation is ineffective even if the regime learns about fundamentals faster than the agents. In these cases, the agents' knowledge that the regime is manipulating their information may actually intensify attacks against the regime. This occurs if the agents are significantly more uncertain about fundamentals and believe that the regime's actions actually increase the probability that the regime collapses in period two.

11 Conclusion

In this chapter, I have theoretically examined the implications of information manipulation for the equilibrium outcome of global games. A central feature of the coordination games I consider is that both the agents and the regime face uncertainty about the outcome of the game. This causes the regime to sometimes take a costly action and manipulate the information of the agents even if the effectiveness of this action is uncertain.

The main result of my analysis is that if agents learn about fundamentals faster than the regime, then information manipulation is ineffective at preventing the agents from coordinating an attack against the regime. Because the regime incurs a cost when it biases the private signals of the agents, in these cases the regime would like to commit to take no action but is unable to do so. Furthermore, I have shown that even if the regime learns faster than the agents, there are still cases in which information manipulation is ineffective or even counterproductive. If the regime learns too much faster than the agents, then agents may fear that the regime's hidden actions are insufficient so that their attacks actually intensify in the limit.

These results are a first step towards understanding the effects of information manipulation in global games in which both the regime and the coordinating agents face some uncertainty. The games I present are stylized and feature a number of simplifying assumptions intended to keep the analysis tractable. A natural and important next step is to examine how my results might change once these assumptions are modified or dropped altogether. In particular, how often is a regime with superior information able to effectively thwart the ability of economic agents to coordinate their actions? Is it always true that agents can properly filter out manipulative actions if the regime has inferior information? Extending this chapter's games to include more of the sources of endogenous information that exist in coordination settings is likely to help answer questions of this kind.

12 Proofs

Before proving the main results, it is useful to present some preliminary results that appear throughout many of the proofs in this section. A monotone equilibrium is given by thresholds x^* and $\theta^*(\delta)$ such that any agent who observes a signal $x_i < x^*$ prefers to join the attack against the regime and the regime collapses in period two if and only if $\theta < \theta^*(\delta)$. These thresholds are determined by the solution to the equations (10.8) and (10.9). Because the event $\{\theta < \theta^*(\delta)\}$ is equivalent to the event $\{\delta < \hat{\delta}(\theta, \nu(\theta))\}$, it follows that

$$P(\theta < \theta^*(\delta) \mid x_i, \nu(\cdot)) = P(\delta < \hat{\delta}(\theta, \nu(\theta)) \mid x_i, \nu(\cdot)),$$

so that, if an agent observes the private signal x_i in period one, then her posterior probability of the regime collapsing in period two is given by

$$\begin{aligned} P(\delta < \hat{\delta}(\theta, \nu(\theta)) \mid x_i, \nu(\cdot)) &= \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\hat{\delta}(\theta,\nu(\theta))} \sigma_\epsilon^{-1} \sigma_a^{-1} \phi\left(\frac{x_i - \theta - \nu(\theta) - \delta}{\sigma_\epsilon}\right) \phi\left(\frac{\delta}{\sigma_a}\right) \, d\delta d\theta}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_\epsilon^{-1} \sigma_a^{-1} \phi\left(\frac{x_i - \theta - \nu(\theta) - \delta}{\sigma_\epsilon}\right) \phi\left(\frac{\delta}{\sigma_a}\right) \, d\delta d\theta} \\ &= \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\hat{\delta}(\theta,\nu(\theta))} \sigma_\epsilon^{-1} \sigma_a^{-1} \phi\left(\frac{x_i - \theta - \nu(\theta)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) \phi\left(\frac{\frac{\sigma_a^2}{\sigma_\epsilon^2 + \sigma_a^2}(x_i - \theta - \nu(\theta)) - \delta}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}}\right) \, d\delta d\theta}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_\epsilon^{-1} \sigma_a^{-1} \phi\left(\frac{x_i - \theta - \nu(\theta)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) \phi\left(\frac{\frac{\sigma_a^2}{\sigma_\epsilon^2 + \sigma_a^2}(x_i - \theta - \nu(\theta)) - \delta}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}}\right) \, d\delta d\theta}. \end{aligned} \tag{12.1}$$

Note that the second equality follows by Lemma 12.3. Equation (12.1) implies that the indifference condition for an agent who observes private signal x^* is given by

$$\frac{\int_{-\infty}^{\infty} \sqrt{\sigma_\epsilon^2+\sigma_a^2}^{-1} \phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta-\nu(\theta))-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) d\theta}{\int_{-\infty}^{\infty} \sqrt{\sigma_\epsilon^2+\sigma_a^2}^{-1} \phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) d\theta} = r. \tag{12.2}$$

This indifference condition states simply that the posterior probability that $\delta < \hat{\delta}(\theta, \nu(\theta))$ (which is equivalent to the probability that $\theta < \theta^*(\delta)$) and the regime collapses in period two is equal to the cost of joining the attack, as in equation (10.9) from Section 10.

Lemma 12.1. *Suppose that $\bar{\nu} \leq x^* \leq 1$ and $C(\bar{\nu}) < 1$. There exists $\hat{\sigma} > 0$ such that if $\sigma_r, \sigma_\epsilon < \hat{\sigma}$, then the regime's hidden actions satisfy*

$$\nu(\theta) = \begin{cases} 0 & \text{if } \theta \leq \theta_l, \\ \bar{\nu} & \text{if } \theta_l < \theta < \theta_m, \\ x^* - \theta - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu)) + \sigma_r \gamma(\theta, \nu) & \text{if } \theta_m \leq \theta < \theta_h, \\ 0 & \text{if } \theta_h \leq \theta, \end{cases} \tag{12.3}$$

where

$$\gamma(\theta, \nu) = \left[-2 \ln \left\{\sigma_r \sqrt{2\pi} C'(\nu) \left(1 + \sigma_\epsilon \Phi^{-1'}(\theta + \hat{\delta}(\theta, \nu))\right)\right\}\right]^{0.5}. \tag{12.4}$$

The thresholds $\theta_l < \theta_m < \theta_h$ are given by the solution to the equations

$$C(\bar{\nu}) + \Phi\left(\frac{\hat{\delta}(\theta_l, \bar{\nu})}{\sigma_r}\right) = \Phi\left(\frac{\hat{\delta}(\theta_l, 0)}{\sigma_r}\right), \tag{12.5}$$

$$\sigma_r \gamma(\theta_m, \bar{\nu}) = -\hat{\delta}(\theta_m, \bar{\nu}), \tag{12.6}$$

$$\sigma_r \gamma(\theta_h, 0) = -\hat{\delta}(\theta_h, 0). \tag{12.7}$$

Proof. The first-order condition for the regime's minimization problem is given by

$$\sigma_r C'(\nu) = -\frac{\partial \hat{\delta}(\theta, \nu)}{\partial \nu} \phi\left(\frac{\hat{\delta}(\theta, \nu)}{\sigma_r}\right),$$

which, by equation (10.3), is equivalent to

$$\sigma_r C'(\nu)\left(1+\sigma_\epsilon \Phi^{-1'}(\theta+\hat{\delta}(\theta,\nu))\right)=\phi\left(\frac{\hat{\delta}(\theta,\nu)}{\sigma_r}\right). \tag{12.8}$$

