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Selling Random Energy

by

Eilyan Yamen Bitar

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in

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in the

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Committee in charge:

Professor Kameshwar Poolla, Chair
Professor Pravin P. Varaiya
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Abstract

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Global warming now poses one of the most serious challenges to the well-being of humanity at large. The projected increase in the Earth's mean surface and ocean temperatures will have a severe impact on human health in both the developed and developing regions of the world. As the burning of fossil fuels contributes significantly to worldwide greenhouse gas emissions, there has been a concerted *policy reform* effort in both the US and abroad to transform the electricity sector by increasing the displacement of conventional fossil fuel-based thermal generation with *clean renewable generation* such as wind and solar. California, for example, has set a target of 33% renewable energy penetration by the year 2020. Wind and solar energy will play a key role in realizing such aggressive targets. However, at these deep penetration levels, the inherent *variability* of wind and solar power production poses serious engineering and market challenges. These are due to the *uncertainty, intermittency, and uncontrollability* of wind and solar power. They are essentially *random* – a stark contrast to conventional thermal power generation.

How is variability in wind an solar power production dealt with today? Today, wind and solar energy are assimilated into the grid through *legislative mandates, feed-in tariffs, lenient imbalance penalty pricing, guaranteed grid access, tax relief, and/or construction subsidies.* Specifically, in California, the *Participating Intermittent Renewable Program* (PIRP) legislation compels the independent system operator (ISO) to accept all produced wind power subject to certain contractual constraints. This amounts to a *system take-all-wind* modus operandi in which *wind power is treated as a negative load* and the subsequent increase in the variability of *net-load* is absorbed by a portfolio of reserve generation capacity, whose cost is allocated amongst the load serving entities (LSE). Moreover, this socialization of added reserve costs amongst the LSEs constitutes an *implicit subsidy for variability costs* to participating wind power producers. We submit to the reader that the current *extra-market* approach to renewable energy integration will become untenable as

wind and solar energy penetration increases. Under this *system-take-all-power* regime, the attendant intermittency and limited forecastability of wind and solar power production will lead to a significant increase in the reserve generation requirements necessary to maintain system balance – an unacceptable consequence. It is *too expensive*. Ergo, it will rapidly become infeasible to continue the implicit subsidization of the variability costs among the load serving entities. Moreover, it *severely mitigates the net greenhouse gas benefit* of renewable energy, as regulating reserves are normally supplied by fast-acting, fossil fuel based thermal generators such as natural gas turbines. *The current strategy cannot scale.* Clearly, strategies that mitigate the need for additional reserve requirements will be an essential means to supporting deep integration of variable renewable energy. Throughout this dissertation, we will focus explicitly on wind, however, much of the analysis is directly applicable to solar power generation as well.

How will variability be dealt with tomorrow? In the near term, we argue that wind power producers will be forced to participate in conventional electricity markets alongside traditional dispatchable generation, where they will face ex-post financial penalties for deviations from contracts offered ex-ante in forward markets – thus eliminating the implicit subsidy for variability costs. In response to the financial risk emanating from uncertainty in wind power production, a rational wind power producer will be *forced to curtail* its projected output, thus decreasing the amount of variability that has to be compensated for with reserve generation by the system operator. However, such a removal of the *implicit subsidy for variability cost* may result in significant profit loss to the wind power producer. Consequently, it will become necessary for the wind power producer to *develop* and *evaluate* strategies that aid in the mitigation of wind power output variability. In this dissertation, we quantify, within the setting of a *perfectly competitive market*, the maximal expected profit achievable by a wind power producer through optimal bidding. Moreover, as wind is an inherently *variable* source of energy, we explore the sensitivity of optimal expected profit to uncertainty in the underlying wind process and quantify the *marginal economic value* of various *firming* mechanisms that aid in the mitigation of power output variability. Specifically, we appraise the benefit of *improved forecasting* and quantify the added value of *recourse opportunities* afforded by the co-location of an *energy storage* system and/or *fast-acting thermal generation* with the wind power producer. Further, we explore the extent to which a group of N independent wind power producers can exploit the statistical benefits of *aggregation* and *risk sharing* by *forming a willing coalition* to pool their variable power to jointly offer the aggregate output as single entity into a forward energy market.

For my Family, Tony, and Adrienne.

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Eilyan Bitar
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CHAPTER 1

Introduction

Global warming now poses one of the most serious challenges to the well-being of humanity at large. The rising average temperatures of the Earth's atmosphere, surface, and oceans threaten to undermine food and water security in tropical and subtropical regions around the World as these regions are particularly sensitive to the growing risk of drought and floods [6, 104].

The dramatic increase in global average temperature, as depicted in Figure 1.1 (a), is attributed largely to the sharp rate of increase of greenhouse gas (GHG) accumulation in the Earth's atmosphere – a process that is driven predominantly by human activity associated with deforestation and the burning of fossil fuels [71]. For a range of climate models, Figure 1.1 (b) depicts the projected global mean temperature increase over the next 100 years, if no actions are taken to mitigate GHG emissions under sustained economic growth (*SRES A2* emissions scenario). These models predict a significant temperature increase ranging from 2° C to 5° C. The global scale of such a projected increase in mean temperature will likely lead to global food shortages, reduced fresh water supply, increased coastal flooding, and increased malnutrition – all of which will have a significant impact on human health in both the developed and developing regions of the world [51].

The burning of fossil fuels contributes significantly to US greenhouse gas emissions, as fossil fuel resources comprise 78% of total US energy consumption – measured at 94.6 Quads in 2009 (Figure 1.3). In particular, 38% of total US energy consumption is used for electricity generation. Clearly, any serious effort to reduce the rate of global warming will necessarily require a transfiguration of the electricity sector.

1.1 Renewable Power Generation

As the electric energy generation fleet is comprised predominantly (68%, Figure 1.3) of fossil-fuel based thermal generation (e.g., coal, natural gas), there has been a recent effort in the US, and worldwide, to increase the displacement of conventional thermal generation with *clean renewable generation* such wind, solar, geothermal, and hydroelectric in order to make significant reductions in total GHG emissions. In addition, nuclear generation has also been espoused as a source of clean energy, however, concerns regarding the disposal

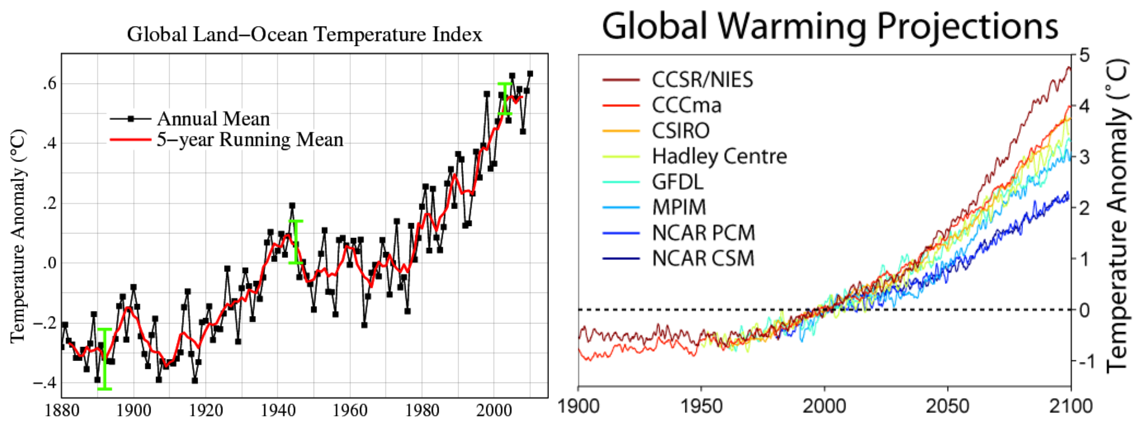


Figure 1.1. (a) A plot of global mean land-ocean temperature from 1880 to 2010. The black line depicts the annual mean, while the red line depicts the five year running mean. *Source: http://data.giss.nasa.gov/gistemp/* (b) Projected global mean temperature increase, for a range of climate models, over the next 100 years if no actions are taken to mitigate GHG emmisions under nominal economic growth (*SRES A2* emissions scenario). *Source: Robert Rohde, Global Warming Art*.

of spent fuel and safety issues are likely to limit its growth in the near term. For example, the recent meltdown disaster at the Fukushima Daiichi nuclear power plant in Japan has largely steered public opinion against the proliferation of nuclear power in many regions of the world. Although nuclear power will play a necessary role in supporting the transition away from fossil-fuel based generation, it is likely that the addition of renewable generation, such as wind and solar, will continue to rise at a much sharper rate over the proceeding decades. Moreover, the conversion of our electricity generation fleet to renewable resources will have a twofold effect on GHG mitigation as the nation's transportation sector increases adoption of plug-in electric and plug-in hybrid electric vehicles. These vehicles will extract the majority of their energy directly from the electricity grid, thus *displacing petroleum as a primary resource* for transportation.

For both environmental and economic reasons, time is of the essence. However, the US electric power industry is notoriously slow to adopt new technologies requiring significant capital investment with uncertain returns. In response – exercising the power of legislative mandate – 30 of the 50 states [62] in the US have adopted *Renewable Portfolio Standards* (RPS) that require increased production of electric energy to come from renewable resources. RPS mechanisms generally require load-serving entities to serve a specified portion of their load demand from clean renewable energy resources. California, for example, has one the most ambitious Renewable Portfolio Standards with a target of 33% renewable energy penetration by the year 2020 [23]. As of 2010, California's three largest utilities have collectively served 18% of their load with renewable energy [23]. In comparison, renewable energy resources nationwide presently comprise only 15% of total electric energy consumed in the US. Moreover, as indicated by Figure 1.3, over 50% of the renewable energy production comes from hydroelectric and geothermal plants, which is due to their competitive capital/production costs. However, the ability to construct hydroelectric and geothermal plants is largely constrained by geography and the availability of limited resources.

For example, the construction of hydroelectric plants, in various modalities (e.g., conventional dam, pumped storage, run-of-the-river), generally requires a geography with a massive river flowing through a moderate elevation change. The inherent potential energy of the water flowing upstream of the elevation change can be used to generate power as it changes elevation. Hydroelectric power plants represent clean, renewable sources of energy with rapid return on investment. In fact, among all sources of electric energy, hydroelectricity produces the least amount of GHG emissions and negative externality – surpassing both wind and solar photovoltaic power plants [41]. Moreover, the Three Gorges Dam in China is estimated to recoup its capital investment from electricity sales within a five to eight year window following its construction [8]. Unfortunately, the United States, along with many European countries, has nearly exhausted the land resources capable of supporting large-scale hydroelectric dams [103].

The expansion of geothermal power is similarly constrained by geography. Essentially, geothermal power plants operate on the same principle as conventional *steam-turbine* thermal power plants. Where they differ is the source of heat – a geothermal plant extracts heat from the Earth's core at depths approaching six miles below the Earth's crust [22]. This represents a *renewable* and, to a large extent, *clean* source of energy. Although the heated fluid drawn from deep within the earth often carries with it a mixture of GHG gases, the associated emission intensity is *small* when compared to conventional fossil fuel based plants [7]. The challenge in siting geothermal plants arises from the prohibitive cost of drilling at significant depths. Hence, there is need to identify locations with hot spots near the Earth's surface. As shallow heat sources (e.g., volcanoes, hot springs, fault lines) are limited in number and can be difficult to locate, the process of exploration can take as long as 15 years and can account for more than half of the plant's capital investment. Consequently, unless there occurs a drastic reduction in the cost of drilling at depths beyond five miles, it is unlikely that geothermal power plants will proliferate at the rate necessary to meet near and long-term RPS targets. Thus, it looks as if we have begun to approach fundamental limitations – both geographic and economic – on the construction of cost-competitive hydroelectric and geothermal renewable generation. There are however alternative sources of clean renewable energy in the form of wind and solar power plants.

Although wind and solar energy resources currently comprise only a small percentage of electric energy generation ($<1\%$), such resources represent a viable alternative to meeting RPS targets and beyond, as they are markedly more plentiful in terms of raw energy availability (see Figure 1.4) that can be easily accessed. Figure 1.4 presents wind power and solar energy density maps for the United States. According to the empirical studies conducted by Archer et al. [3],

> "Global wind power generated at locations with mean annual wind speeds $\geq$ 6.9 m/s at 80 m is found to be $\sim$72 TW for the year 2000. Even if only $\sim$20% of this power could be captured, it could satisfy 100% of the World's energy

demand for all purposes (6995 – 10177 Million Tonnes of Oil Equivalent) and over seven times the World's electricity needs (1.6 – 1.8 TW)."

1.1.1 Challenges to Integration

Although the raw power and energy availability of renewable energy resources such as wind and solar is essentially inexhaustible, there exist serious engineering and market challenges to their large-scale integration into the electricity grid.

Variability: Wind and solar energy are fundamentally different from conventional generation such as coal, nuclear, and natural gas. Such renewable resources are

1. *Non-dispatchable* [cannot be controlled on demand]
2. *Intermittent* [exhibit large fluctuations on short time-scales]
3. *Uncertain* [random and difficult to predict into the future]

We will use the term *variable generation* [73] to encompass these characteristics of wind and solar power. The most serious manifestation of generator output variability is the occurrence of large unpredicted ramps in power output, as these disturb the balance of supply and demand. With respect to solar power generation, the passage of cloud formations over a solar farm can cause significant ramps in solar insolation and subsequent electric power output. Moreover, as cloud formations are patchy in nature, the subsequent intermittency of solar insolation can lead to highly non-stationary behavior of the solar power process. Specifically, solar insolation can change by more than 80% of the peak insolation in a matter of seconds, and vice versa [63].

A wind power plant's output can also exhibit significant ramps up or down within minutes [54]. For example, on December 29-30, 2008, wind power output experienced 95% drop in two hours in the Bonneville Power Administration (BPA) [80]. In addition, severe weather events can cause wind speed to become dangerously high making operation of wind turbines unsafe, forcing shutdown. More commonly, the output of a wind farm ranges from 10% to 90% of nameplate capacity over a typical day and even the best wind forecasting technologies are only able to predict wind power 24 hours ahead to within 20% (mean-absolute error).

Given the prospect of such large disturbances arising from difficult-to-predict ramps in wind or solar power output, the system operator must schedule costly reserve generation to ensure the availability sufficient generating capacity necessary to maintain balance of generation and load in real time. Consequently, the timing of large wind and solar ramps in power is a central problem, as it is critical to improving the efficiency of reserve procurement. *In our opinion, variability is the single most challenging obstacle to the large-scale integration of variable renewable resources.*

Capital Cost: Wind and solar generation resources require significantly higher capital investment than conventional fossil fuel based generators. The Energy Information Administration estimates the levelized capital costs of onshore wind and solar photovoltaic generation resources to fall to $83/MWh and $195/Mwh, respectively, by 2016 (in 2009 dollars) [37] – significantly higher than that of conventional coal ($65/MWh) and natural gas conventional combined cycle ($17/MWh) [37]. Consequently, in the near term, subsidy mechanisms will remain integral to spurring investment in renewable generating resources such as wind and solar.

Limited Access to Transmission: As is apparent from Figure 1.4, areas of high quality wind and solar energy tend to be distant from major load centers and, thus, have limited access to the bulk transmission system. Consequently, the construction of new wind a solar generation resources will require the construction of new high voltage transmission lines – *a costly proposal*. Moreover, as the added transmission lines will be principally carrying wind and solar power, they will likely be acutely *underutilized* as the the capacity factors of wind and solar PV generation resources are approximately 34% and 25%, respectively [31].

Environmentalist Opposition: Ironically, there is growing opposition from environmental conservation groups to the construction of large-scale wind farms in particular locations, because of the potential collateral environmental damage to local wildlife habitats and disruption of migratory patterns for certain avian species. For example, just recently, the Central Huron Council in the Province of Ontario passed a resolution declaring a *moratorium* on all current and future projects for the development of on and off-shore wind farms, until an independent third party can guarantee that "human and animal populations are protected from the direct and indirect negative effects of being in proximity to those wind-energy facilities" [14] – an interesting contention pitting environmentalist against environmentalist.

The preceding points are all central issues that must be resolved in order to realize the renewable energy penetration goals of various RPS portfolios. In the discussion to follow, we will focus primarily on the impact of *generator output variability* on central system operations and the corresponding implications to *competitive trading of variable energy* in electricity markets.

1.2 The Variability Challenge

As indicated in the previous section, wind and solar power plants are inherently *variable in their power output*. They are *non-dispatchable, highly intermittent*, and *difficult to forecast* on horizons beyond five minutes. At levels of deep renewable penetration, these generation variability characteristics will pose formidable challenges to the preservation of *instantaneous balance between supply and demand*, while simultaneously respecting system security constraints.

In contrast to variable renewable generators, the majority of conventional thermal generators in use today are fundamentally *dispatchable* and, to a large extent, *predictable*. Nonetheless, the power system architecture and attending operations have been designed to explicitly deal with the variability emanating from *natural fluctuations in load* and *unplanned contingency events* such as branch and/or generation outages. Generally speaking, imbalances arising between generation and load are compensated for by *reserve generation capacity* procured by the independent system operator (ISO) through *ancillary services* (AS) markets. The various aforementioned phenomena responsible for system imbalances occur on differing time scales and thus require the procurement of reserve resources with a variety of response capabilities [74, 85]. The subsequent cost of the procured reserve capacity is then socialized among the participating participating load serving entities (LSE) based on each LSE's relative contribution to the total demand [47]. To a large extent, the current approach to compensating variability amounts to a paradigm in which *generation is tailored to follow load*. Given the relative success of the modus operandi, it is tempting to presume that the added variability of renewable generation can be similarly compensated for with existing reserve mechanisms. In fact, this is the approach taken by many balancing authorities within the United States: *all wind and solar power production is taken* by the system operator and the attendant variability is compensated for with existing reserve margins. As we will see in the following section, this approach to renewable energy integration will become untenable at deep penetration levels.

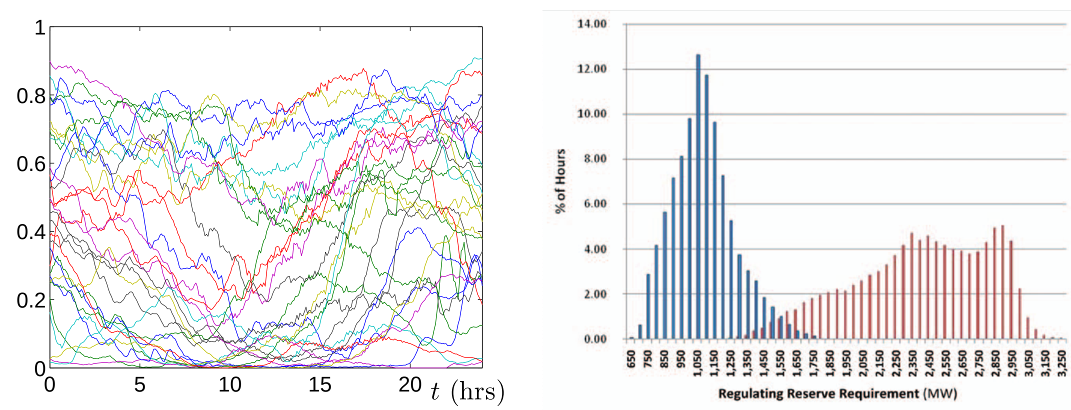


Figure 1.2. (a) Plot of 31 consecutive daily wind power trajectories for aggregate wind power production in the BPA balancing authority. The output is normalized to one by the aggregate nameplate capacity of the wind farms. *Source: BPA, 2008* (b) Histograms depicting simulated projection of the increase in PJM regulating reserve requirements under 20% wind power penetration. *Source:NREL, "EWITS," Final Report 2010.*

1.2.1 Dealing With Variability *Today*

Today, wind and solar energy are assimilated into the grid through *legislative mandates, feed-in tariffs, lenient penalty pricing, guaranteed grid access, and/or construction subsidies.* Specifically, in California, the *Participating Intermittent Renewable Program* (PIRP) legislation compels the independent system operator (ISO) to accept all produced wind power subject to certain contractual constraints. This amounts to a *system take-all-wind* scenario in which *wind power is treated as a negative load* and the subsequent increase in the variability of *net-load* is absorbed by a portfolio of reserve generation capacity, whose cost is allocated among the load serving entities (LSE). This socialization of added reserve costs among the LSEs can be interpreted as an *implicit subsidy for variability costs* to participating wind power producers. Accordingly, there are ongoing public policy and operational procedure debates regarding the *fair allocation* of the costs of these increased reserves [100, 42].

Even at today's modest levels of penetration, the added variability due to wind results in systemic operational problems. For example, on February 26, 2008, the Electric Reliability Council of Texas (ERCOT) had to declare an emergency load curtailment plan due in part to an inaccurate forecast of wind power production [38]. The impact of intermittency and inaccurate forecasting on reserve margins will only become more pronounced as wind energy penetration increases [44, 45, 40]. In order to quantify this statement, several wind integration studies have computed detailed estimates of the increase in reserve requirements needed to compensate the added variability due to wind under a *system-take-all* wind regime. For example, the 2010 EWITS report [40] by NREL projects that regulating reserve requirements will increase by 1500 MW (on average) under a 20% percent wind energy penetration scenario in the PJM interconnection (see Figure 1.2). Such an increase in reserve requirements is unacceptable. It is *too expensive*. Ergo, it will rapidly become infeasible to continue the implicit subsidization of the variability costs among the load serving entities. Moreover, it *severely mitigates the net greenhouse gas benefit* of renewable energy, as regulating reserves are normally supplied by fast-acting, fossil fuel based thermal generators such as natural gas turbines. *Clearly, the current strategy cannot scale.*

1.2.2 Dealing With Variability *Tomorrow*

As renewable energy penetration increases, how must the assimilation of variable power evolve, so as to minimize integration costs, while maximizing the net environmental benefit? Clearly, strategies that mitigate additional reserve requirements will be an essential means to this end. Such strategies will fundamentally fall into one of two categories. Those which (1) lead to direct *reduction of variability in generation* and those which (2) utilize demand-side flexibility to *adapt to variability in generation.*

In the near term, it is likely that wind and solar power producers will be faced with increased exposure to market signals that incentivize reduction in output variability – a stark contrast to the California Participating Intermittent Renewable Program (PIRP). For example, in the United Kingdom, large wind power producers are forced to participate in conventional wholesale electricity markets where they are subject to ex-post financial penalties for deviations from contracts offered ex-ante in forward markets [1] – thus eliminating the implicit subsidy for variability costs. The implementation of imbalance penalty mechanisms represents an initial departure from the system-take-all-wind approach. In response to the financial risk emanating from uncertainty in wind power production, a rational wind power producer will be *forced to curtail* its projected output, thus decreasing the amount of variability that has to be compensated for with reserve generation by the system operator. However, such a removal of the *implicit subsidy for variability cost* may result in significant profit loss to the wind power producer. Consequently, it will become necessary for the wind power producer to *develop* and *evaluate* strategies that aid in the mitigation of generation variability. We refer to this as *firming* of variable renewable power. Potential approaches to firming include:

- *Improved Forecasting*
- *Storage*
- *Renewable Resource Aggregation*
- *Local Generation*

A fundamental question arises in this setting: What is the *marginal value* of a given firming strategy? For example, what is the subsequent increase in expected profit to a renewable power producer per MWh of co-located storage capacity? What is the sensitivity of expected profit to forecast uncertainty? Such a *monetization* of the aforementioned firming strategies will play a central role in shaping investment decisions.

Firming mechanisms, alone, may prove impactful in the near term. However, if we are to transition to a power system with $>$50% of the energy supply coming from variable renewable resources, we will have to fundamentally rethink the way the electricity grid is operated. In the long-term, we argue that *new market systems and instruments* to explicitly address the difficulties with variable generation are required. For example, wind power forecast accuracy tends to steadily decrease with the shortening of the prediction horizon below five hours. Hence, the creation of *intra-day energy and ancillary service markets* (spanning the day-ahead and real-time markets) will introduce additional trading opportunities to leverage on improved forecast accuracy on shorter horizons. In this manner, renewable power producers will have the opportunity to incrementally offer their variable energy in a sequence of intra-day markets – allowing for a more efficient balance of marginal risk and return.

More radically, we envision that market systems will evolve to allow for *price differentiated quality of supply*. Traditionally, the electric grid has been operated such that generation is tailored to counteract the variability in load. Load is largely treated as *inelastic*. However, there exists significant *flexibility inherent to load* that is currently not utilized. The power requirements of many commercial and residential loads are such that a fraction of instantaneous power demand at any given moment is inherently deferrable in time (subject to certain deadlines on delivery). Examples include *thermal systems* such as refrigerators, water heaters, HVAC systems, and, assuming mass adoption of plug-in electric vehicles, *batteries*. Given the inherent deferrability of such loads, certain customers may be willing to accept varying degrees of interruptible power supply in exchange for a lower price, all without experiencing significant loss of utility. Clearly, there is an opportunity and need to design novel market systems that provide flexible consumers the option to purchase multiple quantities of energy – with varying degrees of reliability – from variable generators. Initial work, along these lines, on efficient pricing of interruptible power service contracts can be found in Tan et al. [98]. Further, in addition to participating in such novel market systems, flexible load devices – if intelligently controlled in aggregate – may be capable of providing various ancillary services (e.g., regulation, load-following) to the system operator at a much lower cost than conventional reserve generation, [24].

As the penetration of variable renewable generation continues to increase, we must necessarily transition to a modus operandi in which load is elevated to the equivalent status of dispatchable generation. In this way, *load can be tailored to absorb variability in supply.*

1.3 Summary of Contributions and Dissertation Organization

In this dissertation, we analyze the setting in which a wind power producer is forced to participate in a two-settlement wholesale market for energy, where it faces financial penalties for unscheduled deviations from offered contracts. As wind power production is inherently variable, we identify and analyze various *firming mechanisms* that in aid in mitigating power output variability so as to attenuate market risk, while increasing expected profits. Firming mechanisms considered in this dissertation include *improved forecasting, energy storage, local generation, and aggregation strategies.* The dissertation is organized as follows.

In Chapter 2, Electricity Markets Background , we provide the reader with a brief introduction to electricity markets, with an emphasis on wholesale markets for energy. We consider a *two-settlement market system* operated as a centrally managed *power exchange* and provide a brief description of the welfare properties associated with *locational marginal pricing*.

In Chapter 3, Selling Random Energy in a Two-Settlement System, we explore how an independent wind power producer might *optimally offer* it's variable power so as to maximize its expected profit in a two-settlement market energy. We start with a general stochastic model for wind power production and a model for a perfectly competitive two-settlement market. With these models, we derive explicit formulae for optimal contract offerings and the corresponding optimal expected profit – results that make explicit the trade-off between imbalance prices and the need to *spill* some of the wind energy to increase the probability of meeting the contract. Our analytical characterization of the optimal contract offering is a generalization of the *quantile rule* presented in [81, 32], as it holds on the entire space of expected imbalance prices. We also provide analytical expressions for optimal contract offerings in a multi-period setting in which the WPP has a *recourse* *opportunity* to adjust its DA commitment in an additional intra-day market – offering greater analytical tractability than the LP characterization in [68]. Moreover, we show that extra information from meteorological models and data increases expected optimal profit. We also make explicit the relationship between penalty for contract shortfall and the marginal impact of wind uncertainty on optimal expected profit. For a *uniform* characterization of wind uncertainty, we show that the optimal expected profit is *affine* in the forecast standard deviation. We consider the scenario in which the WPP has installed a fast-acting co-located thermal generator to "hedge" against potential shortfalls corresponding to offered contracts and derive a formula for optimal contract size. In this setting, we also explore the role of local generation in managing the operational and financial risk driven by the uncertainty in generation and obtain analytical expressions for marginal profits from investing in local generation. The formulae make explicit the relationship between price signals and the value of various firming strategies. A treatment of energy storage in this context is discussed in Chapter 5.

