Title
Development of a rapid design procedure for emergency repair of bridge columns using fiber-reinforced polymers

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Publication Date
2008

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA, SAN DIEGO

Development of a Rapid Design Procedure for Emergency Repair of Bridge Columns Using Fiber-Reinforced Polymers

A Thesis submitted in partial satisfaction of the requirements for the degree

Master of Science

in

Structural Engineering

by

Susan E. Slater

Committee in charge:

Professor Vistasp Karbhari, Chair
Professor Gilbert Hegemier
Professor Yu Qiao

2008

The Thesis of Susan E. Slater is approved and it is acceptable in quality and form for publication on microfilm and electronically:

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Chair

University of California, San Diego

2008

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ACKNOWLEDGEMENTS

I am extremely grateful for the opportunity to conduct this research under the guidance of my advisor, Dr. Vistasp Karbhari. Without his support and academic passion, I would not have achieved what I have today. I would like to thank my committee members Prof. Gilbert Hegemier and Prof. Yu Qiao for their personal commitments and technical contributions.

Thanks to all of members of my research group, friends and family for support and help throughout my studies.

The research for this thesis was funded by Caltrans, for which I am thoroughly appreciative.

ABSTRACT OF THE THESIS

Development of a Rapid Design Procedure for Emergency Repair of Bridge Columns Using Fiber-Reinforced Polymers

by

Susan E. Slater

Master of Science in Structural Engineering

University of California, San Diego, 2008

Professor Vistasp Karbhari, Chair

Fiber reinforced polymers (FRP) are increasingly used for seismic retrofit of undamaged bridge columns. The addition of a confining jacket increases overall strength and ductility of the column. However, FRPs can also be used for repair of a damaged bridge column after a seismic event. This research project explores the application of FRPs for retrofit of damaged circular columns. A decision tree for the emergency repair of damaged columns is presented. The decision tree aids field engineers first in assessing the extent of damage to a column from a seismic event, impact or any other source of damage. Once the level of damage is specified, the decision tree can be used to determine the feasibility of FRP retrofitting. If an FRP jacket is a viable option, suggested simplified design guidelines can be used to determine the design thickness and type of required FRP jacket that should be used for the emergency repair of the column. The project included detailed evaluation of existing confining models and a proposition of a new model based on a large database of previously tested specimen columns. Several design equations were also evaluated in order to find the most accurate yet simple approach for the design thickness. The end result is a step-by-step simplified guide for engineers to use in the field for emergency repair of damaged circular bridge columns.

Chapter 1

Introduction

1.1 Introduction

Fiber reinforced polymers (FRPs) are used extensively for seismic retrofit of columns to enhance the ductility of the column and increase the axial load carrying capacity. The effectiveness of this method has been widely studied and proven through full scale laboratory testing and field installation. A variety of fabrication techniques have been implemented with success including wet layup, winding with wet tow or prepreg tow, wrapping of prepregs and adhesive bonding of prefabricated sections.

Seismic retrofit focuses on bridges that are currently undamaged, but deemed deficient for seismic events. There has been little research into the repair of already deficient columns using FRP systems. There is a need for rapid methods that enable emergency repair in order to maintain use of high priority bridges after a seismic event. It is noted that although the seismic retrofit concepts are applicable for emergency repair, there is a lack of availability of guidelines and testing for this application.

1.2 Research Objectives and Motivations

The main objective for this research is to create guidelines for the emergency repair of damaged bridge columns. This project will lead to guidelines that can be used by field engineers to assess whether emergency repair using FRP materials is a viable option, and if so, steps to be taken to design and implement the repair system. The motivation for the guidelines is to generate a rapid response for repair of damaged columns in order to maintain access to key bridges after a seismic event.

There are also several motivations for specifically using FRP as opposed to the more traditional steel jackets for repair. FRP allows for a more rapid installation process. Steel jackets would need to be manufactured and then installed; whereas, wet layup based fabrication of FRP jackets enables the agency to be ready to implement the repair at any moment. Rapid installation at any time is crucial for emergency repair. FRP wet layup also allows for ease of talorability to specific column geometries, since the fabric can conform to any shape of column. The installation process also does not require large machinery, which allows for implementation in more restricted areas when compared with steel jacketing. There are also potential long term advantages for using FRP. There is a potential for enhanced durability and decreased maintenance, due to the materials non corrosive quality.

1.3 Literature Review

It is well known the effectiveness of using FRP jackets for seismic retrofit and the concept has been used with success in the field extensively. However, there has been considerably less research of the use of FRP jackets for repair of damaged or corroded columns. This section will review important research that has been conducted in the topic of column repair with FRP jackets. There will also be a review of confinement models for FRP jackets in Chapter 3 and a review of guidelines for design of FRP jackets in Chapter 4.

1.3.1 Saadatmanesh *et al.* (1997)

Saadatmanesh *et al.* [37] performed experimental tests on reinforced concrete columns with earthquake like damage repaired with GRRP jackets. Unidirectional E-glass fiber in a polyester matrix were wrapped around damaged sections of concrete columns. Four 1/5 scale columns with single bend and a strong footing were used. The specimens consisted of two circular columns and two rectangular columns. Each column corresponded to certain design deficiencies often seen in the field, such as inadequate transverse reinforcement or started bar lap length. The original columns were tested to failure and damaged included debonding of the starter bars, spalling and crushing of concrete, local buckling of the longitudinal reinforcement, yielding of the stirrups and separation of the main bars from the concrete core. The columns were then repaired using 0.8 mm of FRP jacket. This thickness was chosen based on the amount of confining pressure needed to return the column to current design standards.

The repaired columns were subjected to the same loading cycle as the original columns. It was observed that the repaired columns performed at least as well or better than the original columns. In particular, the repaired columns showed a significant increase in the level of strength and ductility achievable. The repaired columns exhibited a ductility level of between 4 and 6, whereas the original columns had ductility levels as low as 1.5. In addition, the repaired columns showed a slower rate of stiffness degradation with each cycle when compared with the original column stiffness degradation rate. It was also observed that the repaired columns showed much larger lateral displacements at low load levels, which is most likely due to the existing damage.

Saadatmanesh *et al.* [37] concluded that FRP wraps are effective in restoring the flexural strength and ductility capacity of concrete columns.

1.3.2 Ma and Xiao (1999)

Ma and Xiao [29] investigated the use of prefabricated GFRP jackets to repair circular bridge columns that have poor lap slice details. A ½ scale column was used for testing, which was built to the pre-1971 design standards; and therefore, possessed certain deficiency for seismic design, including inadequate lap splice lengths. The as-built column was tested to failure, with a failure mode of bond failure in the plastic hinge region. The column was then repaired by injecting epoxy in the damaged plastic hinge region and then wrapping the column with five layers of prefabricated GFRP shells, which has a thickness of 2.5 mm per layer.

The repaired column was subjected to a simulated earthquake load. An increase in ductility from 4 to 6 was observed. Ma and Xiao [29] concluded that GFRP prefabricated shells are effective in repairing columns and show an improved stiffness, as well as ductility and energy absorption capacities.

1.3.3 Lee *et al.* (2000)

Lee *et al.* [25] conducted experimental tests on large scale reinforced concrete columns, subjected to an accelerated corrosive environment. A total of seven large scale concrete columns were tested. Five of the columns were subjected to accelerated corrosion. The accelerated corrosion was applied by exposing the concrete columns to sodium chloride, applying an electrical current to the steel reinforcement and subjecting the columns to wet-dry cycles. The accelerated corrosion resulted in steel loss and cracking and spalling of the concrete. Three columns were repaired using CFRP sheets and the remaining four columns were used as control specimens.

The corrosion damaged columns were repaired using a CFRP jacket. The repaired columns were then tested to failure or subjected to further accelerated corrosion.

Lee *et al.* [25] observed large improvements in the repaired column strength and in the rate of additional corrosion. The repaired columns showed an increase in load carrying capacity of up to 28% as compared to the un-repaired corroded column. It was also observed that the rate of additional corrosion was decreased by 50%. In addition, the columns subjected to further corrosion showed no loss of strength or stiffness and only a small reduction in ductility as compared to the repaired column.

1.3.4 Li *et al.* (2003)

Li *et al.* [26] tested eight circular reinforced concrete columns repaired with FRP jackets. The columns were designed to conform to all design recommendations put forth in ACI 318 [2]. The columns were then subjected to tensile tests until failure occurred. A tensile test was used as opposed to a more realistic bending and compression test because the damage and crack are more controllable with the tensile test.

Three damaged columns were then repaired using wet layup of four layers of GFRP fabric with an epoxy resin. Another three columns were repaired using a prepreg GFRP jacket. Two columns were left un-repaired to use as control specimens. The columns were then subjected to a uniaxial compression test to determine the modulus of elasticity and the compressive strength of each column. It was concluded that the FRP jackets significantly increased the compressive strength of the damaged column. Li *et al.*

[26] also concluded that the prepreg jacketing system obtains a higher modulus of elasticity than the wet layup approach.

1.3.5 Tastani *et al.* (2006)

Tastani *et al.* [46] tested a total of 12 square columns. Nine of the columns were constructed according to the old standards with inadequate transverse reinforcement. The remaining three columns were designed with modern practices. The columns were subjected to compression loads until the concrete cover spalled away. The columns were then repaired using either a GFRP or CFRP jacket of 1-2 layers thickness.

Tastani *et al.* [46] observed that the axial load increased in the repaired columns when compared with the control columns that were un-repaired. There was also determined to be an increase in the deformation capability. Tastani *et al.* [46] also suggested that the effect of increase in strength and ductility was more pronounced in the CFRP jackets as opposed to the GFRP jackets. In all cases, the column failure mode was rupture of the FRP jacket at the corners of the column. This was followed by buckling of the longitudinal reinforcement in the columns constructed from old design standards.

1.3.6 Vosooghi and Saiidi (2008)

Vosooghi and Saiidi [50] tested a damaged ¼ scale two span bridge, comprised of two circular columns. The columns were damaged to a level corresponding to damage state 4, which will be discussed in greater detail in Chapter 2. At this damage state, concrete has spalled severely and reinforcing bars are visible. This damage state also corresponded to an approximate drift of 8%.

The concrete columns were repaired using CFRP jackets. The design thickness was determined using the Caltrans Memo 20-4 [5] design approach, which will be discussed in more detail in Chapter 4. The end design used 2 layers of CFRP within the plastic hinge region and one layer on the remaining column.

The repaired columns were tested using three dynamic tests, eight static tests and four white noise tests. The dynamic tests were the same as on the original column prior to repair.

Vosooghi and Saiidi [50] concluded that repair design was effective in restoring strength and ductility to the damaged column. It was observed that the strength, ductility capacity and drift capacity were restored to at least that of the original column. It was determined that 87% of the stiffness was restored.

1.4 Scope of Research

This research is aimed at three specific tasks. The first task is to characterize the level of damage to a column and establish which degrees of damage can be repaired using FRP jackets. This task will allow field engineers to easily assess and characterize the amount of damage to the column through visual inspection.

The second task is to develop detailed guidelines for the emergency repair of the column. This includes a comprehensive review and analysis of existing guidelines including confining models and design codes.

The final result will be the development of the bases for a decision tree, which will guide the engineer step-by-step in assessing the damage in the column and then choosing an appropriate thickness of the FRP jacket that would be required to sufficiently repair the column.

This research is focused on seismic damage, but is also applicable with minor modifications to a wide range of damage causes including impact damage, design flaws or corrosion damage. Although the general procedure is applicable to the full-range of geometries, this research will only consider circular columns due to their frequency in existing structures and their relative efficiency of confinement.

Chapter 2

Damage Analysis

2.1 Introduction

The current design philosophy for seismic design of bridges is a "no-collapse" approach. The designs allow for significant nonlinear response, which can result in considerable amounts of damage including concrete cracking and spalling, yielding of reinforcing bars and possibly rupture of some reinforcing bars. Therefore, after seismic events, the column is damaged, but theoretically repairable. Visual classification of the damage to a column is an important first step in the repair process. Classification of the extent of damage can correspond to the status of the column and the method of repair.

Typically field engineers use visual inspections to determine the degree of damage to the column. Five damage states characterized from their associated visual characteristics are presented in this chapter. In addition, average response parameters associated with each damage state, such as strain in the reinforcing bars are presented and compared with experimental results.

2.2 Damage States

The American Concrete Institute (ACI), Building Research Establishment, Ltd. (BRE), Concrete Society and International Concrete Repair Institute jointly publish standards for concrete repair. The result is the Concrete Repair Manual, which is currently in the second edition released in 2001 [3]. Within the manual are standards for visually inspecting damaged concrete and the corresponding damage states. The same damage states are adopted for this research in order to maintain consistency within standards for inspection and characterization.

The manual divides the damages states into five distinct damages states (DS), DS-1 through DS-5. Damage state DS-1 corresponds to very slight damage with minimal cracking, characterized by flexural cracks and no visible spalling. Damage state DS-2 has slight damage and occurs when first spalling and shear cracks are visible. Damage state DS-3 is moderate damage and is associated with extensive cracks and spalling. Damage state DS-4 is severe damage, in which the spiral and longitudinal reinforcing bars become visible and large cracks, holes and spalled areas are observed. Damage states DS-5 corresponds to very severe damage and is associated with imminent failure and cracks propagating within the concrete core. Table 2.1 details the specific visual characteristics associated with each damage state as referenced in the Concrete Repair Manual, Second Edition. [3]

Table 2.1- Visual characteristic of damage states.

Damage	Damage State				
	1 (very slight)	2 (slight)	3 (moderate)	4 (severe)	5 (very severe)
Cracks in concrete	Width < 0.1 mm	Width 0.1-0.3 mm	Width 0.3-1 mm	Width 1-3 mm	Width > 5 mm
Pop-outs	Barely Noticeable	Noticeable	Holes up to 10 mm in diameter	Holes between 10 and 50 mm in diameter	Holes > 50 mm in diameter
Spalling	Barely Noticeable	Noticeable	Larger than coarse aggregate	Areas up to 150 mm across	Areas larger than 150 mm

In conjunction with this project, UNR conducted tests on circular reinforced concrete columns. Columns were damaged to levels corresponding to each damage state through the visual inspection descriptions. Figures 2.1 through 2.5 show the visual characteristics associated with each damage state.

Figure 2.1- Damage state DS-1, flexural cracks.

Figure 2.2- Damage state DS-2, spalling and shear cracks.

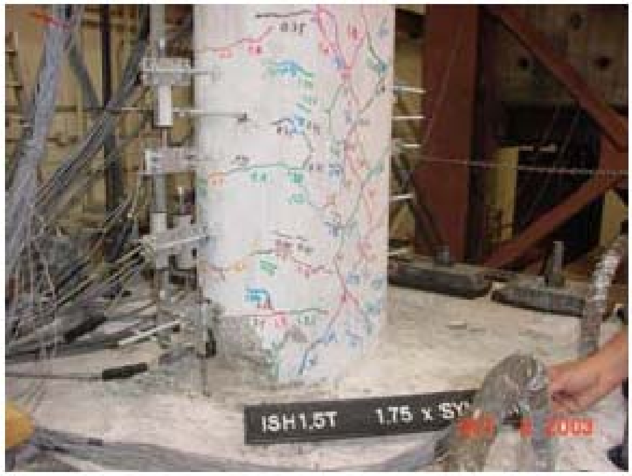

Figure 2.3- Damage state DS-3, extensive cracking and spalling.

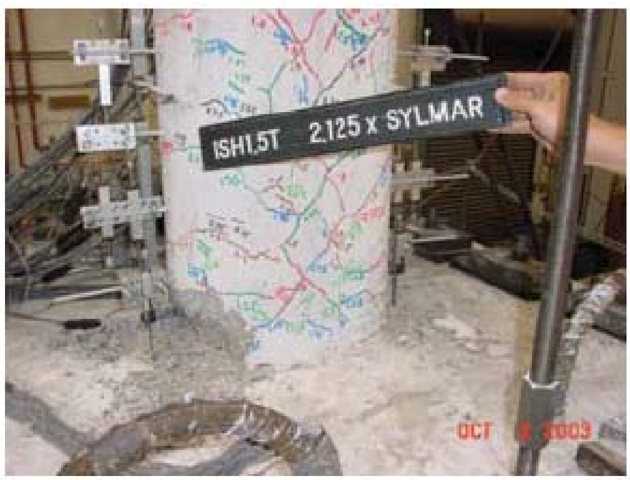

Figure 2.4- Damage state DS-4, spiral and longitudinal reinforcing bars visible.

Figure 2.5- Damage state DS-5, imminent failure, cracking inside concrete core.

2.3 Estimated Response Parameters

The two important response parameters considered for this project are the strain the in longitudinal reinforcing bars and the strain in the spiral hoop reinforcing bars. Strain in the reinforcing bars is important in determining possible repair methods because if the longitudinal bars reach a strain at which they could rupture, repair via FRP jackets is not an appropriate means of repair. Likewise, if the spiral hoops are at a strain level at which they yield, there is likely damage to the concrete core; therefore, repair with an FRP jacket is not appropriate.

The strain levels in the reinforcing bars can be estimated based on the visual classifications associated with each damage state. It is expected that the longitudinal reinforcing bars will yield once flexural cracks are visible, corresponding to DS-1. The longitudinal bars yield at this point because a plastic hinge is formed causing the bars to yield. For Grade 50 steel, the yield strain is between 0.002 and 0.0023. The longitudinal bars are not expected to reach the rupture strain, since ruptured bars are not observed in any of the damage state classifications. The rupture strain for Grade 50 steel is approximately 0.35.

Similarly, the spiral reinforcing bars would be expected to yield once the cracks in the concrete propagate into the column core, which corresponds to DS-5. Once the spiral reinforcement yields, FRP jacket repair is not possible.

The collaborators for this project at UNR measured the response parameters at each damage state level. Nineteen columns, containing Grade 50 steel reinforcement were subjected to seismic forces, which caused varying degrees of damage. Each column was visual inspected and classified into one of the five damage states described above. Response parameters, including maximum strain in longitudinal reinforcing bars and maximum strain in spiral reinforcing bars were measured in correlation with each damage state. Table 2.2 shows the range of strains associated with each damage state. The range was determined using the average value measured plus and minus the standard deviation.

Table 2.2- Damage state response parameters corresponding to each damage state.

Damage State Response Parameters						
Damage State	**Levels**	**DS-1**	**DS-2**	**DS-3**	**DS-4**	**DS-5**
Maximum Strain in Longitudinal Reinforcement	Lower Limit	0.00214	0.0132	0.0194	0.0231	0.0289
	Average	0.00917	0.0183	0.0263	0.0348	0.0425
	Upper Limit	0.0162	0.0235	0.0332	0.0466	0.0561
Maximum Strain in Spiral Reinforcement (microstrains)	Lower Limit	0.000172	0.000419	0.000632	0.00113	0.00143
	Average	0.000307	0.000694	0.00108	0.00167	0.00307
	Upper Limit	0.000442	0.000970	0.00152	0.00222	0.00472
FRP Jacket Application Appropriate?		No	Yes			No

The yellow highlighted strains correspond to steel that has yielded (i.e. strain greater than 0.002). These tests verify that the longitudinal reinforcement yields at DS-2 and that the spiral reinforcement yields at DS-5. Therefore, repair using FRP wet layup jackets is not an appropriate option if the column is characterized as DS-5.

Chapter 3

Confinement Models

3.1 Introduction

FRPs are gaining popularity for seismic retrofitting purposes. Even still a reliable and accurate model of concrete confinement is not agreed upon among experts. Many models of confined strength of concrete and associated strain have been proposed dating back to Richart *et al.*'s [35] original model in 1928 to the present day. FRP is an attractive alternative to steel jacketing because of its high strength-to-weight ratio and resistance to corrosion. FRPs are also appealing due to the relatively ease of construction through wet-layup and versatility of column geometry that can be easily wrapped.

The objective of this chapter is to review existing models for concrete confinement, compare results with experimental data, and finally propose a new model based on the experimental data.

A confined column has increased strength due to the passive restraint provided by the jacket, which restricts the transverse dilation of the column. Circular columns have the most efficient section for confining; therefore, this paper will focus on models and experimental data relating to circular columns only. For a circular column, the lateral confining pressure from the FRP jacket, f_l can be evaluated using a free body diagram as shown in Figure 3.1,

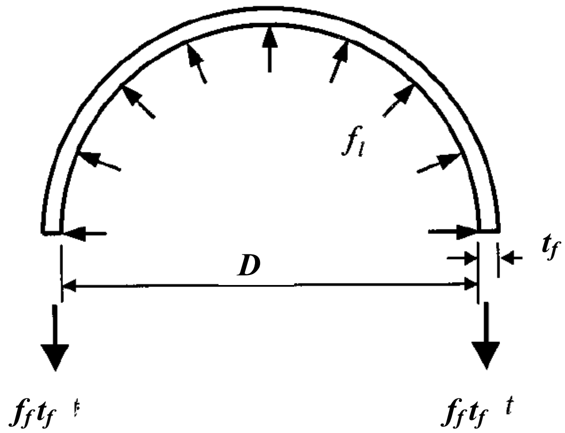

Figure 3.1- Free body diagram of forces from FRP jacket.

The forces from the free body diagram can be solved for, which results in the following relation:

$$f_l = \frac{2 f_f t_f}{D} \qquad \text{Equation 3.1}$$

where f_f is the tensile strength of the FRP in the hoop direction, t_f is the thickness of the FRP jacket, and *D* is the diameter of the confined concrete.

Since FRP behaves elastically, the inward pressure due to a jacket increases continuously. In contrast, steel provides a constant lateral confining pressure. Many early confinement models are based on the response of steel jacketed specimens. These models inaccurately model the elastic behavior of FRP. [35, 30, 9] In this paper, several models will be evaluated and compared to experimental data.

3.2 Review of Existing Models

3.2.1 Richart et al. (1928)

Richart *et al.* [35] performed tests on concrete cylinders with steel spiral reinforcement. The cylinders were subjected to confining hydrostatic pressure. Experimental results showed that the confining action increased the peak strength of the concrete as well as the ultimate strain. It was concluded that the strength of confined concrete at failure, f'_{cc}, could be expressed as a linear function of the lateral confining pressure, f_l, as follows:

$$f'_{cc} = f'_{co} + k_1 f_l \qquad \text{Equation 3.2}$$

where f'_{co} is the strength of the unconfined concrete and k_1 is the confinement effectiveness coefficient. Richart *et al.* [35] recommend using $k_1 = 4.1$.

Richart *et al.* [35] also proposed a model for the longitudinal strain of the confined concrete at failure, ε_{cc}:

$$\varepsilon_{cc} = \varepsilon_{co}\left(1 + k_2 \frac{f_l}{f'_{co}}\right) \qquad \text{Equation 3.3}$$

where $k_2 = 5k_1$ and ε_{co} is the longitudinal strain of the unconfined concrete at failure, which is typically assumed to be 0.002.

3.2.2 Fardis and Khalili (1982)

Fardis and Khalili [12] were the first to consider effects of concrete confinement due to FRP. Concrete cylinders were wrapped with glass FRP (GFRP) and then subjected to compression tests. Fardis and Khalili [12] adopted the general formula for strength of concrete as follows:

$$f'_{cc} = f'_{co}\left[1 + k_1 \frac{f_l}{f'_{co}}\right] \qquad \text{Equation 3.4}$$

From the test results, it was proposed that the coefficient of confinement, k_1, have the following value for a specimen wrapped in FRP:

$$k_1 = 3.7\left(\frac{f_l}{f'_{co}}\right)^{-0.14} \qquad \text{Equation 3.5}$$

This confinement coefficient yielded the following expression for the strength of confined concrete:

$$f'_{cc} = f'_{co} + 3.7\left(\frac{f_l}{f'_{co}}\right)^{0.86} \qquad \text{Equation 3.6}$$

For the corresponding ultimate strain at failure, Fardis and Khalili [12] suggested:

$$\varepsilon_{cc} = \varepsilon_{co} + 0.0005\left(\frac{E_f t_f}{D \cdot f'_{co}}\right) \qquad \text{Equation 3.7}$$

where E_f is the modulus of elasticity of the FRP jacket.

