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UNIVERSITY OF CALIFORNIA  
SANTA CRUZ

**RETROFITTED SUPERSYMMETRIC MODELS**

A dissertation submitted in partial satisfaction of the  
requirements for the degree of

DOCTOR OF PHILOSOPHY

in

PHYSICS

by

**Manatosh Bose**

June 2015

The Dissertation of Manatosh Bose  
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# Abstract

## Retrofitted Supersymmetric Models

by

Manatosh Bose

This thesis explores several models of metastable dynamic supersymmetry breaking (MDSB) and a supersymmetric model of hybrid inflation. All of these models possess discrete R-symmetries. We specially focus on the retrofitted models for supersymmetry breaking models. At first we construct retrofitted models of gravity mediation. In these models we explore the genericity of the so-called “split supersymmetry.” We show that with the simplest models, where the goldstino multiplet is neutral under the discrete R-symmetry, a split spectrum is not generic. However if the the goldstino superfield is charged under some symmetry other than the R-symmetry, then a split spectrum is achievable but not generic. We also present a gravity mediated model where the fine tuning of the Z-boson mass is dictated by a discrete choice rather than a continuous tuning. Then we construct retrofitted models of gauge mediated SUSY breaking. We show that, in these models, if the approximate R-symmetry of the theory is spontaneously broken, the messenger scale is fixed; if explicitly broken by retrofitted couplings, a very small dimensionless number is required; if supergravity corrections are responsible for the symmetry breaking, at least two moderately small couplings are required, and that there is a large range of possible messenger scales. Finally we switch our attention to small field hybrid inflation. We construct a model that yields a spectral index  $n_s = 0.96$ . Here, we also briefly discuss the possibility of relating the scale of inflation with the dynamics responsible for supersymmetry breaking.





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Finally, I would like to thank Lauren Porter for showing me how I can successfully transfer my physics training to data-driven model building.

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Theories,” [arXiv:hep-ph/1212.4369](#) [hep-ph][2], M. Bose, M. Dine, A. Monteux, and L. S. Haskins, “Small Field Inflation and the Spectral Index,” *JCAP* **1401** (2014) 038, [arXiv:1310.2609](#) [hep-ph][3].

# Chapter 1

## Introduction

Much of the development of the modern particle physics has been guided by *the principle of naturalness* as described by 't Hooft. According to this principle a physical quantity should be small only if the underlying theory becomes more symmetric as the quantity tends to zero[4]. For example, the Standard Model (SM) fermions are small compared to the Planck Mass ( $M_p \sim 10^{18}$  GeV) since if they are massless, the theory possesses exact chiral symmetry. It might be that the chiral symmetry is spontaneously broken at a short distance scale, generating fermion masses. In fact, all of the parameters of the SM are natural in the sense of 't Hooft except for the Higgs boson mass which causes the electroweak symmetry breaking that gives masses to the W and Z bosons ( $M_W \sim 80$  GeV,  $M_Z \sim 90$  GeV).

There is no symmetry in the Standard Model that can explain the smallness of the Higgs mass. However, this problem can be resolved by extending the Standard Model to incorporate supersymmetry. In this framework every fermion has a partner boson and if

the supersymmetry is exact, fermion and boson masses are degenerate. Supersymmetry is broken to generate mass hierarchies between the fermions and the bosons. Moreover, in the Minimal Supersymmetric Standard Model (MSSM), supersymmetry breaking can trigger the electroweak symmetry breaking. Thus as long as the scale of supersymmetry breaking is comparable to the electroweak scale, the naturalness problem is solved.

However the recent discovery of the Higgs boson at 125 GeV implies that model parameters need to be fine tuned to achieve a low scale SUSY breaking. In addition to the Higgs mass, there are exclusions at the 1 TeV scale of squarks and gluinos, except in narrow slices of the parameter space. These exclusion limits on the squark and gluino masses along with the heavy Higgs boson make it difficult to build low energy SUSY breaking models that are not fine-tuned.

In this thesis, we explore a few generic metastable dynamical SUSY breaking (MDSB) models with a relatively high scale of SUSY breaking. By generic models we mean models in which the Lagrangian contains all possible terms consistent with the symmetry and no finely tuned parameters[5]. All our models possess discrete R-symmetry as opposed to continuous R-symmetry, since string theory (and possibly quantum gravity) does not have any continuous global symmetry[6]. We mainly focus on the retrofitted models where discrete R-symmetry is an essential feature. We construct models where the SUSY breaking is communicated through gravity mediation and gauge mediation. In the case of gravity mediation, we propose a plausible reason for the tuning in the Higgs sector. Finally we also present a model of hybrid inflation where we attempt to connect the scale of inflation with the dynamics of SUSY breaking.

In Chapter 2, we give a brief introduction to supersymmetry and supersymmetry breaking. We focus on dynamical SUSY breaking and describe the technique of retrofitting. In the next two chapters, Chapter 3 and Chapter 4, we explore a few retrofitted models of gravity mediated and gauge mediated supersymmetry breaking. Work presented in these two chapters was previously published as M. Bose, M. Dine, “Gravity Mediation Retrofitted,” *JHEP* **1303** (2013) 057, [arXiv:hep-ph/1209.2488](#) [[hep-ph](#)][1], and M. Bose, M. Dine, “Discrete Symmetries/Discrete Theories,” [arXiv:hep-ph/1212.4369](#) [[hep-ph](#)][2]. Finally, in Chapter 5, we switch our attention to small field hybrid inflation and discuss the possibility of relating the scale of inflation with the dynamics responsible for supersymmetry breaking. This work was previously published as M. Bose, M. Dine, A. Monteux, and L. S. Haskins, “Small Field Inflation and the Spectral Index,” *JCAP* **1401** (2014) 038, [arXiv:1310.2609](#) [[hep-ph](#)][3].

## Chapter 2

# Supersymmetry and Its Dynamical Breaking

### 2.1 Supersymmetry

Supersymmetry is a vast topic. There are many excellent textbooks and reviews on this topic; in particular we found the textbooks[7, 8, 9], and the reviews[10, 11, 12] to be very useful for our work. In this section, we give a lightning review of the ingredients of a globally supersymmetric theory and describe how to construct one.

Supersymmetric theories are most elegantly formulated using the superspace formalism. In this formulation, the usual space-time coordinate is supplemented by two extra Grassmann variables  $\theta_\alpha$  and  $\bar{\theta}_{\dot{\alpha}} = (\theta_\alpha)^\dagger$ . Here  $\alpha$  and  $\dot{\alpha}$  are spinor indices. Derivatives are now superderivatives which contain derivatives with respect to the Grassmann coordinates

as well, as defined below:

$$D_\alpha = \frac{\partial}{\partial \theta_\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \quad (2.1)$$

In superspace, fields appear as part of superfields. In global supersymmetry, there are two types of superfields: chiral and vector superfields. A chiral superfield,  $\Phi$ , obeys the following condition:

$$\bar{D}_{\dot{\alpha}} \Phi = 0 \quad (2.2)$$

The simplest chiral superfield,  $\Phi$ , contains a complex scalar,  $\phi$ , a Weyl fermion,  $\psi$ , and an auxiliary complex scalar,  $F$ . Using the superspace coordinates,  $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$ , this superfield has the following expansion:

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \quad (2.3)$$

The auxiliary field  $F$  is non-dynamical and is there to match the total fermionic and total bosonic degrees of freedom.

A vector superfield satisfies:

$$V^a = V^{a\dagger} \quad (2.4)$$

Thus, this is invariant under the gauge transformation  $V^a \rightarrow V^a + \Lambda^a + \Lambda^{a\dagger}$ . Here  $\Lambda$  is some chiral superfield. Under this gauge transformation a chiral superfield  $\Phi$  transforms as  $\Phi \rightarrow e^{-gT^a\Lambda^a}\Phi$ . In the Wess-Zumino gauge the vector superfields have a very simple expansion:

$$V^a = -\theta\sigma^\mu\bar{\theta}A_\mu^a(x) + i\theta^2\bar{\theta}\lambda^{\dagger a}(x) - i\theta^2\bar{\theta}^2 D^a(x) \quad (2.5)$$



Here  $A^a$  is a gauge boson,  $\lambda^a$  is a chiral fermion and  $D^a$  is another auxiliary field.

One can form a chiral superfield,  $W_\alpha$  by taking the superderivative of the vector superfield:

$$W_\alpha^a = -\frac{1}{4}\bar{D}^2 e^{T^a V^a} D_\alpha e^{T^a V^a} \quad (2.6)$$

In the Wess-Zumino gauge, it is given by

$$W_\alpha^a = -i\lambda_\alpha^a(y) + \theta_\alpha D^a(y) - (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}^a(y) - \theta^2 \sigma^\mu D_\mu \lambda^{\dagger a}(y) \quad (2.7)$$

$W_\alpha^a$  is called the “field strength superfield,” since it contains the ordinary field strength tensor  $F_{\mu\nu}^a$ .

Now we can write down the Lagrangian for a supersymmetric action as follows

$$\mathcal{L} = \int d^4\theta K(\Phi^\dagger, \Phi, V) + \int d^2\theta \left[ \frac{\tau(\Phi)}{32\pi^2} W^\alpha W_\alpha + W(\Phi) \right] + \text{h.c.} \quad (2.8)$$

The first term is the Kähler potential. It contains the gauge invariant kinetic terms as well as the gauge interactions for the chiral superfields. The second term is holomorphic, i.e. a function of  $\Phi$  and  $W_\alpha$  and not  $\Phi^\dagger$  and  $W_\alpha^\dagger$ . The first piece of the second term contains the gauge field strength terms, while the second piece is the superpotential that contains all the non-gauge interactions of the chiral superfields.

Solving the equations of motion, the non-dynamical auxiliary fields  $F_i$  and  $D^a$  can easily be eliminated.

$$F_i^* = \frac{\partial W}{\partial \Phi^i} = W_i \quad (2.9)$$

$$D^a = -g\phi_i^*(T^a)^{ij}\phi_j \quad (2.10)$$

These lead to the scalar potential

$$V = |F|^2 + \frac{1}{2}D^2 = W_i W^{i*} + \frac{1}{2}g^2 [\phi_i^*(T^a)^{ij}\phi_j]^2 \quad (2.11)$$

If any of  $\langle F \rangle$  or  $\langle D \rangle$  is not zero then supersymmetry is broken; non-zero vacuum energy is a signal of supersymmetry breaking. In this thesis, we focus on models where SUSY is spontaneously broken due to non-zero  $F$ -terms.

## 2.2 Dynamical SUSY breaking

As mentioned earlier, supersymmetry is spontaneously broken by a non-zero vacuum expectation value (vev). This vev is the order parameter of the supersymmetry breaking. Thus if supersymmetry has any hope of solving the hierarchy problem, there must be a dynamical mechanism for spontaneous SUSY breaking. In other words, the order parameter of supersymmetry breaking ( $M_s$ ) should be the result of an exponentially small effect.

$$M_s = M_{cutoff} e^{-\frac{c}{g(M_{cutoff})}} \quad (2.12)$$

In this section, we introduce the concept of dynamical supersymmetry breaking (DSB) and the retrofitting technique using a simple example. Curious readers may consult these lectures[11, 12] and the references therein for a detailed introduction to this topic.

Consider this simplest example of global supersymmetry breaking with a single chiral superfield  $X$  with canonical Kähler potential

$$K = X^\dagger X \quad (2.13)$$

and a linear superpotential

$$W = fX \quad (2.14)$$

Here  $f$  is a constant with mass dimension two. Using (2.9) we get a non-zero vev of the  $F$ -term  $f$ . Thus supersymmetry is spontaneously broken with vev  $V = |f|^2$ .

This model does not give any dynamical explanation for the origin of the parameter  $f$ . In the absence of symmetries or special dynamics, we expect  $f$  to be  $\sim M_p^2$ . However, in a model of dynamical SUSY breaking a very small  $f$ , as suggested in (2.12), can be generated.

### 2.2.1 Retrofitting Technique

The technique of retrofitting [13] is the simplest way to dynamically generate small mass scales. By retrofitting, we can dynamically generate the scale  $f$  in the above SUSY breaking model at low energy. Suppose we add to the above model a  $SU(N)$  gauge group under which the  $X$  is neutral. Then we can replace the superpotential with  $W = \frac{W_\alpha^2}{M_p} X$ . So

$$\mathcal{L} \supset \int \frac{d^2\theta}{32\pi^2} \left( \frac{8\pi^2}{g^2} + \frac{X}{M_p} \right) W^{a\alpha} W_\alpha^a \quad (2.15)$$

At low energy we get an effective superpotential

$$W_{eff} = N\Lambda^3 e^{\frac{-X}{NM_p}} \approx N\Lambda^3 - X \frac{\Lambda^3}{M_p} \quad (2.16)$$

Here  $\Lambda$  is generated by gaugino condensation such that

$$\Lambda^3 = \frac{1}{N} \left\langle \int \frac{d^2\theta}{32\pi^2} W_\alpha W^\alpha \right\rangle \quad (2.17)$$

The effective superpotential is exactly the same as the superpotential of the original model except for the constant term.<sup>1</sup>

This model has an issue with vacuum stability, which we will address shortly. But it demonstrates the key ideas behind the retrofitting technique: take a model of perturbative SUSY breaking and then replace the dimensionful parameters with dynamically small

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<sup>1</sup>The constant term in a superpotential is inconsequential in the case of global supersymmetry breaking but plays an important role in the local supersymmetry breaking

operator vevs[13]. Despite this apparent simplicity of building retrofitted models, these models are quite restrictive and often have interesting predictions. In the next two chapters we analyze several of these models.