Let

$$\gamma(\theta,\nu)=\left[-2\ln\left\{\sigma_r\sqrt{2\pi}C'(\nu)\left(1+\sigma_\epsilon\Phi^{-1'}(\theta+\hat{\delta}(\theta,\nu))\right)\right\}\right]^{0.5}, \tag{12.9}$$

and note that $\gamma(\theta,\nu)\to\infty$ while $\sigma_r\gamma(\theta,\nu)\to 0$ as $\sigma_r\to 0$ for all $\theta\in\mathbb{R}$ and $\nu\in[0,\bar{\nu}]$. Equation (10.3) implies that both $\theta+\hat{\delta}(\theta,0)\to x^*$ and $\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,0))\to 0$ as $\sigma_\epsilon\to 0$ for all $\bar{\nu}\leq x^*\leq 1$. As a consequence, it is possible to choose σ_r and σ_ϵ small enough so that there exists $x^*<\theta_h$ satisfying

$$\theta_h=x^*-\sigma_\epsilon\Phi^{-1}(\theta_h+\hat{\delta}(\theta_h,0))+\sigma_r\gamma(\theta_h,0). \tag{12.10}$$

Any interior solution for the regime's minimization problem satisfies (note that there can be two solutions to the first-order condition, but only the larger solution satisfies the second-order condition as well)

$$\nu=x^*-\theta-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu))+\sigma_r\gamma(\theta,\nu), \tag{12.11}$$

and the right-hand side of the first-order condition (12.8) is decreasing in θ and ν (because $\hat{\delta}(\theta,\nu)$ is decreasing in θ and ν), so it follows from equation (12.10) that all regimes with $\theta\geq\theta_h$ choose $\nu=0$.

Because $\theta+\hat{\delta}(\theta,\bar{\nu})=x^*-\bar{\nu}-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\bar{\nu}))\geq 0$ for any $\theta\in\mathbb{R}$, it follows that there exists $0<\theta_m<x^*$ such that

$$\theta_m=x^*-\bar{\nu}-\sigma_\epsilon\Phi^{-1}(\theta_m+\hat{\delta}(\theta_m,\bar{\nu}))+\sigma_r\gamma(\theta_m,\bar{\nu}).$$

The assumption that $C(\bar{\nu})<1$ is important here. Since $\Phi\left(\frac{\hat{\delta}(\theta_m,0)}{\sigma_r}\right)\to 1$ as $\sigma_r\to 0$ while

$$\Phi\left(\frac{\hat{\delta}(\theta_m,\bar{\nu})}{\sigma_r}\right)=\Phi\left(\frac{-\sigma_r\gamma(\theta_m,\bar{\nu})}{\sigma_r}\right)\to 0,$$

as $\sigma_r\to 0$, the implication is that a regime with fundamentals θ_m has a smaller expected loss if it chooses $\nu=\bar{\nu}$ rather than $\nu=0$, provided that σ_r is sufficiently close to zero. Because all regimes with $\theta_m\leq\theta<\theta_h$ satisfy the first-order condition (12.8) (and the second-order condition for an interior minimum) with $\nu \in [0, \bar{\nu}]$, it follows that those regimes choose ν as given by equation (12.11).

The last step is to consider the minimization problem that faces those regimes for which $\theta < \theta_m$. Although many regimes in this set eventually prefer to choose $\nu = 0$, there are always some regimes for which choosing $\nu = \bar{\nu}$ achieves a smaller expected loss than choosing $\nu = 0$. As before, this relies on the fact that $C(\bar{\nu}) < 1$. A regime with $\theta < \theta_m$ will prefer to choose $\nu = \bar{\nu}$ rather than $\nu = 0$ as long as

$$C(\bar{\nu}) + \Phi\left(\frac{\hat{\delta}(\theta, \bar{\nu})}{\sigma_r}\right) < \Phi\left(\frac{\hat{\delta}(\theta, 0)}{\sigma_r}\right), \tag{12.12}$$

and both $\Phi\left(\frac{\hat{\delta}(\theta,\bar{\nu})}{\sigma_r}\right)$ and $\Phi\left(\frac{\hat{\delta}(\theta,0)}{\sigma_r}\right)$ grow to one as θ decreases, so it follows that there exists some $\theta_l < \theta_m$, given by the solution to

$$C(\bar{\nu}) + \Phi\left(\frac{\hat{\delta}(\theta_l, \bar{\nu})}{\sigma_r}\right) = \Phi\left(\frac{\hat{\delta}(\theta_l, 0)}{\sigma_r}\right),$$

such that the regime chooses $\nu = 0$ for all $\theta \leq \theta_l$ and $\nu = \bar{\nu}$ for all $\theta_l < \theta < \theta_m$. □

Lemma 12.2. *Suppose that $0 \leq x^* < \bar{\nu} < 1$ and $C(\bar{\nu}) < 1/2$. There exists $\hat{\sigma} > 0$ such that if $\sigma_r, \sigma_\epsilon < \hat{\sigma}$, then the regime's hidden actions satisfy*

$$\nu(\theta) = \begin{cases} 0 & \text{if } \theta \leq \theta_{ll}, \\ x^* - \theta - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu)) - \sigma_r \gamma(\theta, \nu) & \text{if } \theta_{ll} < \theta < \theta_{mm}, \\ x^* - \theta - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu)) + \sigma_r \gamma(\theta, \nu) & \text{if } \theta_{mm} \leq \theta < \theta_{hh}, \\ 0 & \text{if } \theta_{hh} \leq \theta, \end{cases} \tag{12.13}$$

where

$$\gamma(\theta, \nu) = \left[-2 \ln\left\{\sigma_r \sqrt{2\pi} C'(\nu) \left(1 + \sigma_\epsilon \Phi^{-1'}(\theta + \hat{\delta}(\theta, \nu))\right)\right\}\right]^{0.5}. \tag{12.14}$$

The thresholds $\theta_{ll} < \theta_{mm} < \theta_{hh}$ are given by the solution to the equations

$$C(\nu_{ll}) + \Phi\left(\frac{\hat{\delta}(\theta_{ll}, \nu_{ll})}{\sigma_r}\right) = \Phi\left(\frac{\hat{\delta}(\theta_{ll}, 0)}{\sigma_r}\right), \tag{12.15}$$

$$\gamma(\theta_{mm}, \nu_{mm}) = 0, \tag{12.16}$$

$$\sigma_r \gamma(\theta_{hh}, 0) = -\hat{\delta}(\theta_{hh}, 0), \tag{12.17}$$

with ν_{ll} and ν_{mm} satisfying

$$\nu_{ll} = x^* - \theta_{ll} - \sigma_\epsilon \Phi^{-1}(\theta_{ll} + \hat{\delta}(\theta_{ll}, \nu_{ll})) - \sigma_r \gamma(\theta_{ll}, \nu_{ll}), \tag{12.18}$$

$$\nu_{mm} = x^* - \theta_{mm} - \sigma_\epsilon \Phi^{-1}(\theta_{mm}). \tag{12.19}$$