In Chapter 4, Wind Energy Aggregation and Profit Sharing, we explore the extent to which a group of N independent wind power producers can exploit the statistical benefits of *aggregation* and *risk sharing* by *forming a willing coalition* to pool their variable power to jointly offer the aggregate output as single entity into a forward energy market. Assuming that coalitional bidding results in profit increase beyond that achievable through individual market participation, a central question arise in this setting. *What are fair sharing mechanisms to allocate the additional profit among the coalition members?* We formalize this question in the setting of cooperative games using tools from coalitional game theory [77, 79]. We define the *value of a coalition* of WPPs as the maximum expected profit achievable joint bidding of the aggregate wind power in a two-settlement market. Using this *value function*, it can be shown that, except for degenerate cases, coalition formation always results in a net increase in expected profit and that there always exist stabilizing rules for sharing the profit. Moreover, via a counterexample, we show that this game is *not convex* and that the famous *Shapley mechanism* is not satisfactory. Alternatively, we propose the use of the *imputation*, which minimizes the worst-case dissatisfaction (excess), as a profit sharing mechanism and show that it is satisfactory for every coalition member in that it satisfies certain *fairness axioms*. As the value function, associated with our coalitional game for wind energy aggregation, is defined in the metric of optimal *expected* profit, an imputation belonging to the corresponding *core*, represents the payment that each wind power producer should receive *in expectation*. In practice, however, the realized profit for will vary day to day, as the profit is inherently a random variable given its explicit dependence on the stochastic wind power production and imbalance prices. To account for this issue, in Section 4.4.3 we propose a *daily* payoff allocation mechanism to distribute the realized profit among the coalition members, such that the payment that each member receives – averaged over an increasing number of days – approaches an imputation in the core, *almost surely*.

In Chapter 5, The Role of Co-located Energy Storage, we extend the results in Chapter 3 by exploring the extent to which *co-located energy storage* can be used to mitigate the inherent financial risk associated with contract imbalances emanating from fluctuations in wind power output. Our goal is to formulate and solve problems of optimal contract sizing for wind power producers with dedicated co-located electric energy storage capacity. Using a simple first order model for the storage system dynamics and a stochastic model for wind power production, we analyze the impact of optimal storage operation on contract sizing and profit. We show that the problem of determining optimal contract offerings for WPP with co-located energy storage reduces to *convex programming*. Moreover, we also show that the expected profit acquired by the wind power producer for optimal contract offerings is concave, non-decreasing in the parameter of energy storage capacity – revealing that greatest marginal benefit from energy storage is derived for a *small* amount of storage capacity. In fact, we show that the marginal optimal expected profit with respect to the energy storage capacity can be analytically computed for *small* capacities – an expression that is closely related to the spectral properties of the underlying wind process. Such results provide a mechanism for empirical calculation of return on investment – an important quantity, as the capital cost of electrical energy storage can be quite large

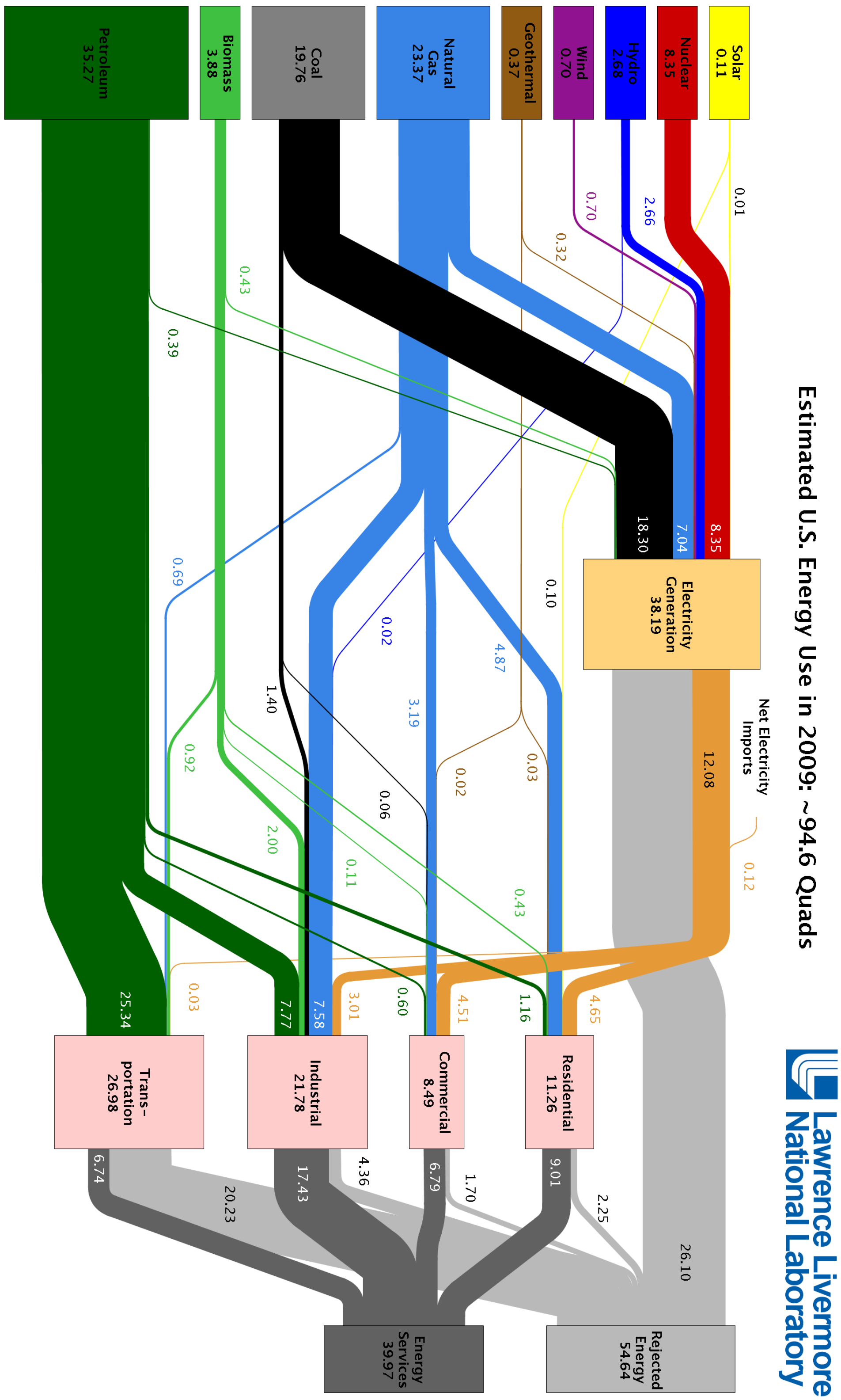
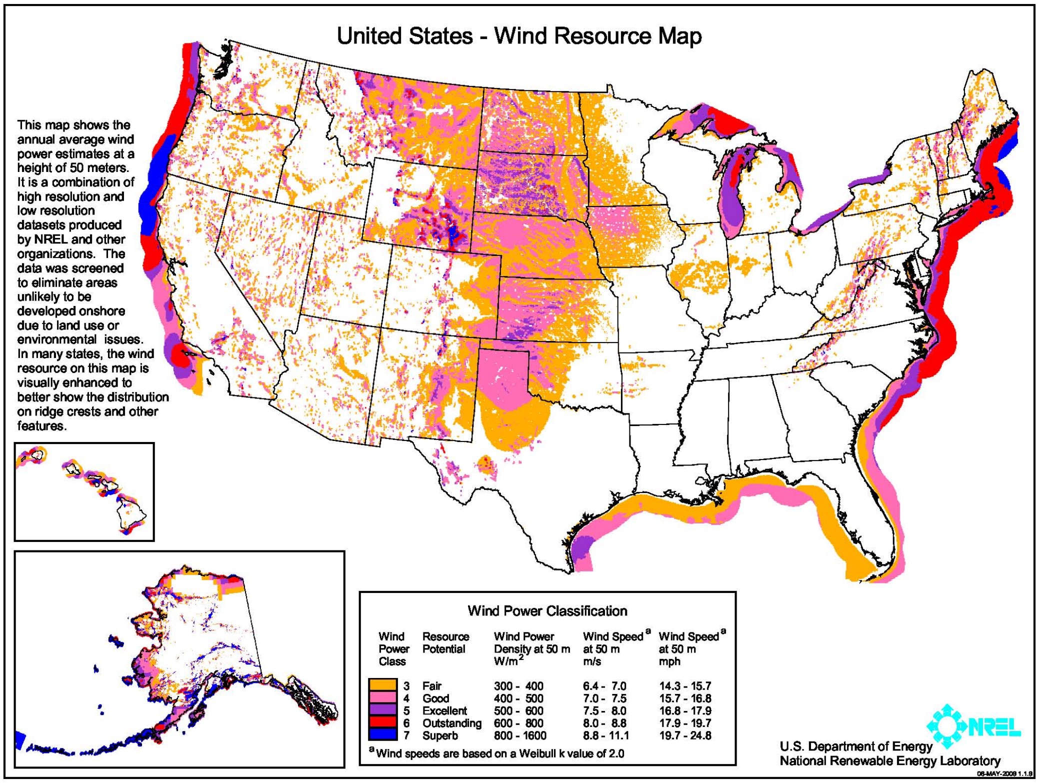


Figure 1.3. Breakdown of U.S. Energy consumption in 2009. A quad is 1015 BTU, or 1.055 1018 joules. *Source:LLNL, 2010*



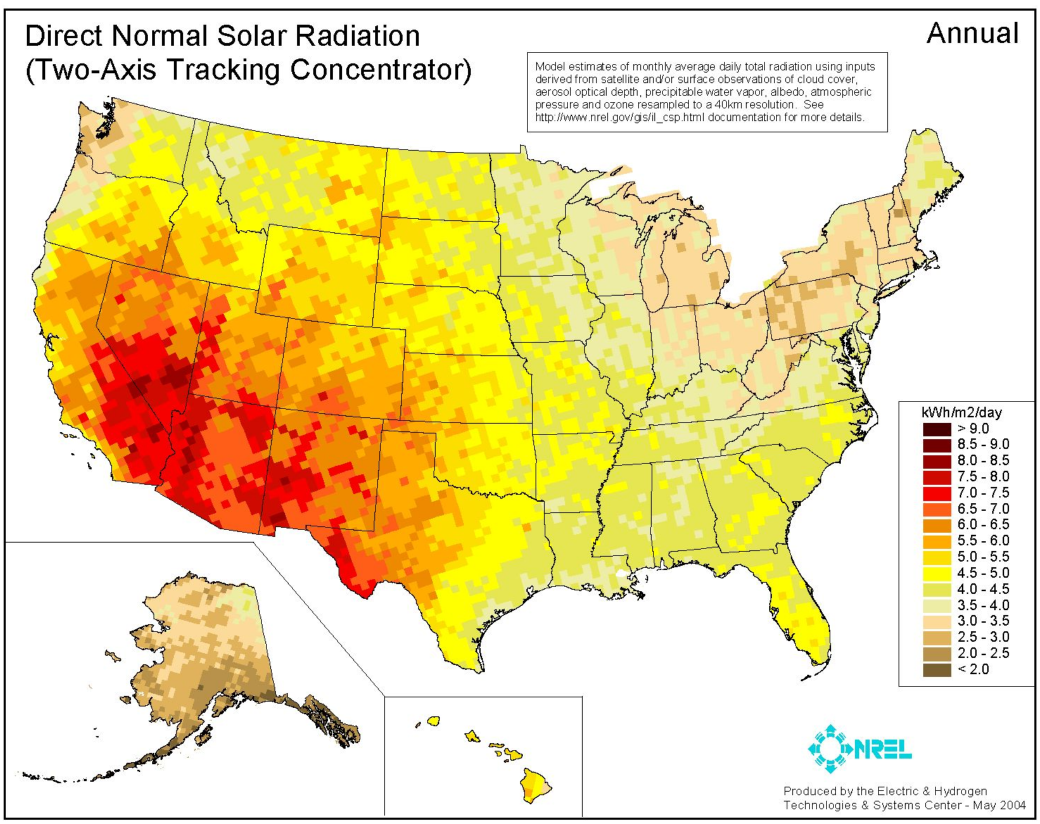


Figure 1.4. Wind power and solar energy density maps for the United States. *Source:NREL, 2004.*

CHAPTER 2

Electricity Markets Background

2.1 Introduction

The deregulation of the electric power industry in various countries has led to the development of *open markets* through which electric power and energy are traded. The design of contemporary electricity markets is based on the assumption that both electrical power and energy can be treated as a commodity, as electrons are inherently uniform across suppliers. However, unlike traditional commodities (e.g., agricultural products, petroleum, copper, etc.), electrical energy cannot be efficiently stored in large quantities and its physical transferal between suppliers and consumers is constrained by the physical laws (e.g., Kirchoff's Current and Voltage Laws) that govern the electrical network through which it flows. Moreover, reliable and safe operation of the electricity grid necessitates nearly *instantaneous* balance of power supply (generation) and demand (load) subject to system capacity constraints. These unique characteristics and constraints imposed by the underlying physical system play a central role in the organization of electricity markets.

2.2 Fundamentals of Electricity Markets

Generally speaking, an electricity market is defined as a system through which suppliers (i.e., generators) and consumers (i.e., load-serving entities) – through *offers to sell* and *bids to buy* energy – enter into contractual obligations for the physical production and consumption of power over pre-specified intervals of time at their respective buses (e.g., typical interval lengths = 1 hr, 15 min, 5 min). The majority of these transactions within a particular *control area* are mediated – to varying degrees – by an *independent system operator* (ISO) whose primary function is to operate the transmission system and maintain instantaneous balance between generation and load across the control area. These transactions can be executed in various market types and occur with various leads times on delivery – from years to minutes. The market types range over the spectrum – from *decentralized* bilateral trading structures to *centrally mediated* power exchanges and pools. Generally, decentralized markets tend to exhibit greater flexibility in contract specification and price determination, while centrally mediated markets permit more rapid, lower cost transactions with increased price transparency. In practice, many electricity market systems are comprised of a mix these differing market types enacted at different timescales.

In addition to *energy markets*, the ISO also implements and manages *ancillary service* (AS) *markets* to procure reserve capacity from various system resources to ensure that the system remains in continuous balance despite uncertainty in demand and the occurrence of unplanned contingency events such as transmission line loss or generation failure. These inherently random phenomena result in system imbalances that occur at differing time-scales with varying degrees of uncertainty – requiring the ISO to procure a broad range of ancillary services to cope with this heterogeneity in variability. These include: *regulation, load-following, spinning reserve, non-spinning reserve, voltage, reactive power,* and *installed capacity.* Alongside the AS markets for reliability, the ISO also manages markets for the trading of transmission congestion and electricity derivatives with the aim of improving liquidity and efficiency of spot markets.

In the sequel, we will focus our exposition on the structure and functioning of energy markets in a *two-settlement system*. For a more detailed description of AS and derivatives markets, see [57, 95].

2.2.1 Bilateral Markets

In a *bilateral market*, two parties (a seller and a buyer) independently negotiate a price, quantity, and auxiliary conditions for the physical production and consumption of energy at their respective bus locations over a future interval of time. Because of the significant time-overhead associated with contract negotiation, bilateral markets are not well suited for near *real-time* operation and balancing. Lead times on delivery for bilateral contracts range from hours to years. From the perspective of the parties involved, the primary benefit of bilateral trading resides in the flexibility in negotiation of the contract terms – price in particular. However, as the transmission network is capacity constrained, these contracted power quantities must be communicated to the ISO to be approved and incorporated into operational schedules and planning in order to maintain secure and balanced operation of the electrical network. For example, the ISO may have to limit the amount of power that a generator can inject at a particular bus, because of insufficient transmission capacity. In certain bilateral markets, two parties entering into a bilateral contract can purchase *physical transmission rights* (PTR) at a public auction to guarantee the right to transmit a specified amount of power through various transmission links in the network over a particular time interval. A potential drawback is that PTRs can lead the exercise of market power by their owners. For a detailed analysis of PTRs, see [52, 57].

2.2.2 Power Exchange Markets: Single Bus Formulation

In contrast to bilateral markets, a centrally managed *power exchange market* consists of numerous buyers and sellers participating in a single market in which bids and offers for energy are cleared and settled by the ISO in the form of a *uniform price auction* [95]. To delineate the basic operation of a power exchange, consider a simplified power network in which all generators and loads are connected to a single bus – or equivalently, a power network with a lossless transmission system of infinite capacity. Working under this idealized setting, we now delineate the basic operation and properties of a power exchange.

Market Bids and Offers

We consider a system with $N_s \in \mathbb{N}$ and $N_c \in \mathbb{N}$ *suppliers* and *consumers*, respectively. Each supplier submits to the ISO, an *offer curve* relating the minimum price p (\$/MWh) required to produce at a power level x (MW) over a pre-specified interval of time. Similarly, each consumer submits to the ISO, a *bid curve* relating the maximum price p (\$/MWh) that the consumer is willing to pay for the consumption of power at a level x (MW) over a pre-specified interval of time. These offer and bid curves can be interpreted as the inverse of the market participants' supply and demand functions, which are defined as follows.

Definition 2.2.1 (Demand function)**.** *For each consumer $n \in N_c$, the* demand function

$$D_n \;:\; \mathbb{R}_+ \longrightarrow [\,0, \overline{d}_n\,]$$

is a monotone, non-increasing *function that relates the quantity of power $x \in [0, \overline{d}_n]$ that consumer n is able and willing to purchase at a price $p \in \mathbb{R}_+$, where $\overline{d}_n \in \mathbb{R}_+$ represents the maximum consumption capacity. The* inverse demand function*,*

$$D_n^{-1}(x) = \sup\{\, p \in \mathbb{R}_+ \mid D_n(p) \geq x\},$$

is correspondingly defined as the maximum price *p that consumer n is willing to pay for a quantity of power x.*

Definition 2.2.2 (Supply function)**.** *For each supplier $n \in N_s$, the* supply function

$$S_n \;:\; \mathbb{R}_+ \longrightarrow [\,0, \overline{s}_n\,]$$

is a monotone, non-decreasing *function that relates the quantity of power $x \in [0, \overline{s}_n]$ that supplier n is able and willing to sell at a price $p \in \mathbb{R}_+$, where $\overline{s}_n \in \mathbb{R}_+$ represents the maximum production capacity. The* inverse supply function*,*

$$S_n^{-1}(x) = \inf\{\, p \in \mathbb{R}_+ \mid S_n(p) \geq x\},$$

is correspondingly defined as the minimum price *p at which supplier n is willing to sell a quantity of power x.*

Remark 2.2.3. (Block Bids and Continuity). It is important to note that in many electricity markets, the suppliers and consumers are required to provide their offer and bid curves in a *block format* that consists of, at most, M price/quantity pairs [93]. This block format amounts to a piecewise constant approximation of the inverse supply and demand functions. However, from the perspective of our work, it suffices to restrict our attention to continuous supply and demand functions, as issues arising from discontinuity do not pertain materially to our analysis. □

Market Clearing

The ISO then combines these bids and offers to construct aggregate supply and demand curves, respectively. The aggregate supply curve is constructed by stacking energy offers in order of increasing offer price. The aggregate demand curve is similarly constructed by stacking energy bids in order of decreasing bid price. Equivalently, the aggregate supply $S(p)$ and aggregate demand $D(p)$ curves can be constructed as sums of the individual supply and demand functions, respectively.

$$S(p) = \sum_{n=1}^{N_s} S_n(p) \tag{1}$$

$$D(p) = \sum_{n=1}^{N_c} D_n(p) \tag{2}$$

Finally, the ISO determines a price-quantity pair (p^*, x^*) to *clear the market* such that the the quantity that consumers are willing to consume is equal the quantity that suppliers are willing to produce, in aggregate.

Definition 2.2.4 (Market equilibrium)**.** *A* market clearing price *or* equilibrium price *is defined as a price p^* at which the quantity that consumers are willing to consume is equal to the quantity that suppliers are willing to produce. An* equilibrium price-quantity pair (p^*, x^*) *is given by*

$$p^* \in \{\ p \in \mathbb{R}_+ \mid D(p) = S(p)\ \} \tag{3}$$

$$x^* \in \{\ x \in \mathbb{R}_+ \mid D^{-1}(x) = S^{-1}(x)\ \} \tag{4}$$

All suppliers that submitted offers at prices at or below the MCP are scheduled. Conversely, all consumers that submitted bids at prices at or above the MCP get scheduled. In accordance with a uniform price auction, all parties scheduled pay or are paid at the MCP.

Assuming *strictly monotone* supply $\{S_n\}$ and demand $\{D_n\}$ functions, individual consumption $y^* \in \mathbb{R}^{N_c}$ and supply $z^* \in \mathbb{R}^{N_s}$ quantity allocations, associated with the market clearing price p^* in equation (3), are given by

$$y_n^* \ = \ D_n(p^*), \quad \text{for all } \ n \in \{1, \cdots, N_c\} \tag{5}$$

$$z_n^* \ = \ S_n(p^*), \quad \text{for all } \ n \in \{1, \cdots, N_s\}. \tag{6}$$

It is straightforward to see that the allocations (y^*, z^*) clear the market at the aggregate quantity allocation x^*, i.e.,

$$\sum_{n=1}^{N_c} y_n^* \ = \ \sum_{n=1}^{N_s} z_n^* \ = \ x^*.$$

2.2.3 Welfare Properties of the Market Clearing Equilibrium

We now show that, under the additional assumption of a *perfectly competitive market*, the equilibrium price-quantity pair (p^*, x^*) as defined by equations (3) and (4) constitutes a *competitive equilibrium* and, in accordance with the *first welfare theorem of economics*, maximizes the *total social welfare* of the suppliers and consumers. To aid in the formalization of this statement, we now present a set of definitions. Generally speaking, a *perfectly competitive market* describes a market in which none of the participants are large enough to set the price. In such a market environment, participants are assumed to behave as *price takers*.

Definition 2.2.5 (Price taker)**.** *A market participant is defined to be a* price taker *if he treats the market price as being independent of his bid/offer actions.*

Definition 2.2.6 (Consumer utility)**.** *For each consumer $n \in \{1, \cdots, N_c\}$, we define the* utility $U_n(y)$ *derived from the consumption of a quantity of power y as a continuous, concave, monotone non-decreasing function, $U_n : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$.*

In order to derive consumer demand preferences $\{D_n\}$ under the presumption of a perfectly competitive market, we must first characterize the net benefit to each consumer derived from purchasing a particular quantity of power at a price p – commonly referred to as *consumer surplus*.

Definition 2.2.7 (Consumer surplus)**.** *For each consumer $n \in \{1, \cdots, N_c\}$, the associated* surplus h_n *is defined as the* utility $U_n(y)$ *derived from consumption of power at a level $y \in \mathbb{R}_+$, less the cost of purchasing that quantity at a price $p \in \mathbb{R}_+$. More specifically,*

$$h_n(y, p) \ = \ U_n(y) \ - \ py. \tag{7}$$

We are now in a position to compute each consumer's ($n = 1, \cdots, N_c$) demand function D_n. It follows that, under the supposition of a perfectly competitive market and rational behavior, consumer n will act as a price taker and will demand a quantity $y^*(p)$ to maximize his surplus h_n for a given price p.

$$D_n(p) \ := \ y^*(p) = \ \arg\max_{y \in \mathbb{R}_+} \ h_n(y, p). \tag{8}$$

To simplify analysis, we assume no upper constraint on consumption capacity (i.e., $\overline{d}_n = \infty$ for all n). It follows from concavity of h_n that the a demand quantity $y \in \mathbb{R}_+$ is optimal if and only if the following first order condition is satisfied:

$$(d - y)\left(\frac{dU_n}{dy} - p\right) \leq 0 \quad \text{for all} \ \ d \in \mathbb{R}_+. \tag{9}$$

Further, assuming that the consumer surplus h_n exhibits a *stationary point* on its domain $\mathbb{R}_+$, we have that in a competitive market setting, consumer n will demand energy up to the point at which the buy price p equals his *marginal utility* of consumption dU_n/d_y – yielding an *inverse demand function* given by

$$D_n^{-1}(y) = \frac{dU_n}{dy}. \tag{10}$$

In an analogous fashion, the derivation of generator supply schedules $\{S_n\}$ requires the characterization of the net benefit derived from producing and selling a particular quantity of power at a price p.

Definition 2.2.8 (Supplier production cost)**.** *For each supplier $n \in \{1, \cdots, N_s\}$, we define the* cost $C_n(z)$ *of producing a quantity of power z as a continuous, convex, monotone non-decreasing function, $C_n : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$.*

Definition 2.2.9 (Supplier profit)**.** *For each supplier $n \in \{1, \cdots, N_s\}$, the* profit π_n *derived from producing and selling a quantity of power z is defined as the revenue acquired from selling a quantity of power z at a price p less the cost $C_n(z)$ of producing that quantity of power. More specifically,*

$$\pi_n(z, p) \; = \; pz \; - \; C_n(z). \tag{11}$$

For a detailed description of generator production costs, see [59]. In accordance with the assumption of a perfectly competitive market, a rational supplier n will sell a quantity $z^*(p)$ to maximize his profit π_n for a given price p. More specifically, the supply schedule is given by

$$S_n(p) \; := \; z^*(p) \; = \; \arg\max_{z \in \mathbb{R}_+} \; \pi_n(z, p). \tag{12}$$

Again, in the interest of technical clarity, we assume no upper bound on production capacity (i.e., $\overline{s}_n = \infty$ for all n). This assumption can be easily relaxed, at the expense of increased notation. It is straightforward to prove concavity of the profit function π_n in the variable z. Hence, it follows that the supply quantity $z \in \mathbb{R}_+$ is optimal if and only if

$$(s - z)\left(p - \frac{dC_n}{dz}\right) \leq 0 \quad \text{for all} \;\; s \in \mathbb{R}_+. \tag{13}$$

For a profit function π_n exhibiting a *stationary point* on its domain $\mathbb{R}_+$, we have that in a competitive market setting, supplier n will supply energy up to the point at which the sell price p equals his *marginal cost of production* dC_n/dz – resulting in an inverse supply function given by

$$S_n^{-1}(z) = \frac{dC_n}{dz}. \tag{14}$$

Definition 2.2.10 (Competitive equilibrium)**.** *The quantity allocations* $z^* = \left[z_1^*, \cdots, z_{N_s}^*\right] \in \mathbb{R}_+^{N_s}$, $y^* = \left[y_1^*, \cdots, y_{N_c}^*\right] \in \mathbb{R}_+^{N_c}$ *and price* $p^* \in \mathbb{R}_+$ *constitute a* competitive equilibirium *if the following conditions are satisfied:*

(a) (Market clearing) *The total consumption of power by consumers is equal to the total production of power by suppliers, i.e.,*

$$\mathbf{1}^T z^* \;=\; \mathbf{1}^T y^*.$$

(b) (Individual profit maximization) *For each supplier* n*, the allocation* z_n^* *is profit maximizing (12) with respect to the price* p^**.*

(c) (Individual surplus maximization) *For each consumer* n*, the allocation* y_n^* *is surplus maximizing (8) with respect to the price* p^**.*

Remark 2.2.11. (Competitive Equilibrium and Total Social Welfare). It is straightforward to see that under the assumption of a perfectly competitive market, the market clearing equilibrium defined by the price $p^* \in \mathbb{R}_+$ (3) and allocations $y \in \mathbb{R}_+^{N_c}$ (5) and $z \in \mathbb{R}_+^{N_s}$ (6) constitutes a *competitive equilibrium*. In addition, as indicated by Theorem 2.2.13, said market equilibrium also maximizes the *total social welfare* defined as the sum of the consumers' surplus and suppliers' profit. □

Definition 2.2.12 (Social Welfare)**.** *The* total social welfare $\mathcal{W}$ *is defined as the sum of the consumers' surplus and suppliers' profit. More specifically,*

$$\mathcal{W}(p, z, y) \;=\; \sum_{n=1}^{N_c} h_n(y_n, p) \;+\; \sum_{n=1}^{N_s} \pi_n(z_n, p) \tag{15}$$

where $y \in \mathbb{R}_+^{N_c}$ *and* $z \in \mathbb{R}_+^{N_s}$ *are vector representations of individual consumer consumption and supplier production quantities.*

Theorem 2.2.13. (First welfare theorem of economics). *Assuming a perfectly competitive market, the market equilibrium defined by the price* $p^* \in \mathbb{R}_+$ *(3) and allocations* $y \in \mathbb{R}_+^{N_c}$ *(5) and* $z \in \mathbb{R}_+^{N_s}$ *(6) maximizes the total social welfare subject to a market clearing constraint, i.e.,*

$$(p^*, z^*, y^*) \;=\; \arg\max_{p,z,y} \; \mathcal{W}(p, z, y) \quad \text{subject to} \quad \mathbf{1}^T z = \mathbf{1}^T y \tag{16}$$

Proof: The result follows immediately when the first order conditions associated with welfare maximization problem (16)

$$\frac{\partial \mathcal{W}}{\partial z_n} = -\frac{dC_n}{dz_n} + p = 0 \qquad \text{for } n = 1, \cdots, N_s \tag{17}$$

$$\frac{\partial \mathcal{W}}{\partial y_n} = \frac{dU_n}{dy_n} - p = 0 \qquad \text{for } n = 1, \cdots, N_c \tag{18}$$

$$\frac{\partial \mathcal{W}}{\partial p} = \mathbf{1}^T z - \mathbf{1}^T y = 0 \tag{19}$$

are combined with our behavioral assumptions of a perfectly competitive market, in which consumers and suppliers were shown to bid and offer their marginal utilities (10) and production costs (14), respectively. Note that the equilibrium price p^* is given by the KKT multiplier associated with the market clearing constraint $\mathbf{1}^T z - \mathbf{1}^T y$. A more detailed proof can be found in Chapter 5 of [102]. ■

Clearly then, under the behavioral assumption of a competitive market, the independent system operator (ISO) – in clearing the market in such a way – is implicitly maximizing the total social welfare of the generators (suppliers) and loads (consumers). In the following section, we consider an extension of the total social welfare formulation to the network setting, in which generators and loads are located at different buses connected by a lossy, capacity constrained transmission network. Essentially, the network capacity constraints *limit* the volume of energy that can be traded between generators and loads connected to distinct buses in the network. This, coupled with heterogeneous generator production cost functions, naturally results in the marginal cost of production to depend on bus location. Consequently, many system operators (e.g., CAISO, PJM, MISO) have adopted the market framework of *locational marginal pricing* (LMP) or *nodal pricing* [95] – a pricing mechanism that efficiently prices the incremental demand of energy at each bus in the network while accounting for transmission constraints and loss effects. The primary idea behind LMPs is as follows. Consider a lossless power network in which some of the transmission lines have constraints on the permissible amount of power flow. Now, consider two distinct nodes in the network A and B and two scenarios where demand at either of these nodes increases incrementally (by 1 MWh). The least cost generation increase to meet the increased demand at A vs. B may well be different as the cheapest producer for supplying increased demand at A may be different from that for B. The incremental cost of supplying an additional MWh of energy at a given node is called the *locational marginal (or nodal) price* at that node. In the next Section 2.2.4, using a general model for active power flow, we systematically describe how nodal prices are computed in a power exchange and provide a simple argument demonstrating *efficiency* of such prices.