3.2.3 Mander et al. (1988)

Mander *et al.* [30] derived a model for confinement of concrete from experimental values of 17 steel spiral reinforced concrete cylinders with varying ratios of reinforcement. The following relation for strength of confined concrete was proposed:

$$f'_{cc} = f'_{co}\left(-1.254 + 2.254\sqrt{1 + 7.94\frac{f_{le}}{f'_{co}}} - 2\frac{f_{le}}{f'_{co}}\right) \quad \text{Equation 3.8}$$

where f_{le} is the effective lateral confining stress, which is affected by the steel reinforcement. For the purposes of this study, Mander *et al.* [30] suggested that f_{le} be assumed equal to f_l, the lateral confining stress, since steel reinforcement is not considered.

The longitudinal strain in the confined concrete was proposed as:

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + 5\left(\frac{f'_{cc}}{f'_{co}} - 1\right)\right] \quad \text{Equation 3.9}$$

3.2.4 Cusson and Paultre (1995)

Cusson and Paultre [9] developed a model based on tests conducted on 27 large scale square specimens made with high strength concrete with steel reinforcement. It was concluded that as the compressive strength of concrete increased, the effectiveness of the confining action and corresponding ductility decreased compared to normal strength concrete specimens. The model proposed was based on the 27 large scale specimens and 23 previously tested by the authors [9]. The regression analysis yielded the following expression for strength of confined concrete:

$$f'_{cc} = f'_{co}\left(1 + 2.1\left(\frac{f_{le}}{f'_{co}}\right)^{0.7}\right) \quad \text{Equation 3.10}$$

where f_{le} is the effective lateral confining stress, which is affected by the steel reinforcement. As previously discussed, f_{le} is assumed to equal f_l.

It was also determined that the ultimate strain in the confined concrete using steel ties could be expressed as:

$$\varepsilon_{cc} = \varepsilon_{co} + 0.21\left(\frac{f_{le}}{f'_{co}}\right)^{1.7} \quad \text{Equation 3.11}$$

3.2.5 Karbhari and Gao (1997)

Karbhari and Gao [20] developed two models to predict the effect of concrete confined by FRP. The first model, an empirical approach, modified Richart *et al.*'s [35] original model for concrete confined by steel in order to apply to FRP confined concrete. This approach yielded the following expressions for strength of confined concrete and ultimate strain:

$$f'_{cc} = f'_{co}\left(1 + 2.1\left(\frac{f_l}{f'_{co}}\right)^{0.87}\right) \quad \text{Equation 3.12}$$

$$\varepsilon_{cc} = \varepsilon_{co} + 0.01\left(\frac{f_l}{f'_{co}}\right) \quad \text{Equation 3.13}$$

An alternate approach Karbhari and Gao [20] took was a simplistic composite analysis. This analysis generated the following expressions:

$$f'_{cc} = f'_{co} + 3.1 f'_{co} \nu_c \frac{2t}{D}\frac{E_f}{E_c} + \frac{2 f_f t_f}{D} \qquad \text{Equation 3.14}$$

where E_f is the modulus of elasticity of the FRP in the hoop direction and E_c is the modulus of elasticity of the concrete and:

$$\varepsilon_{cc} = 1 - \frac{1.004\left[1 - \frac{f'_{co}}{E_{eff}} - 4.1 f'_{co} \nu_c \frac{2t_f}{D}\frac{E_f}{E_c E_{eff}}\right]}{\left(1 + \varepsilon_f\right)^2} \qquad \text{Equation 3.15}$$

where υ_c is Poisson's ratio, ε_f is the strain in the FRP jacket and E_{eff} is the effective modulus of elasticity of the FRP in the hoop direction, which can be calculated using the following equation,

$$E_{eff} = \frac{E_f A_f + E_c A_c}{A_f + A_c} \qquad \text{Equation 3.16}$$

where A_f and A_c are the cross sectional areas of the FRP jacket and the concrete, respectively. ε_{co} was assumed to equal 0.002.

3.2.6 Miyauchi et al. (1998)

Miyauchi *et al.* [34] conducted compression tests on 10 concrete cylinders wrapped in Carbon FRP (CFRP). The specimens were comprised of two unconfined concrete strengths, f'_{co} = 30 MPa and f'_{co} = 50 MPa. Richart *et al.*'s [35] model was altered by adding an "effectiveness coefficient", k_e with the following relationship:

$$f'_{cc} = f'_{co} + 4.1 k_e f_l \qquad \text{Equation 3.17}$$

Through calibration from the experimental data obtained, k_e was found to equal 0.85. Substituting k_e into Equation 3.17 yielded:

$$f'_{cc} = f'_{co} + 3.485 f_l \qquad \text{Equation 3.18}$$

Using the same set of experimental data, two models were derived for ultimate strain, dependent on the unconfined concrete strength, f'_{co}.

$$\varepsilon_{cc} = \varepsilon_{co} + 10.6\left(\frac{f_l}{f'_{co}}\right)^{0.373} \qquad \text{Equation 3.19}$$

for f'_{co} = 30 MPa and:

$$\varepsilon_{cc} = \varepsilon_{co} + 10.5\left(\frac{f_l}{f'_{co}}\right)^{0.525} \qquad \text{Equation 3.20}$$

for f'_{co} = 50 MPa.

The authors provided no suggestions for situations of unconfined concrete strengths other than 30 MPa or 50 MPa.

3.2.7 Kono et al. (1998)

Kono *et al.*'s [21] models differ from most confinement models because they linearly relate the strengthening ratio, $\frac{f'_{cc}}{f'_{co}}$ directly to the confining pressure, f_l. Most other models relate $\frac{f'_{cc}}{f'_{co}}$ to the confinement ratio, $\frac{f_l}{f'_{co}}$. The authors performed a regression analysis on the results from the tests conducted to obtain the following expression for confined concrete strength:

$$f'_{cc} = f'_{co} + 0.0572 f'_{co} f_l \qquad \text{Equation 3.21}$$

Likewise, for the ultimate strain at failure, the authors linearly relate the strain ratio, $\frac{\varepsilon_{cc}}{\varepsilon_{co}}$ to f_l, whereas, other models relate $\frac{\varepsilon_{cc}}{\varepsilon_{co}}$ to the strengthening ratio, $\frac{f_l}{f'_{co}}$. The regression analysis yielded the following for ultimate strain:

$$\varepsilon_{cc} = \varepsilon_{co} + 0.28 \varepsilon_{co} f_l \qquad \text{Equation 3.22}$$

3.2.8 Samaan et al. (1998)

Samaan *et al.* [39] performed compression tests on 30 cylinders wrapped in FRP sheets of various thicknesses. Richart *et al.*'s [35] original model for steel confined concrete was used as a fundamental model. Based on the experimental data, the authors recommended a coefficient of confinement, $k_1 = 6.0 f_l^{-0.3}$. This yielded the following relationship:

$$f'_{cc} = f'_{co} + 6.0 f_l^{0.7} \qquad \text{Equation 3.23}$$

The ultimate strain at failure was determined from the bilinear nature of the stress-strain curve developed by the authors, where E_2 is the slope of the second portion of the curve. Ultimate strain could be calculated as:

$$\varepsilon_{cc} = \frac{\left(f'_{cc} - f_0\right)}{E_2} \qquad \text{Equation 3.24}$$

where $f_0 = 0.872 f'_{co} + 0.371 f_l + 6.258$ and $E_2 = 245.61 f'^{\,0.2}_{co} + 1.3456 \dfrac{E_f t_f}{D}$.

3.2.9 Spoelstra and Monti (1999)

Spoelstra and Monti [45] modified Mander *et al.*'s [30] model to account for the increasing inward pressure caused by an FRP jacket. The resulting model for circular specimens wrapped with FRP used an iterative procedure. Spoelstra and Monti [45] derived an approximation of the iterative model by identifying key independent parameters, f_l, E_c, and ε_{com}. Then a regression analysis was applied to incremental results from the iterative model. The resulting approximation assumed that the strain at unconfined peak stress, ε_{co} = 0.002. This assumption yielded:

$$f'_{cc} = f'_{co}\left[0.2 + 3.0\sqrt{\frac{f_l}{f'_{co}}}\right] \qquad \text{Equation 3.25}$$

and for the ultimate strain:

$$\varepsilon_{cc} = \varepsilon_{co}\left(2 + 1.25\frac{E_c}{f'_{co}}\varepsilon_f\sqrt{\frac{f_l}{f'_{co}}}\right) \qquad \text{Equation 3.26}$$

where ε_f is the ultimate strain in the FRP jacket.

3.2.10 Toutanji (1999)

Toutanji [48] conducted confinement test on 18 cylindrical specimens wrapped in CFRP and GFRP. A regression analysis was run on the experimental data obtained and resulted in the confining coefficient *k* to be the following:

$$k_1 = 3.5\left(\frac{f_l}{f'_{co}}\right)^{-0.15}$$ Equation 3.27

By substituting k_1 into Richart *et al.*'s [35] original model, Equation 3.2, Toutanji [48] concluded that:

$$f'_{cc} = f'_{co}\left(1 + 3.5\left(\frac{f_l}{f'_{co}}\right)^{0.85}\right)$$ Equation 3.28

For modeling of the ultimate strain in the confined concrete, Toutanji [48] suggested an expression which is derived from Richart *et al.*'s [35] model, but accounts for the dependent relationship on lateral strain in the composite, ε_f. Toutanji [48] expressed the constant, k_2 as:

$$k_2 = 310.57\varepsilon_f + 1.90$$ Equation 3.29

When substituted into Richart *et al.*'s [35] model, the following expression was obtained:

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + (310.57\varepsilon_f + 1.90)\left(\frac{f'_{cc}}{f'_{co}} - 1\right)\right]$$ Equation 3.30

where ε_f ranges from the unconfined value of 0.002 to the ultimate value at failure.

3.2.11 Saafi et al. (1999)

Saafi *et al.* [38] proposed a model exactly the same as Toutanji's [48] except that it was calibrated based on tests performed on FRP tube encased specimens instead of

FRP wrapped specimens. 18 concrete cylinders were tested with CFRP or GFRP tubes. Different coefficients than the FRP wrapped case were obtained because of the difference in bond strength between the FRP and concrete. The bond to the concrete cylinder with a wrapped specimen is stronger than that of a tube encased specimen. The following confining coefficients were obtained from the calibrations of the test results from the tube encased specimens:

$$k_1 = 2.2\left(\frac{f_l}{f'_{co}}\right)^{-0.16} \qquad \text{Equation 3.30}$$

which yielded the following for the ultimate strength of confined concrete:

$$f'_{cc} = f'_{co}\left(1 + 2.2\left(\frac{f_l}{f'_{co}}\right)^{0.84}\right) \qquad \text{Equation 3.31}$$

Similarly, the coefficient of confinement for the ultimate strain expression was determined to follow the equation:

$$k_2 = 537\varepsilon_f + 2.6 \qquad \text{Equation 3.32}$$

which yielded the following for the ultimate strain in the confined concrete:

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + (537\varepsilon_f + 2.60)\left(\frac{f'_{cc}}{f'_{co}} - 1\right)\right] \qquad \text{Equation 3.33}$$

3.2.12 Xiao and Wu (2000)

Xiao and Wu [53] tested 36 concrete cylinders wrapped in CFRP jackets. Richart *et al.*'s [35] model was used as a base relationship as follows:

$$f'_{cc} = \alpha \cdot f'_{co} + k_1 f_l \qquad \text{Equation 3.34}$$

where in Richart *et al.*'s [35] model, $\alpha = 1$ and $k_1 = 4.1$. Based on the experimental data obtained, Xiao and Wu [53] suggested the average value of 1.1 for α and the following expression for k_1:

$$k_1 = 4.1 - 0.75\frac{f_c'^2}{C_j} \qquad \text{Equation 3.35}$$

where C_j is the confinement modulus and could be expressed as:

$$C_j = \frac{2tE_f}{D} \qquad \text{Equation 3.36}$$

By substituting into the original equation for strength of confined concrete from Richart *et al.* [35], the following relation was obtained:

$$f'_{cc} = f'_{co}\left[1.1 + \left(4.1 - 0.75\left(\frac{f_{co}'^{2} D}{2E_f t_f}\right)\right)\frac{f_l}{f'_{co}}\right] \qquad \text{Equation 3.37}$$

3.2.13 Lam and Teng (2002)

Lam and Teng [24] conducted over 200 tests on concrete cylinders to obtain strength data. The strength database included specimens of varying concrete strength, diameter, length to diameter ratios, and FRP type (CFRP, GFRP, and AFRP). Based on the strength database, Lam and Teng [24] recommended the following simple model for strength of confined concrete:

$$f'_{cc} = f'_{co} + 2.0 f_l \qquad \text{Equation 3.38}$$

For modeling of the ultimate strain of confined concrete, Lam and Teng [24] compiled a strain database. The strain database was made up of 171 specimens. From this strain database, Lam and Teng [24] developed the following relation for the strain at ultimate stress:

$$\varepsilon_{cc} = \varepsilon_{co}\left[2 + k_2\left(\frac{f_l}{f'_{co}}\right)\right] \qquad \text{Equation 3.39}$$

where k_2 is the strain enhancement coefficient, which depends on the type of FRP confinement. For CFRP, k_2 was recommended as 15.

3.2.14 De Lorenzis and Tepfers (2003)

De Lorenzis and Tepfers [10] compiled a data set of about 180 experimental tests from a variety of authors. All specimens were concrete cylinders of varying size. The data set included specimens with CFRP, GFRP, and aramid FRP (AFRP) wrapped and tube encased cylinders of varying thicknesses. Using the collected data, De Lorenzis and Tepfers [10] proposed two new models for the ultimate strain in confined concrete. The models followed the form:

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + c_1\left(\frac{f_l}{f'_{co}}\right)^{c_2} E_l^{\,c_3}\right] \qquad \text{Equation 3.40}$$

where c_1, c_2, and c_3 were chosen by minimizing the absolute error associated with the predicted values and the experimental results. E_l is the confinement modulus and was calculated as:

$$E_l = \frac{2E_f t_f}{D}$$ Equation 3.41

For FRP wrapped specimens, the following expression was proposed:

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + 26.2\left(\frac{f_l}{f'_{co}}\right)^{0.8}\left[\frac{E_f t_f}{D}\right]^{-0.148}\right]$$ Equation 3.42

and for FRP tube encased specimens the following expression was proposed:

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + 26.2\left(\frac{f_l}{f'_{co}}\right)^{0.68}\left(\frac{E_f t_f}{D}\right)^{-0.127}\right]$$ Equation 3.43

3.2.15 Wu et al. (2003)

Wu *et al.* [52] compared data from over 200 specimens comprised of concrete cylinders confined with FRP wraps and tubes. The data set included many varying parameter including concrete strength, specimen dimensions, type of FRP (CFRP, GFRP, and AFRP), and tensile strength of FRP. Wu *et al.* [52] proposed a set of equations for the strength of confined concrete, f'_{cc}, which is dependent on the type of jacket and the method used to determine the strength of the FRP, f_f.

For concrete confined by FRP sheets, where the strength of FRP is determined through tensile coupon tests, Wu *et al.* [52] suggested the following model:

$$f'_{cc} = f'_{co} + 2.0 f_l$$ Equaiton 3.44

For concrete confined by FRP sheets, where the strength of FRP was given by the manufacturer, the following relation was suggested:

$$f'_{cc} = f'_{co} + 3.0 f_l$$ Equation 3.45

and for concrete confined by FRP tubes, where the strength of FRP was determined through tensile coupon tests, Wu *et al.* [52] recommended the following relation:

$$f'_{cc} = f'_{co} + 2.5 f_l$$ Equation 3.46

To determine the strain in the confined concrete, Wu *et al.* [52] recommended a relation between the strain and Poisson's ratio of the confined concrete:

$$\varepsilon_{cc} = \frac{\varepsilon_f}{\nu_u}$$ Equation 3.47

where ν_u is Poisson's ratio, which Wu *et al*. [52] suggested was dependent on the type of FRP and jacket (i.e. wrap or tube). For normal modulus CFRP, GFRP, and AFRP sheet confined concrete, Poisson's ratio was expressed as:

$$\nu_u = 0.56\left(\frac{f_l}{f'_{co}}\right)^{-0.66}$$ Equaiton 3.48

For GFRP or CFRP tube confined concrete, Poisson's ratio was:

$$\nu_u = 0.31\left(\frac{f_l}{f'_{co}}\right)^{-0.44}$$

Equation 3.49

and for high modulus FRP, Poisson's ratio was described as:

$$\nu_u = 0.56k_f\left(\frac{f_l}{f'_{co}}\right)^{-0.66}$$ Equation 3.50

where k_f is the influence coefficient for high modulus FRP and is equal to 1.0, when $E_f \leq$ 250 GPa and $k_f = \sqrt{\frac{250}{E_f}}$ when $E_f > 250$ GPa.

3.2.16 Xiao and Wu (2003)

Xiao and Wu [54] analyzed the results from 243 experimental specimens that were comprised of nine different types of FRP confinement, including CFRP and GFRP wrapped and tube encased specimens of varying thicknesses and unconfined concrete strengths. The model proposed is the same structure as previously proposed by Xiao and Wu [53]. The new model has more accurately calibrated constants based on the increase database of experimental specimens. Richart *et al.*'s [35] model was used again as a base relationship as follows:

$$f'_{cc} = \alpha \cdot f'_{co} + k_1 f_l$$ Equation 3.51

where in Richart *et al.*'s [35] model, $\alpha = 1$ and $k_1 = 4.1$. Based on the experimental data obtained, Xiao and Wu [54] suggest the average value of 1.1 for α and the following expression for k_1:

$$k_1 = 4.1 - 0.45\frac{f_c'^{1.4}}{C_j}$$ Equation 3.52

This resulted in the following equation for strength of confined concrete:

$$f'_{cc} = f'_{co}\left[1.1 + \left(4.1 - 0.45\left(\frac{f'^{2}_{co} D}{2E_f t_f}\right)^{1.4}\right)\frac{f_l}{f'_{co}}\right] \qquad \text{Equation 3.53}$$

3.2.17 Bisby et al. (2005)

Bisby *et al.* [5] analyzed several existing models based on an experimental data set of approximately 200 specimens from 20 different authors. The data set contained all three types of FRP (CFRP, GFRP, and AFRP). Specimens varied in size and concrete strength. Three models for strength of confined concrete were suggested based on regression analysis of the existing data sets. The three models express the coefficient of confinement, k_1, as either a constant, an exponential function of the confining stress ratio, $\frac{f_l}{f'_{co}}$, or an exponential function of the ultimate confining stress, f_l. The three proposed models are as follows:

$$f'_{cc} = f'_{co}\left[1 + 2.425\frac{f_l}{f'_{co}}\right] \qquad \text{Equation 3.54}$$

$$f'_{cc} = f'_{co}\left[1 + 2.217\left(\frac{f_l}{f'_{co}}\right)^{0.911}\right] \qquad \text{Equation 3.55}$$

$$f'_{cc} = f'_{co} + 3.587 f_l^{0.840} \qquad \text{Equation 3.56}$$

Bisby *et al*. [5] recommend that all three models accurately predict the confining effect of FRP jackets. To model the ultimate strain of FRP confined concrete, Bisby *et al*. [5] recommend a model that is dependent on the type of FRP used:

$$\varepsilon_{cc} = \varepsilon_{co} + k_2 \left(\frac{f_l}{f'_{co}} \right)$$ Equation 3.57

where $k_2 = 0.0240$, 0.0137, or 0.0536 for CFRP, GFRP, or AFRP, respectively.

3.2.18 Guralnick and Gunawan (2006)

Guralnick and Gunawan [14] developed a model for strength of confined concrete analytically by modifying Mohr's strength theory. From previous tests conducted by the authors [14], it was determined that FRP confined concrete, which is classified as passive confinement was of the order of 2 for the Mohr's strength envelope. By analyzing Mohr's strength envelope circles at n = 2, Guralnick and Gunawan [14] developed an expression for f'_{cc} as follows:

$$f'_{cc} = f'_{co} \left[0.616 + \frac{f_l}{f'_{co}} + 1.57 \left(\frac{f_l}{f'_{co}} + 0.06 \right)^{0.5} \right]$$ Equation 3.58

The above model assumed the uniaxial tension of concrete, $f'_t \cong 0.06 f'_{co}$ for a range of concrete strength of 3000 psi to 8000 psi.

3.2.19 Youssef et al. (2007)

Youssef *et al.* [55] performed tests on 87 large scale specimens (16 inch by 32 inch) and 30 standard sized specimens (6 inch by 12 inch) with CFRP and GFRP jackets. Based on 63 specimens for circular columns, Youssef *et al.* [55] calibrated an equation for confinement as follows:

$$f'_{cc} = f'_{co}\left(1 + 2.109\left(\frac{f_l}{f'_{co}}\right)^{0.783}\right)$$ Equation 3.59

The model for ultimate strain was calibrated as:

$$\varepsilon_{cc} = 0.003368 + 0.259\left(\frac{f_l}{f'_{co}}\right)\left(\frac{f_f}{E_f}\right)^{0.5}$$ Equation 3.60

3.2.20 Girgin (unpublished)

Girgin [13] assembled a wide range of experimental data from multiple sources. Data associated with lower levels of confinement, such as specimens wrapped in only one layer of FRP was disregarded. This yielded a set of 102 data points. Girgin [13] first evaluated the data with regards to Richart *et al.*'s [35] original model for strength of confined concrete, which was based on the Mohr-Coulomb failure criterion. Girgin [13] proposed the following expression for the confinement coefficient, *k*:

$$k = 2.109\left(\frac{f_l}{f'_{co}}\right)^{-0.217}$$ Equation 3.61

This resulted in the following equation for ultimate strength:

$$f'_{cc} = f'_{co} + 2.109 f'_{co}\left(\frac{f_l}{f'_{co}}\right)^{0.783}$$ Equation 3.62

Girgin [13] proposed an alternate model, which was based on the Hoek-Brown strength criteria. The Hoek-Brown failure for confined concrete could be expressed as:

$$f'_{cc} = f_l + \sqrt{s \cdot f'^{2}_{co} + m \cdot f'_{co} f_l} \qquad \text{Equation 3.63}$$

Based on the experimental data set, the constants *s* and *m* were determined to equal 1 and 3.5, respectively for FRP wrapped concrete cylinders.

The following Table 3.1 summarizes the confinement models discussed.

Table 3.1- Summary of existing confinement models.