Proof. Recall that the first-order condition for the regime's minimization problem is given by

$$\sigma_r C'(\nu) \left(1 + \sigma_\epsilon \Phi^{-1'}(\theta + \hat{\delta}(\theta, \nu))\right) = \phi\left(\frac{\hat{\delta}(\theta, \nu)}{\sigma_r}\right). \tag{12.20}$$

The same argument from the proof of Lemma 12.1 implies that it is possible to choose σ_r and σ_ϵ small enough so that there exists $x^* < \theta_{hh} < 1$ satisfying

$$\theta_{hh} = x^* - \sigma_\epsilon \Phi^{-1}(\theta_{hh} + \hat{\delta}(\theta_{hh}, 0)) + \sigma_r \gamma(\theta_{hh}, 0). \tag{12.21}$$

It is also true that any interior solution for the regime's minimization problem satisfies

$$\nu = x^* - \theta - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu)) + \sigma_r \gamma(\theta, \nu), \tag{12.22}$$

and the right-hand side of the first-order condition (12.20) is decreasing in θ and ν (because $\hat{\delta}(\theta, \nu)$ is decreasing in θ and ν), so it follows from equation (12.21) that all regimes with $\theta \geq \theta_{hh}$ choose $\nu = 0$.

Because $x^* < \bar{\nu}$, equation (12.22) implies that, once σ_ϵ and σ_r are small enough, there exists no interior solution for the regime's minimization problem such that $\theta > \frac{x^* - \bar{\nu}}{2}$ and $\nu = \bar{\nu}$. If instead $\theta \leq \frac{x^* - \bar{\nu}}{2} < 0$, then $\theta + \hat{\delta}(\theta, \bar{\nu}) \to 0$ as $\sigma_\epsilon \to 0$ and hence

$$\Phi\left(\frac{\hat{\delta}(\theta, \bar{\nu})}{\sigma_r}\right) \to 1$$

as $\sigma_r, \sigma_\epsilon \to 0$. This implies that for σ_ϵ and σ_r small enough, any regime of type $\theta \leq \frac{x^* - \bar{\nu}}{2} < 0$ prefers not to incur the costs of choosing $\nu = \bar{\nu}$. For any regime with $\theta > \frac{x^* - \bar{\nu}}{2}$, before ν reaches $\bar{\nu}$, the left-hand side of the first-order condition (12.20) grows to infinity as $\theta + \hat{\delta}(\theta, \nu)$ decreases to zero. By continuity, then, there exists some $\theta_{mm} < \theta_{hh}$ such that

$$\sigma_r C'(\nu_{mm}) \left(1 + \sigma_\epsilon \Phi^{-1'}(\theta_{mm})\right) = \phi(0), \tag{12.23}$$

where $\nu_{mm} < \bar{\nu}$ is given by

$$\nu_{mm} = x^* - \theta_{mm} - \sigma_\epsilon \Phi^{-1}(\theta_{mm} + \hat{\delta}(\theta_{mm}, \nu_{mm})) = x^* - \theta_{mm} - \sigma_\epsilon \Phi^{-1}(\theta_{mm}).$$

By definition, equation (12.23) implies that $\gamma(\theta_{mm}, \nu_{mm}) = 0$. Furthermore, the fact that $C(\bar{\nu}) < \frac{1}{2}$ implies that for σ_r small enough, a regime with fundamentals θ_{mm} prefers to choose $\nu = \nu_{mm}$ rather than $\nu = 0$ (since the regime's loss converges to $C(\nu_{mm}) + \frac{1}{2}$ with manipulation and to one without manipulation).

Finally, the regime will prefer to choose ν as given by the first-order condition (12.20) up until the decrease in the regime's expected loss with manipulation is outweighed by the cost of manipulating. This occurs for θ_{ll} such that

$$C(\nu_{ll}) + \Phi\left(\frac{\hat{\delta}(\theta_{ll}, \nu_{ll})}{\sigma_r}\right) = \Phi\left(\frac{\hat{\delta}(\theta_{ll}, 0)}{\sigma_r}\right),$$

where ν_{ll} is given by

$$\nu_{ll} = x^* - \theta_{ll} - \sigma_\epsilon \Phi^{-1}(\theta_{ll} + \hat{\delta}(\theta_{ll}, \nu_{ll})) - \sigma_r \gamma(\theta_{ll}, \nu_{ll}).$$

All regimes with $\theta \leq \theta_{ll}$ prefer to choose $\nu = 0$. □

Proof of Lemma 10.4 Suppose that $x^* > 1$. For any regime of type $1 < \theta < x^*$, there exists a value of the standard deviation parameter σ_r that is small enough so that that regime prefers to choose $\nu = 0$. This follows because all regimes with $\theta > 1$ learn that they will survive the agents' attack in the limit as $\sigma_r \to 0$.

Equation (10.3) implies that if $1 < \theta < x^*$, then $\hat{\delta}(\theta, 0) \to 1 - \theta$ as $\sigma_\epsilon \to 0$, so it follows that $\hat{\delta}(\theta, \nu(\theta)) \to 1 - \theta < 0$ as $\boldsymbol{\sigma} \to \mathbf{0}$. Let $1 < x_i < x^*$, and note that

$$P(\delta \leq \hat{\delta}(\theta, \nu(\theta)) \mid x_i, \nu(\cdot)) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\hat{\delta}(\theta,\nu(\theta))} \sigma_\epsilon^{-1} \sigma_a^{-1} \phi\left(\frac{x_i - \theta - \nu(\theta)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) \phi\left(\frac{\frac{\sigma_a^2}{\sigma_\epsilon^2 + \sigma_a^2}(x_i - \theta - \nu(\theta)) - \delta}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}}\right) d\delta d\theta}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_\epsilon^{-1} \sigma_a^{-1} \phi\left(\frac{x_i - \theta - \nu(\theta)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) \phi\left(\frac{\frac{\sigma_a^2}{\sigma_\epsilon^2 + \sigma_a^2}(x_i - \theta - \nu(\theta)) - \delta}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}}\right) d\delta d\theta}.$$

Because $\nu(\theta) \to 0$ as $\sigma_r \to 0$ for all $1 < \theta < x^*$, the previous equation simplifies in the limit as $\boldsymbol{\sigma} \to \mathbf{0}$ yielding

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} P(\delta \leq \hat{\delta}(\theta,\nu(\theta)) \mid x_i,\nu(\cdot)) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{-\infty}^{\infty} \frac{\phi\left(\frac{x_i-\theta}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \Phi\left(\frac{1-\theta-\frac{\sigma_a^2}{\sigma_\epsilon^2+\sigma_a^2}(x_i-\theta)}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) d\theta.$$