Remark 2.2.14. (Ancillary Services). Beyond the procurement of energy to satisfy bulk load forecasts, the system operator is also obliged to procure ancillary services (AS) to hedge against uncertainty emanating from fluctuations in load/supply and unplanned contingency events such as branch and/or generator outages. These various phenomena responsible for system imbalances occur on differing time scales and thus require the procurement of reserve resources with a variety of response capabilities [74, 85]. The California Independent System Operator (CAISO), for example, co-optimizes the procurement of energy with three ancillary service resources in the day-ahead forward market: *regulation* (dispatched every four seconds under automatic generation control), *spinning reserve* (available in ten minutes for synchronized resources) , and *non-spinning reserve* (available in ten minutes). These AS resources are priced as call options in the sense that participating generators receive ex-ante a *capacity payment* for making available a certain amount of power capacity (MW); if said generators are dispatched in real-time by the system operator, they receive an additional energy payment proportional to the amount of energy (MWh) they provide. Moreover, because certain dispatchable generators can offer both energy and ancillary services, there exists arbitrage opportunities between energy and AS markets. However, as much of the analysis in this dissertation is concerned with the competitive integration of wind – an inherently variable resource not suitable for AS – we'll focus on energy markets as the primary medium for integration. □

2.2.4 Power Exchange Markets: Locational Marginal Pricing

The previous characterization of a power network with a lossless transmission system of infinite capacity is useful for basic intuition, but is unrealistic. A more general networked formulation of the power exchange market involves generators and loads participating in the market at different buses connected by lossy capacity constrained transmission lines. In a manner analogous to the single bus setting described above, the ISO forms aggregate supply and demand curves, based on generator offers and load bids, at each bus in the network. Using a *social welfare maximization approach*, the ISO computes optimal consumption/production decisions and a *nodal price* at each bus in the network.

Remark 2.2.15. (Centralized vs. Decentralized). Take note that a centrally managed power exchange or pool, in which the ISO centrally determines energy prices, is not the only path to *efficient nodal pricing*. Decentralized multilateral trading has also been proven to result in *efficient nodal prices* under certain assumptions [106]. However, in practice, a centrally managed power exchange can be cleared more rapidly than a decentralized bilateral market, which necessarily implies reduced minimum lead times on delivery of power for *real-time* markets. Additionally, the centralized approach leads to reduced complexity of trading for buyers and sellers, while increasing the burden on the ISO. For a detailed comparison between *decentralized* bilateral and *centralized* power exchange trading, see [95, 57]. □

Power Network Model

Consider the setting in which generators and loads are connected to differing buses in a capacity constrained transmission network. In our analysis, we will consider an *AC power* *network* with $(N+1) \in \mathbb{N}$ *buses* connected by $M \in \mathbb{N}$ *transmission lines*. Moreover, we assume that the system is operating in sinusoidal steady state, where the voltage phasor at each bus is denoted by $V_n e^{j\theta_n}$ for $n = 0, 1, \cdots, N$. We denote bus 0 as the *slack bus* and set its voltage phasor angle to $\theta_0 = 0$. The array of all remaining bus voltage phasor angles is denoted by

$$\theta \;=\; \big[\theta_1, \cdots, \theta_N\big] \in \mathbb{R}^N.$$

In an analogous manner to [106], we restrict our attention to *active power flows* by assuming the bus voltages $\{V_n\}$ to be held constant through unlimited reactive power support and assume lossless lines [53]. Accordingly, the active power flow from I_{ij} node i to node j is i is equal to

$$I_{ij} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j), \tag{20}$$

where $Y_{ij} \in \mathbb{R}_+$ is the *electrical admittance* of the line connecting node i and j. Note that $Y_{ij} = -Y_{ji}$. Moreover, we take I_{ij} to be positive if power flows from node i to j. It follows then, that we can express the active *net power injection* at each bus as

$$I_n = \sum_{i=0}^{N} I_{ni} = \sum_{i=0}^{N} V_n V_i Y_{ni} \sin(\theta_n - \theta_i) \quad \text{for all} \quad n = 0, 1, \cdots, N. \tag{21}$$

The sign convention is such that I_n is *positive* if the active power generation exceeds load at bus n. The array of all net power injections is denoted by the vector

$$I \;=\; \big[I_0, I_1, \cdots, I_N\big] \in \mathbb{R}^{N+1}.$$

Remark 2.2.16. Note that only N of these equations are independent, because of the network balance constraint $\sum_{n=0}^{N} I_n = 0$. □

We additionally assume that the active power flow across a line connecting node i and node j cannot exceed the capacity $l_{ij} = l_{ji} \in \mathbb{R}_+$ – a constraint which is predominantly determined by the line's thermal limitation. Accordingly, we express the thermal capacity constraint on each line as

$$I_{ij} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j) \leq l_{ij} \quad \text{for all} \quad i, j = 0, 1, \cdots, N. \tag{22}$$

Working within our assumption of unlimited reactive power support [53], for the remainder of the section, we set bus voltage magnitudes to $V_n = 1$ for all n, without loss of generality.

Consumption Model

In our formulation of the power exchange market, we treat the consumers as *perfectly inelastic*. In other words, consumption decisions are independent of price – a characteristic feature of many market systems. As a result, we denote the aggregate *active power demand* at each bus in the network as

$$L_n \in \mathbb{R}_+, \quad \text{for all} \quad n = 0, 1, \cdots, N.$$

Production Model

Although there may be several generators connected to each bus, without loss of generality we consider active power production *in aggregate* at each bus in the network. Let $C_n : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ denote the *aggregate cost* of active power production at bus n. Denote the active power produced at bus n by $x_n \in \mathbb{R}_+$. The array of active power injections across all buses in the network is denoted by the vector

$$x \ = \ \big[x_0, x_1, \cdots, x_N\big] \in \mathbb{R}_+^{N+1}.$$

It follows then that the net injection at bus n is given by $I_n = x_n - L_n$.

Total Social Welfare and Locational Marginal Pricing

As consumer active power demand is treated as *perfectly inelastic* to variations in price and *fixed* at $\{L_n \mid n = 0, 1, \cdots, N\}$, it follows that consumer surplus is *infinite* for any choice of market price. Consequently, aggregate consumer surplus is omitted from *total social welfare* objective. It follows that the social welfare objective is given by the total cost of producing active power sufficient to satisfy the inelastic load. More specifically, the social welfare associated with a given production allocation x is given by

$$\mathcal{W}(x) \ = \ -\sum_{n=0}^{N} C_n(x_n) \tag{23}$$

The independent system operator's (ISO) objective is to maximize total social welfare (or equivalently, minimize total production costs) subject to *power balance* (21) and transmission capacity (22) constraints.

$$\min_{x,\theta} \ -\mathcal{W}(x) \quad : \quad \begin{cases} x_i - L_i \ = \ \sum_{j=0}^{N} Y_{ij} \sin(\theta_i - \theta_j), & \text{for all} \quad i = 0, \cdots, N \\ Y_{ij} \sin(\theta_i - \theta_j) \ \le \ l_{ij}, & \text{for all} \quad i, j = 0, 1, \cdots, N \end{cases} \tag{24}$$

Note that were are omitting $(N-1)$ contingency based constraints. In its more general form, this *nonlinear programming* (NL) problem is referred to as *security constrained economic dispatch* (SCED) or *optimal power flow* (OPF).

Now, if we consider the *Karush-Kuhn-Tucker* (KKT) optimality conditions associated with NL programming problem (24) in conjunction with our assumption of a *perfectly competitive market*, we can readily derive a set of allocations and prices that comprise a *competitive equilibrium*. The Lagrangian associated with problem (24), is given by

$$\mathcal{L}(x,\theta,\lambda,\mu) = \tag{25}$$
$$\sum_{i=0}^{N} C_i(x_i) + \lambda_i \left[\sum_{j=0}^{N} Y_{ij}\sin(\theta_i - \theta_j) - x_i + L_i \right] + \sum_{i,j=0}^{N} \mu_{ij}\left[Y_{ij}\sin(\theta_i-\theta_j) - l_{ij}\right].$$

It follows that if (x^*, θ^*) represent a local minimum satisfying some regularity conditions [19], then there exist KKT multipliers (λ, μ) satisfying:

$$\textit{Stationarity} \qquad \frac{dC_i(x_i^*)}{dx_i} - \lambda_i = 0 \tag{26}$$

$$\sum_{j=0}^{N} Y_{ij}\cos(\theta_i^* - \theta_j^*)[\lambda_i - \lambda_j + \mu_{ij} - \mu_{ji}] = 0 \tag{27}$$

$$\textit{Primal feasibility} \qquad x_i - L_i = \sum_{j=0}^{N} Y_{ij}\sin(\theta_i^* - \theta_j^*) \tag{28}$$

$$Y_{ik}\sin(\theta_i^* - \theta_k^*) \le l_{ik} \tag{29}$$

$$\textit{Dual feasibility} \qquad \mu_{ik} \ge 0 \tag{30}$$

$$\textit{Complementary slackness} \qquad \mu_{ik}\left[Y_{ik}\sin(\theta_i^* - \theta_k^*) - l_{ik}\right] = 0 \tag{31}$$

for all $i, k = 0, \cdots, N$.

Remark 2.2.17. (Competitive Equilibrium). Under the assumption of a perfectly competitive market, each supplier will behave as a price taker and will necessarily offer its *marginal production cost* as its supply function to the ISO. Hence, it follows from the stationarity condition (26) that the quantity x_i^* maximizes supplier i's profit with respect to the price given by the KKT multiplier, λ_i, associated with the corresponding nodal power balance constraint (21) at bus i. It follows then that the quantity allocation x^* and nodal prices given by the associated KKT multipliers $\{\lambda_i\}$ comprise a *competitive equilibrium*. Not that the competitive equilibrium is not necessarily unique. □

Remark 2.2.18. (Efficiency). If (x^*, θ^*) represents a global minimum, then such a price-quantity allocation (x^*, θ^*, λ) is also an *efficient equilibrium* – assuming the generators offer up their true marginal costs to the system operator. We don't consider issues of global optimality here. See [97] for analytical characterizations of power network topologies that guarantee a *zero duality gap*. □

Remark 2.2.19. (Congestion Effects). If for any local minimum (x^*, θ^*), the line loading constraints are non-binding for all lines – i.e., $Y_{ij} \sin(\theta_i^* - \theta_j^*) \ < \ l_{ij}$ for all i, j – then then corresponding KKT multipliers $\{\mu_{ij}\}$ are necessarily zero by the complementarity slackness condition. It follows then by the stationarity condition (27) that the multipliers $\{\lambda_i\}$ associated with the power balance constraints are all equal:

$$\lambda_0 = \lambda_1 = \cdots = \lambda_N$$

In other words, in an uncongested network, the nodal prices are uniform. However, a single congested line can lead to different prices at each node in the network. □

2.3 The Two-Settlement System

Throughout much of the dissertation, we'll be exploring how a variable wind power producer might participate in conventional electricity markets alongside traditional dispatchable generation. To structure our analysis, we consider a *competitive two-settlement energy market* operated as a centrally managed power exchange employing a nodal pricing mechanism [95] described in the previous section. Generally, the two-settlement structure consists of two *ex-ante* markets – a day-ahead (DA) forward market and a real-time (RT) spot market – in which generators can offer power for sale with various lead times on delivery. *Ex-post*, an imbalance settlement mechanism is employed to penalize uninstructed deviations from contracts scheduled ex-ante.

2.3.1 Day-Ahead Market

The DA market permits suppliers to submit *offers to sell* energy for delivery the following day as constant power over time intervals, typically, of length one hour. As the supplier must submit an individual offer for *each* hour of the following day, accepted offers are scheduled as constant power levels over their corresponding hour-long time intervals. Note that in the absence of energy storage capabilities for possible price arbitrage, the decision of how much constant power to offer over any individual hour-long time interval is independent of the decision for every other time interval. However, the presence of generator ramping constraints, start-up costs, and no-load costs result in an inter-temporal coupling of generator production cost functions across contract intervals. For a discussion of such effects on competition in the DA forward market, see [95].

Depending on the region, the DA market closes for bids and offers by 10 AM and clears by 1 PM on the day prior to the operating day. We denote the nodal clearing price in the DA forward market at the bus of interest as $p_1 \in \mathbb{R}_+$ ($/MWh).

2.3.2 Real-Time Market

As the offers submitted to the DA market are cleared well in advance of the operating day, a real-time (RT) spot market is employed to ensure the balance of supply and demand in real-time. Consequently, market participants can adjust their DA schedules based on current (and more accurate) supply, load, and price forecasts. The RT market is cleared five to 15 minutes before the operating interval, which is on the order of five minutes. We denote the nodal clearing price in the RT market at the bus of interest as $p_2 \in \mathbb{R}_+$ (\$/MWh).

2.3.3 Ex-Post Settlement

The resulting contracts are financially binding and are subject to imbalance penalties for uninstructed deviations. For those market participants who deviate from their scheduled transactions agreed upon in the *ex-ante* markets, the independent system operator (ISO) normally employs an *ex-post* settlement mechanism to compute imbalance prices for positive and negative deviations from the generator's offered contract. Generally, the pricing scheme for penalizing contract deviations is a function of the energy imbalance of the control area as a whole and the spot price of balancing energy in the RT market. For example, if the overall system imbalance is negative, those power producers with a positive imbalance with respect to their particular schedules will receive a more favorable price than those producers who have negatively deviated from their schedules, and vice-versa. Depending on the region, imbalance prices may be symmetric or asymmetric. Negative deviations are charged at a price $q \in \mathbb{R}$ (\$/MWh) and positive deviations are charged at price $\lambda \in \mathbb{R}$ (\$/MWh). The imbalance prices (q, λ) are assumed unknown during the DA forward market and are not revealed until the RT spot market, on which they are based, is cleared.

2.3.4 Profit Criterion

The profit derived by a wind power producer (WPP) participating in such a two-settlement market system is comprised of revenue extracted from the ex-ante sale of energy in the DA and RT markets less the ex-post penalties incurred for deviating from contracted forward positions. The recourse opportunity afforded by the RT market allows the WPP to more effectively manage quantity risk by leveraging improved forecasts to incrementally offer a cumulative contract $C = C_1 + C_2$ across the DA and RT markets. More specifically, the WPP initially offers a constant power contract C_1 at a price $p_1 \geq 0$ in the DA market. In the successive RT market, the WPP can leverage improved forecasts of then ensuing wind power production to provide an additional offer C_2 at a price p_2. A basic assumption that we'll maintain throughout the dissertation, is that the wind power producer is taken to behave as a *price taker*. This assumption is reasonable as the individual WPP capacity is assumed small relative to the whole market. Moreover, we assume that the WPP has a *zero marginal cost of production.* It follows then that the profit Π derived from a contract offering (C_1, C_2), is given by

$$\Pi(C_1, C_2) = \int_{t_0}^{t_f} p_1 C_1 + p_2 C_2 - q[C_1 + C_2 - w(t)]^+ - \lambda[w(t) - C_1 - C_2]^+ \, dt, \quad (32)$$

where $[t_0, t_f]$ denotes the time interval over which the contracted power $C_1 + C_2$ is to be delivered and $w(t)$ denotes the realized wind power at time t. Notice that the profit criterion is inherently uncertain as is depends on the realization of wind power production and imbalance prices, which are assumed unknown during ex-ante market transactions. Much of the analysis to follow in the dissertation will be based on variants of (32).

Remark 2.3.1. (Notation). Note that the collection of prices (p_1, p_2, q, λ) are, in LMP-based markets, bus specific. However, as we will be taking the perspective of a single *price taking* producer participating at a *single bus*, we will omit the bus index in the price notation. □

CHAPTER 3

Selling Random Energy in a Two-Settlement System

3.1 Introduction

Global warming, widely regarded as one of the most critical problems we face, has led to great emphasis on clean renewable energy resources such as solar, wind, and geothermal. Many nations have set ambitious goals for increasing the share of renewable energy in electric power generation – wind energy is expected to be a major contributor to the realization of these goals. However, at deep penetration levels, the significant uncertainty and inherent variability in wind power pose major challenges to its integration into the electricity grid.

In many regions around the world, wind power receives extra-market treatment in the form of feed-in tariffs which guarantee grid access and favorable fixed feed-in prices. Specifically, in California, the *Participating Intermittent Renewable Program* (PIRP) legislation compels the system operator to accept all produced wind power subject to certain contractual constraints. This amounts to a *system take-all-wind* scenario in which wind power is treated as a negative load and the burden of balancing costs falls largely on the shoulders of the load serving entities (LSE). This socialization of added reserve costs among the LSEs will become untenable at levels of deep wind energy penetration. Hence, as penetration continues to increase, it is likely that wind power producers (WPP) will be faced with increased exposure to market signals that incentivize reduction in output variability. For example, in the United Kingdom, large WPPs are forced to participate in conventional wholesale electricity markets where they are subject to ex-post financial penalties for deviations from contracts offered ex-ante in forward markets [1].

Motivated by this transition away from a system take-all-wind regulatory environment, we study the setting in which wind power producers (WPP) must sell their variable energy using contract mechanisms in *competitive* two-settlement electricity markets. Our goal is to formulate and solve problems of optimal contract sizing and analytically quantify the relationship between market price signals and the value of improved forecasting, value of local auxiliary generation, value of storage, and the cost of increased reserves needed to accommodate uncertainty in wind power production.

3.2 Related Work and Contributions

There is a considerable literature covering many important aspects of wind power ranging from comprehensive integration studies, forecasting methods, and technology issues. Representative references include [17, 18, 29, 35, 40, 44, 45, 46, 49, 65]. In the narrower context of this chapter, there exist multiple contributions that study the problem of optimizing contract offering strategies in a competitive two-settlement market system in the face of wind power production uncertainty. A scenario-based stochastic programming approach to numerically compute DA contract offers that maximize expected profit in a two-settlement market setting is studied in [5, 64, 68]. Two papers [68, 16] go a step further by introducing a risk-sensitive term (conditional value-at-risk) to the profit function to control variability in expected profit. Additionally, Morales et al. [68] extend these formulations by considering a market system with an additional intra-day market that allows for contract adjustment (i.e., recourse) before the delivery time – resulting in a two-stage stochastic programming formulation, which they show reduces to linear programming (LP). The aforementioned results are primarily computational in nature. In a more general formulation, Sethi et al. [89] consider an analogous problem in which a load-serving entity (LSE) must incrementally purchase a bundle of energy ex-ante in a sequence of N staged markets (with increasing information) subject to consumption uncertainty ex-post. They use stochastic dynamic programming to derive a closed form condition for optimality of purchase amounts. Additionally, in the single forward market setting, Pinson et al. [81] show that the optimal forward contract can be analytically expressed as a probabilistic quantile on prices – a well established result in the field of inventory theory [83]. Also, it has recently come to our attention that Dent et al [32] extend the quantile result of [81] by allowing for stochastic correlation between the wind and imbalance prices. Our work is in the spirit of these latter papers. Our aim is to derive provably optimal analytical expressions to explain the interplay between production uncertainty and profitability.

Our contributions are as follows. We start with a general stochastic model for wind power production and a model for a competitive two-settlement electricity market for energy, as described in Chapter 2. With these models, we derive explicit formulae for optimal contract offerings and the corresponding optimal expected profit – results that make explicit the trade-off between imbalance prices and the need to *spill* some of the wind energy to increase the probability of meeting the contract. Our analytical characterization of the optimal contract offering is a generalization of that in [81, 32] as it holds on the entire space of expected imbalance prices. We also provide analytical expressions for optimal contract offerings in a multi-period setting in which the WPP has a *recourse opportunity* to adjust its DA commitment in an intra-day market – offering greater analytical tractability than the LP characterization in [68]. Moreover, we show that extra information from meteorological models and data increases the expected optimal profit. We also make explicit the relationship between penalty for contract shortfall and the marginal impact of wind uncertainty on optimal expected profit. For a *uniform* characterization of wind uncertainty, we show that the optimal expected profit is *affine* in the forecast standard deviation. We consider the scenario in which the WPP has installed a fast-acting co-located thermal generator to "hedge" against potential shortfalls corresponding to offered contracts and derive a formula for optimal contract size. In this setting, we also explore the role of local generation in managing the operational and financial risk driven by the uncertainty in generation and obtain analytical expressions for marginal profits from investing in local generation. The formulae make explicit the relationship between price signals and the value of various firming strategies. A treatment of energy storage in this context can be found in Chapter 5 and Bitar et al. [11].

The remainder of this chapter is organized as follows. In Section 3.3, we present a stochastic model for wind power production along with a model for the energy market. Our main results are contained in Sections 3.4 through 3.7. In section 3.8, we conduct an empirical study of our strategies on wind power data obtained from the Bonneville Power Administration. Concluding comments and discussion of current and future research are contained in Section 3.9.

3.3 Problem Formulation

3.3.1 Wind Power Model

Wind power $w(t)$ is modeled as a scalar-valued stochastic process. We normalize $w(t)$ by the nameplate capacity of the wind power plant, so $w(t) \in [0, 1]$. For a fixed $t \in \mathbb{R}$, $w(t)$ is a random variable (RV) whose cumulative distribution function (CDF) is assumed known and defined as

$$\Phi(w;t) = \mathbb{P}\{w(t) \leq w\}. \tag{1}$$

The corresponding density function is denoted by $\phi(w;t)$. In this paper, we will work with marginal distributions defined on the time interval $[t_0, t_f]$ of width $T = t_f - t_0$. Of particular importance are the *time-averaged* density and distribution defined as

$$f(w) = \frac{1}{T}\int_{t_0}^{t_f} \phi(w;t)\, dt \tag{2}$$

$$F(w) = \frac{1}{T}\int_{t_0}^{t_f} \Phi(w;t)\, dt = \int_0^w f(x)\, dx \tag{3}$$

Let μ_w denote the mean of the time-averaged distribution. Also, define $F^{-1} : [0,1] \to [0,1]$ as the *quantile function* corresponding to the CDF F. More precisely, for $\beta \in [0,1]$, the β-quantile of F is given by

$$F^{-1}(\beta) = \inf\{x \in [0,1] \;:\; \beta \leq F(x)\} \tag{4}$$

The quantile function corresponding to the time-averaged CDF will play a central role in our results.

3.3.2 Market Model and Metrics

Market Description

We assume that the wind power producer (WPP) is participating in a *competitive two-settlement market system* operated as a power exchange (see Chapter 2, Section 2.3 for a detailed description of such markets). Generally, the two-settlement system consists of two *ex-ante* markets (a day-ahead (DA) forward market and a real-time (RT) spot market) and an *ex-post* imbalance settlement mechanism to penalize uninstructed deviations from contracts scheduled ex-ante. Negative deviations are charged at a price $q \in \mathbb{R}$ ($/MWh) and positive deviations are charged at price $\lambda \in \mathbb{R}$ ($/MWh). The imbalance prices can be positive or negative depending on system conditions.

This pricing scheme for penalizing contract deviations reflects the energy imbalance of the control area as a whole and the spot price of balancing energy in the RT market. Hence, the imbalance prices (q, λ) are assumed unknown during the DA forward market and are not revealed until the RT spot market, on which they are based, is cleared.

Market Model

In the bulk of this chapter, we analyze the problem of optimizing the offering of a constant power contract C in a *single* ex-ante *DA forward market*, scheduled to be delivered continuously over a *single* time interval $[t_0, t_f]$ (typically of length one hour). As the WPP has no energy storage capabilities for possible price arbitrage, the decision of how much constant power to offer over any individual hour-long time interval is independent of the decision for every other time interval. Hence, the problems decouple with respect to contract intervals.

We assume that deviations from said contract C are penalized ex-post according to the imbalance prices (q, λ). Market prices ($/MWh) are denoted as follows.

p : clearing price in the DA forward market

q : ex-post settlement price for negative imbalance $(w(t) \leq C)$

λ : ex-post settlement price for positive imbalance $(w(t) > C)$

We make the following *assumptions* regarding prices and production costs.

A1 The WPP is assumed to be a *price taker* in the forward market, as the individual WPP capacity is assumed small relative to the whole market. As such, the forward settlement price p is assumed *fixed* and *known*.

A2 The WPP is assumed to have a *zero marginal cost of production.*

A3 As imbalance prices $(q, \lambda) \in \mathbb{R}^2$ tend to exhibit volatility and are difficult to forecast, they are modeled as *random* variables, with expectations denoted by

$$\mu_q = \mathbb{E}[\, q \,]$$
$$\mu_\lambda = \mathbb{E}[\, \lambda \,]$$

The imbalance prices (q, λ) are assumed to be statistically independent of the wind $w(t)$.

Remark 3.3.1. *(Value of excess wind)* The positive imbalance price λ can be alternatively interpreted to represent the economic value of surplus wind power in different markets (e.g., ancillary services) or as a stored commodity – assuming the WPP has energy storage capabilities. □

Metrics

The profit acquired, the energy shortfall, and the energy surplus associated with the contract C on the time interval $[t_0, t_f]$ are defined respectively as

$$\Pi(C, \mathbf{w}, q, \lambda) = pCT - q\,\Sigma_-(C, \mathbf{w}) - \lambda\,\Sigma_+(C, \mathbf{w}) \quad (5)$$

$$\Sigma_-(C, \mathbf{w}) = \int_{t_0}^{t_f} [C - w(t)]^+ \, dt \quad (6)$$

$$\Sigma_+(C, \mathbf{w}) = \int_{t_0}^{t_f} [w(t) - C]^+ \, dt \quad (7)$$

where $x^+ := \max\{x, 0\}$ for all $x \in \mathbb{R}$. As wind power $w(t)$ is modeled as a random process, we will be concerned with the *expected* profit $J(C)$, the *expected* energy shortfall $S_-(C)$ and the *expected* surplus wind energy, $S_+(C)$:

$$J(C) = \mathbb{E}\,\Pi(C, \mathbf{w}, q, \lambda) \quad (8)$$

$$S_-(C) = \mathbb{E}\,\Sigma_-(C, \mathbf{w}) \quad (9)$$

$$S_+(C) = \mathbb{E}\,\Sigma_+(C, \mathbf{w}) \quad (10)$$

Here, the expectation is taken with respect to the random prices (q, λ) and wind power process

$$\mathbf{w} = \{w(t) \mid t_0 \le t \le t_f\}\,.$$

3.4 Optimal Contract Offerings

We begin by defining a profit maximizing contract C^* as

$$C^* = \arg\max_{C \geq 0} J(C). \tag{11}$$

In the following theorem, we show that C^* can be expressed analytically using a partition of the space of expected imbalance prices $\pi = (\mu_q, \mu_\lambda) \in \mathbb{R}^2$. Consider the disjoint partion of $\mathbb{R}^2$ defined by

$$\begin{aligned}
\mathcal{M}_1 &= \{\, (x, y) \in \mathbb{R}^2 \mid x(\mu_w - 1) + y\mu_w \leq -p, \;\; y < -p \,\} \\
\mathcal{M}_2 &= \{\, (x, y) \in \mathbb{R}^2 \mid x \geq p, \;\; y \geq -p \,\} \\
\mathcal{M}_3 &= \{\, (x, y) \in \mathbb{R}^2 \mid x(\mu_w - 1) + y\mu_w > -p, \;\; x < p \,\}
\end{aligned}$$

where μ_w is the mean of the time-averaged distribution (3).