Author	Predicted f'_{cc}	Predicted ε_{cc}	Comments
Richart *et al.* (1928)	$f'_{cc}=f'_{co}+4.1f_l$	$\varepsilon_{cc}=\varepsilon_{co}\left(1+20.5\frac{f_l}{f'_{co}}\right)$	
Fardis and Khalili (1982)	$f'_{cc}=f'_{co}+3.7\left(\frac{f_l}{f'_{co}}\right)^{0.86}$	$\varepsilon_{cc}=\varepsilon_{co}+0.0005\left(\frac{E_f t_f}{D\cdot f'_{co}}\right)$	
Mander *et al.* (1988)	$f'_{cc}=f'_{co}\left(-1.254+2.254\sqrt{1+7.94\frac{f_l}{f'_{co}}}-2\frac{f_l}{f'_{co}}\right)$	$\varepsilon_{cc}=\varepsilon_{co}\left[1+5\left(\frac{f'_{cc}}{f'_{co}}-1\right)\right]$	
Cusson and Paultre (1995)	$f'_{cc}=f'_{co}\left(1+2.1\left(\frac{f_l}{f'_{co}}\right)^{0.7}\right)$	$\varepsilon_{cc}=\varepsilon_{co}+0.21\left(\frac{f_l}{f'_{co}}\right)^{1.7}$	
Karbhari and Gao (1997)	$f'_{cc}=f'_{co}\left(1+2.1\left(\frac{f_l}{f'_{co}}\right)^{0.87}\right)$	$\varepsilon_{cc}=\varepsilon_{co}+0.01\left(\frac{f_l}{f'_{co}}\right)$	Empirical Form
	$f'_{cc}=f'_{co}+3.1f'_{co}\nu_c\frac{2t_f}{D}\frac{E_f}{E_c}+\frac{2\sigma_t t_f}{D}$	$\varepsilon_{cc}=1-\frac{1.004\left[1-\frac{f'_{co}}{E_{eff}}-4.1f'_{co}\nu_c\frac{2t_f}{D}\frac{E_f}{E_c E_{eff}}\right]}{\left(1+\varepsilon_f\right)^2}$	Simplistic Composite Analysis
Miyauchi *et al.* (1997)	$f'_{cc}=f'_{co}+3.485f_l$	$\varepsilon_{cc}=\varepsilon_{co}+10.6\left(\frac{f_l}{f'_{co}}\right)^{0.373}$	For $f'_{co}=30$ MPa
		$\varepsilon_{cc}=\varepsilon_{co}+10.5\left(\frac{f_l}{f'_{co}}\right)^{0.525}$	For $f'_{co}=50$ MPa

Table 3.1- Summary of existing confinement models. Continued.

Author	Predicted f'_{cc}	Predicted ε_{cc}	Comments
Kono *et al.* (1998)	$f'_{cc} = f'_{co} + 0.0572 f'_{co} f_l$	$\varepsilon_{cc} = \varepsilon_{co} + 0.28 \varepsilon_{co} f_l$	
Samaan *et al.* (1998)	$f'_{cc} = f'_{co} + 6.0 f_l^{0.7}$	$\varepsilon_{cc} = \dfrac{\left(f'_{cc} - f_0\right)}{E_2}$ Where: $f_0 = 0.872 f'_{co} + 0.371 f_l + 6.258$ $E_2 = 245.61 {f'_{co}}^{0.2} + 1.3456 \dfrac{E_f t_f}{D}$	
Spoelstra and Monti (1999)	$f'_{cc} = f'_{co}\left[0.2 + 3.0\sqrt{\dfrac{f_l}{f'_{co}}}\right]$	$\varepsilon_{cc} = \varepsilon_{co}\left(2 + 1.25 \dfrac{E_c}{f'_{co}} \varepsilon_f \sqrt{\dfrac{f_l}{f'_{co}}}\right)$	Approximation using assumption that ε_{co} = 0.002
Toutanji (1999)	$f'_{cc} = f'_{co}\left(1 + 3.5\left(\dfrac{f_l}{f'_{co}}\right)^{0.85}\right)$	$\varepsilon_{cc} = \varepsilon_{co}\left[1 + \left(310.57 \varepsilon_f + 1.90\right)\left(\dfrac{f'_{cc}}{f'_{co}} - 1\right)\right]$	
Saafi *et al.* (1999)	$f'_{cc} = f'_{co}\left(1 + 2.2\left(\dfrac{f_l}{f'_{co}}\right)^{0.84}\right)$	$\varepsilon_{cc} = \varepsilon_{co}\left[1 + \left(537 \varepsilon_f + 2.60\right)\left(\dfrac{f'_{cc}}{f'_{co}} - 1\right)\right]$	
Xiao and Wu (2000)	$f'_{cc} = f'_{co}\left[1.1 + \left(4.1 - 0.75\left(\dfrac{{f'_{co}}^2 D}{2 E_f t_f}\right)\right)\dfrac{f_l}{f'_{co}}\right]$	--	
Lam and Teng (2002)	$f'_{cc} = f'_{co} + 2.0 f_l$	$\varepsilon_{cc} = \varepsilon_{co}\left[2 + k_2\left(\dfrac{f_l}{f'_{co}}\right)\right]$	k_2 depends on type of FRP. For CFRP, k_2 = 15

Table 3.1- Summary of existing confinement models. Continued.

Author	**Predicted f'_{cc}**	**Predicted ε_{cc}**	**Comments**
De Lorenzis and Tepfers (2003)	--	$\varepsilon_{cc} = \varepsilon_{co}\left[1+26.2\left(\frac{f_l}{f'_{co}}\right)^{0.8}\left[\frac{E_f t_f}{D}\right]^{-0.148}\right]$	For FRP wraps
		$\varepsilon_{cc} = \varepsilon_{co}\left[1+26.2\left(\frac{f_l}{f'_{co}}\right)^{0.68}\left(\frac{E_f t_f}{D}\right)^{-0.127}\right]$	For FRP tubes
Wu *et al.* (2003)	$f'_{cc} = f'_{co} + 2.0 f_l$	$\varepsilon_{cc} = \frac{\varepsilon_f}{\nu_u}$ For normal modulus CFRP, GFRP and AFRP sheets: $\nu_u = 0.56\left(\frac{f_l}{f'_{co}}\right)^{-0.66}$ For GFRP or CFRP tubes: $\nu_u = 0.31\left(\frac{f_l}{f'_{co}}\right)^{-0.44}$ For high modulus FRP: $\nu_u = 0.56 k_f \left(\frac{f_l}{f'_{co}}\right)^{-0.66}$ Where, $k_f = 1$ if $E_f \leq 250$ GPa $k_f = \sqrt{\frac{250}{E_f}}$ if $E_f > 250$ GPa	f'_{cc} for wraps where strength of FRP is determined by tensile coupon test
	$f'_{cc} = f'_{co} + 3.0 f_l$		f'_{cc} for wraps where strength of FRP is obtained by value provided by manufacturer
	$f'_{cc} = f'_{co} + 2.5 f_l$		f'_{cc} for tubes where strength of FRP is determined by tensile coupon test

Table 3.1- Summary of existing confinement models. Continued.

Author	Predicted f'_{cc}	Predicted ε_{cc}	Comments
Xiao and Wu (2003)	$f'_{cc} = f'_{co}\left[1.1 + \left(4.1 - 0.45\left(\frac{f'^{\,2}_{co} D}{2E_f t_f}\right)^{1.4}\right)\frac{f_l}{f'_{co}}\right]$	--	
Bisby *et al.* (2005)	$f'_{cc} = f'_{co}\left[1 + 2.425\frac{f_l}{f'_{co}}\right]$	$\varepsilon_{cc} = \varepsilon_{co} + k_2\left(\frac{f_l}{f'_{co}}\right)$ CFRP: $k_2 = 0.0240$ GFRP: $k_2 = 0.0137$ AFRP: $k_2 = 0.0536$	
	$f'_{cc} = f'_{co}\left[1 + 2.217\left(\frac{f_l}{f'_{co}}\right)^{0.911}\right]$		
	$f'_{cc} = f'_{co} + 3.587 f_l^{\,0.840}$		
Guralnick and Gunawan (2006)	$f'_{cc} = f'_{co}\left[0.616 + \frac{f_l}{f'_{co}} + 1.57\left(\frac{f_l}{f'_{co}} + 0.06\right)^{0.5}\right]$	--	
Youssef *et al.* (2007)	$f'_{cc} = f'_{co}\left(1 + 2.25\left(\frac{f_l}{f'_{co}}\right)^{1.25}\right)$	$\varepsilon_{cc} = 0.003368 + 0.259\left(\frac{f_l}{f'_{co}}\right)\left(\frac{f_f}{E_f}\right)^{0.5}$	
Girgin	$f'_{cc} = f'_{co} + 2.109 f'_{co}\left(\frac{f_l}{f'_{co}}\right)^{0.783}$	--	Mohr-Coulomb Model
	$f'_{cc} = f_l + \sqrt{f'^{\,2}_{co} + 3.5 f'_{co} f_l}$	--	Hoek-Brown Strength Model

3.3 Experimental Data

The experimental data set was comprised of over 300 specimen results previously conducted by several authors [6, 11, 15, 16, 17, 19, 20, 22, 23, 27, 28, 31, 33, 34, 38, 39, 44, 47, 49, 51, 53, 56]. The data set represented varying concrete strengths, specimen dimensions, jacket thicknesses, and types of FRP. Table 3.2 summarizes the range of data used. For all data sets, unless otherwise stated in the study, the elastic modulus of concrete, E is assumed to equal $4700\sqrt{f'_{co}}$, where f'_{co} is the compressive strength of unconfined concrete in MPa.

Table 3.2- Summary of experimental data.

Author	Number of Specimens	Type of FRP	Type of Jacket	Range of Concrete Strength (MPa)	Range of Jacket Thickness (mm)	Diameter of Specimen (mm)	Length of Specimen (mm)
Harmon and Slattery (1992) [15]	4	CFRP	Wrap	41.0	0.09-0.69	51	102
Demers and Neale (1994) [23]	8	GFRP/CFRP	Wrap	32.2-43.7	0.35-1.05	152	305
Howie and Karbhari (1994) [16]	21	CFRP	Wrap	38.6	0.31-1.22	152	305
Howie and Karbhari (1995) [17]	9	CFRP	Wrap	38.38	0.33-1.32	152	305
Nanni and Bradford (1995) [28]	35	AFRP/GFRP	Wrap/Tube	35.6-45.5	0.30-2.4	150	300
Soudki and Green (1996) [23]	2	CFRP	Wrap	46.0	0.16-0.32	152	305
Karbhari and Gao (1997) [20]	4	CFRP	Wrap	18.01	1.55-5.31	152	305
Mastrapa (1997) [23]	10	GFRP	Wrap/Tube	29.8-31.2	0.61-3.07	153	305
Miyauchi *et al.* (1997) [34]	10	CFRP	Wrap	31.2-51.9	0.11-0.33	150	300
Wantanable *et al.* (1997) [51]	9	CFRP/AFRP	Wrap	30.2	0.14-0.67	100	200
Harries *et al.* (1998) [23]	6	GFRP/CFRP	Wrap	26.2	1.0-2.0	152	610
Demers and Neale (1999) [11]	15	CFRP	Wrap	33.9-47.7	0.9	300	1200
Mirmiran *et al.* (1998) [33]	22	GFRP	Wrap	29.6-32.0	1.45-2.97	153	305
Samaan *et al.* (1998) [39]	22	GFRP	Tube	29.64-31.97	1.44-2.97	153	305

Table 3.2- Summary of experimental data. Continued.

Author	Number of Specimens	Type of FRP	Type of Jacket	Range of Concrete Strength (MPa)	Range of Jacket Thickness (mm)	Diameter of Specimen (mm)	Length of Specimen (mm)
Toutanji and Balaguru (1998) [47]	3	GFRP/CFRP	Wrap	31.8	0.11-0.165	76	305
Matthys *et al.* (1999) [31]	4	CFRP	Wrap	34.9	0.12-0.24	150	300
Miyauchi *et al.* (1999) [23]	9	CFRP	Wrap	23.6-33.7	0.11-0.33	150	300
Saafi *et al.* (1999) [38]	6	GFRP/CFRP	Tube	35.0	0.11-2.4	152	435
Le Tegola and Manni (1999) [23]	10	GFRP	Tube	25.6	3.34-5.03	150	300
Kshirasgar *et al.* (2000) [22]	3	GFRP	Wrap	38.0-39.5	1.42	102	204
Shahawy *et al.* (2000) [44]	9	CFRP	Wrap	19.4-49.0	0.5-2.0	153	305
Xiao and Wu (2000) [53]	27	CFRP	Wrap	33.7-55.2	0.38-1.14	152	305
Zhang *et al.* (2000) [56]	5	GFRP/CFRP	Wrap	34.3	1.00-2.83	152	305
Campione and Miraglia (2001) [6]	18	CFRP	Wrap	20.05-60.00	0.13-5.04	150-300	305
Lin and Chen (2001) [27]	11	GFRP/CFRP	Wrap	32.7	0.5-1.8	120	240
Karabinis and Rousakis (2002) [19]	18	CFRP	Wrap	43.5	0.117-0.351	200	320
Theriault *et al.* (2004) [49]	8	GFRP/CFRP	Wrap	37	0.165-3.9	51-304	102-1824

3.4 Strength Evaluation

Figures 3.2 through 3.5 show the experimental data versus the predicted data from each strength model considered. The straight line represents perfect agreement between predicted and experimental strengths. Any data points that fall below this line are considered conservative (i.e. the predicted strength is less than the experimental strength). Likewise, any data points that fall above this line are considered non-conservative (i.e. the predicted strength is greater than the experimental strength).

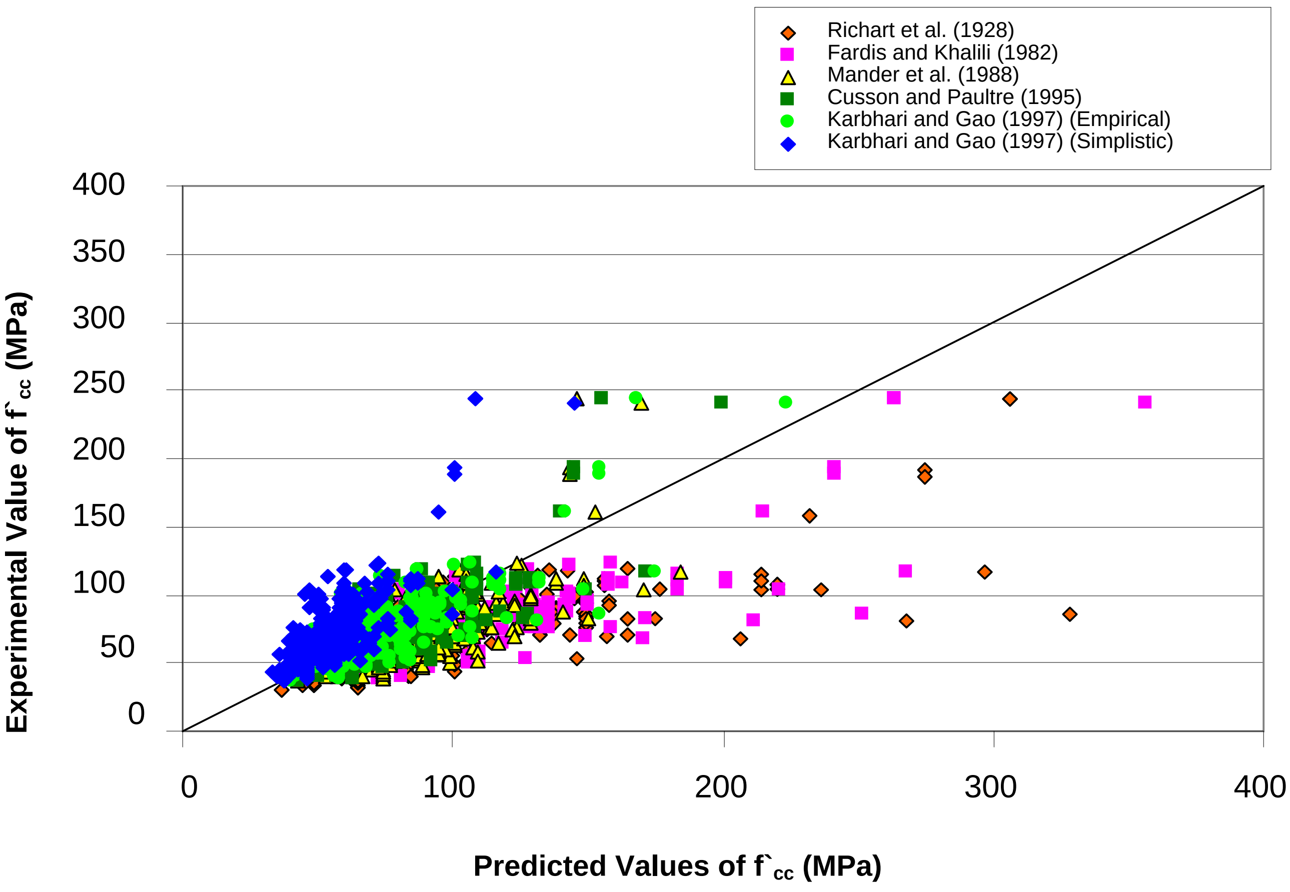

Figure 3.2- Comparison of predictive strength models and experimental data.

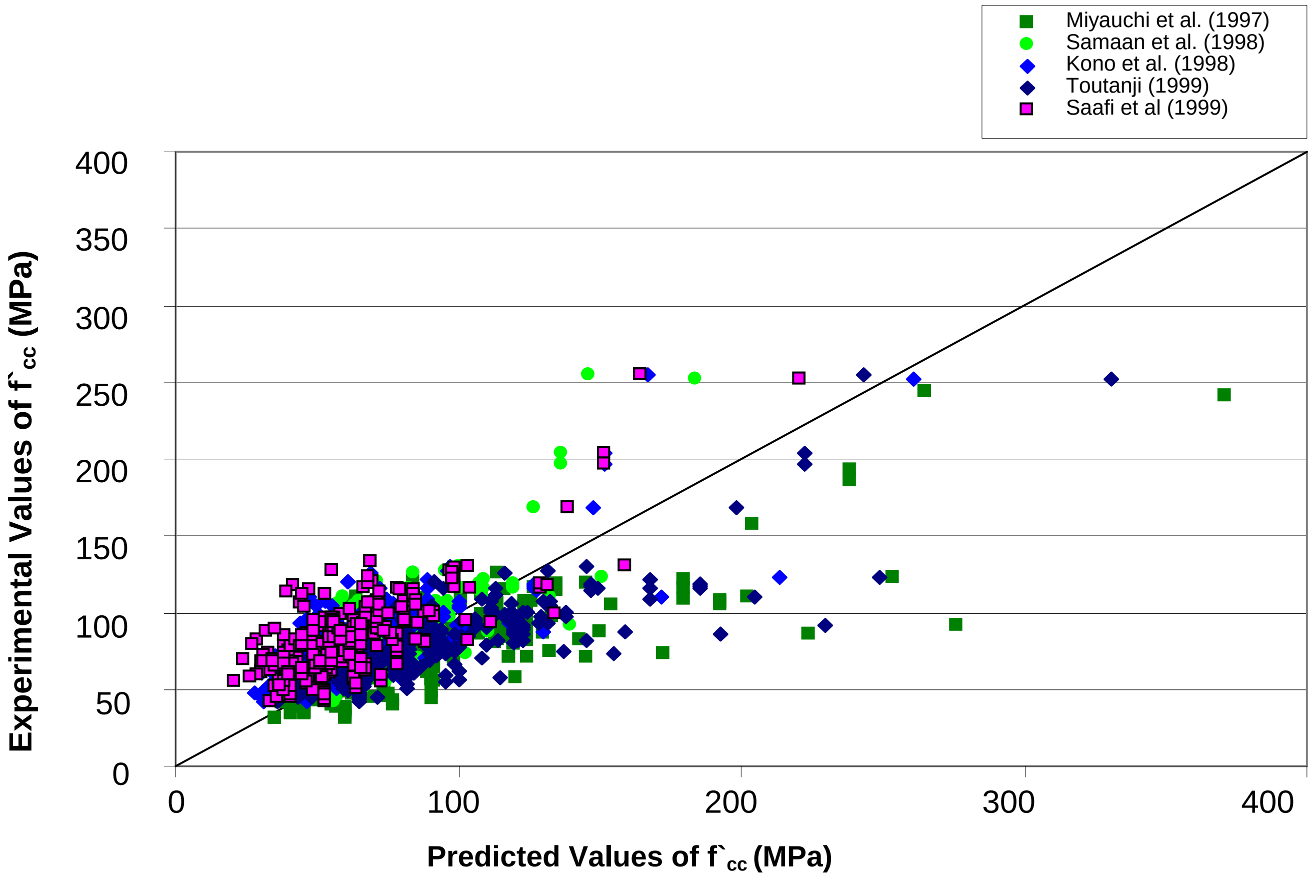

Figure 3.3- Comparison of predictive strength models and experimental data.

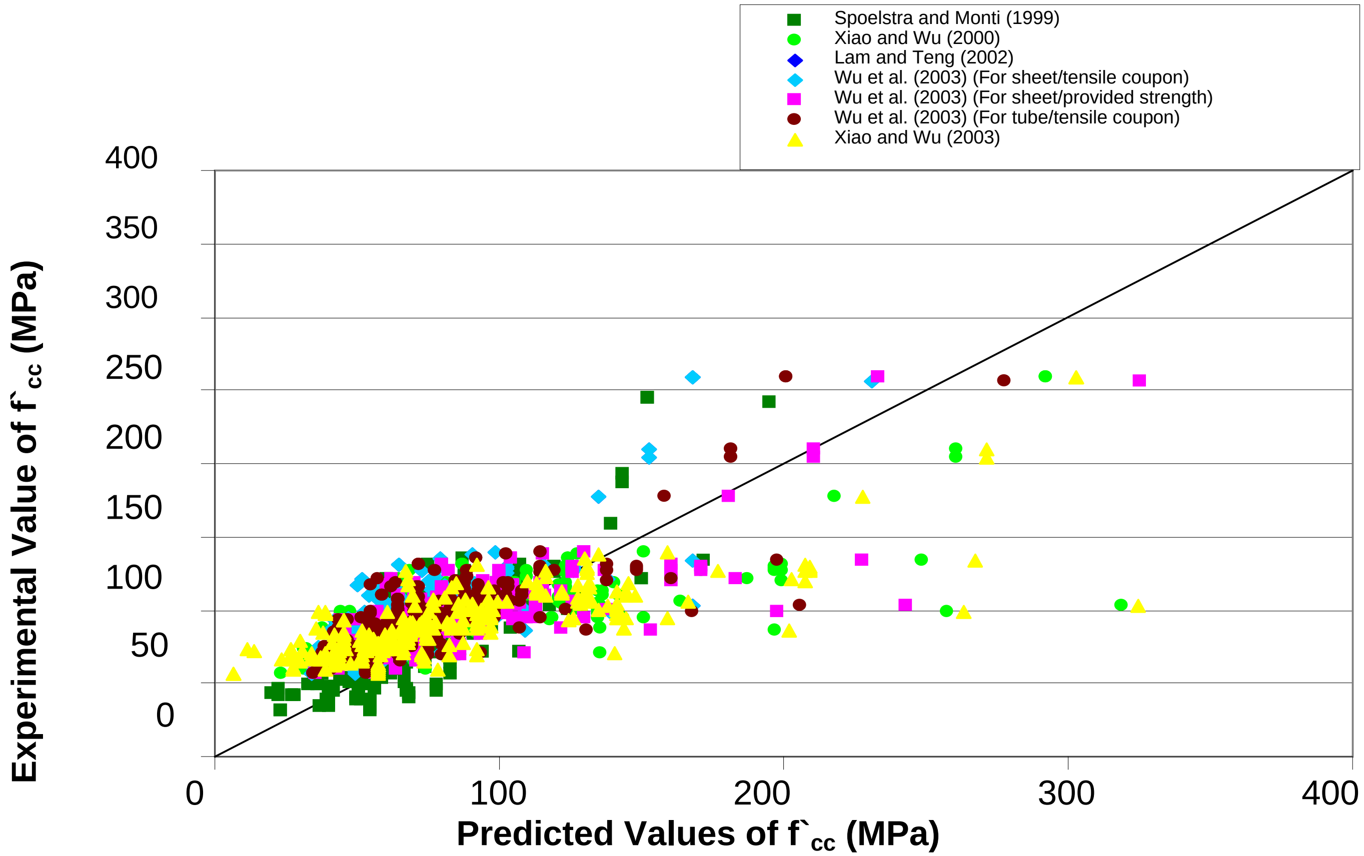

Figure 3.4- Comparison of predictive strength models and experimental data.