The quantity $\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}$ converges to zero while $\frac{\sigma_a^2}{\sigma_\epsilon^2+\sigma_a^2}$ converges to a value between zero and one as $\boldsymbol{\sigma} \to \mathbf{0}$. As a consequence, there exists some $1 < x_i < x^*$ such that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} P(\delta \leq \hat{\delta}(\theta,\nu(\theta)) \mid x_i,\nu(\cdot)) = 0 < r.$$

In any monotone equilibrium, $P(\delta \leq \hat{\delta}(\theta,\nu(\theta)) \mid x_i,\nu(\cdot)) \geq r$ for all $x_i < x^*$, so there is a contradiction and hence there exist no monotone equilibria with $x^* > 1$. $\square$

Lemma 12.3. *For any $x, y \in \mathbb{R}$, $\sigma_x, \sigma_y > 0$,*

$$\phi\left(\frac{x-\theta}{\sigma_x}\right)\phi\left(\frac{y-\theta}{\sigma_y}\right) = \phi\left(\frac{x-y}{\sqrt{\sigma_x^2+\sigma_y^2}}\right)\phi\left(\frac{\frac{\sigma_y^2}{\sigma_x^2+\sigma_y^2}x+\frac{\sigma_x^2}{\sigma_x^2+\sigma_y^2}y-\theta}{\frac{\sigma_x\sigma_y}{\sqrt{\sigma_x^2+\sigma_y^2}}}\right) \tag{12.24}$$

Proof. For any $x, y \in \mathbb{R}$, $\sigma_x, \sigma_y > 0$, the definition of the density function for the standard normal distribution implies that

$$\begin{aligned}
\phi\left(\frac{x-\theta}{\sigma_x}\right)\phi\left(\frac{y-\theta}{\sigma_y}\right) &= \frac{1}{2\pi}\exp\left\{-\frac{(x-\theta)^2}{2\sigma_x^2}\right\}\exp\left\{-\frac{(y-\theta)^2}{2\sigma_y^2}\right\} \\
&= \frac{1}{2\pi}\exp\left\{-\frac{\sigma_y^2x^2-2\sigma_y^2x\theta+\sigma_y^2\theta^2+\sigma_x^2y^2-2\sigma_x^2y\theta+\sigma_x^2\theta^2}{2\sigma_x^2\sigma_y^2}\right\} \\
&= \frac{1}{2\pi}\exp\left\{-\frac{\frac{\sigma_y^2(\sigma_x^2+\sigma_y^2)}{(\sigma_x^2+\sigma_y^2)^2}x^2+\frac{\sigma_x^2(\sigma_x^2+\sigma_y^2)}{(\sigma_x^2+\sigma_y^2)^2}y^2+\theta^2-2\frac{\sigma_y^2}{\sigma_x^2+\sigma_y^2}x\theta-2\frac{\sigma_x^2}{\sigma_x^2+\sigma_y^2}y\theta}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_x^2+\sigma_y^2}}\right\} \\
&= \frac{1}{2\pi}\exp\left\{-\frac{(x-y)^2}{2(\sigma_x^2+\sigma_y^2)}\right\}\exp\left\{-\frac{\left(\frac{\sigma_y^2}{\sigma_x^2+\sigma_y^2}x+\frac{\sigma_x^2}{\sigma_x^2+\sigma_y^2}y-\theta\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_x^2+\sigma_y^2}}\right\} \\
&= \frac{1}{2\pi}\phi\left(\frac{x-y}{\sqrt{\sigma_x^2+\sigma_y^2}}\right)\phi\left(\frac{\frac{\sigma_y^2}{\sigma_x^2+\sigma_y^2}x+\frac{\sigma_x^2}{\sigma_x^2+\sigma_y^2}y-\theta}{\frac{\sigma_x\sigma_y}{\sqrt{\sigma_x^2+\sigma_y^2}}}\right).
\end{aligned}$$

$\square$

Proof of Theorem 10.2 The first step of the proof is to split the numerator and denominator of the indifference condition (12.2) into separate parts. Each part corresponds to a different chosen manipulation by the regime. In particular, let

$$\Psi(\theta_l,\theta_h)=\int_{(-\infty,\theta_l)\cup(\theta_h,\infty)}\frac{\phi\left(\frac{x^*-\theta}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta)-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,0))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right)d\theta, \tag{12.25}$$

$$\Psi_l(\theta_l,\theta_m)=\int_{\theta_l}^{\theta_m}\frac{\phi\left(\frac{x^*-\theta-\bar{\nu}}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta-\bar{\nu})-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\bar{\nu}))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right)d\theta, \tag{12.26}$$

$$\Psi_h(\theta_m,\theta_h)=\int_{\theta_m}^{\theta_h}\frac{\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta-\nu(\theta))-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right)d\theta, \tag{12.27}$$

and

$$\Lambda(\theta_l,\theta_h)=\int_{(-\infty,\theta_l)\cup(\theta_h,\infty)}\frac{\phi\left(\frac{x^*-\theta}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}d\theta=\Phi\left(\frac{\theta_l-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)+\Phi\left(\frac{x^*-\theta_h}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right),$$

$$\Lambda_l(\theta_l,\theta_m)=\int_{\theta_l}^{\theta_m}\frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\phi\left(\frac{x^*-\theta-\bar{\nu}}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)d\theta=\Phi\left(\frac{\theta_m+\bar{\nu}-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)-\Phi\left(\frac{\theta_l+\bar{\nu}-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right),$$

$$\Lambda_h(\theta_m,\theta_h)=\int_{\theta_m}^{\theta_h}\frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)d\theta,$$

where θ_l,θ_m, and θ_h are given by equations (12.5), (12.6), and (12.7) from Lemma 12.1. Given these definitions, it follows by Lemma 12.1 that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}}P(\delta<\hat{\delta}(\theta,\nu(\theta))\mid x^*,\nu(\cdot))=\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\frac{\Psi(\theta_l,\theta_h)+\Psi_l(\theta_l,\theta_m)+\Psi_h(\theta_m,\theta_h)}{\Lambda(\theta_l,\theta_h)+\Lambda_l(\theta_l,\theta_m)+\Lambda_h(\theta_m,\theta_h)}. \tag{12.28}$$

The fact that $\theta_l<\theta_m$ implies that $\Phi\left(\frac{\theta_l-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\leq\Phi\left(\frac{\theta_m-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)$, so by equation (12.3) from the lemma it follows that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\Phi\left(\frac{\theta_l-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\leq\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\Phi\left(\frac{\sigma_r\gamma(\theta_m,\bar{\nu})-\bar{\nu}-\sigma_\epsilon\Phi^{-1}(\theta_m+\hat{\delta}(\theta_m))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)=0.$$

As consequence, $\Lambda(\theta_l, \theta_h)$ simplifies so that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Lambda(\theta_l, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{x^* - \theta_h}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{\sigma_\epsilon \Phi^{-1}(\theta_h + \hat{\delta}(\theta_h, 0)) - \sigma_r \gamma(\theta_h, 0)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right), \tag{12.29}$$

and $\Psi(\theta_l, \theta_h)$ simplifies so that (this uses the fact that $0 \le \Psi(\theta_l, \theta_h) \le \Lambda(\theta_l, \theta_h)$)