## Chapter 3

# Retrofitting Gravity Mediation

Most discussion of metastable dynamical supersymmetry breaking (MDSB) has focussed on low energy breaking, as in gauge mediation. It is of interest to consider possible implications for intermediate scale breaking (“gravity mediation”), especially as the early LHC results suggest the possibility that supersymmetry, if broken at relatively low energies, might be tuned. A somewhat high scale for SUSY breaking could ameliorate the usual flavor problems of gravity mediation, resolve the question of cosmological moduli, and account for a Higgs with mass well above  $M_Z$ . We study MDSB in gravity mediation, especially retrofitted models in which discrete  $R$  symmetries play an important role, considering questions including implications of symmetries for  $B$  and  $A$  terms, and the genericity of split supersymmetry.

### 3.1 Introduction

If supersymmetry has something to do with the hierarchy problem, it is almost certainly dynamically broken. First, this is necessary to naturally account for a large hierarchy. Second, the “landscape”, whatever its limitations, provides a *model* for considering questions of naturalness, and in this context, if supersymmetry breaking is not dynamical, breaking at the highest scales is favored[14, 15, 16].

Until the work of Intriligator, Shih and Seiberg (ISS)[17], however, dynamical supersymmetry breaking appeared to be a special, almost singular, phenomenon<sup>1</sup>. With ISS, the focus shifted to metastable, dynamical breaking (MDSB), and this appears generic. The simplest implementation of such breaking occurs in retrofitted models[13]. Much of the

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<sup>1</sup>In the context of dynamical models, metastability had earlier been exploited in in [18]

work on models of MDSB has focussed on gauge mediation. In the framework of retrofitted models, a number of interesting results have been obtained[19]:

1. Models can be constructed with a range of SUSY breaking scales
2. One can account for the breaking of the approximate  $R$  symmetry of the low energy theory, generating suitable gaugino masses.
3. One can naturally account for a small  $\mu$  term.
4. The size of the superpotential is parametrically of the correct order of magnitude to account for the smallness of the cosmological constant, if the Planck scale controls the size of higher dimension operators.

As the LHC has already excluded significant parts of the supersymmetry parameter space, conventional ideas of naturalness are arguably under some stress. While it is possible that we will find evidence for a natural structure, such as supersymmetry with light stops and an additional singlet field, it is also possible that naive notions of naturalness are simply not correct. At an extreme level, the cosmological constant problem, coupled with a landscape hypothesis, suggests that perhaps one should abandon notions of naturalness entirely. Even within a landscape framework, however, features of parameter distributions and possible competing anthropic pressures might yield a more moderate degree of tuning, perhaps accounting for scales of supersymmetry breaking of order 10's to 1000's of TeV. This would be consistent with the mass of the (candidate) Higgs and current supersymmetry exclusions. High scales might ameliorate or eliminate the problem of flavor changing neutral currents at low energies and the cosmological moduli problem<sup>2</sup>. It is then interesting to

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<sup>2</sup>The virtues of high scales, and their associated tuning, for the moduli problem were noted in [20, 21],

reconsider models of “gravity mediation”. It is hard to see how such models could account for hierarchies unless the breaking is dynamical. The possibility that supersymmetry is dynamically broken in supergravity models in a stable vacuum has been considered for some time[25, 26, 27], and more recently in [28, 29, 30]. We will review features of such models, and their virtues and limitations, in section 3.2.

Our focus in this paper, however, will be on metastable dynamical supersymmetry breaking in the context of supergravity, and especially on retrofitted models. In retrofitted models, symmetries and their dynamical breaking play a central role, and it is possible that considering such theories will lead to new insights into longstanding puzzles. Questions such as the notion of an approximate, continuous  $R$  symmetry, the generation of gaugino masses, and the  $\mu$  term may be seen in a different light than in the past. It is also possible that these sorts of considerations might point in new directions for a more natural phenomenology. Exploring these possibilities is the goal of the present paper.

Discrete  $R$  symmetries, spontaneously broken, are a feature of retrofitted models. Such symmetries can account for the approximate, continuous  $R$  symmetries required by the Nelson-Seiberg theorem; they can also help account for the smallness of the cosmological constant. So we will assume such symmetries throughout this paper. We will ask whether these lead to restrictions on the soft breaking parameters at low energies. These symmetries can forbid, not only a large  $\mu$  term, but also the Giudice-Masiero coupling[31]; at the same time, other sources of  $\mu$  can arise naturally within this structure. For the  $A$  and  $B$  terms we will see that there are significant constraints in certain circumstances and not in others.

Perhaps most interesting, these symmetries control whether gaugino masses are generated

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and more recently have been stressed in the connections noted here by [22],[23] and [24]



at tree level or in loops. This is particularly relevant to assessing the genericity of “split supersymmetry” [32]. It is often argued that split supersymmetry is natural, as symmetries can protect gaugino masses, but not scalar masses. A symmetry under which the gauginos transform is necessarily an  $R$  symmetry, however, and the scale of  $R$  symmetry breaking is tied to the cosmological constant. In retrofitted models, this correlation, at the order of magnitude level, can be natural. In these cases, the Goldstino supermultiplet (the chiral multiplet whose fermionic component is the longitudinal mode of the gravitino<sup>3</sup>) is neutral under the  $R$  symmetry, and (except in special circumstances which we will describe) is allowed by all symmetries to couple directly to the gauge fields. In such cases, the gaugino masses are typically of order the gravitino mass (as are those of the scalars). These couplings may vanish by accident. Alternatively, as we will describe, there are theories in which the Goldstino supermultiplet transforms under a non- $R$  discrete symmetry (and is neutral under the  $R$  symmetry). These theories require additional features (fields and interactions) to account for moduli stabilization. Finally, the Goldstino may transform under the  $R$  symmetry so that the gaugino masses are suppressed, at the price, again, of additional interactions and now also *very* small parameters. So we will see that “split supersymmetry”, while plausible, does not appear particularly generic. Even if not, such a phenomenon still may arise by accident, or as a consequence of anthropic tunings.

In section 3.2, we review features of supergravity models which exhibit stable dynamical supersymmetry breaking, and contrast with MDSB, both for the ISS models and retrofitted theories. In section 3.3, we turn to MDSB, explaining in more detail why one might expect a role for discrete  $R$  symmetries. We discuss why the breaking should

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<sup>3</sup>As we will discuss, this notion is not always sharp; we will clarify when needed.

be small, and why there is, as a result, a low energy effective field theory, which to a first approximation is globally supersymmetric and  $R$  symmetric. In section 3.4, we discuss models in which the “goldstino multiplet” is neutral under the  $R$  symmetry. We will consider mechanisms for stabilization of the moduli. We will see that the  $B_\mu$  and  $A$  terms are not predicted in such models (though in some cases  $A$  parameters are proportional to Yukawa couplings). In these models, gaugino and scalar masses are typically of the same order. In section 3.5, we consider the case that the goldstino multiplet is charged (as we describe, any  $R$ -neutral moduli might be fixed at some high scale by additional dynamics). We will explain the need for additional interactions and small parameters. In such a model, the gaugino masses are automatically suppressed by a loop factor. Predictions for  $A$  and  $B$  terms can emerge in such a framework. Possible origins for fine tuning are discussed in the concluding section (5.7).

## 3.2 Stable vs. Metastable Dynamical Breaking and Supergravity

Models of stable, dynamical supersymmetry breaking have been known for some time[25]. They have certain characteristic features:

1. At the level of the lowest dimension operators, they exhibit continuous global symmetries, which are spontaneously broken. Typically, fields with non-zero  $F$  components carry charges under the corresponding symmetries.
2. There are no approximate flat directions (pseudo moduli).

3. In renormalizable theories, there is one characteristic scale.

When coupled to supergravity, these theories have the features that:

1. Because of point 1 above, gauginos cannot gain mass from dimension five couplings to fields with non-vanishing  $F$  components. The leading masses are “anomaly mediated” [26].
2. Because of point 2, there is no moduli problem in these theories.
3. One requires a large constant in the superpotential,  $W_0$ . One can imagine that this is added by hand. A more principled position is that it arises in a landscape, where there is some continuous distribution of such constants, and anthropically selected. Alternative (again in a landscape) it might be generated by some additional dynamics (and again anthropically selected).
4. One needs additional features to understand  $\mu$ . The Giudice-Masiero mechanism is not operative in these theories.  $\mu$  might be added by hand (again, perhaps, anthropically selected) or be generated by some additional dynamics<sup>4</sup>

Reference [30] revisits these questions, *assuming* that in fact there is a tuned hierarchy of scales, and studies the phenomenology of these models.

The retrofitted models discussed in the following sections provide a different viewpoint on many of these issues. The retrofitted theories typically *do* involve supersymmetry breaking by pseudo moduli. Symmetries (generally discrete  $R$ ) are inherent in these models.

They have several promising features:

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<sup>4</sup>In [28],  $\mu$  is generated by a term in the Kahler potential,  $c H_u H_d$ . In the presence of a non-zero  $W_0$ , and for  $c \sim 1$ , this is equivalent to a bare  $\mu$  term, as can be seen by performing a Kahler transformation. If the smallness of  $\mu$  is accounted for by a spontaneously broken  $R$  symmetry, say, due to gaugino condensation in another group (also accounting for  $W_0$ ), this is equivalent to a  $W_\alpha^2 H_u H_d$  coupling.

1. In a broad class of models, the dynamics automatically generate a constant in the superpotential of the required order of magnitude (as we will review in the next section) to yield a small cosmological constant.
2. They contain symmetries which suppress the  $\mu$  term.
3. The same dynamics which generates supersymmetry breaking and the constant in the superpotential can generate a  $\mu$  term; alternatively, the Giudice-Masiero mechanism may be operative.
4. The models suffer from a moduli problem, but this may be a positive feature: the required large mass for the modulus may have an anthropic origin (accounting for tuning – and a lower bound on the SUSY-breaking scale).

In the context of gravity mediation, the retrofitted models are distinguished from the ISS models. Indeed, the original ISS models are closer to the models with stable supersymmetry breaking in structure. They don't possess moduli; they require an additional constant in the superpotential, or some new dynamics, to account for the smallness of the cosmological constant; absent the constant in the superpotential, they typically have approximate  $R$  symmetries which prevent a gaugino mass, so the anomaly mediated contributions dominate. Typically additional dynamics is necessary to account for the  $\mu$  term.

### 3.3 The Role of Discrete $R$ Symmetries

There are several reasons why we might expect discrete  $R$  symmetries to play a role in any understanding of supersymmetry breaking. The first has to do with the cosmological

constant. In order to understand the smallness of the cosmological constant, it is necessary that

$$W_0 = \langle W \rangle \tag{3.1}$$

be small. The only *natural* way to understand this is to suppose that there is an underlying, *discrete*,  $R$  symmetry. Of course, we do not have a natural understanding of the dark energy overall, and one might simply view the smallness of  $W$  as arising as a part of some anthropic selection of small cosmological constant. This is the assumption of most landscape analyses[14]. But in a landscape, if both  $W_0$  and the scale of supersymmetry breaking are dynamically generated, the overall level of fine tuning might be significantly reduced[16]. In retrofitted models, the small breaking of the  $R$  symmetry induces SUSY breaking of the order required to give small c.c. A second reason to consider discrete  $R$  symmetries is the requirement of an (approximate) continuous  $R$  symmetry to account for the spontaneous breaking of supersymmetry (in a metastable vacuum). Such a symmetry might arise as a result of accidents involving the structure of the gauge-invariant renormalizable couplings in a theory, but it could also arise from the restrictions on the structure of low dimension operators imposed by an  $R$  symmetry; for example, the discrete symmetry might be a subgroup of the approximate continuous symmetry.

Such symmetries might be relevant, as well, to understanding proton stability and other issues in supersymmetric theories. Issues with understanding such symmetries in a landscape context have been discussed in [33, 34], with counting of states in explicit models performed in several explicit constructions[35, 36]. There it was noted that in flux landscapes, discrete symmetries are rare, but a picture in which cosmology might favor

such symmetries was put forward. We will assume the presence of such symmetries in what follows.

Given the assumption that there is an underlying discrete  $R$  symmetry, the first question we might ask is: should we impose anomaly constraints? Model builders often demand satisfaction of some putative set of discrete anomaly constraints. It is well known, from studies of string theory[37] that, until one commits oneself to the structure of the microscopic theory (e.g. a conventional grand unified theory) one can demand, at most, the cancellation of anomalies associated with non-abelian gauge groups. But even for these, if there are light scalars, anomalies can be canceled by a Green-Schwarz mechanism. In heterotic string examples, when this occurs, one often finds that all anomalies can be cancelled by couplings to a single field[38]. A priori, in the presence of multiple moduli, anomalies not only need not vanish, but need not be equal[39, 40]. But there is a simpler, more macroscopic reason, that one should not impose anomaly constraints. Any such  $R$  symmetry is necessarily broken at a high scale, given the small value of the observed cosmological constant. It is possible that fields transforming under the discrete symmetry gain mass at this scale. If the breaking of the symmetry is dynamical, and if, in first approximation, supersymmetry is unbroken, possible order parameters for this breaking include, as discussed in [19], gaugino condensates, of dimension three, and scalar fields, of dimension one, associated with some new gauge group. So such masses can be far larger than  $m_{3/2}$ . As a result of these considerations, we do not view anomalies as constraining[40].

