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Essays on Labor Market Mechanisms

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Essays on Labor Market Mechanisms

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by

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Abstract of the Dissertation

Essays on Labor Market Mechanisms

by

Ana Luisa Toledo Piza Pessoa Araujo

Doctor of Philosophy in Economics

University of California, Los Angeles, 2015

Professor Hugo Andres Hopenhayn, Chair

This dissertation studies the interaction between job stability and labor markets. Chapter 1 studies the impact of firm turnover and job recall on wages. I start this chapter with an empirical contribution. I demonstrate the importance of recall and turnover for employment dynamics and wages using matched employer-employee data from Brazil. First, I document large dispersion of job-destruction rates and recall rates across sectors. Second, I show that after controlling for worker and firm characteristics, sectors with greater job instability (an inverse measure of tenure that controls for recall) pay more. To explain this finding I construct a multi-sector closed economy version of Mortensen and Pissarides (1994) with directed search and heterogeneous recall rates as well as heterogeneous layoff rates across sectors. Once I have estimated the model's parameters using the data, I then conduct the main experiment which is to assess the impact of Brazil's recall restrictions on employment dynamics. The Brazilian government bans recalls within 3 months of the date of firing. I simulate the imposition of this law, and I find that this restriction on recall activity decreases the employment rate significantly in aggregate, but the impact is very heterogeneous across sectors. Chapter 2, studies wage inequality and job stability. I start this chapter with a data motivation where I use matched employer-employee data to establish that separations are disproportionately comprised of layoffs for low wage workers, not separations due to job-to-job transitions. Secondly, I show that existing models such as Burdett and Coles (2003) and Shi (2009) are *inconsistent* with this fact since they assume that the exogenous component of job destruction is constant across firms, and low wage workers search on-the-job much more intensely than high wage workers. To explain this fact, I develop a new model that takes into account the disproportionate share of layoffs among low wage workers. I show that after correctly matching the wage/layoff relationship, the introduction of a 30% 'firing cost' results in nearly twice as much wage inequality as compared to a model with a homogeneous firing rate across firms.

The dissertation of Ana Luisa Toledo Piza Pessoa Araujo is approved.

Leonardo A. Bursztyn

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Pierre-Olivier Weill

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University of California, Los Angeles

2015

This dissertation was possible because of the love and support that I received from a few very important people in my life. It all started with my passion for economics that I inherited from my father, also an economist. I began the process with the strong encouragement of my parents, Cristina & Aloisio and my sister Clarinha. At the beginning of my studies at UCLA I met the love of my life, Kyle, that along the way always supported me. Everything was essential during those years, and it all came together in the end.

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CHAPTER 1

The Impact of Turnover and Recall on Wages

1.1 Introduction

In this paper, I make two contributions, one empirical and one theoretical. In the empirical part of the paper, I demonstrate the importance of recall and turnover for employment dynamics and wages using matched employer-employee data from Brazil for the years of 2004 to 2010. Firstly, I document large the dispersion of job-destruction rates and recall rates across sectors. I find that the sector with the highest yearly turnover rate (construction) has a turnover rate of more than 200% which is ten times higher than the sector with the lowest rate (baking services with a rate of 19%). Also I find that the recall rate varies a lot among sectors, the sector with the highest yearly recall rate (machinery and electronics manufacturing) have a recall rate of 10% which is ten times larger than the recall rate of the sector with the lowest yearly recall rate, automobile manufacturing. Secondly, I show that after controlling for worker and firm characteristics, sectors with greater job instability (an inverse measure of tenure that controls for recall) pay more. Establishing this fact requires three steps. The first step is to use a tenure regression to show that even after controlling for a host of worker and firm characteristics, workers in unstable sectors such as construction have a 50% shorter job tenure than workers in the most stable sectors, like textile manufacturing. The second step is to use the same detailed individual level worker and firm controls to run wage regressions. From these regressions I isolate sector specific wage premia. The third step is to show that wage premia in these sectors with significant instability (i.e. sectors with strong negative effects on expected tenure) are, on average, greater. In other words, workers are compensated for higher uncertainty over their job tenure: for example, the construction sector pays a wage rate which is 12% greater than textile manufacturing.

To explain this finding I construct a multi-sector closed economy version of Mortensen and Pissarides (1994) with directed search and both heterogeneous recall and layoff rates across sectors. I find that in equilibrium all sectors have the same market tightness and all sectors give the worker the same value. However, the equilibrium wages vary across sectors. Sectors with higher layoff or turnover rates must pay higher wages to the worker. Once I have estimated the model's parameters using the data, I then conduct the main experiment which is to assess the impact of Brazil's recall restrictions on employment dynamics. The Brazilian government does not allow for recalls within 3 months of the date of firing. I simulate the case were the imposition of this law is retracted, and I find that if this restriction on recall activity cease to exist the employment rate would increase significantly. The mechanism is quite simple: when recall rates increase, mechanically the unemployment rate falls, and when the unemployment rate falls, there is less congestion in the labor market enabling other workers to find jobs quickly. The offsetting general equilibrium effect is that firms now face lower probabilities of facing vacancies, and so fewer firms enter. But, ultimately this effect only partially offsets the main positive mechanical impact of recall on employment.

Because layoff rates and recall rates are heterogeneous across sectors, it is possible to decompose the gains and losses of such a policy on sectoral employment. Sectors with very low recall rates are essentially unaffected by such a recall policy, and therefore only respond via the general equilibrium effect on firms. On the other hand, sectors in which recall is important have large employment gains from eliminating the recall restrictions.

In Section 1.2 I discuss the related literature. In Section 1.3 I establish new facts in the data. In Section 1.4 I build the model and explore its properties theoretically. In 1.5 I conduct the main quantitative experiment.

1.2 Literature Review

Several authors measured and documented the importance of recall, however most of the literature on recall is microeconomic based. Fujita and Moscarini (2013) was one of the first to use a search and matching model to analyze the connection between recall and macroeconomic cyclical unemployment. Using the SIPP data they estimate that in the US over 40% of all workers separating into unemployment get recalled by their previous employer, implying that most of the difference in the unemployment duration across workers is associated with recall.

Fujita and Moscarini (2013)'s duration dependence result has been discussed previously by Katz (1986) and Katz and Meyer (1988). Using a sample of unemployment insurance data for the states of Missouri and Pennsylvania, the authors (in both papers) estimate recall and it's importance when analyzing the determinants of unemployment spell durations. Katz (1986) show that the declining hazard rate of leaving unemployment is due to a declining recall rate. Katz and Meyer (1988) discuss the importance of recall when analyzing the determinants of unemployment duration and the impacts of unemployment insurance. They find that workers that expect to be recalled and end up not being recalled, have long unemployment spells.

Using these data findings of Kataz and Meyer, and Fujita and Moscarini, Fernández-Blanco (2013) proposes a model and theoretical analyzes an economy were separations are not permanent. He finds that reactivated firms prefer to recall former workers instead of seeking for new one. Furthermore, if firms could commit to wages contingent on recall, they would provide wage incentives for workers to reduce their search effort in case they are temporary layoff.

In terms of wage results, Bonhomme and Jolivet (2009) use the European Community Household Panel (ECHP) to estimate a dynamic model of wages, amenities and labor mobility. They find that there is a near-zero wage/amenity correlation meaning that low wage workers are not given compensating differentials. They argue that this is because there is heterogeneity in mobility costs, even though workers have a large marginal willingness to pay for amenities. Mukoyama (2014) constructs job flows that correct for recall, and he find turnover is an important determinant of productivity growth in the US.

In terms of other countries, Røed and Nordberg (2003) document similar trends to Fujita and Moscarini (2013). Using Norwegian micro-data they estimate that unemployment spells have different explanations depending if the worker was permanently or temporary dismissed. For permanently dismissed workers the unemployment duration is explained by individual economic resources and incentives, while for temporary dismissed workers the unemployment duration is explained by firm incentives. Alba-Ramírez et al. (2007) document similar trends for Spain. Due to the nature of employment contracts (particularly the prevalence of temporary employment contracts and other mechanisms to avoid permanent employment contracts) in Spain, recall rates are extremely important for employment flows.

1.3 Data Motivation

In this section, I conduct several empirical exercises to motivate the model that is proposed in Section 1.4. In particular, I document large amounts of heterogeneity in both recall and turnover rates in Brazil, and I show that this heterogeneity is an important component of wages. This stands in contrast to earlier results by Bonhomme and Jolivet (2009) who find a lack of compensating differentials in European data.

For these empirical exercises and the estimation of the model in later sections I use a matched employer/employee Brazilian dataset called RAIS. RAIS is a monthly panel that covers every formal worker in Brazil, and includes worker characteristics (such as age, education, sex, tenure, average annual wage and the wage in December) and firm characteristics (such as sector). Because it is possible to follow workers and firms using their respective id numbers, it is possible to isolate the number of workers that were recalled every monthly. Because of the size of the data, I chose to analyze one state of Brazil, Bahia for the years of 2004 to 2010. Also, because I assume that there is competition across sectors, I choose to analyze only the private sectors, excluding public administration jobs etc.

1.3.1 Sectoral Heterogeneity of Job-Destruction and Recall Rates

First let me define some statistics that are used in the analysis that follows. Let $t = 2004, 2005, ..., 2010$ denote the year of the data (within each year, I observe months of employment at each firm, so effectively this is a monthly panel), and $s = 1, 2, .., S$ denote the sector of the economic activity. Let $size_{st}$ denote the average monthly size of sector s at year t, $inflow_{st}$ denote the gross hires[1] of sector s in period t, and $outflow_{st}$ denote the gross separations of sector s at time t. Let $layoff_{st}$ be the gross involuntary layoffs of sector s in year t, which includes any of the three "involuntary" types of job separation[2]: (i) firing, (ii) end of a temporary contract and (iii) retirement (looking through the lens of my model, retirement is involuntary). A recall occurs when a worker that was involuntary separated from an establishment returns to the same establishment within six months of separation (without having any other formal job in between), let $re\text{-}hire_{st}$ denote the gross hires that via recall of sector s in year t. I will define the yearly turnover rate and recall rate to be,

$$turnover_{st} = \frac{inflow_{st} + outflow_{st}}{size_{st}}. \qquad recall_{st} = \frac{re\text{-}hire_{st}}{layoff_{st}}$$

In Table 1.1 below I document the share that each sector represents of the total private sector, and the average turnover and recall rate for each sector between 2004 and 2010 in Bahia, Brazil. In Table 1.1 we can see that there exists wide dispersion of turnover rates and recall rates across sectors. For example, the construction sector has a high turnover rate, 210%, and at the same time a high recall rate, 6.1%. The wholesale trade sector has almost the same recall rate as the construction sector, 5.8%, however it also a much lower turnover rate, 95%. This heterogeneity of turnover rates and recall rates across sectors must be taken into consideration when analyzing job durations across sectors (see Mukoyama (2014) for an alternate method of correction).

[1] I exclude hires due to transfer among establishments of the same firm.
[2] I exclude the separations due to worker death.

Table 1.1: Brazil, Bahia Summary Statistics of Private Sector 2004-2010* (RAIS)

Sector	**Size**** (%)	**Turnover Rate**** (%)	**Recall Rate**** (%)
Textile manufacturing	3.80	81.46	3.13
Food & drink manufacturing	3.18	101.57	9.49
Metal & mineral manufacturing	3.03	75.58	3.37
Chemical manufacturing	1.97	60.80	3.11
Public utility manufacturing	1.32	35.16	4.40
Machinery & electronics manufact.	0.93	136.51	10.31
Automobile manufacturing	0.51	34.04	1.09
Others manufacturing	2.12	76.23	5.03
Construction	7.82	209.96	6.14
Retail trade	21.11	95.00	2.07
Wholesale trade	3.78	94.83	5.77
Banking services	1.35	19.03	0.98
Other financial & insurance serv.	0.34	74.15	0.92
Real state services	14.19	120.98	5.24
Traffic & transportation services	6.36	72.44	2.79
Accommodation and food services	10.99	89.02	2.61
Health & social services	4.85	46.38	2.00
Educational services	4.64	52.69	2.35
Agriculture, etc	7.72	197.27	7.14
All sectors	100.00	105.41	4.65

*Do not include separations and hires due to transfer.

** Average rates.

1.3.2 Sector Effect on Wages and Tenure

In the spirit of Bonhomme and Jolivet (2009), I now consider the role of compensating differentials: are workers that work in sectors with high turnover rates and lower recall rates compensated for their employment risk? In this Section I show that after controlling for worker and firm characteristics, sectors with greater job instability (an inverse measure of tenure that controls for recall) pay more. Existing studies find mixed results with respect to compensating differentials (see Bonhomme and Jolivet (2009) and citations therein), with some recent studies documenting an absence of compensating differentials in advanced economies.

1.3.3 Sector Effect on Wages

The importance of this section, which estimates the wage regression given by Equation (1.1), is to describe how I estimate the sector fixed effects for *wages*, the coefficients on $\mathbb{I}(\text{sector}_s)$, which I will use in combination with results below to assess the importance of compensating differentials. The general wage regression in Equation (1.1) has been analyzed and implemented across many countries over many time periods, and so I relegate the results to the Appendix 1.7.2, Table 1.8.

Let me describe how I implement the wage regression below. In Brazil, unions are still very important, and so I must augment my data with union density numbers in each sector. To do this, I construct a union density measure using the Brazilian national household survey, *Pesquisa Nacional de Amostra por Domiclio* (PNAD), from 2004 to 2008. For each worker education level and for each sector I estimate the proportion of workers that belong to a union (see Table 1.6 in the Appendix 1.7.1). To assess the impact of sector fixed effects on wages, I consider the following regression:

$$\begin{aligned} &Ln(wage)_{ist} = \\ &f\Bigg(\mathbb{I}(\text{semester working}_{it}), \mathbb{I}(\text{sector}_s), \mathbb{I}(\text{union level}_{is}), \mathbb{I}(\text{year}_t), X_{it}\Bigg) + \alpha_i + \epsilon_{it} \end{aligned} \tag{1.1}$$

We have $i = 1, ..., I$ individuals working on sector $s = 1, ..., S$ observed over $t = 1, ..., T$ time periods (years). $Ln(wage)_{ist}$ is the dependent variable and it is the log of the average hourly wage of individual i working in sector s in year t. Because the wage is the average of the year, I must control for the semester that the worker was being paid. $\mathbb{I}$(semester working$_{it}$ is a dummy corresponding to the semester(s) that the worker was employed (it can be the first semester, both semesters or only the second semester). $\mathbb{I}$(union level$_{is}$) is a dummy corresponding to the percentage level of union participation for workers in sector s with the same level of education of individual i (hence the dependence on i). α_i is the person i fixed-effect and ϵ_{it} is the error term. X_{it} is a vector of worker characteristics that change with time, age and age squared, and $\mathbb{I}$(year$_t$) is a dummy for the year t. The main objects of interest are the coefficients on $\mathbb{I}$(sector$_s$) which are dummies corresponding to sector s.

1.3.4 Sector Effect on Tenure

The importance of this section, which estimates the tenure regression given by Equation (1.2), is to describe how I estimate the sector fixed effects for *tenure*, the coefficients on $\mathbb{I}$(sector$_s$), which I will use in combination with the previous results for sector fixed effects on *wages* to assess the importance of compensating differentials. The general tenure regression in Equation (1.2) has been analyzed before, and so, as in the previous section, I relegate the results to the Appendix 1.7.2, Table 1.9.

From 2004 to 2010 in Brazil, Bahia, the number of private sector employees in RAIS increased 34.5% (see Table 1.7 in Appendix 1.7.1). This growth was not equally distributed across sectors, for instance the construction sector grew 65% between 2004 and 2010 while the size of the wholesale trade sector didn't change. Some sectors like agriculture even shrank, decreasing its size by 28%. Therefore to analyze the impact that sectors have on tenure I have to take into the sector growth in the year.

Because I am interested in the sectoral component of job stability, I need to control for the duration of the job for recall. As shown before in Section 1.3.1, there much heterogeneity in turnover and recall rates across sectors and this needs to be taken into account when considering tenure. Therefore I use worker tenure adjusted for recall as a dependent variable.

The adjusted tenure is the total time that the worker worked in the firm. If the worker was recalled the new measure of tenure (tenure adjusted) after the recall starts counting right where is left-off prior to layoff, using the previous tenure as the new starting point[3]. To assess the impact of sector on tenure consider the following regression:

$$Ln(tenure)_{ist} = f\left(growth_{st}, \mathbb{I}(\text{sector}_s), \mathbb{I}(\text{year}_t), X_{it}\right) + \alpha_i + \epsilon_{it} \tag{1.2}$$

We have $i = 1, ..., I$ individuals working in sector $s = 1, ..., S$ observed over $t = 1, ..., T$ time periods (years). $Ln(tenure)_{ist}$ is the dependent variable and it is the log of adjusted tenure measured in months of individual i working on sector s at year t. The variable $growth_{st}$ is the growth rate of sector s in the year t. $\mathbb{I}(\text{year}_t)$ is a dummy for the year. X_{it} is a vector of worker characteristics that changes with time, age and age squared. α_i is the person i fixed-effect and ϵ_{it} is the error term. The main object of interest from this section is the sector fixed effects, the coefficients on the sector dummies $\mathbb{I}(\text{sector}_s)$.

1.3.5 Sectors With More Instable Jobs Pay Higher Wages

Figure 1.1 below plots the sector fixed effects from the wage regression, Section 1.3.3, and the sector fixed effects from the tenure regression, Section 1.3.4. The line labeled 'Wage Premium by Sector' is the coefficient of the sector dummy from the estimation of wages using Equation (1.1). The line labeled 'Sector Effect on Tenure' is the coefficient of the sector dummy from the estimation of tenure using Equation (1.2). The line in Figure 1.1 is the fitted regression line which weights sectors by their size. In this graph we can see a negative relationship between sector effects on wages and tenure; however, there are two clear outliers: (1) banking services and (2) automobile manufacturing. Those two sectors have a much higher union rates than the other sectors. In Figure 1.2, I exclude these two sectors. When I exclude these outliers the line of the fitted values becomes much more steep. Even though the banking services sector and the automobile manufacturing sector combined

[3] For example, a worker that was dismissed of a job when he had a tenure of 25 months, when he is recalled for the same job his new tenure starts at 25 months.

represent less than 2% of all private sectors, because the line is fitted with OLS, they still have a disproportionate impact on the weighted fitted values in Figure 1.1. When those sectors are excluded, Figure 1.2, the negative relationship between the sector effect on wages and the sector effect on tenure becomes much more evident.

Figure 1.1: Effects that sectors have on wages and job duration, Bahia-Brazil (RAIS) 2004-2010

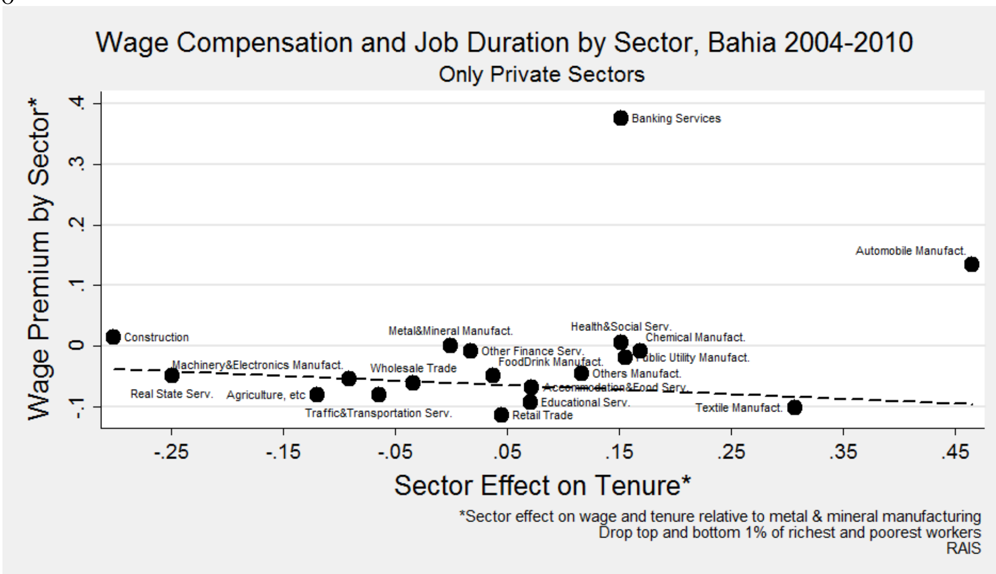
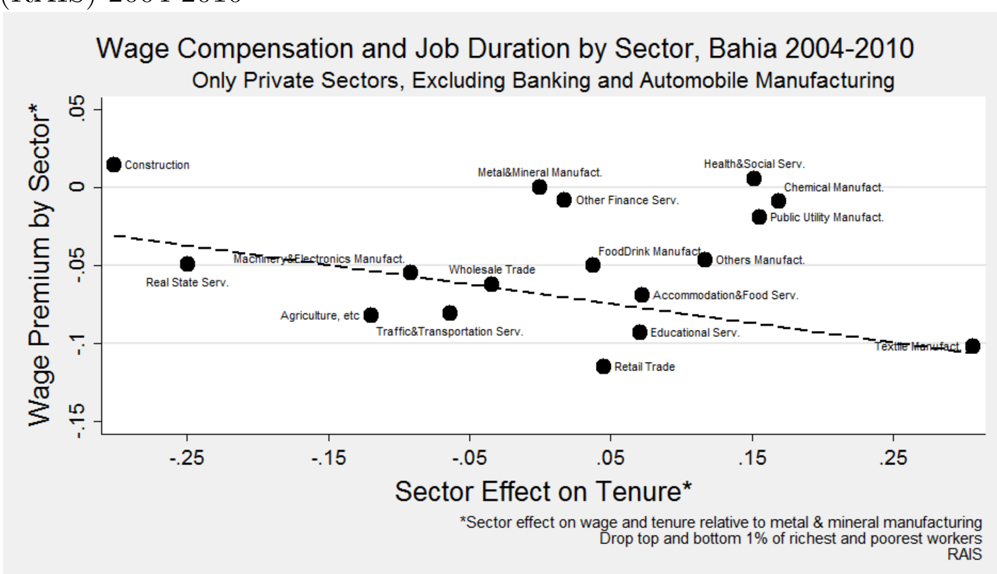


Figure 1.2: Effects that sectors have on wages and job duration, excluding outliers, Bahia-Brazil (RAIS) 2004-2010



1.4 Model

Consider a discrete time, infinite horizon economy. There are two types of agents in this economy, workers and firms. Firms operate in one of I sectors in the economy, and each sector produces a different good. Workers can either be employed or unemployed, and while unemployed workers direct their search freely among sectors. Within a sector, matches occur randomly. On the other side of the labor market, firms post vacancies freely across sectors. The vacancy posting cost is paid in terms of a bundle of goods across the I sectors. The economy closed and all goods produced within the period are used either by firms posting vacancies or workers consuming.

For simplicity, I assume that all firms have the same productivity across all sectors (i.e. every employed worker produces 1 unit per time period if matched). There are two features that differentiate sectors from one another. The first feature is that each sector produces a unique good, and the second feature is that the contiguous match duration, $1/\delta_i$, differs exogenously by sector. With probability δ_i, the match becomes "dormant" and the firm's productivity temporarily becomes zero. The match does not end after receiving a negative productivity shock (δ_i); instead the firm becomes temporarily inactive, and with some probability λ_i, the firm's productivity recovers. If the worker who was previously producing with the firm is still unemployed when the firm's productivity recovers, the firm and worker optimally rematch immediately.[4] The final assumption is that dormant/inactive matches are permanently destroyed at an exogenous rate η_i. If the permanent destruction of the match occurs (η_i), both the worker and firm must find new matching partners since there is no longer any prospect of recall. To summarize, workers and firms can be in three states: 1) matched/active; 2) matched/inactive (dormant); 3) unmatched.