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi(\theta_l, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_h}^{\infty} \frac{\phi\left(\frac{x^*-\theta}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}} \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^* - \theta) - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, 0))}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) d\theta. \tag{12.30}$$

Finally, equation (12.3) from Lemma 12.1 also implies by substitution that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Lambda_l(\theta_l, \theta_m) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{\sigma_r \gamma(\theta_m, \bar{\nu}) - \sigma_\epsilon \Phi^{-1}(\theta_m + \hat{\delta}(\theta_m, \bar{\nu}))}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) - \Phi\left(\frac{\theta_l - x^* + \bar{\nu}}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right), \tag{12.31}$$

and that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Lambda_h(\theta_m, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_m}^{\theta_h} \frac{1}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}} \phi\left(\frac{\sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta))) - \sigma_r \gamma(\theta, \nu(\theta))}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) d\theta. \tag{12.32}$$

Now, suppose that $\frac{\sigma_\epsilon}{\sigma_a^2} \to 0$. In this case, the fact that $\frac{\sigma_\epsilon}{\sigma_\epsilon^2+\sigma_a^2} \to 0$ implies that

$$\begin{aligned}\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^* - \theta - \nu(\theta)) - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta)))}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) &= \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(-\Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta)))\right) \\ &= \lim_{\boldsymbol{\sigma}\to\mathbf{0}} 1 - \theta - \hat{\delta}(\theta, \nu(\theta)).\end{aligned}$$

By equation (10.3), the quantity $1 - \theta - \hat{\delta}(\theta, \nu(\theta))$ is equal to $1 - x^* + \nu(\theta) + \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta)))$, so it follows by equations (12.26), (12.27), and (12.30) that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi(\theta_l, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_h}^{\infty} \frac{1}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}} \phi\left(\frac{x^* - \theta}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) (1 - x^*)\, d\theta, \tag{12.33}$$

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi_l(\theta_l, \theta_m) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_l}^{\theta_m} \frac{1}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}} \phi\left(\frac{x^* - \theta - \bar{\nu}}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) (1 - x^* + \bar{\nu})\, d\theta, \tag{12.34}$$

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi_h(\theta_m, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_m}^{\theta_h} \frac{1}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}} \phi\left(\frac{x^* - \theta - \nu(\theta)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) (1 - \theta + \sigma_r \gamma(\theta, \nu(\theta)))\, d\theta. \tag{12.35}$$

Note that I can use the dominated convergence theorem to take the limits of integrals in these equalities since

$$\begin{aligned}
\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta-\nu(\theta))-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) \\
\leq \phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right),
\end{aligned}$$

and the right-hand side of this inequality forms a monotonically decreasing sequence of functions as $\boldsymbol{\sigma}\to\mathbf{0}$.

If also $\frac{1}{\sigma_a}\phi\left(\frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a}\right)\to 0$, then for all $\theta\in\mathbb{R}$, $\frac{\sigma_r\gamma(\theta,\nu(\theta))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\to\infty$ as $\boldsymbol{\sigma}\to\mathbf{0}$, so that

$$\Phi\left(\frac{\sigma_r\gamma(\theta_m,\bar{\nu})-\sigma_\epsilon\Phi^{-1}(\theta_m+\hat{\delta}(\theta_m,\bar{\nu}))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\to 1 \tag{12.36}$$

as $\boldsymbol{\sigma}\to\mathbf{0}$ as well. Consider the quantity $\theta_l-x^*+\bar{\nu}$. Note that $\hat{\delta}(\theta_l,0)<0$ because $\theta_l<x^*$ (recall equation (10.3)), so equation (12.5) from Lemma 12.1 implies that

$$\begin{aligned}
\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\theta_l-x^*+\bar{\nu}+\sigma_\epsilon\Phi^{-1}(\theta_l+\hat{\delta}(\theta_l,\bar{\nu})) &= \lim_{\boldsymbol{\sigma}\to\mathbf{0}}\sigma_r\Phi^{-1}\left(1-\Phi\left(\frac{\hat{\delta}(\theta_l,0)}{\sigma_r}\right)+C(\bar{\nu})\right) \\
&= \lim_{\boldsymbol{\sigma}\to\mathbf{0}}\sigma_r\Phi^{-1}(C(\bar{\nu})),
\end{aligned}$$

and hence also that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\frac{\theta_l-x^*+\bar{\nu}+\sigma_\epsilon\Phi^{-1}(\theta_l+\hat{\delta}(\theta_l,\bar{\nu}))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}=\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\frac{\sigma_r}{\sigma_a}\Phi^{-1}(C(\bar{\nu}))=-\infty. \tag{12.37}$$

The last equality follows because $C(\bar{\nu})<1/2$ and $\frac{\sigma_r}{\sigma_a}\to\infty$. Together, equations (12.36) and (12.37) imply that $\Lambda_l(\theta_l,\theta_m)\to 1$ and $\Psi_l(\theta_l,\theta_m)\to 1-x^*+\bar{\nu}$ as $\boldsymbol{\sigma}\to\mathbf{0}$.

The next step is to show that $\Psi_h(\theta_m,\theta_h),\Psi(\theta_l,\theta_h),\Lambda_h(\theta_m,\theta_h)$, $\Lambda(\theta_l,\theta_h)$ all converge to zero as $\boldsymbol{\sigma}\to\mathbf{0}$. By assumption $\frac{1}{\sigma_a}\phi\left(\frac{\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right)\to 0$ as $\boldsymbol{\sigma}\to\mathbf{0}$, so it follows that

$$\frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\phi\left(\frac{\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))-\sigma_r\gamma(\theta,\nu(\theta))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\to 0$$

as $\boldsymbol{\sigma}\to\mathbf{0}$, and hence by equations (12.32) and (12.35), that both $\Psi_h(\theta_l,\theta_h)$ and $\Lambda_h(\theta_m,\theta_h)$ converge to zero as $\boldsymbol{\sigma}\to\mathbf{0}$ as well (as before, the dominated convergence theorem can be used to evaluate limits of integrals). This same argument also implies, by equation (12.29), that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Lambda(\theta_l, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{\sigma_\epsilon \Phi^{-1}(\theta_h + \hat{\delta}(\theta_h, 0)) - \sigma_r \gamma(\theta_h, 0)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right) = 0.$$