### 3.4 Retrofitting the Polonyi Model: Neutral Fields

In this section we will make the simplifying assumption that the only scale in the microscopic theory is  $M_p$ . We will also assume that the theory consists of a gauge theory which breaks a  $Z_N$   $R$  symmetry without breaking supersymmetry; an  $SU(N)$  gauge theory without matter fields provides a simple example, but others have been explored in [19, 41].

A simple model for supersymmetry breaking in supergravity then consists of a single field,  $X$ , neutral under the  $R$  symmetry, and coupled to a supersymmetric gauge theory, with coupling

$$W = -\frac{1}{4}f\left(\frac{X}{M_p}\right)W_\alpha^2, \quad (3.2)$$

where  $W_\alpha$  are the gauge fields of the  $R$ -breaking sector. By a holomorphic redefinition of the fields, we can take

$$W = -\frac{1}{4}\left(\frac{1}{g^2} + a\frac{X}{M_p}\right)W_\alpha^2, \quad (3.3)$$

Because  $X$  is neutral under *any* symmetry of the theory, the definition of the origin is arbitrary. Moreover,  $X$  is a pseudo modulus, in that no couplings of the form  $X^n$  are permitted by the symmetries. String theory models would suggest that  $X$  might transform under an approximate shift (Peccei-Quinn) symmetry,  $X \rightarrow X + i\alpha$ . Non-perturbative effects would generate a small, non-perturbative (explicit) breaking of the symmetry.

The interaction of eqn. 3.3 leads to a superpotential for  $X$ :

$$W(X) = \Lambda^3 e^{-\frac{X}{bM_p}} \quad (3.4)$$

for some constant  $b$ .  $\Lambda$  is the scale of the hidden sector dynamics, at the (arbitrarily chosen)

0 of  $X$ . An alternative is to define  $X$  so that

$$\langle X \rangle = -\frac{1}{g^2} + \dots \quad (3.5)$$

( $g^2 = g^2(M_p)$ ). This allows us to write

$$W(X) = M_p^3 e^{-X/b} \quad (3.6)$$

$W(X)$  yields a potential for  $X$ , which vanishes for large  $X$  as  $e^{-2\text{Re } X/b}$ . By assumption, the potential has a (metastable) minimum.  $X$  may be stabilized by features of the Kahler potential (“Kahler stabilization”), described in section 3.4.1, or as a result of couplings to fields which became massless at points on the moduli space. The latter is necessary in models of low energy (gauge-mediated) supersymmetry breaking[19].

As noted in [19], with these choices of scalings, vanishing of the cosmological constant can arise if  $a$  is an  $\mathcal{O}(1)$  number (albeit adjusted to many decimal places). We will see that this is not the case for other possible mechanisms for supersymmetry breaking.

The underlying theory may contain multiple fields like  $X$ , neutral under the  $R$  symmetry. It might contain charged moduli as well. Ignoring the latter, for the moment, we can label the neutral fields by  $X_i$ ,  $i = 0, \dots, N$ , and define  $X_0$  so that

$$\langle F_i \rangle = 0 \quad i > 0. \quad (3.7)$$

From the perspective of symmetries,  $X_0$  is not distinguished in any particular way.

### 3.4.1 Soft Breakings: Moduli Stabilization

When considering soft breakings, the first question one needs to address is the stabilization of the modulus (moduli)  $X$ . Neutral moduli might be stabilized by features of



the Kahler potential, “Kahler stabilization”[42]. It will be useful to be explicit about what this means. For a single field, we can simply define  $X = 0$  as the stationary point of the potential, as in eqn. 3.3. Then we can write a Taylor series expansion of  $K$ :

$$K = k_0 + k_1 X + \text{c.c.} + k_2 X^\dagger X + \tilde{k}_2 X^2 + \text{c.c.} + k_3 X X^{\dagger\,2} + \tilde{k}_3 X^3 + \text{c.c.} \quad (3.8)$$

We impose the conditions

$$V'(0) = V(0) = 0. \quad (3.9)$$

These are two algebraic conditions on the  $k_i$ ’s; they have a multi-parameter set of solutions. There is no small parameter in these equations, and the  $k_i$ ’s (in Planck units) generically are comparable.

For the question of gaugino masses, we will be interested in

$$\langle F_X \rangle = \frac{\partial W}{\partial X} + \frac{\partial K}{\partial X} W = \Lambda^3 \left( -\frac{1}{b} + k_1 \right). \quad (3.10)$$

### 3.4.2 Soft Breakings: Gaugino Masses

It is often remarked that gaugino masses may be small compared to squark and slepton masses, as a result of the chiral symmetries which can protect fermion masses. Any symmetry under which gauginos transform would necessarily be an  $R$  symmetry, and this symmetry, in turn, is necessarily broken, given the smallness of the cosmological constant, which requires a non-zero expectation value of the superpotential,  $W_0 = \langle W \rangle$ . The scale of this breaking is tied to the scale of supersymmetry breaking,  $W_0 = m_{3/2} M_p^2$ .  $W_0$ , at the very

least, gives rise to the anomaly mediated contribution to the masses[43, 44]. The corresponding loop suppression, in such a case, gives rise to the idea of “split supersymmetry” [32].<sup>5</sup>

But given that the  $R$  symmetry must be broken by some dynamics, there are potentially other contributions, which may not be loop suppressed. Gaugino masses can arise from a  $XW_\alpha^2$  coupling, where  $W_\alpha$  now refers to the standard model fields. Such couplings to the hidden sector are typical of retrofitted models (eqn. 3.3); it would be surprising if similar couplings to the standard model gauge fields were absent. We will glean some insight into this question when we consider unification, below.

The coupling  $XW_\alpha^2$  leads to a gaugino mass

$$m_{1/2} = F_X k_2^{-1}. \quad (3.11)$$

We have seen that, once  $X$  is stabilized, its  $F$ -component is of order  $m_{3/2}M_p$ , and  $k_2 = \mathcal{O}(1)$ . In general. If  $X$  is neutral, this coupling can not be forbidden by symmetries, the gaugino masses at the high scale are of order  $m_{3/2}$ . As we have remarked, it is possible that couplings of  $X$  to the standard model gauge groups vanish; in this case, the “anomaly-mediated” contribution dominates for the standard model gauginos. Still, split supersymmetry, in the framework of a goldstino multiplet neutral under the  $R$  symmetry, would not appear generic. It might, of course, simply arise by accident, or it might be selected by requirements for suitable dark matter or other (anthropic?) constraints. We will see in section 3.5 that under special circumstances, the Goldstino multiplet may be charged under non- $R$  symmetries, allowing a natural suppression of gaugino masses.

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<sup>5</sup>The authors of [32] contemplated very large hierarchies between scalar and gaugino masses; they have dubbed this one-loop hierarchy “mini-split”.

## Unification

If there is one such field, defining  $X$  as above, unification requires that  $X$  couple in the same way to each of the Standard Model groups; one expects that it couples to the additional strong group, with a coupling which might differ by an order one factor. So in the retrofitted framework with neutral fields, one expects gaugino masses of order scalar masses. If there are multiple neutral moduli, as is familiar in string theory, then unification would seem to be a significant constraint. One possibility, simple to describe but not necessarily to realize, is the following: two moduli,  $X_0$  and  $X_1$ , where  $X_0$  couples only to the hidden sector gauge group, while  $X_1$  couples only to standard model fields.  $X_0$  is the Goldstino multiplet;  $X_1$  is another neutral multiplet. This would be consistent with unification, and with an “anomaly mediated” origin for the gaugino masses for the MSSM. Any of these scenarios has implications for the moduli problem of supersymmetric cosmology, as we will discuss in the conclusions.

It is worth noting that in the heterotic string theory compactified on an  $R$  symmetric space, familiar,  $R$  neutral moduli are the model-independent dilaton and the radion. The radion typically couples in loops to the gauge fields, in a non-universal fashion.

### 3.4.3 Soft Breakings: $\mu$ , $B_\mu$ and $A$ Terms

As in general supergravity models, there are a variety of sources for masses for the scalar partners of the quarks and leptons, as well as the Higgs scalars. We will denote these fields generically by  $\phi_i$ . Writing the terms in the Kahler potential in the form

$$K(X, X^\dagger, \phi^i, \phi^{i\dagger}) = f(X, X^\dagger) + g(X, X^\dagger)_{ij} \phi^i \phi^{j\dagger} + \dots \quad (3.12)$$

allows a completely general matrix for the  $\phi_i^* \phi_j$  soft breakings.

For the  $\mu$ ,  $B_\mu$ , and  $A$  terms, one might ask whether the  $R$  symmetries yield interesting restrictions. If the product  $H_u H_d$  is neutral under the symmetry, then a  $\mu$  term is forbidden above the scale of  $R$  breaking. A  $\mu$  term *can* arise from the familiar Giudice-Masiero mechanism[31]:

$$\mathcal{L}_\mu = \frac{1}{M} \int d^4\theta f(X, X^\dagger) H_u H_d \quad (3.13)$$

of order  $m_{3/2}$ . By rescaling of the Higgs fields, we can take the coefficient of  $H_u^* H_u$  and  $H_D^* H_D$  in the Kahler potential to be unity. In this case, certain universal contributions to  $B_\mu$  terms arise from the terms in the supergravity action:

$$V_{sugra} = e^K \left( \frac{\partial W_{eff}}{\partial H_u} \frac{\partial K}{\partial H_u^*} W_0^* - 3|W|^2 \right) \quad (3.14)$$

However, the term

$$\frac{\partial W^*}{\partial X^*} g^{X, X^*} \frac{\partial K}{\partial X} W + \text{c.c.} \quad (3.15)$$

depends on  $\frac{\partial K}{\partial X}$ , which is not constrained by the symmetries. So there is no prediction for the relation between  $B_\mu$  and  $\mu$ .

Similar issues arise for the  $A$  terms. Non-calculable contributions arise from

$$\frac{\partial W^*}{\partial X^*} \frac{\partial K}{\partial X} W g^{XX^*}. \quad (3.16)$$

While these are proportional to the Yukawa couplings in the superpotential, non-proportional terms would arise from terms in the Kahler potential of the form:

$$\delta K = \gamma_{ij} X^\dagger \phi_i^* \phi_j + \text{c.c.} \quad (3.17)$$

These and similar terms might be forbidden by symmetries, yielding  $A$  terms proportional to Yukawa couplings.

If  $H_u H_d$  carries a non-trivial  $R$  charge, not equal to that of the superpotential, the  $\mu$  term must be generated in a different fashion. If there are singlet fields,  $S$ , of suitable charge, with  $S \neq 0$ , couplings

$$\kappa \frac{S^n}{M^{n-1}} H_u H_d \quad (3.18)$$

Give rise to a  $\mu$  term[19]. Again, however,  $B_\mu$  is not predicted without further knowledge of the microscopic theory; there is a contribution proportional to  $\frac{\partial K}{\partial X} W$ . Similar remarks apply to the  $A$  parameter in this case.

### 3.5 Generalizations of the Polonyi Model: Charged fields

If the Goldstino field were charged under a symmetry, one could suppress the gaugino mass. Given that, at least in known string models, there are usually moduli of  $R$  charge zero, it is first necessary to ask how theses might be stabilized. Given our basic assumption of an underlying  $R$  symmetry, the “KKLT” mechanism[45] is not available to us, but Kahler stabilization again can provide a solution. For example, if

$$W \approx e^{-N/b} \quad (3.19)$$

then supersymmetry is unbroken for  $N_0$  such that

$$\frac{\partial K}{\partial N_0} = -1/b. \quad (3.20)$$

For self consistency, this must occur for large  $N$ .

Suppose that the Goldstino field,  $X$ , carries a charge under the discrete symmetries. Then  $X$  does not couple directly to the gauge fields. The leading contribution to gaugino masses arises from the so-called anomaly mediated effects at one loop. This is, indeed, an implementation of the slogan of [32], that gaugino masses can be suppressed relative to scalar masses as a result of symmetries. Interestingly, it is not the  $R$  symmetry or any symmetry carried by gauginos which is responsible – the suppression of the coupling arises precisely because the gauginos are neutral under the symmetry.

In order to achieve a model of this type, we must suppose that the  $R$  symmetry is broken by a model such as that of [19], where there are order parameters of dimension one,  $\Phi$ , with less trivial discrete charges. For the models of [19],  $\Phi^3$  carries charge 2 under the  $R$  symmetry.  $\Phi = \mathcal{O}(\Lambda)$ , the scale of the underlying gauge dynamics, and  $W_0 \sim \Lambda^3$ . Then, for example, there may be a superpotential:

$$W = \kappa X \frac{\Phi^{3+n}}{M_p^{n+1}} \quad (3.21)$$

Consider, first,  $n \neq 0$ .  $X$  now carries a non-trivial  $R$  charge. There is a well-defined notion of origin, and there is a meaningful sense in which  $X$  may be small. We will assume for the moment that  $X$  is stabilized near the origin; we will consider the problem of stabilization shortly. If this is the entire content of the theory, the cosmological constant is problematic. The scale of supersymmetry breaking is

$$F_X = \kappa \frac{\Lambda^3}{M_p} \left( \frac{\Lambda}{M_p} \right)^n. \quad (3.22)$$

But  $\langle W \rangle$  is also of order  $\Lambda^3$ , so

$$\kappa \sim \left( \frac{M_p}{\Lambda} \right)^n. \quad (3.23)$$

This makes sense for  $n < 0$ , but requires that  $\kappa$  is *extremely* small. For  $n > 0$ , additional dynamics are required to break the  $R$  symmetry in a way that can yield a small cosmological constant.