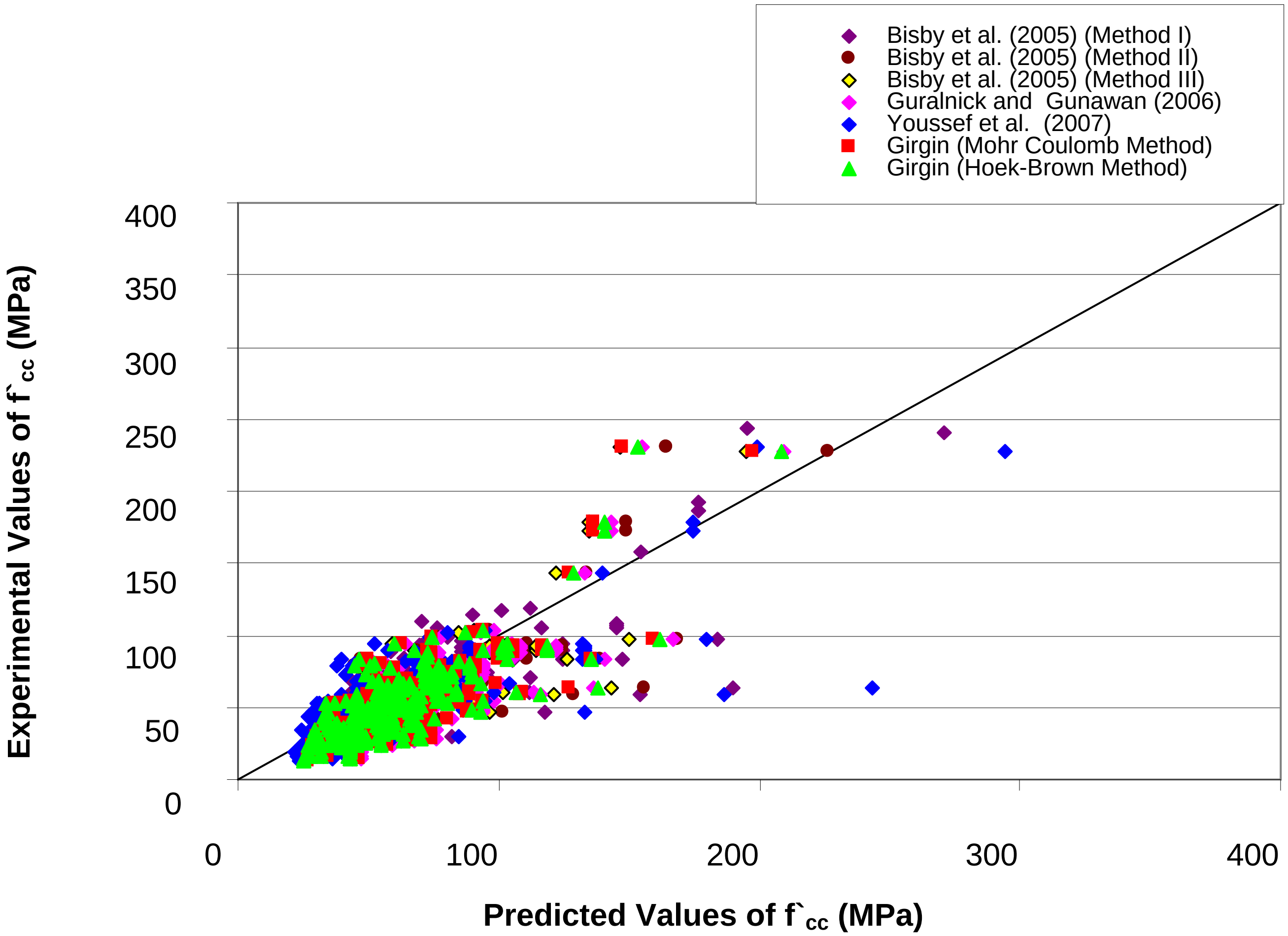

Figure 3.5- Comparison of predictive strength models and experimental data.

3.5 Strain Evaluation

Figures 3.6 through 3.9 show plots of the experimental data versus the predicted data from each strain model considered in this study. Similar to the confined strength graphs, the straight line represents perfect agreement between predicted and experimental strain. Any data points that fall below this line are considered conservative (i.e. the predicted strain is less than the experimental strain). Likewise, any data points that fall above this line are considered non-conservative (i.e. the predicted strain is greater than the experimental strain).

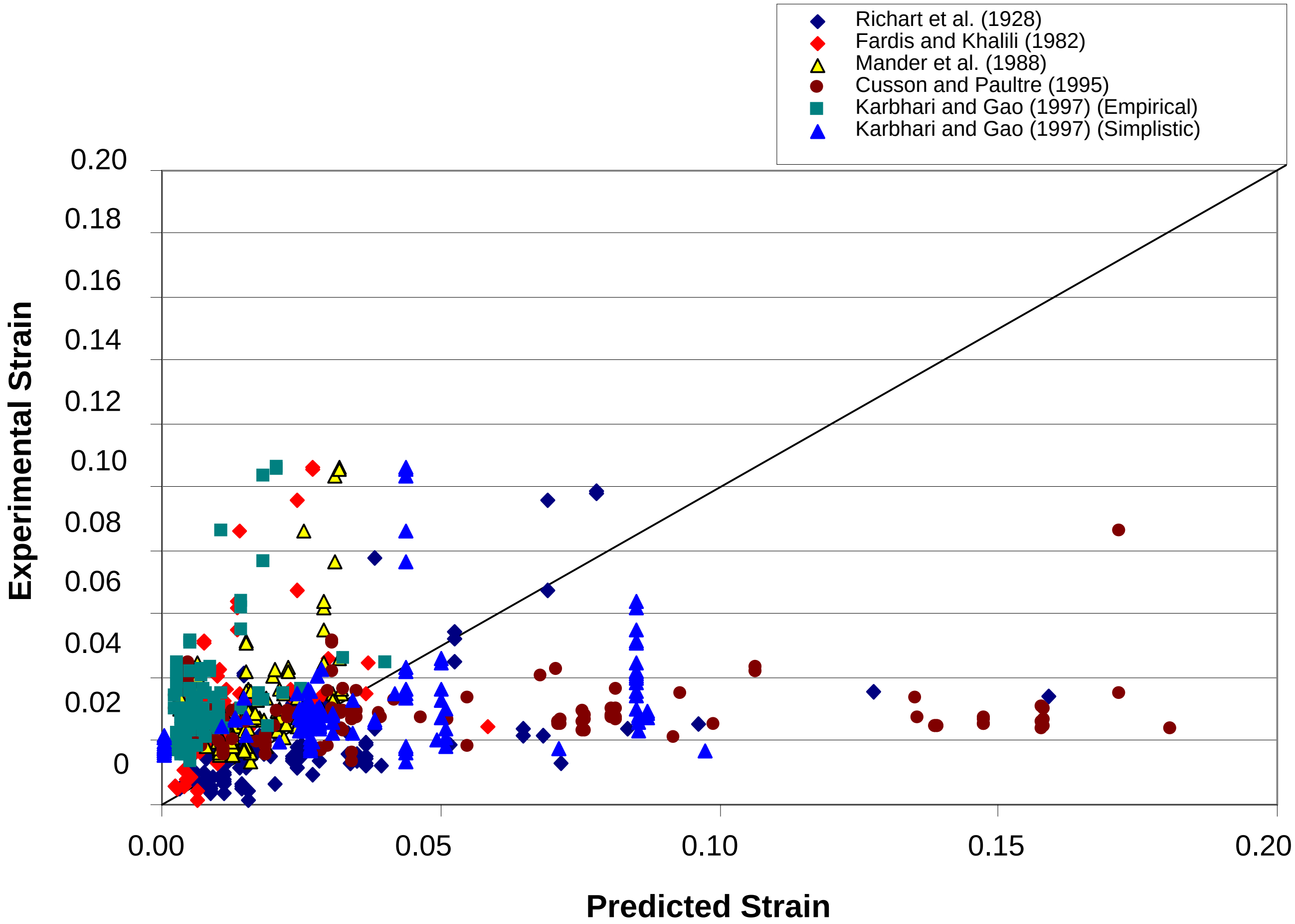

Figure 3.6- Comparison of predictive strain models and experimental data.

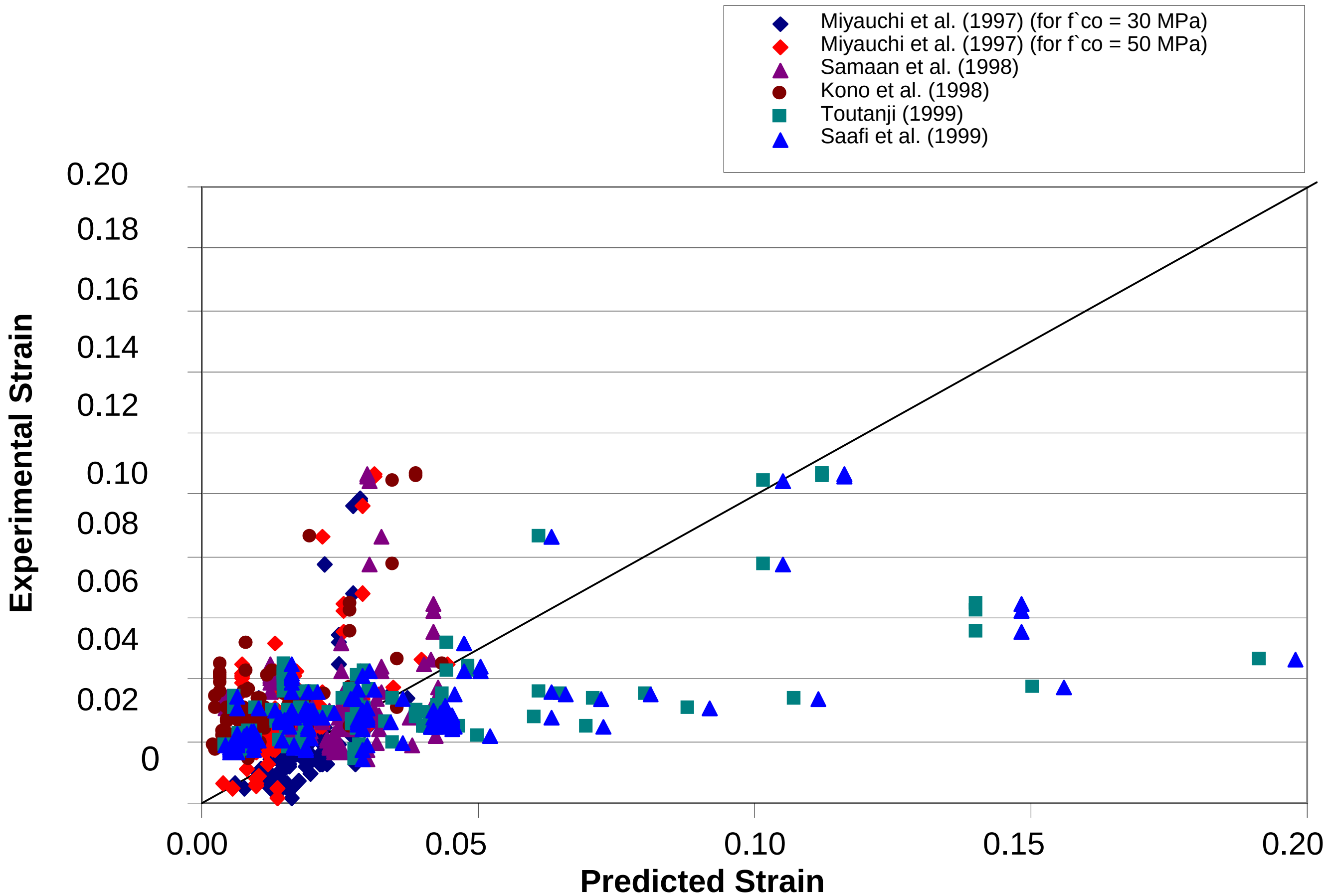

Figure 3.7- Comparison of predictive strain models and experimental data.

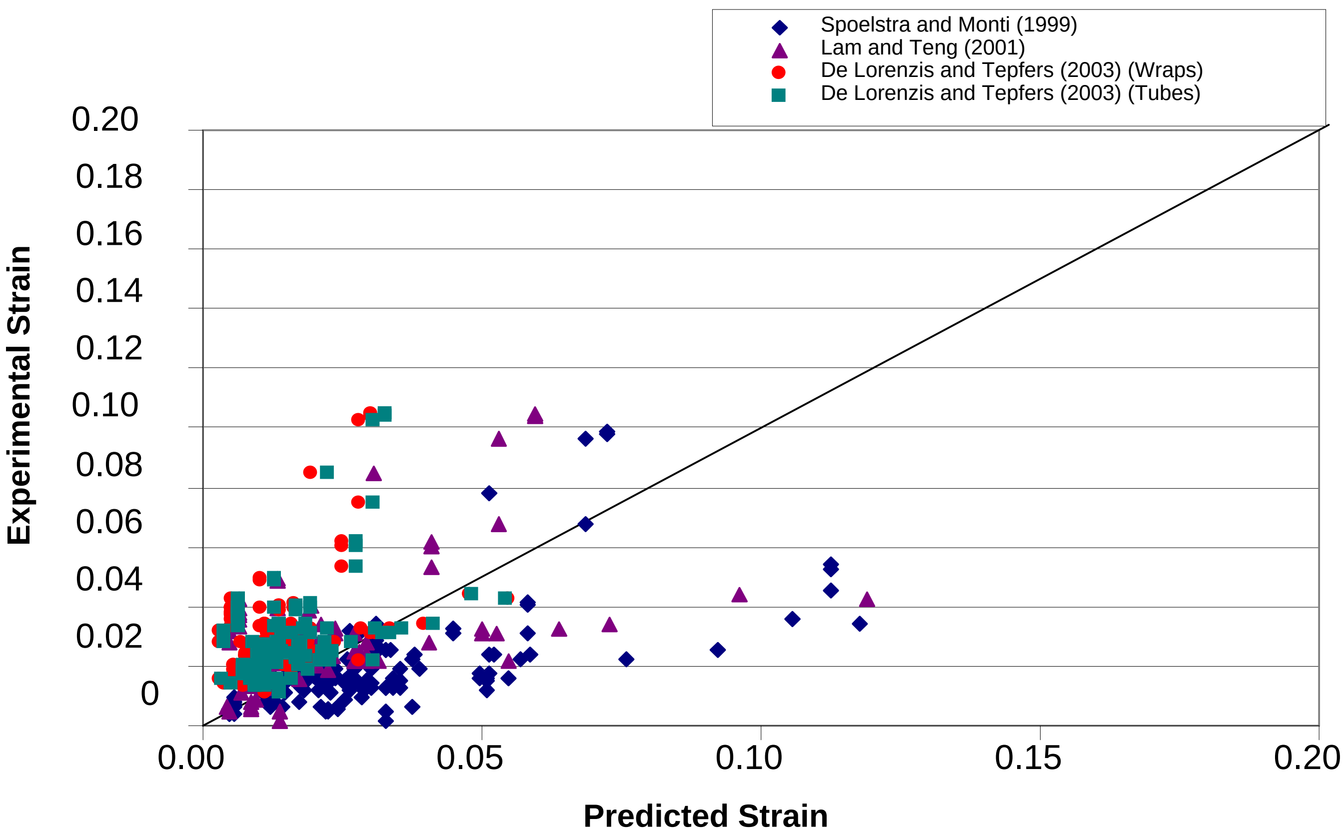

Figure 3.8- Comparison of predictive strain models and experimental data.

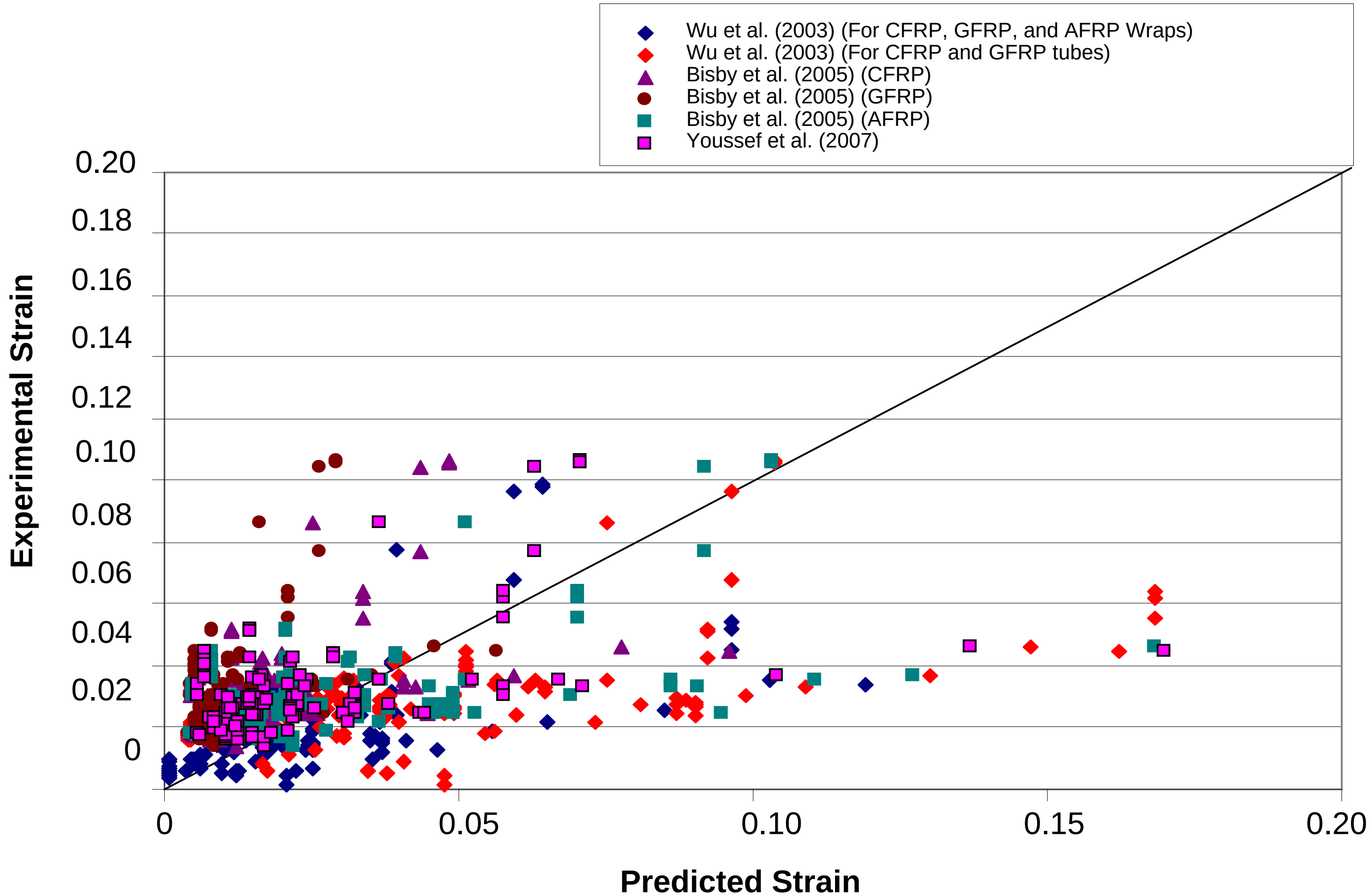

Figure 3.9- Comparison of predictive strain models and experimental data.

3.6 Proposed Model

Using the experimental data set, a regression analysis was performed to create a proposed model. The regression analysis yielded the following models for confined concrete strength and the corresponding ultimate strain:

$$f'_{cc} = f'_{co} \cdot \left[-0.503 \left(\frac{f_l}{f'_{co}} \right)^2 + 2.7798 \frac{f_l}{f'_{co}} + 0.9469 \right] \qquad \text{Equation 3.64}$$

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1.0427 \left(\frac{f'_{cc}}{f'_{co}} \right)^2 - 1.1181 \frac{f'_{cc}}{f'_{co}} + 6.1949 \right] \qquad \text{Equation 3.65}$$

where f_l, f'_{co} and f'_{cc} are in MPa.

The above models for confined concrete strength and ultimate strain have a coefficient of correlation value, R-squared of 0.7449 and 0.5701, respectively. Figures 3.10 and 3.11 depict the predicted values of the proposed model versus the experimental data.

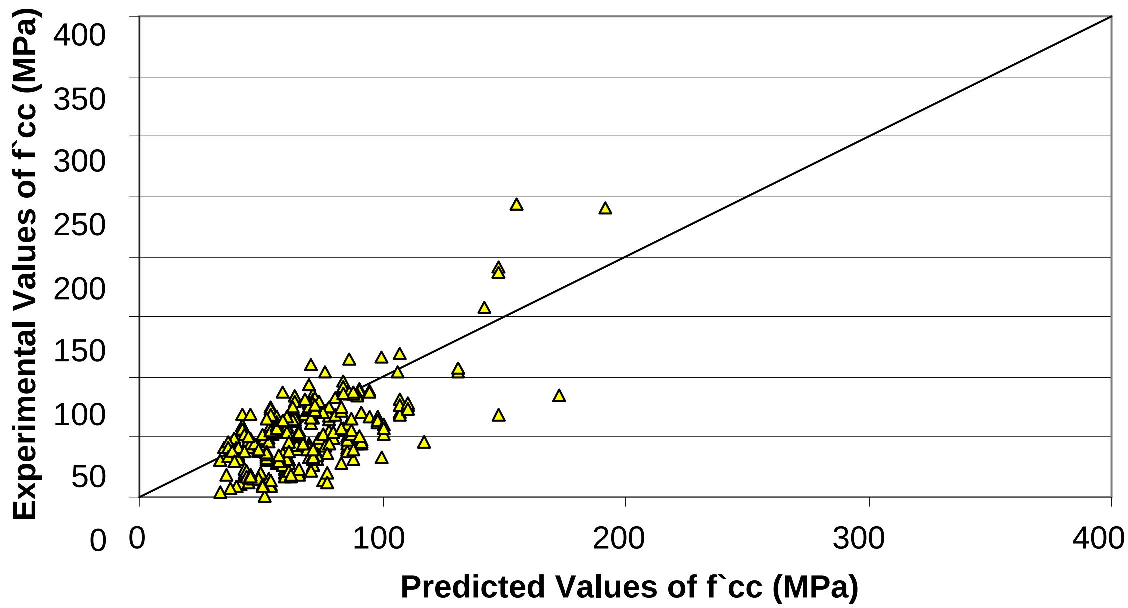

Figure 3.10- Comparison of proposed strength model and experimental data.