Furthermore, because

$$0 \leq \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi(\theta_l, \theta_h) \leq \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Lambda(\theta_l, \theta_h),$$

it follows that $\Psi(\theta_l, \theta_h) \to 0$ as $\boldsymbol{\sigma} \to \mathbf{0}$ as well. The implication is that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} P(\delta < \hat{\delta}(\theta, \nu(\theta)) \mid x^*, \nu(\cdot)) = 1 - x^* + \bar{\nu},$$

and hence that $x^* \to 1 - r + \bar{\nu}$ as $\boldsymbol{\sigma} \to \mathbf{0}$. Equation (10.8) then implies that $\theta^*(\delta) \to x^* - \delta - \nu(\theta^*(\delta))$ as $\sigma_\epsilon \to 0$, so it follows that $\theta^*(0) \to 1 - r$ in the case of $\frac{1}{\sigma_a}\phi\left(\frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a}\right) \to 0$. The final step is to show that there does not exist a monotone equilibrium with $x^* < \bar{\nu}$. This is accomplished in the same manner as the above argument, but appealing now to Lemma 12.2. In particular, it is not difficult to show that $P(\delta < \hat{\delta}(\theta, \nu(\theta)) \mid x^*, \nu(\cdot)) \to 1$ as $\boldsymbol{\sigma} \to \mathbf{0}$ in this case. $\square$

Proof of Theorem 10.3 Suppose that $\frac{\sigma_\epsilon}{\sigma_a} \to \infty$, and that $\frac{1}{\sigma_\epsilon}\phi\left(\frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_\epsilon}\right) \to \infty$ and $\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a} > \Phi^{-1}(1-r)$. If $\frac{\sigma_a}{\sigma_\epsilon} \to 0$, then it follows that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^* - \theta - \nu(\theta)) - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta)))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{x^* - \theta - \nu(\theta) - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta)))}{\sigma_a}\right).$$

Suppose that $\bar{\nu} \leq x^* \leq 1$. As in the proof of Theorem 10.2 (recall equations (12.25), (12.26), and (12.27)), Lemma 12.1 implies that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi(\theta_l, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_h}^{\infty} \frac{\phi\left(\frac{x^*-\theta}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \Phi\left(\frac{x^*-\theta-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,0))}{\sigma_a}\right) d\theta, \tag{12.38}$$

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi_l(\theta_l, \theta_m) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_l}^{\theta_m} \frac{\phi\left(\frac{x^*-\theta-\bar{\nu}}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \Phi\left(\frac{x^*-\theta-\bar{\nu}-\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\bar{\nu}))}{\sigma_a}\right) d\theta, \tag{12.39}$$

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Psi_h(\theta_m, \theta_h) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_m}^{\theta_h} \frac{\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \Phi\left(\frac{-\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right) d\theta, \tag{12.40}$$

and that $\Lambda(\theta_l, \theta_h), \Lambda_l(\theta_l, \theta_m)$, and $\Lambda_h(\theta_l, \theta_h)$ are as in equations (12.29), (12.31), and (12.32) from above. If $\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a} > \Phi^{-1}(1-r)$, then it follows that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{-\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right) < r \tag{12.41}$$

for all θ. Lemma 12.1 implies that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\phi\left(\frac{\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))-\sigma_r\gamma(\theta,\nu(\theta))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right),$$

so if $\frac{1}{\sigma_\epsilon}\phi\left(\frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_\epsilon}\right) \to \infty$, then equation (12.32) implies that $\Lambda_h(\theta_m, \theta_h) \to \infty$ as $\boldsymbol{\sigma} \to \mathbf{0}$ since the lemma also implies that $\theta_h - \theta_m \to \bar{\nu} > 0$ as $\boldsymbol{\sigma} \to \mathbf{0}$. A similar argument proves that both $\Lambda(\theta_l, \theta_h)$ and $\Lambda_l(\theta_l, \theta_m)$ do not diverge to infinity in the limit, as well as that both $\Psi(\theta_l, \theta_h)$ and $\Psi_l(\theta_l, \theta_m)$ also do not diverge to infinity in the limit. Conversely, the previous argument together with (12.40) implies that $\Psi_h(\theta_m, \theta_h)$ may diverge to infinity in the limit as $\boldsymbol{\sigma} \to \mathbf{0}$, so it follows by equation (12.41) that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} P(\delta \le \hat{\delta}(\theta,\nu(\theta)) \mid x^*, \nu(\cdot)) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\Psi_h(\theta_m,\theta_h)}{\Lambda_h(\theta_m,\theta_h)} = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{-\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right) < r. \tag{12.42}$$

Because the preceding argument is valid for any $\bar{\nu} \le x^* \le 1$ by Lemma 12.1, the implication is that there does not exist a monotone equilibrium of this game with $\bar{\nu} \le x^* \le 1$.

By Lemma 10.4, there exist no monotone equilibria with $x^* > 1$ in the limit as $\boldsymbol{\sigma} \to \mathbf{0}$. It follows that the only possibility left to consider is that $0 \le x^* < \bar{\nu}$. According to Lemma 12.2, if $C(\bar{\nu}) < \frac{1}{2}$, then $\theta_{ll} \to 0$ and $\theta_{ll} + \nu_{ll} \to x^*$ as $\boldsymbol{\sigma} \to \mathbf{0}$ so that in any monotone equilibrium with $0 \le x^* < \bar{\nu}$ it must be that $\theta^*(0) \to 0$ as $\boldsymbol{\sigma} \to \mathbf{0}$ (since $\theta^*(\delta) \to x^* - \delta - \nu(\theta^*(\delta))$ as $\sigma_\epsilon \to 0$

by equation 10.3). Consequently, if I can show that there exists a monotone equilibrium with $0 \leq x^* < \bar{\nu}$, then the implication is that in the limit there exists a monotone equilibrium with $\theta^*(0) \to 0$ and there exist no monotone equilibria with $\theta^*(0) > 0$.

Suppose that $x^* = 0$. I first split the numerator and the denominator of the indifference condition (12.2) into separate parts as in the proof of Theorem 10.2. In particular, let

$$\Omega(\theta_{ll}, \theta_{hh}) = \int_{(-\infty,\theta_{ll})\cup(\theta_{hh},\infty)} \frac{\phi\left(\frac{x^*-\theta}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta) - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, 0))}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) d\theta,$$

$$\Omega_l(\theta_{ll}, \theta_{mm}) = \int_{\theta_{ll}}^{\theta_{mm}} \frac{\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta-\nu(\theta)) - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta)))}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) d\theta,$$

$$\Omega_h(\theta_{mm}, \theta_{hh}) = \int_{\theta_{mm}}^{\theta_{hh}} \frac{\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta-\nu(\theta)) - \sigma_\epsilon \Phi^{-1}(\theta + \hat{\delta}(\theta, \nu(\theta)))}{\frac{\sigma_\epsilon \sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) d\theta,$$

and

$$\Gamma(\theta_{ll}, \theta_{hh}) = \int_{(-\infty,\theta_{ll})\cup(\theta_{hh},\infty)} \frac{\phi\left(\frac{x^*-\theta}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} d\theta = \Phi\left(\frac{\theta_{ll}-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) + \Phi\left(\frac{x^*-\theta_{hh}}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right),$$

$$\Gamma_l(\theta_{ll}, \theta_{mm}) = \int_{\theta_{ll}}^{\theta_{mm}} \frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) d\theta,$$

$$\Gamma_h(\theta_{mm}, \theta_{hh}) = \int_{\theta_{mm}}^{\theta_{hh}} \frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}} \phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) d\theta,$$

where θ_{ll}, θ_{mm}, and θ_{hh} are given by equations (12.15), (12.16), and (12.17) from Lemma 12.2. Given these definitions, it follows by Lemma 12.2 that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} P(\delta \leq \hat{\delta}(\theta, \nu(\theta)) \mid x^*, \nu(\cdot)) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\Omega(\theta_{ll}, \theta_{hh}) + \Omega_l(\theta_{ll}, \theta_{mm}) + \Omega_h(\theta_{mm}, \theta_{hh})}{\Gamma(\theta_{ll}, \theta_{hh}) + \Gamma_l(\theta_{ll}, \theta_{mm}) + \Gamma_h(\theta_{mm}, \theta_{hh})}. \tag{12.43}$$