The case  $n = 0$  is similar to the  $XW_\alpha^2$  case of the previous section.  $X$  is neutral under  $R$  symmetries. One now has a natural understanding of the order of magnitude of  $W_0$  (i.e.  $\kappa \sim 1$ ). But one would like to explain the absence of the  $XW_\alpha^2$  coupling. This requires that  $S$  couple to a combination of fields carrying some charge, preferably a discrete (non- $R$ ) charge. The models of [19] have the feature that they may exhibit such symmetries. In particular, these models have multiple singlet fields,  $S_i$ . These appear coupled to “quark” and “antiquark”, fields,  $Q_f$  and  $\bar{Q}_f$ , transforming as  $N$  and  $\bar{N}$  of  $SU(N)$ . A simple model with an additional symmetry is

$$W = y_1 S_{-2} \bar{Q}_1 Q_1 + S_1 (y_2 \bar{Q}_0 Q_{-1} + y_3 \bar{Q}_{-1} Q_0) + \lambda S_{-2} S_1 S_1. \quad (3.24)$$

In the limit that  $\lambda$  is smaller than the other couplings,  $y_i$ , one can integrate out the  $Q$ ’s, obtaining an effective superpotential:

$$W_{eff} = (y_1 y_2 y_3 S_{-2} S_1 S_1)^{1/N} \Lambda^{3-3/N} + \lambda S_{-2} S_1 S_1. \quad (3.25)$$

This model has, at the level of dimension four couplings, a  $U(1)$  symmetry under which  $S_i$ ’s have charges corresponding to their subscripts. The problem is rather general; it is difficult, with only dimension four couplings, to obtain discrete symmetries apart from  $Z_3$ . In the event that there are approximate  $U(1)$  symmetries, one obtains a one (complex) dimensional set of vacua and corresponding pseudo moduli. These directions may be lifted by higher dimension operators or supersymmetry-breaking effects but they certainly yield

new complications for model building. So, while it is possible to construct models of this type, they don't appear especially generic.

So far we have not discussed the stabilization of the modulus  $X$ . We require, not only stability and small c.c., but also that  $X \ll M_p$ , in order that the symmetry be effective. This last aspect is problematic, requiring that the models possess additional features. The difficulty is that the potential for  $X$  is inherently symmetry breaking; there is no small scale unless it arises from some *other* dynamics. If we suppose that  $X$  is stabilized by features of the Kahler potential, then, unless there are large dimensionless ratios,  $X \sim M_p$ . This can be avoided if  $X$  couples to other light fields, providing, essentially, a retrofitted O'Raifeartaigh (as opposed to Polonyi) model. This requires new fields and additional mass terms. It is possible to build models along these lines and we will assume such a structure in what follows.

With a superpotential of the form of eqn. 3.21, and with  $F_x \sim \frac{W}{M_p}$ , scalar masses are of order  $m_{3/2}$ .  $\phi_i^* \phi_j$  type terms of a completely general form arise due to the terms in the Kahler potential

$$\Gamma_{ij} X^* X \phi^{i*} \phi^j. \quad (3.26)$$

Parametrically, these masses are of order  $m_{3/2}^2$ .

The Guidice-Masiero Kahler potential gives rise to a  $\mu$  term, again, if the charges of  $H_u$  and  $H_d$  are suitable. Now, because  $\langle X \rangle$  is small (as is  $\frac{\partial K}{\partial X}$ ), there are only a few sources of  $B_\mu$  and  $A$  terms in the supergravity lagrangian.  $B_\mu$  is then determined in terms of  $\mu$  and  $m_{3/2}$ :

$$B_\mu = -m_{3/2}\mu. \quad (3.27)$$



Similarly, because of the symmetries, the  $A$  terms vanish at tree level and leading order in  $\Lambda/M_p$ .

Alternatively, the Giudice-Masiero term in the Kahler potential might be forbidden by symmetries, and the  $\mu$  term arise as a result of retrofitting or some other mechanism[19].

In that case, one again has a prediction for  $B_\mu$  and  $A$ . Again,

$$B_\mu = -m_{3/2}\mu \quad A = 0. \quad (3.28)$$

To summarize, the possibility of charged moduli is interesting from the perspective of relatively light gauginos. It comes at a price, however.

1. Additional dynamics are required to fix any neutral moduli without breaking supersymmetry. (This is similar to the DSB and ISS theories).
2. *Extremely* small couplings are required to fix the charged moduli, while at the same time obtaining small cosmological constant, if  $n < 0$ .
3. New dynamics (possibly related to those which fix neutral moduli) are required to obtain a small cosmological constant if  $n > 0$ . (This is similar to the stable DSB and ISS theories).
4. The case  $n = 0$  requires additional symmetries beyond the  $R$  symmetries, and, except for  $Z_3$ 's, these introduce additional pseudo moduli and associated complications.
5. In all of these cases, additional degrees of freedom (similar to those of O’Raifeartaigh models) are required to stabilize  $X$  near the symmetric point.

If these features are present, this structure has predictive features: gaugino masses are dominated by the anomaly mediated contributions, while  $B_\mu$  and  $A$  terms are universal,

$B_\mu = -m_{3/2}\mu$ ;  $A = 0$ . These models have possible implications for the moduli problem. Because the origin is a point of enhanced symmetry, it is natural that the minimum of the  $X$  potential lie at the origin, and that  $X$  sit at the origin both immediately after inflation ends. The latter point may be viewed as a virtue relative to the models of the previous section; alternatively, it is possible that anthropic issues related to light moduli select for the tuning needed in the supergravity models. Finally, it should be noted, again, that in addition to the discrete  $R$  symmetries, these models, to be natural, require a discrete non- $R$  symmetry.

### 3.6 Conclusions: Origins of Tuning

We have seen that supergravity models with metastable dynamical supersymmetry breaking are readily constructed in the framework of retrofitting. We have argued that discrete  $R$  symmetries are likely to be an important feature of these models, and we have focussed particularly on their consequences. In the simplest models, the goldstino supermultiplet,  $X$ , is neutral under the  $R$  symmetry. In these cases, we have seen that split supersymmetry is not generic. More generally, this framework is not particularly predictive; one can make statements even about the  $A$  and  $B_\mu$  parameters only in restrictive circumstances. We have seen that stable supersymmetry breaking and the ISS models are similar in that, while gaugino mediation may dominate, additional elements are required to understand  $\mu$  and the cosmological constant.

We have considered the alternative possibility that the goldstino superfield is charged under the discrete  $R$  symmetry or other symmetries. The structure of the theory

is distinctly more restricted, and, for example, suppression of gaugino masses is automatic. But understanding the smallness of the cosmological constant requires unattractive features: extremely small couplings or additional dynamics, introduced only for this purpose. Further fields and dynamics are necessary to stabilize the moduli. Somewhat more interesting is the possibility, which can be achieved in actual models, that  $X$  is neutral under the  $R$  symmetry, but charged under some other discrete symmetry. In this special set of circumstances, split supersymmetry is automatic, there are predictions for  $A$  and  $B_\mu$ , and the superpotential is automatically of the correct order of magnitude to cancel the c.c. Still, additional structure is required for moduli stabilization and there are generally additional pseudo moduli. Because of the additional structure required, this scenario does not appear generic.

The main issue with all of these theories is one of tuning. Indeed, it has long been argued that a high scale for supersymmetry breaking, of order 30 TeV or so, would:

1. Resolve the cosmological moduli problem
2. Ameliorate the flavor problems of supersymmetry (including CP)

A Higgs with mass of order 125 GeV, it has been widely noted, would also point to such a scale. But why a tuning of a part in  $10^5$ ? And if that large, why not larger. These questions might be related. Recently, there have been suggestions that perhaps a large mass scale for the moduli is an anthropic requirement. The observed light element abundances have little anthropic significance, so if there is an anthropic selection, it must arise for other reasons. Possibilities include dark matter and formation of structure. To address this, one needs a framework capable of producing the observed baryon to photon ratio, dark matter density,

baryon number density and perturbation spectrum, somewhere within its parameter space. Given such a model, one can ask whether something like our observed universe is selected, with a high scale of moduli masses (supersymmetry breaking). This question is under study.

## Chapter 4

# Retrofitting Gauge Mediation

Dynamical, metastable supersymmetry breaking appears to be a generic phenomena in supersymmetric field theories. It’s simplest implementation is within the so-called “retrofitted O’Raifeartaigh Models”. While seemingly flexible, model building in these theories is significantly constrained. In gauge-mediated versions, if the approximate  $R$  symmetry of the theory is spontaneously broken, the messenger scale is fixed; if explicitly broken by retrofitted couplings, a very small dimensionless number is required; if supergravity corrections are responsible for the symmetry breaking, at least two moderately small couplings are required, and there is a large range of possible messenger scales. In gravity mediated versions, achieving  $m_{3/2} \approx M_Z$  is a problem of discrete tuning. With plausible assumptions, one can’t achieve this to better than a factor of 100, perhaps accounting for a *little hierarchy* and the surprisingly large value of the Higgs mass.

## 4.1 Introduction: The Genericity of Metastable DSB

As Nelson and Seiberg pointed out[5], generic, stable spontaneous supersymmetry breaking requires a continuous  $R$  symmetry. If we insist that there should be no exact continuous  $R$  symmetries in nature, then we expect that, at some level, any continuous  $R$  symmetry should be explicitly broken, leading, generically, to restoration of supersymmetry somewhere in the space of fields. Discrete symmetries, on the other hand, are plausible in generally covariant theories, and indeed frequently arise in string constructions.<sup>1</sup> A simple possibility is that the discrete symmetry is a subgroup of the required continuous  $R$  symmetry. This can readily be implemented to generate metastable O’Raifeartaigh

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<sup>1</sup>Whether they are “typical”, and might emerge in a landscape context, is another question[33, 46, 34].

models. For example, in a theory with fields  $X, Y, A$  transforming, under a  $Z_N$  symmetry, as:

$$X \rightarrow \alpha^2 X \quad Y \rightarrow \alpha^2 Y \quad A \rightarrow A \quad (4.1)$$

with  $\alpha = e^{\frac{2\pi i}{N}}$ , and also a  $Z_2$  under which  $A$  and  $Y$  are odd, the superpotential has the structure

$$W = X(A^2 - f) + mAY + \left( \frac{YA^3}{M} + \frac{X^{N+1}}{M^{N-2}} + \dots \right) \quad (4.2)$$

Ignoring the non-renormalizable couplings, the theory possesses a supersymmetry-breaking ground state at the origin of  $X, Y$ . Including these couplings, there is a supersymmetric ground state at large  $X, Y$ . The supersymmetry-breaking state is metastable. It exhibits an approximate, continuous  $R$  symmetry. This would seem a generic phenomenon.

One would like to understand the breaking of supersymmetry dynamically. Models with stable dynamical supersymmetry breaking (DSB) were discovered some time ago[25]; they seem quite special, and pose challenges for model building. Models of metastable DSB (MDSB) were considered by Intriligator, Shih and Seiberg[17] exhibited strongly coupled models which exhibit metastable dynamical supersymmetry breaking. The ISS class of models are a rich and interesting set of theories, but they pose challenges for building models. An even broader class of theories is obtained by studying the O’Raifeartaigh models, and rendering the scales ( $f$  and  $m$ ) in eqn. 4.2, for example) dynamical[13, 47, 19]. In these “retrofitted” models, the discrete  $R$  symmetry is spontaneously broken by gaugino condensation or its generalizations[19]. This symmetry breaking can also readily generate a  $\mu$  term. If one retrofits an O’Raifeartaigh model in which all fields have  $R$  charge 0 or

2, one has a problem (also typical of ISS models) that the approximate  $R$  symmetry is not spontaneously broken. A simple approach, adapted in [19], is to retrofit one of the models of Shih[48], in which not all fields have such charges. But given the seeming freedom of the retrofitted approach, it is interesting to ask whether one can break the continuous  $R$  symmetry explicitly. In particular, if there is a distinct, messenger sector, it would seem possible that retrofitting a breaking of the approximate  $R$  symmetry might not spoil supersymmetry breaking. This might allow construction of classes of models of *General Gauge Mediation*[49]. Alternatively, supergravity corrections might dominate, as has been discussed by Kitano[50]. We'll see in this case one can obtain the structure of minimal gauge mediation (MGM).

There is another interesting feature of the retrofitted models, stressed first in [19]. If one assumes that higher dimension operators are controlled by the Planck scale,  $M_P$ , then the expectation value of the superpotential,  $\langle W \rangle$  is readily of the correct order of magnitude to cancel the cosmological constant. This is remarkable; it means that one neither has to introduce a peculiar,  $R$ -breaking constant in the superpotential, nor introduce additional dynamics (e.g. additional gaugino condensates) to account for the observed dark energy (of course, one must still tune an order one constant to incredible accuracy). This is in contrast to the viewpoint, for example, of KKLT[45], that the constant in the superpotential is to be thought of as a random number, selected as part of the anthropic determination of the cosmological constant.

If we insist on this relation, there are striking restrictions on the allowed theories. We will see, in particular, that the underlying scale of supersymmetry breaking (as measured



by  $m_{3/2}$ ), sometimes takes on discrete values. In such theories, the usual questions of fine tuning become a question of selection of discrete, rather than continuous, parameters.