There is a fixed measure of workers normalized to m, and the number of firms in equilibrium will be given by the free-entry condition. Let the number of workers employed in sector i be denoted by e_i, and let the number of unemployed workers with and without a

[4]The reason workers and firms optimally rematch immediately is simple: all firms produce with the same productivity (when non-dormant), and if the worker can avoid the search costs of finding another firm in another sector (a subset of firms over which the worker should be indifferent), then the worker strictly prefers rematching with the old firm.

dormant match be denoted by u_i^D and u_i^{ND}, respectively. Let the number of firms with a vacancy in sector i be denoted by v_i. When there are $u_i = u_i^D + u_i^{ND}$ unemployed workers looking for jobs, and there are v_i firms with a vacancy on sector i, the market tightness is denoted by $\theta_i = \frac{v_i}{u_i}$. The total number of matches is given by a matching technology $m(u_i, v_i)$, the frequency that a firm is matched with a worker in sector i is denoted by $q(\theta_i)$ where $q(\theta_i) = \frac{m(u_i,v_i)}{v_i}$. Assume for example that the matching function is Cobb-Douglas, then $m(u_i, v_i) = (u_i^D + u_i^{ND})^\zeta v_i^{1-\zeta}$ and $q(\theta_i) = \theta_i^{-\zeta}$.

Both workers and firms discount the future at a constant rate r. Unemployed workers have preferences over each of the I types of goods (i.e. workers may potentially be risk averse if their preferences over the consumption bundle are concave), but throughout the paper, I assume that unemployed workers cannot save. While this assumption may seem stark, even in countries with well developed consumer saving vehicles such as the United States, Gruber (2001) and Kaplan et al. (2014) have shown that a significant fraction of the unemployed are hand-to-mouth. While out of a job (either awaiting recall or searching for a new employer), unemployed workers receive a constant flow utility of leisure, z.

Let U_0 denote the value of an unemployed worker without a match, U_i^D denote the value of an unemployed worker with a dormant match in sector i, and let W_i denote the value of a employed worker in sector i. Similarly, let V_{0i} denote the value of a firm with an unmatched vacancy in sector i, let V_i^D denote the value of firm with a dormant match in sector i, and let J_i denote the value of a active firm in sector i.

Workers employed in sector i receive a wage, w_i, and their share of the net profit of the firms, π. Since the measure of workers is normalized to 1, each worker receives π. With their income, $y_i = w_i + \pi$, employed workers consume goods $x^i = (x_1^i, ..., x_N^i)$ at prices $p = (p_1, ..., p_N)$. The continuation value of a worker employed in sector i is given by:

$$\max_{x^i} \ rW_i = u(x^i) + \delta_i \left(U_i^D - W_i\right)$$
$$s.t. \quad px^i \leq w_i + \pi$$

The workers utility of consuming bundle x^i is $u(x^i) = \prod_n (x_n^i)^{\alpha_n}$. Workers maximize their continuation value subject to their budget constraint. The worker continuation value, as described in the equation above, is the utility flow of consuming goods plus the expected net utility flow of entering a dormant match. The worker enters a dormant match with probability δ_i and receives utility U_i^D in continuation, giving up W_i; the expected net utility flow of entering a dormant match is therefore $U_i^D - W_i$.

Solving the problem above, the demand for each good n of a worker employed in sector i is:

$$x_n^{*i} = \frac{\alpha_n}{p_n} y_i = \frac{\alpha_n}{p_n}(w_i + \pi)$$

Solving the maximization problem above, the worker continuation value is a function of his income multiplied by his marginal utility of income. Let $\mu(p) = \prod_i \left(\frac{\alpha_i}{p_i}\right)^{\alpha_i}$ denote the worker marginal utility for income, therefore at the optimum the continuation value of the worker is given by,

$$rW_i = \mu(p)(w_i + \pi) + \delta_i \left(U_i^D - W_i\right)$$

An unemployed worker receives a flow utility of leisure, z, plus an income corresponding his share of the net profit of firms, π. Using income π, the unemployed worker consumes the bundle x^{ui}. The continuation value of an unemployed worker searching for a job in sector i is given by,

$$\max_{x^{ui}} \ rU_{0i} = z + u(x^{ui}) + \theta_i q(\theta_i) \left(W_i - U_{0i}\right) \qquad s.t. \quad px^{ui} \leq \pi$$

The continuation value of a unemployed worker is given by the intra-period worker utility flow, $z + u(x^{ui})$, plus the net expected gain of finding a job, which occurs with intensity $\theta_i q(\theta_i)$. At optimum, the continuation value of an unemployed worker searching for a job in sector i is:

$$rU_{0i} = z + \mu(p)\pi + \theta_i q(\theta_i) \left(W_i - U_{0i}\right) \tag{1.3}$$

The continuation value of an unemployed worker that is in a dormant match in sector i is given by,

$$\begin{aligned} \max_{x^{ui}} \; rU_i^D = z + u(x^{ui}) + \lambda_i \left(W_i - U_i^D\right) + \eta_i \left(U_0 - U_i^D\right) \\ + \max\left\{ \; \theta_1 q(\theta_1) \left(W_1 - U_i^D\right), \; ... \; , \; \theta_N q(\theta_N) \left(W_N - U_i^D\right), \; 0 \; \right\} \\ s.t. \quad px^{ui} \leq \pi \end{aligned}$$

An unemployed worker with a dormant match can either (i) be recalled by his old job with probability λ_i, and receive continuation utility W_i upon being recalled, (ii) have his dormant match destroyed with probability η_i, becoming unemployed without a dormant match, and receive continuation utility U_0, or (iii) the worker can find a new job, which occurs with intensity $\theta_i q(\theta_i)$, and receive continuation utility W_i. At the optimum, the continuation value of a worker unemployed that is in a dormant match in sector i is given by,

$$\begin{aligned} rU_i^D = z + \mu(p)\pi + \lambda_i \left(W_i - U_i^D\right) + \eta_i \left(U_0 - U_i^D\right) \\ + \max\left\{ \; \theta_1 q(\theta_1) \left(W_1 - U_i^D\right), \; ... \; , \; \theta_N q(\theta_N) \left(W_N - U_i^D\right), \; 0 \; \right\} \end{aligned}$$

On the firm side, the continuation value of a firm with a match in sector i is given by:

$$rJ_i = p_i - w_i + \delta_i \left(V_i^D - J_i\right) \tag{1.4}$$

An active firm generate a profit flow of $p_i - w_i$. With probability δ_i, an active firm's productivity becomes zero. If an active firm's productivity becomes zero, the firm is inactive/dormant, and has a continuation utility of V_i^D instead of J_i. The continuation value of a firm that is in a dormant match in sector i is:

$$rV_i^D = \lambda_i \left(J_i - V_i^D\right) + \theta_i q(\theta_i) \left(V_0 - V_i^D\right) + \eta_i \left(0 - V_i^D\right) \tag{1.5}$$

In the equation above we can see that a firm with a dormant match will either (i) recover with probability λ_i, (ii) lose the worker because the worker found a new job with probability $\theta_i q(\theta_i)$, or (iii) the firm can receive a permanent match destruction shock and exit with probability η_i. If the firm recovers and recalls a worker, the firms receives a net flow of $J_i - V_i^D$, if the firm loses the worker because the worker found a new job, the firm must post a new vacancy and therefore has a net flow of $V_0 - V_i^D$. And finally, if the firm exits, the firm receives a net flow of $-V_i^D$.

The cost of posting a vacancy is the same for all sectors. To post a vacancy the firm must purchase a combination of all goods.[5] In particular, the "production function" of a vacancy is $f(x^f)$. An unmatched firm wants to minimize the cost of posting a vacancy:

$$\min_{x^f} \; px^f \qquad s.t. \quad f(x^f) = 1$$

Solving the firm minimization problem, the cost of posting one vacancy is $c = f(p)^{-1}$. The continuation value of a firm with a vacancy is equal to the flow cost of posting a vacancy, c, plus the expected new gain of filling the vacancy, $J_i - V_{0i}$. An unmatched firm in sector i fills the vacancy with intensity $q(\theta_i)$, therefore the continuation value of a firm posting a vacancy in sector i is:

$$rV_{0i} = -c + q(\theta_i)\left(J_i - V_0\right)$$

[5] There are two ways of interpreting this vacancy cost: (i) in units of a final composite good, or (ii) the firm must buy many different types of goods from each sector in order to enter a market including construction goods (a building), secretarial services, cleaning services, IT goods, office furniture, etc.

The net profit of all firms combined is given by:

$$\pi = \sum_i e_i(p_i - w_i) - \sum_i v_i c$$

1.4.1 Equilibrium

1.4.1.1 Definition of Equilibrium

A Stationary Recursive Competitive Equilibrium for this economy is a vector of commodity prices $\{p\}$, a vector of market tightnesses $\{\theta\}$ and allocations, $\{x^i, x^{ui^D}, xi^{ui^{ND}}, x^f\}$ such that

- Given prices and market tightnesses, agents optimal consumption is consistent with the equilibrium allocation of good.
- Free entry holds.
- The market for each type i commodity clears.

1.4.1.2 Equilibrium Properties

The total demand from workers for each good i is given by the sum of the demands for this good among both the employed and unemployed workers, with or without a dormant match:

$$X_i^{\text{demand workers}} = \sum_n (e_n x_n^n + u_i x_i^{un}) = \frac{\alpha_i}{p_i} \sum_n (e_n(w_n + \pi) + u_n \pi) = \frac{\alpha_i}{p_i} \left(\pi + \sum_n e_n w_n \right)$$

To post vacancies firms also consumes goods, and so the total demand of the firms for each good i is given by,

$$X_i^{\text{demand firms}} = \frac{\alpha_i}{p_i} \sum_n v_n c$$

Therefore, summing across households and firms, the total demand for good xi is given by,[6]

$$X_i^{\text{demand}} = \frac{\alpha_n}{p_n} \sum_n e_n p_n$$

On the supply side, since each employed worker in sector i produces one unit of good i, the total amount produced of each good i is e_i. In equilibrium, all markets must clear,

$$e_i = \frac{\alpha_i}{p_i} \sum_n e_n p_n \quad ^7$$

Assumption 1 *When a worker and a new firm meet, the worker may not be recalled by the old firm.*

This assumption 1 simplifies the bargaining process enormously. As a result, when a firm meets a new worker, it does not need to know if the worker previously had a dormant match since the worker's outside option is to break off negotiations and remain unemployed without a dormant match, i.e. the workers outside option is U_{0i}.

In Theorem 2, I prove that the equilibrium market tightness must be the same across all markets.

Theorem 2 *The market tightness is the same across sectors, i.e.,* $\theta_i = \theta_j = \theta$.

Proof.

The free-entry condition implies that in equilibrium the value of a vacancy must be zero, therefore, $J_i = \frac{c}{q(\theta_i)}$. When workers and firms meet, the wage is determined by Nash-bargaining. By assumption 1, a firm that meets a new worker will bargain with the worker knowing that the worker's outside option is to remain unemployed without a dormant match, U_{0i}. Therefore the Nash-bargaining solution for sector i is:

$$W_i - U_{0i} = \frac{\beta}{1-\beta} J_i$$

[6] Substituting π into $X_i^{\text{demand}} = \frac{\alpha_i}{p_i}\left(\sum_n (e_n w_n) + \pi + \sum_n v_n c\right)$

[7] Implying that: $\frac{e_i}{e_{i'}} = \frac{\alpha_i}{\alpha_{i'}} \frac{p_{i'}}{p_i}$ and $e_i = e_0 \frac{\alpha_i}{\alpha_0} \frac{p_0}{p_i}$.

On the worker side, because workers can choose in which sector to search, unemployed workers without a dormant match must be indifferent between searching across sectors, i.e. $U_{0i} = U_{0j} = U_0$:

$$rU_0 = z + \mu(p)\pi + \theta_i q(\theta_i)\left(W_i - U_0\right) = z + \mu(p)\pi + \theta_j q(\theta_j)\left(W_i - U_0\right) \Rightarrow$$
$$z + \mu(p)\pi + \theta_i c\frac{\beta}{1-\beta} = z + \mu(p)\pi + \theta_j c\frac{\beta}{1-\beta} \Rightarrow \quad \theta_i = \theta_j = \theta$$

■

In Corollary 1 below, I show that since the market tightness is the same across sectors and since unemployed workers must be indifferent across sectors, the continuation value for an employed worker must also be the same across sectors:

Corollary 1 *The worker continuation value is the same across sectors, $W_i = W_j = W$.*

Proof. The proof follows from substituting the worker free mobility condition across sectors ($U_{0i} = U_{0j} = U_0$), and the result from Theorem 2 ($\theta_i = \theta_j = \theta$), into the continuation value for an unemployed worker, (Equation(1.3)):

$$rU_0 = z + \tilde{\mu}(p)\pi + \theta q(\theta)(W_i - U_0) = z + \tilde{\mu}(p)\pi + \theta q(\theta)(W_j - U_0) \quad \Rightarrow \quad W_i = W_j = W$$

■

In particular, in equilibrium the value of an unemployed worker is:

$$U_0^* = \frac{1}{r}\left(z + \mu(p)\pi + \theta c\frac{\beta}{1-\beta}\right) \tag{1.6}$$

And, in equilibrium the value of an employed worker is [8]:

$$W^* = \frac{1}{\theta q(\theta)}\Bigg(- z - \mu(p)\pi + \left(r + \theta q(\theta)\right) U_0\Bigg) \tag{1.7}$$

[8] Substituting U_0 from Equation 1.6 into $rU_0 = z + \mu(p)\pi + \theta q(\theta)(W_i - U_0)$

To find the wage of equilibrium, first substitute the fact that in equilibrium the market tightness and the employed worker continuation value are the same across sectors (Theorem 2 and Corollary 1), into the continuation value of a worker with a dormant match in sector i:

$$rU_i^D = z + \mu(p)\pi + \lambda_i \left(W - U_i^D\right) + \eta_i \left(U_0 - U_i^D\right) + \theta q(\theta) \left(W - U_i^D\right)$$

Isolating U_i^D and re-arranging[9], we can write the worker surplus used on the Nash-Bargaining as:

$$W - U_0 = W - U_i^D + \frac{z + \mu(p)\pi + (\lambda_i + \theta q(\theta)) (W - U_0) - rU_0}{r + \lambda_i + \eta_i + \theta q(\theta)}$$

Substituting $W - U_i^D$ from the equilibrium continuation value of an employed worker [10], we can write the worker surplus used on the Nash-Bargaining solution as a function of W, U_0 and wage:

$$W - U_0 = \frac{\mu(p)(w_i + \pi) - rW}{\delta_i} + \frac{z + \mu(p)\pi + (\lambda_i + \theta q(\theta)) (W - U_0) - rU_0}{r + \lambda_i + \eta_i + \theta q(\theta)}$$

Substituting U_0^* and W^* of equilibrium (Equations (1.6) and (1.7)) into the equation above, we can write the Nash-Bargaining solution as:

$$\frac{\mu(p)(w_i + \pi) - rW^*}{\delta_i} + \frac{z + \mu(p)\pi + (\lambda_i + \theta q(\theta)) (W^* - U_0^*) - rU_0^*}{r + \lambda_i + \eta_i + \theta q(\theta)} = \frac{\beta}{1-\beta}\frac{c}{q(\theta)}$$

Therefore the equilibrium wage is:

$$w_i^* = \frac{1}{\mu(p)}\left(z + \frac{\beta}{1-\beta}\frac{c}{q(\theta)}\left(r + \theta q(\theta)\right)\right) + \frac{1}{\mu(p)}\frac{\beta}{1-\beta}\frac{c}{q(\theta)}\delta_i\left(\frac{r + \eta_i + \theta q(\theta)}{r + \lambda_i + \eta_i + \theta q(\theta)}\right) \quad (1.8)$$

In the Corollary 2 bellow we can see that the equilibrium wage is increasing with the instability of a job. Sectors that have a high probability of a negative shock, and sectors

[9] $U_i^D = U_0 + \left(z + \mu(p)\pi + (\lambda_i + \theta q(\theta)) (W - U_0) - rU_0\right) / \left(r + \lambda_i + \eta_i + \theta q(\theta)\right)$

[10] $rW = \mu(p)(w_i + \pi) + \delta_i \left(U_i^D - W\right)$

that have a high probability of a permanent destruction of a dormant match, must pay their workers higher wages to compensate the uncertainty. We can also see in Corollary 2 that sectors that have a high recall rate pays their workers lower wages than sectors that have a low recall rate.

Corollary 2 *The wage in equilibrium is increasing with the probability of a negative shock and with the probability of a permanent destruction of the match, ie,* $\frac{\partial w_i^*}{\partial \delta_i} > 0$ *and* $\frac{\partial w_i^*}{\partial \eta_i} > 0$.

Also, the wage in equilibrium is decreasing with the probability of a recall in case the match is dissolved, ie, $\frac{\partial w_i^*}{\partial \lambda_i} < 0$

Proof. See appendix 2.7.5. ■

To recover p_i first isolate V_i^D from Equations (1.4) and (1.5), then:

$$V_i^D = \frac{(r + \delta_i)\, J - (p_i - w_i)}{\delta_i} = \frac{\lambda_i J}{r + \lambda_i + \eta_i + \theta q(\theta)}$$

Substituting $J = \frac{c}{q(\theta)}$ and w_i^*, Equation (1.8), into the equation above we can recover the equilibrium prices:

$$p_i^* = w_i^* + \frac{c}{q(\theta)} \left[r + \delta_i \left(\frac{r + \eta_i + \theta q(\theta)}{r + \lambda_i + \eta_i + \theta q(\theta)} \right) \right] \tag{1.9}$$

Corollary 3 *The price in equilibrium is increasing with the probability of a negative shock and with the probability of a permanent destruction of the match, ie,* $\frac{\partial p_i^*}{\partial \delta_i} > 0$ *and* $\frac{\partial p_i^*}{\partial \eta_i} > 0$.

Also, the price in equilibrium is decreasing with the probability of a recall in case the match is dissolved, ie, $\frac{\partial p_i^*}{\partial \lambda_i} < 0$

Proof. See appendix 2.7.5. ■

1.4.1.3 Equilibrium of Workers Flows

The total number of workers is normalized to m:

$$\sum_i (e_i + u_i) = m \ , \qquad \text{where} \ \ u_i = u_i^{ND} + u_i^D$$

And the total number of employed workers is normalized to 1:

$$\sum_i e_i = 1$$

The number of employed workers in sector i evolves according to:

$$e_i' = e_i - \delta_i e_i + \theta_i q(\theta_i) u_i + \lambda_i u_i^D$$

The number of employed workers in sector i tomorrow is equal to the number of workers employed in sector i today, minus the number of matches that became dormant due to exogenous shocks ($\delta_i e_i$), plus the number of workers that started working in this sector. A worker can start working in sector i if the worker finds a new job or is recalled by an old job. The total number of unemployed workers, with and without a dormant match, that find a new job is $\theta_i q(\theta_i) u_i$, and the number of workers recalled in the sector i is $\lambda_i u_i^D$ at every period.

The pool of workers with a dormant match in sector i evolves according to:

$$u_i^{D'} = u_i^D - (\theta_i q(\theta_i) + \lambda_i)\, u_i^D - \eta_i u_i^D + \delta_i e_i$$

The number of workers with a dormant match in the next period is equal the number of workers with a dormant match in the previous period, minus the number of workers with a dormant match that found a new job or was recalled $((\theta_i q(\theta_i) + \lambda_i)\, u_i^D)$, minus the number of dormant matches that ended because the firm exited the market $(\eta_i u_i^D)$, plus the new dormant matches $(\delta_i e_i)$. The pool of unemployed workers without a dormant match searching for a job in sector i evolves according to:

$$u_i^{ND'} = u_i^{ND} - \theta_i q(\theta_i) u_i^{ND} + \eta_i u_i^D$$

The number of workers unemployed without a dormant match tomorrow is equal to the number of unemployed workers without a dormant match today, minus the measure of these workers that found a job, $\theta_i q(\theta_i) u_i^{ND}$, plus the workers with a dormant match that had their inactive match ended, $\eta_i u_i^D$.

The number of firms with a vacancy in sector i evolves according to:

$$v_i' = v_i - q(\theta_i) v_i + v_{i0}$$

The number of firms tomorrow is equal to the number of firms today, minus the firms that had their vacancy filled, $q(\theta_i) v_i$, plus the new entrants in sector i, v_{i0}. Note that for each sector in equilibrium the number of entrants must be the same as the number of firms that had a dormant match destroyed.

$$v_{i0} = \eta_i u_i^D + \theta_i q(\theta_i) u_i^D$$

A firm can have a dormant match destroyed for two reasons. The first one is when the firms cannot recall the worker any more because the worker found another job $(\theta_i q(\theta_i) u_i^D)$, and the second reason is when the dormant firm received a permanently negative shock $(\eta_i u_i^D)$.

The number of firms operating in each sector must the same as the number of workers employed in that sector, so we can also call e_i the number of firms active in sector i. Also, for each sector, the number of firms with a dormant match must be the same as the number of workers with a dormant match, $v_i^{D'} = u_i^{D'}$. Finally, the number of firms operating in sector i evolves according to:

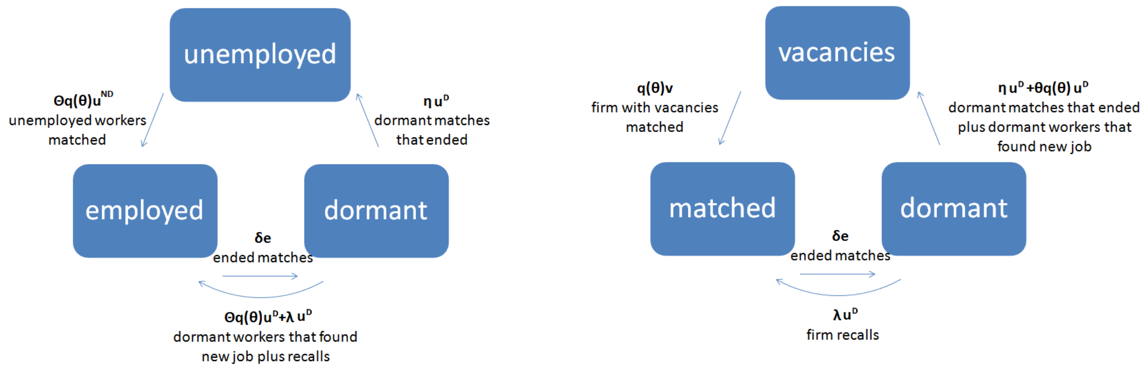
$$e_i' = e_i + q(\theta_i) v_i + \lambda_i u_i^D - \delta_i e_i$$

The number of firms active in each sector tomorrow is equal to the number of active firms today, plus the unmatched firms that had their vacancy filled ($q(\theta_i)v_i$), plus the dormant firms that recovered ($\lambda_i u_i^D$), minus the active firms that had their match destroyed ($\delta_i e_i$) on that period.

Rearranging all worker flow equation and using the result from Theorem 2 we have:

$$\boxed{u_i^{ND} = u_i^D \frac{\eta_i}{\theta q(\theta)} \qquad e_i = u_i^D \frac{(\lambda_i + \eta_i + \theta q(\theta))}{\delta_i} \qquad v_i = u_i^D \frac{(\eta_i + \theta q(\theta))}{q(\theta)}}$$

Figure 1.3: Worker And Firm Flows



In the Figure 1.3 below we can see the dynamics of the workers and firms flows.

1.5 Quantitative Exercise

In this section, I conduct the main exercise of the paper. I estimate the layoff rates, recall rates, and permanent separation rates of the model using Brazilian RAIS data. I then consider how an increase in recall rates, designed to simulate the abolition of the 3-month recall restriction in Brazil, impacts employment, output, wages, and prices.