Consider the value of $\Phi\left(\frac{\theta_{ll}-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)$ in the limit as $\boldsymbol{\sigma} \to \mathbf{0}$. If $x^* = 0$, then equation (10.3) implies that $\theta + \hat{\delta}(\theta, \nu) \to 0$ as $\sigma_\epsilon \to 0$ for all $\theta \in \mathbb{R}$ and $\nu \in [0, \bar{\nu}]$. It follows that $\Phi^{-1}\left(\theta + \hat{\delta}(\theta, \nu)\right) \to -\infty$ in the limit as well. Equations (12.13) and (12.15) together imply

that
$$-\frac{\sigma_r}{\sigma_\epsilon}\Phi^{-1}\left(C(\nu_{ll})+\Phi\left(\gamma(\theta_{ll},\nu_{ll})\right)\right)-\Phi^{-1}\left(\theta_{ll}+\hat{\delta}(\theta_{ll},0)\right)=\frac{\theta_{ll}-x^*}{\sigma_\epsilon},$$
which, because $\nu_{ll}\to 0$, $\frac{\sigma_r}{\sigma_\epsilon}<1$, and $\gamma(\theta_{ll},\nu_{ll})\to\infty$ as $\boldsymbol{\sigma}\to\mathbf{0}$, also implies that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}}-\Phi^{-1}\left(C(\nu_{ll})+\Phi\left(\gamma(\theta_{ll},\nu_{ll})\right)\right)-\Phi^{-1}\left(\theta_{ll}+\hat{\delta}(\theta_{ll},0)\right)\le\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\frac{\theta_{ll}-x^*}{\sigma_\epsilon}. \tag{12.44}$$

In order to determine how the quantity $\frac{\theta_{ll}-x^*}{\sigma_\epsilon}$ behaves in the limit, it is necessary to find the limit of $\frac{\Phi^{-1}\left(\theta_{ll}+\hat{\delta}(\theta_{ll},0)\right)}{\sqrt{-\ln\sigma_r}}$ as $\boldsymbol{\sigma}\to\mathbf{0}$. This is accomplished by appealing to equation (10.3), which implies that

$$\frac{\partial\Phi^{-1}\left(\theta_{ll}+\hat{\delta}(\theta_{ll},0)\right)}{\partial\left(\frac{1}{\sigma_\epsilon}\right)}=\sigma_\epsilon\Phi^{-1}\left(\theta_{ll}+\hat{\delta}(\theta_{ll},0)\right)-\frac{1}{\sigma_\epsilon}\frac{\partial\hat{\delta}(\theta_{ll},0)}{\partial\left(\frac{1}{\sigma_\epsilon}\right)}, \tag{12.45}$$

and hence by l'Hôpital's rule that $\frac{\Phi^{-1}\left(\theta_{ll}+\hat{\delta}(\theta_{ll},0)\right)}{\sqrt{-\ln\sigma_r}}\to-\infty$ as $\boldsymbol{\sigma}\to\mathbf{0}$. Note that this last step follows because $\frac{1}{\sigma_\epsilon}\frac{\partial\hat{\delta}(\theta_{ll},0)}{\partial\left(\frac{1}{\sigma_\epsilon}\right)}\to 0$ as $\boldsymbol{\sigma}\to\mathbf{0}$ (this is not difficult to prove) and $\frac{\partial\sqrt{-\ln\sigma_r}}{\partial\left(\frac{1}{\sigma_r}\right)}=\frac{\sigma_r}{2\sqrt{-\ln\sigma_r}}\to 0$ as $\boldsymbol{\sigma}\to\mathbf{0}$. The implication is that the left-hand side of equation (12.44) diverges to infinity in the limit as $\boldsymbol{\sigma}\to\mathbf{0}$, so that also $\Phi\left(\frac{\theta_{ll}-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\to 1$ as $\boldsymbol{\sigma}\to\mathbf{0}$. Lemma 12.2 also implies that the limit of $\Phi\left(\frac{x^*-\theta_{hh}}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)$ as $\boldsymbol{\sigma}\to\mathbf{0}$ is less than the limit of $\theta_{hh}+\hat{\delta}(\theta_{hh},0)$, which converges to zero. This proves that $\Gamma(\theta_{ll},\theta_{hh})\to 1$ as $\boldsymbol{\sigma}\to\mathbf{0}$, and a similar argument also proves that $\Omega(\theta_{ll},\theta_{hh})\to 1$ as $\boldsymbol{\sigma}\to\mathbf{0}$.

Equation (12.13) from Lemma 12.2 implies that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\Omega_l(\theta_{ll},\theta_{mm})=\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\int_{\theta_{ll}}^{\theta_{mm}}\frac{1}{\sigma_\epsilon}\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\Phi\left(\frac{\sigma_r\gamma(\theta,\nu(\theta))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right)d\theta,$$
$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\Gamma_l(\theta_{ll},\theta_{mm})=\lim_{\boldsymbol{\sigma}\to\mathbf{0}}\int_{\theta_{ll}}^{\theta_{mm}}\frac{1}{\sigma_\epsilon}\phi\left(\frac{\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))+\sigma_r\gamma(\theta,\nu(\theta))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)d\theta,$$

and that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Omega_h(\theta_{mm},\theta_{hh}) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_{mm}}^{\theta_{hh}} \frac{1}{\sigma_\epsilon}\phi\left(\frac{x^* - \theta - \nu(\theta)}{\sqrt{\sigma_\epsilon^2 + \sigma_a^2}}\right)\Phi\left(\frac{-\sigma_r\gamma(\theta,\nu(\theta))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right)\,d\theta,$$

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Gamma_h(\theta_{mm},\theta_{hh}) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \int_{\theta_{mm}}^{\theta_{hh}} \frac{1}{\sigma_\epsilon}\phi\left(\frac{\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta))) - \sigma_r\gamma(\theta,\nu(\theta))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)\,d\theta.$$