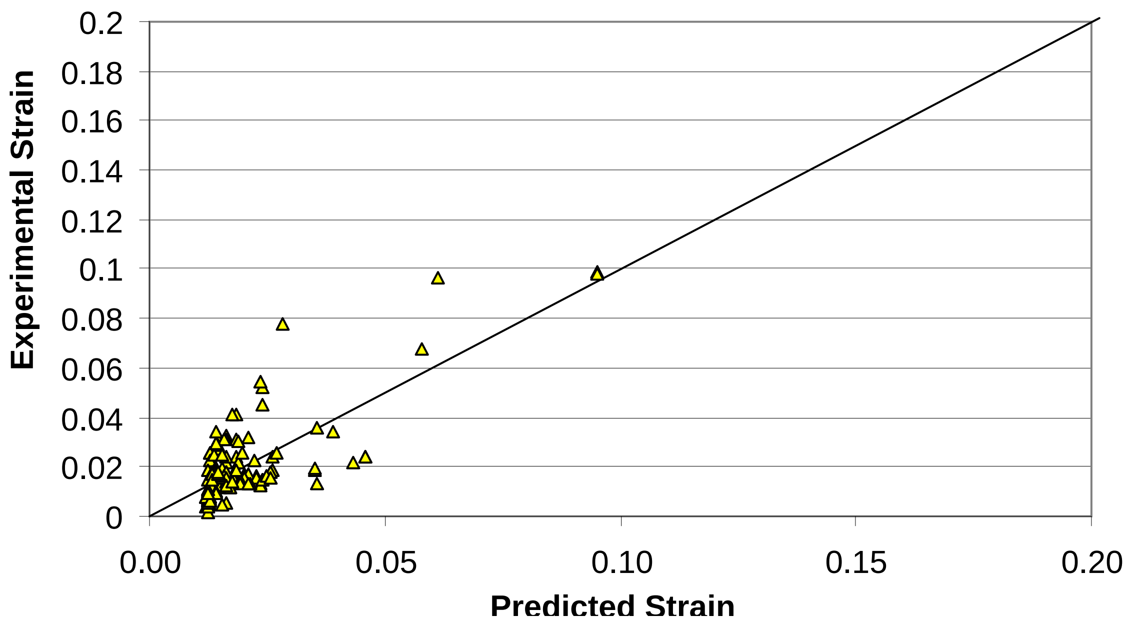

Figure 3.11- Comparison of proposed strength model and experimental data.

3.7 Discussion

Each model was evaluated using the root mean squared error (RMSE), as well as the coefficient of variance (R^2). Table 3.3 shows the rankings of the confined concrete strength models based on the lowest RMSE values. Similarly, Table 3.4 shows the rankings of the confined concrete strain models based on the lowest RMSE values. The strength model analysis was based on 311 data points and the strain model was based on 149 points.

Table 3.3- Ranking of strength models based on RMSE values.

All Data, f'_{cc} (311 data points)				
Rank	**RMSE**	**R^2**	**Author**	**Notes**
1	16.43	0.74491	Proposed Model	
2	16.66	0.69386	Girgin	Based on Hoek-Brown for wraps
3	16.75	0.68776	Karbhari and Gao (1997)	Empirical Method
4	16.86	0.67356	Bisby *et al.* (2005)	Method III
5	16.98	0.68010	Bisby *et al*. (2005)	Method II
6	17.08	0.69286	Saafi *et al*. (1999)	
7	17.15	0.66097	Lam and Teng (2001)	
7	17.15	0.66097	Wu *et al*. (2003)	For FRP sheet with tensile test
9	17.21	0.70131	Girgin	Based on Mohr Coulomb
10	17.42	0.71512	Spoelstra and Monti (1999)	
11	17.52	0.69907	Guralnick and Gunawan (2006)	
12	17.82	0.67693	Samaan *et al.* (1998)	
13	18.19	0.71046	Cusson and Paultre (1995)	
14	18.47	0.66097	Bisby *et al*. (2005)	Method I
15	19.08	0.66097	Wu *et al.* (2003)	For FRP tube with tensile test
16	19.28	0.66364	Kono *et al*. (1998)	
17	21.15	0.59328	Youssef *et al*. (2007)	
18	25.11	0.66097	Wu *et al*. (2003)	For FRP sheet with given strength
19	25.83	0.68378	Mander *et al.* (1988)	
20	26.50	0.66658	Karbhari and Gao (1997)	Simplistic Method
21	32.80	0.66097	Miyauchi *et al*. (1997)	
22	33.50	0.69121	Toutanji (1999)	
23	35.60	0.67507	Xiao and Wu (2000)	
24	36.89	0.68951	Fardis and Khalili (1982)	
25	39.30	0.68716	Xiao and Wu (2003)	
26	43.66	0.66097	Richart *et al.* (1928)	

Table 3.4- Ranking of strain models based on RMSE values.

All Data, strain (149 data points)				
Rank	RMSE	R-Squared	Author	Notes
1	0.0104	0.5695	Proposed Model	
2	0.01426	0.2032	Mander *et al.* (1988)	
3	0.01428	0.2215	Miyauchi *et al.* (1997)	For $f`_{co}$=30MPa
4	0.01432	0.2383	Miyauchi *et al*. (1997)	For $f`_{co}$=50MPa
5	0.0144	0.2196	De Lorenzis and Tepfers (2003)	For FRP tubes
6	0.0148	0.4002	Kono *et al*. (1998)	
7	0.0151	0.2217	De Lorenzis and Tepfers (2003)	For FRP wraps
8	0.0155	0.2451	Bisby *et al*. (2005)	For CFRP
9	0.0163	0.2451	Lam and Teng (2001)	
10	0.0170	0.2451	Bisby *et al*. (2005)	For GFRP
11	0.01708	0.3706	Wu *et al*. (2003)	For wraps
12	0.01713	0.1699	Fardis and Khalili (1982)	
13	0.0173	0.0422	Samaan *et al*. (1998)	
14	0.0184	0.2451	Karbhari and Gao (1997)	Empirical Method
15	0.0203	0.2258	Youssef *et al.* (2007)	
16	0.0204	0.4065	Spoelstra and Monti (1999)	
17	0.0207	0.2451	Richart *et al*. (1928)	
18	0.0281	0.2451	Bisby *et al.* (2005)	For AFRP
19	0.0296	0.1245	Karbhari and Gao (1997)	Simplistic Method
20	0.0325	0.3434	Toutanji (1999)	
21	0.0341	0.3463	Saafi *et al.* (1999)	
22	0.0369	0.3643	Wu *et al*. (2003)	For tubes
23	0.2518	0.1760	Cusson and Paultre (1995)	

The results show that the proposed model, Girgin's Hoek-Brown model [13], and Karbhari and Gao's [20] empirical model yield the best results for strength of confined concrete. Likewise, the proposed model, Mander *et al.*'s [30], and Miyauchi *et al.*'s [34] model most accurately predict the corresponding strain.

3.8 Conclusion

Several confinement models were evaluated and compared with a proposed model, which was developed using a regression analysis on the experimental data set. When evaluated, it was shown that the most accurate models for confined concrete strength were the proposed model, Girgin's Hoek-Brown model [13], and Karbhari and Gao's [20] empirical model. Likewise for ultimate strain, the most accurate models were the proposed model, Mander *et al.*'s [30], and Miyauchi *et al.*'s [34] model most accurately predict the corresponding strain. However, the strain models in general are much less accurate than the strength models.

Chapter 4

Design Models

4.1 Introduction

There is currently no universally accepted design code for the seismic retrofit of concrete columns using FRP jackets. This chapter will review several design approaches commonly used in design for seismic retrofit and then compare the resulting jacket thicknesses recommended by each approach through a sample column design.

4.2 Background

A confined column has increased strength and ductility due to the passive restraint provided by the jacket, which restricts the lateral dilation of the column. The jacket provides a confining pressure on the concrete column, which suppresses micro-cracking and dilation of the concrete. Circular columns have the most efficient section for retrofitting; therefore, this paper will focus on circular columns only. For a circular column, the lateral confining pressure exerted on the concrete in the radial direction from the FRP jacket, f_l can be evaluated using a free body diagram as shown in Figure 3.1, which results in the following relation:

$$f_l = \frac{2t_f f_f}{D} = \frac{2t_f E_f \varepsilon_f}{D} \qquad \text{Equation 4.1}$$

where f_f is the tensile strength of the FRP in the hoop direction, t_f is the thickness of the FRP jacket, D is the diameter of the confined concrete, E_f is the modulus of elasticity of the FRP in the hoop direction and ε_f is the strain in the FRP in the hoop direction.

Since FRP behaves elastically, the inward pressure due to a jacket increases continuously. In contrast, steel provides a constant lateral confining pressure. Therefore, confinement models and design approaches for steel jackets inaccurately model the elastic behavior of FRP.

The purpose of seismic retrofit is to increase the columns seismic response by increasing the columns ductility and axial strength. There are four major failure modes that FRP jackets can restrain: flexural, shear, lap slice and longitudinal bar buckling.

Flexural failure in the plastic hinge occurs near the column ends, where the behavior is that of a plastic hinge. Flexural failure can be observed through concrete spalling, failure of transverse steel reinforcement and buckling of longitudinal steel reinforcement, which cause deterioration of the plastic hinge. There is extensive deformation associated with this failure mode; therefore, it is considered a ductile and desirable failure mode as compared to shear failure. An FRP jacket provides confinement for the flexural failure mode, which increases the overall strength and ultimate strain and provides lateral support, which can hinder flexural failure.

Shear failure, in contrast to flexural failure, is a very brittle failure mode. It is; therefore, an extremely undesirable failure mode. Shear failure is characterized by diagonal shear cracks in the column and subsequent rupture of the transverse steel reinforcement. Shear failure is very prevalent in structures built prior to the 1971 San Fernando earthquake, which have insufficient transverse steel reinforcement corresponding to inadequate confinement. An FRP jacket can contribute to the overall shear strength of the column and thereby inhibit the failure.

Lap splice failure is a result of inadequate confinement in the splice in the longitudinal steel reinforcement. The splice region is confined to a small region near the column ends, where the starter bars from the footing are spliced with the column longitudinal reinforcement. The failure occurs because of dilation of the column, which results in debonding of the splice. Lap splice failure can be observed through concrete spalling, vertical cracks, rupture of transverse steel reinforcement and buckling of the longitudinal steel reinforcement. Similar to flexural failure, there is extensive deformation associated with lap splice failure; therefore, it is considered a ductile and desirable failure mode when compared to shear failure. An FRP jacket inhibits the lap splice failure mode through the application of confining pressure in the splice region, which creates a lateral clamping force and prevents debonding from occurring.

The last failure mode is buckling of the longitudinal steel reinforcement. This is considered to be a rare failure mode and is usually not considered by most design approaches.

4.3 Review of Design Models

There are a few existing design codes and guidelines for the seismic retrofit of concrete columns using FRP materials. Those codes and/or guidelines that will be reviewed include documents from the American Concrete Institute (ACI), California Department of Transportation (Caltrans), Concrete Society, and the Intelligent Sensing for Innovative Structures (ISIS) Canada Network of Center of Excellence. In addition to the official documents in place, there are a number of other approaches put forth. To contrast these approaches, an approach by Seible *et al.* [43] will be reviewed.

4.3.1 ACI 440.2R-02 (2002)

The ACI 440.2R-02 [1] guideline utilizes the confinement model originally proposed by Mander *et al.* [30] derived for steel hoop reinforcement. The confinement model from Mander *et al.* [30] is slightly modified to account for the linearly elastic behavior of FRP. The confinement model predicts the confined strength of concrete, f'_{cc} through the following relation:

$$f'_{cc} = f'_{co} \cdot \left[2.25\sqrt{1 + 7.9\frac{f_l}{f'_{co}}} - 2\frac{f_l}{f'_{co}} - 1.25 \right] \qquad \text{Equation 4.2}$$

where f'_{co} is the unconfined concrete strength and f_l is the confining pressure, which is defined as:

$$f_l = \frac{k_s \rho_f \varepsilon_f E_f}{2} \qquad \text{Equation 4.3}$$

where k_s is the efficiency coefficient, which depends on the column geometry. For a circular column $k_s = 1.0$. ε_f is defined as the lesser of 0.004 or 0.75 ε_{fu}, where ε_{fu} is the ultimate strain in the FRP. This strain limit was chosen to avoid loss of interlock in the concrete aggregate and was determined through the study of experimental data. However, the limiting strain was determined from pure axial tests and is; therefore, an approximation for combined axial and bending situations induced by seismic forces. ρ_f is the FRP volumetric ratio and is equal to:

$$\rho_f = \frac{4t_f}{D} \qquad \text{Equation 4.4}$$

The increased axial load carrying capacity of a strengthened column can be expressed as:

$$\phi P_n = k_e \phi \left[0.85 \psi_f f'_{cc} \left(A_g - A_{st} \right) + f_y A_{st} \right] \qquad \text{Equation 4.5}$$

where k_e is the resistance factor (= 0.85 for spiral reinforced columns and 0.8 for tie reinforced columns), P_n is the nominal axial load carrying capacity, ϕ is the strength reduction factor and equal to 0.75, ψ_f is an additional reduction coefficient for FRP wrapped columns, which is equal to 0.95, A_{st} is the longitudinal steel area and f_y is the steel yield strength.

The above equation only considers the axial increase in strength. For retrofit in seismic areas, ACI 440 [1] suggests using Mander *et al.* [30] model for the confined strain, ε_{cc}:

$$\varepsilon_{cc} = \frac{1.71 \left(5 f'_{cc} - 4 f'_{co} \right)}{E_c} \qquad \text{Equation 4.6}$$

where E_c is the elastic modulus of the concrete. ACI 440 [1] suggests that the design should be developed to have sufficient strains associated with the desired displacement demands.

4.3.2 Caltrans Memo 20-4 (2000)

Caltrans Memo 20-4 [5] is specifically aimed at preventing flexural failure. Caltrans Memo 20-4 [5] recommends the use of a target confining stress, f_l, of 2,068 kPa (300 psi) and radial dilating strain, ε_{cc}, of 0.004 inside the plastic hinge zone. These limiting values were determined experimentally at a corresponding displacement ductility, μ, of 6. For regions outside the plastic hinge zone, Caltrans Memo 20-4 [5] recommends f_l = 1,034 kPa (150 psi) and ε_{cc} = 0.004. The thickness can be calculated using the following equation, which was derived from equilibrium expression from Figure 4.1:

$$2t_f E_f \varepsilon_{cc} = f_l D \qquad \text{Equation 4.7}$$

where E_f is the elastic modulus of FRP multiplied by a reduction safety factor of 0.9. This approach is solely based on target confinement pressures and does not specifically account for the various failure modes possible.

4.3.3 Concrete Society Technical Report No. 55 (2004)

The Concrete Society Technical Report No. 55 [8], similar to the Caltrans Memo 20-4 [5], sets a target confining pressure. The design sets a minimum value of 0.183 mm^2/N for the following ratio, which was determined through experimental testing:

$$\frac{2t_f E_f}{D\left(f'_{co}\right)^2} > 0.183\,\frac{mm^2}{N} \qquad \text{Equation 4.8}$$

4.3.4 ISIS Canada Network of Centers of Excellence (2001)

ISIS Canada [18] set up design guidelines based on limiting levels of confining pressure. According to this approach, the confined strength of concrete can be found through:

$$f'_{cc} = f'_{co}\left(1 + \alpha_{pc} w_w\right) \qquad \text{Equation 4.9}$$

where α_{pc} is a performance coefficient that depends on column geometry and is equal to 1.0 for circular columns and w_w is the volumetric strength ratio, which is expressed by the following relation:

$$w_w = \frac{2 f_l}{\phi_c f'_{co}} \qquad \text{Equation 4.10}$$

where ϕ_c is the concrete resistance reduction factor and is equal to 0.6 and the confining pressure f_l is determined from:

$$f_l = \frac{2 N_b \phi_{frp} E_f \varepsilon_f t_f}{D} \qquad \text{Equation 4.11}$$

where N_b is the number of FRP layers, ϕ_{frp} is a reduction factor for FRP, which is equal to 0.75.

To ensure a certain level of ductility a minimum amount of confining pressure imposed is equal to 4 MPa. This minimum confining pressure is four times the target pressure of 1 MPa for Caltrans Memo 20-4 [5]. Also a maximum level of confining pressure is imposed to limit amount of axial strains.

$$f_{l\max} \leq \frac{0.29 f'_{co}}{\alpha_{pc}}$$ Equation 4.12

4.3.5 Seible et al. (1997)

The approach proposed by Seible *et al.* [43] differs from most of the aforementioned documents because it considers each failure mode separately and calculates a design thickness associated with each of the failure modes. For shear failure, the required thickness is calculated as follows:

$$t_j = \frac{\frac{V_o}{\phi_v} - \left(V_c + V_s + V_p\right)}{\frac{\pi}{2} \cdot 0.004 E_f D}$$ Equation 4.13

where V_o is the column shear demand based on full flexural over-strength in the potential plastic hinge region, ϕ_v is the shear capacity reduction factor, assumed to be 0.85, V_c is the shear capacity contribution from concrete, V_s is the shear capacity contribution from horizontal steel reinforcement and V_p is the shear capacity contribution from axial load. The shear demand, V_o is calculated from the as built moment capacity, M_{yi}. Therefore, the shear demand is expressed as:

$$V_o = \frac{1.5 M_{yi}}{H}$$ Equation 4.14

where H is the height of the column.

The shear contributions from concrete can be calculated through the following expressions:

$$V_c = k\sqrt{f'_{co}}A_e \qquad \text{Equation 4.15}$$

where k is 0.5 inside the plastic hinge and 3.0 outside the plastic hinge and A_e is the effective area, which is equal to 0.8 times the gross area of the column. However, for damaged columns the contribution due to concrete should be neglected, since cracks large enough to lose aggregate interlock are most likely present.

The shear contribution from the steel hoop reinforcement is calculated from:

$$V_s = \frac{\pi}{2}\frac{A_h f_y D' \cot(\theta)}{s} \qquad \text{Equation 4.16}$$

where A_h is the area of hoop steel reinforcement and θ is the angle of shear crack, which can conservatively be assumed as 45°, s is the spacing between steel hoops and D\` is the effective diameter, which is equal to the overall diameter, D minus twice the concrete cover, cc minus the diameter of a steel hoop bar, d_h.

The shear contribution from the axial load is determined by:

$$V_p = \frac{P(D-c)}{H} \qquad \text{Equation 4.17}$$

where P is the axial load applied and c is the neutral axis depth.

The shear thickness is applied over the shear reinforcement length, L_v from either end of the column, where L_v is equal to 1.5 times the diameter of the column, D.

For flexural hinge failure mode, the thickness in the flexural hinge region is calculated as follows:

$$t_j = 0.09 \frac{D(\varepsilon_{cu} - 0.004) f'_{cc}}{\phi_f f_f \varepsilon_f} \qquad \text{Equation 4.18}$$

where ϕ_f is the flexural capacity reduction factor, assumed as 0.9, f'_{cc} is conservatively assumed as 1.5 f'_{co} and ε_{cu} is the ultimate concrete strain that depends on the level of confinement and is calculated as:

$$\varepsilon_{cu} = 0.004 + \frac{2.8 \rho_j f_f \varepsilon_f}{f'_{cc}} \qquad \text{Equation 4.19}$$

where ρ_j is the volumetric jacket reinforcement ratio, ε_{cu} can be obtained from:

$$\varepsilon_{cu} = \Phi_u c_u \qquad \text{Equation 4.20}$$

where c_u is the neutral axis depth and Φ_u is the ultimate section curvature can be obtained from the ductility factor equation:

$$\mu_\Delta = 1 + 3\left(\frac{\Phi_u}{\Phi_y} - 1\right)\frac{L_p}{L}\left(1 - 0.5\frac{L_p}{L}\right) \qquad \text{Equation 4.21}$$

where L_p is the plastic hinge length and determined from:

$$L_p = 0.08L + 0.022 f_y d_b \qquad \text{Equation 4.22}$$

and Φ_y = section yield curvature.

The thickness required for the flexural reinforcement jacket is applied over the flexural plastic hinge region, where L_{c1} is the primary flexural hinge region and is equal to the greater of 0.5D or L/8, and L_{c2} is the secondary flexural hinge region and is equal to the greater of 0.5D or L/8. The secondary flexural hinge region has $t_j/2$ thickness.

The lap splice failure mode is prevented through applying a thickness in the lap-splice region of:

$$t_j = 500\frac{D(f_l - f_h)}{E_f} \qquad \text{Equation 4.23}$$

where f_h is the horizontal stress provided by the existing hoop reinforcement at a strain of 0.1% and is calculated through the following relation:

$$f_h = \frac{0.002A_h E_s}{D \cdot s} \qquad \text{Equation 4.24}$$

and f_l is the lateral clamping pressure, which can be found from:

$$f_l = \frac{A_s f_y}{\left[\frac{p}{2n} + 2(d_b + cc)\right]L_s} \qquad \text{Equation 4.25}$$

where p is the perimeter line in the column cross section along the lap-spliced bar locations, n is the number of spliced bars along p, A_s is the area of one main column reinforcing bar, cc is the concrete cover to the main column reinforcement and d_b is the diameter of main column reinforcement bars. The length to which the lap splice reinforcement is applied, L_s must be greater than the lap length itself.

4.4 Design Example

Two sample columns were chosen based on existing testing data from Sieble and Innamorato [41] and Seible *et al.* [40]. Seible and Innamorato [41] report on an investigation wherein a reinforced concrete column, which was deficient in shear was wrapped with carbon fiber FRP and then subjected to the given loads. Similarly, Sieble *et al.* [40] reported on a flexural deficient column with a lap splice, which was wrapped with a carbon fiber FRP jacket. Table 4.1 shows the properties and loads for the sample shear column and Table 4.2 shows the properties and loads for the sample flexural column.

Table 4.1- Properties of sample column with deficiency in shear.

Column Section Properties	Column Height, H	2.438 m
	Column Diameter, D	0.610 m
	Concrete Cover, cc	20.32 mm
	Concrete Strength, $f`_c$	34.45 MPa
Longitudinal Reinforcement (Grade 40)	Bar Diameter, d_b	19 mm (26 total)
	Bar Area, A_s	284 mm^2
	Yield Strength, f_y	303.4 MPa
Transverse Reinforcement (Grade 40)	Bar Diameter, d_h	6.35 mm
	Bar Area, A_h	31.7 mm^2
	Spacing, s	127 mm
Column Load Properties	Axial Load, P	591.6 kN
	Moment Capacity, M_{yi}	646.7 kN-m
	Yield Curvature, ϕ_y	0.005984 1/m
	Neutral Axis Depth, c_u	152.4 mm
Jacket Material Properties	Jacket Modulus, E_f	124.1 GPa
	Ultimate Strength, f_f	1.31 GPa
	Ultimate Strain, ε_f	1.10%

Table 4.2- Properties of sample column with deficiency in flexure.