In this note after reviewing generalized gaugino condensation in section 4.2, we briefly revisit the problem of retrofitting gravity mediation, focussing especially on the discrete choices required (particularly in the sector responsible for discrete  $R$  symmetry breaking) in section 4.3. Here the observation concerning the cosmological constant relates the scale of the new interactions to  $m_{3/2}$ ; with some plausible assumptions about unification, this scale is determined, once one makes a (discrete) choice of the underlying gauge group.  $m_{3/2}$  is then exponentially dependent on the leading beta function of the underlying theory, and one can ask how closely one can (discretely) tune the gravitino mass to  $M_Z$ . We will see that with some plausible assumptions about coupling unification (more precisely, a plausible *model* for coupling unification), one typically misses by factors of order 100, perhaps providing an explanation of a little hierarchy.

We then consider the problem of retrofitting models of gauge mediation in sections 4.4-4.6. We will take the observation above about the cosmological constant as a guiding principle. We will see that this is a significant constraint. All of the models possess an approximate, continuous  $R$  symmetry. We will consider the possibilities that this symmetry is spontaneously broken, or explicitly broken. Given the current experimental constraints, we will accept a significant degree of tuning, and take this scale to be large, of order  $10^6$  GeV. Tuned models of gauge mediation have been considered in [51].

Apart from the fact that one can readily build realistic models, there are several striking features which emerge from these studies.

1. In models with spontaneous breaking of the  $R$  symmetry, the scalings are fixed by discrete choices. Quite generally,

$$\sqrt{F} \approx 10^9 \text{ GeV} \tag{4.3}$$

corresponding to a messenger scale of order  $10^{12}$  GeV (an interesting number, for example, from the perspective of axion physics) and  $m_{3/2} = 1$  GeV.

2. In models in which one retrofits an explicit breaking of the  $R$  symmetry, small couplings are required in order that the graviton mass be small, and that the gauge-mediated contributions dominate.
3. In models in which the breaking of the  $R$  symmetry arises from supergravity corrections (i.e. the low dimension terms in the theory respect the  $R$  symmetry), one can obtain acceptable models without exceptionally small dimensionless parameters. The messenger scale can range over a broad range of scales; in the simplest cases, the superparticle spectrum is that of mgm.
4. As has been noted previously[19], a suitable  $\mu$  term can readily be obtained, though this typically requires the introduction of a small, dimensionless number.
5. As has been discussed elsewhere, if the  $\mu$  term arises as a result of retrofitting,  $B_\mu$  is small, so  $\tan \beta$  is large[47].
6. With the assumption of a large scale,  $\Lambda_{gm}$ , CP constraints are weakened. In some of the models we will describe, however, CP conservation is automatic.

In section 5.7, we present our conclusions.

## 4.2 Brief Review of Discrete R Symmetries and Generalized Gaugino Condensation

Crucial to most discussions of supersymmetry dynamics is gaugino condensation. Gaugino condensation can be defined, in a general way, as dynamical breaking of a discrete  $R$  symmetry, accompanied by dimensional transmutation. As such, it occurs in a wider variety of theories than just pure (supersymmetric) gauge theories. For example, an  $SU(N)$  gauge theory with  $N_f$  flavors, and a singlet,  $S$ , with superpotential

$$W = y_f S \bar{Q}_f Q_f + \frac{\gamma}{3} S^3 \quad (4.4)$$

has a  $Z_{3N-N_f}$   $R$  symmetry. This is broken by  $\langle \lambda\lambda \rangle \sim 32\pi^2 \Lambda^3$ , and by  $\langle S \rangle$ . In the limit  $|\gamma| \ll |y_f|$ ,  $S$  is large, and one can integrate out the quark fields, obtaining an effective superpotential:

$$W = N \left( \prod y_f \right)^{1/N} S^{N_f/N} \Lambda^{3-N_f/N} + \frac{\gamma}{3} S^3. \quad (4.5)$$

This has supersymmetric stationary points with

$$S \sim \Lambda \left[ \left( \prod y_f \right)^{\frac{1}{N}} \frac{N_f}{\gamma} \right]^{\frac{N}{3N-N_f}} \quad (4.6)$$

(this model also has a disconnected, runaway branch; this can be avoided, if desired, by adding additional scalars). The low energy superpotential has a constant term,

$$W_0 = \left\langle -\frac{1}{4g^2} W_\alpha^2 \right\rangle \sim N \Lambda^3 \quad (4.7)$$

With these ingredients we can readily “retrofit” any O’Raifeartaigh model. For example, we can take

$$W = X(A^2 - \mu^2) + mAY \quad (4.8)$$

and replace it by

$$W = X(A^2 - c \frac{W_\alpha^2}{M_p}) + \kappa S A Y. \quad (4.9)$$

This model has a metastable minimum near the origin, as seen from the standard Coleman-Weinberg calculation. It has a runaway to a supersymmetric vacuum at  $\infty$ , separated by a barrier from the (metastable) minimum at the origin. Under the discrete  $R$ ,  $X$  is neutral, while  $A$  transforms like the gauginos,  $S$  has charge  $2/3$ , and  $Y$  charge  $1/3$ . Various higher dimension terms are allowed, which lead to (faraway) supersymmetric vacua.

Clearly any dimensional coupling can be generated in this way, and the possibilities for model building are vast. This type of construction will be the basic ingredient of all of the models of this paper. One striking feature of this model is that, for  $c$  an order 1 constant, the cosmological constant can be very small; upon coupling to supergravity, the terms  $|\frac{\partial W}{\partial X}|^2$  and  $-\frac{3}{M_p^2}|W|^2$  are automatically of the same order of magnitude. We view this remarkable coincidence as a potential clue, and will largely insist that it hold in the models we describe in this paper. This will greatly restrict possibilities for model building.

### 4.3 Retrofitted Gravity Mediation: Discrete Choices

In gravity mediated models, we can make do with less structure than the O’Raifeartaigh models; higher order supergravity and Kahler potential corrections can stabilize  $X$ , without additional fields like  $A$ . With

$$W = -\frac{1}{4}W_\alpha^2(g^{-2} + cX) \quad (4.10)$$

we have a Polonyi-type model. If we simply define  $X = 0$  as the location of the minimum of the potential, we can expand the Kahler potential about this point, and impose the conditions of a stable minimum at the origin with (nearly) vanishing  $V[1]$ . Note, in particular, that  $X$  is neutral under the  $R$  symmetry, so the origin is not a distinguished point.

If we take the gravitino mass to be of order 10 TeV, we expect stop masses of this order, and can really account for the apparent observed Higgs mass. But such a choice leaves several questions.

1. Raising the scale ameliorates, but does not resolve, the problems of flavor of supergravity models. This has led to the suggestion, in [52], that the scale of supersymmetry breaking should be much higher, even 1000's of TeV. Alternatively, one might invoke some model for flavor, e.g. those of [53]. (Other aspects of these questions are under study[54]. For 10 TeV squarks, such models are easily compatible with existing data on flavor-changing processes.
2. 10 TeV represents a significant tuning. Even allowing, say, anthropic selection among approximately supersymmetric states in a landscape, where might such a little hierarchy come from? In this subsection, we will offer a possibility. Others have been suggested in [55, 56].
3. Are there observable consequences of such a picture? The authors of [52] invoke unification and dark matter to argue that some gauginos should be relatively light. In [1], however, the genericity of light gauginos was questioned.

Once one has allowed for the possibility that there may be some degree of tuning,

the question which immediately follows is: how much tuning is reasonable. A part in  $10^3 - 10^4$ ? This would lead to squarks in the  $3 - 10$  TeV range. A part in  $10^6 - 10^7$ ? This would allow squarks in the  $10^3 - 10^4$  TeV range. Here we suggest one possible origin for tuning, which points towards the former.

Suppose, for the moment, that we take the  $R$  breaking sector to be a pure gauge theory, and we require vanishing of the cosmological constant. Then we have, as parameters, the choice of gauge group, the value of the gauge coupling at some fixed large scale, and a small number of order one terms in the Kahler potential. Up to order one numbers, the choice of gauge group and the value of the coupling fix  $m_{3/2}$ . We can ask whether we can achieve, among possible groups,  $m_{3/2} \approx M_Z$ . To make sense of this question, we need to make further assumptions. We will assume that *all* of the gauge couplings unify at  $M_p$ , and employ the standard results for unification within the MSSM. Then, given a choice of gauge group in the  $R$  breaking sector, the scale of that sector, *and the value of the gravitino mass*,  $m_{3/2}$ , are determined. Confining our attention, for simplicity, to  $SU(N)$  theories, we have that

$$\Lambda = M_p e^{-\frac{1}{b_0} \frac{8\pi^2}{g^2(M_p)}} \quad (4.11)$$

and

$$m_{3/2} = \frac{N\Lambda^3}{M_p^2} \quad (4.12)$$

For  $N$  such that  $b_0 = 3N$  gives a gravitino mass in the TeV range, a change in  $N$  by 1 results in a change in the gravitino mass of order  $10^4$ . So, accounting for threshold and other effects, one would expect, typically, to have a gravitino mass of order 100 times  $M_Z$

(or  $.01 M_Z$ ). This might well account for the sort of tuning needed to account for the Higgs mass, and not much more! This is, of course, just one possible model; other models might make significantly different predictions.

## 4.4 Retrofitting Gauge Mediation: Spontaneous (Continuous) R Symmetry Breaking

In broad classes of O’Raifeartaigh models, one finds that the (continuous)  $R$  symmetry is unbroken at the minimum of the potential when one performs the requisite Coleman-Weinberg calculation. In retrofitting such models, and in building gauge-mediated theories, we need to explicitly break the symmetry, or to insure that there is no such symmetry in the messenger sector. Instead, in this section, we consider retrofitting in models in which the  $R$  symmetry is spontaneously broken. The simplest such model has superpotential[48]:

$$W = X(\phi_1\phi_{-1} - f) + m_1\phi_1\phi_1 + m_2\phi_{-3}\phi_1 \quad (4.13)$$

We have not explicitly indicated dimensionless couplings. This model has a metastable minimum at  $X \sim m_1, m_2$ , provided

$$|f| < |m_1 m_2| \quad (4.14)$$

When this bound is not satisfied, the model exhibits runaway behavior. When it is,  $F_X = f$  is the order parameter for supersymmetry breaking.

Given these remarks, and the constraint of the cosmological constant, the only possibilities for retrofitting are

1. Comparable  $m_1, m_2$ :

$$f \rightarrow \frac{W_\alpha^2}{M_p}, \frac{S^3}{M_p}; m_1, m_2 \rightarrow S \quad (4.15)$$

with coefficients of order one.

2. Hierarchy of  $m_1, m_2$ :

$$f \rightarrow \frac{W_\alpha^2}{M_p}, \frac{S^3}{M_p}; m_1 \sim S, m_2 \sim \frac{S^2}{M_p} \quad (4.16)$$

or

$$f \rightarrow \frac{W_\alpha^2}{M_p}, \frac{S^3}{M_p}; m_1 \sim \frac{S^2}{M_p}, m_2 \sim S \quad (4.17)$$

with suitable order one constants, in each case, so that eqn. 4.14 is satisfied.

The latter case, however, is problematic if there are no very small dimensionless numbers. First, unless  $m_1 \gg m_2$ , the  $R$  symmetry is unbroken[48]. Following the analysis of [48], if this condition is satisfied, the vev of  $X$  is:

$$|\langle X \rangle|^2 \approx \frac{m_1^2}{9\lambda^2} \sim \Lambda^2. \quad (4.18)$$

if the couplings in the superpotential are of order one. So the scalar component of  $X$  is of order  $\Lambda$  (up to dimensionless constants), as in the previous case.

#### 4.4.1 Couplings to Messengers

In the first case, if we couple  $X$  to messengers, with coupling

$$X \widetilde{M} M \quad (4.19)$$



we have the usual sorts of gauge-mediated relations, but with scales that are now, essentially, fixed. In particular, the scale that sets the masses of squarks, leptons and gauginos is:

$$\Lambda_{gm} = \frac{F_X}{X} = \frac{\Lambda^2}{M_p} \quad (4.20)$$

(up to dimensionless coupling constants). Requiring

$$\Lambda_{gm} = 10^6 \text{ GeV} \quad (4.21)$$

(consistent with current experimental constraints, but, needless to say, demanding significant tuning) gives

$$\Lambda = 10^{12} \text{ GeV}; \quad m_{3/2} \sim 1 \text{ GeV}. \quad (4.22)$$

The scales here are close to those considered in [51], who have discussed some of the issues associated with possible detection and dark matter. These will be further considered elsewhere, but it should be noted that the lightest of the new supersymmetric particles are in the TeV range, and these do not carry color, so their discovery will be challenging, if these ideas are correct.

#### 4.4.2 The R Axion

Models of this type, where the approximate  $R$  symmetry is spontaneously broken, possess an  $R$  axion. To determine its mass, we must examine sources of  $R$  symmetry breaking. These will arise from higher dimension terms in the superpotential, and also from coupling the low dimension terms to supergravity. These latter are always present, so we content ourselves with estimating these.

As in the estimate of Bagger, Poppitz and Randall[57], the  $R$  breaking arises from terms such as  $-3|W|^2$  in the potential. For the retrofitted versions of Shih’s model, writing

$$X \approx \langle X \rangle e^{ia/\langle X \rangle} \quad (4.23)$$

yields a mass of order

$$m_a^2 \approx m_{3/2}^2 \frac{f}{X} \quad (4.24)$$

or about 1 TeV, in the present case. This is heavy enough so as not to be astrophysically problematic, and, of course, is difficult to see in accelerator experiments.