1.5.1 Parametrization and Estimation of the Model

The general strategy I take when disciplining the model is to take as many parameters as I can from the literature, such as the matching function elasticity etc., and estimate the remaining parameters, such as the recall rates etc., from Brazilian matched employer-employee data.

Since the data used for the estimation is in discrete time (months), I must solve the model using a discrete time approximation. The derivation of the model in discrete time and the computational strategy is described in Appendix 1.7.4. I estimate the model based on data from 2004 to 2010 in one state of Brazil, Bahia. I use a Cobb-Douglas matching function $m(u;v) = u^{\zeta}v^{1-\zeta}$ where I set the matching function parameter ζ such that $\zeta = 0.5$, a common assumption in the literature (e.g. Shimer and Smith (2000)). I assume that firms and workers have the same bargaining power, $\beta = .5$ (i.e. the Hosios condition holds, Shimer (2005)). To estimate the real interest rate in Brazil from 2004 to 2010, I subtract the average interest rate by the cumulative inflation in those years (source: Central Bank of Brazil). The result is a yearly average real interest rate of 8% that corresponds to a monthly interest rate of .7%. Using the unemployment benefit rules in Brazil, I estimate the benefit replacement rate from the data, $z = 0.424$. To estimate the average employment replacement rate in Bahia, Brazil, 2004 to 2010, I use the following equation ($w_{\min}$ is the minimum wage):

$$z_{data} = \frac{mean\left(\mathbb{I}_{(w \leq 1.5 w_{\min})} 0.8w + \mathbb{I}_{(1.5 w_{\min} < w \leq 2.5 w_{\min})} 0.5w + \mathbb{I}_{(w > 2.5 w_{\min})} 1.7 w_{\min}\right)}{mean(w)} \tag{1.10}$$

For simplicity, I assume that the vacancy cost function for the firms is the same as the worker preferences: $f(x^f) = \prod_i \left(x_n^f\right)^{\alpha_n}$. Therefore, solving the firm problem, the vacancy cost is equal to the inverse of the worker marginal utility: $c = \prod \left(\frac{p_i}{\alpha_i}\right)^{\alpha_i} = \frac{1}{\mu(p)}$

For the estimation of λ's, η's and θ I use the following notation (notice that i is now the *sector*):

$\varepsilon =$ Poisson arrival rate of exiting the labor force;

$\theta q(\theta) = \rho =$ Poisson arrival rate of finding a new job;

$\lambda_i =$ Poisson arrival rate of recall in sector i;

$\eta_i =$ Poisson arrival rate of a permanently termination of a dormant match in sector i;

Let u_{i0} denote the number of workers that involuntary left their job in sector i. Let u_{it} denote the number of these workers that are still unemployed and *actively looking* for a job after t periods. Note that we cannot tell if these workers that involuntary left their jobs are still actively looking for a job after t periods or if they left the labor force. We can only observe the number of workers that left jobs involuntary in sector i and that did not get re-employed after t periods, call $u_{it}^{observe}$.

The number of workers that involuntary were separated from a job in sector i and that are still unemployed after t periods evolves according to:

$$u_{it} = u_{i0} \exp\left(-(\rho+\varepsilon)t + \frac{\lambda_i}{\eta_i}(e^{-\eta_i t} - 1)\right)$$

What we observe is the number of workers that were involuntary separated from a job in sector i and that did not start a new job after t periods, independently if they are still looking for a job or not. This number evolves according to:

$$u_{it}^{observe} = u_{i0} \exp\left(-\rho t + \frac{\lambda_i}{\eta_i}(e^{-\eta_i t} - 1)\right) = u_{it} e^{\varepsilon t}$$

Let n_{it} denote the number of workers that were involuntary separated from a job in sector i and found a new job after t periods, and let r_{it} represent the number of these workers that were recalled after t periods in sector i by their old job. The pool of unemployed workers that became unemployed in sector i and that found a new job or got recalled evolves respectively according to:

$$n_{it} = \int_{t}^{t+\Delta t} \rho u_{is} ds = \lim_{\Delta t \to 0} \rho u_{it} \qquad \text{and} \qquad r_{it} = \lambda_i e^{-\eta_i t} u_{it}$$

The hazard rate of finding a job after t periods is:

$$\frac{n_{it}}{u_{it}} = \frac{\rho u_{it}}{u_{it}} = \rho, \quad \text{however we observe:} \quad \frac{n_{it}}{u_{it}^{observe}} = \frac{\rho u_{it}}{u_{it} e^{\varepsilon t}} = \rho e^{-\varepsilon t}$$

The hazard rate of being recalled in sector i after t periods is given by:

$$\frac{r_{it}}{u_{it}} = \frac{\lambda_i e^{-\eta_i t} u_{it}}{u_{it}} = \lambda_i e^{-\eta_i t}, \quad \text{however we observe:} \quad \frac{r_{it}}{u_{it}^{observe}} = \frac{\lambda_i e^{-\eta_i t} u_{it}}{u_{it} e^{\varepsilon t}} = \lambda_i e^{-\eta_i t - \varepsilon t}$$

For the results of the estimations of $\frac{n_{it}}{u_{it}^{observe}}$ and $\frac{r_{it}}{u_{it}^{observe}}$ for each sector see Tables 1.10 and 1.11 respectively, in Appendix 1.7.5.

From Theorem 2 all the θ's (or equivalently ρ's), must be the same across sectors. Using $\frac{n_{it}}{u_{it}^{observe}}$ as the dependent variable, we can recover ρ and ϵ. The results are described in Table 1.2 below. Since the 'other financial and insurance services' sector is very small, I will exclude this sector from my estimations. In Table 1.3 below we can see the results of the estimation of δ's for each sector. To estimate δ_{iy} for sector i for year y I use the following equation:

$$\hat{\delta}_{iy} = \frac{(average\ montly\ involuntary\ job\ separations_{iy})}{average\ montly\ employment_{iy}}$$

Through the lens of my model, I define involuntary job separations to be: (i) layoff, (ii) retirement (this is purely a function of the fact that my model does not allow for voluntary retirement) or (iii) end of a temporary contract. Let $\hat{\delta}_i$ be the average of the δ's within sector i for all years, 2004 to 2010, in Bahia, Brazil.

Table 1.2: Estimation* of ρ, ε, λ's and η's.

$\hat{\boldsymbol{\rho}}^{**}$	0.0547258	
$\hat{\boldsymbol{\varepsilon}}$	0.0258396	
Sector	$\hat{\boldsymbol{\lambda}}_{\boldsymbol{i}}$	$\hat{\boldsymbol{\eta}}_{\boldsymbol{i}}$
Metal and mineral manufacturing	0.003	0.263
Machinery and electronics manufacturing	0.029	0.000
Others manufacturing	0.005	0.165
Chemical manufacturing	0.035	0.000
Textile manufacturing	0.004	0.106
Food and drink manufacturing	0.001	0.688
Construction	0.008	0.161
Retail trade	0.001	0.378
Wholesale trade	0.002	0.448
Real state services	0.011	0.030
Traffic and transportation services	0.001	0.390
Accommodation and food services	0.002	0.268
Health and social services	0.002	0.151
Educational services	0.002	0.185
Agriculture, etc	0.005	0.256
Public utility manufacturing	0.013	0.000
Banking services	0.002	0.141
Automobile manufacturing	0.003	0.141

*Weighted for sector size.

**$\hat{\rho} = \theta q(\theta)$.

Table 1.3: Estimation of $\hat{\delta}$'s per sector i: $\hat{\delta}_i$=(involuntary job separations$_i$) / employment$_i$

Sector	$\boldsymbol{\hat{\delta}_i}$ (Average for years: 2004-2010)
Metal and mineral manufacturing	0.025
Machinery and electronics manufacturing	0.049
Others manufacturing	0.025
Chemical manufacturing	0.020
Textile manufacturing	0.025
Food and drink manufacturing	0.035
Construction	0.076
Retail trade	0.031
Wholesale trade	0.031
Real state services	0.041
Traffic and transportation services	0.023
Accommodation and food services	0.029
Health and social services	0.013
Educational services	0.016
Agriculture, etc	0.066
Public utility manufacturing	0.012
Banking services	0.005
Automobile manufacturing	0.009
Total weighted by sector size	0.035

Table 1.4: Non-Estimated Parameters

		VALUE	SOURCE
Average Employment Benefit	z	0.424	Unemployment Benefit Rule in Bahia, Brazil 2004-2010 (see Equation (1.10)).
Real Trimester Interest Rate	r	0.007	Central Bank of Brazil,2004-2010.
Matching Function Parameter	ζ	0.5	
Wage Bargaining Parameter	β	0.5	

1.5.2 Main Experiment Results

The Table 1.5 below shows the results of the calibration. When there is an increase of 5% on the probability of recall (λ's are multiplied by 1.05), the employment rate increases 10% and the market tightness also increases. Because there is more recall, there will be more production overall. And also because firms became active with a higher frequency, the unemployment rate decreases, the number of vacancies increases causing the market tightness (*vacancies/unemployment*) to increase.

We can see that the sectors with no probability of having a dormant match permanently destroyed, $\eta_i = 0$, were the sectors least affected by the increase on the recall rate (the increase of λ's). Since in those sectors, machinery and electronics, chemical and public utility manufacturing, the dormant match never ends, they are not affected by the increase in the recall rates, except through general equilibrium (GE) effects on market tightness. The GE effects in these sectors do reduce the unemployment rate, but the decline is lower than the average sector.

The sectors that had the largest increase in the employment level, therefore on the respective production level, were the sectors with the highest $\hat{\alpha}_i$. This means that sectors which give workers higher marginal utility (wholesale trade, real state and health and social services, and automobile manufacturing), grow proportionally more than other sectors with weaker recall rates.

Table 1.5: Comparing Baseline Model With an Increase on the Recall Rates, λ's

	Baseline Model			**Model with 5% Larger λ's**		**Data**
Sector	$\hat{\boldsymbol{\alpha}}$'s	$\hat{\boldsymbol{e}}_{\boldsymbol{i}}$	$\hat{\boldsymbol{u}}_{\boldsymbol{i}}^{*}$	$\hat{\boldsymbol{e}}_{\boldsymbol{i}}$	$\hat{\boldsymbol{u}}_{\boldsymbol{i}}^{*}$	$\boldsymbol{e}_{\boldsymbol{i}}$
Metal & mineral manufact.	0.069	0.04	0.014	0.04	0.010	0.04
Machinery & electronics m.	0.056	0.03	0.015	0.03	0.012	0.03
Others manufacturing	0.052	0.03	0.010	0.03	0.008	0.03
Chemical manufacturing	0.031	0.02	0.004	0.02	0.003	0.02
Textile manufacturing	0.017	0.01	0.003	0.01	0.003	0.01
Food & drink manufact.	0.019	0.01	0.005	0.01	0.004	0.01
Construction	0.025	0.01	0.010	0.01	0.007	0.01
Retail trade	0.036	0.02	0.009	0.02	0.006	0.02
Wholesale trade	0.144	0.08	0.035	0.09	0.026	0.08
Real state services	0.396	0.21	0.108	0.23	0.084	0.21
Traffic & transportation s.	0.068	0.04	0.013	0.04	0.010	0.04
Accommodation & food s.	0.018	0.01	0.004	0.01	0.003	0.01
Health & social services	0.216	0.14	0.025	0.15	0.019	0.14
Educational services	0.095	0.06	0.013	0.07	0.010	0.06
Agriculture, etc	0.251	0.11	0.100	0.12	0.075	0.11
Public utility manufact.	0.075	0.05	0.007	0.05	0.006	0.05
Banking services	0.072	0.05	0.004	0.05	0.003	0.05
Automobile manufact.	0.119	0.08	0.010	0.09	0.008	0.08
Total		1.000	0.388	1.094	0.295	1.000
	Baseline Model			**Higher λ's**		**Data**
Market tightness ($\boldsymbol{\theta}$)	0.0051			0.0109		0.0031

$^{*}u_i = u_i^D + u_i^{ND}$

1.6 Conclusion

In this paper I empirically demonstrate the importance of recall and turnover for employment dynamics and wages. I document large the dispersion of job-destruction rates and recall rates across sectors, and I show that sectors with greater job instability pay more. To explain this finding I construct a multi-sector closed economy version of Mortensen and Pissarides (1994) with directed search and heterogeneous recall rates as well as heterogeneous layoff rates across sectors. Once I have estimated the model's parameters using data, I then conduct the main experiment which is to assess the impact of Brazil's recall restrictions on employment dynamics. The Brazilian government does not allow for recalls within 3 months of the date of firing. I simulate the case were the imposition of this law is retracted, and I find that if this restriction on recall activity ceases to exist the employment rate would increase significantly.

One aspect to be explored in the future is the impact that unemployment insurance has on different sectors. Higher unemployment insurance will make unemployment less painful, and sectors with higher turnover rates might benefit more from this. Also, one might explore an economy were there is a cost for a worker to change sectors. Maybe there is over-employment in sectors with high turnover because the worker entry cost in those sectors is lower. Government policies that reduce the cost of changing sectors, such as a subsidy to education, may then subsequently increase employment in low turnover sectors.

1.7 Appendix

1.7.1 Tables

Table 1.6: Ratio of Union Participation by Worker Education Level and Sector 2004-2008, (PNAD)

	Union Participation (%)			
	No Formal Education	Less than High School	High School Degree	College Degree
Textile manufacturing	34.8	29.7	29.6	34.1
Metal & mineral manufacturing	29.6	28.7	37.2	44.2
Chemical manufacturing	42.3	31.7	33.3	40.4
Food & Drink Manufacturing	33.3	25.3	27.5	37.5
Public utility manufacturing	47.6	39.0	46.0	60.2
Machinery & electronics manufact.	25.0	32.7	31.5	41.5
Automobile manufacturing	25.0	39.9	42.5	44.5
Others manufacturing	24.1	24.3	31.6	37.1
Construction	25.9	20.0	23.6	35.8
Trade	22.0	18.0	21.1	28.3
Banking services	52.6	47.9	42.3	59.3
Other financial & insurance serv.	34.6	27.8	24.8	31.9
Real state services	25.2	26.8	25.4	30.7
Traffic & transportation services	35.5	36.2	37.1	39.7
Accommodation & food services	23.7	22.2	23.1	34.5
Health & social services	37.3	24.7	26.7	39.4
Educational services	44.0	27.4	23.2	41.1
Agriculture, etc	34.9	21.7	24.8	29.3

Table 1.7: Brazil, Bahia Summary Statistics of Increase in Size of Private Sectors 2004-2010 (RAIS)

	Size of Sector in 2004 (%)	Size of Sector in 2010 (%)	Change from 2004 to 2010(%)
Textile manufacturing	3.5	4.1	17.9
Food & drink Manufacturing	3.2	3.1	-5.0
Metal & mineral manufacturing	2.7	3.1	13.3
Chemical manufacturing	2.2	1.8	-16.7
Public utility manufacturing	1.6	1.2	-23.2
Machinery & electronics manufact.	0.8	0.9	9.6
Automobile manufacturing	0.5	0.4	-2.1
Others manufacturing	2.1	2.0	-0.9
Construction	6.4	10.5	65.3
Retail trade	20.9	21.3	2.1
Wholesale trade	3.7	3.8	0.9
Banking services	1.5	1.2	-19.1
Other financial & insurance services	0.3	0.4	24.2
Real state services	14.6	14.5	-0.6
Traffic & transportation services	6.5	6.3	-3.1
Accommodation & food services	11.5	10.3	-10.3
Health & social services	5.1	4.4	-13.3
Educational services	4.3	4.4	1.9
Agriculture, etc	8.8	6.4	-28.1
All sectors			34.4

Table 1.8: Estimation results: Dependent Variable Log of Hourly Wage, Controlling for Worker Fixed Effect

Variable	**Coefficient**	**(Std. Err.)**
Age	0.052	(0.000)
Age square	-0.001	(0.000)
Year (2004)	0	(0.000)
2005	0.108	(0.000)
2006	0.221	(0.000)
2007	0.315	(0.000)
2008	0.416	(0.000)
2009	0.531	(0.001)
2010	0.64	(0.001)
Sector (Metal and mineral manufacturing)	0	(0.000)
Machinery and electronics manufacturing	-0.055	(0.002)
Chemical manufacturing	-0.009	(0.002)
Textile manufacturing	-0.102	(0.001)
Food and drink manufacturing	-0.05	(0.001)
Automobile manufacturing	0.133	(0.003)
Public utility manufacturing	-0.019	(0.002)
Others manufacturing	-0.046	(0.001)
Construction	0.015	(0.001)
Retail trade	-0.115	(0.001)
Wholesale trade	-0.062	(0.001)
Banking services	0.375	(0.003)
Other financial and Insurance services	-0.008	(0.003)
Real state services	-0.049	(0.001)
Traffic and transportation services	-0.081	(0.001)
Accommodation and food services	-0.069	(0.001)
Health and social services	0.006	(0.001)
Educational services	-0.093	(0.002)
Agriculture	-0.082	(0.001)
Working 1^{st} semester	0	(0.000)
Working 1^{st} and 2^{nd} semester	0.017	(0.000)
Working 2^{nd} semester	0.004	(0.000)
Union* (10-20%)	0	(0.000)
Union 20-30%	0.002	(0.001)
Union 30-40%	0.063	(0.001)
Union 40-50%	0.119	(0.001)
Union 50-60%	0.248	(0.003)
Intercept	0.033	(0.004)

* Union level by sector and education, calculated using PNAD.

Table 1.9: Estimation results: Dependent Variable Log of Worker Tenure*, Controlling for Worker Fixed Effect

Variable	**Coefficient**	**(Std. Err.)**
Age	0.121	(0.001)
Age square	-0.001	(0.000)
Year (2004)	0	(0.000)
2005	0.124	(0.001)
2006	0.19	(0.001)
2007	0.241	(0.002)
2008	0.282	(0.002)
2009	0.372	(0.002)
2010	0.518	(0.002)
Sector (Metal and mineral manufacturing)	0	(0.000)
Machinery and electronics manufacturing	-0.091	(0.005)
Chemical manufacturing	0.117	(0.005)
Textile manufacturing	0.169	(0.005)
Food and Drink manufacturing	0.307	(0.005)
Automobile manufacturing	0.037	(0.004)
Public utility manufacturing	-0.302	(0.004)
Others manufacturing	0.045	(0.004)
Construction	-0.034	(0.004)
Retail trade	0.018	(0.009)
Wholesale trade	-0.249	(0.003)
Banking services	-0.064	(0.004)
Other Financial and Insurance services	0.072	(0.004)
Real state services	0.152	(0.005)
Traffic and transportation services	0.071	(0.005)
Accommodation and food services	-0.12	(0.004)
Health and social services	0.156	(0.007)
Educational services	0.152	(0.010)
Agriculture	0.466	(0.010)
Sector growth rate	-0.431	(0.008)
Intercept	0.01	(0.014)

*Worker Tenure is adjusted for recall, the tenure represents the total amount of time that the worker is employed on the firm.

1.7.2 Regressions of Wage and Tenure on Sector

1.7.3 Proofs

Proof. of Corollary 2 Taking the derivative of the equilibrium wage given by Equation (1.8) with respect to δ_i, η_i and λ_i, respectively:

$$
\begin{aligned}
\frac{\partial w_i^*}{\partial \delta_i} &= \frac{1}{\mu(p)} \frac{\beta}{(1-\beta)} \frac{c}{q(\theta)} \frac{r+\eta_i+\theta q(\theta)}{(r+\lambda_i+\eta_i+\theta q(\theta))} & > 0 \\
\frac{\partial w_i^*}{\partial \eta_i} &= \frac{1}{\mu(p)} \frac{\beta}{(1-\beta)} \frac{c\delta_i}{q(\theta)} \frac{\lambda_i}{(r+\lambda_i+\eta_i+\theta q(\theta))^2} & > 0 \\
\frac{\partial w_i^*}{\partial \lambda_i} &= -\frac{1}{\mu(p)} \frac{\beta}{(1-\beta)} \frac{c\delta_i}{q(\theta)} \frac{r+\eta_i+\theta q(\theta)}{(r+\lambda_i+\eta_i+\theta q(\theta))^2} & < 0
\end{aligned}
$$

■ ■

Proof. of Corollary 3 Taking the derivative of the equilibrium price given by Equation (1.9) with respect to δ_i, η_i and λ_i, respectively:

$$
\begin{aligned}
\frac{\partial p_i^*}{\partial \delta_i} &= \frac{\partial w_i^*}{\partial \delta_i} + \frac{c}{q(\theta)} \frac{r+\eta_i+\theta q(\theta)}{(r+\lambda_i+\eta_i+\theta q(\theta))} & > 0 \\
\frac{\partial p_i^*}{\partial \eta_i} &= \frac{\partial w_i^*}{\partial \eta_i} + \frac{c\delta_i}{q(\theta)} \frac{\lambda_i}{(r+\lambda_i+\eta_i+\theta q(\theta))^2} & > 0 \\
\frac{\partial p_i^*}{\partial \lambda_i} &= \frac{\partial w_i^*}{\partial \lambda_i} - \frac{c\delta_i}{q(\theta)} \frac{r+\eta_i+\theta q(\theta)}{(r+\lambda_i+\eta_i+\theta q(\theta))^2} & < 0
\end{aligned}
$$

■ ■

1.7.4 Estimation With Discrete Time

Since the data is in discrete time, I will use discrete time for the estimation of the model. For this part I will use the results from Theorem 2 from Corollary 1, θ and W is the same across sectors.