Column Section Properties	Column Height, H	3.658 m
	Column Diameter, D	0.610 m
	Concrete Cover, cc	19.05 mm
	Concrete Strength, $f`_c$	34.45 MPa
Longitudinal Reinforcement (Grade 40)	Bar Diameter, d_b	19 mm (26 total)
	Bar Area, A_s	284 mm^2
	Yield Strength, f_y	303.4 MPa
Transverse Reinforcement (Grade 40)	Bar Diameter, d_h	6.35 mm
	Bar Area, A_h	31.7 mm^2
	Spacing, s	127 mm
Column Load Properties	Axial Load, P	1800 kN
	Moment Capacity, M_{yi}	518.6 kN-m
	Yield Curvature, ϕ_y	0.008196 1/m
	Neutral Axis Depth, c_u	136.4 mm
Jacket Material Properties	Jacket Modulus, E_f	124.1 GPa
	Ultimate Strength, f_f	1.31 GPa
	Ultimate Strain, ε_f	1.10%

Using the properties from the shear and flexural examples, design thicknesses were calculated using each approach described above. In addition, the approaches that included the confined strength of concrete or the confined strain of concrete within the design calculations were modified to include the two best models from Chapter 2. The best two models for the confined strength of concrete were determined in Chapter 2 as the proposed model and Girgin [13]. The two best models for the associated strain were the proposed model and Mander *et al.* [30]. Only two design models incorporated the confined strength of concrete and/or the associated strain into the design procedure. ACI-440.2R [1] used both the confined strength and the associated strain; therefore, there were a total of ten modified approaches using combinations of the various confinement models. Figure 4.1 shows the different design approaches that were considered.

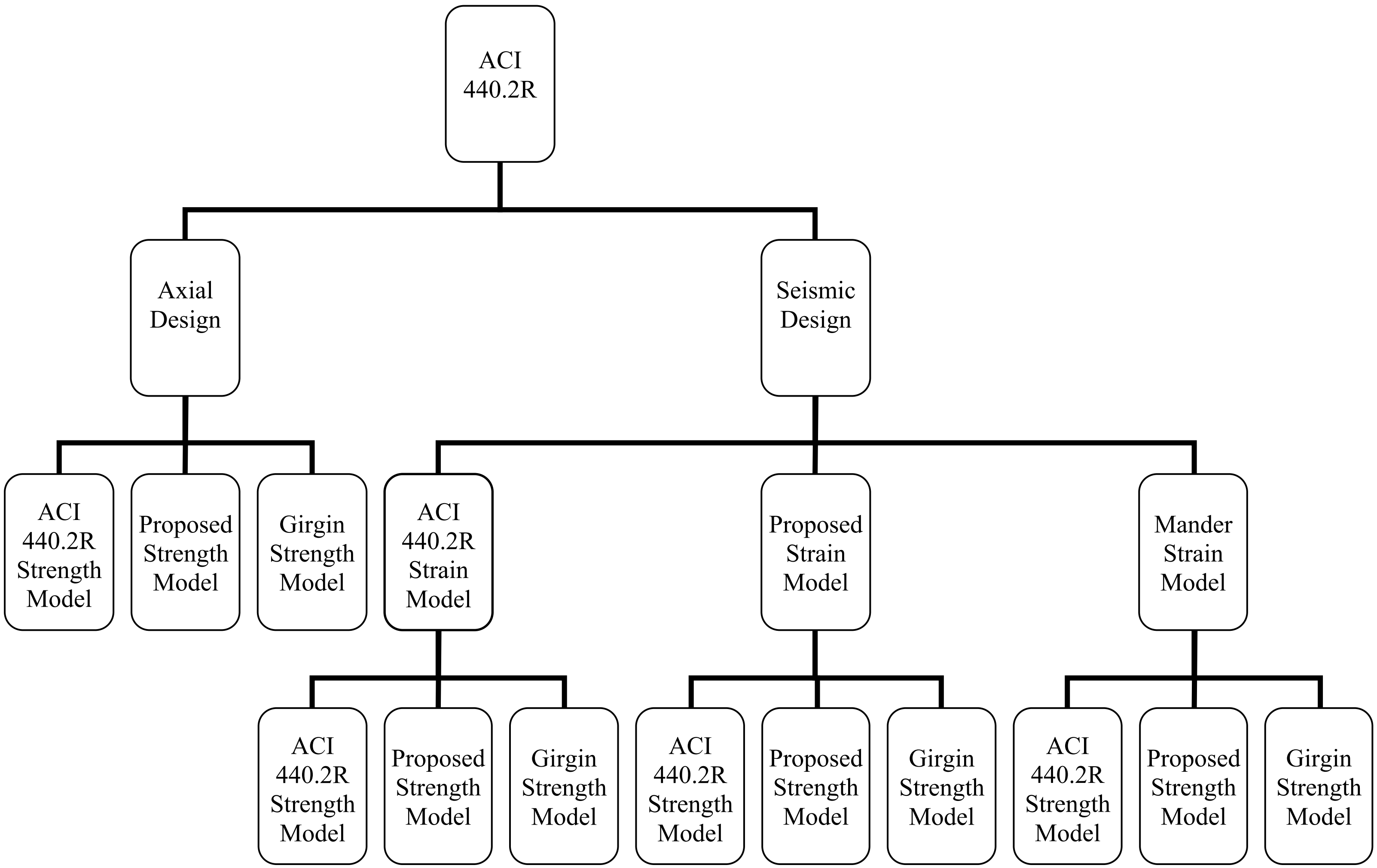

Figure 4.1- Variations of ACI 440.2R [1] design code considered.

Similarly, the Seible *et al.* [43] design approach incorporated the confined strength of concrete into the flexural strengthening calculations; therefore, alternate design thicknesses were determined using each of the different strength models considered. Figure 4.2 shows the alternate approaches considered for the Seible *et al.* [43] design calculations.

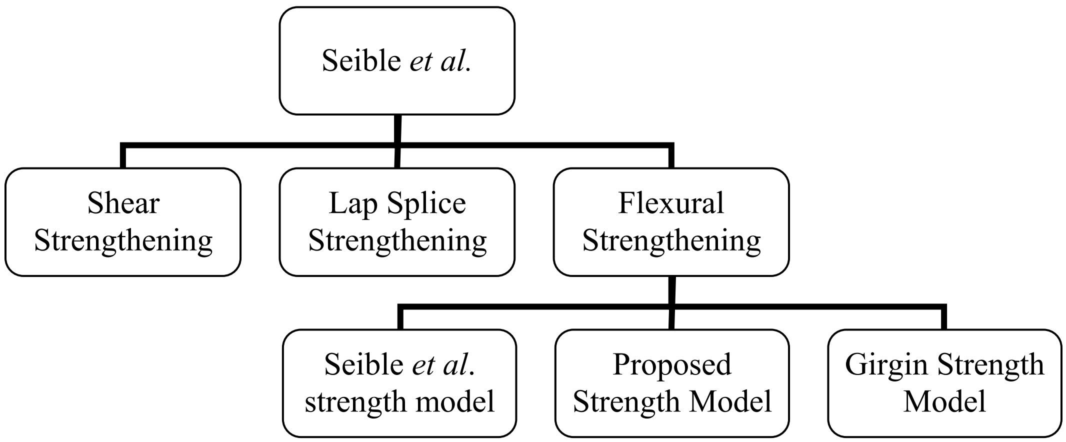

Figure 4.2- Variations of Seible *et al.* [43] considered.

Each design approach was analyzed using the sample column and the theoretical design jacket thickness required was determined. The design thickness was determined for a ductility of 8 and 10. Table 4.3 shows the resulting required thicknesses for each design approach for the shear column example. The table also shows any modifications made to the design approach through the confined strength model or the confined strain model. Similarly, Table 4.4 shows the thicknesses and variations for the flexural column example. Individual calculations for each approach are detailed in Appendix A.

Table 4.3- Summary of required design thicknesses for shear deficient column.

Required Design Thickness for Shear Deficient Column						
Model	**μ = 8 Thickness mm. [in.]**	**μ=10 Thickness mm. [in.]**	**Length**	**Strength Model**	**Strain Model**	**Notes**
ACI 440.2R [1]	N/A	N/A	N/A	ACI 440.2R [1]	N/A	Axial Design
	N/A	N/A	N/A	Proposed	N/A	
	N/A	N/A	N/A	Girgin [13]	N/A	
	4.97 [0.196]	7.47 [0.294]	Full	ACI 440.2R [1]	ACI 440.2R	Seismic Design
	3.56 [0.140]	4.77 [0.188]	Full	Proposed		
	3.71 [0.146]	5.07 [0.20]	Full	Girgin [13]		
	1.82 [0.072]	5.5 [0.217]	Full	ACI 440.2R [1]	Proposed	
	1.64 [0.065]	3.84 [0.151]	Full	Proposed		
	1.57 [0.062]	4.02 [0.158]	Full	Girgin [13]		
	5.56 [0.219]	8.39 [0.33]	Full	ACI 440.2R [1]	Mander *et al.* [*30*]	
	3.87 [0.152]	5.17 [0.204]	Full	Proposed		
	4.06 [0.16]	5.50 [0.217]	Full	Girgin [13]		

Table 4.3- Summary of required design thicknesses for shear deficient column. Continued.

Model	μ = 8 Thickness mm. [in.]	μ=10 Thickness mm. [in.]	Length	Strength Model	Strain Model	Notes
Caltrans Memo 20-4 [5]	1.412 [0.056]	1.412 [0.056]	223.5 mm from ends	N/A	N/A	Inside Plastic Hinge Region
	0.706 [0.028]	0.706 [0.028]	Remaining Column	N/A	N/A	Outside Plastic Hinge Region
Concrete Society Technical Report No. 55 [8]	0.534 [0.021]	0.534 [0.021]	Full	N/A	N/A	
ISIS Canada [18]	3.10 [0.122]	3.10 [0.122]	Full	N/A	N/A	
Seible *et al.* [*43*]	1.60 [0.063]	1.60 [0.063]	Full	N/A	N/A	Shear Strength
	N/A	N/A	N/A	N/A	N/A	Lap Splice Clamping
	2.33 [0.092]	3.21 [0.126]	223.5 mm from ends	Seible *et al.* [*43*]	N/A	Primary Plastic Hinge Region
	1.17 [0.046]	1.61 [0.063]	223.5 mm from primary region			Secondary Plastic Hinge Region
	2.97 [0.117]	5.69 [0.224]	233.5 mm from ends	Proposed	N/A	Primary Plastic Hinge Region
	1.49 [0.058]	2.89 [0.112]	223.5 mm from primary region			Secondary Plastic Hinge Region
	2.91 [0.115]	5.28 [0.208]	223.5 mm from ends	Girgin [13]	N/A	Primary Plastic Hinge Region

Table 4.3- Summary of required design thicknesses for shear deficient column. Continued.

Model	μ = 8 Thickness mm. [in.]	μ = 10 Thickness mm. [in.]	Length	Strength Model	Strain Model	Notes
Seible *et al.* [*43*]	1.455 [0.057]	2.64 [0.104]	223.5 mm from primary region	Girgin [13]	N/A	Secondary Plastic Hinge Region

Table 4.4- Summary of required design thicknesses for flexural deficient column.

Required Design Thickness for Flexural Deficient Column						
Model	**μ = 8** **Thickness mm. [in.]**	**μ = 10** **Thickness mm. [in.]**	**Length**	**Strength Model**	**Strain Model**	**Notes**
ACI 440.2R [1]	N/A	N/A	N/A	ACI 440.2R [1]	N/A	Axial Design
	N/A	N/A	N/A	Proposed	N/A	
	N/A	N/A	N/A	Girgin [13]	N/A	
	15.77 [0.621]	31.49 [1.24]	Full	ACI 440.2R [1]	ACI 440.2R	Seismic Design
	7.75 [0.305]	10.98 [0.432]	Full	Proposed		
	8.21 [0.323]	11.14 [0.439]	Full	Girgin [13]		
	14.43 [0.568]	26.11 [1.028]	Full	ACI 440.2R [1]	Proposed	
	7.35 [0.289]	10.14 [0.399]	Full	Proposed		
	7.80 [0.307]	10.43 [0.411]	Full	Girgin [13]		
	18.35 [0.722]	28.06 [1.105]	Full	ACI 440.2R [1]	Mander *et al.* [*30*]	
	8.46 [0.333]	12.19 [0.480]	Full	Proposed		
	8.90 [0.350]	12.08 [0.476]	Full	Girgin [13]		

Table 4.4- Summary of required design thicknesses for flexural deficient column. Continued.

Model	μ = 8 Thickness mm. [in.]	μ = 10 Thickness mm. [in.]	Length	Strength Model	Strain Model	Notes
Caltrans Memo 20-4 [5]	1.412 [0.056]	1.412 [0.056]	419.5 mm from ends	N/A		N/A
	0.706 [0.028]	0.706 [0.028]	Remaining Column	N/A		N/A
Concrete Society Technical Report No. 55 [8]	0.534 [0.021]	0.534 [0.021]	Full	N/A		N/A
ISIS Canada [18]	3.10 [0.122]	3.10 [0.122]	Full	N/A		N/A
Seible *et al.* [*43*]	N/A	N/A	N/A	N/A	N/A	Shear Strength
	3.632 [0.143]	3.632 [0.143]	381 mm from ends	N/A	N/A	Lap Splice Clamping
	5.11 [0.201]	6.77 [0.267]	419.5 mm from ends	Seible *et al.* [*43*]	N/A	Primary Plastic Hinge Region
	2.56 [0.101]	3.39 [0.133]	419.5 mm from primary region			Secondary Plastic Hinge Region
	14.87 [0.585]	21.57 [0.849]	419.5 mm from ends	Proposed	N/A	Primary Plastic Hinge Region
	7.44 [0.293]	10.79 [0.425]	419.5 mm from primary region			Secondary Plastic Hinge Region
	17.23 [0.678]	48.41 [1.906]	419.5 mm from ends	Girgin [13]	N/A	Primary Plastic Hinge Region
	8.62 [0.339]	24.21 [0.953]	419.5 mm from primary region			Secondary Plastic Hinge Region

4.5 Discussion

As can be seen from Tables 4.3 and 4.4, there is a wide range of recommended design thicknesses from the various approaches. It is notes that these sample columns were retrofitted and tested to determine the level of ductility achievable and therefore an experimental data set can be used for assessment of the results.

For the shear column, experimental tests conducted by Seible and Innamorato [41] used a carbon fiber jacket which consisted of 1.524 mm for the first 305 mm, 1.041 mm for the next 914 mm and 0.61 mm for the middle 610 mm of the specimen. These design thicknesses correspond to the Seible *et al.* [43] design approaches with a desired ductility of 8, but were slightly knocked down to account for conservativeness. Table 4.5 shows the percent error of each approach when comparing the design models for a desired ductility of 10 and the experimental thickness, which was proven to provide a ductility of 10 from the tests.

Table 4.5- Comparison of required design and experimental thicknesses for shear deficient column.

Model	μ = 10 Thickness mm. [in.]	Percent Error	Strength Model	Strain Model	Notes
ACI 440.2R [1]	N/A	N/A	ACI 440.2R [1]	N/A	Axial Design
	N/A	N/A	Proposed	N/A	
	N/A	N/A	Girgin [13]	N/A	
	7.47 [0.294]	390%	ACI 440.2R [1]	ACI 440.2R [1]	Seismic Design
	4.77 [0.188]	213%	Proposed		
	5.07 [0.20]	233%	Girgin [13]		
	5.5 [0.217]	262%	ACI 440.2R [1]	Proposed	
	3.84 [0.151]	152%	Proposed		
	4.02 [0.158]	163%	Girgin [13]		
	8.39 [0.33]	450%	ACI 440.2R [1]	Mander *et al.* [30]	
	5.17 [0.204]	240%	Proposed		
	5.50 [0.217]	262%	Girgin [13]		
Caltrans Memo 20-4 [5]	1.412 [0.056]	-7%	N/A	N/A	Inside Plastic Hinge Region
	0.706 [0.028]	-53%	N/A	N/A	Outside Plastic Hinge Region
Concrete Society Technical Report No. 55 [8]	0.534 [0.021]	-65%	N/A	N/A	
ISIS Canada [18]	3.10 [0.122]	103%	N/A	N/A	

Table 4.5- Comparison of required design and experimental thicknesses for shear deficient column. Continued.

Model	μ = 10 Thickness mm. [in.]	Percent Error	Strength Model	Strain Model	Notes
Seible *et al.* [43]	1.60 [0.063]	5%	N/A	N/A	Shear Strength
	N/A	N/A	N/A	N/A	Lap Splice Clamping
	3.21 [0.126]	110%	Seible *et al.* [43]	N/A	Primary Plastic Hinge Region
	1.61 [0.063]	5%			Secondary Plastic Hinge Region
	5.69 [0.224]	273%	Proposed	N/A	Primary Plastic Hinge Region
	2.89 [0.112]	87%			Secondary Plastic Hinge Region
	5.28 [0.208]	247%	Girgin [13]	N/A	Primary Plastic Hinge Region
	2.64 [0.104]	73%			Secondary Plastic Hinge Region

The tests concluded that the carbon FRP wraps significantly increased the ductility of the column from 3 to well over the desired target ductility of 8 to a level around 10. This suggests that even the slightly knocked down design values used for the test were significantly conservative as well. Therefore, a less conservative design approach could be used.

It can be seen that the ACI 440.2R [1] and ISIS Canada [18] design approaches are extremely conservative and yield design thicknesses approximately twice than that from tested specimen. Therefore, these two approaches will be discarded from further consideration in this study. In contrast, the Concrete Society Technical Report No. 55 [8] yields much smaller design thicknesses than the tested specimen, approximately half the value. While this may be a viable design approach that could eliminate the conservatism in the current design approaches, more research would be needed to instill confidence in this method for design.

Alternatively, there is the Caltrans Memo 20-4 [5], which yields design thicknesses slightly less that used for the specimen. The suggested design thicknesses from Caltrans Memo 20-4 [5] are almost exactly the same as the actual thicknesses used in the Seible and Innamorato [41] tests, which yielded very good, conservative results. Combined with the relative ease of this approach in comparison with Seible *et al.* [43] makes Caltrans Memo 20-4 [5] a very practical design approach.

However, the Caltrans Memo 20-4 [5] does not consider the desired level of ductility in the design. Therefore, the design thickness may not be appropriate for ductility levels lower than 10. Figure 4.3 shows the required design thicknesses for various ductilities for the Caltrans Memo 20-4 [5] approach and the Seible *et al.* [43] approach.

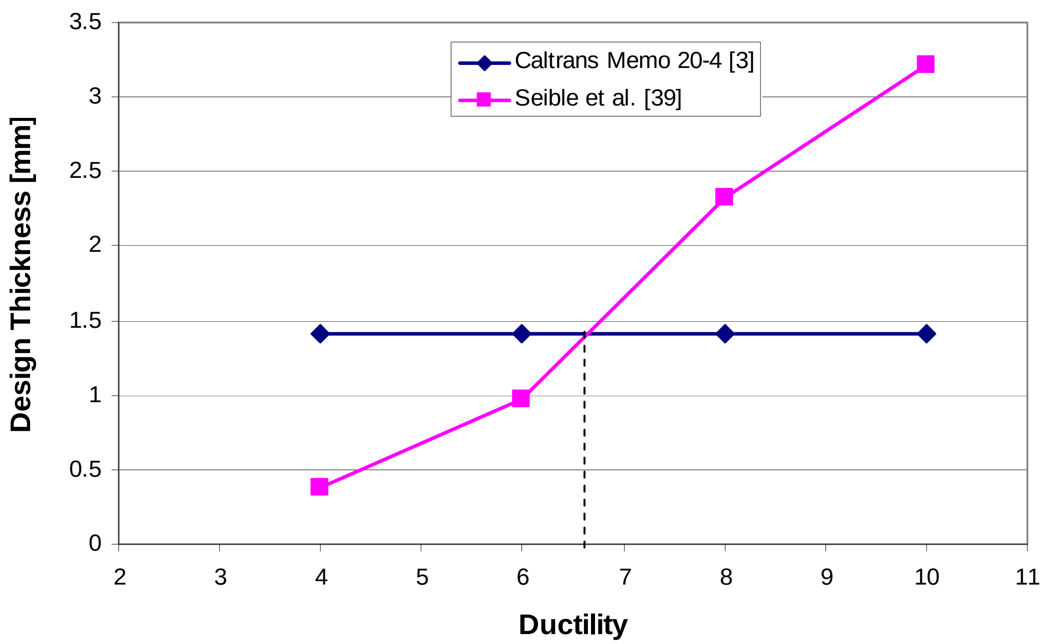

Figure 4.3- Comparison of models based on ductility.

It can be seen from Figure 4.3 that Caltrans Memo 20-4 [5] requires a constant thickness regardless of the desired ductility level; whereas, in the Seible *et al.* [43] approach the thickness decreases significantly as the desired ductility decreases. The Caltrans Memo 20-4 [5] approach was verified through the previously discussed experimental tests at high desired ductilities. Therefore, for design purposes for a shear column Caltrans Memo 20-4 [5] should be used when the desired ductility is above 6.5 to minimize material used and the Seible *et al.* [43] approach should be used for ductility level below 6.5. However, since the Seible *et al.* [43] is very complex and mathematically in depth, it is possible based on Figure 4.4 to use a reduction factor along with the Caltrans Memo 20-4 [5] approach when the desired ductility is below 6.5. If the Seible *et al.* [43] is approximated as linear with ductility, then to calculate the required thickness when the desired ductility is less than 6.5 the following expression could be used derived from the Caltrans Memo 20-4 [5] approach:

$$t_f = \frac{f_l D}{2E_f \varepsilon_{cc}} - \left(\frac{f_l D}{2E_f \varepsilon_{cc}} - 0.4925\,\mu - 1.725 \right) \qquad \text{Equation 4.26}$$

where f_l = 2,068 kPa (300 psi) and radial dilating strain, ε_{cc}, of 0.004 inside the plastic hinge zone and f_l = 1,034 kPa (150 psi) and ε_{cc} = 0.004 outside the plastic hinge zone. When the desired ductility is greater than 6.5, the regular Caltrans Memo 20-4 [5] could be used.

For the flexural column, experimental tests conducted by Seible *et al.* [40] used a carbon fiber jacket, which consisted of 5.08 mm for the first 457 mm, 2.54 mm for the next 457 mm of the specimen. These design thicknesses correspond to the Seible *et al.* [43] design approaches with a desired ductility of 8. Table 4.6 shows the percent error of each approach when comparing the design models for a desired ductility of 10 and the experimental thickness, which is proven to provide a ductility of 10 from the tests.

Table 4.6- Comparison of required design and experimental thicknesses for flexural deficient column.

Model	μ = 10 Thickness mm. [in.]	Percent Error	Strength Model	Strain Model	Notes
ACI 440.2R [1]	N/A	N/A	ACI 440.2R [1]	N/A	Axial Design
	N/A	N/A	Proposed	N/A	
	N/A	N/A	Girgin [13]	N/A	
	31.49 [1.24]	520%	ACI 440.2R [1]	ACI 440.2R [1]	Seismic Design
	10.98 [0.432]	116%	Proposed		
	11.14 [0.439]	120%	Girgin [13]		
	26.11 [1.028]	414%	ACI 440.2R [1]	Proposed	
	10.14 [0.399]	100%	Proposed		
	10.43 [0.411]	106%	Girgin [13]		
	28.06 [1.105]	453%	ACI 440.2R [1]	Mander *et al.* [30]	
	12.19 [0.480]	140%	Proposed		
	12.08 [0.476]	138%	Girgin [13]		
Caltrans Memo 20-4 [5]	1.412 [0.056]	-72%	N/A	N/A	N/A
	0.706 [0.028]	-86%	N/A	N/A	N/A
Concrete Society Technical Report No. 55 [8]	0.534 [0.021]	-90%	N/A	N/A	N/A
ISIS Canada [18]	3.10 [0.122]	-39%	N/A	N/A	N/A
Seible *et al.* [43]	N/A	N/A	N/A	N/A	Shear Strength
	3.632 [0.143]	-29%	N/A	N/A	Lap Splice Clamping

Table 4.6- Comparison of required design and experimental thicknesses for flexural deficient column. Continued.