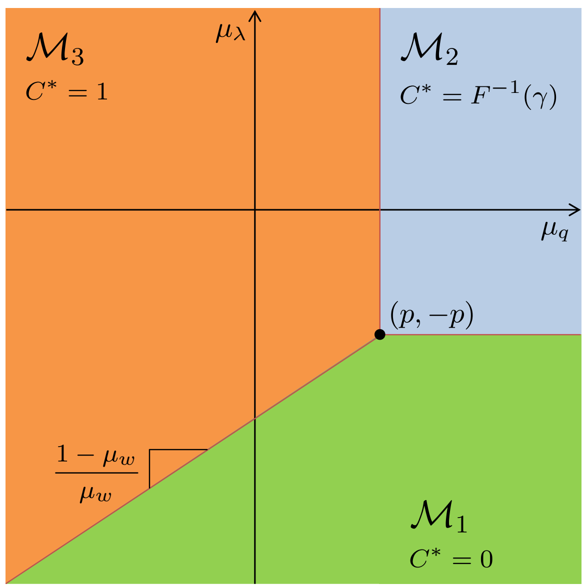


Figure 3.1. Graphical illustration of the optimal bidding policy C^* as a function of the expected imbalance prices (μ_q, μ_λ).

Theorem 3.4.1. *Define the time-averaged distribution $F(w)$ as in (3). For an expected imbalance price pair $\pi = (\mu_q, \mu_\lambda)$,*

(a) an optimal contract *C^* is given by*

$$C^* = \begin{cases} 0, & \pi \in \mathcal{M}_1 \\ F^{-1}(\gamma), & \pi \in \mathcal{M}_2 \\ 1, & \pi \in \mathcal{M}_3 \end{cases} \quad \textit{where} \;\; \gamma = \frac{p + \mu_\lambda}{\mu_q + \mu_\lambda} \tag{12}$$

(b) *The optimal* expected profit *is given by*

$$\frac{J\left(C^*\right)}{T} = \frac{J^*}{T} = \begin{cases} -\mu_\lambda \mu_w, & \pi \in \mathcal{M}_1 \\ \mu_q \displaystyle\int_0^\gamma F^{-1}(x)dx \; - \; \mu_\lambda \int_\gamma^1 F^{-1}(x)dx, & \pi \in \mathcal{M}_2 \\ p - \mu_q(1-\mu_w), & \pi \in \mathcal{M}_3 \end{cases} \tag{13}$$

(c) *The optimal* expected shortfall *and* surplus *are*

$$S_-\left(C^*\right) = S_-^* = T\int_0^{F(C^*)} \left[C^* - F^{-1}(x)\right] dx \tag{14}$$

$$S_+\left(C^*\right) = S_+^* = T\int_{F(C^*)}^1 \left[F^{-1}(x) - C^*\right] dx. \tag{15}$$

Proof: Part (a): Using the assumption of independence between the imbalance prices (q, λ) and wind power $w(t)$, notice that $J(C)$ can be rewritten in terms of the time-averaged density $f(w)$ as defined in (2).

$$\begin{aligned} \frac{J(C)}{T} &= pC \; - \; \frac{1}{T}\int_0^1 \int_{t_0}^{t_f} \left(\mathbb{E}[q][C-w]^+ \; + \; \mathbb{E}[\lambda][w-C]^+\right) \phi(w;t)\, dt dw \\ &= pC \; - \; \mu_q \int_0^C (C-w) f(w)\, dw \; - \; \mu_\lambda \int_C^1 (w-C) f(w)\, dw \end{aligned}$$

Clearly $J(C)$ is continuous in C on $[0,1]$ for any probability density function $f(w)$. For technical simplicity in the proof, we additionally assume that the density $f(w)$ is continuous on $[0,1]$ – from which it follows that $J(C)$ is also differentiable in C on $[0,1]$. Straightforward application of the Leibniz integral rule yields the first and second derivatives of $J(C)$.

$$\frac{dJ}{dC} = T\left(p + \mu_\lambda - (\mu_q + \mu_\lambda)F(C)\right) \tag{16}$$

$$\frac{d^2J}{dC^2} = -T(\mu_q + \mu_\lambda) f(C) \tag{17}$$

As $f(C) \geq 0$ for all $C \in [0,1]$, $J(C)$ is concave $\iff \mu_q + \mu_\lambda \geq 0$. Similarly, $J(C)$ is convex $\iff \mu_q + \mu_\lambda \leq 0$. We now consider the optimization problem on each half-space of expected imbalance prices, separately.

(i): Assume $\mu_q + \mu_\lambda > 0$. As $J(C)$ is concave on this half-space of expected imbalance prices, it follows that $C^* \in [0,1]$ is optimal if and only if

$$(x - C^*) \left.\frac{dJ}{dC}\right|_{C=C^*} \leq 0 \quad \forall\, x \in [0,1] \tag{18}$$

We now evaluate this optimality criterion on three subsets $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\}$ that partition the half-space $\{\mu_q + \mu_\lambda > 0\}$.

$$\begin{aligned}
\mathcal{B}_1 &= \{\mu_q < p\} \cap \{\mu_q + \mu_\lambda > 0\} \\
\mathcal{B}_2 &= \{\mu_q \geq p\} \cap \{\mu_\lambda \geq -p\} \\
\mathcal{B}_3 &= \{\mu_\lambda < -p\} \cap \{\mu_q + \mu_\lambda > 0\}
\end{aligned}$$

For all $(\mu_q, \mu_\lambda) \in \mathcal{B}_1$, it is straightforward to show that $J(C)$ is *non-decreasing* on $[0, 1]$, which yields an optimal point $C^* = 1$ satisfying (18). Conversely, $J(C)$ is *non-increasing* on $[0, 1]$ for all $(\mu_q, \mu_\lambda) \in \mathcal{B}_3$ – yielding $C^* = 0$. Finally, for $(\mu_q, \mu_\lambda) \in \mathcal{B}_2$, the expected profit exhibits a *stationary point* given by $C = F^{-1}(\gamma)$. Notice that $\mathcal{B}_2 = \mathcal{M}_2$.

(ii): Now, assume that $\mu_q + \mu_\lambda \leq 0$. As $J(C)$ is convex on this half-space of expected imbalance prices, the optimum will necessarily occur at the boundary of the feasible set $[0, 1]$. This leads to a simple test for optimality:

$$C^* = \begin{cases} 1, & J(1) \geq J(0) \\ 0, & \text{otherwise} \end{cases}$$

Evaluation of the expected profit criterion $J(C)$ at the boundary points yields

$$J(1) - J(0) \;=\; T(p - \mu_q(1 - \mu_w) + \mu_\lambda \mu_w)$$

Hence, for $\mu_q + \mu_\lambda \leq 0$, we have

$$C^* = \begin{cases} 1, & \mu_q(\mu_w - 1) + \mu_\lambda \mu_w \geq -p \\ 0, & \mu_q(\mu_w - 1) + \mu_\lambda \mu_w < -p \end{cases}$$

Combining this threshold result with those of part (i), we recover the desired result in Theorem 3.4.1-(a).

Part (b): For $(\mu_q, \mu_\lambda) \in \mathcal{M}_1, \mathcal{M}_3$, the result is easily proven by direct substitution of C^* into the expected profit criterion (8). For $(\mu_q, \mu_\lambda) \in \mathcal{M}_2$, consider the change of variables $\theta = F(w)$.

$$\begin{aligned}
\frac{J^*}{T} &= pC^* \;-\; \mu_q \int_0^{C^*} (C^* - w) f(w) dw \;-\; \mu_\lambda \int_{C^*}^{1} (w - C^*) f(w) dw \\
&= pC^* \;-\; \mu_q \int_0^{\gamma} (C^* - F^{-1}(\theta)) d\theta \;-\; \mu_\lambda \int_{\gamma}^{1} (F^{-1}(\theta) - C^*) d\theta \\
&= \underbrace{(p + \mu_\lambda - (\mu_q + \mu_\lambda)\gamma)}_{=0} C^* \;+\; \mu_q \int_0^{\gamma} F^{-1}(\theta) d\theta \;-\; \mu_\lambda \int_{\gamma}^{1} F^{-1}(\theta) d\theta
\end{aligned}$$

which gives us the desired result.

Part (c): The proof is analogous to that of part (b). ■

Remark 3.4.2. (*Newsvendor*) The profit criterion and quantile structure of the optimal policy (12) are closely related to the classical *Newsvendor* inventory problem in operations research [83]. □

Remark 3.4.3. (*Graphical Interpretation*) Parts (b) and (c) of Theorem 3.4.1 provide explicit characterizations of the optimal expected profit J^*, energy shortfall S_-^*, and energy surplus S_+^*. These three quantities can be graphically represented as areas bounded by the time-averaged CDF $F(w)$ as illustrated in Figure 3.2 for $F(C^*) = \gamma = 0.5$. An equivalent characterization of these quantities in terms of the areas $\{A_1, A_2, A_3, A_4\}$ is given by

$$J^*/T = \begin{cases} \mu_q A_1 - \mu_\lambda(A_3 + A_4), & \pi \in \mathcal{M}_1, \mathcal{M}_2 \\ p - \mu_q + \mu_q A_1, & \pi \in \mathcal{M}_3 \end{cases}$$

$$S_-^*/T = A_2$$
$$S_+^*/T = A_3$$

From Figure (3.2), it is apparent that a reduction of statistical dispersion in the time-averaged distribution $F(w)$ will generally result in an increase in area (A_1) and a decrease in areas $\{A_2,\ A_3\}$ – all of which are favorable consequences that result in an increase in optimal expected profit and decrease in expected energy shortfall and surplus. This intuition will be made more precise the section 3.5 where we derive an analytical expression for the marginal value of uncertainty reduction (i.e. improved forecasting). □

Remark 3.4.4. (*Price Elasticity of Supply*) Under certain assumptions, the quantile rule (12) in Theorem 3.4.1 can be interpreted as the optimal *supply curve* for the WPP. Of primary importance is the assumption that the WPP is a price taker in the forward market, ensuring that it wields no influence over the market price p. For a fixed a pair of expected imbalance prices (μ_q, μ_λ), one can interpret the optimal quantile rule (12) as indicating the amount of energy that the WPP is willing to supply at a price p. Specifically, for imbalance prices $\mu_q + \mu_\lambda \geq 0$, the WPP's supply curve is given by

$$C(p) = \begin{cases} 0, & p < -\mu_\lambda \\ F^{-1}(\gamma), & \mu_\lambda \leq p \leq \mu_q \\ 1, & p > \mu_q \end{cases} \quad \text{where} \quad \gamma = \frac{p + \mu_\lambda}{\mu_q + \mu_\lambda}$$

With this explicit characterization of the WPP's supply curve, it's straightforward to see that the WPP is perfectly inelastic for prices $p \notin [-\mu_\lambda, \mu_q]$. Conversely, for prices $p \in [-\mu_\lambda, \mu_q]$,

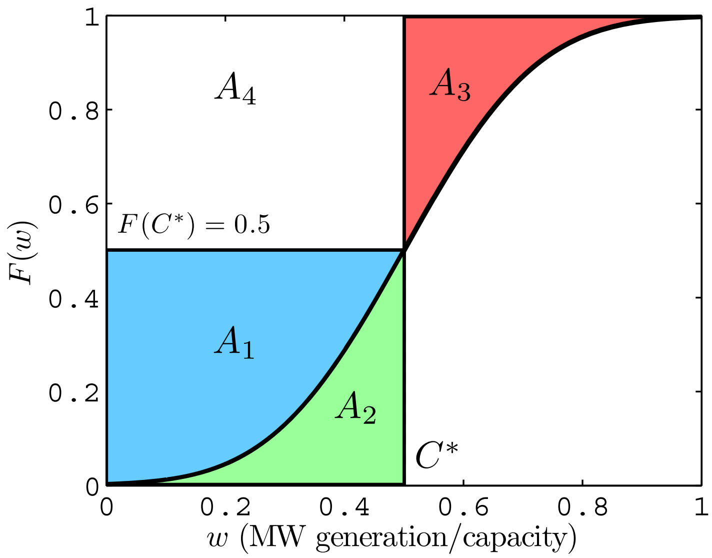


Figure 3.2. Graphical interpretation of (A_1) proportional to optimal profit J^*, (A_2) deficit S_-^*, and (A_3) surplus S_+^* for $F(C^*) = 0.5$.

the price elasticity of supply, E_C, can be readily derived as

$$E_C := \frac{d \ln C(p)}{d \ln p} = \frac{\gamma}{F^{-1}(\gamma)} \frac{dF^{-1}(\gamma)}{d\gamma} = \frac{\gamma}{C f(C)}.$$

□

Remark 3.4.5. (*Non-uniqueness of C^* for $\pi \in \mathcal{M}_2$*) For any pair of expected imbalance prices $\pi = (\mu_q, \mu_\lambda) \in \mathcal{M}_2$, it follows that any contract C that solves $\gamma = F(C)$ is profit maximizing with respect to problem (13). Because the CDF F is only guaranteed to be monotone non-decreasing on it's domain $[0, 1]$, it may have intervals in its domain on which it is constant, which allows for non-uniqueness of the optimizer C^*. Hence, it is straight forward to see that C^* is unique if and only if the set

$$\Gamma(F, \gamma) := \{x \in [0, 1] \ : \ \gamma = F(x)\}$$

is a singleton. As stated in Theorem 3.4.1-(a), a particular choice for an optimal contract is

$$C^* = F^{-1}(\gamma), \qquad \gamma = \frac{p + \mu_\lambda}{\mu_q + \mu_\lambda}$$

the γ-quantile of F, as specified in equation (4). Although the optimal expected profit J^* is independent of the choice of $C^* \in \Gamma(F, \gamma)$, it is straightforward to see that $C^* = F^{-1}(\gamma)$ is the minimizer of the expected optimal shortfall S_-^* among all contracts $C \in \Gamma(F, \gamma)$. The opposite is true for S_+^*. The effect of alternative choices of C^* from $\Gamma(F,\gamma)$ on S_+^* and S_-^* is quantified as

$$S_-\left(C^*\right) = S_-\left(F^{-1}(\gamma)\right) + \gamma\left(C^* - F^{-1}(\gamma)\right) \tag{19}$$

$$S_+\left(C^*\right) = S_+\left(F^{-1}(\gamma)\right) - (1-\gamma)\left(C^* - F^{-1}(\gamma)\right) \tag{20}$$

for $C^* \in \Gamma(F,\gamma)$. □

Remark 3.4.6. (*Role of* γ) Consider the set of expected imbalance prices $(\mu_q, \mu_\lambda) \in \mathcal{M}_2$. It follows from the quantile structure (12) of the optimal solution that C^* is chosen to be the largest contract C such that the probability of *shorting* on said contract – with respect to the time averaged distribution $F(w)$ – is less than or equal to $\gamma = (p+\mu_\lambda)/(\mu_q+\mu_\lambda)$. $1-\gamma$ is interpreted as a confidence level.

Clearly then, the imbalance price ratio γ plays a critical role in implicitly controlling the probability of shortfall associated with optimal contracts $C^* = F^{-1}(\gamma)$. Consider the scenario in which the ISO has direct control over the shortfall imbalance price price q. As the expected short price μ_q becomes more *harsh,* (i.e., larger), the price ratio γ decreases – resulting in smaller offered contracts C^*. This follows from the fact that the quantile function $F^{-1}(\gamma)$ is non-decreasing in γ (non-increasing in μ_q). Consequently, the probability of shortfall $\Phi(C^*;t)$ with respect to the optimal contract C^*, is non-increasing in μ_q. □

3.4.1 Market Simplifying Assumption

We have thus far considered all possible combinations of forward prices p and expected imbalance prices (μ_q, μ_λ), as depicted in Figure 3.1. In the remainder of this paper, we assume that

A4 the WPP has *curtailment capability* and restrict our attention to the case $\lambda = 0$.

We of course realize that in some circumstances surplus wind has economic value ($\lambda < 0$), while in cases of systemic overproduction, excess wind must be curtailed or has negative value ($\lambda > 0$). Nevertheless, in the remaining sections we assume $\lambda = 0$. We make this choice to make our exposition more transparent, and to isolate our studies to one issue at a time. We remark that all of our results generalize to the case $\lambda \neq 0$ at the expense of clarity in exposition. Note that under our assumption $\lambda = 0$, the price penalty ratio simplifies to

$$\gamma = \frac{p}{\mu_q}$$

– a quantity that will play a central role in the interpretation of the results to follow.

3.4.2 Market Based Curtailment

A key consequence of Theorem 3.4.1 is that the price penalty ratio, $\gamma = p/\mu_q$ – in addition to controlling the *reliability of offered contracts* – also controls the degree to which wind power is curtailed in a competitive market.

It is natural then to consider a scenario in which the ISO has direct control over the deviation penalty price q and can directly communicate a statistical description of the shortfall price q (e.g., CDF of q) to the WPP during the DA forward market. As the ISO increases the expected penalty price μ_q, the price ratio γ decreases – resulting in increased *wind energy curtailment* through a smaller optimal contract C^*. From a public policy perspective, it is informative to interpret the shortfall imbalance price q as a continuous knob that controls the degree to which WPPs receive extra-market support. For example, setting $q = p$ with probability one (i.e. $\gamma = 1$) recovers the *system-take-all-wind* scenario in which the WPPs pay no penalty for contract imbalances. Moreover, as governments begins to transition away from legislation guaranteeing extra-market support for WPPs, Theorem 3.4.1 provides the machinery key to understanding how an incremental increase in the penalty price will impact WPP profitability.

Figure 3.2 offers a graphical illustration that makes explicit the trade-off between the optimal expected profit J^*, wind energy shortfall S_-^*, and wind energy curtailment S_+^* – as controlled by the price-penalty ratio γ. A straightforward corollary of Theorem 3.4.1 is that the expected optimal shortfall S_-^* and surplus S_+^* are monotonically non-decreasing and non-increasing in γ, respectively. This makes explicit the claim that some wind energy must be curtailed in order to reduce the amount of operational reserve capacity needed to hedge against uncertainty in the wind power. Moreover, it opens the possibility of *shaping* operational reserves requirements through more advanced penalty mechanisms.

Remark 3.4.7. *(Imbalance Penalty Design)* How can we design *intelligent deviation penalty mechanisms* that incentivize the WPP to limit their injected variability? The conventional imbalance penalty mechanisms considered in this chapter impose a financial penalty that is linear in the cumulative *energy imbalance*. An inherent shortcoming of this approach is that penalty mechanisms of this type do not accurately represent the cost associated with the rate of deviation (i.e., *power*). Generally speaking, more rapid excursions from scheduled contracts require more costly (fast-acting) spinning reserves to maintain system balance. The general message is that imbalances of equivalent energy deviation may incur drastically different balancing costs from the ISO's perspective.

We suggest that deviation penalties that penalize both deviation magnitude (energy) and rate (power) would offer an effective mechanism to more accurately represent the stratified operational cost incurred by the system operator as financial risk to the WPP. In future work, we plan to explore these choices using the framework developed in this chapter. □

3.5 Role of Information

Intuitively, an increase in uncertainty in future wind power production will increase contract sensitivity to imbalance prices. Hence, it is of vital importance to understand the effect of information [such as available implicitly through forecasts] on expected optimal profit. More explicitly, consider a simple scenario in which the WPP observes a random variable Y that is statistically correlated to the wind process $w(t)$. The random variable Y can be interpreted as an observation of a meteorological variable relevant to the wind. As Y is observed prior to the contract offering, the WPP will naturally base its contract on the distribution of the wind $F(w|y)$ conditional on the observation $Y = y$, which is defined as

$$F(w|y) = \frac{1}{T}\int_{t_0}^{t_f} \Phi(w;t|y)dt \tag{21}$$

where $\Phi(w;t|y)$ is the CDF of $w(t)$ conditioned on the realization $Y = y$. It follows then from Theorem 3.4.1 that the optimal contract offering and corresponding expected profit are given by

$$C^*(y) = F^{-1}(\gamma|y), \quad \text{where } \gamma = \frac{p}{\mu_q}$$
$$J^*(y) = \mu_q T \int_0^{\gamma} F^{-1}(w|y)dw.$$

Using this construction, it is straightforward to show that information has economic value.

Lemma 3.5.1. *Information in the form of observations Y helps in the metric of expected optimal profit.*

$$\mathbb{E}[\, J^*(Y)\,] \;\geq\; J^*, \tag{22}$$

where J^* denotes the unconditional optimal expected profit. ■

Proof: Define $C^*(y)$ as a profit maximizing contract conditional on the observation $Y = y$. More precisely,

$$C^*(y) = \arg\max_{C\in[0,1]} \; \mathbb{E}[\Pi(C, \mathbf{w}, q) \mid Y = y],$$

where expectation is taken with respect to the time-averaged conditional distribution. The following inequality holds for all $C_0 \in [0, 1]$ by optimality of $C^*(y)$.

$$\begin{aligned} J^*(y) &= pC^*(y) - \mu_q \int_0^1 [C^*(y) - w]^+ f(w|y)dw \\ &\geq pC_0 - \mu_q \int_0^1 [C_0 - w]^+ f(w|y)dw \end{aligned}$$

Taking expectation with respect to y of both sides of the inequality yields

$$\begin{aligned} \mathbb{E}[J^*(Y)] &\geq pC_0 - \mu_q \int_y \int_0^1 [C_0 - w]^+ f(w|y)dwf(y)dy \\ &= pC_0 - \mu_q \int_0^1 [C_0 - w]^+ \int_y f(w, y)dy\, dw \\ &= pC_0 - \mu_q \int_0^1 [C_0 - w]^+ f(w)dw. \end{aligned}$$

The first equality follows from a straightforward application of Bayes rule and the second equality comes from a marginalization over y. Finally, setting $C_0 = F^{-1}(\gamma)$ yields the desired result, as this corresponds to the unconditional optimal expected profit J^*. ■

Clearly then, information helps in the metric of expected profit. Moreover, Figure 3.2 offers some intuition as to how a reduction in "statistical dispersion" of the CDF F results in increased expected optimal profit. In the limit as the time-averaged distribution $F(w)$ approaches the Heaviside function $H(w - \overline{w})$ (i.e. the associated random variable takes on the realization $\overline{w}$ with probability one, which is equivalent to no uncertainty), we have that the optimal expected profit goes to $J^* = \mu_q T(\gamma \overline{w}) = pT\overline{w}$.

3.5.1 Quantifying the Effect of Uncertainty

It is of interest to more generally quantify the marginal improvement of expected optimal profit with respect to information increase in various metrics of dispersion. In practice, there are numerous *deviation measures* of dispersion of probability distributions (e.g. standard deviation, mean absolute deviation). In [86], the authors take an axiomatic approach to construct a class of *deviation measures* for which there is a one-to-one correspondence with a well known class of functionals known as *expectation-bounded risk measures*.

Definition 3.5.2 (General Deviation Measures)**.** *A* deviation measure *is any functional* $\mathcal{D} : \mathcal{L}^2 \to [0, \infty)$ *satisfying*

1. $\mathcal{D}(X + C) = \mathcal{D}(X)$ *for constant* C
2. $\mathcal{D}(\lambda X) = \lambda \mathcal{D}(X)$ *for all* $\lambda > 0$.
3. $\mathcal{D}(X + Y) \leq \mathcal{D}(X) + \mathcal{D}(Y)$
4. $\mathcal{D}(X) \geq 0$

for all $X, Y \in \mathcal{L}^2$.

Standard deviation and mean-absolute deviation are examples that belong to this class of general deviation measures. We refer the reader to [4, 86] for a detailed exposition. For our purposes, it suffices to realize that

$$\mathcal{D}_\gamma(X) = \mathbb{E}[X] - \mathbb{E}[X \mid X \leq F^{-1}(\gamma)] \tag{23}$$

$$= \mathbb{E}[X] - \frac{1}{\gamma} \int_0^\gamma F^{-1}(x) dx \tag{24}$$

is a *general deviation measure* for all square-integrable random variables X with CDF $F(x)$ and $\gamma \in (0, 1)$. It is sometimes referred to as the *conditional value-at-risk* (CVaR) deviation measure. It measures the distance between the unconditional mean and the mean in the γ probability tail of the distribution.

The particular choice of the CVaR deviation measure $\mathcal{D}_\gamma$ is special in that it permits the analytical characterization of the marginal improvement of optimal expect profit J^* with respect to the wind uncertainty, as measured by $\mathcal{D}_\gamma$. Simple algebraic manipulation of the formula for optimal expected profit (13) reveals J^* to be an affine function in $\mathcal{D}_\gamma(W)$, where W is distributed according to the time averaged distribution $F(w)$.

$$J^* \; = \; \underbrace{pT\,\mathbb{E}[W]}_{\text{expected revenue}} \; - \; \underbrace{pT\,\mathcal{D}_\gamma(W)}_{\substack{\text{loss due to uncertainty} \\ \text{and deviation penalty}}} \tag{25}$$

This result quantifies the increase in expected profit that results from a reduction in $\mathcal{D}_\gamma(W)$ using sensors and forecasts. Further, it makes explicit the joint sensitivity of optimal expected profit J^* to uncertainty and prices. Essentially, the loss term $pT\,\mathcal{D}_\gamma(W)$ can be interpreted as the *price of uncertainty*.

Remark 3.5.3. (*Role of* γ) As we discovered earlier, the price-penalty ratio, $\gamma = p/\mu_q$, plays a central role in controlling the shortfall probability associated with optimal contracts $C^* = F^{-1}(\gamma)$. In a related capacity, the price-penalty ratio γ also acts to discount the impact of uncertainty in the underlying wind process, $w(t)$, on optimal expected profit J^*. This assertion is made rigorous by the fact that $\mathcal{D}_\gamma(W)$ is monotone non-increasing in γ. Its limiting values are given by

$$\begin{aligned} \lim_{\gamma \to 0} \mathcal{D}_\gamma(W) &= \mathbb{E}[W] \\ \lim_{\gamma \to 1} \mathcal{D}_\gamma(W) &= 0. \end{aligned}$$

Hence, as the expected short price μ_q approaches the forward price p from above, we have that $\gamma \to 1$, which attenuates the sensitivity of expected profit to uncertainty in the underlying wind process, as measured by $\mathcal{D}_\gamma(W)$. □

Example 3.5.4. (*Uniform Distribution*) It is informative to consider the case in which $F(w)$ is taken be a uniform distribution having support on a subset of $[0, 1]$. Under this assumption, it is straightforward to compute the optimal expected profit as

$$J^* = pT\left(\,\mathbb{E}[W] - \sigma\sqrt{3}(1-\gamma)\,\right), \tag{26}$$

where σ is the *standard deviation* of W – the most commonly used measure of statistical dispersion. Note that the expected profit is linear in the standard deviation σ. The marginal expected profit with respect to wind uncertainty, as measured by σ, is

$$\frac{dJ^*}{d\sigma} \; = \; -pT\sqrt{3}(1-\gamma). \tag{27}$$

A direct consequence is that the expected profit's sensitivity to uncertainty, σ, increases as the expected short price μ_q becomes more harsh – or equivalently, as $\gamma \to 0$.

Remark 3.5.5. *(Wind Farm Siting)* This result has a useful implication with respect to the siting of wind farms, as both the statistics on the wind process (as described by the distribution $F(w)$) and the market prices (p, q) are location dependent (e.g., locational marginal prices). Using empirically identified distributions on both the wind and price processes, one can use the profit criterion (25) to rank the potential profitability of various wind farm locations. The benefit of the characterization of profit in equation (25) is the ability to separate the effects of imbalance pricing from wind uncertainty on the expected optimal profit. □

3.6 Role of Reserve Margins and Local Generation

In order to maintain reliable operation of the electric grid, the ISO is responsible for procuring *ancillary services* (AS) to balance potential deviations between generation and load. The various underlying phenomena responsible for these deviations result in system imbalances with varying degrees of uncertainty on differing time scales. In order to absorb this variability on the different time scales, multiple ancillary services must be procured. Broadly, these services consist of *regulation, load-following, reserve (spinning and non-spinning), voltage control, and reactive power compensation*.

Based on the scheduled energy, the ISO first determines the total reserve requirement for the entire control area needed to satisfy pre-specified reliability criteria. The ISO then assigns to each participating load serving entity (LSE) a share of the total reserve requirement based roughly on its scheduled demand, because of the uncertainty in load [47]. Each LSE has the option to procure all or a portion of its reserve requirement through bilateral contracts or forward markets. The remaining portion of the reserve requirement not provided by the LSE is procured by the ISO through ancillary service markets. A detailed exposition on ancillary services can be found in [57].