On the worker side we have the equations:

$$
\begin{aligned}
W &= \mu(p)(w_i+\pi)\Delta t + \frac{1}{1+r\Delta t}\Big(\delta_i \Delta t U_i^D + (1-\delta_i\Delta t)W\Big)\\
U_0 &= (z+\mu(p)\pi)\,\Delta t + \frac{1}{1+r\Delta t}\Big(\theta q(\theta)\Delta t W + (1-\theta q(\theta)\Delta t)U_0\Big)\\
U_i^D &= (z+\mu(p)\pi)\,\Delta t + \frac{\Big(\big((\lambda_i+\theta q(\theta))\,W + \eta_i U_0\big)\Delta t + (1-(\lambda_i+\theta q(\theta)+\eta_i)\,\Delta t)\,U_i^D\Big)}{1+r\Delta t}
\end{aligned}
$$

And on the firm side we have the equations:

$$
\begin{aligned}
J &= (p_i - w_i)\,\Delta t + \frac{1}{1+r\Delta t}\Big(\delta_i\Delta t V_i^D + (1-\delta_i\Delta t)J\Big)\\
V_i^D &= \frac{1}{1+r\Delta t}\Big(\lambda_i\Delta t J + (1-(\lambda_i+\theta q(\theta)+\eta_i)\,\Delta t)\,V_i^D\Big)\\
0 &= -c\Delta t + \frac{1}{1+r\Delta t} q(\theta)\Delta t J
\end{aligned}
$$

The wages and prices of equilibrium are the same as in the discrete case, given by Equations (1.8) and (1.9):

$$
w_i^* = \frac{1}{\mu(p)}\left(z + \frac{\beta}{1-\beta}\frac{c}{q(\theta)}\,(r+\theta q(\theta))\right) + \frac{1}{\mu(p)}\frac{\beta}{1-\beta}\frac{c}{q(\theta)}\delta_i\left(\frac{r+\eta_i+\theta q(\theta)}{r+\lambda_i+\eta_i+\theta q(\theta)}\right) \tag{1.11}
$$

$$
p_i^* = w_i^* + \frac{c}{q(\theta)}\left(r+\delta_i\left(\frac{r+\eta_i+\theta q(\theta)}{r+\lambda_i+\eta_i+\theta q(\theta)}\right)\right) \tag{1.12}
$$

1.7.4.1 Estimation Strategy

The estimation strategy is to estimate the preferences over the good of each sector,α's, to target the employment rate across sectors, e's. And estimate the market tightness, θ, to target the total recall rate observed on the economy, *recalls / involuntary separations.*

- Guess: θ_{guess}, $\alpha_{guess} = (\alpha_{1guess}, \alpha_{2guess}, ..., \alpha_{Nguess})$ and μ_{guess}
- Use parameters estimated from the data: $\hat{\delta}$'s, $\hat{\lambda}$'s, $\hat{\eta}$'s and z, r. And choose β and ζ.
- Recover $\hat{c} = 1/\mu_{guess}$

- Recover $\hat{w}_i$ using Equation (1.11) and $\hat{p}_i$ using Equation (1.12)

- Recover $\hat{\mu}(p) = \prod_i (\alpha_{iguess}/\hat{p}_i)^{\alpha_{iguess}}$ and check if $\hat{\mu}(p) \cong \mu(p)_{guess}$

- Set $e_0 = e_0^{observed}$ and recover $\hat{e}_i = e_0 \frac{\alpha_{iguess}}{\alpha_{0guess}} \frac{\hat{p}_0}{\hat{p}_i}$

- Repeats until:

  – $e_i^{observed} \cong \hat{e}_i$ for all $i's$

  – $total\ economy\ recall\ rate^{observed} \cong \frac{\sum_i \hat{u}_i^D \lambda_i}{\sum_i \hat{e}_i \delta_i}$ for all $i's$

1.7.5 Estimation of Parameters

Table 1.10: Estimation results: ratio of unemployed workers that found a new job and the workers that still did not found a job after t periods by sector, $n_{it}/u_{it}^{observed}$

Sector	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10	t=11	t=12
Metal&mineral manufact.	0.089	0.102	0.054	0.048	0.049	0.055	0.068	0.095	0.090	0.091	0.086	0.083	0.084
Machinery&electronics m.	0.109	0.107	0.065	0.060	0.057	0.056	0.064	0.081	0.066	0.066	0.060	0.059	0.057
Others manufacturing	0.071	0.072	0.039	0.038	0.041	0.047	0.063	0.088	0.090	0.082	0.075	0.071	0.067
Chemical manufacturing	0.138	0.110	0.062	0.062	0.067	0.075	0.095	0.138	0.133	0.121	0.116	0.106	0.112
Textile manufacturing	0.035	0.033	0.022	0.020	0.024	0.026	0.036	0.053	0.049	0.044	0.037	0.032	0.030
Food&drink manufact.	0.068	0.070	0.039	0.036	0.037	0.045	0.050	0.089	0.099	0.078	0.073	0.063	0.058
Construction	0.054	0.096	0.072	0.063	0.060	0.064	0.067	0.087	0.083	0.080	0.076	0.074	0.072
Retail trade	0.039	0.059	0.039	0.036	0.039	0.049	0.070	0.095	0.088	0.082	0.078	0.072	0.068
Wholesale trade	0.074	0.098	0.058	0.053	0.058	0.070	0.096	0.124	0.128	0.114	0.109	0.104	0.096
Other finance&insurance	0.135	0.126	0.074	0.059	0.064	0.064	0.086	0.112	0.091	0.088	0.090	0.091	0.096
Real state services	0.117	0.109	0.059	0.054	0.053	0.064	0.068	0.083	0.080	0.073	0.070	0.067	0.067
Traffic&transportation s.	0.139	0.136	0.079	0.074	0.075	0.085	0.104	0.139	0.132	0.130	0.126	0.127	0.125
Accommodation&food s.	0.077	0.075	0.042	0.036	0.038	0.043	0.053	0.073	0.067	0.062	0.059	0.054	0.050
Health and social s.	0.092	0.092	0.051	0.046	0.047	0.049	0.064	0.081	0.082	0.072	0.070	0.066	0.062
Educational services	0.053	0.053	0.034	0.027	0.025	0.029	0.036	0.049	0.043	0.038	0.036	0.032	0.033
Agriculture, etc	0.039	0.080	0.052	0.041	0.041	0.045	0.053	0.100	0.115	0.127	0.121	0.097	0.067
Public utility manufact.	0.134	0.132	0.069	0.072	0.061	0.058	0.062	0.075	0.068	0.072	0.074	0.068	0.062
Banking services	0.393	0.414	0.206	0.162	0.187	0.219	0.266	0.492	0.704	2.416	2.234	2.390	1.649
Automobile manufact.	0.317	0.264	0.138	0.145	0.150	0.193	0.177	0.247	0.286	0.351	0.457	0.671	1.852

Table 1.11: Estimation results: ratio of unemployed workers that were recalled and the workers that still did not found a job after t periods, $r_{it}/u_{it}^{observed}$

Sector	t=0	t=1	t=2	t=3	t=4	t=5	t=6
Metal & mineral manufact.	0.0025	0.0078	0.0062	0.0066	0.0096	0.0112	0.0156
Machinery & electronics manufact.	0.0172	0.0358	0.0329	0.0247	0.0209	0.0168	0.0199
Others manufacturing	0.0126	0.0045	0.0028	0.0124	0.0098	0.0101	0.0178
Chemical manufacturing	0.0211	0.0455	0.0425	0.0321	0.0275	0.0224	0.0274
Textile manufacturing	0.0029	0.0084	0.0056	0.0055	0.0058	0.0045	0.0092
Food & drink manufacturing	0.0011	0.0049	0.0041	0.0063	0.0227	0.0378	0.0774
Construction	0.0049	0.0160	0.0152	0.0158	0.0166	0.0160	0.0168
Retail trade	0.0009	0.0027	0.0024	0.0033	0.0039	0.0062	0.0118
Wholesale trade	0.0012	0.0056	0.0119	0.0138	0.0238	0.0203	0.0208
Other finance & insurance serv.	0.0013	0.0023	0.0008	0.0013	0.0034	0.0015	0.0058
Real state services	0.0101	0.0165	0.0107	0.0109	0.0101	0.0108	0.0143
Traffic & transportation services	0.0014	0.0046	0.0040	0.0083	0.0098	0.0095	0.0181
Accommodation & food services	0.0012	0.0067	0.0037	0.0053	0.0058	0.0064	0.0103
Health & social services	0.0020	0.0073	0.0027	0.0028	0.0036	0.0040	0.0088
Educational services	0.0015	0.0068	0.0038	0.0033	0.0052	0.0050	0.0074
Agriculture, etc	0.0047	0.0149	0.0101	0.0124	0.0154	0.0189	0.0300
Public utility manufact.	0.0063	0.0171	0.0193	0.0125	0.0152	0.0083	0.0078
Banking services	0.0014	0.0060	0.0064	0.0074	0.0035	0.0077	0.0099
Automobile manufact.	0.0000	0.0041	0.0046	0.0070	0.0064	0.0030	0.0122
Total weighted by sector size	0.00397	0.00958	0.00751	0.0084	0.00979	0.01081	0.0167

Table 1.12: Estimation results: Dependent Variable Log of Worker Tenure, Controlling for Worker Fixed Effect

Variable	**Coefficient**	**(Std. Err.)**
Age	0.118	(0.001)
Age square	-0.001	(0.000)
Year (2004)	0	(0.000)
2005	0.126	(0.001)
2006	0.189	(0.001)
2007	0.234	(0.002)
2008	0.277	(0.002)
2009	0.364	(0.002)
2010	0.504	(0.002)
Sector (Metal and mineral manufacturing)	0	(0.000)
Machinery and electronics manufacturing	-0.105	(0.005)
Chemical manufacturing	0.184	(0.005)
Textile manufacturing	0.309	(0.005)
Food and Drink manufacturing	0.043	(0.004)
Automobile manufacturing	0.481	(0.010)
Public utility manufacturing	0.133	(0.008)
Others manufacturing	0.126	(0.005)
Construction	-0.307	(0.004)
Retail trade	0.052	(0.004)
Wholesale trade	-0.024	(0.004)
Banking services	0.112	(0.010)
Other Financial and Insurance services	0.037	(0.009)
Real state services	-0.241	(0.004)
Traffic and transportation services	-0.056	(0.004)
Accommodation and food services	0.082	(0.004)
Health and social services	0.175	(0.005)
Educational services	0.093	(0.005)
Agriculture	-0.124	(0.004)
Sector growth rate	-0.437	(0.008)
Intercept	0.033	(0.015)

CHAPTER 2

Wage Inequality and Job Stability

2.1 Introduction

In this paper, I make three contributions. Firstly, I use matched employer-employee data to establish two stylized facts: (i) separations are disproportionately comprised of layoffs for low wage workers, not separations due to job-to-job transitions, and (ii) controlling for worker characteristics (tenure, education, etc.) and firm characteristics (sector, size, etc.), workers in jobs with higher turnover rates, defined to be the ratio of total gross worker flows to firm size, have lower wages. To my knowledge, the first stylized fact is new. Secondly, I show that existing models such as Burdett and Coles (2003) and Shi (2009) are consistent with the second fact that low wage jobs have higher turnover.[1] However, these models are *inconsistent* with the first fact. Burdett and Coles (2003) and Shi (2009) assume that the exogenous component of job destruction is constant across firms, and low wage workers optimally search on-the-job much more intensely than high wage workers. Lastly, I solve a Burdett and Mortensen (1998) model with general equilibrium in which firms are heterogeneous with respect to their exogenous layoff rates. I demonstrate theoretically that firms with greater exogenous layoff rates offer lower wages to workers in equilibrium. I show that after one takes into account the disproportionate share of layoffs among low wage workers, the burden of firing taxes shifts to low wage workers, and as a result, such labor market interventions can increase wage inequality nearly twice as much as models such as Burdett and Coles (2003) and Shi (2009) would predict.

[1] In those models, firms are indifferent between paying low wages to short tenure workers (who will leave because of OJS) and high wages to long tenure workers (who are unlikely to leave for a longer period of time) as these two contracts will result in the same profits for the firm. In that framework, all firms and workers are identical.

Models with constant exogenous layoff rates among firms such as Burdett and Coles (2003) and Shi (2009) imply that the burden of firing taxes is equally shared among both high wage workers and low wage workers. I depart from these existing frameworks by incorporating heterogeneous layoff rates into an otherwise standard Burdett and Mortensen (1998) model. Workers are risk neutral, receive a constant unemployment while unemployed, and are allowed to search on-the-job. Upon entry, firms draw an exogenous layoff rate and must post wages to attract workers. In equilibrium there is a non-degenerate distribution of wages offered by firms. Firms that match with workers operate at a constant returns to scale technology and are homogeneous with respect to productivity.

The key features of the model that deliver the correct negative turnover-wage relationship and the fact that separation rates among low wage workers are primarily layoffs are (i) heterogeneity with respect to the exogenous job destruction component and (ii) on-the-job search (OJS). The intuition behind this results is as follows: Imagine a firm that has an exogenous job destruction rate of 1 – every period the firm separates from the worker for exogenous reasons. In this case, the firm will offer workers the lowest per-period wage possible (set equal to the worker's outside option) ignoring any implications that such a low wage would have for the workers' on-the-job search behavior (since the worker separates for sure, the presence of OJS is irrelevant for this firm). Now consider a firm with an intermediate job destruction rate of .1 – every 10 periods on average the firm separates from its worker for exogenous reasons. In this case, when the firm makes its wage offer to workers, it must take into account the probability the worker will leave the firm by finding a better paying job. In equilibrium, low-wage jobs are higher turnover jobs in which the majority of separations are layoffs, and high-wage jobs are low turnover jobs in which the majority of separations are due to on-the-job search.

After theoretically characterizing the model and exploring its equilibrium properties, I use a matched employer-employee dataset from Brazil to estimate the key parameters of my model. What allows me to estimate the model and differentiate between layoffs versus separation due to on-the-job search is that the data explicitly records layoffs separately from quits. I can then use the data to estimate the distribution of layoff rates across firms.[2] For the estimation and subsequent exercise to make sense, since Brazil has relatively large firing costs in place throughout my sample period, I estimate a version of model in which firms must pay a fraction of prior wages to a worker when the worker is laid off. This type of firing cost mimics the salient features of the 'FGTS' program in Brazil well enough that I can directly map the statutory 'FGTS' severance payments to my model. The calibrated firing cost is approximately equal to 30% of the workers' prior wages.

With the estimated model in hand, I conduct my main experiment. I compare the implications of Brazil's firing costs on wage inequality in two economies, one with heterogeneous layoff rates (my benchmark model) versus one with homogeneous layoff rates. I find that introducing a 30% firing cost in an economy with heterogeneous layoff rates increases wage inequality 2× more than in an economy with homogeneous layoff rates. The intuition is that low wage workers are disproportionately fired more often in my model, and therefore a firing tax is primarily borne by low wage workers. As a results, with a firing tax, low wage workers are offered even lower wages, exacerbating inequality.

Because the firing cost is modeled as a transfer, and since both firms and workers have linear returns, the model's allocation of workers across firms is unchanged. The dispersion in utility levels and overall welfare level in the economy is unaffected by the firing cost. All that changes in equilibrium is the posted wages offered by firms, and I show this both theoretically and quantitatively.

This paper relates to a large literature on search that tries to understand labor market flows, employment in equilibrium, and wage dispersion. Diamond (1971) started this literature by showing how the introduction of a small search friction can drastically impact equilibrium outcomes. The present-day search literature is composed of two sides: (i) matching and (ii) contract-posting. The matching literature started with Pissarides (1990) and Mortensen and Pissarides (1994), and the contract-posting literature was initiated with Burdett and Judd (1983) and Burdett and Mortensen (1998) (BM). BM propose a model

[2]Using worker and firm fixed effects, I can actually parse out what fraction of a firms layoff rate is due to worker characteristics versus firm characteristics

with random search that also allows workers to do on-the-job search (OJS). Coles (2001) extended the BM framework allowing for firms to have no future wage pre-commitment, predicting that in equilibrium the wage will depend on the firm size. More recently Coles and Mortensen (2012) extended the BM framework by introducing a hiring margin, as well as firm entry/exit and private-information firm specific productivity shocks into the matching framework with on-the-job search, implying that in equilibrium the wage will only depend on the aggregate state of the economy and the firm productivity. Menzio and Shi (2009) argue that random search makes it difficult to solve for equilibrium outside the steady state, and so they introduce a directed search framework with risk neutral agents and full information, allowing for on-the-job search. Menzio and Shi (2009) prove that with directed search the model with on-the-job search can be solved outside the steady state using a block recursive equilibrium. Moscarini and Postel-Vinay (2013) analyze the impact of aggregate stochastic dynamics in wage-posting models with random search, proving the existence of a rank preserving equilibrium path. The main findings of the authors in this paper corroborate with their empirical findings in a previous article. Moscarini and Postel-Vinay (2012) show empirically that large employers have more cyclical job creation. Other authors also contributed to the more empirical side of this literature, such as Nagypál (2008). This last author estimated that employer-to-employer transitions are responsible to 49% of all separations from employers in the United States. Cahuc et al. (2006) propose and empirically estimate a model were the wage is determined by a combination of productivity, the worker's bargaining power, and between-firm competition resulting from OJS.

My policy experiments also relate to the literature on firing costs, such as Hopenhayn and Rogerson (1993), Castex and Ricaurte (2011), Jaef (2011), among others. In those models, the decision of firms to fire workers is endogenous, but there is typically no on-the-job search. Very recent work by Pinheiro and Visschers (2013) theoretically analyzes wage-turnover relationships in a partial equilibrium Burdett and Mortensen (1998) model, finding that low wage workers have higher turnover rates. What differentiates my work from theirs is that (i) my model is in general equilibrium with an endogenous job finding rate (and is therefore useful for the subsequent policy analysis I conduct), (ii) I use the model to assess the role of firing taxes on wage inequality, and (iii) I estimate the model with matched employer-employee data.

On the empirical side, both Blau and Kahn (1996) and Koeniger et al. (2004) analyze labor market institutions and wage inequality. Blau and Kahn (1996) conduct a cross-country regression of employment protection on wage inequality for OECD countries. In general, they find that more employment protection (such as strong unions) leads to less wage inequality. Their analysis however omits the role of extensive margin of work, and ignores the endogeneity of labor market protection. Taking a structural approach, Bonhomme and Jolivet (2009) estimate a dynamic model of wages, amenities and labor mobility. They find that there is a near-zero wage/amenity correlation meaning that low wage workers are not given compensating differentials. They argue that this is because there is heterogeneity in mobility costs, even though workers have a large marginal willingness to pay for amenities.

In the next Section 2.2 I briefly motivate this paper with some new data facts. After, in Section 2.3 I describe the model, the optimal quit behavior of workers and firms, and derive the optimal contract a firm with a particular exogenous destruction rate offer to its workers. In Section 2.4 I extend the model and analyze the impact of a firing tax on wage inequality. Then in Section 2.5 I simulate the model and calibrate to match some moments of the data. Finally, in the last Section 2.6 I conclude and discusses some implications of the results and propose some extensions.

Lane (2000) discuss the impact of turnover on low-wage workers in detail. In particular, she argues that less-educated workers are less likely to jump jobs and more likely to be pushed out. In Section 2.2.1, I show that, controlling for workers characteristics, this is true for low wage workers.

2.2 Data Motivation

The statistical analysis in this section and the estimation of the model in later sections are both based on a Brazilian dataset called RAIS. RAIS is a matched employer/employee panel for every formal worker in Brazil. Because of the size of the data, I chose to analyze one state of Brazil, Bahia. The RAIS dataset includes worker characteristics such as age, education, sex, tenure, average annual wage and the wage in December. RAIS also provides some firm characteristics such as sector. Although there are not a lot of firm characteristics, since it is possible to follow firms and workers using their respective id numbers, I can construct several additional firm characteristics such as size, turnover rate, etc.

First let me define several statistical measures for the data analysis that follows. Let $t = 1998, 1999, \ldots, 2010$ denote the year of the data. Let $inflow_t$ be the gross hires of a firm, and $outflow_t$ be the gross separations of a firm (including separations due to on-the-job search, quits, and layoffs). I will define the annual turnover rate, the annual separation rate and the annual growth rate to be,

$$turnover_t = \frac{inflow_t + outflow_t}{size_t}. \quad separation_t = \frac{outflow_t}{size_t}. \quad growth_t = \frac{size_{December\ t}}{size_{January\ t}}.$$

The hourly wage, *hourly wage*$_t$, is computed as the average monthly salary in a year divided by the number of hours worked per month.

$$w_t = log(hourly\ wage_t)$$

The following Table 2.1 contains a summary of the most relevant variables from Bahia, Brazil 2010. Table 2.1 shows that the turnover and separation rate vary considerably across sectors. In the construction sector, in which workers are typically bound by shorter contracts, turnover is much greater. Also looking to the standard deviations we can see that there is a lot of dispersion in these rates across sectors.

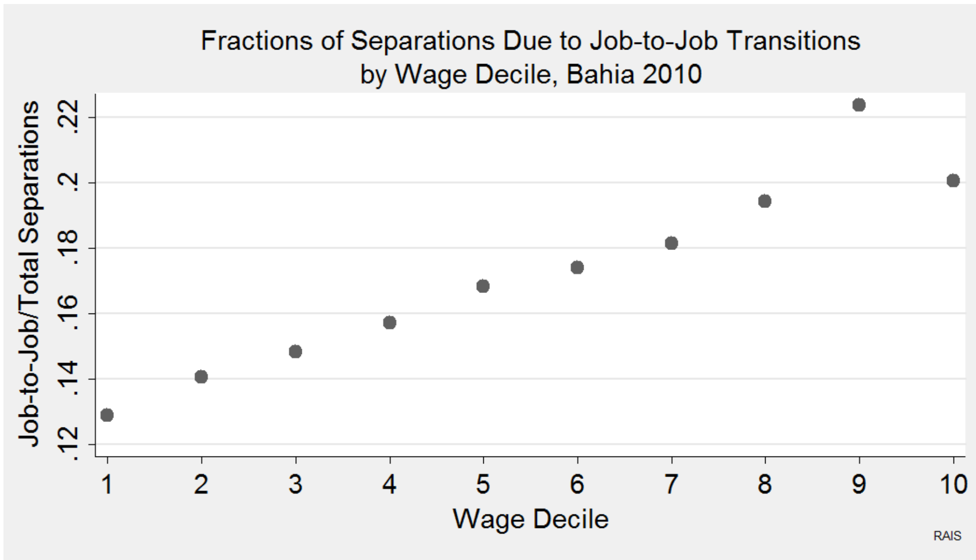
Table 2.1: Brazil, Bahia Summary Statistics 2010 (RAIS)

	Mean	**S.D.**	**p25**	**p50**	**p75**	**N**
Wage	6.80	0.751	6.2	6.5	7.1	3,016,662
Size	2567	8944.8	17	170	1003	3,016,662
Turnover	1.69	8.102	0.35	0.85	1.96	3,009,957
Separation	0.78	4.045	0.12	0.35	0.80	3,009,957
Growth	2.46	34.106	0.90	1.00	1.14	2,896,137
Tenure	50.91	81.458	5.9	16.9	51.9	3,016,662
	Others	**Manufacturing**	**Construction**	**Government**	**Agriculture**	
Turnover	1.56	1.16	3.67	0.33	5.13	
(S.D.)	(8.9)	(2.7)	(7.7)	(0.4)	(15.5)	
Separation	0.71	0.51	1.69	0.15	2.51	
(S.D.)	(4.4)	(1.3)	(3.8)	(0.2)	(7.7)	

2.2.1 High Wage Workers Have More Job-to-Job Transitions Than Low Wage Workers

Most models of on-the-job search, including Burdett and Coles (2003) and Shi (2009), assume that layoff rate is exogenous and constant cross firms, and that low wage workers optimally search on-the-job much more intensely than high wage workers. As a result, if one were to observe a job separation by a low wage worker in models like Burdett and Coles (2003) and Shi (2009), it is much more likely the result of on-the-job search. In contrast, Figure 2.1 demonstrates that the proportion of job separations that is due to worker job-to-job transitions is *increasing* with wages. I find that in the lowest wage decile, the fraction of separations due to job to job transitions is 13%, whereas for the highest wage decile, it is more than 20%, nearly double that of the lowest wage workers in Bahia, Brazil in 2010. In Appendix 2.7.1, Figure 2.6 illustrates that the proportion of job separations that is due to job-to-job transitions by educations levels looks quite similar.

Figure 2.1: Fraction of job-to-job transitions and total separations by wages, Bahia-Brazil (RAIS) 2010



To formalize the result in Figure 2.1, I will turn to a logistic model which allows me to control for worker characteristics. Table 2.2 demonstrates results from a logistic regression of the probability of a worker move from one job to another given that the worker quit or was fired (he was separated from the job). Controlling for firms and worker characteristics, the probability of job to job transition is increasing with the wage decile of the worker. Also most educated workers have a higher probability of moving from one job to another job.

Table 2.2: Logistic regression: P(Job to Job Transition|Separated from Job), RAIS, Bahia 2010

	(1)	(2)	(3)
Tenure	-0.016***	-0.016***	
	(0.000)	(0.000)	
Wage Decile			
2	0.090***	0.097***	0.066***
	(0.013)	(0.013)	(0.013)
3	0.151***	0.166***	0.109***
	(0.013)	(0.013)	(0.013)
4	0.210***	0.229***	0.146***
	(0.013)	(0.013)	(0.013)
5	0.281***	0.308***	0.213***
	(0.013)	(0.013)	(0.013)
6	0.324***	0.355***	0.259***
	(0.013)	(0.013)	(0.013)
7	0.347***	0.386***	0.286***
	(0.013)	(0.013)	(0.013)
8	0.426***	0.491***	0.338***
	(0.014)	(0.014)	(0.014)
9	0.530***	0.640***	0.433***
	(0.015)	(0.015)	(0.015)
10	0.370***	0.524***	0.130***
	(0.017)	(0.016)	(0.016)
Some Education			
Less than HS	0.062***		0.052***
	(0.011)		(0.011)
HS	0.271***		0.291***
	(0.009)		(0.009)
College	0.450***		0.494***
Demographic Controls	Yes	Yes	Yes
Industry Controls	Yes	Yes	Yes
N	792,430	792,430	792,430

$^{***}p < 0.001$. Sample: 2010 Bahia. Demographic controls include a quadratic in age and dummies for sex. Tenure expressed in years. Omitted education group is those with no formal education.