Model	μ = 10 Thickness mm. [in.]	Percent Error	Strength Model	Strain Model	Notes
Seible *et al.* [43]	6.77 [0.267]	34%	Seible *et al.* [43]	N/A	Primary Plastic Hinge Region
	3.39 [0.133]	-34%			Secondary Plastic Hinge Region
	21.57 [0.849]	325%	Proposed	N/A	Primary Plastic Hinge Region
	10.79 [0.425]	113%			Secondary Plastic Hinge Region
	48.41 [1.906]	853%	Girgin [13]	N/A	Primary Plastic Hinge Region
	24.21 [0.953]	377%			Secondary Plastic Hinge Region

The tests concluded that the carbon FRP wraps significantly increased the ductility of the column from 3 to well over the desired target ductility of 8 to a level around 10. This suggests that even the slightly knocked down design values used for the test were significantly conservative as well. Therefore, a less conservative design approach could be used.

It can be seen that the ACI 440.2R [1] and the Concrete Society Technical Report No. 55 [8] and the variations of Seible *et al.* [43] design approaches are extremely conservative and yield design thicknesses much thicker than the tested specimen. In contrast, the Caltrans Memo 20-4 [5] and the ISIS Canada [18] approaches yield much smaller design thicknesses than that used in the tested specimen. This may be a viable design approach that could eliminate the conservatism in the current design approaches, more research would be needed to instill confidence on this method for design.

The two models that produce design thicknesses with the least amount of error are the ACI 440.2R [1] approach modified using the proposed strength of confined concrete and the proposed confined strain models and the Seible *et al.* [43] design approach. The modified ACI 440.2R [1] approach yields a conservative design thickness; whereas, the Seible *et al.* [43] approach is very slightly non-conservative in the secondary plastic hinge region. However, the test specimen thickness was determined to be conservative itself, so a very slightly non-conservative design approach may be acceptable in this context.

4.6 Conclusion

It can be seen from the example design problems above that there are significant differences in the various design approaches. The design thicknesses ranged from 0.534 mm to 7.47 mm for the shear column example and from 0.534 mm to 48.41 mm for the flexural column example. This proves the need for more research in the design code approaches used and the need to unify and find a universally accepted approach. One major problem with the design guidelines today is the over conservatism of most codes and guideline due to the lack of knowledge and confidence with the current design codes and guidelines. With more research, that confidence will go up and the overall conservatism will decrease, which will lead to more efficient designs and a decrease in overall cost of material, which will make FRP jackets even more desirable than the steel alternative.

For this research, it was determined that the Caltrans Memo 20-4 [5] design approach is the most practical model for a shear column with a desired ductility greater than 6.5, due to its slight conservatism and simplicity. Simplicity of the design calculations will be very important for this application in order to help speed and ease the process of emergency repair. However, when the desired ductility is less than 6.5, the Caltrans Memo 20-4 [5] is much more conservative, since it does not account for the desired ductility in the design. Therefore, for shear column with a desired ductility below 6.5, it is more practical to use the Seible *et al.* [43] design approach. However, the Caltrans Memo 20-4 [5] could be used at desired levels of ductility less than 6.5 with a reduction factor using Equation 4.26. This approach allows one to use the more simple approach at lower desired ductilities without sacrificing the level of conservatism.

A shear column or a short column is defined by the ACI 318-08 [2] in section 10.13.2 of the building code from the ratio of the unbraced length, l and the radius of gyration, r. The code states that the slenderness effect may be neglected is the ratio of unbraced length to the radius of gyration, also known as the slenderness ratio, $\frac{l}{r}$ is less than 22, where the radius of gyration is equal to $\sqrt{\frac{I}{A}}$, where I is the moment of inertia and A is the cross sectional area. For a circular column the slenderness ratio is equal to $\frac{4H}{D}$, where H is the height of the column and D is the diameter.

Similarly, for flexural columns it was determined that the Seible *et al.* [43] approach is the most practical for design. These models are only validated for carbon/epoxy FRP jackets. There would need to be additional research conducted to validate these aspects for other FRP systems.

Chapter 5

Strain Based Design

5.1 Introduction

The design models discussed in Chapter 4 can be used for repair of damaged columns; however, it would be beneficial to have a more rapid, simple approach that can be implemented in the field in emergency situations.

This chapter will discuss an alternate design approach, which will determine a design thickness based on the amount of strain in the spiral hoop reinforcement or the longitudinal reinforcement, determined in Chapter 2, and the associated loss of axial capacity in the concrete column.

5.2 Axial Capacity Loss

As presented in Chapter 2, each damage state (DS-1 through DS-5) has a strain range associated with it for the spiral hoop reinforcement and the longitudinal reinforcement. Each strain value can be associated with an axial capacity from a sectional analysis. The axial capacity at a specific strain level can be compared to the original theoretical capacity of the column prior to damage, which can be determined using the ACI 318 [2] equations for axial capacity as follows:

$$\phi P_n = 0.8\phi\left[0.85\left(f_c' A_g + f_{ccs}' A_{cc}\right) + f_y A_s\right] \qquad \text{Equation 5.1}$$

where ϕP_n is the axial capacity, ϕ is the reduction factor for the transverse reinforcement, which is equal to 0.75 for spiral hoops, A_g is the gross cross sectional area of the column, A_{cc} is the area confined by the transverse reinforcement, f_y is the yield strength of the longitudinal reinforcement, A_s is the area of longitudinal reinforcement and f'_{ccs} is the confined strength of concrete due to the transverse reinforcement. The confined strength of concrete due to the transverse reinforcement f'_{ccs} can be determined using Richart *et al.*'s [35] relation for steel spiral hoops:

$$f'_{ccs} = f'_c + 4.1 f_l \qquad \text{Equation 5.2}$$

where f_l is the confining pressure provided by the spiral hoops and is calculated as:

$$f_l = \frac{2 A_h f_{sy}}{d \cdot s} \qquad \text{Equation 5.3}$$

where A_h is the cross sectional area of the spiral hoop, f_{sy} is the yield strength of the spiral hoop steel, d is the diameter of the column that is confined by the hoops, which is equal to the gross column diameter, D minus twice the concrete cover, cc minus twice the diameter of the spiral hoop, d_h and s is the spacing of the spiral hoops.

The loss in axial capacity for each damage state is simply the axial capacity calculated from the as-built using Equation 5.1 minus the axial capacity from the sectional analysis at the given strain value for that damage state.

5.3 Design Approach

Using the loss of axial capacity at a given damage state, the required confined strength of the FRP jacket needed to restore the original axial capacity of the undamaged column can be determined. The confining effect of the FRP jacket can be added into the expression for axial capacity, Equation 5.1. The new expression for axial capacity is expressed as:

$$\phi P_n = 0.8\phi\left[0.85\left(f'_c A_g + f'_{ccs} A_{cc} + f'_{cc} A_g\right)\right] \qquad \text{Equation 5.4}$$

The effect of the longitudinal bars is neglected since they can show some level of yielding even at damage state DS-1. Since the loss of axial capacity is known, the required effect of the FRP jacket can be determined. Therefore, the required confined strength of concrete due to the FRP jacket, f'_{cc} can be determined as:

$$f'_{cc} = \frac{\phi P_{no} - \phi P_{new}}{0.8\phi 0.85 A_g} \qquad \text{Equation 5.5}$$

where ϕP_{no} is the initial axial capacity determined from Equation 5.1 and the as-built drawings and ϕP_{new} is the reduced axial capacity for the given damage state from the sectional analysis and the associated strain level.

Once the required confined strength due to the FRP jacket is determined, a design thickness can be calculated using the Caltrans Memo 20-4 [5] approach along with Mander *et al.*'s [30] confined strength model or the confinement models in Chapter 3, so as to compare various approaches.

5.4 Design Example

A sample column was used to demonstrate how this approach can be implemented. Table 5.1 shows the properties of the sample column and the CFRP jacket used to repair the deficiency.

Table 5.1- Sample column and jacket properties.

Column Section Properties	Column Diameter, D	305 mm
	Concrete Cover, cc	19 mm
	Concrete Strength, $f`_c$	34.5 MPa
Longitudinal Reinforcement (Grade 40)	Bar Diameter, d_b	9.5 mm (16 total)
	Bar Area, A_s	71 mm^2
	Yield Strength, f_y	468 MPa
Transverse Reinforcement (Grade 40)	Bar Diameter, d_h	4.92 mm
	Bar Area, A_h	19 mm^2
	Spacing, s	32 mm
Jacket Material Properties	Jacket Modulus, E_f	59.36 GPa
	Ultimate Strength, f_f	752 MPa

The sample column was classified as a DS-4 based on visual examination. The initial axial capacity was determined to equal 2722 kN using Equation 5.1. The associated strain, as determined in Chapter 2 for DS-4 is 0.0348 for the longitudinal reinforcement and 0.00167 for the transverse reinforcement. Using these strain levels, an axial capacity was determined from a sectional analysis using Response 2000 software program. The new axial capacity was 174.9 kN. Using Equation 5.5 the required confined strength due to the FRP jacket was determined to equal 68.36 MPa.

Three approaches were then evaluated using the Caltrans Memo 20-4 [5] approach along with Mander *et al.*'s [30] confined strength model and the two best strength models from Chapter 3, the proposed strength model and Girgin's [13] strength model. The resulting design thicknesses were 5.0 mm, 2.81 mm and 5.67 mm using the Caltrans Memo 20-4 [5] approach, the proposed strength model and Girgin's [13] strength model, respectively. Individual calculations for each approach are detailed in Appendix B. The Caltrans Memo 20-4 [5] and Girgin's [13] strength model approaches are very conservative in comparison with the proposed strength model approach.

5.5 Discussion

The column described in the design example above was tested by Vosooghi and Saiidi [50]. The Caltrans Memo 20-4 [5] approach from Chapter 4 was used, not the modified approach described within this chapter. The design thickness was 2.03 mm. Vosooghi and Saiidi [50] concluded that repair design was effective in restoring strength and ductility to the damaged column. It was observed that the strength, ductility capacity and drift capacity were restored to at least that of the original column. It was also determined that 87% of the stiffness was restored.

When the column design that was tested is compared to the design thicknesses produced from the approaches discussed in this chapter, it is concluded that the proposed strength model approach is most accurate in determining design thickness. The Caltrans Memo 20-4 [5] approach and Girgin's [13] strength model are much too conservative. The proposed strength model approach is slightly conservative for the DS-4 design. However, since the approach used by Vosooghi and Saiidi [50] is constant for all damage states and the proposed model approach design thickness decreases as the damage state decreases, the proposed strength model approach should be less conservative for lower damage state levels and is actually based on changed in damage rather that use of a "one-size-fits-all" approach. The proposed strength model can be simplified in terms of thickness as follows:

$$t_f = \frac{1}{2} \cdot \left[\frac{13899}{5030} \cdot f_c' - \frac{1}{5030} \cdot \left(240811270 \cdot f_c' - 5.03 \cdot 10^7 \cdot f_{cc}' \cdot' f_c' \right)^{\frac{1}{2}} \right] \cdot \frac{D}{f_f}$$

Equation 5.6

5.6 Conclusion

A design approach for each damage state was derived using the strain levels expected for each damage state. The strain was related to a loss in axial capacity using sectional analysis, which was used to design an FRP jacket for repair.

Three design approaches were considered and compared. The Caltrans Memo 20-4 [5], the proposed strength model and Girgin's [13] strength model were used to calculate a jacket thickness for an experimental column. It was determined that the proposed strength model is a more accurate design approach. This design approach provides a simple method to determine a jacket thickness based on the level of damage observed in the column.

Chapter 6

Decision Tree

6.1 Decision Tree

Figure 6.1 shows the final basis for the decision tree using the proposed strain based design discussed in Chapter 5. This decision tree will aide engineers in determining the level of damage to a column through visual inspection, determine whether repair using an FRP jacket is appropriate, and then design the FRP jacket if repair is appropriate.

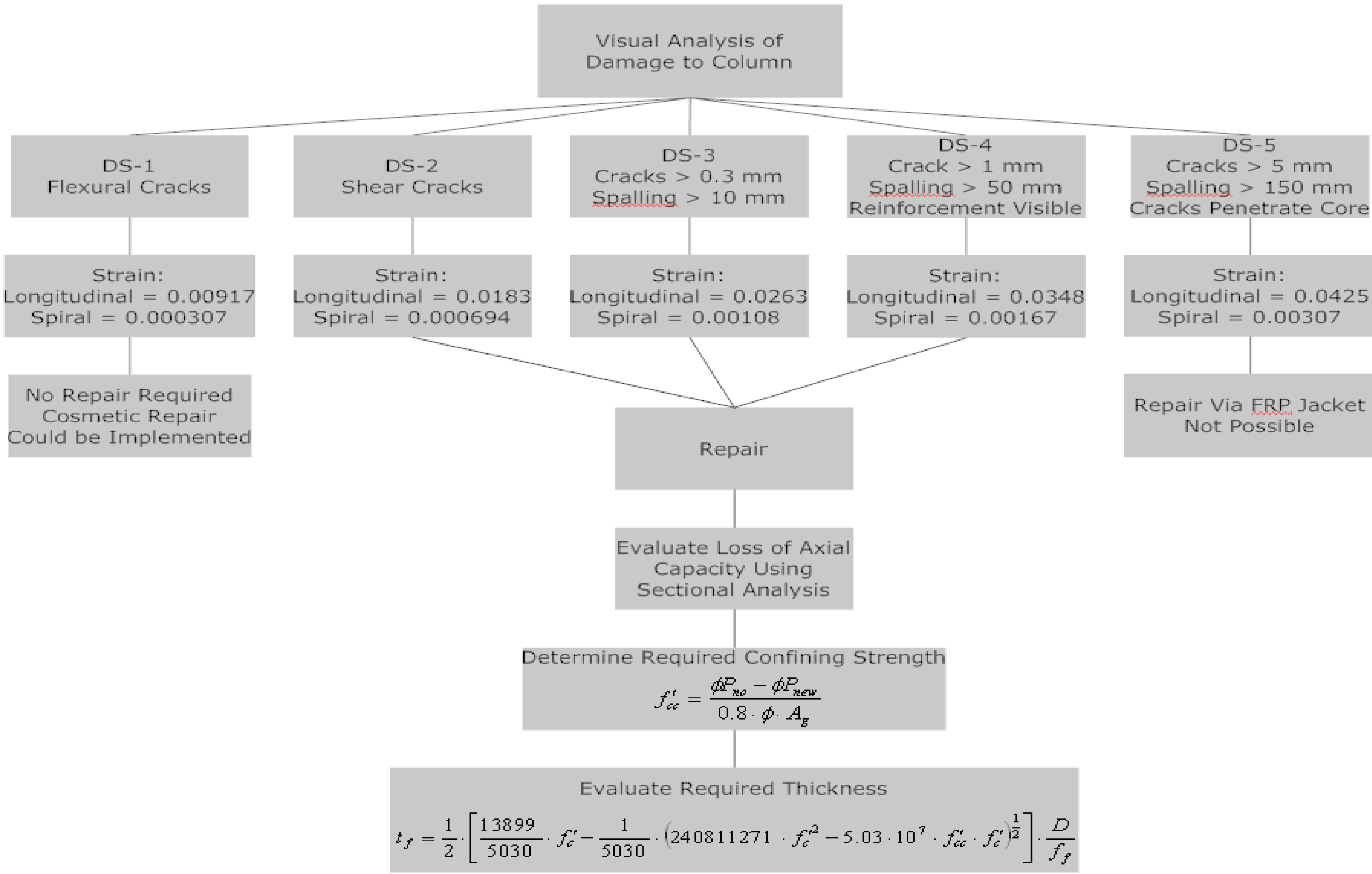

Figure 6.1- Decision tree for emergency repair.

Chapter 7

Example

7.1 Example

The following example is a reinforced concrete column with a diameter of 305 mm. The step-by-step design example shows how to calculate the different design thicknesses for each damage state level.

Column Properties:

$D := 305mm$

$cc := 19mm$

$A_s := 16 \cdot 71mm^2 \qquad A_s = 1.136 \times 10^3 mm^2$

$d_h := 4.877mm \qquad d_h = 0.192 in$

$d_s := D - 2 \cdot cc - 2 \cdot d_h \qquad d_s = 257.246 mm$

$f'_c := 34.48MPa$

$A_h := \frac{\pi \cdot (d_h)^2}{4} \qquad A_h = 18.681 mm^2 \qquad A_h = 0.029 in^2$

$E_s := 29000ksi \qquad E_s = 1.999 \times 10^5 MPa$

$s := 32mm$

$f_{yh} := 413.685MPa \qquad f_{yh} = 60 ksi$

$f_{ys} := 468.43MPa \qquad f_{ys} = 67.94 ksi$

Composite Properties:

$f_f := 752MPa \qquad f_f = 109.068 ksi$

$E_f := 59360MPa \qquad E_f = 8.609 \times 10^3 ksi$

Confining Pressure Provided by Steel Hoops:

$f_h := \frac{2A_h \cdot f_{yh}}{d_s \cdot s}$

$f_h = 272.319 psi \qquad f_l := f_h \qquad f_l = 1.878 MPa$

Initial Axial Capacity:

$f'_{ccs} := f'_c + 4.1f_l \qquad f'_{ccs} = 42.178 MPa$

$A_g := \frac{\pi \cdot D^2}{4} \qquad A_g = 7.306 \times 10^4 mm^2$

$A_{cc} := \frac{(d_s)^2 \cdot \pi}{4} \qquad A_{cc} = 5.197 \times 10^4 mm^2$

$\phi_s := 0.75$ for spiral hoops

$\phi P_{no} := 0.8 \cdot 0.85 \cdot \phi_s (f'_c \cdot A_g + f'_{ccs} \cdot A_{cc}) + 0.8 \cdot \phi_s \cdot A_s \cdot f_{ys}$

$\phi P_{no} = 2.722 \times 10^3 kN$ Initial Axial Capacity

Proposed Stregth Model:

$$f'_{cc} \equiv f'_c \cdot \left[\left[-0.503 \cdot \left(\frac{2 \cdot f_f \cdot t_f}{D \cdot f'_c}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot f_f \cdot t_f}{D \cdot f'_c}\right) + 0.9469\right]\right]$$

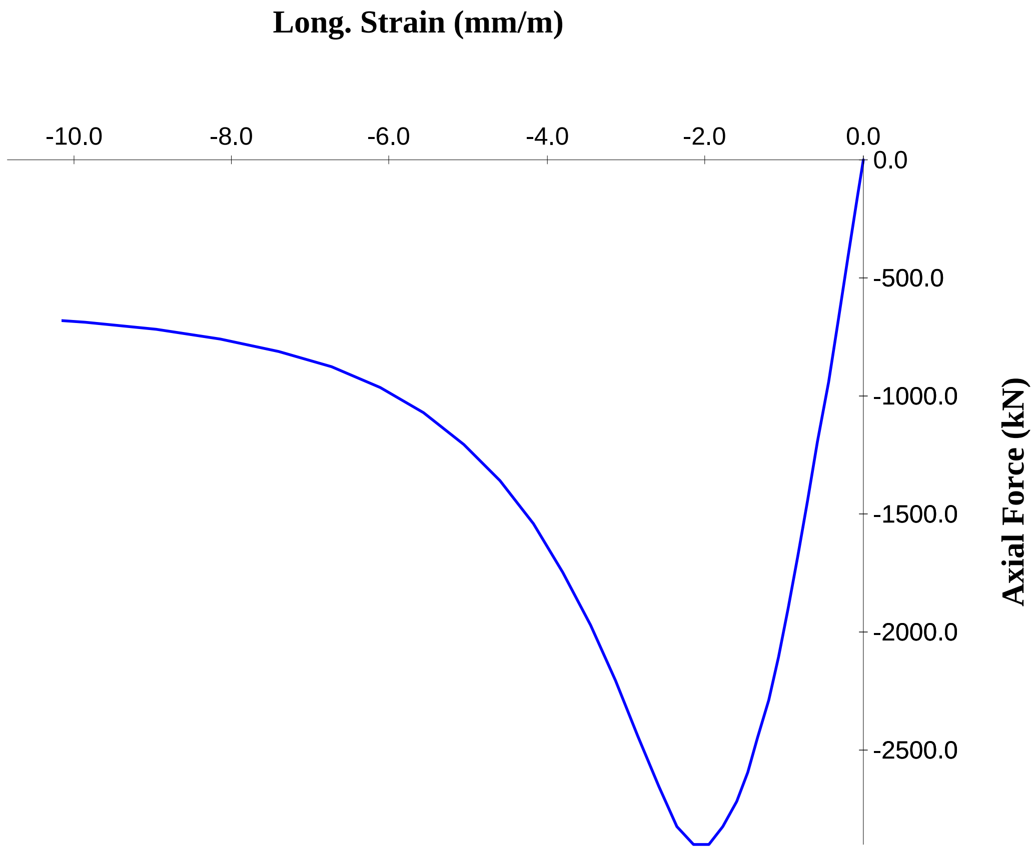

Figure 7.1- Axial capacity of column versus longitudinal strain from sectional analysis program, Response 2000 [4].

DS-1

New column capacity:

$$\phi P_{new} := 689.2kN$$

Required strength from jacket:

$$f'_{cc} := \frac{\phi P_{no} - \phi P_{new}}{0.8 \cdot 0.85 \cdot \phi_s A_g}$$

$$f'_{cc} = 54.557\,MPa$$

Proposed Strength Model

$$54.557 = 34.48 \cdot \left[-0.503 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48}\right) + 0.9469\right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} 1.6704386609774765960 \\ 36.972026653518526111 \end{pmatrix}$$

$t_f := 1.67mm$ $\qquad$ $t_f = 0.066\,in$

DS-2

New column capacity:

$$\phi P_{new} := 338.7kN$$

Required strength from jacket:

$$f'_{cc} := \frac{\phi P_{no} - \phi P_{new}}{0.8 \cdot 0.85 \cdot \phi_s A_g}$$

$$f'_{cc} = 63.963\,MPa$$

Proposed Strength Model

$$63.963 = 34.48 \cdot \left[-0.503 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48}\right) + 0.9469\right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} 2.4382674584921689135 \\ 36.204197856003833794 \end{pmatrix}$$

$t_f := 2.44mm$ $\qquad$ $t_f = 0.096\,in$

DS-3

New column capacity:

$$\phi P_{new} := 233.3kN$$

Required strength from jacket:

$$f'_{cc} := \frac{\phi P_{no} - \phi P_{new}}{0.8 \cdot 0.85 \cdot \phi_s A_g}$$

$$f'_{cc} = 66.792\,MPa$$

Proposed Strength Model

$$66.792 = 34.48 \cdot \left[-0.503 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48}\right) + 0.9469\right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} 2.6761309068788307940 \\ 35.966334407617171913 \end{pmatrix}$$

$t_f := 2.68mm$ $\qquad$ $t_f = 0.106\,in$

DS-4

New column capacity:

$$\phi P_{new} := 174.9kN$$

Required strength from jacket:

$$f'_{cc} := \frac{\phi P_{no} - \phi P_{new}}{0.8 \cdot \phi_s \cdot 0.85 A_g}$$

$$f'_{cc} = 68.359\,MPa$$

Proposed Strength Model

$$68.359 = 34.48 \cdot \left[-0.503 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48} \right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 752 \cdot t_{fl}}{305 \cdot 34.48} \right) + 0.9469 \right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} 2.8093594791505366243 \\ 35.833105835345466083 \end{pmatrix}$$

$t_f := 2.81mm$ $\qquad$ $t_f = 0.111\,in$

It can be seen from this simple design example that the design thickness will increase as the damage state becomes more severe. The resulting design thicknesses are 1.67 mm, 2.44 mm, 2.68 mm and 2.81 mm for DS-1, DS-2, DS-3 and DS-4, respectively.