#### 4.4.3 Discrete Tunings

In the gravity mediated case, we saw that, with a model for unification of couplings, discrete changes of theory lead to large changes in  $m_{3/2}$ . This arose, in part, because we assumed the simplest possibility for gaugino condensation: a gauge theory without matter fields. In the gauge-mediated case, we require a theory with matter, and, while this may represent an increase in complication, smaller steps in the beta function (one instead of three for the pure gauge case) are inherent to this class of models. As a result, the difficulties of tuning do not appear to be as pronounced as in the gravity mediated case we described earlier. A “natural” model of gauge mediation would have

$$\Lambda_{gm} \sim \Lambda_{gm}^{natural} \equiv 4 \times 10^4 \text{GeV}. \quad (4.25)$$

If we take the  $R$ -breaking sector to be an  $SU(N)$  gauge theory with  $N_f$  flavors and no particularly small dimensional parameters and makes the same unification assumptions we made in the gravity-mediated case, it is easy to choose the number of flavors and colors,

so as to obtain  $\Lambda_{gm}$  within a factor of three of  $\Lambda_{gm}^{natural}$ . So if nature is gauge mediated, understanding the little hierarchy will require additional elements. For example, if there is an underlying landscape, and  $N$  and  $N_f$  are not uniformly distributed, one might easily account for a hierarchy of several orders of magnitude.

## 4.5 Retrofitted Gauge Mediation: Explicit R Symmetry Breaking

Given the seemingly unlimited ability to introduce scales through retrofitting, one is led to consider models in which the O’Raifeartaigh sector has an approximate, unbroken continuous  $R$  symmetry, while the would-be  $R$  symmetry of the messenger sector is broken by explicit mass terms or couplings in the superpotential. This would be interesting in itself, but especially because, even with the simplest messenger structure, the spectrum would be that of general gauge mediation (as opposed to MGM). But, as we will see in this section, this possibility is remarkably constrained. It is difficult to construct realistic models, without very small dimensionless parameters, subject to the following rules:

1.  $M_p$  sets the overall energy scale of the theory.
2. The cosmological constant should vanish at the level of the dynamics responsible for supersymmetry breaking.

A simple model illustrates the main issue. We consider a retrofitted O’Raifeartaigh model with a field,  $X$ , neutral under the  $R$  symmetry and with  $F$ -component  $\Lambda^3/M_p$ . For

the coupling to the messengers we take

$$\left(yX \frac{S^m}{M_p^m} + \lambda \frac{S^m}{M_p^{m-1}}\right) \widetilde{M} M \quad (4.26)$$

The problem is that, for any choice of  $m$ ,

$$m_{3/2} \approx \frac{\lambda}{y} \Lambda_{gm} \quad (4.27)$$

If  $\Lambda_{gm} \approx 10^6$  GeV, it is necessary that  $\frac{\lambda}{y}$  be quite small if the gauge-mediated contributions are to dominate.

The difficulty here arises because  $X$  is invariant under the symmetry. One might try to avoid this by considering a different type of O’Raifeartaigh model, in which  $|f| \gg |m|^2$ . For example,

$$W = X \left( \frac{S^{2m}}{M_p^{2m-2}} - A^2 \right) + \frac{S^n}{M_p^{n-1}} AY. \quad (4.28)$$

If  $m < n$ ,  $A$  acquires a vev, and

$$F_Y \approx \frac{S^{m+n}}{M_p^{m+n-2}}. \quad (4.29)$$

Requiring vanishing of the cosmological constant gives

$$m + n = 3. \quad (4.30)$$

So there are a limited set of possibilities; indeed, we need  $n = 2, m = 1$ . But if the fields  $S$  transform with  $\alpha^{2/3}$  under discrete R-symmetry, then  $Y$  is again neutral, and we encounter exactly the difficulty of the previous model.

Given these difficulties, one might try to construct a model in which  $X$  transforms non-trivially under the  $R$  symmetry. In a model like

$$W = y_f \frac{S^k}{M_p^{k-1}} \widetilde{Q}_f Q_f - \frac{\gamma}{p} \frac{S^p}{M_p^{p-3}} \quad (4.31)$$

$S$  transforms as  $\alpha^{2/p}$ . But now if we are to replicate our “cosmological constant coincidence”, we require that  $X$  couple to  $\frac{S^p}{M_p^{p-4}}$ . But then  $X$  is neutral again.

There are other strategies one might try, but it seems difficult, in general, to break the  $R$  symmetry subject to our rules. Needless to say, relaxing these would open up additional possibilities.

## 4.6 Explicit R Breaking By Supergravity

Finally, one might wonder whether simply coupling one of these systems to supergravity might provide an adequate breaking of the continuous  $R$  symmetry<sup>2</sup>.

In the simplest OR model, coupled to messengers:

$$W = Xf + \lambda XA^2 + mAY + c f M_p |\gamma X M \bar{M}|. \quad (4.32)$$

(with  $c$  an  $\mathcal{O}(1)$  constant), the tadpole (linear term in the potential) for  $X$  is of order

$$\Gamma \approx \frac{f^2}{M_p}. \quad m_X^2 = \frac{\lambda^4 f^2}{16\pi^2 m^2}. \quad (4.33)$$

So, if  $f \sim \frac{\Lambda^3}{M_p}$  and  $m \sim \Lambda$ ,

$$X \approx \frac{\Gamma}{m_X^2} \sim \frac{\Lambda^2}{M_p} \left( \frac{\lambda^4}{16\pi^2} \right)^{-1} \quad (4.34)$$

The simplest coupling to messengers again has the MGM form:

$$\gamma X M \bar{M} \quad (4.35)$$

There are now two conditions on  $\gamma$  and  $\lambda$ . First, we require that the messenger masses not be tachyonic:

$$|\gamma X| > |F_X| \quad (4.36)$$

---

<sup>2</sup>Supergravity corrections of this type in gauge mediation have been considered by Kitano[58].

and second that the corrections to the  $X$  potential due to the messengers be small compared to those from the  $X$  interactions with the massive field  $A$ :

$$\frac{\gamma^2}{X^2} \ll \frac{\lambda^4}{m^2}. \quad (4.37)$$

These conditions require that both  $\lambda$  and  $\gamma$  be small, but they do not have to be *extremely* small. For example, they are satisfied with

$$\lambda = 0.08; \quad \gamma = 0.01; \quad \gamma X \approx 10^{12} \text{ GeV}. \quad (4.38)$$

A slightly smaller  $\lambda$  yields  $X$  at the maximum scale for gauge mediation, while allowing a larger  $\gamma$ :

$$\lambda = .05; \quad \gamma = 0.10; \quad \gamma X \approx 10^{15} \text{ GeV}. \quad (4.39)$$

On the other hand, once  $\lambda$  is larger than about 0.18,  $\gamma$  becomes non-perturbatively large.

So overall, one can achieve a realistic model in this manner, with  $\lambda$  and  $\gamma$  which are small but not extremely so. The gauge mediated scale can range over the full range normally considered for gauge mediated models; the simplest models have the spectrum of MGM.

## 4.7 Conclusions

It seems likely that our cherished ideas about naturalness and supersymmetry are not correct. Supersymmetry, if present at low energies, appears somewhat tuned and may be hard, or impossible, to find. The apparent value of the Higgs mass suggests that the supersymmetry breaking scale might be in the 10 – 100 TeV range.

In this paper, we have reexamined the question of dynamical supersymmetry breaking in the framework of retrofitted models. These models appear to have a rather generic character, and allow one to address easily questions ranging from the  $\mu$  term to the cosmological constant. With plausible assumptions, they are highly constrained. We have considered gravity mediated models (extending slightly the work of [1]) and gauge mediated models. In both cases, the requirement of small cosmological constant strongly constrains the underlying theory. In the supergravity case, the question of fine tuning, i.e. of how close  $m_{3/2}$  lies to  $M_Z$ , is a question of discrete choices. With plausible assumptions about the microscopic theory, the apparent degree of tuning is typically a part in thousands or tens of thousands, perhaps explaining the tuning we see. It is still necessary, in this case, that there be some suppression of low energy flavor violation. Models along the lines of [53] which achieve this will be considered elsewhere.

Our principle focus, however, was on gauge mediated models. We constrained our constructions, again, by requiring the possibility of small cosmological constant in the effective theory, and a fixed supersymmetry breaking scale (corresponding to stops at 10 TeV, or  $\Lambda_{gm} = 10^6$  GeV). We explored the question of whether one might break the approximate, continuous  $R$  symmetry explicitly, taking advantage of the freedom apparently implied by the retrofitted constructions. While we cannot claim that our survey of possible constructions are complete, in broad classes of theories:

1. If the  $R$  symmetry is spontaneously broken, and absent very small dimensionless couplings, the underlying scale of supersymmetry breaking is fixed, with a gravitino mass of order 1 GeV.

2. If the  $R$  symmetry is explicitly broken through retrofitted couplings in the superpotential, a very small dimensionless number, of order  $10^{-6}$ , is required in order that the gauge-mediated contributions dominate.
3. If the  $R$  symmetry is explicitly broken by supergravity effects, two small, but not exceptionally small couplings, are required. The has scale of the messengers ranges, in simple cases, from  $10^7$  to  $10^{15}$  GeV.

We draw from these observations the conclusions:

1. If supersymmetry breaking is gravity mediated, the relatively high scale may result from the limited effectiveness of required discrete tuning. Flavor symmetries, associated with quark and lepton masses, readily can provide adequate alignment of soft breakings to suppress low energy flavor changing processes[25].
2. If supersymmetry breaking is gauge mediated, the approximate  $R$  symmetry may be spontaneously broken, in which case the underlying scale of supersymmetry breaking corresponds to a gravitino mass of order 1 GeV, and the mass of the corresponding  $R$  axion is similar. Simple models of Minimal Gauge Mediation can be realized in this framework.
3. The breaking may be explicit. In the most compelling models, the breaking of the  $R$  symmetry arises from supergravity effects. The messenger scale may be small or large, and again MGM can be realized.

There remains the most important question: is there anything one might hope to see[51]. In a subsequent publication, we will focus on this issue, considering questions such



as dark matter and its implications for possible light states, electric dipole moments, and rare processes.

## Chapter 5

# Small Field Inflation and Spectral

## Index

It is sometimes stated that  $n_s = 0.98$  in hybrid inflation; sometimes that it predicts  $n_s > 1$ . A number of authors have considered aspects of Planck scale corrections and argued that they affect these predictions. Here we consider these systematically, describing the situations which can yield  $n_s = 0.96$ , and the extent to which this result requires additional tuning.

## 5.1 Introduction

In [56], it was argued that, with some very mild assumptions about genericity, we can characterize small field inflation quite simply. First, it was argued that the effective theory should exhibit an approximate (global) supersymmetry in order that there be fields light on the scale of the Hubble constant during inflation,  $H_I$ . Then, assuming  $H_I \gg m_{3/2}$ :

1. The inflaton is a pseudomodulus, labeling a set of approximate ground states with spontaneously broken supersymmetry.
2. The effective theory should obey a discrete  $R$  symmetry in order that the cosmological constant (c.c.) be approximately zero at the end of inflation.
3. At the end of inflation, the inflaton must couple through relevant or marginal operators to fields which are light with respect to the scale of the energy density during inflation, in order that the cosmological constant be small at the end of inflation. In particular, it was stressed that inflation typically ends, in the hybrid case, *before the inflaton reaches the waterfall region*.

So-called models of hybrid inflation[59, 60, 61, 62, 63] have in common the last feature above; in [56] it was argued that this full set of conditions should be taken as the definition of hybrid inflation.

Within such models, these authors noted general features:

1. The (approximate) goldstino may or may not lie in a multiplet with the inflaton.
2. The effective theory exhibits an approximate, continuous  $R$  symmetry.
3. Terms allowed by the discrete symmetry break the accidental continuous global symmetry and spoil inflation, unless the inflationary scale (the square of the Goldstino decay constant) is sufficiently small.
4. There are further requirements on the Kähler potential in order to obtain slow roll inflation with adequate  $e$ -foldings. This sets an *irreducible* minimum amount of fine tuning necessary to achieve acceptable inflation. This tuning grows in severity with the number of Hubble mass fields.
5. In order that inflation ends with small c.c., the inflaton must couple, as noted above, to other light degrees of freedom, or must have appreciable self-couplings in the final ground state. The coupling to this extra field, or the self couplings, are fixed by the density perturbations  $\mathcal{P}_{\mathcal{R}}$  and the inflationary scale. In the case of extra fields, the resulting structure is necessarily what is called “hybrid inflation”[59, 60, 61, 62, 63]. The spectral index, quite generally, is less than one.

In [56], it was noted that for a broad range of parameters,  $n_s = 0.98$  was typical; this is widely considered a general result of hybrid models. Recently, considering the Planck

CMB temperature data supplemented by the WMAP large-scale polarization data, the Planck collaboration has reported a value [?]:

$$n_s = 0.9603 \pm 0.0073. \quad (5.1)$$

And indeed, the authors of the Planck papers argued that their data excludes hybrid inflation. Within the definition outlined above, it is interesting to look more carefully at the range of allowed values of  $n_s$ .