2.2.2 High Turnover Jobs Pay Lower Wages

It is intuitive that the wage is influenced by firm and worker characteristics. In this section I show that high turnover firms offers lower wages to workers. Consider the panel regression:

$$Ln(wage)_{ijt} =$$

$$f\left(\mathbb{I}(\text{firm turnover quintile}_{ij}), \mathbb{I}(\text{firm size 6th-tile}_{ijt}), \mathbb{I}(\text{sector}_{ij}), \text{tenure}_{it}, \mathbb{I}(\text{year}_t), X_j\right)$$

We have $i = 1, ..., I$ individuals and $j = 1, ..., J$ firms observed over $t = 1, ..., T$ time periods (years). $Ln(wage)_{ijt}$ is the dependent variable and it is the log of the wage of individual i working for firm j in the period t. ϵ_{ijt} is the error term. $\mathbb{I}(\text{firm turnover quintile}_{ij})$ is a dummy corresponding to the quintile that of the firm j's average turnover rate across all years. Any given firm j only has one turnover rate (the average across all years), but workers may switch firms, hence the dependence on i. $\mathbb{I}(\text{firm size 6th-tile}_{ijt})$ and $\mathbb{I}(\text{sector}_{ij})$ are dummies corresponding respectively to the $6th$-tile of the size and sector of the firm j in period t that employs worker i. Tenure_{it} is the tenure of the worker i at time t and $\mathbb{I}(\text{year}_t)$ is a dummy for the year. Finally, X_j is a vector of worker characteristics: age, age squared, education dummies and gender dummies.

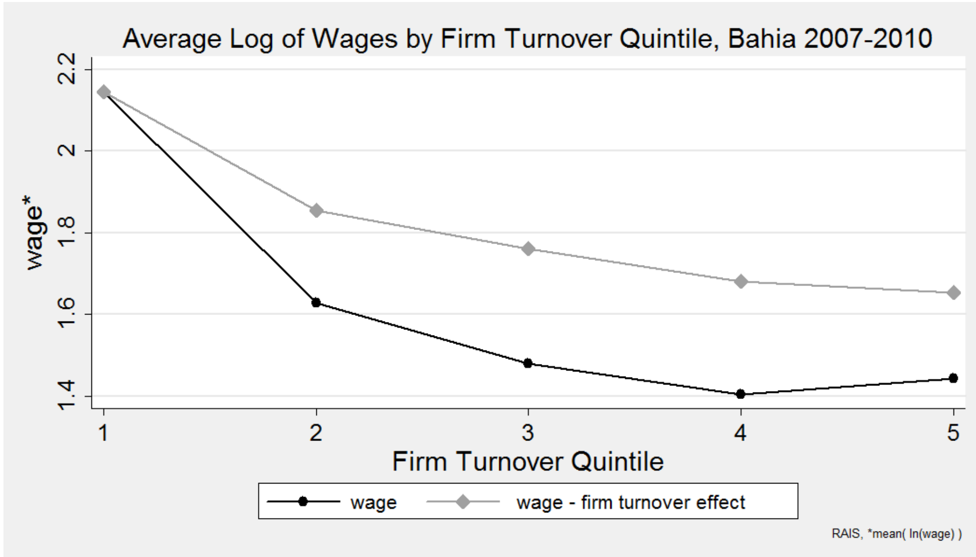
The results from the panel model described by Equation (2.1) are summarized in Appendix 2.7.3. The results show that wages are decreasing with firm turnover, even without controls for firm size or sector. The wage is also increasing with firm size. Turning to the regression that includes all available control variables[3], *ceteris paribus*, a worker that works in a firm in the highest turnover quintile will have in expectation a 20% lower wage than if the worker was working for a firm in the lowest quintile. Figure 2.2 summarizes this relationship. The line labelled 'wage' is the average observed log of the wage of workers in any given turnover quintile. The counter-factual line labelled 'wage-Firm Turnover Effect' depicts the average wage a worker would receive if the worker were moved from their actual firm to another firm in the lowest turnover quintile. *Ceteris paribus*, this graph shows that the average wage of workers would increase by close to 20%, express in log points here.

2.2.3 Firms Differ in Their Firing Rates

The main assumption underlying the model is that firms differ exogenously with respect to their firing rates. To provide suggestive evidence in support of this modelling assumption, I consider a sample of workers who were reported as being laid-off on one of their jobs over a period of 10 years (this layoff indicator is not inferred, it is directly reported). Among

[3]Appendix 2.7.3 regression (1)

Figure 2.2: Wages by firm turnover quintile, Bahia-Brazil (RAIS) 2007-2010



this subset of households, I ask the following question: what fraction of the job duration is attributable to the worker (a person fixed effect) and what fraction is attributable to the firm (a firm fixed effect). Let y_{ijt} be the duration of the job that ended at time t of individual i in firm j, let ϕ_j be the firm j fixed-effect, and let α_i be the person i fixed-effect. I also include controls such as a_{ijt} which is the age of individual i when hired for the job that ended at time t. To decompose the job duration into worker and firm components, I first estimate the following equation:

$$y_{ijt} = \beta a_{ijt} + \phi_j + \alpha_i + u_{ijt}$$

The next step is to use a variance decomposition to recover the importance of the firm fixed effect. More specifically, I decompose the variance into the following components:

$$1 = \frac{Cov(y, \beta age)}{Var(y)} + \frac{Cov(y, PersonFE)}{Var(y)} + \frac{Cov(y, FirmFE)}{Var(y)} + \frac{Cov(y, \epsilon)}{Var(y)}$$

In the following Table 2.3 we can note that for workers with lower levels of education, the firm fixed effect and the person fixed effect have almost the same importance in explaining the variance of the duration of a job: about 30%. However, for more educated workers, with at least a high school degree, the firm fixed effect explain very little of the duration of a job. Therefore, for the following estimations in these paper I only use workers with less than a high school degree, and for those workers, I take some component of the job destruction rate as exogenous.

Table 2.3: Comparing linear regression of duration of the match on person and firm fixed–effects (RAIS 2000-2010), by education level (only male, age from 25 to 50)

Education Level	**None**	**Less than HS**	**HS**	**College**
Variance Decomposition				
Initial Age	0.30	0.33	0.37	0.43
person FE	0.35	0.35	0.56	0.52
Firm FE	0.33	0.30	0.05	0.03
Residual	0.02	0.02	0.03	0.02
SDs of FEs				
Person	64.13	76.71	77.97	79.62
Firm	47.62	58.32	43.29	11.87
Correlation of Job Length				
and Initial Age	0.35	0.38	0.37	0.44
and Person FE	0.30	0.27	0.44	0.51
and Firm FE	0.38	0.31	0.08	0.18
and Residual	0.16	0.14	0.16	0.14
Observations	1,215,473	956,910	2,223,920	353,151
Person Categories	537,787	458,151	850,928	118,103
Firm Categories	63,875	71,990	103,976	18,564
Adj R squared	0.95	0.95	0.96	0.97

2.3 Model

2.3.1 Basic Framework

Consider a continuous time environment in which there is a unit mass of ex-ante identical workers and firms. Workers are risk neutral, infinitely lived, and seek to maximize preferences over non-durable consumption c_t, discounting the future at a rate of r.

At each instant, a worker can either be unemployed, in which case the worker receives unemployment benefit b, or employed, in which case the worker works for a firm that promises the worker a lifetime utility of at least $\tilde{U}$. Let $G(\tilde{U})$ denote the measure of workers employed at firms that offer utility $\tilde{U}$ to their employees. Also let $N(\delta)$ denote the number of workers employed in firms with separation rate no greater than δ. Workers are free to search on-the-job. Unemployed workers' search effort is normalized to unity whereas employed workers' search effort is given by $s < 1$.

Firms operate a constant returns to scale technology and differ with respect to the rate at which matches become unproductive. Let δ denote the firm specific job destruction rate. δ is the Poisson intensity with which a job becomes unproductive and a worker is laid off. Every period, firms that are already matched with existing employees are free to post a vacancy to attract a new worker.

Firms hire workers by posting vacancies. Let v denote vacancies posted by firms, and let u denote the mass of unemployed workers and e is the measure of employed workers. There is recruiting by entrant and incumbent firms. Let c_e denote the cost of posting a vacancy for an entrant. The recruitment of the existing firms happens when one of her employees meets another worker. Therefore, the total number of vacancies is given by the sum of new entrants (v_e) and existing firms (e), $v = v_e + e$.

Both, unemployed and employed workers look for jobs. However the employed workers are less effective at searching per unit of time. Their search effort is therefore scaled by $s < 1$. This is equivalent of having se employed workers searching for jobs. Let x be the effective number of workers searching at any instant in time were $x = u + se$. There is a matching technology denoted by $M(x,v)$. I assume random matching as in Mortensen and Pissarides (1994). The labor market tightness is given by: $\theta \equiv \frac{total\ vacancies}{workers\ looking\ for\ jobs} = \frac{v_e+e}{u+se} = \frac{v_e+e}{x}$. The matching function between workers and firms is given by $q(\theta) = \frac{M(x,v)}{v} = \frac{x^{\zeta}v^{1-\zeta}}{v}$.

Firm post wages that are constant through a lifetime of an employed worker. And as are going to show below, there is a one-to-one mapping from the wage to the utility level of a worker for a given δ.

An entrant firm draws a value for expected duration δ from some distribution $\Gamma(\delta)$. In equilibrium each δ is associated with a utility level offered to the worker, $U(\delta)$. Let $H(U)$ represent the distribution of promised continuation values among entrants, $h : \delta \mapsto U$. The continuation value of a worker employed in a firm with destruction rate δ and that offers him wage of w is given by,

$$\begin{aligned} rU(\delta,w) =& w + \delta\left(U_0 - U(\delta,w)\right) + \\ & s\theta q(\theta)\left(\int_{U(\delta,w)}^{\infty}\left(\tilde{U} - U(\delta,w)\right)\left(\frac{e}{v}dG(\tilde{U}) + \frac{v_e}{v}dH(\tilde{U})\right)\right) \end{aligned} \tag{2.2}$$

Note that when an employed worker matches with a firm (arrival rate $s\theta q(\theta)$), he can be matched with a new firm or an existing firm. The probability of being matched with an existing firm is $\frac{e}{v}$ while the probability of being matched with a new firm is $\frac{v_e}{v}$. The distributions of existing firms with a vacancy offering a utility level of U is given by some distribution function $G(U)$. And the distributions of entrant firms with a vacancy offering a utility level of U is given by $H(U)$. Note that the equation above (2.2) defines the wage as a function of the job destruction rate and the promised utility.

The continuation value of an unemployed worker is given by the utility flow of being unemployed plus the expected utility that he will receive if he receives an offer. This offer can be from an existing firm or from a new firm.

$$rU_0 = b + \theta q(\theta)\left(\int_{U_0}^{\infty}\left(\tilde{U} - U_0\right)\left(\frac{e}{v}dG(\tilde{U}) + \frac{v_e}{v}dH(\tilde{U})\right)\right)$$

Lemma 1 *The worker utility is increasing with wages and decreasing with the job destruction rate, ie* $\frac{\partial U(w,\delta)}{\partial w} > 0$ *and* $\frac{\partial U(w,\delta)}{\partial \delta} \leq 0$

Proof. See appendix 2.7.5. ■ A firm with δ, offering a utility U to the worker, that has $n_i(\delta)$ employees, has a value of:

$$
\begin{aligned}
&n_i(\delta) rV(\delta, U) = n_i(\delta) \times \\
&\Bigg(1 - w - \delta V(\delta, U) \\
&- s\theta q(\theta) \left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right) V(\delta, U) + q(\theta) \left(\frac{se}{x} G(U) + \frac{u}{x}\right) V(\delta, U) \Bigg)
\end{aligned}
$$

Because the production function is constant returns to scale, the firm profit can be expressed as the multiplication of the number of workers employed and the value per worker. The continuation value per worker for a firm with exogenous layoff rate δ that promises a utility level U for its workers is given by,

$$
\begin{aligned}
rV(\delta, U) =& 1 - w - \delta V(\delta, U) - s\theta q(\theta) \left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right) V(\delta, U) \\
&+ q(\theta) \left(\frac{se}{x} G(U) + \frac{u}{x}\right) V(\delta, U)
\end{aligned}
$$

The value per worker is equal to the profit, $1 - w$, minus the capital loss if the match is dissolved, plus the gain that the firm has if it hires a new worker. The match can be dissolved if it becomes unproductive, with probability δ, or if the firm loses the worker to another firm. The loss that the firm has if it loses the worker is represented by the last term on the first line of the equation above. The job seeking rate for a employed worker is $s\theta q(\theta)$. An employed worker will leave his current job if he meets another existing firm that offers him a higher utility than his current one, $\frac{e}{v}(1 - G(U))$, or if he meets an entrant firm that also offers him a utility higher than his current one, $\frac{v_e}{v}(1 - H(U))$. The gain that the firm has if it hires a new worker is represented by the term on the second line of the equation above. The probability that an existing firms meet another worker is given by $q(\theta)$. The firm will hire the new worker that it meets if the worker is currently employed in a firm that offers the worker a lower utility, $\frac{se}{x} G(U)$, or if the worker is unemployed, $\frac{u}{x}$. Unemployed workers accept any job. Therefore, the firm value per worker is given by,

$$V(\delta, U) = \frac{1-w}{r+\delta-q(\theta)\left(\frac{se}{x}G(U)+\frac{u}{x}\right)+s\theta q(\theta)\left(\frac{e}{v}(1-H(U))+\frac{v_e}{v}(1-H(U))\right)} \tag{2.3}$$

The firm wants to maximize its value per worker subject to a Promise-Keeping constraint (PK) to deliver a promised U. The Promise-Keeping constraint guarantees that the wage offered by the firm, given the firm's expected match duration, yield promised utility U to the worker. The firm problem is therefore given by,

$$\begin{aligned} V(\delta, U) = \max_{w} \quad & \frac{1-w}{r+\delta-q(\theta)\left(\frac{se}{x}G(U)+\frac{u}{x}\right)+s\theta q(\theta)\left(\frac{e}{v}(1-H(U))+\frac{v_e}{v}(1-H(U))\right)} \\ s.t. \quad rU(\delta, w) =& w+\delta\left(U_0-U(\delta, w)\right)+ \\ & s\theta q(\theta)\left(\int_{U(\delta,w)}^{\infty}\left(\tilde{U}-U(\delta, w)\right)\left(\frac{e}{v}dG(\tilde{U})+\frac{v_e}{v}dH(\tilde{U})\right)\right) \quad (PK) \end{aligned}$$

Substituting the wage from the promise keeping constraint into the firm value, the maximization problem of the firm is equivalent to,

$$V(\delta, U) = \max_{U} \frac{1-rU+\delta\left(U_0-U\right)+s\theta q(\theta)\left(\int\limits_{U(\delta,w)}^{\infty}\left(\tilde{U}-U(\delta, w)\right)\left(\frac{e}{v}dG(\tilde{U})+\frac{v_e}{v}dH(\tilde{U})\right)\right)}{r+\delta-q(\theta)\left(\frac{se}{x}G(U)+\frac{u}{x}\right)+s\theta q(\theta)\left(\frac{e}{v}(1-H(U))+\frac{v_e}{v}(1-H(U))\right)} \tag{2.4}$$

Taking the first order condition with respect to U and setting it equal to zero:

$$\begin{aligned} & FOC: \quad \frac{\partial V(\delta, U)}{\partial U} = 0 \;\Rightarrow \\ & -r-\delta-s\theta q(\theta)\left(\frac{e}{v}(1-G(U))\frac{v_e}{v}(1-H(U))\right)+\frac{sq(\theta)}{x}\left(2eg(U)+v_e h(U)\right)V(\delta, U) \\ & = 0 \end{aligned} \tag{2.5}$$

Rearranging:

$$V(\delta, U) = \frac{r + \delta + s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right)}{\frac{sq(\theta)}{x}\left(2eg(U) + v_e h(U)\right)} > 0 \tag{2.6}$$

The Equation (2.6) above proves that the firm always has a positive continuation value, independent of δ. This shows that all firms, independent of the draw of δ will enter the market.

Equating the optimal value for the firm (Equation (2.6)) with the value of the firm given by Equation (2.3) we can pin down the optimal wage for each δ.

Following Mortensen and Pissarides (1994) , before a firm enters the market, it must pay a fixed cost c_e. Before matching with a worker, the firm draws δ from some distribution $\delta \sim \Gamma(\underline{\delta}, \overline{\delta})$ and decides whether or not to exit. Since $V(\delta, U)$ is always positive, which is given by equation (2.6), all firms will choose to enter the market regardless of δ. Assuming that there is free-entry:

$$c_e = E_\delta\left[V(\delta, U)\right] = \int_{\underline{\delta}}^{\overline{\delta}} q(\theta)V(\delta)\left(\frac{u}{x} + \frac{se}{x}G(U(\delta))\right)d\Gamma(\delta)$$

2.3.2 Definition of Stationary Recursive Competitive Equilibrium

A Stationary Recursive Competitive Equilibrium for this economy is wage policy function for firms $w(\delta)$, a set of distributions for promised utility levels among incumbents $G(U)$, the number of workers at each utility level $N(U)$, and the distribution of promised utilities among entrants $H(U)$ such that:

- Given the distributions of workers across firms, the firm policy function for wages is optimal and satisfies the promise keeping constraint.
- The distributions $G(U)$, $N(U)$, and $H(U)$ are time-invariant and consistent with firm policy functions.

2.3.3 Equilibrium Properties

In Lemma 2, I prove that the equilibrium utility of the worker is decreasing in the firm exogenous separation shock. The economic intuition behind this lemma is straight forward: Why do low layoff firms choose to offer the worker a higher utility? If the firm offers a higher utility they are going to lose fewer workers due to OJS, and since the firm is a low layoff (low δ) firm, the worker is more durable and therefore more valuable to retain.

Lemma 2 *The utility level of the worker is decreasing in the exogenous layoff rate δ (i.e. job uncertainty):*

$$\frac{\partial U}{\partial \delta} \quad < 0$$

Proof. Applying the implicit function theorem into the FOC:

$$\frac{\partial U}{\partial \delta} = -\frac{\frac{\partial^2 V(\delta,U)}{\partial U \partial \delta}}{\frac{\partial^2 V(\delta,U)}{\partial^2 U}} \tag{2.7}$$

From the FOC I know that the optimal U is decreasing with δ. Using the maximization problem of the firm in equation (2.4), I can show that the derivative of $V(\delta, U(\delta))$ with respect to δ is given by,

$$\begin{aligned}
\frac{\partial V\left(\delta, U(\delta)\right)}{\partial \delta} =& \frac{dV\left(\delta, U(\delta)\right)}{d\delta} + \overbrace{\frac{\partial V\left(\delta, U(\delta)\right)}{\partial U(\delta)}}^{=0 \text{ at optimum}} \frac{dU(\delta)}{d\delta} \\
=& \frac{(U_0 - U)}{\left(r + \delta - q(\theta)\left(\frac{se}{x} G(U) + \frac{u}{x}\right) + s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right)\right)} \\
&- \frac{V\left(\delta, U(\delta)\right)}{\left(r + \delta - q(\theta)\left(\frac{se}{x} G(U) + \frac{u}{x}\right) + s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right)\right)}
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial V\left(\delta, U(\delta)\right)}{\partial \delta} = \\
&\frac{U_0 - U - V(\delta, U)}{\left(r + \delta - q(\theta)\left(\frac{se}{x} G(U) + \frac{u}{x}\right) + s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right)\right)} \leq 0
\end{aligned} \tag{2.8}$$

Now take the derivative of both sides of equation (2.5). The second derivative of $V(\delta, U)$ with respect to U and δ is therefore given by,

$$\frac{\partial^2 V(\delta, U)}{\partial^2 U} < 0 \quad \text{guarantees that } U^* \text{ is a maximum (SOC)}$$
$$\frac{\partial^2 V(\delta, U)}{\partial U \partial \delta} = -1 + \frac{sq(\theta)}{x}\left(2eg\left(U^*\right) + v_e h\left(U^*\right)\right) \underbrace{\frac{\partial V\left(\delta, U^*\right)}{\partial \delta}}_{\leq 0 \text{ Equation (2.8)}} < 0$$

Substituting the derivatives found above into the equation 2.7 (derived from the implicit function theorem) we have that the worker utility of equilibrium is decreasing with δ:

$$\frac{\partial U}{\partial \delta} = -\frac{\overset{(-)}{\frac{\partial^2 V(\delta,U)}{\partial U \partial \delta}}}{\underset{(-)}{\frac{\partial^2 V(\delta,U)}{\partial^2 U}}} < 0$$

■ Lemma 3 demonstrates that the wage is decreasing with the layoff rate, δ, for high turnover firms. The intuition behind the Lemma is that the promise keeping constraint for worker utility has two opposing effects. Wages are function of δ and U. Wages increase with δ because in order to deliver a higher promised utility, a higher turnover firm must pay a higher wage to compensate for unemployment risk. On the other hand, the firm optimally chooses to deliver a lower promised utility since the worker is less durable, which tends to lower the wage. The later effect dominates for high δ firms.

Lemma 3 *The optimal wage that firms offer is decreasing with the layoff rate, δ, for firms with high δ's:*

$$\frac{\partial w}{\partial \delta} = < 0 \textit{ if } \delta \textit{ is high enough}$$

Proof. See Appendix 2.7.5. ▌■ In the following Lemma 4 we can see that firms with smaller exogenous turnover, that offers a higher utility to the worker, grows faster.