Wind power is inherently difficult to forecast. Moreover, it exhibits variability on multiple time scales ranging from single-minute to hourly. It follows then that regulation, load-following, and reserve services will be necessary to compensate imbalances resulting from fluctuations in wind [44]. Several wind integration studies have computed detailed estimates of the increase in additional reserves of various types needed to compensate the added variability due to wind [40]. To simplify our analysis, we will lump these ancillary services into a single service that we refer to as "reserve margin." Under the current *low* capacity penetration levels of wind power ($\sim 1\%$), the added variability of wind is largely absorbed by existing reserve margins used to cover fluctuations in the load. As the capacity penetration of wind increases, its affect on operating reserve margins will become more pronounced [44, 45]. Moreover, it will become economically infeasible to continue the socialization of the added reserve costs, stemming from wind variability, among participating LSEs. Hence, it is likely that the wind power producer will have to bear the added cost of reserve margins [42].

This transfer of financial burden to the WPP has already begun to emerge in the Pacific Northwest. The Bonneville Power Administration (BPA) in cooperation with Iberdrola Renewables, has deployed a pilot program in which the WPP is responsible for *self-supplying* its own balancing services – from owned and/or contracted dispatchable generation capacity – to satisfy certain reliability criteria (on imbalances) imposed by the BPA [76].

Motivated by this change, consider now the scenario in which the WPP is capable of procuring its reserve margin from a small fast-acting power generator co-located with its wind farm. In addition to assumptions A1-A3 in Section 4.2.2, we also assume the following.

A5 The co-located generator has fixed power capacity L (MW) and fixed and known operational cost $q_L > 0$ (\$/MWh).

A6 The co-located generator operational cost is greater than the forward price (i.e. $q_L \geq p$), to avoid trivial solutions.

A7 Ex-ante, at the time of the contract offering, the *sign of the random shortfall imbalance price* q (relative to q_L) is revealed to the WPP. The corresponding mean of the shortfall price q conditioned on either event is denoted by

$$\mu_q^+ = \mathbb{E}[q \mid q > q_L] \quad (28)$$

$$\mu_q^- = \mathbb{E}[q \mid q \leq q_L] \quad (29)$$

In the event that $\{q \leq q_L\}$, the WPP derives no financial benefit from the co-located generator, and we revert back to our orginal market setting without local generation support – as outlined in Section 3.4. It follows from Theorem 3.4.1 that an optimal contract offering conditioned on the information $\{q \leq q_L\}$ is given by

$$C^* = \begin{cases} F^{-1}\left(p/\mu_q^-\right), & \mu_q^- > p \\ 1, & \mu_q^- \leq p \end{cases}$$

In the complementary event that $\{q > q_L\}$, it follows that the co-located generator can be used to mitigate shortfall risk by covering contract shortfalls $[C - w(t)]^+$ up to a limit L at a cost q_L. For shortfalls larger than L, the WPP pays at the shorfall imbalance price q. This *augmented penalty mechanism* corresponding to the event $\{q > q_L\}$ is captured by the following penalty function $\phi : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}_+$.

$$\phi(x, L) = \begin{cases} qx - (q - q_L)L & x \in (L, \infty) \\ q_L x & x \in [0, L] \\ 0 & x \in (-\infty, 0) \end{cases} \quad (30)$$

It follows that the fiscal cost and benefit of local generation capacity to the WPP can be explicitly accounted for in the following expected profit criterion

$$J_L(C) = \mathbb{E} \int_{t_0}^{t_f} pC - \phi\left(C - w(t),\ L\right) dt \tag{31}$$

where expectation is taken with respect to $(\mathbf{w}, q)$ conditioned on the sign event $\{q > q_L\}$. As before, we define a profit maximizing contract C_L^* as

$$C_L^* = \arg\max_{C \geq 0} J_L(C) \tag{32}$$

Because of the *significant capital costs* associated with the investment in local generation, it's important to quantify the marginal improvement in profit resulting from the investment in a generator with *small* power capacity L. The following theorem distills this notion.

Theorem 3.6.1. *Define the time-averaged distribution $F(w)$ as in (3).*

(a) An optimal contract C_L^* *is given by any solution C of*

$$p = q_L F(C) + (\mu_q^+ - q_L) F(C - L). \tag{33}$$

(b) The marginal expected optimal profit *with respect to power capacity L is given by*

$$\left.\frac{dJ_L^*}{dL}\right|_{L=0} = \left(1 - \frac{q_L}{\mu_q^+}\right) pT \tag{34}$$

Proof: The proof for part (a) follows from direct application of the proof technique for Theorem 3.4.1-(a).

Part (b) is proven as follows. It is straightforward to show that the expected profit is given by

$$\begin{aligned} J_L(C) = pCT - T \int_0^{C-L} \left[\mu_q^+(C - w) - (\mu_q^+ - q_L)L\right] f(w)dw \\ - T \int_{C-L}^{C} q_L(C - w) f(w)dw \end{aligned}$$

for any choice of C and L. Taking the derivative with respect to L yields

$$\frac{dJ_L(C)}{dL} = (\mu_q^+ - q_L) F(C - L) T.$$

Taking the limit as L goes to zero and substituting for the first order optimality condition (a) yields the desired result. ∎

Remark 3.6.2. According to the optimality condition in Theorem 3.6.1-(a), as the expected shortfall price $\mu_q^+ \to \infty$, we have that the optimal $C_L \to 0$, as is consistent with intuition. Moreover, as $\mu_q^+ \to q_L$, we have that $C_L \to F^{-1}(p/q_L)$ – recovering the optimal policy in Theorem 3.4.1. □

Remark 3.6.3. (*Capacity Reservation*) Note that this framework is easily extendable to model the setting in which the WPP does not physically posses a co-located thermal generator, but rather can purchase, ex-ante, reserve generation of any power capacity L at a capacity price q_c (\$/MW). If the reserve capacity is called on, the WPP must pay at the energy cost q_e (\$/MWh), which is analogous to the operational cost q_L in the co-located generation setting. □

3.7 Markets with Recourse

Until now, we have operated under the assumption that the WPP has no market recourse – i.e, the WPP has no opportunity to use improved forecasts to adjust its offer made in the day-ahead (DA) market. We now relax this assumption by augmenting our market model to include an *intra-day* market permitting contract recourse *ex-ante*. More specifically, the market system consists of two sequential *ex-ante* markets in which the WPP can incrementally offer a cumulative contract $C = C_1 + C_2$ for the delivery of power on some future time interval $[t_0, t_f]$. The WPP initially offers a contract C_1 at a price $p_1 \geq 0$ in the DA market (stage-1). In a successive intra-day market (stage-2), the WPP observes a random variable Y (ex: weather conditions, wind speed and direction) that is statistically correlated to the wind power process $\mathbf{w}$. Using this information (Y), the WPP has the option to make an additional offer C_2 at a price p_2 in the intra-day market. Note that the information structure of the recourse decision is taken as

$$C_2 = C_2(Y).$$

We maintain the price taking assumption, as before. Moreover, to avoid trivial solutions, we assume that

$$p_1 \geq p_2 \geq 0.$$

If it were the case that $p_2 > p_1$, there would be no incentive for the WPP to make an offer in the DA market. Ex-post, the WPP is penalized at a price q for shortfalls with respect to the cumulative offered contract $C = C_1 + C_2$. As before, the shortfall price q is assumed unknown ex-ante and is modeled as a random variable that is statistically uncorrelated to the wind, with mean $\mathbb{E}[q] = \mu_q$. We additionally assume that $\lambda = 0$ to isolate the effect of an additional intra-day market on DA contract offerings. *Our objective* is to find explicit characterizations of contracts C_1^* and $C_2^*(Y)$ that optimize the expected profit criterion:

$$J(C) = \mathbb{E} \int_{t_0}^{t_f} p_1 C_1 + p_2 C_2(Y) - q\left[C_1 + C_2(Y) - w(t)\right]^+ dt$$

where expectation is taken with respect to $(\mathbf{w}, Y, q)$. For brevity and clarity in exposition, we focus our analyis on the important case of

$$\mu_q \geq p_1.$$

Optimal contract offerings under the alternative assumption are easily derived. Related work can be found in [84, 89].

Theorem 3.7.1. *Let $\gamma_1 = p_1/\mu_q$ and $\gamma_2 = p_2/\mu_q$. Define the random variable W to be distributed according to the time-averaged distribution $F(w)$ as in (3). Correspondingly, define the conditional distribution $F(w|y) = \mathbb{P}\{W \leq w \mid Y = y\}$, from which we can define the derived random variable Z given by its γ_2 quantile,*

$$Z = F^{-1}(\gamma_2|Y).$$

The portfolio of profit maximizing contracts $\{C_1^*, C_2^*(Y)\}$ *is given by the following. The stage-1 optimal contract C_1^* is given by a solution to*

$$\gamma_1 \; - \; \gamma_2 \mathbb{P}\{Z \geq C_1^*\} \; - \; \mathbb{P}\{Z \leq C_1^*, W \leq C_1^*\} \; = \; 0. \tag{35}$$

The stage-2 optimal contract $C_2^*(Y)$ is given by the threshold rule

$$C_2^*(Y) = \left[Z - C_1^*\right]^+. \tag{36}$$

Proof: Proof is analogous to that of Theorem 3.4.1. ■

3.8 Empirical Studies

Using a wind power time series data set provided by the Bonneville Power Administration (BPA), we are in a position to illustrate the utility and impact of the theory developed in this chapter.

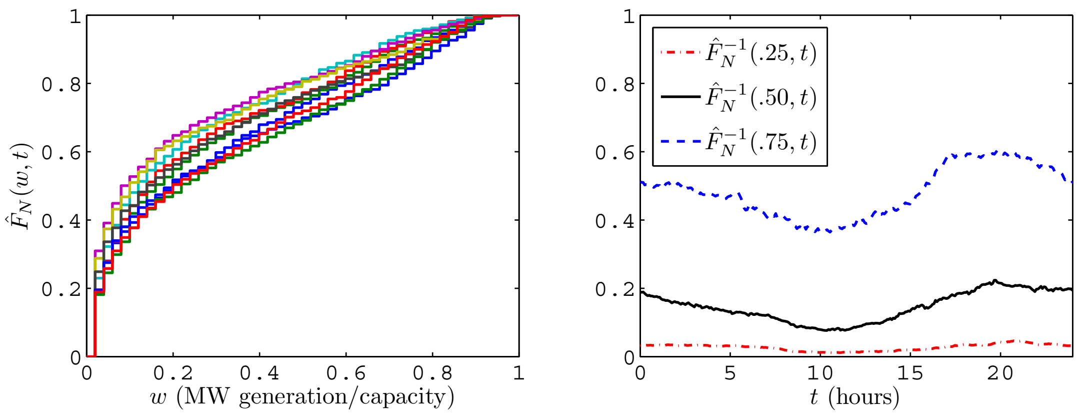


Figure 3.3. (a) Empirical CDFs $\hat{\Phi}_N(w;\tau)$ for nine equally spaced times throughout the day, (b) Trajectory of the empirical median $\hat{\Phi}_N^{-1}(.5,t)$ and its corresponding interquartile range $[\hat{\Phi}_N^{-1}(.25;t), \hat{\Phi}_N^{-1}(.75;t)]$.

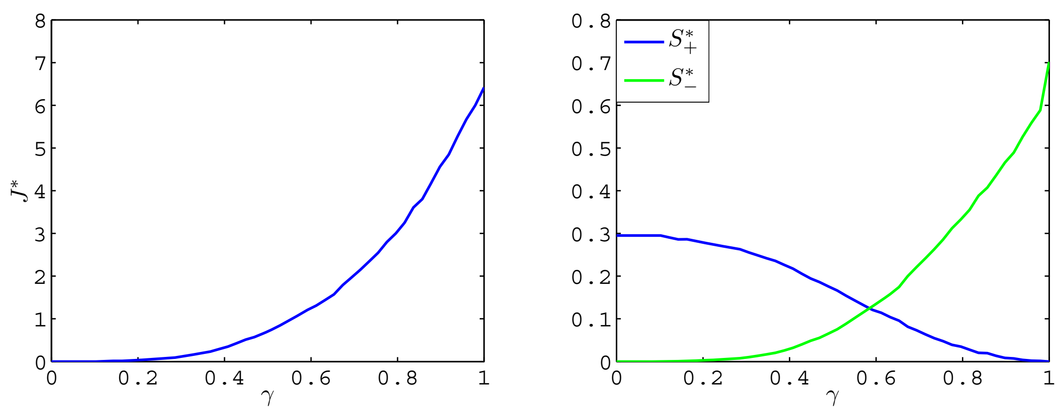


Figure 3.4. (a) Optimal expected profit J^* as a function of γ, (b) Optimal expected energy shortfall and surplus for the 12^{th} hour interval, as a function of γ,

3.8.1 Data Description

The data set consists of a time series of measured wind power aggregated over the 14 wind power generation sites in the BPA control area [9]. The wind power is sampled every 5 minutes and covers the 2008 and 2009 calendar years. To account for additional wind power capacity coming online at various points in time over the 2-year horizon, all of the data are normalized by the aggregate nameplate wind power capacity as a function of time.

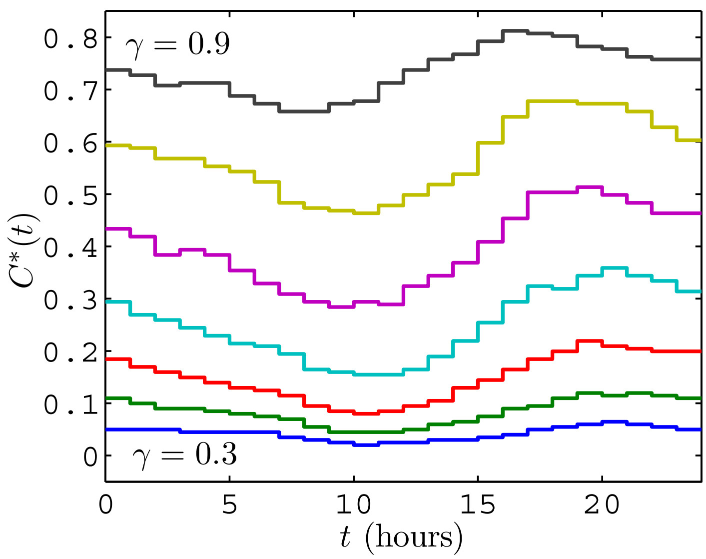


Figure 3.5. Optimal contracts offered in the DA market for various values of $\gamma = 0.3, 0.4, \cdots, 0.9$.

3.8.2 Empirical Probability Model

As stated earlier, wind power is modeled as a continuous time stochastic process whose marginal cumulative distribution is denoted by $\Phi(w;t)$. While the identification of stochastic models that accurately capture the statistical variability in wind power is of critical importance, that is not the focus of this chapter. We will make some simplifying assumptions on the underlying physical wind process to facilitate our analysis.

A8 The wind process $w(t)$ is assumed to be first-order cyclostationary in the strict sense with period $T_0 = 24$ hours [99, 43]. More specifically,

$$\Phi(w;t) \;=\; \Phi(w;t+T_0) \quad \text{for all } \; t$$

We are thus ignoring the effect of seasonal variability.

A9 For a fixed time τ, the discrete time stochastic process $\{w(\tau + nT_0) \mid n \in \mathbb{N}\}$ is independent across days indexed by (n).

Fix a time $\tau \in [0, T_0]$ and consider a finite length sample realization of the discrete time process:

$$z_\tau(n) := w(\tau + nT_0) \quad \text{for all } \; n = 1, \cdots, N.$$

Using this data set, we take the empirical distribution $\hat{\Phi}_N(w;\tau)$ as an approximation of the underlying distribution $\Phi(w;\tau)$:

$$\hat{\Phi}_N(w;\tau) = \frac{1}{N}\sum_{i=n}^{N} \mathbf{1}\left\{z_\tau(n) \leq w\right\} \tag{37}$$

Invoking the Strong Law of Large Numbers under the working assumptions, it can be shown that the $\hat{\Phi}_N(w;\tau)$ is consistent with respect to $\Phi(w;\tau)$ [10]. Figure 3.3 (a) depicts nine representative marginal empirical distributions identified from the BPA data set described earlier. Note that the times corresponding to the nine distributions are equally spaced throughout the day to provide a representative sample. Figure 3.3 (b) depicts the trajectory of the empirical median $\hat{\Phi}_N^{-1}(0.5;t)$ and its corresponding inter-quartile range $[\hat{\Phi}_N^{-1}(0.25;t), \hat{\Phi}_N^{-1}(0.75;t)]$.

3.8.3 Optimal Contracts in Conventional Markets

Using empirical wind power distributions identified from the BPA wind power data set, we are now in a position to compute and appraise optimal day-ahead (DA) contracts offered by a hypothetical Oregon wind power producer (WPP) participating in the idealized market system described in Sections 3.3 and 3.4. For a the set of expected imbalance prices $(\mu_q, \mu_\lambda) \in \mathcal{M}_2$, we are also able to examine the effect of the *price-penalty ratio* $\gamma = (p+\mu_\lambda)/(\mu_q+\mu_\lambda)$ on J^*, S_-^*, and S_+^*. The following empirical studies assume a contract structure

$$\{C_i,\ \ [t_{i-1}, t_i)\}_{i=1}^{24}$$

where $[t_{i-1}, t_i)$ is of length one hour for all i.

Remark 3.8.1. (*Optimal DA Contracts*) Figure 3.5 depicts optimal contracts $(C_1^*, \cdots C_{24}^*)$ for various ratios $\gamma = 0.3, 0.4, \cdots, 0.9$. As expected, as the price-penalty ratio γ decreases, the optimal contract C^* decreases. From Figure 3.5, it is evident that WPPs will tend to offer larger contracts during morning/night periods when wind speed is typically higher than during mid-day (as indicated by Figure 3.3 (b)). □

Remark 3.8.2. (*Profit, Shortfall, and Surplus*) Figures 3.4 (a) and (b) demonstrate the effect of the price-penalty ratio γ on the optimal expected profit, energy shortfall, and energy surplus. The units of S_-^* and S_+^* are (MWh)/(nameplate capacity). In computing the optimal expected profit, we assumed the WPP to have curtailment capability, which is equivalent to $\mu_\lambda = 0$. The units of J^* are in \$/(μ_q · nameplate capacity). When $\mu_q = p$, we have that $\gamma = 1$ and the WPP sells all of its energy production at price $p = \mu_q$. This is equivalent to the current policy of system-take-all-wind. In this situation, the expected profit per hour (see Figure 3.4 at $\gamma = 1$) of approximately $\frac{6.4}{24}$ equals the ratio of average production to nameplate capacity. This number is consistent with typical values of the wind production *capacity factor* (≈25%). The energy surplus S_+^* and shortfall S_-^* are relatively insensitive to variations in γ (for $\gamma \in [0, 0.1]$), because the marginal empirical distributions are steep there. □

3.9 Discussion

In this chapter we have formulated and solved a variety of problems related to optimal contract sizing for a wind power producer offering power in a two-settlement electricity market. Our results have the merit of providing key insights into the trade-offs between a variety of factors such as expected imbalance penalties, cost of local generation, value of information, etc. In our current and future work, we will investigate a number of intimately connected research directions: improved forecasting of wind power, development of probabilistic reliability criteria for reserve margin, dynamic optimization of reserve capacity procurement, improved dispatchability of wind power, network aspects of renewable energy aggregation and profit sharing, and the development of novel market systems that price-differentiate *quality of supply* to facilitate the integration of renewable sources. We are also studying the important case of markets with recourse where the producer has opportunities to adjust bids in multiple successive intra-day markets. We are also developing large scale computational simulations which can be used to test the behavior of of simplified analytically tractable models and suggest new avenues for research applicable to real-world grid-scale problems.

CHAPTER 4

Wind Energy Aggregation and Profit Sharing

4.1 Introduction and Contributions

Motivated by ensuing effects of climate change, there are efforts on a global scale to increase the penetration of renewable energy resources serving electrical loads. Wind and solar electric energy resources posses tremendous potential to reduce the use of carbon emitting fuel sources such as coal, oil, and natural gas [28]. However, wind and solar power generation differ from these traditional sources of electric power, because they are inherently *variable*. Due to natural variations in wind speed, wind power output from a wind turbine exhibits major fluctuations over various time scales. Additionally, wind resources have limited dispatchability and are extremely difficult to forecast. Because of the need to maintain instantaneous balance between load and generation, this inherent variability presents a central challenge to large-scale integration of renewable energy into the electric grid. The interested reader is referred to [73, 40, 45, 49] for a thorough review of the challenges posed by renewable output variability.

It has been widely suggested [40, 73] that the aggregation of geographically diverse wind energy resources offers compelling potential to mitigate wind power variability. Indeed this approach has been successfully monetized by aggregators such as Iberdrola Renewables [50]. For example, the EWITS report [40] states,

> "Both variability and uncertainty of aggregate wind decrease percentage wise with more wind and larger geographic areas."

This attenuation of output variability of wind resources aggregated over large spatial regions is derived from the the tendency of wind speed at different geographic locations to decorrelate with increasing spatial separation. In this chapter, we analyze and quantify the financial benefit of wind power aggregation through *coalitional bidding* in a competitive two-settlement market setting. The central idea is that a set of independent wind power producers (WPP) can exploit the statistical benefits of aggregation by forming a willing coalition to pool their variable power to jointly offer the aggregate output as a single entity into a forward energy market. As deviations from offered contracts are penalized, this amounts to an act of *risk sharing* among the members of the coalition. Assuming that coalitional bidding results in profit increase beyond that achievable through individual market participation, a central question arises in this setting. *What are fair sharing mechanisms to allocate the additional profit among the coalition members?*

We formalize this question in the setting of cooperative games using tools from coalitional game theory [77, 79]. We define the *value of a coalition* of WPPs as the maximum expected profit achievable by joint bidding of the aggregate wind power in a two-settlement market. Using this *value function*, it can be shown that, except for degenerate cases, coalition formation always results in a net increase in expected profit and that there always exist stabilizing rules for sharing the profit. Moreover, via a counterexample, we show that this game is *not convex* and that the famous *Shapley mechanism* is not satisfactory. Alternatively, we propose the use of the *imputation*, which minimizes the worst-case dissatisfaction (excess), as a profit sharing mechanism and show that it is satisfactory for every coalition member in that it satisfies certain *fairness axioms*.

As the value function, associated with our coalitional game for wind energy aggregation, is defined in the metric of optimal *expected* profit, an imputation belonging to the corresponding *core*, represents the payment that each wind power producer should receive *in expectation*. In practice, however, the realized profit for will vary day to day, as the profit is inherently a random variable given its explicit dependence on the stochastic wind power production and imbalance prices. To account for this issue, in Section 4.4.3 we propose a *daily* payoff allocation mechanism to distribute the realized profit among the coalition members, such that the payment that each member receives – averaged over an increasing number of days – approaches an imputation in the core, *almost surely*.

The results presented in this chapter are limited to the setting in which all WPPs are connected to a common single bus (or equivalently, a power network with a lossless transmission system of infinite capacity). We are currently working on extensions of these results to the multi-bus network setting with a capacity constrained transmission system. The transmission network severely constrains the coalition's ability to directly aggregate wind power generated at different buses in the network through (a) real and reactive power conservation, and (b) capacity constraints on the transmission lines. These physical constraints manifest themselves as market rules which can influence the ability and willingness of individuals to form a coalition. We refer to this effect as *network mediated aggregation*. In order to minimize the impact of the intervening network on the *aggregability of wind*, we are currently exploring the development of a market system for *coordinated multilateral trading* [106] to facilitate coalitional trading amongst WPPs connected to distinct buses in the transmission network.

Although different in application and formulation, our problem has significant connections with the classical newsvendor problem [105] in operations research. In both cases, the optimal contract offering is given in terms of a probabilistic quantile. Moreover, coalitional game theory has also been applied in the newsvendor setting [39] where it has been shown that the *core is nonempty* [67].

The chapter is organized as follows. In Section 4.2, we begin with a formulation of the WPP coalitional bidding problem in a two-settlement market setting. We follow this, in Section 4.3 with a brief review of certain key results from coalitional game theory. Finally, in Section 4.4, we state our main results and provide illustrations with some numerical examples.

4.2 Problem Formulation

4.2.1 Aggregate Wind Power Model

Consider a group of N independent wind power producers (WPP) indexed by $i \in \mathcal{N} := \{1, 2, \ldots, N\}$. The power $w_i(t) \in [0, W_i]$ produced at wind farm i is modeled as a scalar valued random process. Denote the collection of wind power production as a vector-valued random process

$$w(t) = [\ w_1(t), \cdots, w_N(t)\ ]^T$$

whose cumulative distribution function (CDF) at each time t is given by

$$\Phi(w;t) = \mathbb{P}\{w(t) \leq w\}. \tag{1}$$

The distribution $\Phi(w;t)$ has support $[0, W_1] \times [0, W_2] \times \ldots \times [0, W_n]$ where W_i is the nameplate capacity of the wind power plant i. The corresponding probability density function is denoted by $\phi(w;t)$. We assume that

A1 the group $\mathcal{N}$ of WPPs are connected to a common bus in the power network.

Consequently, the group $\mathcal{N}$ of WPPs face common market prices and can directly aggregate the power without regard to transmission capacity constraints. Accordingly, it is natural to consider scenarios in which individual wind power producers form willing coalitions $\mathcal{S} \subseteq \mathcal{N}$ to aggregate their wind power production and jointly bid into electricity markets for energy. The aggregate output corresponding to a coalition $\mathcal{S} \subseteq \mathcal{N}$ is denoted by

$$w_{\mathcal{S}}(t) = \sum_{i \in \mathcal{S}} w_i(t). \tag{2}$$

Similarly, the CDF corresponding to the aggregate wind power $w_{\mathcal{S}}(t)$ at time t is defined as

$$\Phi_{\mathcal{S}}(w;t) = \mathbb{P}\left\{w_{\mathcal{S}}(t) \leq w\right\}. \tag{3}$$

with support $[0, \sum_{i \in \mathcal{S}} W_i]$. Note that the coalitional CDF (3) can be computed using the joint distribution (1). Throughout the chapter, we will work with distributions defined on the interval $[t_0, t_f]$ of width $T = t_f - t_0$. Of critical importance are the *time-averaged* density and distribution corresponding to the coalition $\mathcal{S} \subseteq \mathcal{N}$.

$$f_{\mathcal{S}}(w) = \frac{1}{T} \int_{t_0}^{t_f} \phi_{\mathcal{S}}(w;t) dt \tag{4}$$

$$F_{\mathcal{S}}(w) = \frac{1}{T} \int_{t_0}^{t_f} \Phi_{\mathcal{S}}(w;t) dt \tag{5}$$

The stochastic process corresponding to the aggregate power output of a coalition $\mathcal{S} \subseteq \mathcal{N}$ is denoted by

$$\mathbf{w}_{\mathcal{S}} = \{ w_{\mathcal{S}}(t) \mid t \in [t_0, t_f] \}.$$

Also, define $F_{\mathcal{S}}^{-1} : [0,1] \to [0, \sum_{i \in \mathcal{S}} W_i]$ as the *quantile function* corresponding to the coalitional CDF $F_{\mathcal{S}}$. More precisely, for $\beta \in [0,1]$, the β-quantile of $F_{\mathcal{S}}$ is given by

$$F_{\mathcal{S}}^{-1}(\beta) = \inf \{ x \in [0,1] \ : \ \beta \leq F_{\mathcal{S}}(x) \}. \tag{6}$$

4.2.2 Market Model and Metrics

Market Description

We assume that the coalition $\mathcal{S} \subseteq \mathcal{N}$ of wind power producers (WPP) is participating in a *competitive two-settlement market system* operated as a power exchange. See Chapter 2, Section 2.3 for a detailed description of such markets. Generally, the two-settlement system consists of two *ex-ante* markets (a day-ahead (DA) forward market and a real-time (RT) spot market) and an *ex-post* imbalance settlement mechanism to penalize uninstructed deviations from contracts scheduled ex-ante. Negative deviations are charged at a price $q \in \mathbb{R}$ (\$/MWh) and positive deviations are charged at price $\lambda \in \mathbb{R}$ (\$/MWh).

This pricing scheme for penalizing contract deviations reflects the energy imbalance of the control area as a whole and the spot price of balancing energy in the RT market. Hence, the imbalance prices (q, λ) are assumed unknown during the DA forward market and are not revealed until the RT spot market, on which they are based, is cleared.