In this paper, we systematically consider various Planck scale corrections to the simplest version of hybrid inflation. We explain why (parametrically) the most important are the quartic corrections to the Kähler potential, and certain power law corrections to the superpotential. The former must be suppressed by an amount of order  $1/\mathcal{N}$ , where  $\mathcal{N}$  is the number of  $e$ -foldings. The latter lead to an approximately zero c.c., supersymmetric minimum for large fields; in turn this means that the potential has a local maximum (saddle). This gives rise to a variant of “hilltop inflation” [65]; we will see that the initial conditions need not be substantially tuned in order that one obtain adequate  $e$ -foldings and  $n_s \approx 0.96$ . If the superpotential has coefficient scaled by a suitable power of  $M_P$  and a dimensionless coefficient of order one, one obtains a *prediction* of the scale of inflation. The scale depends on the index  $N$  of a  $Z_N$   $R$  symmetry, and ranges from about  $10^{11}$  GeV to  $10^{15}$  GeV.

In the next section, we review the simplest hybrid model, and recall the prediction  $n_s = 0.98$ . In section 5.3, we classify the various Planck scale corrections to the simplest hybrid model. In section 5.4, we consider the implications of the leading superpotential corrections for inflation, explaining why one obtains the structure of hilltop inflation. In section 5.5, we present numerical results for these models. In section 5.6, we suggest that

predictions might arise if inflation is connected with supersymmetry breaking. In section 5.7, we conclude by considering possible observable consequences of this picture.

## 5.2 Hybrid Models and $M_P$ Effects

The simplest model of hybrid inflation contains two chiral superfields,  $S$  and  $\phi$ , with superpotential

$$W = S(\kappa\phi^2 - \mu^2). \quad (5.2)$$

If one imposes, as is usually done, a *continuous*  $R$  symmetry under which the charges of  $S$  and  $\phi$  are respectively 2 and zero, this superpotential is the most general permitted by symmetries. Classically, the theory has a moduli space,

$$|S|^2 > \left| \frac{\mu^2}{\kappa} \right|, \quad (5.3)$$

on which

$$V = V_0 = |\mu|^4. \quad (5.4)$$

At one loop, the potential receives corrections. In the global limit:

$$V = V_0 \left( 1 + \frac{\kappa^2}{8\pi^2} \log |S| \right). \quad (5.5)$$

If one considers only this term, one has, for the number of  $e$ -foldings:

$$\mathcal{N} = \frac{1}{2} \frac{8\pi^2}{\kappa^2} |S|^2. \quad (5.6)$$

In this simple model, the  $\epsilon$  parameter is negligible, and

$$\eta = -\frac{1}{2\mathcal{N}}. \quad (5.7)$$

This yields

$$n_s = 1 - \frac{1}{\mathcal{N}}. \quad (5.8)$$

This is the origin of the prediction that  $n_s \approx 0.98$ .

In this model,  $\kappa$  is related to  $\mu$  by the fluctuation spectrum:

$$\kappa = 0.17 \times \left( \frac{\mu}{10^{15} \text{GeV}} \right)^2 = 7.1 \times 10^5 \times \left( \frac{\mu}{M_P} \right)^2. \quad (5.9)$$

### 5.3 Hierarchy of Corrections

This treatment, however, is oversimplified. Already, in [61, 59], the role of higher order terms in the Kähler potential was considered. More recently, in [66], the effects of a linear term in the potential for  $S$ , arising from the constant term in the superpotential (needed to account for the small cosmological constant of the present universe) has been considered. In [56], this particular contribution was treated as small, but a number of other effects were considered. So it is first worthwhile to consider the various possible corrections in powers of  $1/M_P$ , and their relative importance.

First, it is generally believed that theories of gravity should not exhibit continuous global symmetries; in string theories, this is a theorem. Replacing the continuous  $R$  symmetry by a discrete  $Z_N$  symmetry allows corrections of the form

$$W_R = \frac{\lambda}{2(N+1)} \frac{S^{N+1}}{M_P^{N-2}}. \quad (5.10)$$

More generally, our viewpoint will be that all terms allowed in the effective action below  $M_P$  should appear with order one coefficients; we will assume that smaller coefficients

represent a “fine tuning” of parameters. We can systematically consider types of corrections, ordered in powers of  $1/M_P$ :

1. Kähler potential corrections:  $\frac{\alpha}{M_P^2}(S^\dagger S)^2, \frac{\beta}{M_P^4}(S^\dagger S)^3$ .
2. Superpotential corrections: in addition to  $W_R$  (and higher powers of  $S$ , other fields), at some level there must be a constant in the superpotential,  $W_0$ , to account for the smallness of the cosmological constant now.
3. Supersymmetry breaking effects.

The term

$$\delta K = \frac{\alpha}{M_P^2}(S^\dagger S)^2 \tag{5.11}$$

has been noted already in [59]. In [56], precise limits on  $\alpha$  (of order  $1/\mathcal{N}$ , where  $\mathcal{N}$  is the number of  $e$ -foldings) were discussed. It was noted that the quantum corrections of eqn. (5.5) only dominate over this Kähler potential correction for sufficiently small  $S$ . In fact, as we will review shortly, for the simplest model, the quantum corrections *never* dominate unless  $\mu$  is quite small.

Terms of sixth order or higher in  $S$  in the Kähler potential are irrelevant. They lead to highly suppressed contributions to  $\eta$  and  $\epsilon$ , for example. We will be more quantitative about this question when we turn to models that can reproduce the Planck value of  $n_s$ .

Now we turn to the various superpotential corrections. Our definition of hybrid inflation is motivated by the hypothesis that the scale of inflation is large compare to scales of supersymmetry breaking. This means, in particular, that

$$m_{3/2} = \frac{|W_0|}{M_P^2} \ll H_I \tag{5.12}$$



with  $H_I = \frac{\mu^2}{M_P}$  the Hubble scale during inflation. As a result, terms in the potential arising from  $W_0$  can be neglected during inflation. If, in fact, the actual value of  $m_{3/2}$  is comparable to  $H_I$ , then this term, and terms associated with supersymmetry breaking, would be important. Even for  $m_{3/2} = 10^2$  TeV, this corresponds to an inflationary energy scale well below  $10^{12}$  GeV.

So finally we turn, again, to  $W_R$ . The presence of this term in the superpotential gives rise to a supersymmetric minimum of the potential at  $S$  large but parametrically smaller than  $M_P$ . This is unlike the case, for example, of higher order corrections to the Kähler potential. As a result, this term qualitatively alters the behavior of the system, for large but not Planck scale fields. In [56] this term was used to constrain features of inflation. Requiring that it was not important during inflation constrained the scale of inflation, and lead to the prediction  $n_s \approx 0.98$ . To be compatible with the results from Planck, however, it is clearly necessary that inflation occur in a region near the local maximum (as in “Hilltop inflation” [65]). We will explore this in the next section.

## 5.4 Hybrid Inflation and $W_R$

Including  $W_R$ , it is first important that the system not flow towards the supersymmetric minimum. Indeed, for an intermediate range of field values, there are corrections to the potential (5.5) of the form

$$\delta V_R = \lambda \mu^2 \frac{S^N}{M_P^{N-2}} + \text{c.c.} \quad (5.13)$$

For negative  $\lambda$ , this leads to a maximum, for

$$S^N \approx \frac{\kappa^2}{8\pi^2} \frac{\mu^2}{|\lambda|} M_P^{N-2}. \quad (5.14)$$

To obtain suitable inflation, it is necessary that  $S$  be smaller than this at the beginning.

But, given eqn. (5.9), except for very large  $N$ ,  $S$  is smaller than the “waterfall value”,

$$S_w = \frac{\mu}{\sqrt{\kappa}}. \quad (5.15)$$

As a result of these considerations, the simplest (and rather standard) model of hybrid inflation (allowing for  $W_R$ ) does not appear suitable. In [56], a simple modification was suggested with two fields,  $S$  and  $I$ , with couplings at the renormalizable level:

$$W = S(\kappa\phi^2 - \mu^2) + \lambda I\phi\phi' + \dots \quad (5.16)$$

The theory, classically, has two flat directions, one with large  $S$ , one with large  $I$ . As in the previous model, in order that inflation occur, the Kähler potential must be tuned so that at least one of the fields  $S$  or  $I$ , has mass small compared to the Hubble constant during inflation,  $H_I = \frac{\mu^2}{M_P}$ . To obtain a workable model, we require that  $I$  be the light field. This amounts to requiring that in the Kähler potential term

$$\delta K = \alpha \frac{S^\dagger S I^\dagger I}{M_P^2} \quad (5.17)$$

$\alpha$  should be close to unity. The waterfall regime is now at smaller value of the inflaton field  $I$ ,  $I_w = \sqrt{k} \frac{\mu}{\lambda}$ , and hybrid inflation can be driven by the quantum and discrete symmetry corrections.

Assuming a discrete  $R$  symmetry, there are a variety of possible higher dimension terms which might appear in  $W$  depending on the transformation properties of the fields.

We will consider a term of the form

$$\delta W = -\gamma \frac{SI^N}{M_P^{N-2}}. \quad (5.18)$$

Alternatively, a term proportional to  $I^M$ , for example, corrects the potential for  $I$  if there is a term in the Kähler potential

$$\delta K = \frac{SI^{*M}}{M_P^{M-1}}. \quad (5.19)$$

The allowed values of  $M$  depend on the discrete charge assignments of the fields. If  $M$  is not too large, its effects are dramatic.

Such terms, again, lead to a supersymmetric minimum of the potential at large  $I$  (with  $\phi = \phi' = 0$ ), and again give rise, for positive  $\gamma$ , to a *maximum* of the potential for  $I$  at field strength generically large compared to  $\mu$  but small compared to  $M_P$ .

Proceeding as before, using the superpotential and Kähler corrections in eqns. (5.16)–(5.18), we can compute the number of  $e$ -foldings and the slow roll parameters (and hence  $n_s$ ). The potential for  $I$  is now, approximately,

$$V(I) = \mu^4 \left( 1 + \frac{\kappa^2}{16\pi^2} \log(I^\dagger I) - (\alpha - 1) \frac{I^\dagger I}{M_P^2} \right) - \gamma \mu^2 M_P^2 \left( \frac{I}{M_P} \right)^N + \text{c.c.} \quad (5.20)$$

The fluctuation spectrum relates  $\kappa$  and  $\mu$ , as before. For a given value of  $\mu$ , the initial value of the field at  $\mathcal{N} = 60$   $e$ -foldings is fixed. So, then, is  $n_s$ .

To get a rough sense of scalings, we can suppose that  $I$  starts very near the maximum of the potential, and that  $\eta = -0.02$  (in order to achieve  $n_s = 0.96$ ). Because  $V' \sim 0$  at the hilltop, we will simply use the formula for normal hybrid inflation in our estimate; shortly we will check the accuracy of this numerically, and see that this leads to

an order one error. Then one finds that

$$\frac{\mu}{M_P} = \left( \left( \frac{.02}{N} \right)^N \cdot (6.4 \times 10^9)^{2-N} (N\gamma)^{-2} \right)^{\frac{1}{4N-12}}. \quad (5.21)$$

For particular values of  $N$ , we can compute  $\mu$  and  $\kappa$ : taking  $\gamma \approx 1$  and  $N = 4$ , this gives  $\mu \approx 10^{11}$  GeV and  $\kappa \approx 10^{-10}$ . For  $N = 5$ , one obtains  $\mu \approx 10^{13}$  GeV, and  $\kappa \approx 10^{-5}$ . The scale  $\mu$  grows slowly with  $N$ , reaching  $10^{14}$  GeV at  $N = 7$  and  $10^{15}$  GeV for  $N = 12$ . In general, these results scale with  $\gamma$  as:

$$\gamma^{-\frac{1}{2(N-3)}}. \quad (5.22)$$

We discuss numerical studies of this problem in the next section. But the lesson here is that, for fixed values of  $\gamma$ , and for a given  $N$ , the scale of inflation,  $\mu$ , is fixed to a narrow range.

## 5.5 Numerical Studies of Small Field Inflation

Denoting the real part of the field  $I$  by  $\sigma$ , the potential in eqn. (5.20) becomes

$$V(\sigma) = \mu^4 \left( 1 + \frac{\kappa^2}{16\pi^2} \log(\sigma^2) - (\alpha - 1) \frac{\sigma^2}{M_P^2} \right) - \gamma \mu^2 M_P^2 \left( \frac{\sigma}{M_P} \right)^N, \quad (5.23)$$

where we have included in  $\gamma$  the numerical factor  $2^{N/2-1}$  coming from the field redefinition.

It will be handy to denote the hilltop position by  $\sigma_h$ , and to investigate how close  $\sigma$  has to be to  $\sigma_h$  in order to successfully have  $\mathcal{N} = 50$ –60  $e$ -foldings of inflation.

For a given  $N$ , the parameters of the two field model are readily enumerated:  $\mu$ ,  $\kappa$ ,  $\alpha$ , and  $\gamma$ . Given knowledge of these, we can compute the observable predictions of the inflationary model, to be compared with the Planck collaboration results [?]:

1. The number of  $e$ -foldings  $\mathcal{N}$ . To solve the horizon and flatness problems, it must be  $\mathcal{N} \geq 50$ . In our numerical treatment, we will assume the range of  $\mathcal{N} = 50\text{--}60$   $e$ -foldings.
2. The slow roll parameters  $\eta, \epsilon$ , which result in the spectral index  $n_s = 1 - 6\epsilon + 2\eta$ . The measured value by the Planck collaboration is  $n_s = 0.9603 \pm 0.0073$ .
3. The density perturbation spectrum  $\mathcal{P}_{\mathcal{R}}$ , whose amplitude is a function of  $V^{3/2}/V'$ . Planck measurements translate to  $V^{3/2}/V' = (5.10 \pm 0.07) \times 10^{-4} M_P^3$ .