Lemma 4 *Firms with lower δ grows faster.*

Proof. See Appendix 2.7.5. ▌■

2.3.4 Equilibrium Worker Flows

Since Lemma 2 demonstrated that worker utility is decreasing with δ, an employed worker always searches for jobs in firms with lower δ's than that of their present employer. On the firm side, firms can always recruit workers employed in other firms with higher δ's. Let $N(\delta)$ denote the number of workers working in a firm with an exogenous layoff rate no greater than δ. The law of motion for the distribution of employed workers is therefore given by,

$$\frac{dN(\delta)}{dt} =$$
$$-\int_{\underline{\delta}}^{\delta} \tilde{\delta} n(\tilde{\delta}) d\tilde{\delta} + \quad s\theta q(\theta)(e - N(\delta)) \left(\frac{e}{v}\frac{N(\delta)}{e} + \frac{v_e}{v}\Gamma(\delta)\right) + \theta q(\theta) u \left(\frac{e}{v}\frac{N(\delta)}{e} + \frac{v_e}{v}\Gamma(\delta)\right)$$

The first term on the right side of the equation above represents the flow of workers that had their job terminated in firms with $\tilde{\delta} \leq \delta$. The second term of the right side of the equation above represents the flow of employed workers that firms with $\tilde{\delta} \leq \delta$ poached from firms that offer a lower utility than $U(\delta)$. There are $e - N(\delta)$ workers employed in these lower utility firms, these workers have a match intensity of $s\theta q(\theta)$, and they will be poached by incumbents and entrant firms. The probabilities of meeting an incumbent and an entrant firm with a job destruction rate less than δ are respectively given by $\frac{e}{v}\frac{N(\delta)}{e}$ and $\frac{v_e}{v}\Gamma(\delta)$. The last term of the right side of the equation above represents the flow of unemployed workers hired by firms with $\tilde{\delta} \leq \delta$. There are u unemployed workers that have a match intensity of $\theta q(\theta)$, and they can be hired by incumbents and entrant firms. Note that no worker in the pool of employed workers in firms with $\tilde{\delta} \leq \delta$ leave their job because of on-the-job search. Workers can move inside this pool, but never leave it since firms with job destruction rates greater than δ will offer a lower utility. In equilibrium $\frac{dN(\delta)}{dt} = 0$: $\int_{\underline{\delta}}^{\delta} \tilde{\delta} n(\tilde{\delta}) d\tilde{\delta} = \theta q(\theta) \left(\frac{N(\delta)}{v} + \frac{v_e \Gamma(\delta)}{v}\right) (s(e - N(\delta)) + u)$

Integrating among all firms, it must be that case in a stationary recursive competitive equilibrium that the expected number of workers that had their jobs terminated must be equal the flow of unemployed workers that find a job at every instant: $E\left(\theta | \text{worker employed}\right) = \int_{\underline{\delta}}^{\overline{\delta}} \tilde{\delta} n(\tilde{\delta}) d\tilde{\delta} = \theta q(\theta) u$

Taking the derivative of the worker flow with respect to δ it is possible to derive the equilibrium number of workers employed in δ type firms:

$$dN(\delta) = v_e \gamma(\delta) \frac{x - sN(\delta)}{2sN(\delta) + sv_e\Gamma(\delta) + \frac{x}{q(\theta)}\delta - x} \tag{2.9}$$

We know that $N(\bar{\delta}) = e$ and $N(\underline{\delta}) = 0$. Therefore if there is free entry, $v_e > 0$, then numerator of the above quotient, $v_e\gamma(\delta)(x - sN(\delta))$, is always positive.

Assumption 3 *The firm match intensity is low enough such that* $\underline{\delta} > q(\theta)$.

Assumption 3 implies that $dN(\delta)$ is positive for all δ. In particular, this assumption implies that the denominator of equation (2.9) is always positive. The first two terms of the denominator are always non-negative, and the rest is always positive: $\frac{x}{q(\theta)}\left(\delta - q(\theta)\right) \geq \frac{x}{q(\theta)}\left(\underline{\delta} - q(\theta)\right) > 0$.

Therefore, since $N(\underline{\delta}) = 0$, $N(\bar{\delta}) = e$ and $dN(\delta) > 0$, it must be the case that the distribution of workers across δ's is well defined and continuous.

On-the-Job Search Rate

The total number of separations that occur at every instant of time among employees working in firms of type δ is due to one of two causes in the model. The first is that the match is dissolved because of some exogenous shock. And the second is that the worker leaves the job to go to a better job because of on-the-job search. The total number of separations can be described as:

$$Total\ Separations_{(\delta)} = \underbrace{\delta n(\delta)}_{\text{Separation Due To Exogenous Shock}} + \underbrace{s\theta q(\theta) n(\delta)\left(\frac{N(\delta)}{v} + \frac{v_e\Gamma(\delta)}{v}\right)}_{\text{Separation Due To OJS}}$$

The fraction of total separation due to OJS of workers employed in firms of type δ is given by,

$$\frac{\textit{Separation Due To OJS}}{\textit{Total Separation}}(\delta) = \frac{\frac{s\theta q(\theta)}{v}\left(N(\delta) + v_e\Gamma(\delta)\right)}{\delta + \frac{s\theta q(\theta)}{v}\left(N(\delta) + v_e\Gamma(\delta)\right)}$$

$$\frac{\partial\left(\frac{\textit{Separation Due To OJS}}{\textit{Total Separation}}(\delta)\right)}{\partial\delta} = \frac{s\theta q(\theta)}{v}\frac{\left(\delta n(\delta) - N(\delta) + \delta v_e\gamma(\delta) - v_e\Gamma(\delta)\right)}{\left(\delta + \frac{s\theta q(\theta)}{v}\left(N(\delta) + v_e\Gamma(\delta)\right)\right)^2}$$

2.4 Theoretic Analysis of Firing Costs

2.4.1 Severance Payment

Suppose that the government introduces a firing cost to the firm in the form of a severance payment. If the firm fires the worker (which occurs with probability δ), then the firm must pay a lump sum transfer to the worker that is proportional to the wage of the worker. This is an approximation to the "FGTS" unemployment insurance system in Brazil. A worker with a wage of w that gets fired will receive τw from the firm. Therefore, it is possible to interpret this type of firing cost as a change of variable: the effective flow compensation of the worker is not w, but $(1 + \delta\tau)w$. The worker receives the wage plus some expected compensation in the case he is fired. Proposition 1 shows that such a tax is neutral in the sense that it does not change the equilibrium.

Proposition 1 *There is no change in the promised utilities, firm value, worker flows and the distributions of equilibrium with the introduction of a firing cost.*

Proof. See Appendix 2.7.5. ■ ■ However, in equilibrium, the wages does change. Wages adjust according to the firm turnover rate of the firm. Wages decrease more so for high turnover firms (who were already paying low wages) relative to low turnover firms. The new equilibrium wage is given by,

$$w^{\tau}(\delta) = w^{\text{no firing cost}}(\delta)\frac{1}{1 + \tau\delta}$$

2.4.1.1 Equilibrium Properties with Severance Payment

Thought the following Lemma 5, we can conclude that, as before, the utility is still decreasing with δ and so is the wage fro δ high enough. And, obviously, the wage is decreasing with the firing cost.

Lemma 5 *The utility level of the worker is decreasing with the layoff rate, δ, in a model with a wage-proportional firing cost. Also the wage is decreasing with δ for high δs.*

$$\frac{\partial U^{\tau}(\delta)}{\partial \delta} < 0. \qquad \frac{\partial w^{\tau}(\delta)}{\partial \delta} < 0 \ \ \textit{if } \delta \textit{ is high enough.} \qquad \frac{\partial w(\delta)^{\tau}}{\partial \tau} < 0.$$

Proof. Since the maximization problem of the firm without the firing cost is analogous to the one with the firing cost, the proof is analogous to that of Lemma 2. See Appendix 2.7.5.

▮ ■

2.4.1.2 The Effect of a Severance Payment on Wage Inequality

Lemma 6 states that the higher is the layoff rate, δ, the more the wage will be distorted with the firing cost τ:

Lemma 6 *The higher is δ, the more the wage will de distorted with the firing cost.*

$$\frac{\partial\left(\frac{w^{\tau}(\delta)}{w^{no\ firing\ cost}(\delta)}\right)}{\partial \delta} = -\frac{\tau}{(1+\tau\delta)^2} < 0$$

Proof. From Equation (2.16): $\frac{w^{\tau}(\delta)}{w^{\text{no firing cost}}(\delta)} = \frac{1}{1+\tau\delta}$ ■

Can you say why these lemmas are important? Lemma shows that wage inequality will increase when there is a firing cost. This may be particularly important for Brazil which witnessed a large increase in wage inequality when labor market protection was introduced, and a subsequent decline in wage inequality when labor market protection was phased out. In the following section, I quantitatively analyze the importance of labor market protection for wage inequality.

2.4.2 Severance Payment and Firing Tax

Now, suppose that besides the severance payment, the government introduces a firing tax to the firm. If the firm fires the worker the firm must pay a lump sum transfer to the worker and a tax to the government that are proportional to the wage paid to the worker, plus a . This is an more accurate approximation to the "FGTS" unemployment insurance system in Brazil. A firm that pays a wage of w to fire an employee must pay $\tau_W w$ to the worker and ϵw to the government. Now the firm value of equilibrium is given by:

The worker value is:

$$
\begin{aligned}
&rU(\delta, w) = \\
&w + \delta\left(U_0 + \tau_W w - U(\delta, w)\right) + s\theta q(\theta)\left(\int_{U(\delta,w)}^{\infty}\left(\tilde{U} - U(\delta, w)\right)\left(\frac{e}{v}dG(\tilde{U}) + \frac{v_e}{v}dH(\tilde{U})\right)\right)
\end{aligned}
$$

The continuation value per worker for a firm (δ, U) is given by:

$$
\begin{aligned}
rV(\delta, U) =& 1 - w - \delta(\tau_W + \epsilon)w \\
&+ \left(-\delta - s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right) + q(\theta)\left(\frac{se}{x}G(U) + \frac{u}{x}\right)\right)V(\delta, U)
\end{aligned}
$$

Solving the firm problem and we get the firm value as a function of (δ, U) given θ and ϵ:

$$
V(\delta, U; \theta, \epsilon) = \frac{r + \delta + s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right)}{\frac{sq(\theta)}{x}\left(2eg(U) + v_e h(U)\right)}\left(\frac{1 + (\tau_W + \epsilon)\delta}{1 + \tau_W \delta}\right)
$$

Now the free-entry condition is given by,

$$
c_e = E_\delta\left[V(\delta, U; \theta, \epsilon)\right] = \int_{\underline{\delta}}^{\overline{\delta}} q(\theta)V(\delta; \theta, \epsilon)\left(\frac{u}{x} + \frac{se}{x}G(U(\delta; \theta\epsilon))\right)d\Gamma(\delta)
$$

Lemma 7 *The utility level of the worker does not change with the market tightness and the government tax:* $\frac{\partial U}{\partial \theta} = \frac{\partial U}{\partial \epsilon} = 0$

Proof. See Appendix 2.7.5. ■

Lemma 8 *The market tightness decrease when the government tax increase:* $\frac{\partial \theta}{\partial \epsilon} < 0$

Proof. See Appendix 2.7.5. ■

2.5 Results

2.5.1 Estimation of the Parameters

I solve the model using a discrete state space approximation. I approximate the continuous distribution of δ with an extremely fine grid, and I use discrete time, were one period of time corresponds to one month. In the Appendix 2.7.6, I provide the derivation of the discrete model and details of the computational method. I estimate the model for the year of 2010 using data of one state in Brazil, Bahia.

I use a Cobb-Douglas matching function $M(x, v) = u^{\zeta} v^{1-\zeta}$ where I set the matching function parameter ζ such that $\zeta = 0.5$. I will assume that one period on the model corresponds to one month. To estimate the real interest rate in Brazil in the period analyzed, 2010, I subtract the average interest rate by the cumulative inflation on that year (source: Central Bank of Brazil). The result is a yearly real interest rate of 3.5% that corresponds to a monthly interest rate of .287%.

Using the unemployment benefit rules in Brazil, I estimate the benefit replacement rate from the data, $b = .660$. To estimate the average employment replacement in Bahia, Brazil, 2010, I use the following equation[4]:

$$b_{data} = \frac{mean\left(\mathbb{I}_{(w \leq 850)} 0.8w + \mathbb{I}_{(850 < w \leq 1{,}400)} 0.5w + \mathbb{I}_{(w > 1{,}400)} 955\right)}{mean(w)} \tag{2.10}$$

The average unemployment rate in Bahia, Brazil in 2010 was 10.73%, so $u = 0.1073$. I use the estimated firm entry cost from *doingbusiness.org*. They estimate that the firm entry-cost in Brazil is about 46% of the yearly income per-capita. Since in my model I assume a time period corresponds to one month and I that one employed worker produces one unit of

[4]The numerator of b_{data} is the unemployment benefit rule in Brazil.

output per period, than the monthly income per-capita is $\frac{e}{1}$ and the yearly income per-capita is $12e$. Therefore, I use $c_e = .046 * 12 * e$. Finally, I will consider a firing tax of 30% of prior wages, that is consistent with the policy that is done in Brazil, so that $\tau = 0.3$.

2.5.2 Estimation of δ Distribution

To recover the distribution of deltas across firms from the data I first estimate the δ for all firms f and all months m, and then take the average δ for each firm:

$$\hat{\delta_f} \quad = mean\left(\hat{\delta}_{fm}\right) = mean\left(\frac{\text{Total Separations}_{f_m} - \text{Separations Due to Job-to-Job}_{f_m}}{\text{Firm Size}_{f_m}}\right) \tag{2.11}$$

Then it is naturally that the number of workers employed in firms with a certain δ is equal to the sum of the size of the firms that have this estimated δ:

$$\hat{n}(\delta_i) = \sum_f \mathbb{I}(\hat{\delta_f} = \delta_i) \times MeanSize_f$$

Since the number of employed workers is very large, I normalize such that the total number of employed across all types of firms is equal to the employment rate of the economy in the year estimated.

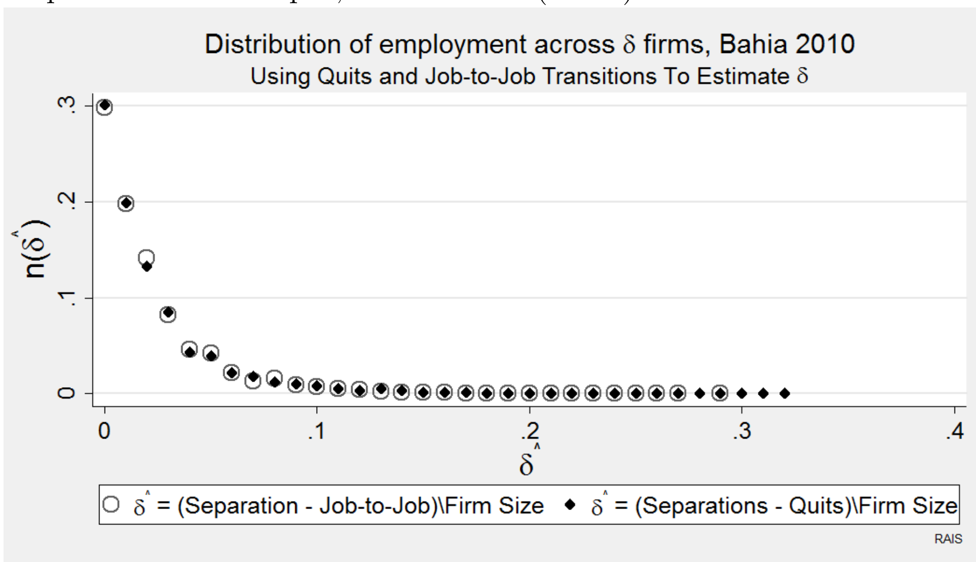
$$\hat{n}(\delta_i)_{Normalized} = \frac{\text{Employment Rate}}{\text{Total Employment}} \times \hat{n}(\delta_i)$$

I do the estimation of $n(\delta)$ considering the total separations due to job-to-job transitions, as described in Equation 2.11, and also using total separations due to quits. In Figure 2.3 we can see that the estimated distribution of δ in both cases is very similar.

2.5.3 Model Fit

Table 2.4 below shows the parameters that are not estimated, but taken directly from the data or from the literature. Table 2.5 below illustrates how well the model does at matching

Figure 2.3: Comparing estimations of $n(\delta)$ using the difference between total number of separation minus the total number of separations due to job-to-job transitions or the total number of separations due to quit, Bahia-Brazil (RAIS) 2010



its moments. The model exactly replicates the mass of workers at each δ_i grid point. The model also does quite well at matching non-targeted moments such as the variance of the wage and the ratio of the 90th percentile to the 10th percentile. In the Appendix 2.7.7 I describe the solution algorithm in great detail.

Table 2.4: Non-Estimated Parameters

		VALUE	**SOURCE**
Average Employment Replacement*	b	0.65	Unemployment Benefit Rule in Bahia, Brazil 2010 (see Equation (2.10)).
Real Trimester Interest Rate	r	0.078	Central Bank of Brazil, 2010.
Matching Function Parameter	ζ	0.5	
Unemployment Rate	u	0.107	Government of Bahia, Brazil 2010.
Wage Proportional Firing Cost	τ	0.3	Approximation of the Brazilian Firing Cost ‘FGTS’
Entry Cost	c_e	$0.046y$	Doing Business Index.

**Only for Males, 25-55, High School or Less*

Table 2.5: Estimated Parameters

Target	Model	Data	Source
Fraction Employed	0.89	0.89	Government of Bahia, Brazil 2010.
OJS Fraction	0.1563	0.166	RAIS 2010
dN of			
δ =0.03	0.445	0.445	RAIS 2010
δ =0.09	0.234	0.234	RAIS 2010
δ =0.15	0.101	0.101	RAIS 2010
δ =0.21	0.044	0.044	RAIS 2010
δ =0.27	0.036	0.036	RAIS 2010
δ =0.33	0.017	0.017	RAIS 2010
δ =0.39	0.007	0.007	RAIS 2010
δ =0.45	0.004	0.004	RAIS 2010
δ =0.51	0.002	0.002	RAIS 2010
δ =0.57	0.003	0.003	RAIS 2010
Non-Target Parameters			
Var(wage)	0.003	0.0027	RAIS 2010
p90-p10	1.216	1.246	RAIS 2010
p75-p25	1.100	1.066	RAIS 2010

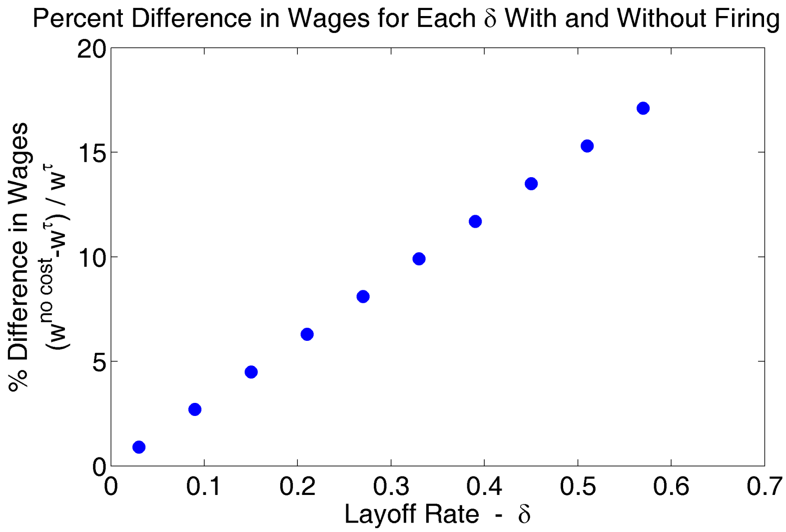
* *Only for males, 25-55, high school or less, period of 3 months.*

2.5.4 Numerical Experiment: Firing Cost and Wage Inequality

The main experiment is to increase the firing cost from τ=0 to τ=.3 and compare the implications of such a reform across two economies: (i) an economy with heterogeneous firing rates that I call the *benchmark heterogeneous layoff economy* (ii) an economy with very little dispersion in firing rates (similar to Shi (2009)) that I call the *homogeneous layoff economy.* To be precise, the *homogeneous layoff economy* has the same employment level as the heterogeneous layoff economy, except the δ grid is a mean preserving *contraction.*

Figure 2.4 is central to the main result of the paper. It plots the impact of firing costs on wages across each possible exogenous layoff rate δ in the *benchmark heterogeneous layoff economy.* The graph illustrates that for workers in low wage jobs with greater layoff rates (higher δ's), wages drop disproportionately more than for workers with more secure jobs (lower δ's). As a result, the variance in wages in the benchmark heterogeneous layoff economy almost triples when the firing cost, τ, increases from 0 to 30%. In the *homogeneous layoff economy* with very little dispersion in δ, the introduction of the same sized firing cost

Figure 2.4: Difference in the equilibrium wage for each δ with and without firing cost



increases the variance of wages by twice (see Table 2.6).

Table 2.6 includes the main result of the paper. In the heterogeneous layoff economy, an increase in the firing cost increases the p90-p10 wage ratio by more than 10%. In the homogeneous layoff economy, an equivalent increase in the firing cost increases the p90-p10 wage ratio only by 5%. Therefore, taking into account layoff heterogeneity amplifies wage inequality from a firing tax by a factor of 2.

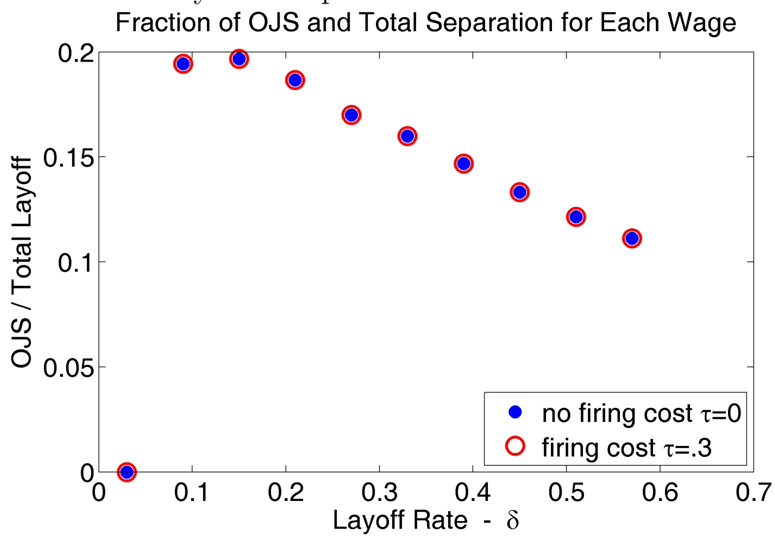
Turning to the other rows of Table 2.6, it is clear that the quantitative results are in line with Proposition 1. The market tightness, employment, and other variables in the economy remain unchanged. The only variable impact is *measured* wage payments during the job spell. Why is this the case? The firm knows that it must make the firing tax transfer to the worker upon separation. The firm therefore lowers the contemporaneous promised wage taking this transfer into account. The firm is able to maintain the same utility level promised to the worker since this is a lump sum transfer between risk neutral agents. As a result, wage inequality increases with a firing cost, but welfare remains unchanged. Obviously welfare will be lower if the ability to transfer funds between the firm and worker is imperfect. In reality the imperfect transferability of funds occurs because of the type of firing cost. In Brazil, the economy studied in this paper, to fire a worker the firm must also pay a fee to

Table 2.6: Comparing baseline model with the introduction of a firing cost

	Baseline Model		**Model with small dispersion of δs**		**Data**
	τ=0	τ=.3	τ=0	τ=.3	
Employment (e)	0.89	0.89	0.89	0.89	0.89
Market Tightness (θ)	8.78	8.78	24.88	24.88	
Var(wage)	0.0013	0.0030	0.0012	0.0022	0.0027
p90-p10	1.099	1.216	1.115	1.150	1.246
OJS Rate	0.1563	0.1563	0.1681	0.1681	0.166
dN of					
δ =0.03	0.445	0.445	0.045	0.045	0.445
δ =0.09	0.234	0.234	0.018	0.018	0.234
δ =0.15	0.101	0.101	0.009	0.009	0.101
δ =0.21	0.044	0.044	0.018	0.018	0.044
δ =0.27	0.036	0.036			0.036
δ =0.33	0.017	0.017			0.017
δ =0.39	0.007	0.007			0.007
δ =0.45	0.004	0.004			0.004
δ =0.51	0.002	0.002			0.002
δ =0.57	0.003	0.003			0.003
A estimated		0.35		0.05	
s estimated		0.07		0.15	

Notes: Data on var(wage), p90-p10 wage, OJS rate, and employment comes from RAIS 2010. Only for males, 25-55, high school or less.

Figure 2.5: Ratio of OJS by total separations for each δ with and without firing cost



the government of 10% of the amount paid to the worker. I could incorporate this to the model by assuming that τ_f of the firm is higher than the τ_w of the worker.

Lastly, Figure 2.5 plots the ratio of on-the-job search to total layoffs in the model with heterogeneous layoff rates with and without firing costs. Over some region of the state space, for extremely high turnover jobs, this ratio *decreases* as the exogenous component of turnover increases (as δ increases). The model still predicts that the highest wage workers do not engage in on-the-job search, as is typical in these models. However, the bulk of employment in Brazil is in the region of the state space in which the model correctly matches the fact that lower wage jobs have higher layoff rates related to OJS rates.