Market Model

In order to identify conditions under which *coalitions form* and *fair profit sharing mechanisms*, we first analyze the problem of optimizing the offering of a coalition constant power contract C in a *single* ex-ante *DA forward market*, scheduled to be delivered continuously over a *single* time interval $[t_0, t_f]$ (typically of length one hour). As the WPP has no energy storage capabilities for possible price arbitrage, the decision of how much constant power to offer over any individual hour-long time interval is independent of the decision for every other time interval. Hence, the problems decouple with respect to contract intervals.

We assume that deviations from said contract C are penalized ex-post according to the imbalance prices (q, λ). Market prices ($/MWh) are denoted as follows.

p : clearing price in the DA forward market

q : ex-post settlement price for negative imbalance $(w_{\mathcal{S}}(t) \leq C)$

λ : ex-post settlement price for positive imbalance $(w_{\mathcal{S}}(t) > C)$

We make the following *assumptions* regarding prices and production costs.

A2 The WPP is assumed to be a *price taker* in the forward market, as the individual WPP capacity is assumed small relative to the whole market. As such, the forward settlement price p is assumed *fixed* and *known*.

A3 The WPP is assumed to have a *zero marginal cost of production.*

A4 As imbalance prices $(q, \lambda) \in \mathbb{R}^2$ tend to exhibit volatility and are difficult to forecast, they are modeled as *random* variables, with expectations denoted by

$$\mu_q = \mathbb{E}[\, q \,]$$
$$\mu_\lambda = \mathbb{E}[\, \lambda \,]$$

The imbalance prices (q, λ) are assumed to be *statistically independent* of the wind $w(t)$.

A5 The imbalance prices are assumed to be *non-negative*, i.e., it is never profitable to deviate from offered contracts.

$$(q, \lambda) \in \mathbb{R}^2_+ \quad \text{with probability one}$$

Profit Metric

In accordance with the preceding market rules, it follows that the profit acquired by a coalition $\mathcal{S} \subseteq \mathcal{N}$ for an offered contract C on the time interval $[t_0, t_f]$ is defined as

$$\Pi(C, \mathbf{w}_{\mathcal{S}}, q, \lambda) \;=\; \int_{t_0}^{t_f} pC \;-\; q\; [C - w_{\mathcal{S}}(t)]^+ \;-\; \lambda\; [w_{\mathcal{S}}(t) - C]^+ \; dt \qquad (7)$$

where $x^+ := \max\{x, 0\}$ for all $x \in \mathbb{R}$. As the aggregate wind power process $\mathbf{w}_{\mathcal{S}}(t)$ and imbalance prices are modeled as a random, we will be concerned with the *expected* profit

$$J_{\mathcal{S}}(C) \;=\; \mathbb{E}\, \Pi(C, \mathbf{w}_{\mathcal{S}}, q, \lambda). \qquad (8)$$

4.2.3 Initial Results

The *profit maximizing contract* $C_{\mathcal{S}}^*$ corresponding to a coalition $\mathcal{S} \subseteq \mathcal{N}$ can be obtained by solving the following optimization problem

$$C_{\mathcal{S}}^* = \arg\max_{C \geq 0} \; J_{\mathcal{S}}(C). \qquad (9)$$

The solution to this problem is presented in Theorem 3.4.1 and explored in depth in Chapter 3. For completeness, the main result is restated below for the important case of $\mu_q \geq p$.

Theorem 4.2.1. *Define the time-averaged distribution $F_{\mathcal{S}}(w)$ as in (5). An* optimal contract *$C_{\mathcal{S}}^*$ is given by*

$$C_{\mathcal{S}}^* \;=\; F_{\mathcal{S}}^{-1}(\gamma), \qquad \textit{where} \quad \gamma = \frac{p + \mu_\lambda}{\mu_q + \mu_\lambda}. \qquad (10)$$

The optimal expected profit *is given by*

$$\frac{J_{\mathcal{S}}\,(C^*)}{T} \;=\; \frac{J_{\mathcal{S}}^*}{T} \;=\; \mu_q \int_0^{\gamma} F_{\mathcal{S}}^{-1}(x)\, dx \;-\; \mu_\lambda \int_{\gamma}^{1} F_{\mathcal{S}}^{-1}(x)\, dx. \qquad (11)$$

In this chapter, one of our objectives is to quantify the financial benefit of coalitional bidding in two-settlement markets. As a motivating result, it is straightforward to show that the act of *risk sharing* through coalitional bidding leads to an increase in collective profit *almost surely*.

Theorem 4.2.2. *Let $\{C_1, \cdots, C_N\}$ be a set of N individual contracts. Then,* almost surely

$$\Pi\,(C_{\mathcal{N}},\; \mathbf{w}_{\mathcal{N}},\; q,\; \lambda) \;\geq\; \sum_{i=1}^{N} \Pi\,(C_i,\; \mathbf{w}_i,\; q,\; \lambda) \qquad (12)$$

where $C_{\mathcal{N}} = \sum_{i=1}^{N} C_i$.

Proof:

$$\begin{aligned}\Pi\left(C_{\mathcal{N}},\ \mathbf{w}_{\mathcal{N}},\ q,\ \lambda\right) &= \int_{t_0}^{t_f} pC_{\mathcal{N}} \ - \ q\ \left[C_{\mathcal{N}} - w_{\mathcal{N}}(t)\right]^+ \ - \ \lambda\ \left[w_{\mathcal{N}}(t) - C_{\mathcal{N}}\right]^+ \ dt \\ &\geq \int_{t_0}^{t_f} pC_{\mathcal{N}} \ - \ q\ \sum_{i=1}^{N} \left[C_i - w_i(t)\right]^+ \ - \ \lambda\ \sum_{i=1}^{N} \left[w_i(t) - C_i\right]^+ \ dt \\ &= \sum_{i=1}^{N} \Pi\left(C_i,\ \mathbf{w}_i,\ q,\ \lambda\right),\end{aligned}$$

where the inequality follows from the sub-additivity property of x^+ for all $x \in \mathbb{R}$. Moreover, the inequality holds for any realization of the stochastic process $\mathbf{w}_{\mathcal{S}}$ and random prices $(q, \lambda) \in \mathbb{R}_+^2$. ■

It follows from Theorem 4.2.2 that coalitional bidding will always result in a net profit increase that can be shared between the coalition participants. Unfortunately, the expression for optimal expected profit (11) does not provide any clue as to how the added income should be shared among the coalition participants. Naïve sharing mechanisms, such as equal distribution of the profit among the participants, are not satisfactory, because certain members of the coalition may obtain a greater profit if they were to break up the coalition and form a smaller one. Thus, our primary objective is to identify *fair* payoff allocation mechanisms for wind farm coalitions.

The problem of sharing collective profits has been extensively studied in cooperative game theory [72]. We will show that our problem can be modeled as a coalitional game and we will study its properties and identify sharing mechanisms that are fair from an axiomatic perspective. In the next section we review some of the basic concepts and results of the coalitional game theory. The interested reader may see [72, 77, 69, 79] for a more detailed exposition on the topic of cooperative game theory.

Finally, we close this section introducing a function Ψ that will be vital in analyzing the properties of the coalitional game associated with wind power aggregation. Let $\mathbf{x} = \{x(t) \mid t \in \mathbb{R}\}$ be a scalar random process that takes nonnegative values on the interval $[t_0, t_f]$ and define the functional $\Psi[\mathbf{x}]$ as a mapping from the space of continuous probability density functions to the positive reals. The functional Ψ represents the maximal expected profit achievable under the random process $\mathbf{x}$.

Remark 4.2.3. (Notation). Note that, in order to retain notational simplicity, we designate the random process $\mathbf{x}$ as the input argument to the functional Ψ, rather than the underlying probability law on which the functional directly acts. □

Specifically,

$$\Psi[\mathbf{x}] \ := \ \max_{C \geq 0} \ \mathbb{E}\,\Pi(C, \mathbf{x}, q, \lambda) \tag{13}$$

where Π is defined in equation (7). The following lemma establishes certain properties of the functional Ψ that will be used to characterize the coalitional game in the sequel.

Lemma 4.2.4. *The function Ψ as defined in (13) is* positively homogeneous (of degree one) *and* superadditive *in the underlying random process. For any pair of random processes $\mathbf{x} = \{x(t) \mid t \in \mathbb{R}\}$ and $\mathbf{y} = \{y(t) \mid t \in \mathbb{R}\}$, we have*

(i) (positive homogeneity) $\Psi[\alpha\mathbf{x}] = \alpha\Psi[\mathbf{x}] \;\; \forall \;\; \alpha \geq 0$

(ii) (superadditivity) $\Psi[\mathbf{x}] + \Psi[\mathbf{y}] \leq \Psi[\mathbf{x} + \mathbf{y}]$

where $\alpha\mathbf{x} = \{\alpha x(t) \mid t \in \mathbb{R}\}$ *and* $\mathbf{x} + \mathbf{y} = \{x(t) + y(t) \mid t \in \mathbb{R}\}$.

Proof: Throughout the proof, we restrict ourselves to the set of expected imbalance prices such that $\mu_q \geq p$. The results are similary proven for the complementary case of $\mu_q < p$.

Part (i), *(Positive Homogeneity)*: Fix $\alpha > 0$. For brevity, let the stochastic process $\mathbf{x}$ inherit the properties and distributional notation associated with the wind process defined in Section 4.2.1. Moreover, let $\Gamma_\alpha(x; t)$ denote the marginal CDF associated with the positively scaled stochastic process $\alpha\mathbf{x}$. First observe that

$$\Gamma_\alpha(x; t) \;=\; \mathbb{P}\{\alpha x(t) \leq x\} \;=\; \Phi\left(\frac{x}{\alpha}; t\right).$$

It follows that the time-averaged distribution $G_\alpha(x)$ associated with the scaled process $\alpha\mathbf{x}$ is similarly given by

$$G_\alpha(x) \;=\; \frac{1}{T}\int_{t_0}^{t_f} \Gamma_\alpha(x; t)\, dt \;=\; \frac{1}{T}\int_{t_0}^{t_f} \Phi\left(\frac{x}{\alpha}; t\right)\, dt \;=\; F\left(\frac{x}{\alpha}\right).$$

Using the previous identity $G_\alpha(x) = F(x/\alpha)$, it follows that the β quantile of G_α is given by

$$G_\alpha^{-1}(\beta) = \alpha F^{-1}(\beta).$$

Using the previous identity in conjunction with Theorem 4.2.1, the desired result of positive homogeneity follows immediately. More specifically,

$$\begin{aligned}
\Psi[\alpha\mathbf{x}] &= \mu_q T \int_0^\gamma G_\alpha^{-1}(z)\, dz \;-\; \mu_\lambda \int_\gamma^1 G_\alpha^{-1}(z)\, dz \\
&= \alpha\left(\mu_q T \int_0^\gamma F^{-1}(z)\, dz \;-\; \mu_\lambda \int_\gamma^1 F^{-1}(z)\, dz\right) \\
&= \alpha\Psi[\mathbf{x}].
\end{aligned}$$

The result for $\alpha = 0$ is trivial, as $\Psi[0] = 0$.

Part (ii), *(Superadditivity)*: Consider two stochastic processes $\mathbf{x}$ and $\mathbf{y}$.

$$\begin{aligned}\Psi[\mathbf{x}] + \Psi[\mathbf{y}] &= \max_{C_x \geq 0} \ \mathbb{E}\,\Pi(C_x, \mathbf{x}, q, \lambda) \ + \ \max_{C_y \geq 0} \ \mathbb{E}\,\Pi(C_y, \mathbf{y}, q, \lambda) \\ &= \ \mathbb{E}\,\Pi(C_x^*, \mathbf{x}, q, \lambda) \ + \ \mathbb{E}\,\Pi(C_y^*, \mathbf{y}, q, \lambda)\end{aligned}$$

where C_x^* and C_y^* are the optimizers of their respective maximization problems. It follows directly from Theorem 4.2.2 that

$$\Pi(C_x^*, \mathbf{x}, q, \lambda) \ + \ \Pi(C_y^*, \mathbf{y}, q, \lambda) \ \leq \ \Pi(C_x^* + C_y^*, \ \mathbf{x} + \mathbf{y}, \ q, \lambda)$$

Using this inequality, we can bound the sum $\Psi[\mathbf{x}] + \Psi[\mathbf{y}]$ to obtain the desired result. More specifically,

$$\begin{aligned}\Psi[\mathbf{x}] + \Psi[\mathbf{y}] &\leq \ \mathbb{E}\,\Pi(C_x^* + C_y^*, \ \mathbf{x} + \mathbf{y}, \ q, \lambda) \\ &\leq \ \Pi(C_{x+y}^*, \ \mathbf{x} + \mathbf{y}, \ q, \lambda) \\ &= \ \Psi[\mathbf{x} + \mathbf{y}],\end{aligned}$$

where

$$C_{x+y}^* \ = \ \arg\max_{C \geq 0} \ \mathbb{E}\,\Pi(C, \mathbf{x} + \mathbf{y}, q, \lambda).$$

■

4.3 Background: Results from Coalitional Game Theory

Game theory deals with rational behavior of economic agents in a mutually interactive setting. In a game, interacting agents aim to maximize certain expected utility by making particular decisions. The final payoff of each agent depends on the decisions taken by all the agents. The *game* is specified by the set of participants, the possible decisions taken by each agent and the set of all possible payoffs. The agents in the game are called the players. A game is called *cooperative* if the players are allowed to form alliances or teams. Cooperative games [72] are also known as coalitional games and have been used extensively in diverse disciplines such as social science, economics, philosophy, psychology [69] and more recently in engineering and communication networks [88].

In a *coalitional game*, we are interested in identifying the topology of alliance formation, under which the subsequent groups of players can *improve their payoff* if they decide to play jointly in said alliances. An alliance of players is called a coalition. Let

$$\mathcal{N} := \{1, 2, \ldots, N\}$$

denote the finite collection of players.

Definition 4.3.1 (Coalition)**.** *A* coalition *is any subset* $\mathcal{S} \subset \mathcal{N}$. *The cardinality of the coalition* $\mathcal{S}$ *is its number of players and is denoted by* $|\mathcal{S}|$. *The* set of all possible coalitions *is defined as the power set* $2^{\mathcal{N}}$ *of* $\mathcal{N}$*:*

$$2^{\mathcal{N}} := \{\mathcal{S} : \mathcal{S} \subseteq \mathcal{N}\} \tag{14}$$

The grand coalition $\mathcal{N}$ *is the alliance that comprises every player in the game.*

Definition 4.3.2 (Coalitional game and value)**.** *A coalitional game is defined by a pair* $(\mathcal{N}, v)$ *where*

$$v : 2^{\mathcal{N}} \to \mathbb{R}$$

is the value function *that assigns a real value to each possible coalition* $\mathcal{S} \subseteq \mathcal{N}$. *The* value of the coalition $\mathcal{S}$ *is defined as* $v(\mathcal{S})$.

Definition 4.3.3 (Superadditive game)**.** *A coalitional game* $(\mathcal{N}, v)$ *is* superadditive *if its value function is superadditive,* i.e. *if for any pair of disjoint coalitions* $\mathcal{S}, \mathcal{T} \subset \mathcal{N}$ *with* $\mathcal{S} \cap \mathcal{T} = \emptyset$,

$$v(\mathcal{S}) + v(\mathcal{T}) \ \leq \ v(\mathcal{S} \cup \mathcal{T}) \tag{15}$$

Remark 4.3.4. Superadditivity implies that the value of a coalition cannot be improved by splitting it up into two smaller coalitions. □

A central problem in coalitional game theory is the identification of payoff allocation mechanisms that *fairly* share the coalition value $v(\mathcal{S})$ among all of the members of said coalition $\mathcal{S}$. The use of payoff allocation mechanisms that *do not fairly share* the coalition value among the members may result in certain members exiting the coalition to form more profitable sub-coalitions. We make this more precise by presenting an *axiomatic formulation of fairness* in definition 4.3.8. Additionally, we are interested in the class of coalitional games with *transferable payoff*.

Definition 4.3.5 (Transferable payoff)**.** *A coaltional game with* transferable payoff *is characterized by the property that there is no restriction on the sharing of coalition value between members of the coalition.*

Definition 4.3.6 (Payoff allocation)**.** *A* payoff allocation *for the coalition* $\mathcal{S} \subseteq \mathcal{N}$ *is a vector*

$$x \in \mathbb{R}^{|\mathcal{S}|}$$

whose entries represent payoffs to each member of the coalition.

1. (Efficiency) *An allocation x is said to be* efficient *if the payoffs add up to the value of the coalition,*

$$x^T \mathbf{1} = \sum_{i \in \mathcal{S}} x_i = v(\mathcal{S})$$

2. (Individually rationality) *An allocation is said to be* individually rational *if each player gets a payoff that is at least as good as that obtained by playing alone,*

$$x_i \geq v(\{i\}), \;\; \forall \;\; i \in \mathcal{S}$$

Definition 4.3.7 (Imputation)**.** *A payoff allocation x for the grand coalition $\mathcal{N}$ is said to be an* imputation *if it is simultaneously efficient and individually rational. The set of all imputations $\mathcal{I}$ for the game $(\mathcal{N}, v)$ is defined as follows*

$$\mathcal{I} := \left\{ x \in \mathbb{R}^N \;\middle|\; \sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}), \;\; x_i \geq v(\{i\}), \;\; \textit{for all} \;\; i \in \mathcal{N} \right\} \tag{16}$$

We next define the a fundamental solution concept for coalitional games known as the *core*. It can be interpreted as being analogous to Nash equilibria for non-cooperative games [72].

Definition 4.3.8 (The Core)**.** *Consider a coalitional game $(\mathcal{N}, v)$ with* transferable payoff. *The* core *is defined to be the set of imputations such that no coalition can obtain a payoff which is better than the sum of the members current payoffs. Consequently, for an imputation in the core, no subgroup of players has an incentive to leave the grand coalition to form another coalition $\mathcal{S} \subset \mathcal{N}$. A mathematical expression for the core is given by:*

$$\mathcal{C} := \left\{ x \in \mathbb{R}^N \;\middle|\; \sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}), \;\; \sum_{i \in \mathcal{S}} x_i \geq v(\mathcal{S}), \;\; \textit{for all} \;\; \mathcal{S} \subseteq \mathcal{N} \right\} \tag{17}$$

4.3.1 Existence of a Nonempty Core

Certain coalitional games have a empty cores. Two important classes of games with a nonempty core are *convex games* and *balanced games*.

Definition 4.3.9 (Convex game)**.** *A coalitional game $(\mathcal{N}, v)$ is* convex *if and only if its value function is* supermodular, i.e.

$$v(\mathcal{S}) + v(\mathcal{T}) \;\leq\; v(\mathcal{S} \cup \mathcal{T}) + v(\mathcal{S} \cap \mathcal{T}), \;\; \textit{for all} \;\; \mathcal{S}, \mathcal{T} \subset \mathcal{N} \tag{18}$$

Lemma 4.3.10 (Supermodularity)**.** *Alternatively, a value function v is supermodular if and only if for all $i \in \mathcal{N}$ and every set of coalitions $\mathcal{S} \subset \mathcal{T} \subset \mathcal{N}$ such that $\mathcal{S} \cap \{i\} = \mathcal{T} \cap \{i\} = \emptyset$, the following inequality holds:*

$$v(\mathcal{S} \cup \{i\}) - v(\mathcal{S}) \;\leq\; v(\mathcal{T} \cup \{i\}) - v(\mathcal{T}) \tag{19}$$

Generally speaking, a game is convex if a an individual's marginal contribution to a coalition increases if he joins a larger coalition.

Theorem 4.3.11 ([90])**.** *Every coalitional game has a* nonempty core *if it is* convex.

Convexity of a coalitional game is a strong condition and many real-world games are not convex. A weaker condition is balancedness of a coalitional game. In order to define a balanced coalitional game, we need to introduce the concept of a balanced map.

Definition 4.3.12 (Balanced map)**.** *A map $\alpha : 2^{\mathcal{N}} \to [0, 1]$ is said to be* balanced *if for any $i \in \mathcal{N}$,*

$$\sum_{\mathcal{S} \in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \mathbf{1}\{i \in \mathcal{S}\} = 1 \tag{20}$$

where $\mathbf{1}\{\cdot\}$ denotes the indicator function.

Thus, a balanced map provides a weight for each coalition in the game such that for each player $i \in \mathcal{N}$, the sum of the weights corresponding to all coalitions that contain the player i equals one.

Definition 4.3.13 (Balanced game)**.** *A game $(\mathcal{N}, v)$ is* balanced *if for any balanced map α,*

$$\sum_{\mathcal{S} \in 2^{\mathcal{N}}} \alpha(\mathcal{S}) v(\mathcal{S}) \;\leq\; v(\mathcal{N}). \tag{21}$$

A balanced coalitional game always has a nonempty core. In fact, [15] and [91] independently proved, using duality in linear programming, that balancedness is an equivalent condition to the existence of a nonempty core.

Theorem 4.3.14. (Bondareva-Shapley Theorem [15, 91]) *A coalitional game has a nonempty core if and only if it is balanced.*

However, not every coalitional game is balanced. For such games, alternative solution concepts have been introduced. The most important among these are the *Shapley value* and the *nucleolus*.

4.3.2 Alternative Solution Concepts

The Shapley Value

The Shapley value takes an axiomatic approach to value allocation in a coalitional game. For a coalitional game $(\mathcal{N}, v)$, the *Shapley value* $\chi_i(v)$ denotes the payoff to each player $i \in \mathcal{N}$. The Shapley value must satisfy five basic axioms.

1. *(Individual rationality)* $\chi_i(v) \geq v(\{i\})$ for all $i \in \mathcal{N}$.
2. *(Efficiency)* $\sum_{i \in \mathcal{N}} \chi_i(v) = v(\mathcal{N})$
3. *(Symmetry)* Let $\mathcal{S} \cap \{i, j\} = \emptyset$, if $v(\mathcal{S} \cup \{i\}) = v(\mathcal{S} \cup \{j\})$ then $\chi_i(v) = \chi_j(v)$.
4. *(Dummy action)* Let $\mathcal{S} \cap \{i\} = \emptyset$, if $v(\mathcal{S} \cup \{i\}) = v(\mathcal{S})$ then $\chi_i(v) = 0$.
5. *(Additivity)* If v_1 and v_2 are two value functions then $\chi_i(v_1 + v_2) = \chi_i(v_1) + \chi_i(v_2)$.

Theorem 4.3.15 ([91]). *Consider a coalitional game* $(\mathcal{N}, v)$*. An analytical expression for the corresponding* Shapley value *is given by*

$$\chi_i(v) = \sum_{\mathcal{S} \subset \mathcal{N} \setminus \{i\}} \frac{|\mathcal{S}|!(N - |\mathcal{S}| - 1)!}{N!} \left[v(\mathcal{S} \cup \{i\}) - v(\mathcal{S}) \right] \tag{22}$$

The Shapley value $\chi_i(v)$ can be interpreted as the expected marginal contribution of player i to the grand coalition $\mathcal{N}$ when it joins at a uniformly at random order. The weight is the probability that player i enters right after every player in the coalition $\mathcal{S}$.

Remark 4.3.16. *(Relation to the core)* The Shapley value always exists but is *not necessarily in the core.* If a coalitional game has a nonempty core and if in addition the imputation defined by the Shapley value lies in the core, then this imputation shares the stability properties of the core and the fairness established by the axioms of the Shapley value. As a matter of fact, for a convex game, the imputation corresponding to the Shapley value is always in the core [90]. However, this is not true, in general, for a balanced game. □

The Nucleolus

The *nucleolus* of a coalitional game $(\mathcal{N}, v)$ is an imputation that minimizes the *dissatisfaction* of the players. Let $x \in \mathbb{R}^N$ be an imputation associated with the coalitional game $(\mathcal{N}, v)$. The *dissatisfaction* of a coalition $\mathcal{S}$ with respect to the imputation x is measured by the *excess* defined as follows:

$$e(x, \mathcal{S}) = v(\mathcal{S}) - \sum_{i \in \mathcal{S}} x_i. \tag{23}$$

For a given imputation x, define the associated *excess vector*, $\theta(x) \in \mathbb{R}^{2^N-2}$, as a vector whose entries are the excesses for all coalitions (excluding the grand coalition) arranged in nonincreasing order, *i.e.*

$$\theta_i(x) \leq \theta_j(x) \text{ for all } i, j \in \mathbb{N} \text{ such that } i \geq j.$$

Let Θ denote the set of excess vectors associated with each imputation $x \in \mathcal{I}$ for a coalitional game $(\mathcal{N}, v)$.

$$\Theta = \{\theta(x) : x \in \mathcal{I}\} \tag{24}$$

Definition 4.3.17 (Lexicographic order)**.** *Define a lexicographic order on the elements of Θ as follows: $\theta(x) \leq_{lex} \theta(y)$ if there exists an index $k \in \mathbb{N}$ such that for all $i < k$, $\theta_i(x) = \theta_i(y)$ and $\theta_k(x) \leq \theta_k(y)$.*

Definition 4.3.18 (Nucleolus)**.** *The nucleolus of the game $(\mathcal{N}, v)$ is the* lexicographically minimal imputation *based on this ordering.*

Remark 4.3.19. *(Relation to the core)* The core can be easily related to the nucleolus solution concept [36]. The nucleolus always exists and is unique. Moreover, the nucleolus belongs to the core, if the core is non-empty, as the the core is the set of all imputations with negative or zero excesses. □

4.4 A Coalitional Game for Wind Energy Aggregation

Let $\mathcal{N} = \{1, \cdots, N\}$ denote the set of N independent wind power producers (WPP). As the collection of WPPs are connected to a common bus in the power network, they face common market prices and can directly aggregate their power output without regard to transmission capacity constraints. Ergo, any subset $\mathcal{S} \subseteq \mathcal{N}$ of wind power producers has the option of forming a coalition to participate in the market as a single entity. An act which amounts to one of *risk sharing*. Using the machinery of coalitional game theory, we aim to

1. *prove that independent wind power producers (WPP) can improve their expected optimal profit, in aggregate, by forming a coalition to jointly offer their aggregate power as a single entity.*
2. *identify fair sharing mechanisms to allocate the additional profit among the members of the coalition.*

We model the formation of a willing coalition among wind power producers to jointly offer a contract for energy in a two-settlement market as a *coalitional game* $(\mathcal{N}, v)$, where the *value function* $v(\mathcal{S})$ is defined as the expected profit corresponding to an optimal coalitional offer (Theorem 4.2.1) of the aggregate wind power $\mathbf{w}_{\mathcal{S}}$ associated with the coalition $\mathcal{S} \subseteq \mathcal{N}$.

$$v(\mathcal{S}) = \Psi[\mathbf{w}_{\mathcal{S}}] \;=\; \max_{C \geq 0} \;\; \mathbb{E}\, \Pi(C, \mathbf{w}_{\mathcal{S}}, q, \lambda) \tag{25}$$

In section 4.4.1, we prove that the corresponding coalitional game is *superadditive*, from which it follows that the the formation of a grand coalition $\mathcal{N}$ is optimal from the perspective of maximizing the WPPs collective expected profit. We also prove that the coalitional game is *balanced* and hence has a *nonempty core* (i.e., $\mathcal{C} \neq \emptyset$). This guarantees the existence of a *fair* payoff allocation in the core.

The challenge is to find an *imputation* $x^* \in \mathbb{R}^N$ *in the core* $\mathcal{C}$. Through counterexample, we show in section 4.4.2 that the coalitional game for wind energy aggregation is *not convex* and that the *Shapley value* does not necessarily belong to the core. Although the nucleolus belongs to the core for a balanced game, its calculation can be computationally demanding, as it requires the solution of a sequence of $o(2^N)$ linear programs [87]. As an alternative, we propose the use of a candidate imputation that minimizes the *worst-case excess* for every coalition.

Finally, as the coalitional value function v (25) is defined in the metric of optimal *expected* profit, an imputation $x^* \in \mathbb{R}^N$ belonging to the corresponding *core* $\mathcal{C}$, represents the payment that each WPP (coalition member) should receive in expectation. In practice, the realized profit will vary day by day, as the profit (7) is a random variable. Hence, given any realization of the profit, we propose, in section 4.4.3, a payoff allocation mechanism to distribute the realized profit among the coalition members, such that the payment that each member receives – averaged over an increasing number of days – approaches the imputation $x^* \in \mathcal{C}$.