According to Lemma 12.2, both $\theta_{hh} - \theta_{mm}$ and $\theta_{mm} - \theta_{ll}$ converge to zero as $\boldsymbol{\sigma}\to\mathbf{0}$, so it follows that if the quantity

$$\frac{1}{\sigma_\epsilon}\phi\left(\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta))) + \frac{\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_\epsilon}\right)$$

converges to a finite limit for all $\theta_{ll} < \theta < \theta_{mm}$, and the quantity

$$\frac{1}{\sigma_\epsilon}\phi\left(\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta))) - \frac{\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_\epsilon}\right)$$

converges to a finite limit for all $\theta_{mm} \le \theta < \theta_{hh}$, then $\Omega_l(\theta_{ll},\theta_{mm})$, $\Omega_h(\theta_{mm},\theta_{hh})$, $\Gamma_l(\theta_{ll},\theta_{mm})$, and $\Gamma_h(\theta_{mm},\theta_{hh})$ all converge to zero in the limit as $\boldsymbol{\sigma}\to\mathbf{0}$. One way in which to show that these limits are finite is to prove that the quantity $\frac{\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))}{\sqrt{-\ln\sigma_\epsilon}}$ converges to a nonzero limit for all $\theta_{ll} < \theta < \theta_{hh}$. In a manner similar to equation (12.45) from above, it can be shown that

$$\begin{aligned}\frac{\partial\Phi^{-1}\left(\theta+\hat{\delta}(\theta,\nu(\theta))\right)}{\partial\left(\frac{1}{\sigma_\epsilon}\right)} &= \sigma_\epsilon\Phi^{-1}\left(\theta+\hat{\delta}(\theta,\nu(\theta))\right)\\ &\qquad - \frac{1}{\sigma_\epsilon}\frac{\partial\hat{\delta}(\theta,\nu(\theta))}{\partial\left(\frac{1}{\sigma_\epsilon}\right)} - \frac{1}{\sigma_\epsilon}\frac{\partial\nu(\theta)}{\partial\left(\frac{1}{\sigma_\epsilon}\right)}\left(1+\frac{\partial\hat{\delta}(\theta,\nu(\theta))}{\partial\nu(\theta)}\right),\end{aligned}$$

so that again by l'Hôpital's rule it follows that $\frac{\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))}{\sqrt{-\ln\sigma_\epsilon}} \to -\infty$ as $\boldsymbol{\sigma}\to\mathbf{0}$. By equation (12.43), then, it follows that $P(\delta < \hat{\delta}(\theta,\nu(\theta)) \mid x^*, \nu(\cdot)) \to 1$ as $\boldsymbol{\sigma}\to\mathbf{0}$ whenever $x^* = 0$, so that by continuity there exists some $0 < x^* < \bar{\nu}$ such that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} P(\delta < \hat{\delta}(\theta,\nu(\theta)) \mid x^*, \nu(\cdot)) = r.$$

It is important to note that no part of the previous argument used the assumption from part (i) of the theorem that $\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a} > \Phi^{-1}(1-r)$. It follows, then, that this argument is also true whenever $\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a} < \Phi^{-1}(1-r)$ as assumed in part (ii) of the theorem, and hence that $P(\delta < \hat{\delta}(\theta,\nu(\theta)) \mid x^*, \nu(\cdot)) \to 1$ as $\boldsymbol{\sigma} \to \mathbf{0}$ whenever $x^* = 0$ in this case as well. Suppose now that $\bar{\nu} \leq x^* \leq 1$ and that $\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\sigma_r\sqrt{-\ln\sigma_r}}{\sigma_a} < \Phi^{-1}(1-r)$, so that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{-\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right) > r \tag{12.46}$$

for all $\theta \in \mathbb{R}$. Like in the case in which $x^* = 0$, in this case the previous argument about

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\phi\left(\frac{x^*-\theta-\nu(\theta)}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{1}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\phi\left(\frac{\sigma_\epsilon\Phi^{-1}(\theta+\hat{\delta}(\theta,\nu(\theta)))-\sigma_r\gamma(\theta,\nu(\theta))}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right)$$

diverging to infinity for all $\theta_m \leq \theta < \theta_h$ still applies. By equation (12.46), then, it follows that both $\Psi_h(\theta_m,\theta_h)$ and $\Lambda_h(\theta_m,\theta_h)$ diverge to infinity in the limit while $\Psi(\theta_l,\theta_h)$, $\Psi_l(\theta_l,\theta_m)$, $\Lambda(\theta_l,\theta_h)$, and $\Lambda_l(\theta_l,\theta_m)$ all converge to finite limits, so that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} P(\delta < \hat{\delta}(\theta,\nu(\theta)) \mid x^*,\nu(\cdot)) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\Psi_h(\theta_m,\theta_h)}{\Lambda_h(\theta_m,\theta_h)} = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{-\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right) > r.$$

The implication is that there exists no $\bar{\nu} \leq x^* \leq 1$ such that $P(\delta < \hat{\delta}(\theta,\nu(\theta)) \mid x^*,\nu(\cdot)) \to r$ as $\boldsymbol{\sigma} \to \mathbf{0}$.

The final step in the proof of part (ii) of the theorem is to show that in the limit there exist no monotone equilibria with $0 < x^* < \bar{\nu}$. This follows from the fact that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right) \geq \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{-\sigma_r\gamma(\theta,\nu(\theta))}{\sigma_a}\right) > r,$$

which by Lemma 12.2 guarantees that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\Omega_l(\theta_{ll},\theta_{mm})+\Omega_h(\theta_{mm},\theta_{hh})}{\Gamma_l(\theta_{ll},\theta_{mm})+\Gamma_h(\theta_{mm},\theta_{hh})} > r,$$

whenever $0 < x^* < \bar{\nu}$. Furthermore, it is not difficult to show that a value of x^* in this range implies that $\Phi\left(\frac{\theta_{ll}-x^*}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}\right) \to 0$ as $\boldsymbol{\sigma} \to \mathbf{0}$ and by equation (12.21) that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_a^2}(x^*-\theta_{hh})-\sigma_\epsilon\Phi^{-1}(\theta_{hh}+\hat{\delta}(\theta_{hh},0))}{\frac{\sigma_\epsilon\sigma_a}{\sqrt{\sigma_\epsilon^2+\sigma_a^2}}}\right) = \lim_{\boldsymbol{\sigma}\to\mathbf{0}} \Phi\left(\frac{-\sigma_r\gamma(\theta_{hh},0)}{\sigma_a}\right) > r.$$

Together, these two facts imply that

$$\lim_{\boldsymbol{\sigma}\to\mathbf{0}} \frac{\Omega(\theta_{ll},\theta_{hh}) + \Omega_l(\theta_{ll},\theta_{mm}) + \Omega_h(\theta_{mm},\theta_{hh})}{\Gamma(\theta_{ll},\theta_{hh}) + \Gamma_l(\theta_{ll},\theta_{mm}) + \Gamma_g(\theta_{mm},\theta_{hh})} > r.$$

Equation (12.43) yields the conclusion that a monotone equilibrium with $x^* \leq 1$ (and hence $\theta^*(0) < 1 - \bar{\nu}$) cannot be constructed. □

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