2.6 Conclusion

In this paper, I make three contributions. Firstly, I use matched employer-employee data to establish two stylized facts: (i) separations are disproportionately comprised of layoffs for low wage workers, not separations due to job-to-job transitions, and (ii) controlling for worker characteristics (tenure, education, etc.) and firm characteristics (sector, size, etc.), workers in jobs with higher turnover rates, defined to be the ratio of total gross worker flows to firm size, have lower wages. To my knowledge, the first stylized fact is new. Secondly, I show that existing models such as Burdett and Coles (2003) and Shi (2009) are consistent with the second fact that low wage jobs have higher turnover.[5] However, these models are *inconsistent* with the first fact. Burdett and Coles (2003) and Shi (2009) assume that the exogenous component of job destruction is constant across firms, and low wage workers optimally search on-the-job much more intensely than high wage workers. Lastly, I solve a Burdett and Mortensen (1998) model in which layoff rates are greater for low wage workers. I show that after one takes into account the disproportionate share of layoffs among low wage workers, the burden of firing taxes shifts almost entirely to low wage workers, and as a result, such labor market interventions can increase wage inequality nearly 2x more than what Burdett and Coles (2003) and Shi (2009) would predict.

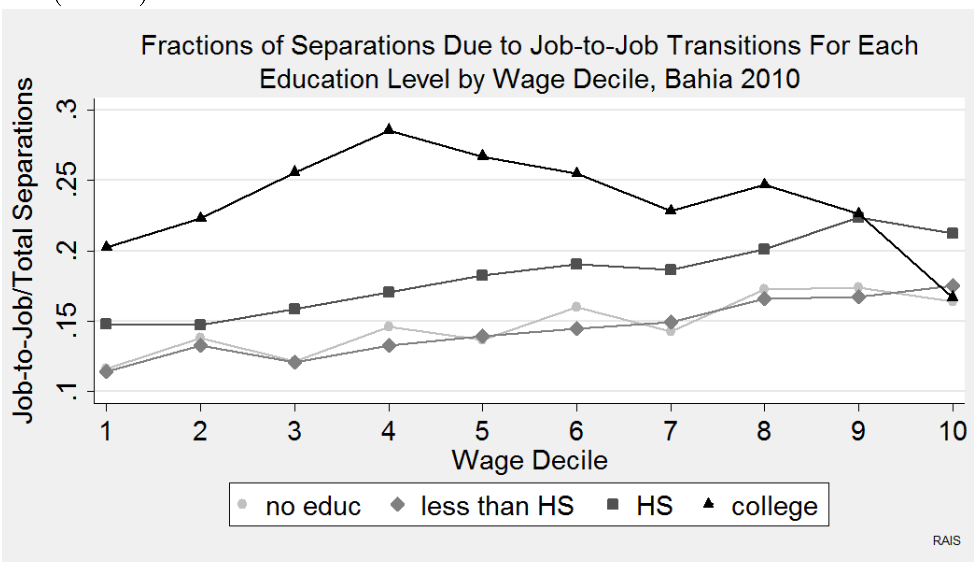
In future research, I plan to consider the role of heterogeneity among workers as well as heterogeneity among firms in terms of productivity. The matched employer-employee data will allow me to consider the implications of labor market regulations on assortative matching and inequality. I also plan to analyze the impact of a minimum wage on equilibrium job flows. Dube et al. (2013) finds that minimum wages do have a sizable negative effect on employment flows, especially among low tenure workers.

[5]In those models, firms are indifferent between paying low wages to short tenure workers (who will leave because of OJS) and high wages to long tenure workers (who are unlikely to leave for a longer period of time) as these two contracts will result in the same profits for the firm. In that framework, all firms and workers are identical.

2.7 Appendix

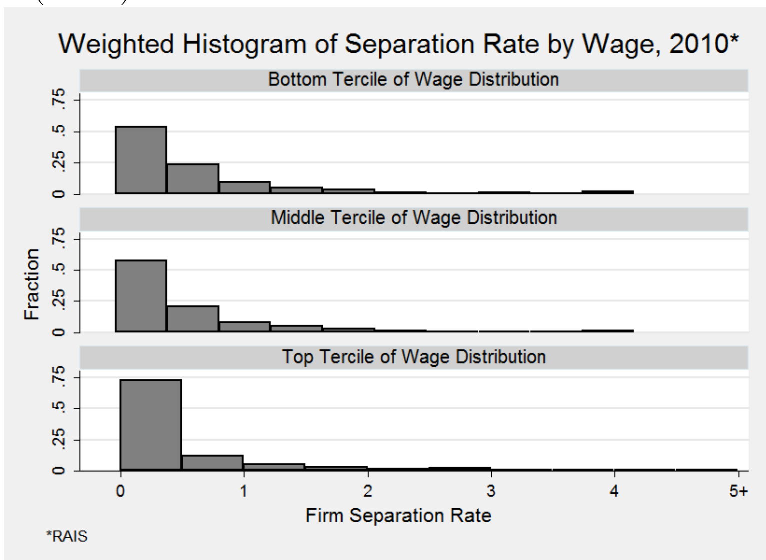
2.7.1 Fraction of Total Separations Due to Job-to-Job Transitions By Wage And Education

Figure 2.6: Fraction of job-to-job transitions and total separations by wages and education, Bahia-Brazil (RAIS) 2010



2.7.2 Wage Tercile And Separation Rate

Figure 2.7: Histogram of the separation rate of the firm that the worker is employed by wage, Bahia-Brazil (RAIS) 2010



2.7.3 Regression of Wages on Firm Turnover

Table 2.7: Panel regression, dependent variable is log of wage (source:Bahia,RAIS 2007-2010)

	(1)	(2)	(3)	(4)
Constant	0.297***	0.478***	0.271***	0.497
	(0.002)	(0.002)	(0.002)	(0.002)
2^{nd} turnover quintile	-0.227***	-0.226***	-0.200***	-0.228***
	(0.001)	(0.001)	(0.001)	(0.001)
3^{rd} turnover quintile	-0.281***	-0.265***	-0.229***	-0.265***
	(0.001)	(0.001)	(0.001)	(0.001)
4^{th} turnover quintile	-0.278***	-0.272***	-0.213***	-0.267***
	(0.001)	(0.001)	(0.001)	(0.001)
5^{th} turnover quintile	-0.209***	-0.189***	-0.105***	-0.137***
	(0.001)	(0.001)	(0.001)	(0.001)
Manufacturing	0.094***	0.173***		
	(0.001)	(0.001)		
Construction	0.168***	0.260***		
	(0.001)	(0.001)		
Government	-0.154***	0.028***		
	(0.001)	(0.001)		
Agriculture	0.004***	-0.018***		
	(0.001)	(0.001)		
2^{nd} firm size 6-tile	0.149***		0.148***	
	(0.001)		(0.001)	
3^{rd} firm size 6-tile	0.307***		0.314***	
	(0.001)		(0.001)	
4^{th} firm size 6-tile	0.319***		0.302***	
	(0.001)		(0.001)	
5^{th} firm size 6-tile	0.370***		0.326***	
	(0.001)		(0.001)	
6^{th} firm size 6-tile	0.454***		0.398***	
	(0.001)		(0.001)	
Worker characteristics	yes	yes	yes	yes
Tenure year	yes	yes	yes	yes
Observations	10230022	10230022	10230022	10230022
Adj R squared	0.464	0.439	0.456	0.429

standard deviation in parentheses ***p<0.01, **p<0.05, *p<0.1

2.7.4 Worker Fixed Effect Regressions

In the Table 2.9 below we can see that controlling for firms and workers fixed effects, the wage is decreasing with the firm separation rate.

Table 2.8: Individual fixed effects panel regression, dependent variable is log hourly wage (source: RAIS 2000-2010)

Variables	(1)	(2)
Firm Turnover		
30-60%	-0.082***	-0.083***
	(-129.88)	(-130.62)
60-120%	-0.142***	-0.140***
	(-218.49)	(-216.77)
120%+	-0.157***	-0.150***
	(-222.44)	(-213.20)
entrant/exit firm	-0.075***	-0.056***
	(-89.15)	(-65.78)
Establishment Size		
10-25	0.053***	0.054***
	(96.20)	(97.60)
25-50	0.099***	0.100***
	(153.88)	(155.45)
50-100	0.144***	0.145***
	(216.16)	(217.89)
100-250	0.186***	0.187***
	(292.20)	(293.96)
250-1000	0.206***	0.207***
	(341.65)	(342.83)
1000+	0.230***	0.231***
	(359.26)	(360.41)
tenure	0.001***	0.001***
	(295.02)	(267.13)
Semester Working		
1st and 2nd equally		0.207***
		(26.67)
only 2nd semester		0.247***
		(31.84)
Year Fixed Effects	Yes	Yes
Individual Fixed Effects	Yes	Yes
Observations	17,665,762	17,665,762
R-squared	0.509	0.510
Number of Individuals	3,994,982	3,994,982

t-statistics in parentheses

***p<0.01, **p<0.05, *p<0.1

Table 2.9: Individual fixed effects panel regression, dependent variable is log hourly wage (source: RAIS 2000-2010)

	(1)	(2)
Firm Separation Rate		
15-30%	-0.068***	-0.068***
	(-116.14)	(-115.88)
30-60%	-0.116***	-0.114***
	(-195.85)	(-193.16)
60%+	-0.122***	-0.116***
	(-188.69)	(-178.40)
entant/exit	-0.038***	-0.019***
	(-49.55)	(-24.74)
Establishment Size		
10-25	0.054***	0.054***
	(97.44)	(98.84)
25-50	0.099***	0.100***
	(154.98)	(156.57)
50-100	0.145***	0.146***
	(217.15)	(218.94)
100-250	0.187***	0.188***
	(293.90)	(295.71)
250-1000	0.208***	0.208***
	(343.85)	(345.08)
1000+	0.235***	0.236***
	(367.30)	(368.44)
Semester Working		
all year		0.206***
		(26.56)
2nd semester		0.247***
		(31.78)
tenure	0.001***	0.001***
	(325.00)	(294.60)
Year Fixed Effects	Yes	Yes
Individual Fixed Effects	Yes	Yes
Observations	17,665,762	17,665,762
R-squared	0.508	0.509
Number of Individuals	3,994,982	3,994,982

t-statistics in parentheses

***p<0.01, **p<0.05, *p<0.1

2.7.5 Proofs

Proof. of Lemma 1 Taking the derivative of the worker utility (Equation 2.2) with respect to w and δ respectively, we have:

$$
\begin{aligned}
& r\frac{dU(\delta,w)}{dw} = \\
& 1-\delta\frac{dU(\delta,w)}{dw} - s\theta q(\theta)\frac{dU(\delta,w)}{dw}\left(U(\delta,w)-U(\delta,w)\right)\left(\frac{e}{v}dG(U(\delta,w))+\frac{v_e}{v}dH(U(\delta,w))\right) \\
& - s\theta q(\theta)\int_{U(\delta,w)}^{\infty}\frac{dU(\delta,w)}{dw}\left(\frac{e}{v}dG(\tilde{U})+\frac{v_e}{v}dH(\tilde{U})\right) \\
& \Rightarrow \frac{dU(\delta,w)}{dw} = \frac{1}{r+\delta+s\theta q(\theta)\left(\frac{e}{v}\left(1-G(U(\delta,w))\right)+\frac{v_e}{v}\left(1-H(U(\delta,w))\right)\right)} > 0 \\
& r\frac{dU(\delta,w)}{d\delta} = \\
& 1-s\theta q(\theta)\frac{dU(\delta,w)}{d\delta}\left(U(\delta,w)-U(\delta,w)\right)\left(\frac{e}{v}dG(U(\delta,w))+\frac{v_e}{v}dH(U(\delta,w))\right) \\
& - s\theta q(\theta)\int_{U(\delta,w)}^{\infty}\frac{dU(\delta,w)}{d\delta}\left(\frac{e}{v}dG(\tilde{U})+\frac{v_e}{v}dH(\tilde{U})\right)+\left(U_0-U(\delta,w)\right)-\delta\frac{dU(\delta,w)}{d\delta} \\
& \Rightarrow \frac{dU(\delta,w)}{d\delta} = \frac{-\left(U(\delta,w)-U_0\right)}{r+\delta+s\theta q(\theta)\left(\frac{e}{v}\left(1-G(U(\delta,w))\right)+\frac{v_e}{v}\left(1-H(U(\delta,w))\right)\right)} \leq 0
\end{aligned}
$$

■

Proof. of Lemma 3 Taking the derivative of the worker continuation value, Equation (2.2), with respect to δ:

$$
\begin{aligned}
& r\frac{dU(\delta)}{d\delta} = \frac{dw(\delta)}{d\delta}+\left(U_0-U(\delta)\right)-\delta\frac{dU(\delta)}{d\delta}-s\theta q(\theta)U(\delta)\left(\frac{e}{v}g(U)+\frac{v_e}{v}h(U)\right)\frac{dU(\delta)}{d\delta} \\
& s\theta q(\theta)\left(-\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)\frac{dU(\delta)}{d\delta}+U(\delta)\left(\frac{e}{v}g(U)+\frac{v_e}{v}h(U)\right)\frac{dU(\delta)}{d\delta}\right) \\
& \Rightarrow \frac{dw(\delta)}{d\delta} = \underbrace{\frac{dU(\delta)}{d\delta}}_{(-)}\left(r+\delta+s\theta q(\theta)\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)\right)+U(\delta)-U_0
\end{aligned}
$$

Since U is decreasing with δ, proved in Lemma 2 above, the highest δ firm will offer the smallest utility. Therefore, for the highest $\overline{\delta}$, $U(\overline{\delta})=U_0$ implying that by continuity $\frac{dw(\delta)}{d\delta}<0$ for high enough *delta*. ■

Proof. of Proposition 4 The firm growth rate is given by:

$$
\begin{aligned}
n_i'(\delta) &= n_i(\delta)\left(1-\delta-s\theta q(\theta)\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)+q(\theta)\left(\frac{se}{x}G(U)+\frac{u}{x}\right)\right)\\
\frac{n_i'(\delta)-n_i(\delta)}{n_i(\delta)} &= -\delta-s\theta q(\theta)\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)+q(\theta)\left(\frac{se}{x}G(U)+\frac{u}{x}\right)\\
\Rightarrow \frac{\partial\left(\frac{n_i'(\delta)-n_i(\delta)}{n_i(\delta)}\right)}{\partial\delta} &= -1+s\theta q(\theta)\left(\frac{e}{v}g(U)+\frac{v_e}{v}h(U)\right)\underset{(-)}{\frac{dU}{d\delta}}+q(\theta)\frac{se}{x}g(U)\underset{(-)}{\frac{dU}{d\delta}}<0
\end{aligned}
$$

$\blacksquare$ $\blacksquare$

Proof. of Proposition 1 With the introduction of a firing cost the worker continuation value can be written as:

$$
rU(\delta,w) = w+\delta\left(U_0+\tau w-U(\delta,w)\right)+s\theta q(\theta)\int\limits_{U(\delta,w)}^{\infty}\left(\tilde{U}-U(\delta,w)\right)\left(\frac{e}{v}g(\tilde{U})+\frac{v_e}{v}h(\tilde{U})\right)d\tilde{U}
$$

The continuation value of an unemployed worker remain the same as before. The continuation value per worker for a firm with δ that promises a utility level U for its workers is:

$$
\begin{aligned}
rV(\delta,U) = 1-w-\delta\left(V(\delta,U)+\tau w\right)+q(\theta)\left(\frac{se}{x}G(U)+\frac{u}{x}\right)V(\delta,U)\\
-s\theta q(\theta)\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)V(\delta,U)
\end{aligned}
\tag{2.12}
$$

The firm maximization problem with the tax is:

$$V(\delta, U) = \max_{w} \tag{2.13}$$

$$\frac{1 - w(1+\tau\delta)}{r + \delta - q(\theta)\left(\frac{se}{x}G(U) + \frac{u}{x}\right) + s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right)}$$

$$s.t. \quad rU(\delta, w) = \tag{2.14}$$

$$w + \delta\left(U_0 + \tau w - U(\delta, w)\right) + s\theta q(\theta) \int\limits_{U(\delta,w)}^{\infty} \left(\tilde{U} - U(\delta, w)\right)\left(\frac{e}{v}g(\tilde{U}) + \frac{v_e}{v}h(\tilde{U})\right) d\tilde{U} \quad (PK)$$

$$\Rightarrow V(\delta, U) = \max_{U}$$

$$\frac{1 - rU + \delta\left(U_0 - U\right) + s\theta q(\theta) \int\limits_{U(\delta,w)}^{\infty} \left(\tilde{U} - U(\delta, w)\right)\left(\frac{e}{v}g(\tilde{U}) + \frac{v_e}{v}h(\tilde{U})\right) d\tilde{U}}{r + \delta - q(\theta)\left(\frac{se}{x}G(U) + \frac{u}{x}\right) + s\theta q(\theta)\left(\frac{e}{v}(1 - G(U)) + \frac{v_e}{v}(1 - H(U))\right)} \tag{2.15}$$

The last line on the equation above implies that the firm maximization problem is analogous to the problem without firing cost showed in the previous section. Therefore, the FOC and the firm optimal value are the same as in the economy with no firing cost, Equations (2.5) and (2.6) respectively. To obtain the equilibrium wage with the firing cost we equate the optimal value for the firm, Equation (2.6), with the new value of the firm, Equation (2.12):

$$w^{\tau}(\delta) = w^{\text{no firing cost}}(\delta)\frac{1}{1 + \tau\delta} \tag{2.16}$$

The firm entry-condition is the same as before. The derivative of $V(\delta, U(\delta))$ with respect to δ, using the maximization problem of the firm is the same as before with no tax since the firm problem is the same:

$$\frac{\partial V\left(\delta, U(\delta)\right)}{\partial \tau} = \frac{dV\left(\delta, U(\delta)\right)}{d\tau} + \overbrace{\frac{\partial V\left(\delta, U(\delta)\right)}{\partial U(\delta)}}^{=0 \text{ at optimum}} \frac{dU(\delta)}{d\tau} = 0$$

Applying the implicit function theorem to the firm optimal solution of the profit maximization, $\frac{\partial V(\delta,U)}{\partial U} = 0$:

$$\frac{dU}{d\tau} = -\frac{\frac{\partial^2 V(\delta,U)}{\partial U \partial \tau}}{\frac{\partial^2 V(\delta,U)}{\partial^2 U}}$$

The FOC =0 is the same as the one without tax:

$$
\begin{aligned}
&\frac{\partial V(\delta,U)}{\partial U}= \\
&-r-\delta+s\theta q(\theta)\left(-\frac{e}{v}(1-G(U))-\frac{v_e}{v}(1-H(U))+\left(2\frac{e}{v}g(U)+\frac{v_e}{v}h(U)\right)V(\delta,U)\right)=0
\end{aligned}
$$

Taking the second derivative of $V(\delta,U)$ with respect to τ:

$$
\frac{\partial^2 V(\delta,U)}{\partial U \partial \tau}=s\theta q(\theta)\left(2\frac{e}{v}g(U)+\frac{v_e}{v}h(U)\right)\underbrace{\frac{\partial V(\delta,U)}{\partial \tau}}_{=0}=0
$$

Substituting the derivatives found above into the equation from the implicit function theorem we have that the worker utility does not change with τ: $\frac{\partial U}{\partial \tau}=0$

■

Proof. of Lemma 5 Taking the derivative of the worker continuation value with respect to τ:

$$
\begin{aligned}
&r\frac{dU(\delta)}{d\tau}=\frac{dw(\delta)}{d\tau}(1+\tau\delta)+\delta w(\delta)-\delta\frac{dU(\delta)}{d\tau} \\
&+s\frac{q(\theta)}{x}\left(U(\delta)\left(eg(U)+v_e h(U)\right)-\left(e\overline{G(U)}+v_e\overline{H(U)}\right)-U(\delta)\left(eg(U)+v_e h(U)\right)\right)\frac{dU(\delta)}{d\tau} \\
&\text{Since: } \frac{dU(\delta)}{d\tau}=0\Rightarrow \qquad \frac{dw(\delta)}{d\tau}=-\frac{\delta w(\delta)}{(1+\tau\delta)}<0
\end{aligned}
$$

Taking the derivative of the worker continuation value with firing cost, with respect to δ:

$$
\begin{aligned}
&r\frac{dU(\delta)}{d\delta}=\frac{dw^\tau(\delta)}{d\delta}(1+\tau\delta)+\tau w^\tau(\delta)+(U_0-U(\delta))-\delta\frac{dU(\delta)}{d\delta}-s\frac{q(\theta)}{x}U(\delta)\left(eg(U)+v_e h(U)\right)\frac{dU(\delta)}{d\delta} \\
&s\theta q(\theta)\left(-\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)\frac{dU(\delta)}{d\delta}+U(\delta)\left(\frac{e}{g}(U)+\frac{v_e}{v}h(U)\right)\frac{dU(\delta)}{d\delta}\right)\Rightarrow \\
&\frac{dw^\tau(\delta)}{d\delta}=\frac{\overbrace{\frac{dU(\delta)}{d\delta}}^{(-)}\left(r+\delta+s\theta q(\theta)\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)\right)+U(\delta)-U_0-\tau w(\delta)}{1+\tau\delta}
\end{aligned}
$$

Since U is decreasing with δ, proved in Lemma 2 above, the highest δ firm will offer the smallest utility. Therefore, for the highest $\overline{\delta}$, $U(\overline{\delta})=U_0$ implying that $\frac{dw^\tau(\delta)}{d\delta}<0$ for this type of firm. By continuity, the inequality also holds for high δ firms. ▌■

Proof. of Lemma 7 First note that

$$\frac{\partial V(\delta,U;\theta,\epsilon)}{\partial\theta} = \frac{\partial(\theta q(\theta))/\partial\theta}{\theta q(\theta)}\left(\frac{e(1-G(U))+v_e(1-H(U))}{(2eg(U)+v_e h(U))}\left(\frac{1+(\tau_W+\epsilon)\delta}{1+\tau_W\delta}\right)-V(\delta,U;\theta,\epsilon)\right)$$
$$\frac{\partial V(\delta,U;\theta,\epsilon)}{\partial\epsilon} = \frac{\delta}{1+(\tau_W+\epsilon)\delta}V(\delta,U;\theta,\epsilon)$$

Applying the implicit function theorem into the FOC:

$$\frac{\partial U}{\partial\theta} = -\frac{\frac{\partial^2 V(\delta,U)}{\partial U\partial\theta}}{\frac{\partial^2 V(\delta,U)}{\partial^2 U}} \qquad and \qquad \frac{\partial U}{\partial\epsilon} = -\frac{\frac{\partial^2 V(\delta,U)}{\partial U\partial\epsilon}}{\frac{\partial^2 V(\delta,U)}{\partial^2 U}}$$