4.4.1 Properties of the Coalitional Game

Theorem 4.4.1. *The coalitional game* $(\mathcal{N}, v)$ *for wind energy aggregation is superadditive, i.e.,*

$$v(\mathcal{S}) + v(\mathcal{T}) \;\leq\; v(\mathcal{S} \cup \mathcal{T}) \quad \textit{for all disjoint coalitions } \; \mathcal{S}, \mathcal{T} \subset \mathcal{N} \tag{26}$$

Proof: As the value function is defined as $v(\mathcal{S}) = \Psi[\mathbf{w}_{\mathcal{S}}]$ for all $\mathcal{S} \subseteq \mathcal{N}$, the result follows directly from the the superadditivity property of Ψ established in Lemma 4.2.4. More specifically, for any disjoint pair $\mathcal{S}, \mathcal{T} \subset \mathcal{N}$, we have

$$\begin{aligned} v(\mathcal{S}) + v(\mathcal{T}) &= \Psi[\mathbf{w}_{\mathcal{S}}] + \Psi[\mathbf{w}_{\mathcal{T}}] \\ &\leq \Psi[\mathbf{w}_{\mathcal{S}} + \mathbf{w}_{\mathcal{T}}] \\ &= v(\mathcal{S} \cup \mathcal{T}). \end{aligned}$$

■

Remark 4.4.2. (*Positively Correlated Wind Processes*) Superadditivity of the game $(\mathcal{N}, v)$ guarantees that coalition formation will never detract from the members' expected profit in aggregate. In the worst case, the coalition optimal expected profit equals the sum of the individuals' optimal expected profits if they were to participate in the market independently.

More specifically, consider two disjoint coalitions $\mathcal{S}$ and $\mathcal{T}$ whose corresponding aggregate wind power processes $\mathbf{w}_{\mathcal{S}}$ and $\mathbf{w}_{\mathcal{T}}$ are *perfectly positively correlated* on the time interval of interest $[t_0, t_f]$. Given such characteristics, it is straightforward to show that the individual coalitions $\mathcal{S}$ and $\mathcal{T}$ have no incentive to form the larger coalition $\mathcal{S} \cup \mathcal{T}$, as $v(\mathcal{S}) + v(\mathcal{T}) \ = \ v(\mathcal{S} \cup \mathcal{T})$. □

Theorem 4.4.1 demonstrates that wind power producers can improve their expected profit by forming coalitions with other producers to jointly offer a contract for their aggregate power. Moreover, the larger the coalition the greater the improvement in the aggregate expected profit – indicating that the most profitable coalition is the *grand coalition*. Superadditivity, however, does not guarantee the existence of a payoff allocation that is satisfactory (fair) from the perspective of every member. Consequently, there may exist an opportunity for certain members to increase their expected profit by defecting from the grand coalition $\mathcal{N}$ to form a smaller one $\mathcal{S} \subset \mathcal{N}$. As outlined in Section 4.3, the core is the set of all imputations that disincentivize the defection of any member from the grand coalition. As indicated in Theorem 4.3.14, balancedness of the game is equivalent to the existence of at least a single imputation in the core.

In Theorem 4.4.3 we prove that the coalitional game $(\mathcal{N}, v)$ has a nonempty core. Homogeneity and superadditivity of the map Ψ, as proven in Lemma 4.2.4, are instrumental in the proof of this theorem.

Theorem 4.4.3. *The coalitional game $(\mathcal{N}, v)$ for wind energy aggregation is* balanced *and therefore has a* nonempty core*.*

Proof: Let $\alpha : 2^{\mathcal{N}} \to [0, 1]$ be an arbitrary balanced map from the space of all coalitions $\mathcal{S} \subseteq \mathcal{N}$ to the unit interval $[0, 1]$ (see (20)). Let $\mathcal{S} \subseteq \mathcal{N}$ be an arbitrary coalition of the game $(\mathcal{N}, v)$. Balancedness of the game is proven by applying the *positive homogeneity* and *supperadditivity* properties of the function Ψ – established in Lemma 4.2.4.

$$
\begin{aligned}
\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})v(\mathcal{S}) &= \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\Psi[\mathbf{w}_{\mathcal{S}}] \\
&= \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \Psi[\alpha(\mathcal{S})\mathbf{w}_{\mathcal{S}}], \quad \text{by positive homogeneity of } \Psi \\
&\leq \Psi\left[\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\mathbf{w}_{\mathcal{S}}\right], \quad \text{by superadditivity of } \Psi \\
&= \Psi\left[\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \sum_{i\in\mathcal{S}} \mathbf{w}_i\right] \\
&= \Psi\left[\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \sum_{i\in\mathcal{N}} \mathbf{1}\{i\in\mathcal{S}\}\mathbf{w}_i\right] \\
&= \Psi\left[\sum_{i\in\mathcal{N}} \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\mathbf{1}\{i\in\mathcal{S}\}\mathbf{w}_i\right] \\
&= \Psi\left[\sum_{i\in\mathcal{N}} \mathbf{w}_i\right], \quad \text{by balancedness of } \alpha \\
&= v(\mathcal{N})
\end{aligned}
$$

and this proves balancedness of the game $(\mathcal{N}, v)$. ■

4.4.2 Sharing of Expected Coalition Profit

As the coalitional game for wind energy aggregation has a nonempty core, there exists an imputation in the core that guarantees that no wind power producer can improve its expected profit by defecting from the grand coalition.

The Shapley Value Is Not in the Core

For convex games, the Shapley value provides a closed-form expression for an imputation that belongs to the core. It can be shown through counterexample, however, that our class of coalitional games is not convex and that the Shapley value does not necessarily specify an imputation belonging to the core.

Example 4.4.4 (Counterexample)**.** Consider a coalitional game involving three independent wind power producers, $\mathcal{N} = \{1, 2, 3\}$, offering contracts on the time interval $[t_0, t_f]$ of length one hour. Each wind power process $\mathbf{w}_i$ ($i = 1, 2, 3$) is assumed to be *stationary in the strict sense* with discrete marginal distributions. The wind power processes $\mathbf{w}_1$ and $\mathbf{w}_2$

are assumed to be *independent* and have *identical marginal distributions* defined by

$$w_i(t) = \begin{cases} 1, & w.p. \quad 0.5 \\ 2, & w.p. \quad 0.5 \end{cases} \qquad i = 1, 2 \quad \text{for all } \ t.$$

The wind power process $\mathbf{w}_3$ is assumed to be *perfectly positively correlated* to $\mathbf{w}_2$, i.e.,

$$w_3(t) = w_2(t) \quad \text{for all } \ t.$$

The forward market clearing price and expected imbalance prices are set at ($p = 0.5, \mu_q = 1, \mu_\lambda = 0$) with units (\$/MWh).

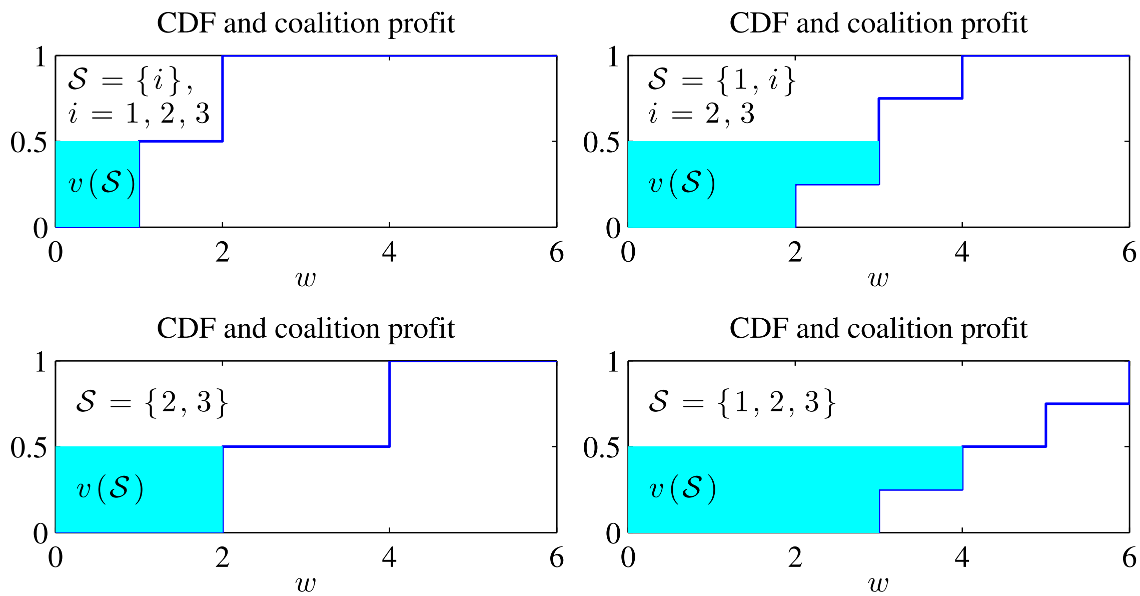


Figure 4.1. This figure depicts the time-averaged cumulative distribution function $\Phi_{\mathcal{S}}(w)$ and value $v(\mathcal{S})$ associated with each coalition $\mathcal{S}$ in the power set of $\mathcal{N}$. The shaded blue area depicts the value $v(\mathcal{S})$ in units of (\$) for each coalition.

Consider the coalitional game $(\mathcal{N}, v)$. The time-averaged cumulative distribution function $F_{\mathcal{S}}(w)$ and value $v(\mathcal{S})$ associated with each coalition $\mathcal{S} \subseteq \mathcal{N}$ are depicted in Figure 4.1. The shaded blue area depicts the value $v(\mathcal{S})$ in units of (\$) for each coalition. The numerical values are given by the following:

$$\begin{aligned} v(\{i\}) &= \Psi[w_i] = 0.5, \ i \in \{1, 2, 3\} \\ v(\{1, i\}) &= \Psi[w_1 + w_i] = 1.25, \ i \in \{2, 3\} \\ v(\{2, 3\}) &= \Psi[w_2 + w_3] = 2v(\{2\}) = 1 \\ v(\{1, 2, 3\}) &= \Psi[w_1 + w_2 + w_3] = 1.75 \end{aligned}$$

As indicated in Theorem 4.4.3, this coalitional game is balanced and, consequently, has a nonempty core. However, this game is *not convex*, as the *value function is not supermodular*. Take for example,

$$\begin{aligned} v(\{1,2,3\}) - v(\{1,2\}) &= 0.50 \\ &< v(\{1,3\}) - v(\{1\}) = 0.75, \end{aligned}$$

which contradicts the supermodularity property defined in equation (19). Hence, the imputation given by the Shapley value $\chi(v)$ does not necessarily lie in the core of the game – as indicated in Section 4.3.2.

We now show that the imputation given by the Shapley value is *not in the core.* An imputation $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ is in the core if it satisfies the following conditions, as defined by equation (17).

$$x_i \geq v(\{i\}) = 0.5, \ i \in \{1,2,3\} \tag{27}$$

$$x_1 + x_i \geq v(\{1,i\}) = 1.25, \ i \in \{2,3\} \tag{28}$$

$$x_2 + x_3 \geq v(\{2,3\}) = 1.0 \tag{29}$$

$$x_1 + x_2 + x_3 = v(\{1,2,3\}) = 1.75 \tag{30}$$

The imputation given by the Shapley value can be easily computed using the closed form expression in equation (22):

$$\chi_1(v) = \frac{2}{3}, \quad \chi_2(v) = \frac{1.625}{3}, \quad \chi_3(v) = \frac{1.625}{3}$$

It is straightforward to see that the Shapley value violates condition (28).

$$\chi_1(v) + \chi_2(v) = \frac{3.625}{3} = 1.2083 < 1.25 = v(\{1,2\})$$

Hence, *the imputation given by the Shapley value is not in the core* for this particular game. ■

The Nucleolus and Minimizing Worst-Case Excess

With respect to the coalitional game for wind energy aggregation, the previous counterexample 4.4.4 proves that the game *not convex* and, consequently, the imputation given by the Shapley value is not guaranteed to belong to the core. The strength in application of the Shapley value resides in its closed form characterization – providing computational efficiency. However, as the Shapley value for a non-convex game is not guaranteed to belong to the core, one must seek alternative solution concepts to extract imputations from the core.

An alternative solution concept to the Shapley value is given by the *nucleolus*, which is guaranteed to belong to the core for a balanced game. However, as the nucleolus is defined as the imputation with the lexicographically minimal excess vector, its computation requires the solution of a sequence of $o(2^N)$ linear programs [87]. This can be computationally demanding. To surmount this difficulty we propose the use of a candidate imputation that *minimizes the worst-case excess* for every coalition. This imputation is defined as follows:

$$e^* \;=\; \min_{x\in\mathbb{R}^N}\max_{\mathcal{S}\in 2^{\mathcal{N}}} e(x,\mathcal{S}) \quad \text{subject to} \quad \begin{cases} e(x,\mathcal{N}) \;=\; 0 \\ v(\{i\}) - x_i \;\leq\; 0 \quad \text{for all } \; i\in\mathcal{N} \end{cases} \tag{31}$$

In contrast to the nucleolus solution concept, computation of the imputation that minimizes the worst-case excess can be recast as a *single linear program*:

$$e^* \;=\; \min_{x\in\mathbb{R}^N,\, e\in\mathbb{R}} e \quad \text{subject to} \quad \begin{cases} v(\mathcal{S}) - \sum_{i\in\mathcal{S}} x_i - e \;\leq\; 0, & \text{for all } \; \mathcal{S}\subset\mathcal{N} \\ v(\mathcal{N}) - \sum_{i\in\mathcal{N}} x_i = 0 & \\ v(\{i\}) - x_i \;\leq\; 0, & \text{for all } \; i\in\mathcal{N} \end{cases} \tag{32}$$

Although the imputation that minimizes the worst-case excess in problem (32) is not guaranteed to belong to the core, it is a simple matter to check feasibility with respect to the core.

Lemma 4.4.5. *A feasible imputation x^* achieving the minimal cost e^* in problem (32) belongs to the core if $e^* \leq 0$.*

Proof: It is clear to see that a feasible imputation x^* achieving the minimal cost e^* is both individually rational and budget balanced. Moreover, if $e^* \leq 0$, we have that

$$v(\mathcal{S}) - \sum_{i\in\mathcal{S}} x_i^* \;\leq\; e^* \;\leq\; 0 \quad \text{for all } \; \mathcal{S}\subset\mathcal{N}$$

which guarantees that no member has any incentive to defect from the grand coalition. ■

The following example depicts an instance where this imputation that minimizes worst-case excess belongs to the core and the Shapley value does not.

Example 4.4.6. Consider again the coalitional game corresponding to the Example 4.4.4 in Section 4.4. Recall that the Shapley value of this game does not belong to the core. However, since the coalitional game is balanced, it has a nonempty core. Using problem formulation (32), we can solve a linear program (LP) to compute an imputation that minimizes the worst-case excess for any possible coalition in the game. Such an imputation is computed by solving the following LP corresponding to our game.

$$\begin{array}{ll} \text{Minimize} & e \\ \text{subject to} & e + x_i - 0.5 \geq 0, \; i\in\{1,2,3\} \\ & e + x_1 + x_i - 1.25 \geq 0, \; i\in\{2,3\} \\ & e + x_2 + x_3 - 1.0 \geq 0 \\ & x_1 + x_2 + x_3 = 1.75 \\ & x_i - 0.5 \geq 0, \; i\in\{1,2,3\} \end{array}$$

The minimal cost e^* and corresponding imputation x^* are given by

$$e^* = 0, \qquad x_1^* = 0.75, \;\; x_2^* = 0.5, \;\; x_3^* = 0.5.$$

Moreover, in contrast to the Shapley value for this game, the imputation x^* *belongs to the core* as $e^* = 0$. ■

4.4.3 Sharing of Realized Coalition Profit

We have thus far focused our attention on the computation of payoff allocations that fairly distribute the *optimal expected profit* among coalition members. This approach stems from our formulation of the coalitional game $(\mathcal{N}, v)$ as having a value function v defined in the metric of optimal *expected* profit,

$$v(\mathcal{S}) \; = \; \max_{C \geq 0} \;\; \mathbb{E}\, \Pi(C, \mathbf{w}_{\mathcal{S}}, q, \lambda) \quad \text{for all } \; \mathcal{S} \subseteq \mathcal{N}.$$

Consequently, an imputation $x^* \in \mathbb{R}^N$ belonging to the corresponding *core* $\mathcal{C}$, represents the payment that each WPP (coalition member) should receive in expectation. In practice, however, the realized profit for the grand coalition will vary day to day, as the profit (7) is inherently a random variable given its dependence on the random wind power process $\mathbf{w}_{\mathcal{S}}$ and imbalance prices (q, λ). A natural question thus arises:

Does there exist a profit allocation mechanism to distribute the realized profit *among the coalition members, such that the payment that each member receives – averaged over an increasing number of days – approaches the imputation $x^* \in \mathcal{C}$?* The answer is yes.

Assumptions

A6 We assume that the wind power process $\mathbf{w}_{\mathcal{S}}^k$ (for all $\mathcal{S} \subseteq \mathcal{N}$) and imbalance prices (q^k, λ^k) are independent and identically distributed (*iid*) across days indexed by k.

$$w_{\mathcal{S}}^k(t) \perp\!\!\!\perp w_{\mathcal{S}}^j(t) \quad \text{for all } \; t \in [t_0, t_f] \tag{33}$$

$$q^k \perp\!\!\!\perp q^j \tag{34}$$

$$\lambda^k \perp\!\!\!\perp \lambda^j \tag{35}$$

for all days $k \neq j$.

It follows that the optimal profit (36), corresponding to any coalition $\mathcal{S} \subseteq \mathcal{N}$, is likewise an *iid* sequence $\left\{\Pi_{\mathcal{S}}^k\right\}$ across days.

$$\Pi_{\mathcal{S}}^k := \Pi(C_{\mathcal{S}}^*, \mathbf{w}_{\mathcal{S}}^k, q^k, \lambda^k), \qquad \text{where} \quad C_{\mathcal{S}}^* = F_{\mathcal{S}}^{-1}(\gamma) \tag{36}$$

Remark 4.4.7. (*Cyclostationarity*) The assumption of distribution stationarity, across days, is motivated by the empirical observation of strong diurnal periodicity in the underlying wind speed and price processes [43, 99]. □

Remark 4.4.8. (*Negative Profit Realization*) Whereas the expected optimal profit is guaranteed to be nonnegative, it is important to note that realized optimal profit can take on negative values. Consequently, there may occur a day such that certain members of the coalition have to *pay* for their contribution to the cost of contract imbalance. □

A Consistent Approach to Daily Profit Allocation

Denote the allocation of the profit realized on day k by

$$\varrho^k = \begin{bmatrix} \varrho_1^k & \cdots & \varrho_N^k \end{bmatrix}^T \in \mathbb{R}^N,$$

where coalition member i receives ϱ_i^k of the realized profit on day k.

Definition 4.4.9 (Budget Balanced)**.** *A profit allocation $\varrho^k \in \mathbb{R}^N$ is* budget balanced *with respect to the profit realized on day k if*

$$\sum_{i=1}^{N} \varrho_i^k \;=\; \Pi_{\mathcal{N}}^k$$

Definition 4.4.10 (Consistency)**.** *A mechanism for daily profit allocation ϱ^k is (strongly)* consistent *with respect to a fixed allocation $x \in \mathbb{R}^N$ if*

$$T_i(K) := \frac{1}{K} \sum_{k=1}^{K} \varrho_i^k \tag{37}$$

converges (almost surely) in probability to x_i as $K \longrightarrow \infty$.

Consider the following naïve mechanism for daily profit allocation. Let $x^* \in \mathbb{R}^N$ be an imputation in the core $\mathcal{C}$ for the coalitional game defined by the value function (25). Given a realization of profit $\Pi_{\mathcal{N}}^k$ on day k for the grand coalition $\mathcal{N}$, *distribute the profit among the coalition members according to the following rule:*

$$\varrho_i^k \;=\; \beta_i \, \Pi_{\mathcal{N}}^k, \qquad \text{where} \quad \beta_i = \frac{x_i^*}{\sum_{j=1}^N x_j^*} \tag{38}$$

Theorem 4.4.11. *The naïve profit allocation mechanism (38) is both* budget balanced *and* strongly consistent *with respect to the corresponding imputation $x^* \in \mathcal{C}$ on which it is based.*

Proof: Budget balancedness follows directly from $\sum_i \beta_i = 1$. Strong consistency follows directly from the strong law of large numbers. ■

Remark 4.4.12. (*Defection in the Short Run*) The sharing of the realized coalition profit in such a manner as (38) – although fair in the long run – may lead to the defection of certain coalition members in the short run if said members consistently receive payments that are below that which would have been attainable through independent participation in the market, i.e., if the event

$$\varrho_i^k < \Pi(C_i^*, \mathbf{w}_i^k, q^k, \lambda^k) \tag{39}$$

occurs with a sufficiently high frequency.

We are currently exploring alternative formulations of the coalitional game to disincentive defection of coalition members in the short run. For example, consider a formulation where the value function is defined as the realized optimal profit (40), rather than the expected optimal profit (25).

$$v(\mathcal{S}) \; = \; \Pi(C_{\mathcal{S}}^*, \mathbf{w}_{\mathcal{S}}, q, \lambda), \quad \text{for all} \quad \mathcal{S} \subseteq \mathcal{N} \tag{40}$$

Working with such a stochastic formulation of the coalition game $(\mathcal{N}, v)$, one can directly compute *fair* profit allocations explicitly as a function of the realized wind power production and imbalance prices. Moreover, assuming the existence of a nonempty core for such a game, a daily payoff allocation given by

$$\varrho^k = x^{*,k},$$

where $x^{*,k}$ is an imputation in the core associated with day k, would guarantee that event (39) never occurs – among other beneficial properties. □

4.5 Discussion

Motivated by the inherent benefits of risk sharing and reduction in output variability achievable through aggregation, we have explored the problem of wind power aggregation in the setting of competitive two-settlement electricity markets. Using coalitional game theory as a vehicle for our analysis, we have analyzed the benefits of aggregation attainable through the formation of a willing coalition among wind power producers (WPP) to pool their variable power to jointly offer the aggregate output as a single entity into a forward energy market. As coalitional bidding necessarily leads to an increase in optimal expected profit beyond that achievable through individual market participation, we have attempted characterize payoff allocation mechanisms to *fairly* distribute the profit among WPPs participating in the coalition.

Having assumed transferable payoff and a value function defined as the maximum expected profit attainable through competitive bidding, we have shown that the associated coalitional game is *superadditive* and *balanced*. Consequently, the *core* of such a game is necessarily *nonempty* – or, more simply, there exists a *fair* profit sharing rule that is satisfactory from the perspective of every coalition participant. To this end, we propose a sharing rule – that minimizes worst-case excess for each coalition in the game – to fairly allocate the expected profit among coalition members.

Our results demonstrate that wind power aggregation and coalitional bidding can serve as an effective means for improving wind power profitability in the face of future production uncertainty. However, our results are limited to the setting in which all WPPs are connected to a common single bus in the network. As the transmission network can severely constrain a coalitions ability to directly aggregate wind power generated at different buses, we are presently working on extensions of these results to the multi-bus network setting to account for transmission effects.

CHAPTER 5

The Role of Co-located Energy Storage

5.1 Introduction

Motivated by the dangers posed by global warming, there is great interest in renewable energy sources. Electric energy is the dominant form of energy consumption (accounting for more than 50 % in the US). Wind and solar energy are expected to become a much larger source of electric energy to meet the renewable energy production targets in many parts of the world [46]. These sources of electricity production are inherently uncertain, variable, and largely uncontrollable. Together, these characteristics constitute major challenges to the integration of these clean energy sources into the electric grid at deep penetration levels [40, 45, 73].

There is a considerable investment and interest in energy storage as a means to deal with the inherent variability of renewable generation resources such as wind and solar. [27, 29, 30, 33, 34, 48, 55, 58, 75, 96]. Indeed, hydro-power has traditionally been used for such purposes [25, 70]. Although pumped hydro represents an efficient and flexible storage modality, resources of this nature are geographically constrained and are consequently of limited capacity to the bulk system. Consequently, as utility-scale renewable energy continues to proliferate on the transmission system, we will witness an increased need for alternative storage modalities (e.g., compressed air, battery, flywheel) that can be deployed near regions of high wind and solar power density to manage the corresponding output variability in order to minimize the *quantity risk exposure* to the independent system operator (ISO).

We again consider the the setting in which a wind power producer (WPP) must offer its *variable power* in a two-settlement market system (see Chapter 2). In this chapter, we extend the results in Chapter 3 by exploring the extent to which *co-located energy storage* can be used to mitigate the inherent financial risk associated with contract imbalances emanating from fluctuations in wind power output. Further, as the capital cost of electrical energy storage can be quite large, we derive analytical expressions that quantify the marginal value of energy storage capacity in the metric of expected profit. Such results provide a mechanism for empirical calculation of return on investment.

5.2 Related Work and Contributions

Energy storage devices such as pumped-hydro, compressed air [55], sodium-sulfur batteries [78], etc. offer the capability to *firm* wind generation power supply through joint optimal dispatch of the storage device in conjunction with the variable resource power output. Cavallo [26] wrote one of the earliest papers to make the case for joint operation of wind energy and storage (see [27] for a utility scale investigation of compressed air energy storage with wind). Greenblatt et al. [48] compared gas turbines and compressed air energy storage in the context of wind as part of baseload electricity generation (see also [96] for a detailed report on CAES and wind energy). Denholm and Sioshansi [31] have studied energy storage and wind power in a transmission constrained system where they compare the economics of siting the storage system near the wind power generation site versus the load site. Electric and plug-in hybrid vehicles also represent potential distributed energy storage devices. Economic viability of compressed air energy storage (CAES) in a wind energy system in Denmark has recently been investigated in [61]. They also investigate the operation of this joint storage-wind energy system in the Nordic spot and regulatory energy markets. They conclude that the economic viability of such a system depends on the monthly payments from the regulation power market. A recent paper by DeCesaro and Porter [29] presents a summary of most wind integration studies to date.

Recently, Angarita et al. [2] have investigated combined wind-hydro bids in an electricity pool market. They formulate a stochastic programming problem that accounts for uncertainty in the wind availability and prices of electricity in various markets. Using a "scenario based" approach to dealing with the uncertainty, they develop a linear programming solution which yields optimal offer curves and limits the risk of profit variability. Our work is related to this investigation, but employs analytical methods to yield computably optimal solutions.

In this chapter, we focus on the scenario in which wind power producers (WPP) must sell their energy using contract mechanisms in *conventional* forward electricity markets. Our goal is to formulate and solve problems of optimal contract sizing for such wind power producers with dedicated co-located electric energy storage capacity. We explore the impact of optimal storage operation on contract sizing and profit. We start with a simple stochastic model for wind power production and a model for the electricity market. We show that the problem of determining optimal contract offerings for WPP with co-located energy storage reduces to convex programming. We also show that the expected profit acquired by the wind power producer for optimal contract offerings is concave, non-decreasing in the parameter of energy storage capacity – revealing that greatest marginal benefit from energy storage is derived for a *small* amount of storage capacity. In fact, we show that the marginal optimal expected profit with respect to the energy storage capacity can be analytically computed for *small* capacities – an expression that is closely related to the spectral properties of the underlying wind process [11, 13].

As storage is currently quite expensive and the level of penetration of renewable energy is not very high, storage is not considered to be necessary for integrating wind into the electric grid for the 20% penetration levels ([40], p.229). A full analysis of the economics of storage in the context of renewable integration can be found in [30]. We note that there are large investments in new technologies for energy storage research and development [75]. The Solar Energy Grid Integration Systems-Energy Storage (SEGIS-ES) project [**?**] represents a recent comprehensive effort along this direction in the U.S. A recent review of battery based storage technologies can be found in [33]. In their report, Denholm et al conclude that

> "It is clear that high penetration of variable generation (VG) increases the need for all flexibility options including storage, and it also creates market opportunities for these technologies. Evaluating the role of storage with VG sources requires continued analysis, improved data, and new techniques to evaluate the operation of a more dynamic and intelligent grid of the future."

While we present our results in the context of joint optimization of wind and storage, we believe they can be generalized and extended to the situation of joint optimization of storage with the composite generation+load uncertainty in the grid.

5.3 Problem Formulation

5.3.1 Wind Power Model

For reasons of technical clarity in exposition, we take a slight detour from the continuous time model presented in Chapter 3 and model wind power production as a *discrete time* random process

$$\mathbf{w} = \{w_n \mid n \in \mathbb{N}\}. \tag{1}$$

The proceeding characterization of the stochastic process is analogous to that of Chapter 3, Section 3.3.1, but is presented here for completeness. For a fixed $n \in \mathbb{N}$, w_n is a continuous random variable whose cumulative distribution function (CDF) is assumed known and defined as

$$\Phi(w; n) = \mathbb{P}\{w_n \leq w\}. \tag{2}$$

The random process $\mathbf{w}$ takes on values in the unit interval $[0, 1]$, as wind power output is assumed to be normalized by the farm's nameplate capacity. Note that the

In the following results, we will be interested in *time-averaged* distributions defined on integer intervals of length $N \in \mathbb{N}$. For example, the time-averaged CDF on the integer interval $\{1, \cdots, N\}$ is defined as

$$F(w) = \frac{1}{N} \sum_{n=1}^{N} \Phi(w; n) \tag{3}$$

Also, define $F^{-1} : [0, 1] \to [0, 1]$ as the *quantile function* corresponding to the CDF F. More precisely, for $\beta \in [0, 1]$, the β-*quantile* of F is given by

$$F^{-1}(\beta) = \inf\{x \in [0,1] \,:\, \beta \leq F(x)\} \tag{4}$$

The quantile function corresponding to the time-averaged CDF will play a central role in our results.