We can, in principle, compute the tensor to scalar ratio  $r$ , but in all such models this will be unobservably small. In general, as said in the previous section,  $(1 - \alpha)$  quantifies the Kähler correction independent from the discrete symmetry, and is already required to be small, while the dependence on  $\gamma$  is weak. In the following we will set  $\alpha \sim 1$ ,  $\gamma = 1$ .

Given the potential (5.23), the expression for the number of  $e$ -foldings  $\mathcal{N}$  involves an integral that can be computed numerically. With a  $\chi^2$  analysis, for each given  $N$ , we set the three remaining parameters  $\mu, \sigma, \kappa$  by fitting the experimental values of  $\mathcal{N}, n_s, V^{3/2}/V'$ . For example, in Fig. 5.1, where we set  $N = 4$  and  $\kappa$  to its best-fit value, we show how the allowed ranges for each experimental quantity intersect at specific values of  $\mu$  and  $\sigma$ .

In Table 5.1, we give the best-fit values of  $\mu, \sigma, \kappa$ , with the corresponding uncertainties for  $N$  from 4 to 12. There is no fine-tuning associated with the inflaton being close to the hilltop value, as the allowed values for  $\sigma/\sigma_h$  are in the range 0.6–0.8. For small  $N$ , the coupling  $\kappa$  is tuned to be small. In the last column, we show how close to unity  $\alpha$  has to be for the Kähler correction not to overcome the discrete symmetry correction. As  $|\alpha - 1|$  is already fine-tuned to be of order  $\frac{1}{\mathcal{N}}$  in order not to spoil inflation, we conclude that there

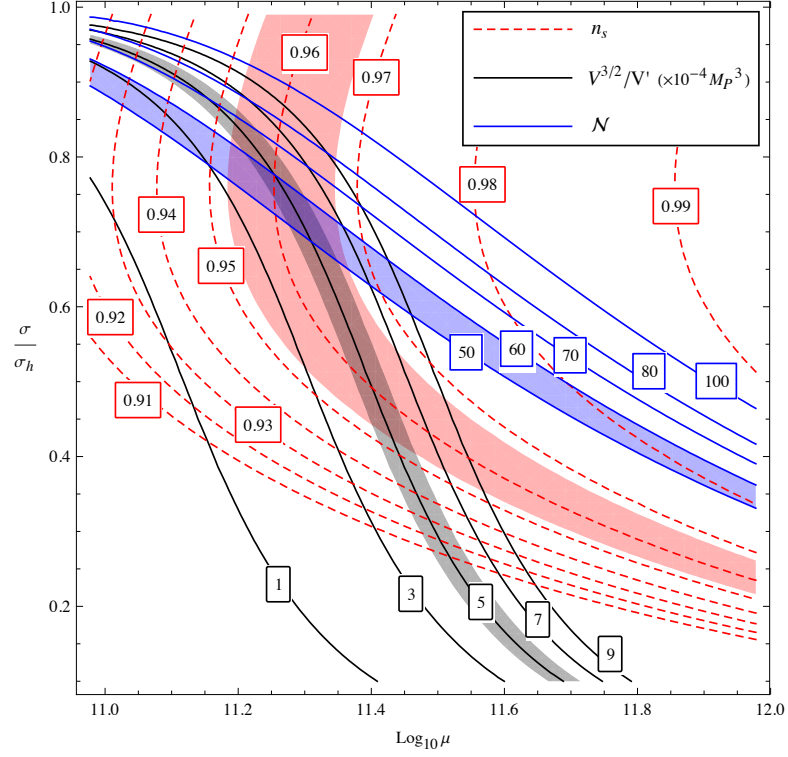


Figure 5.1: Contours for the spectral index  $n_s$  (dashed red), the density perturbation  $V^{3/2}/V'$  (solid black) and the number of  $e$ -foldings  $\mathcal{N}$  (solid blue), for  $N = 4$ . The coupling  $\kappa$  is kept fixed at its best fit value of  $\kappa = 2.1 \times 10^{-9}$ . The shaded zones indicate the 1-sigma regions allowed by the Planck results for  $n_s$  and  $V^{3/2}/V'$ , and the range of 50–60  $e$ -foldings. The  $\chi^2$  is minimized where the bands intersect each other. For each value of  $\kappa$  a specific range of  $\mu$  and  $\sigma$  is allowed. As  $\kappa$  varies, each variable changes independently and the allowed region moves and shrinks, until the three bands do not intersect.

| $N$ | $\mu$<br>( $10^{14}$ GeV) | $\sigma$<br>(GeV)                | $\sigma_h$<br>(GeV)   | $\kappa$                       | $\ \alpha - 1\ _{max}$ |
|-----|---------------------------|----------------------------------|-----------------------|--------------------------------|------------------------|
| 4   | $0.002 \pm 0.0005$        | $(4.8 \pm 2.3) \times 10^9$      | $6.7 \times 10^9$     | $(2.1 \pm 1.2) \times 10^{-9}$ | 0.004                  |
| 5   | $0.25 \pm 0.05$           | $(8 \pm 2) \times 10^{13}$       | $11 \times 10^{13}$   | $(3.5 \pm 1.5) \times 10^{-5}$ | 0.006                  |
| 6   | $1.25 \pm 0.2$            | $(2.1 \pm 0.4) \times 10^{15}$   | $2.8 \times 10^{15}$  | $(9 \pm 1) \times 10^{-4}$     | 0.007                  |
| 7   | $2.9 \pm 0.5$             | $(1.1 \pm 0.16) \times 10^{16}$  | $1.42 \times 10^{16}$ | $(4.7 \pm 0.7) \times 10^{-3}$ | (−)0.008<br>(+)0.02    |
| 8   | $4.8 \pm 0.7$             | $(2.9 \pm 0.3) \times 10^{16}$   | $3.8 \times 10^{16}$  | $(1.3 \pm 0.1) \times 10^{-2}$ | (−)0.01<br>(+)0.016    |
| 12  | $12 \pm 1.5$              | $(1.74 \pm 0.12) \times 10^{17}$ | $2.14 \times 10^{17}$ | $(7.4 \pm 0.6) \times 10^{-2}$ | (−)0.013<br>(+)0.008   |

Table 5.1: Numerical results: central values and  $1\sigma$  allowed ranges for the parameters, for different choices of  $N$ . The central column lists the hilltop value for the central value of the parameters. The last column shows how close to 1 the quartic Khaler correction  $\alpha$  is forced to be (at the 95%CL); for some  $N$ , there is a weak dependence on the sign of  $(\alpha - 1)$ ; these values should be compared to the irreducible tuning of order  $\frac{1}{N} \sim 0.016\text{--}0.020$ .

is another mild tuning which operates to keep  $\alpha$  close to 1.

For  $N = 12$ , the initial value of the field is  $\sigma = 1.7 \times 10^{17}$  GeV, just a factor of 10 below  $M_P$ . For larger  $N$ , it is not possible to accomodate  $n_s = 0.96$  within the framework of small field inflation. Even for this large value of the field, the tensor-to-scalar ratio is predicted to be small:

$$r = 0.12 \frac{V}{(1.94 \times 10^{16} \text{ GeV})^4} \leq 2 \times 10^{-6} \quad (5.24)$$

## 5.6 Incorporating Supersymmetry Breaking

The picture of small field inflation we have developed up to now assumes that the scale of inflation is large compared to the scale of supersymmetry breaking, i.e. that  $H_I \gg m_{3/2}$ . This is the origin of the requirement that the superpotential should vanish and supersymmetry be unbroken, to a good approximation, at the end of inflation. But one might consider the possibility that  $H_I \sim m_{3/2}$ . A higher scale of  $m_{3/2}$  is suggested by the observed Higgs mass and supersymmetry exclusions. In addition, for small values of  $N$ , we have obtained small values of  $H_0$ . So it is interesting to consider the possibility that the scale of inflation is comparable to  $m_{3/2}$ .

For example we can modify the models we have studied, to give them an O’Raifeartaigh like structure, adding to the superpotential of eqn. (5.2) a coupling

$$m\phi\Phi. \tag{5.25}$$

Provided

$$|m^2| > \kappa\mu^2 \tag{5.26}$$

supersymmetry is broken, in a state with  $\Phi = 0$ . It is interesting that in this case, inflation ends without ever passing into a “waterfall” regime. As we have stressed, the so-called waterfall is indeed not the distinguishing feature of hybrid inflation.

A different approach has been pursued in [67]. Again, it is assumed that the scale of inflation is not too much different than the scale of supersymmetry breaking. One writes a theory of a single field,  $\phi$ , and does not require an unbroken  $R$  symmetry at the end of inflation. Instead, one assumes that the negative contribution to the cosmological constant



arising from the vev of the superpotential is cancelled by some supersymmetry breaking dynamics. To constrain the form of the superpotential, one still assumes a discrete  $R$  symmetry. It is necessary, as in hybrid inflation, to tune the Kähler potential so that the  $|\phi|^4$  term is small. The superpotential takes the form:

$$W(\phi) = v^2\phi - \frac{g}{n+1}\phi^{n+1}, \quad (5.27)$$

while the quartic term in the Kähler potential must be quite small. The resulting model is of the hilltop type. The potential exhibits a local maximum at the origin, and the initial value of the field must lie quite close to the maximum (compared to the distance of the origin from the minimum). Inflation occurs in a region very close to the origin in field space (defined by an unbroken  $R$  symmetry). The field then settles into a minimum with small cosmological constant and broken supersymmetry and  $R$  symmetry. The model can produce the requisite number of  $e$ -foldings and fluctuation spectrum, without introducing an extremely small number analogous to  $\kappa$  of eqn. (5.2). However, it predicts too *small* a value of  $n_s$ ,

$$n_s = 0.94. \quad (5.28)$$

To obtain a spectral index consistent with Planck, it is necessary to introduce a small and well-tuned constant in the superpotential, which the authors denote  $c$ , and is of order  $10^{-19}$  (in Planck units). There are other issues, such as a possible gravitino problem and overproduction of dark matter, but these can readily be solved by introducing additional matter coupled to the inflaton.

Both approaches, then, seem viable, and have the potential to relate supersymmetry breaking dynamics to inflationary dynamics. Each requires certain tunings.

## 5.7 Conclusions: Predictions and Observable Consequences for Low Energy Physics

The results from Planck pose challenges for models of small field inflation. It has been said that they rule out “hybrid inflation.” Here, following [56], we have carefully defined models of hybrid inflation as models in which inflation occurs on a pseudomoduli space, with supersymmetry and an  $R$  symmetry approximately restored at the end of inflation. We have assumed a *discrete*  $R$  symmetry, and have considered the importance of corrections to the superpotential and Kähler potential. For initial values of the field far from the local maximum of the potential, one predicts a spectral index inconsistent with Planck. To obtain  $n_s = 0.96$ , it is necessary that the field start near the local maximum, though this condition is not severely tuned. For  $Z_N$  symmetry with  $N = 4$ , the scale of inflation is rather low, and we considered the possibility that  $H_I \approx m_{3/2}$ . In this case, the dynamics of inflation might be closely tied to the scale of supersymmetry breaking, and there is some chance that aspects of the physics of inflation could be studied in accelerator experiments.

We have noted that, in this case, the assumption of an unbroken  $R$  symmetry and unbroken supersymmetry at the end of inflation might be relaxed, and compared the hybrid models with those of [67]. Each of these models can reproduce the data, and involves very small parameters and tunings. The fact that many models with such features can reproduce the basic data of inflation raises, as always, the question of whether there is any way they might be testable or falsifiable. We would argue that the best hope is connecting inflation with the dynamics responsible for supersymmetry breaking. It will be particularly interesting to explore dynamical supersymmetry breaking (and generation of scales) in this

framework.

## Chapter 6

# Conclusion

This is a pivotal time for high energy particle physics; naturalness, the holy grail of particle physics, is under stress[68] by the recent LHC results. In light of these new discoveries, we have analyzed a few supersymmetric models as part of this dissertation. Discrete R-symmetries played an important role in all of these models.

In the case of gravity mediated SUSY breaking, we showed that the simplest generic models with discrete R-symmetries do not have a split spectrum. Kähler potentials in these models are arbitrary. As a result, they do not have generic predictions for various parameters such as A-terms and B-terms. However, if the supersymmetry breaking field is charged under a symmetry other than the R-symmetry, a split spectrum is possible but not generic. We argued that, if we require that the cosmological constant be parametrically small, then in the simplest retrofitted model the tuning of the Z-boson mass depends only on choices of some discrete parameters.

We have also constructed models of gauge mediated SUSY breaking. Here, we

showed that, if the approximate R-symmetry is spontaneously broken, the underlying scale of supersymmetry breaking corresponds to a gravitino mass of order 1 GeV. The R-axion due to spontaneous breaking of R-symmetry is also of order 1 GeV. We also explored the possibility of explicit R-symmetry breaking. Such breaking can arise due to supergravity effects. We showed that in both of these scenarios minimal gauge mediation can be realized.

Lastly, we constructed models of hybrid inflation. In these models, we have assumed a discrete R-symmetry, and have considered the importance of corrections to the superpotential and Kähler potential. We showed that the initial value of the inflaton field must be very close to the local maximum for the spectral index to be consistent with the Planck results. We explored the possibilities that the dynamics of inflation might be closely tied to the scale of supersymmetry breaking, and that there is some chance that aspects of the physics of inflation could be studied in accelerator experiments.

In this thesis, we have frequently classified models as more or less natural. It is not clear whether this is a criterion shared by nature. Even if we discover evidence for supersymmetry in the future LHC runs, the supersymmetry will most likely be broken at a high scale.

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