Taking the second derivative of $V(\delta,U;\theta,\epsilon)$ with respect to U and, θ and ϵ respectively, is given by

$$\begin{aligned}
\frac{\partial^2 V(\delta,U;\theta,\epsilon)}{\partial U\partial\theta} =& \left(\frac{\partial V(\delta,U;\theta,\epsilon)}{\partial\theta}\frac{s\theta q(\theta)}{v}+V(\delta,U;\theta,\epsilon)\frac{s}{v}\frac{\partial\theta q(\theta)}{\partial\theta}\right)(2eg(U)+v_e h(U)) \\
&-\frac{s}{v}\frac{\partial\theta q(\theta)}{\partial\theta}\left(e(1-G(U))+v_e(1-H(U))\right)\left(\frac{1+(\tau_W+\epsilon)\delta}{1+\tau_W\delta}\right) \\
=&\frac{s}{v}\frac{\partial\theta q(\theta)}{\partial\theta}\left(\frac{e(1-G(U))+v_e(1-H(U))}{(2eg(U)+v_e h(U))}\left(\frac{1+(\tau_W+\epsilon)\delta}{1+\tau_W\delta}\right)\right)(2eg(U)+v_e h(U)) \\
&-\frac{s}{v}\frac{\partial\theta q(\theta)}{\partial\theta}\left(e(1-G(U))+v_e(1-H(U))\right)\left(\frac{1+(\tau_W+\epsilon)\delta}{1+\tau_W\delta}\right)=0 \\
\frac{\partial^2 V(\delta,U;\theta,\epsilon)}{\partial U\partial\epsilon} =& \frac{\partial V(\delta,U;\theta,\epsilon)}{\partial\epsilon}\frac{s\theta q(\theta)}{v}(2eg(U)+v_e h(U)) \\
&-\left(r+\delta+s\theta q(\theta)\left(\frac{e}{v}(1-G(U))+\frac{v_e}{v}(1-H(U))\right)\right)\frac{\delta}{1+\tau_W\delta} \\
=&\frac{\delta}{1+(\tau_W+\epsilon)\delta}V(\delta,U;\theta,\epsilon)\frac{s\theta q(\theta)}{v}(2eg(U)+v_e h(U)) \\
&-\delta V(\delta,U;\theta,\epsilon)\frac{\frac{sq(\theta)}{x}\left(2eg\left(U\right)+v_e h\left(U\right)\right)}{1+(\tau_W+\epsilon)\delta}=0
\end{aligned}$$

Substituting the derivatives found above into the equation derived from the implicit function theorem we have that the worker utility of equilibrium does not change with θ and ϵ. ∎

■ **Proof. of Lemma 8** We can write the free-entry condition as:

$$FE(\theta,\epsilon) = \int_{\underline{\delta}}^{\bar{\delta}} \theta q(\theta)V\left(\delta;\theta,\epsilon\right)\left(\frac{u}{v}+\frac{se}{v}G(U(\delta;\theta\epsilon))\right)d\Gamma(\delta)-c_e=0$$

The implicit function theorem implies that:

$$\frac{\partial\theta}{\partial\epsilon} = -\frac{FE_{\epsilon}(\theta,\epsilon)}{FE_{\theta}(\theta,\epsilon)}$$

Where:

$$FE_{\theta}(\theta,\epsilon) = \int_{\underline{\delta}}^{\bar{\delta}} \left(\frac{\partial\theta q(\theta)}{\partial\theta}V(\delta;\theta,\epsilon) + \theta q(\theta)\frac{\partial V(\delta;\theta,\epsilon)}{\partial\theta}\right)\left(\frac{u}{v} + \frac{se}{v}G(U(\delta;\theta\epsilon))\right)d\Gamma(\delta)$$

$$+ \int_{\underline{\delta}}^{\bar{\delta}} \theta q(\theta)V(\delta;\theta,\epsilon)\frac{se}{v}g(U(\delta;\theta\epsilon)) \underbrace{\frac{\partial U(\delta;\theta\epsilon)}{\partial\theta}}_{=0 \text{ by Lemma 7}} d\Gamma(\delta)$$

$$= \int_{\underline{\delta}}^{\bar{\delta}} \frac{\partial\theta q(\theta)}{\partial\theta}\left(\frac{e(1-G(U)) + v_e(1-H(U))}{(2eg(U) + v_e h(U))}\right)\left(\frac{1+(\tau_W+\epsilon)\delta}{1+\tau_W\delta}\right)\left(\frac{u}{v} + \frac{se}{v}G(U(\delta;\theta\epsilon))\right)d\Gamma(\delta)$$

$$FE_{\epsilon}(\theta,\epsilon) =$$

$$\int_{\underline{\delta}}^{\bar{\delta}} \theta q(\theta)\left(\frac{\partial V(\delta;\theta,\epsilon)}{\partial\epsilon}\left(\frac{u}{v} + \frac{se}{v}G(U(\delta;\theta\epsilon))\right) + V(\delta;\theta,\epsilon)\frac{se}{v}g(U(\delta;\theta\epsilon))\frac{\partial U(\delta;\theta\epsilon)}{\partial\epsilon}\right)d\Gamma(\delta)$$

$$= \int_{\underline{\delta}}^{\bar{\delta}} \theta q(\theta)\frac{\delta}{1+(\tau_W+\epsilon)\delta}V(\delta,U;\theta,\epsilon)\left(\frac{u}{v} + \frac{se}{v}G(U(\delta;\theta\epsilon))\right)d\Gamma(\delta)$$

Substituting into the equation using the implicit function theorem:

$$\frac{\partial\theta}{\partial\epsilon} = -\frac{\theta q(\theta)\int_{\underline{\delta}}^{\bar{\delta}} \frac{\delta}{1+(\tau_W+\epsilon)\delta}V(\delta,U;\theta,\epsilon)\left(\frac{u}{v} + \frac{se}{v}G(U(\delta;\theta\epsilon))\right)d\Gamma(\delta)}{\frac{\partial\theta q(\theta)}{\partial\theta}\int_{\underline{\delta}}^{\bar{\delta}}\left(\frac{e(1-G(U))+v_e(1-H(U))}{(2eg(U)+v_e h(U))}\right)\left(\frac{1+(\tau_W+\epsilon)\delta}{1+\tau_W\delta}\right)\left(\frac{u}{v} + \frac{se}{v}G(U(\delta;\theta\epsilon))\right)d\Gamma(\delta)} < 0$$

▮ ■

2.7.6 Estimation With Discrete Case of δ

For the estimation part let time and δ be discrete, with $\delta_i \in \{\delta_1, \delta_2, ..., \delta_N\}$ where $\delta_1 < \delta_2 < ... < \delta_N$. The value of a worker employed in a firm with destruction rate δ_i and that offers him wage of w is given by:

$$U(\delta_i, w) = U_i = w(\delta_i, U_i)\Delta t + \frac{\Delta t \left(\delta_i \left(U_0 + \tau w\right) + s\theta q(\theta) \sum_{\tilde{U}} \max\left\{\tilde{U}, U_i\right\} \left(\frac{e}{v} g(\tilde{U}) + \frac{v_e}{v} h(\tilde{U})\right)\right) + \left(1 - (\delta_i + s\theta q(\theta))\Delta t\right) U_i}{1 + r\Delta t} \tag{2.17}$$

Call $w(\delta_i, U_i) = \mathbf{w_i}$. We can rewrite the Equation (2.17) above as

$$U_i = \frac{\mathbf{w_i}(1 + \tau\delta_i + r\Delta t) + \delta_i U_0 + s\frac{q(\theta)}{x} \sum_{\tilde{U}=U_1}^{U_i} \tilde{U} \left(eg(\tilde{U}) + v_e h(\tilde{U})\right)}{r + \delta_i + s\frac{q(\theta)}{x}(e\overline{G(U_{i+1})} + v_e \overline{H(U_{i+1})})} \tag{2.18}$$

Writing the worker value given by Equation (2.17) for $i+1$ and rearranging we can write the worker expected future utility as a function of U_{i+1} and substituting the expected future utility into Equation (2.18) we can have U_i as a function only of U_{i+1} and U_0:

$$\begin{aligned} U_i = & \frac{\mathbf{w_i}(1 + \tau\delta_i + r\Delta t) - \mathbf{w_{i+1}}(1 + \tau\delta_{i+1} + r\Delta t)}{r + \delta_i + s\frac{q(\theta)}{x}(e\overline{G(U_{i+1})} + v_e \overline{H(U_{i+1})})} \\ & + \frac{(\delta_i - \delta_{i+1})U_0 + U_{i+1}\left(r + \delta_{i+1} + s\frac{q(\theta)}{x}(e\overline{G(U_{i+1})} + v_e \overline{H(U_{i+1})})\right)}{r + \delta_i + s\frac{q(\theta)}{x}(e\overline{G(U_{i+1})} + v_e \overline{H(U_{i+1})})} \end{aligned} \tag{2.19}$$

The value of an unemployed worker is given by:

$$U_0 = b\Delta t + \frac{1}{1 + r\Delta t}\left(\theta q(\theta)\Delta t \sum_{all\tilde{U}} \tilde{U}\left(\frac{e}{v} g(\tilde{U}) + \frac{v_e}{v} h(\tilde{U})\right) + (1 - \theta q(\theta)\Delta t)U_0\right)$$

The value per worker for a firm with δ_i that promises a utility level U_i for its workers is:

$$\begin{aligned} V(\delta_i, U_i) = & \left(1 - \mathbf{w_i}\right)\Delta t + \Delta t \times \\ & \frac{\left(-\delta_i \tau \mathbf{w_i} + q(\theta)\left(2\left(\frac{se}{x} G(U_{i+1}) + \frac{u}{x}\right) + \frac{se}{x}\overline{G(U_{i+1})}\right) V(\delta_i, U_i) + s\theta q(\theta)\left(\frac{e}{v} G(U_i) + \frac{v_e}{v} H(U_i)\right) V(\delta_i, U_i)\right)}{1 + r\Delta t} \\ & + \frac{\left(1 - (\delta_i + q(\theta) + s\theta q(\theta))\Delta t\right) V(\delta_i, U_i)}{1 + r\Delta t} \end{aligned}$$

The firm with the highest δ, $(\bar{\delta})$, will pay a wage such that the workers is indifferent between accepting the job or not ($U_N = U_0$):

$$U_N = \frac{\mathbf{w_N}(1+\tau\delta_N + r\Delta t) + \delta_N U_0 + s\frac{q(\theta)}{x}\sum_{all\tilde{U}} \tilde{U}\left(eg(\tilde{U}) + v_e h(\tilde{U})\right)}{r + \delta_N + s\frac{q(\theta)}{x}(e+v_e)} =$$
$$U_0 = \frac{b(1+r\Delta t) + \frac{q(\theta)}{x}\sum_{all\tilde{U}} \tilde{U}\left(eg(\tilde{U}) + v_e h(\tilde{U})\right)}{r + \theta q(\theta)} \Rightarrow \mathbf{w_N} = \frac{sb(1+r\Delta t) + (1-s)rU_0}{1+\tau\delta_N + r\Delta t} \tag{2.20}$$

The incentive-comparability (IC) constraint of the firm must guarantee that the firm prefers to offer U_i associated with her specific δ_i, than a lower utility level U_{i-1} associated with δ_{i-1}. Therefore the IC is given by:

$$V(\delta_i, U_i) \geq V(\delta_i, U_{i+1}) \quad \Rightarrow$$
$$\frac{1 + r\Delta t - \mathbf{w_i}(1+\tau\delta_i + r\Delta t)}{r + \delta_i - \frac{q(\theta)}{x}\left(seG\left(U_{i+1}\right) + u\right) + s\frac{q(\theta)}{x}\left(e\overline{G\left(U_i\right)} + v_e\overline{H\left(U_i\right)}\right)} \geq$$
$$\frac{1 + r\Delta t - w(\delta_i, U_{i+1})(1+\tau\delta_i + r\Delta t)}{r + \delta_i - \frac{q(\theta)}{x}\left(seG\left(U_{i+2}\right) + u\right) + s\frac{q(\theta)}{x}\left(e\overline{G\left(U_{i+1}\right)} + v_e\overline{H\left(U_{i+1}\right)}\right)}$$

Implying that:

$$\mathbf{w_i} \leq \frac{(1+r\Delta t)(1-Z^i_{i,i+1}) + w(\delta_i, U_{i+1})(1+\tau\delta_i + r\Delta t)Z^i_{i,i+1}}{1+\tau\delta_i + r\Delta t} \tag{2.21}$$

Where $Z^i_{i,i+1} = \frac{r+\delta_i - \frac{q(\theta)}{x}(seG(U_{i+1})+u) + s\frac{q(\theta)}{x}\left(e\overline{G(U_i)} + v_e\overline{H(U_i)}\right)}{r+\delta_i - \frac{q(\theta)}{x}(seG(U_{i+2})+u) + s\frac{q(\theta)}{x}\left(e\overline{G(U_{i+1})} + v_e\overline{H(U_{i+1})}\right)} \in (0,1)$. The wage that guarantees a utility of U_{i+1} for a worker employed in a firm with exogenous destruction rate δ_{i+1}

and δ_i, respectively, is:

$$w(\delta_i, U_{i+1})(1 + \tau\delta_i + r\Delta t) =$$

$$U_{i+1}\left(r + \delta_i + s\frac{q(\theta)}{x}(e\overline{G(U_{i+2})} + v_e\overline{H(U_{i+2})})\right) - \delta_i U_0 - s\frac{q(\theta)}{x}\sum_{\tilde{U}=U_1}^{U_{i+1}} \tilde{U}\left(eg(\tilde{U}) + v_e h(\tilde{U})\right)$$

$$\mathbf{w_{i+1}}(1 + \tau\delta_{i+1} + r\Delta t) =$$

$$U_{i+1}\left(r + \delta_{i+1} + s\frac{q(\theta)}{x}(e\overline{G(U_{i+2})} + v_e\overline{H(U_{i+2})})\right) - \delta_{i+1} U_0 - s\frac{q(\theta)}{x}\sum_{\tilde{U}=U_1}^{U_{i+1}} \tilde{U}\left(eg(\tilde{U}) + v_e h(\tilde{U})\right)$$

Subtracting $w(\delta_i, U_{i+1})(1+\tau\delta_i+r\Delta t)$ from $\mathbf{w_{i+1}}(1+\tau\delta_{i+1}+r\Delta t)$ [6] and substituting $w(U_{i+1}, \delta_i)(1+\tau\delta_i + \Delta t)$ into Equation (2.21) we can write the optimal worker wage in firm δ_i as a function of the optimal wage in firm δ_{i+1}:

$$\mathbf{w_i} \leq \frac{(1 + r\Delta t)\left(1 - Z^i_{i,i+1}\right)}{1 + \tau\delta_i + r\Delta t} + \frac{\left(\mathbf{w_{i+1}}(1 + \tau\delta_{i+1} + r\Delta t) + (\delta_i - \delta_{i+1})(U_{i+1} - U_0)\right) Z^i_{i,i+1}}{1 + \tau\delta_i + r\Delta t} \tag{2.22}$$

And finally, if δ is discrete the firm free-entry condition is given by:

$$E_\delta\left\{V(\delta, \bar{U})\right\} = q(\theta)\sum_{all\ \ \delta_i}\left(V(\delta_i)\left(\frac{se}{x}G(U(\delta_i)) + \frac{u}{x}\right)\gamma(\delta_i)\right) = \frac{c_e(1 + r\Delta t)}{r}$$

Worker Flow

Since we are assuming that time and δ is discrete and $\delta_1 < \delta_2 < ... < \delta_N$, then $n_t(\delta_i) = N_t(\delta_i) - N_t(\delta_{i-1})$, $\gamma(\delta_i) = \Gamma(\delta_i) - \Gamma(\delta_{i-1})$ and the numbers of workers employed in firm with

[6] $w(\delta_i, U_{i+1})(1 + \tau\delta_i + r\Delta t) - \mathbf{w_{i+1}}(1 + \tau\delta_{i+1} + r\Delta t) = (\delta_i - \delta_{i+1})(U_{i+1} - U_0)$

δ evolves according to:

$$
\begin{aligned}
n_{t+1}(\delta_i) - n_t(\delta_i) = & - \underbrace{\delta_i n_t(\delta_i)}_{\substack{\text{matches}\\ \text{that ends}}} + \underbrace{s\theta_t q(\theta_t)(e_t - N_t(\delta_i)) \left(\frac{e_t}{v_t}\frac{n_t(\delta_i)}{e_t} + \frac{v_{et}}{v_t}\gamma(\delta_i)\right)}_{\substack{\text{employed workers hired}\\ \text{from highest } \delta \text{ firms and from new firms}}} \\
& + \underbrace{\theta_t q(\theta_t) u_t \left(\frac{e_t}{v_t}\frac{n_t(\delta_i)}{e_t} + \frac{v_{et}}{v_t}\gamma(\delta_i)\right)}_{\substack{\text{unemployed workers hired}\\ \text{from existing firms and from new firms}}} \\
& - \underbrace{s\theta_t q(\theta_t) n_t(\delta_i) \left(\frac{e_t}{v_t}\frac{N_t(\delta_{i-1})}{e_t} + \frac{v_{et}}{v_t}\Gamma(\delta_{i-1})\right)}_{\substack{\text{workers that left because found}\\ \text{job in existing firms or in new firms}}}
\end{aligned} \tag{2.23}
$$

At equilibrium $n_{t+1}(\delta) = n_t(\delta) = n(\delta) =$, and $\theta_t = \theta$, $e_t = e$, $v_t = v$.... $\forall t$ [7]:

$$
n(\delta_i)\left(sn(\delta_i) + \frac{x\delta_i}{q(\theta)} - x + 2sN(\delta_{i-1})\right) + sn(\delta_i)v_e\Gamma(\delta_i) - \left(x - sN(\delta_{i-1})\right)v_e\gamma(\delta_i) = 0 \tag{2.24}
$$

OJS Rate

The total number of separations that occurs at every instant of time among employees working in firms type δ can be distinguish among two causes. The first is that the match is dissolved because of some exogenous shock (δ_i). And the second cause is that the worker leaves the job to go to a better job, OJS.

$$
Total\ Separations_{(\delta_i)} = \delta_i n(\delta_i) + s\theta q(\theta) n(\delta_i)\left(\frac{N(\delta_{i-1})}{v} + \frac{v_e\Gamma(\delta_{i-1})}{v}\right)
$$

The fraction of total separation due to OJS of workers employed in firms type δ is:

$$
\frac{Separation\ Due\ To\ OJS}{Total\ Separation}(\delta_i) = \frac{\frac{sq(\theta)}{x}\left(N(\delta_{i-1}) + v_e\Gamma(\delta_{i-1})\right)}{\delta_i + \frac{sq(\theta)}{x}\left(N(\delta_{i-1}) + v_e\Gamma(\delta_{i-1})\right)}
$$

[7] $\left(\frac{x}{q(\theta)}\delta_i - x + s(N(\delta_i) + N(\delta_{i-1}) + v_e\Gamma(\delta_{i-1}))\right)n(\delta_i) = \left(x - sN(\delta_i)\right)v_e\gamma(\delta_i)$

2.7.7 Estimation Strategy

***fsolve* to find s:** Solve the following algorithm until:

$$\widehat{OJSRate}_{Data} = \sum_{t=\text{all months}} \frac{\text{Total Job-to-Job Transitions}_t}{\text{Total Job Separations}_t} / T$$

$$\widehat{OJSRate}_{Data} \approxeq OJSRate_{Model} = \frac{\sum_i \left(s\frac{q(\theta)}{x} n(\delta_i) \left(N(\delta_{i-1} + v_e \Gamma(\delta_{i-1})) \right) \right)}{\sum_i \left(\delta_i n(\delta_i) + s\frac{q(\theta)}{x} n(\delta_i) \left(N(\delta_{i-1} + v_e \Gamma(\delta_{i-1})) \right) \right)}$$

Algorithm:

- From the date use:
  - δ and $N(\delta)$ estimated from the data
  - e, u and τ from the economy
  - Assume a monthly period, r from the economy and let $\Delta t = 1$
- Since the firm free-entry condition is decreasing with the market tightness, pick θ_{Low} low enough and θ_{High} high enough such that: $FE(\theta_{Low}) > 0$ and $FE(\theta_{High}) < 0$

Calculate the equilibrium below for $\theta = 0.5 \times \theta_{Low} + 0.5 \times \theta_{High}$:

First Part inside *fsolve*: Recover the entrants distributions for δ, $\Gamma()$ and $\gamma()$, and the utilities distributions $G()$, $g()$, $H()$ and $h()$:

- $\Gamma(\overline{\delta}) = 1$
- $\Gamma(\delta_i) = \Gamma(\delta_{i+1}) - \gamma(\delta_{i+1})$
- From Equation (2.24): $\gamma(\delta_i) = \frac{sn(\delta_i)^2 + \left(\frac{x}{q(\theta)}\delta_i - x + s(2N(\delta_{i-1}) + v_e\Gamma(\delta_i))\right)n(\delta_i)}{(x - sN(\delta_{i-1}))v_e}$
- Since δ is discrete:
  - $H(U(\delta_i)) = H(U_i) = 1 + \frac{1-\gamma(\overline{\delta})}{1-\gamma(\underline{\delta})}\left(\gamma(\underline{\delta}) - \Gamma(\delta)\right)$ and $h(U_i) = H(U_i) - H(U_{i-1})$

- $G(U(\delta_i)) = G(U_i) = e + \frac{e-n(\overline{\delta})}{e-n(\underline{\delta})}\left(n(\underline{\delta}) - N(\delta)\right)$ and $g(U_i) = G(U_i) - G(U_{i-1})$

Second Part inside *fsolve*: Recover U, V, w,... using *fsolve*:

- From Equations (2.19), (2.20) and (2.22):

$$w_N - \frac{sb(1+r\Delta t) + (1-s)rU_0}{1+\tau\delta_N + r\Delta t} = 0$$

$$w_i - \frac{(1+r\Delta t)\left(1 - Z^i_{i,i+1}\right) + \Big(w_{i+1}(1+\tau\delta_{i+1} + r\Delta t) + (\delta_i - \delta_{i+1})(U_{i+1} - U_0)\Big) Z^i_{i,i+1}}{1+\tau\delta_i + r\Delta t} = 0$$

$$U_N - U_0 = 0$$

$$U_i - \frac{w_i(1+\tau\delta_i + r\Delta t) - w_{i+1}(1+\tau\delta_{i+1} + r\Delta t)}{r+\delta_i + s\frac{q(\theta)}{x}(e\overline{G(U_{i+1})} + v_e\overline{H(U_{i+1})})} - \frac{(\delta_i - \delta_{i+1})U_0 + U_{i+1}\Big(r+\delta_{i+1} + s\frac{q(\theta)}{x}(e\overline{G(U_{i+1})} + v_e\overline{H(U_{i+1})})\Big)}{r+\delta_i + s\frac{q(\theta)}{x}(e\overline{G(U_{i+1})} + v_e\overline{H(U_{i+1})})} = 0$$

Where $Z^N_{N-1,N} = \frac{r+\delta_{N-1} + \frac{q(\theta)}{x}\left(-seG(U_N) - u + se\overline{G(U_{N-1})} + sv_e\overline{H(U_{N-1})}\right)}{r+\delta_{N-1} + \frac{q(\theta)}{x}\left(-u + se\overline{G(U_N)} + sv_e\overline{H(U_N)}\right)}$, and

$Z^i_{i,i+1} = \frac{r+\delta_i - \frac{q(\theta)}{x}(seG(U_{i+1}) + u) + s\frac{q(\theta)}{x}\left(e\overline{G(U_i)} + v_e\overline{H(U_i)}\right)}{r+\delta_i - \frac{q(\theta)}{x}(seG(U_{i+2}) + u) + s\frac{q(\theta)}{x}\left(e\overline{G(U_{i+1})} + v_e\overline{H(U_{i+1})}\right)}$ for $i < N-1$.

Check if: $U^{\max} = U(\delta_1) = \frac{w_1(1+\tau\delta_1 + r\Delta t) + \delta_1 U_0}{r+\delta_1}$

Update θs: $FE(\theta) = \sum_{i=1}^{N} q(\theta)V(\delta_i)\left(\frac{u}{x} + \frac{se}{x}G(U(\delta_i))\right)\gamma(\delta_i)$

If $FE(\theta) < 0$ update: $\theta_{High\ new\ guess} = \theta$

If $FE(\theta) > 0$ update: $\theta_{Low\ new\ guess} = \theta$

Repeats Until: $FE(\theta) \approxeq 0$

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