5.3.2 Energy Storage Model

Consider the following linear difference equation as a dynamic model for a generic energy storage system [58].

$$e_{n+1} = (1+\alpha h)e_n + h\left[\eta_{\text{inj}} P_{n,\text{inj}} - \frac{1}{\eta_{\text{ext}}} P_{n,\text{ext}}\right] \tag{5}$$

subject to the following constraints

$$0 \leq \quad e_n \quad \leq \overline{e} \tag{6}$$

$$0 \leq \quad P_{n,\text{inj}} \quad \leq \overline{P}_{\text{inj}} \tag{7}$$

$$0 \leq \quad P_{n,\text{ext}} \quad \leq \overline{P}_{\text{ext}} \tag{8}$$

The energy contained in the storage system at time n is denoted by e_n. The magnitude of the power extracted (injected) from (into) the storage system at time n is denoted by $P_{n,\text{ext}}$ ($P_{n,\text{inj}}$). The parameter $\alpha \leq 0$ is the dissipation coefficient on the stored energy, while $\eta_{\text{inj}}, \eta_{\text{ext}} \in [0,1]$ model power injection and extraction efficiencies, respectively. As this difference equation is derived from a first order ODE, the discretization step size is denoted by h. Note that h should be chosen such that $|1+\alpha h| < 1$ for numerical stability considerations.

5.3.3 Market Model and Metrics

Market Description

We assume that the wind power producer (WPP) is participating in a *competitive two-settlement market system* operated as a power exchange. See Chapter 2, Section 2.3 for a detailed description of such markets. Generally, the two-settlement system consists of two *ex-ante* markets (a day-ahead (DA) forward market and a real-time (RT) spot market) and an *ex-post* imbalance settlement mechanism to penalize uninstructed deviations from contracts scheduled ex-ante. Negative deviations are charged at a price $q \in \mathbb{R}$ (\$/MWh) and positive deviations are charged at price $\lambda \in \mathbb{R}$ (\$/MWh).

This pricing scheme for penalizing contract deviations reflects the energy imbalance of the control area as a whole and the spot price of balancing energy in the RT market. Hence, the imbalance prices (q, λ) are assumed unknown during the DA forward market and are not revealed until the RT spot market, on which they are based, is cleared.

Market Model

We employ a market model that consists of a *single* ex-ante DA forward market with an ex-post financial penalty for deviations from offered contracts. In the DA market, generators offer a portfolio of M time-ordered contracts for the delivery of power the following day. The *contract portfolio* $\mathbf{C} \in \mathbb{R}_+^M$ is structured as a sequence of M power levels that are piecewise constant on intervals, typically, of length one-hour.

$$\mathbf{C} = \left[\ C^{(1)}, \cdots, C^{(M)}\ \right]$$

The time interval corresponding to contract $C^{(m)}$ is defined as the integer interval

$$\mathcal{N}_m = \{N(m-1)+1,\ \cdots,\ Nm\} \tag{9}$$

where $|\mathcal{N}_m| = N$. It follows naturally that the contract value C_n at time n is given by

$$C_n = \sum_{m=1}^{M} \mathbf{1}\{n \in \mathcal{N}_m\} C^{(m)} \tag{10}$$

where $\mathbf{1}\{\cdot\}$ is defined to be the indicator function. See Figure 5.1 for an example of a contract portfolio $\mathbf{C}$ (ex: $M = 24$) offered in a day-ahead forward market.

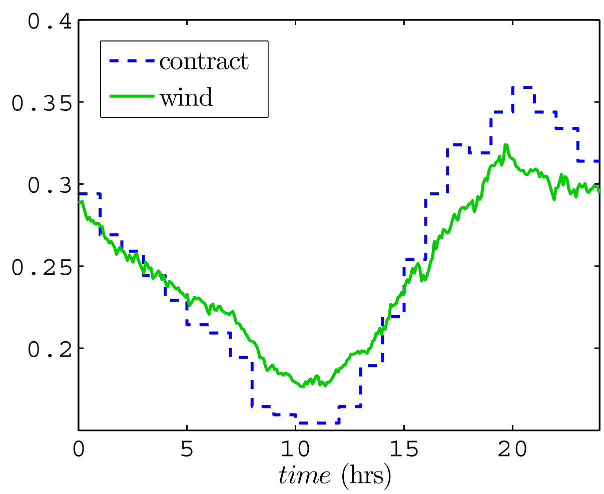


Figure 5.1. Illustrative example of typical contract portfolio (dashed) offered ex-ante in day-ahead (DA) market. Contract intervals are of length one-hour. The wind power producer is subject to financial penalties for generation shortfalls realized ex-post – i.e. when the wind power (solid) dips below the offered contract (dashed).

The WPP receives a price p (\$/MW-hour) for each offered contract $C^{(m)}$. As the power contracts are offered ex-ante, deviations naturally occur between the offered contracts and the realized wind power output. For uninstructed deviations from said contract portfolio $\mathbf{C}$, the WPP is penalized ex-post according to the imbalance prices (q, λ) (\$/MW-hour).

p : clearing price in the DA forward market

q : ex-post settlement price for negative imbalance ($w_n \le C^{(m)}$) for all $m = 1, \cdots, M$

λ : ex-post settlement price for positive imbalance ($w_n > C^{(m)}$) for all $m = 1, \cdots, M$

We make the following *assumptions* regarding prices and production costs:

A1 The WPP is assumed to be a *price taker* in the forward market, as the individual WPP capacity is assumed small relative to the whole market. As such, the forward settlement price p is assumed *fixed* and *known*.

A2 The WPP is assumed to have a *zero marginal cost of production.*

A3 As imbalance prices $(q, \lambda) \in \mathbb{R}^2$ are a function of the realized price of balancing energy in the RT spot market, they are modeled as unknown *random variables* at the time of contract offering in the DA forward market. Their expectations are denoted by

$$\mu_q = \mathbb{E}[\ q\ ]$$
$$\mu_\lambda = \mathbb{E}[\ \lambda\ ]$$

A4 The imbalance prices (q, λ) are assumed to be statistically independent of the wind $w(t)$.

A5 The imbalance prices are assumed to be *nonnegative*, i.e., it is never profitable to deviate from offered contracts.

$$(q, \lambda) \in \mathbb{R}^2_+ \quad \text{with probability one}$$

A6 Finally, the imbalance prices are assumed to be *revealed* shortly before the contract delivery period. Consequently, any recourse action taken during the delivery interval is allowed to depend explicitly on (q, λ).

Metrics

For a given contract portfolio $\mathbf{C}$, the *profit* acquired by the WPP on the interval $\{1, \cdots, NM\}$ is defined as

$$\Pi(\mathbf{C}, \mathbf{w}, q, \lambda) \ = \ h \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} pC^{(m)} \ - \ q \left[C^{(m)} - w_n\right]^+ \ - \ \lambda \left[w_n - C^{(m)}\right]^+ \tag{11}$$

where $x^+ := \max\{x, 0\}$ for all $x \in \mathbb{R}$ and h is the discretization time step. As wind power is modeled as a random process $\mathbf{w}$, we will be concerned with the *expected* profit $J(\mathbf{C})$:

$$J(\mathbf{C}) = \mathbb{E}\,\Pi(\mathbf{C}, \mathbf{w}, q, \lambda) \tag{12}$$

Here, the expectation is taken with respect to the random wind power process $\mathbf{w} = \{w_n \mid n \in \mathbb{N}\}$ and the imbalance prices (q, λ).

Remark 5.3.1. *(Storage)* The introduction of energy storage will manifest in an augmented profit model, because the WPP will have recourse capability to mitigate contract imbalances by drawing on stored energy. This scenario will analyzed in Section 5.5.1. □

5.4 Contract Sizing without Energy Storage

We begin by defining a profit maximizing portfolio $\mathbf{C}^*$ as

$$\mathbf{C}^* = \arg\max_{\mathbf{C} \in \mathbb{R}_+^M} J(\mathbf{C}). \tag{13}$$

In the absence of any energy storage capability, the opportunities for energy arbitrage between contract intervals evaporate and the decision of how much constant power to offer on interval i is independent of the decision on interval j for all $i \neq j$. Hence, the portfolio optimization (13) decouples into M independent optimization problems:

$$C^{(m)*} = \arg\max_{C \in \mathbb{R}_+} J(C) \qquad m = 1, \cdots, M \tag{14}$$

The formulation in (14) has been carefully studied in Chapter 3 and [12] and is closely related to the *newsvendor* problem in operations research [67, 83]. The main result is presented here.

Theorem 5.4.1. *[12] Define the time-averaged distribution*

$$F_m(w) = \frac{1}{N} \sum_{n \in \mathcal{N}_m} \Phi(w; n)$$

An optimal contract $C^{(m)*}$ *is given by*

$$C^{(m)*} = \begin{cases} F_m^{-1}(\gamma), & \mu_q \geq p \\ 1, & \mu_q < p \end{cases} \qquad \textit{where } \gamma = \frac{p + \mu_\lambda}{\mu_q + \mu_\lambda}. \tag{15}$$

Remark 5.4.2. Properties of the optimal quantile rule (15), such as uniqueness, price elasticity of supply, and the effect penalty pricing are explored in detail in Chapter 3. □

5.5 Contract Sizing with Energy Storage

5.5.1 Energy Storage Formulation

As wind energy penetration levels increase, energy storage will play a more dominant role in facilitating the firming of wind power contracts in conventional electricity markets. We now consider the scenario in which the wind power producer (WPP) has a co-located energy storage device at its disposal. As the capital cost of an energy storage system is quite prohibitive, a fundamental question in this context arises: what impact does energy storage capacity have on expected profit and what are optimal contract offerings in this context? We now formalize these questions as a constrained stochastic optimal control problem.

Recall section 5.3.2 and consider the linear difference equation (5) and constraints (6) - (8) as a model for the energy storage system. The energy storage system interfaces with the wind power producer through the power injection (extraction) variables $P_{n,\text{inj}}(P_{n,\text{ext}})$.

Information Structure of Storage Operation Policy

Define the storage decision vector $u_n = [P_{n,\text{ext}},\ P_{n,\text{inj}}]^{\text{T}}$. Recall that, at the outset of the contract delivery interval, we assume that the stochastic imbalance prices (q, λ) are realized and observed by the WPP. Moreover, at each time n during the contract delivery period we additionally assume that storage e_n and wind w_n states are completely observed. For a particular time n, all of the information from the past relevant to the future is contained in the current storage state e_n and all past observed wind power realizations $w^n := \{w_i \mid i = 1, \cdots, n\}$. Hence, we consider storage operation policies of the form

$$u_n = g_n\left(e_n, w^n, q, \lambda\right) = \begin{bmatrix} P_{n,\text{ext}} \\ P_{n,\text{inj}} \end{bmatrix} \tag{16}$$

where g_n is constrained to belong to the set of *feasible* operation policies guaranteeing that constraints (5) - (8) are satisfied. Let $g := \{g_n \mid n = 1, \cdots, NM\}$ and let $\mathcal{G}$ denote the set of all feasible operation policies g.

Expected Profit Criterion

The expected profit corresponding to a particular operational policy and contract portfolio $(g, \mathbf{C})$ is defined as

$$J(g, \mathbf{C}) = \tag{17}$$

$$\mathbb{E}\left[h \sum_{m=1}^{M} \sum_{n\in\mathcal{N}_m} pC^{(m)} - q\left[C^{(m)} - w_n + P^g_{n,\text{in}} - P^g_{n,\text{ext}}\right]^+ - \lambda\left[w_n - P^g_{n,\text{in}} + P^g_{n,\text{ext}} - C^{(m)}\right]^+\right]$$

where expectation is taken with respect to the random wind power process $\mathbf{w}$ and imbalance prices (q, λ). The superscript g is included to indicate the dependence on the control policy g. A profit maximizing storage operation policy and contract portfolio $(g^*, \mathbf{C}^*)$ are given by

$$(g^*, \mathbf{C}^*) = \begin{array}{l} \arg\max_{\substack{\mathbf{C} \geq 0 \\ g \in \mathcal{G}}} \; J(g, \mathbf{C}) \\ \text{subject to } (5) - (8) \end{array} \tag{18}$$

In the proceeding sections (5.5.2) - (5.5.3), we explore various properties of this optimal contract sizing problem for a WPP with co-located storage.

Optimal Storage Operational Policy

For a given contract portfolio $\mathbf{C} \in \mathbb{R}_+^M$, it is straightforward to show that a *greedy* storage operational strategy belongs to the class of feasible optimal policies. The *intuition* is as follows.

- As the surplus price λ is time-invariant and there is no holding cost associated with stored energy, it is optimal to always inject the maximum allowed energy when there is a surplus in generation (i.e. $w_k > C_n$) relative to the offered contract. This attenuates the surplus penalty by an amount directly proportional to the energy injected.
- Similarly, as there are *no price arbitrage* opportunities, because the shortfall price q is also time-invariant – when there is a shortfall in generation (i.e. $w_k \leq C_n$) relative to the offered contract, an optimal policy is to extract the maximum allowable energy from the storage needed to cover the shortfall.

More formally, an *optimal feedback policy* is given by:

If there is a *contract shortfall* (i.e., $\Delta_n = C_n - w_n > 0$),

$$g^*(e_n, w^n) = \begin{bmatrix} \min\left\{\Delta_n, \; \frac{\eta_{\text{ext}}}{h}(e_n), \; \overline{P}_{\text{ext}}\right\} \\ 0 \end{bmatrix} \tag{19}$$

If there is a *contract surplus* (i.e., $\Delta_n = C_n - w_n \leq 0$),

$$g^*(e_n, w^n) = \begin{bmatrix} 0 \\ \min\left\{-\Delta_n, \; \frac{1}{h\eta_{\text{inj}}}(\overline{e} - e_n), \; \overline{P}_{\text{inj}}\right\} \end{bmatrix} \tag{20}$$

Remark 5.5.1. Note that the optimal storage operational policy above is *stationary* and *causal*. Moreover, we have omitted dependence of the feedback policy g on the imbalance prices (q, λ), as the greedy feedback strategy outlined above does not depend on their particular realization. This follows directly from the assumption of imbalance price *nonnegativity* and *time invariance*. □

Given such characterization of an optimal storage feedback policy, we now explore the problem of optimizing contract offerings *ex-ante* in the day ahead forward market.

5.5.2 Convexity of Optimal Contract Sizing

In the day-ahead market, the WPP must offer a contract portfolio $\mathbf{C}$ for the delivery of *uncertain* power during a future time interval, where deviations from offered contracts are penalized. We show that the problem of computing a profit maximizing portfolio $\mathbf{C}^*$ is a *convex optimization problem*.

Theorem 5.5.2. (Convexity Property) *Let g^* be an optimal storage operational policy for a fixed $\mathbf{C} \in [0,1]^M$. Then $J(g^*, \mathbf{C})$ is* concave *in $\mathbf{C}$.*

Proof: Without loss of generality, we prove the result for a single contract interval $(M = 1,\ \mathbf{C} = C \in [0,1])$. Define the random profit criterion

$$\Pi(g, C, \mathbf{w}, q, \lambda) = h \sum_{n=1}^{N} pC - q\left[C - w_n + P^g_{n,\text{inj}} - P^g_{n,\text{ext}}\right]^+ - \lambda\left[w_n - P^g_{n,\text{inj}} + P^g_{n,\text{ext}} - C\right]^+$$

As parameterized by a particular contract $C \in [0,1]$, the expected profit under the optimal feedback policy g^* (19) - (20) is given by

$$J(g^*, C) = \max_{g \in \mathcal{G}} \ \mathbb{E}_{\mathbf{w},q,\lambda}\, \Pi(g, C, \mathbf{w}, q, \lambda),$$

where expectation is taken with respect to the random wind power process $\mathbf{w}$ and imbalance prices (q, λ). As the imbalance prices (q, λ) enter linearly into the profit criterion Π and the optimal feedback policy is independent of their realization, we can equivalently replace them their respective expected values, i.e.,

$$J(g^*, C) = \max_{g \in \mathcal{G}} \ \mathbb{E}_{\mathbf{w}}\, \Pi(g, C, \mathbf{w}, \mu_q, \mu_\lambda),$$

This next step does not hold in general. However in our case, the expectation and maximization operators commute, because – as indicated by equations (19) and (20) – our stationary policy g^* is optimal for each realization of the wind process $\mathbf{w}$. Hence,

$$J(g^*, C) = \mathbb{E}_{\mathbf{w}} \max_{g \in \mathcal{G}} \Pi(g, C, \mathbf{w}, \mu_q, \mu_\lambda).$$

Now, define the optimal value $z(C, \mathbf{w})$, parameterized by the contract C and realized wind process $\mathbf{w}$, as

$$z(C, \mathbf{w}) = \max_{g \in \mathcal{G}} \Pi(g, C, \mathbf{w}, \mu_q, \mu_\lambda)$$

We first prove *concavity* of the optimal value $z(C, \mathbf{w})$ in the parameters $(C, \mathbf{w})$. Consider the following linear programming (LP) formulation

$$y(C,\mathbf{w}) = \max_{r,s,P_{\text{inj}},P_{\text{ext}}} \; h\sum_{n=1}^{N} pC \;-\; \mu_q s_n \;-\; \mu_\lambda r_n \tag{21}$$

$$\begin{aligned}
&\text{subject to}\\
&e_0 = 0\\
&s_n \geq 0\\
&r_n \geq 0\\
&s_n \geq C - w_n + P_{n,\text{inj}} - P_{n,\text{ext}}\\
&r_n \geq w_n - P^g_{n,\text{inj}} + P^g_{n,\text{ext}} - C\\
&(5) - (8)\ \forall\, n
\end{aligned}$$

where we have introduced new *slack decision variables*, $s \in \mathbb{R}^N$ and $r \in \mathbb{R}^N$. It is straightforward to show that

$$z(C,\mathbf{w}) = y(C,\mathbf{w}) \quad \text{for all} \quad (C,\mathbf{w}).$$

Thus, showing concavity of $z(C,\mathbf{w})$ in $(C,\mathbf{w})$ is equivalent to showing concavity of $y(C,\mathbf{w})$. Let $\alpha \in [0,1]$ and define

$$\begin{aligned}
C^\alpha &= \alpha C^1 + (1-\alpha)C^2\\
\mathbf{w}^\alpha &= \alpha \mathbf{w}^1 + (1-\alpha)\mathbf{w}^2
\end{aligned}$$

for arbitrary $C^1, C^2 \in [0,1]$ and $\mathbf{w}^1, \mathbf{w}^2 \in [0,1]^N$. Moreover, for any pair $(C,\mathbf{w})$, denote the optimizing arguments of Problem (21) by

$$\left\{r^*(C,\mathbf{w}),\; s^*(C,\mathbf{w}),\; P^*_{\text{inj}}(C,\mathbf{w}),\; P^*_{\text{ext}}(C,\mathbf{w})\right\}.$$

Concavity of $y(C,\mathbf{w})$ in $(C,\mathbf{w})$ is proven as follows.

$$\begin{aligned}
&y(C^\alpha,\mathbf{w}^\alpha)\\
&\quad = phNC^\alpha - h\sum_{n=1}^{N} \mu_q s_n^*(C^\alpha,\mathbf{w}^\alpha) \;+\; \mu_\lambda r_n^*(C^\alpha,\mathbf{w}^\alpha)\\
&\quad \geq phNC^\alpha - h\sum_{n=1}^{N} \mu_q \left[\alpha s_n^*(C^1,\mathbf{w}^1) + (1-\alpha)s_n^*(C^2,\mathbf{w}^2)\right]\\
&\qquad\qquad + \;\mu_\lambda \left[\alpha r_n^*(C^1,\mathbf{w}^1) + (1-\alpha)r_n^*(C^2,\mathbf{w}^2)\right]\\
&\quad = \alpha y(C^1,\mathbf{w}^1) + (1-\alpha)y(C^2,\mathbf{w}^2)
\end{aligned}$$

The inequality follows from the fact that

$$\alpha \begin{bmatrix} r^*(C^1, \mathbf{w}^1) \\ s^*(C^1, \mathbf{w}^1) \\ P^*_{\text{inj}}(C^1, \mathbf{w}^1) \\ P^*_{\text{ext}}(C^1, \mathbf{w}^1) \end{bmatrix} + (1-\alpha) \begin{bmatrix} r^*(C^2, \mathbf{w}^2) \\ s^*(C^2, \mathbf{w}^2) \\ P^*_{\text{inj}}(C^2, \mathbf{w}^2) \\ P^*_{\text{ext}}(C^2, \mathbf{w}^2) \end{bmatrix}$$

is a *feasible point* for Problem (21) with parameters $(C^\alpha, \mathbf{w}^\alpha)$. This feasibility holds, because the parameters $(C, \mathbf{w})$ enter linearly into the constraints of Problem (21).

We, thus far, have shown that $z(C, \mathbf{w}) = y(C, \mathbf{w})$ is concave in $(C, \mathbf{w})$. It follows immediately that

$$J(g^*, C) = \mathbb{E}_{\mathbf{w}}\, z(C, \mathbf{w})$$

is concave in C, as *expectation preserves concavity*. ■

5.5.3 Marginal Value of Energy Storage Capacity

Energy capacity constitutes a large percentage of the capital cost associated with many storage modalities. Hence, in order to accurately amortize the the capital investment in storage capacity over a period of time, it is of vital importance to quantify the fiscal benefit to the WPP in terms of storage capacity (i.e. J^* as a function of $\overline{e}$). This relation can then be used to optimally size the storage system so as to maximize return on investment. In Theorem 5.5.2, we proved that the problem of computing optimal contract offerings and the corresponding expected profit reduces to convex programming – a result that naturally lends itself to efficient computation of the return on investment curves.

In the following Theorem 5.5.3, we show that the optimal expected profit $(J^* := J(g^*, \mathbf{C}^*))$ derived by the WPP – with co-located storage of capacity $\overline{e}$ – is concave and non-decreasing in the capacity $\overline{e}$. This result reveals that the *greatest marginal benefit* is derived from a *small* amount of storage capacity. In fact, the marginal optimal expected profit with respect to the storage capacity $dJ^*/d\overline{e}$ can be analytically computed for $\overline{e}$ *small*.

Theorem 5.5.3. *The optimal expected profit $J(g^*, \mathbf{C}^*)$ is concave and monotonically non-decreasing in the energy storage capacity $\overline{e}$.*

Proof: Monotonicity is straightforward. Let $\epsilon > 0$. With a slight abuse of notation, let $J^*(\overline{e})$ denote the optimal expected profit corresponding to a system with storage capacity $\overline{e} \geq 0$. Clearly, $J^*(\overline{e} + \epsilon) \geq J^*(\overline{e})$, as the set of feasible solutions for problem (18) with capacity parameter $\overline{e}$ is a subset of the feasible set corresponding to a capacity parameter of $\overline{e} + \epsilon$. Concavity is proved analogously to theorem 5.5.2. ■

Marginal Value of Energy Storage Capacity $\overline{e}$

We now present a result that quantifies in closed-form the marginal expected optimal profit $dJ^*/d\overline{e}$ for *small* capacity $\overline{e}$. We'll see that marginal value of storage capacity at the origin is closely related to the spectral properties of the underlying wind process $\mathbf{w}$ – as represented by the frequency of *energy arbitrage opportunities*.

Definition 5.5.4 (Energy Arbitrage Opportunities, $\xi(\mathbf{C})$)**.** *Consider a contract portfolio* $\mathbf{C} \in \mathbb{R}_+^M$ *with individual contract duration of length* N*. Define the random variable* $\xi(\mathbf{C})$ *to be the number of times that the random process* $\{w_n\}$ *crosses the associated contract sequence* $\{C_n\}$ *from above. The random variable* $\xi(\mathbf{C})$ *can be interpreted as the number of* energy arbitrage opportunities *associated with the contract portfolio* $\mathbf{C}$.

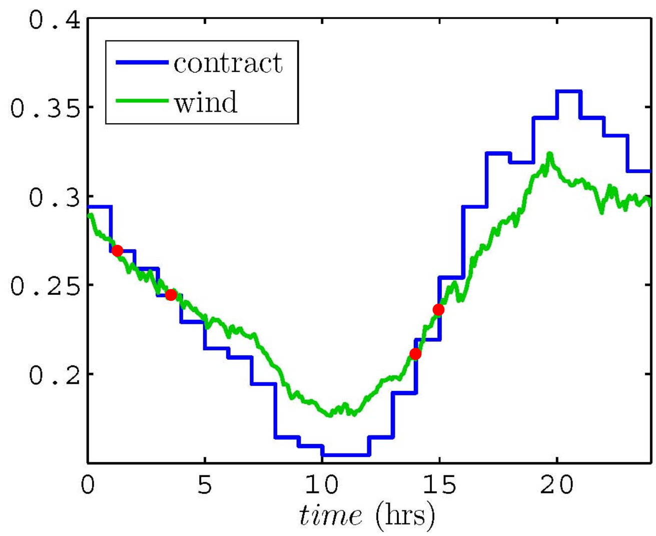


Figure 5.2. Pictorial representation of a collection of *energy arbitrage opportunities* (red dots) associated with a particular realization of the wind process and contract portfolio. As Theorem 5.5.5 indicates, the marginal value of energy storage capacity (for a small amount) is proportional to the expected number of energy arbitrage opportunities.

Theorem 5.5.5. *[13] Let* $\gamma := (p + \mu_\lambda)/(\mu_q + \mu_\lambda)$*. Assume that (1) the energy storage system is non-dissipative (i.e.* $\alpha = 0$*), (2) no constraints on power extraction or injection, and (3)* $e(0) = 0$*. Then the* marginal expected optimal profit *with respect to* $\overline{e}$ *at the origin is given by*

$$\left.\frac{dJ^*}{d\overline{e}}\right|_{\overline{e}=0} \;=\; (\mu_\lambda + \mu_q\, \eta_{\text{inj}}\eta_{\text{ext}})\; \mathbb{E}\,[\,\xi(\mathbf{C}^*)\,] \tag{22}$$

where $C^{(m)*} = F_m^{-1}(\gamma)$ *for* $m = 1, \cdots, M$.

Remark 5.5.6. *(Intuition)* The previous result has an intuitive interpretation in that the marginal value of storage capacity (for small amounts) is proportional to the expected number of *energy arbitrage opportunities*. Consider a system with small storage capacity $\epsilon > 0$. Each time the wind power process crosses the contract from above, the WPP has the opportunity to inject an ϵ amount of energy into the storage system to decrement the surplus penalty by $\lambda\epsilon$. This energy arbitrage event is also accompanied by the additional opportunity to extract ϵ energy from its storage device and thus decrement its shortfall penalty by $q(\eta_{\text{inj}}\eta_{\text{ext}})\epsilon$. Clearly then, the total expected financial benefit for small storage capacity is roughly $\approx (\lambda + q\eta_{\text{inj}}\eta_{\text{ext}})\,\mathbb{E}[\xi]\epsilon$. See Figure 5.2 for an example of a collection of energy arbitrage opportunities associated with a particular realization of the wind process for a given contract portfolio. □

5.6 Discussion

In this chapter we have formulated and solved the problem of optimal contract sizing for a wind power producer with *co-located energy storage* participating in a conventional two-settlement electricity market. We have shown that the problem of determining optimal contract offerings for a WPP with co-located energy storage can be solved using convex programming. Our results have the merit of providing key analytical insight into the trade-offs between a variety of factors such as energy storage *capacity* and optimal expected profit. In fact, we analytically quantify the marginal value of energy storage capacity and demonstrate that the storage capacity value is highly dependent on the spectral properties of the underlying wind power process. Using such results, one can formally study the planning question of optimal storage sizing so as to maximize return on investment.

In the near term, we plan to identify efficient computational methodologies for solving the convex contract sizing problem outlined in this paper. In the long term, we intend to explore alternative revenue streams (e.g. ancillary service and capacity markets) for an independently operated storage system, as wind-firming in of itself may not be sufficient to cover the capital cost of a storage device. An intimately related question is where to locate the storage device? Co-locating the storage system with the wind farm will lead to a direct increase in WPP profit and transmission line utilization. However, as wind sites tend to have limited transmission access and are distant from load centers, the storage device will necessarily face limitations on it's ability to offer ancillary services. Further, who commands the storage?

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