Chapter 8

Conclusions and Future Research

8.1 Conclusions

A basis for a decision tree was presented to help engineers quickly design a CFRP jacket for damaged reinforced concrete columns for emergency repair. The decision tree first determines a level of damage based on visual inspection of the damaged column. The damage states were classified into five levels, DS-1 through DS-5, based on the level of visual damage. Strain ranges for longitudinal and transverse reinforcement associated with each damage state level were determined. From these strain levels, it was determined that DS-1 would not require repair, unless one layer of jacket is desired for aesthetic reasons. It was also concluded that DS-5 would not be appropriate to repair using an FRP wet layup jacket, since the damage has penetrated into the concrete core and transverse reinforcement has yielded.

Modeling concrete confinement from an FRP jacket is considered because FRP does not behave like the traditional steel jackets, since they are perfectly elastic to failure. Therefore, several existing confinement models were analyzed and compared with a large database of existing experimental data. In addition, from the experimental database, a new confinement model for confined strength of concrete and the associated strain in the concrete are proposed, Equations 3.64 and 3.65. It was concluded that the proposed models performed more accurately than the existing models based on the root mean squared error (RMSE) analysis and the coefficient of variance (R^2) analysis. The second best models were Girgin [13] for confined strength and Mander *et al.* [30] for the associated strain.

Several seismic retrofit design guidelines and approaches were analyzed and also modified using the confinement models deemed most accurate earlier. Two sample columns, one deficient in shear, and the other deficient in flexure were used as test cases. It was concluded that, when design thicknesses were compared with experimental data from a column tested that for a shear column, the Caltrans Memo 20-4 [5] approach was most appropriate when the desired ductility was above 6.5 and the Seible *et al.* [43] approach is most suitable when the desired ductility is below 6.5. However, it was proposed that the Caltrans Memo 20-4 [5] could be used for lower ductilities with a reduction factor to account for the over conservativeness. Similarly, for a flexural column, it was determined that the Seible *et al.* [43] approach was most appropriate for all cases.

In contrast to the seismic design approaches, a new approach using the strain levels in the reinforcement at each damage state was proposed. This strain based design, used sectional analysis to determine the decrease in axial load carrying capacity of the column at the strain level for the damage state being considered. Using the loss of axial capacity, a required confined strength that the FRP jacket would need to restore the original axial capacity was determined. From the required confined strength, a design thickness can be found. Three approaches were analyzed, the Caltrans Memo 20-4 [5], the proposed strength model and Girgin's [13] strength model. When compared with an experimental column that was classified as damage state DS-4, which was repaired using a CFRP jacket and then tested to failure, the proposed strength model was the most appropriate approach in determined a design thickness. The main advantage to the strain based approach, as opposed to the seismic retrofit design is that the design thickness decrease as the damage state decreases; therefore, better utilizing the material and creating a more economical design.

8.2 Considerations for Construction

Before this research is implemented, detailed guidelines for construction should be generated. Field conditions and construction approaches can greatly affect the performance of FRP jackets. A few suggestions for construction are presented here.

Column surfaces should be cleaned and loose concrete should be cleared before applying the FRP jacket. Large holes or spalled areas should be filled with an epoxy grout to create a smooth surface for the FRP to bond to. Quality control measures should be employed to ensure the FRP jacket is applied correctly. Measures should be taken to monitor cure conditions to ensure full cure before opening the bridge for use.

8.3 Future Research

The goal of this thesis was to move forward in the research areas discussed within and to present a baseline for a decision tree to be expanded upon. There are several areas that could use more research to validate or expand the research presented here. The following areas could use more research:

1. More extensive research for the strain levels associated with each damage state is needed. The strain levels presented within this thesis are based on a very limited set of tests.

2. Most tests conducted to validate the damage state strain, the design models and the strain based design approach were conducted on small scale specimens. More research to validate using large scale specimens is needed.
3. Design models and strain based design were validated for carbon/epoxy FRP jackets only. Research to validate equations for GFRP and AFRP is needed.
4. Experimental validation of the decision tree is needed to eliminate further conservatism.
5. The effect of bond between the FRP jacket and the concrete needs to be explored.
6. Research to further expand the decision tree for other column geometries is needed.

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Appendix A

Shear Design Example

Column Section Properties:

$H := 2438mm$ Column Height

$D := 610mm$ Column Diameter

$cc := 20.32mm$ Concrete Cover

$f'_c := 34.45MPa$ Unconfined Concrete Strength

$E_c := 27580MPa$ Modulus of Concrete

Longitudinal Reinforcement Properties (Grade 40):

$d_c := 19mm$ Bar Diameter, 26 in total

$A_s := 284mm^2$ Total Bar Area

$f_y := 303.4MPa$ Yield Strength

Transverse Reinforcement Properties (Grade 40):

$d_h := 6.35mm$ Bar Diameter

$A_h := 31.7mm^2$ Bar Area

$s := 127mm$ Spacing

Column Load Properties:

$P := 591.61kN$ Axial Load

$M_{yi} := 646.73kN \cdot m$ Yield Moment

$\Phi_y := 0.005984 \cdot \frac{1}{m}$ Yield Curvature

$c_u := 152.4mm$ Neutral Axis

Jacket Material Properties:

$E_f := 124.1GPa$ Jacket Modulus

$f_f := 1310MPa$ Jacket Ultimate Strength

$\varepsilon_f := \frac{f_f}{E_f}$ $\varepsilon_f = 0.011$ Jacket Ultimate Strain

ACI 440.2R-02 (2002)

Axial Capacity Design (

$k_e := 0.85$ For spiral reinforced columns

$\phi := 0.75$

$\Psi_f := 0.95$ For wrapped column jackets

$A_g := \frac{\pi \cdot D^2}{4}$ $A_g = 0.292\,m^2$

$\phi \cdot P_n = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot (A_g - A_s) + f_y \cdot A_s\right]$

Want ϕPn to equal axial load (591.61kN)

$P = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot (A_g - A_s) + f_y \cdot A_s\right]$

$$f'_{cc} := \frac{\frac{P}{k_e \cdot \phi} - f_y \cdot A_s}{0.85 \cdot \Psi_f \cdot (A_g - A_s)}$$

$f'_{cc} = 3.571\,MPa$ Required confining strength to achieve axial load given

$k_s := 1.0$ For circular columns

$\rho_f = \frac{4 \cdot t_f}{D}$ Volumetric ratio of FRP

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2}$ $f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$ Confining Pressure

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$3.571 \cdot MPa = 34.45 \cdot MPa \cdot \left[-1.25 + 2.25 \cdot \left[\sqrt{1 + 7.9 \cdot \frac{2 \cdot t_f \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_f \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}\right]\right] \text{solve}, t_f \rightarrow \begin{bmatrix} -\left(2.2796747425512636262 \cdot 10^{-3}\right) \cdot m \\ 3.1349756833606863946 \cdot 10^{-2} \cdot m \end{bmatrix}$$

Thickness is negative; therefore, no FRP required for axial capacity. $t_f := 0$

Seismic Design (

Assume given ductility desire is 8

$\mu_\Delta := 8$

$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Double Bending

$L_p := 223.52mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$\mu_\Phi = 15.01$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\quad \varepsilon_{cc} = 0.014$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$f'_{cc} = 71.714MPa$

$\varepsilon_{fd} := \min\left(0.004, 0.75\varepsilon_f\right)$ $\quad \varepsilon_{fd} = 4 \times 10^{-3}$

$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2}$ $\quad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$71.714 \cdot MPa = 34.45 \cdot MPa \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m} - 1.25\right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} 4.9671916430944384411 \cdot 10^{-3} \cdot m \\ .13614733277647647211 \cdot m \end{pmatrix}$$

$t_f := 0.136m$ $\qquad t_f = 136\,mm$

or

$t_f := 0.00497m$ $\qquad t_f = 4.97\,mm$ $\qquad t_f = 0.196\,in$

Assume given ductility desire is 10:

$\mu_\Delta := 10$

$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$ $\qquad$ Plastic Hinge Length for Double Bending

$L_p := 223.52mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$\mu_\Phi = 19.012$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\qquad \varepsilon_{cc} = 0.017$ $\qquad$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$f'_{cc} = 83.489\,MPa$

$\varepsilon_{fd} := \min\left(0.004, 0.75\varepsilon_f\right)$ $\qquad \varepsilon_{fd} = 4 \times 10^{-3}$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$83.489 \cdot \text{MPa} = 34.45 \cdot \text{MPa} \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}}} - 2 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}} - 1.25\right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} .12640816252009413783 \cdot \text{m} \\ 7.4715210453268935931 \cdot 10^{-3} \cdot \text{m} \end{pmatrix}$$

$t_f := 0.126\text{m}$ $\qquad t_f = 126\,\text{mm}$

or

$t_f := 0.00747\text{m}$ $\qquad t_f = 7.47\,\text{mm}$ $\qquad t_f = 0.294\,\text{in}$

Caltrans Memo 20-4 (2000)

To achieve ductility of 6:

Inside Plastic Hinge:

$f_l := 2068 \cdot 10^3 \text{Pa}$ Recommended Confining Pressure

$\varepsilon_{cc} := 0.004$ Recommended Confined Strain of Concrete

$$t_f := \frac{f_l \cdot D}{2 \cdot 0.9 \cdot E_f \cdot \varepsilon_{cc}}$$

$t_f = 1.412\,\text{mm}$ $\qquad t_f = 0.056\,\text{in}$

Outside Plastic Hinge:

$f_l := 1034 \cdot 10^3 \text{Pa}$ Recommended Confining Pressure

$\varepsilon_{cc} := 0.004$ Recommended Confined Strain of Concrete

$$t_f := \frac{f_l \cdot D}{2 \cdot 0.9 \cdot E_f \cdot \varepsilon_{cc}}$$

$t_f = 0.706\,\text{mm}$ $\qquad t_f = 0.028\,\text{in}$

Concrete Society Technical Report No. 55 (2004)

$f_l := 0.183 \frac{mm^2}{N}$ Recommended Confining Pressure

$$t_f := \frac{f_l \cdot D \cdot f'_c{}^2}{2 \cdot E_f}$$

$t_f = 0.534\,mm$ $t_f = 0.021\,in$

ISIS Canada Network of Centers of Excellence (2001)

$\alpha_{pc} := 1.0$ For circular columns

$f_{lmax} := \frac{0.29 f'_c}{\alpha_{pc}}$ Recommended Maximum Confining Pressure

$f_{lmax} = 9.991\,MPa$

$\phi_{frp} := 0.75$

$$f_l = \frac{2 \cdot \phi_{frp} \cdot E_f \cdot \varepsilon_f \cdot t_f}{D}$$

$$t_f := \frac{f_{lmax} \cdot D}{2 \cdot \phi_{frp} \cdot E_f \cdot \varepsilon_f}$$

$t_f = 3.101\,mm$ $t_f = 0.122\,in$

Associated Confined Strength of Concrete, f'cc:

$\phi_c := 0.6$

$w_w := \frac{2 \cdot f_{lmax}}{\phi_c \cdot f'_c}$ $w_w = 0.967$

$$f'_{cc} := f'_c \cdot \left(1 + \alpha_{pc} \cdot w_w\right)$$

$f'_{cc} = 67.752\,MPa$

Seible *et al.* (1997)

Shear Strengthening

$$V_o := \frac{1.5 \cdot M_{yi} \cdot 2}{H}$$ Shear Demand

$V_o = 795.812\,kN$

$V_c := 0kN$ Ignore Shear Contribution from Concrete, since concrete is damaged

$\theta := 45deg$ Assumed angle of shear crack, this is conservative

$D' := D - 2 \cdot cc - d_h$ Effective Column Diameter

$$V_s := \frac{\pi}{2} \cdot \frac{A_h \cdot f_y \cdot D'}{s} \cdot \cot(\theta)$$ Shear Contribution from Steel Reinforcement

$V_s = 66.974\,kN$

$$V_p := \frac{P \cdot (D - c_u)}{H}$$ Shear Contribution from Axial Load

$V_p = 111.042\,kN$

$\phi_v := 0.85$

$$t_v := \frac{\frac{V_o}{\phi_v} - (V_c + V_s + V_p)}{\frac{\pi}{2} \cdot 0.004 \cdot E_f \cdot D}$$

$t_v = 1.594\,mm$ $t_f = 0.122\,in$

Flexural Strengthening

Assume Desired Ductility is 8

$\mu_{\Delta} := 8$

$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Double Bending

$L_p := 223.52\text{mm}$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$\mu_{\Phi} = 15.01$

$\varepsilon_{cu} := \mu_{\Phi} \cdot \Phi_y \cdot c_u$

$\varepsilon_{cu} = 0.014$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$\phi_f := 0.9$

$f'_{cc} := 1.5 \cdot f'_c$ Assumed Confined Strength of Concrete, conservative

$$t_{c1} := \frac{0.09 \cdot D \cdot (\varepsilon_{cu} - 0.004) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$t_{c1} = 2.331\,\text{mm}$ $t_f = 0.122\,\text{in}$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 1.166\,\text{mm}$ $t_f = 0.122\,\text{in}$

Assume Desired Ductility if 10

$\mu_\Delta := 10$

$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Double Bending

$L_p := 223.52mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$\mu_\Phi = 19.012$

$\varepsilon_{cu} := \mu_\Phi \cdot \Phi_y \cdot c_u$

$\varepsilon_{cu} = 0.017$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$\phi_f := 0.9$

$f'_{cc} := 1.5 \cdot f'_c$ Assumed Confined Strength of Concrete, conservative

$$t_{c1} := \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$t_{c1} = 3.21\ mm$ $t_{c1} = 0.126in$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 1.605mm$ $t_{c2} = 0.063in$

Lap Splice Clamping

This column does not contain a lap splice

Modifications using the top two confinement models described in chapter 3

Top Strength Models:

Proposed Model:

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

Girgin's Hoek-Brown Model:

$$f_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

Top Strain Models:

Proposed Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1.0427 \cdot \left(\frac{f'_{cc}}{f'_c}\right)^2 - 1.1181 \cdot \frac{f'_{cc}}{f'_c} + 6.1949\right]$$

Mander *et al.*'s Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc}}{f'_c} - 1\right)\right]$$

ACI 440.2R-02 (2002)

Axial Capacity Design Using Proposed Strength Model (

$k_e := 0.85$ For spiral reinforced columns

$\phi := 0.75$

$\Psi_f := 0.95$ For wrapped column jackets

$A_g := \frac{\pi \cdot D^2}{4}$ $A_g = 0.292\,m^2$

$$\phi \cdot P_n = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot (A_g - A_s) + f_y \cdot A_s\right]$$

Want ϕPn to equal axial load (591.61kN)

$$P = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot (A_g - A_s) + f_y \cdot A_s\right]$$

$$f'_{cc} := \frac{\frac{P}{k_e \cdot \phi} - f_y \cdot A_s}{0.85 \cdot \Psi_f \cdot (A_g - A_s)}$$

$f'_{cc} = 3.571\ \mathrm{MPa}$ Required confining strength to achieve axial load given

$k_s := 1.0$ For circular columns

$\rho_f = \frac{4 \cdot t_f}{D}$ Volumetric ratio of FRP

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2}$ $f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$ Confining Pressure

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469 \right]$$

$$3.571 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{ solve}, t_{f2} \rightarrow \begin{pmatrix} -2.3124451284916043324 \\ 46.638934361035273614 \end{pmatrix}$$

Thickness is negative; therefore, no FRP required for axial capacity. $t_f := 0$

Axial Capacity Design Using Girgin's Strength Model (

Girgin Goek-Brown Model:

$$f'_{cc} = f_l + \sqrt{f'^2_c + 3.5 \cdot f'_c \cdot f_l}$$

$$3.571 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow -2.0050600526625523667$$

Thickness is negative; therefore, no FRP required for axial capacity. $t_f := 0$

Seismic Design Using ACI 440 Strain Model and Proposed Strength Model

Assume given ductility desire is 8

$$\mu_{\Delta} := 8$$

$$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Double Bending

$$L_p := 223.52\text{mm}$$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$$\mu_{\Phi} = 15.01$$

$$\varepsilon_{cc} := \mu_{\Phi} \cdot \Phi_y \cdot c_u \qquad \varepsilon_{cc} = 0.014$$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$$f'_{cc} = 71.714\text{MPa}$$

$$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f) \qquad \varepsilon_{fd} = 4 \times 10^{-3}$$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$71.714 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 3.5602451623889862144 \\ 40.766244070154683067 \end{pmatrix}$$

$t_f := 40.77mm$

or

$t_f := 3.56mm$ $\qquad t_f = 0.14\,in$

Assume given ductility desire is 10:

$\mu_\Delta := 10$

$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$ $\qquad$ Plastic Hinge Length for Double Bending

$L_p := 223.52mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$\mu_\Phi = 19.012$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\qquad \varepsilon_{cc} = 0.017$ $\qquad$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$f'_{cc} = 83.489\,MPa$

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\qquad \varepsilon_{fd} = 4 \times 10^{-3}$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$83.489 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{ solve}, t_{f2} \rightarrow \begin{pmatrix} 4.7748661195058329309 \\ 39.551623113037836351 \end{pmatrix}$$

$t_f := 39.55mm$

or

$t_f := 4.77mm$ $\quad t_f = 0.188in$

Seismic Design Using ACI 440 Strain Model and Proposed Strength Model (

Assume Desired Ductility is 8

$f'_{cc} := 71.714MPa$ From above calculations

$$f'_{cc} = f_l + \sqrt{f'^{2}_{c} + 3.5 \cdot f'_c \cdot f_l}$$

$$71.714 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 3.7131984785569819563$$

$t_f := 3.71mm$ $\quad t_f = 0.146in$

Assume Desired Ductility is 10

$f'_{cc} := 83.489MPa$ From above calculations

$$f'_{cc} = f_l + \sqrt{f'^{2}_{c} + 3.5 \cdot f'_c \cdot f_l}$$

$$83.489 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 5.0662165526023944625$$

$t_f := 5.07mm$ $\quad t_f = 0.2in$

Seismic Design Using Proposed Strain Model and ACI 440 Strength Model

Assume given ductility desire is 8:

$$\mu_\Delta := 8$$

$$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Double Bending

$$L_p := 223.52\text{mm}$$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$$\mu_\Phi = 15.01$$

$$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u \qquad \varepsilon_{cc} = 0.0137$$ Confined Concrete Strain

Proposed Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1.0427 \cdot \left(\frac{f'_{cc}}{f'_c}\right)^2 - 1.1181 \cdot \frac{f'_{cc}}{f'_c} + 6.1949\right]$$

$$0.0137 = 0.002 \cdot \left[6.1949 + 1.0427 \cdot \left(\frac{f'_{cc1}}{34.45}\right)^2 - 1.1181 \cdot \frac{f'_{cc1}}{34.45}\right] \text{solve}, f'_{cc1} \rightarrow \begin{pmatrix} -14.496040014618419466 \\ 51.437197586307304093 \end{pmatrix}$$

$$f'_{cc} := 51.44\text{MPa}$$ Required Confined Strength of Concrete

$$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f) \qquad \varepsilon_{fd} = 4 \times 10^{-3}$$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$51.44\cdot MPa = 34.45\cdot MPa\cdot\left[2.25\cdot\sqrt{1+7.9\cdot\frac{2\cdot t_{f1}\cdot\left(4\cdot 10^{-3}\cdot 124.1\cdot GPa\right)}{34.45\cdot MPa\cdot 0.61\cdot m}}-2\cdot\frac{2\cdot t_{f1}\cdot\left(4\cdot 10^{-3}\cdot 124.1\cdot GPa\right)}{34.45\cdot MPa\cdot 0.61\cdot m}-1.25\right]\text{solve}, t_{f1} \rightarrow \begin{pmatrix}1.817591885211725768 7\cdot 10^{-3}\cdot m\\ .15175376170438335884\cdot m\end{pmatrix}$$

$t_f := 0.152m$ $\quad$ $t_f = 152mm$

or

$t_f := 0.00182m$ $\quad$ $t_f = 1.82mm$ $\quad$ $t_f = 0.072in$

Assume given ductility desire is 10:

$\mu_\Delta := 10$

$L_p = 0.08\cdot\frac{H}{2} + 0.022\cdot f_y\cdot d_b$ $\quad$ Plastic Hinge Length for Double Bending

$L_p := 223.52mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3\cdot\left(\frac{L_p}{\frac{H}{2}}\right)\cdot\left[1-0.5\cdot\left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$\mu_\Phi = 19.012$

$\varepsilon_{cc} := \mu_\Phi\cdot\Phi_y\cdot c_u$ $\quad$ $\varepsilon_{cc} = 0.0173$ $\quad$ Confined Concrete Strain

Proposed Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co}\cdot\left[1.0427\cdot\left(\frac{f_{cc}}{f_c}\right)^2 - 1.1181\cdot\frac{f_{cc}}{f_c} + 6.1949\right]$$

$$0.0173 = 0.002\cdot\left[6.1949 + 1.0427\cdot\left(\frac{f_{cc1}}{34.45}\right)^2 - 1.1181\cdot\frac{f_{cc1}}{34.45}\right]\text{solve}, f_{cc1} \rightarrow \begin{pmatrix}-37.525495209446177463\\ 74.466652781135062090\end{pmatrix}$$

$f_{cc} := 74.47MPa$ $\quad$ Required Confined Strength of Concrete

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\quad$ $\varepsilon_{fd} = 4\times 10^{-3}$

$f_l = \frac{k_s\cdot\rho_f\cdot\varepsilon_{fd}\cdot E_f}{2}$ $\quad$ $f_l = \frac{2\cdot k_s\cdot t_f\cdot\varepsilon_{fd}\cdot E_f}{D}$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$74.47 \cdot MPa = 34.45 \cdot MPa \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m} - 1.25\right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 5.5010053273720839076 \cdot 10^{-3} \cdot m \\ .13392016695682412882 \cdot m \end{pmatrix}$$

$t_f := 0.134m$ $\quad t_f = 134\,mm$

or

$t_f := 0.00550m$ $\quad t_f = 5.5\,mm$ $\quad t_f = 0.217\,in$

Seismic Design Using Proposed Strain Model and Proposed Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 51.44MPa$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$51.44 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 1.6366552688306581444 \\ 42.689833963713011137 \end{pmatrix}$$

$t_f := 42.69mm$

or

$t_f := 1.64mm$ $\quad t_f = 0.065\,in$

Assume given ductility desire is 10:

$f'_{cc} := 74.47MPa$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$74.47 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{ solve}, t_{f2} \rightarrow \begin{pmatrix} 3.8373160144904812162 \\ 40.489173218053188065 \end{pmatrix}$$

$t_f := 40.49\text{mm}$

or

$t_f := 3.84\text{mm}$ $\quad t_f = 0.151\text{ in}$

Seismic Design Using Proposed Strain Model and Girgin's Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 51.44\text{MPa}$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$51.44 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 1.5676976478534047524$$

$t_f := 1.57\text{mm}$ $\quad t_f = 0.062\text{ in}$

Assume given ductility desire is 10:

$f'_{cc} := 74.47\text{MPa}$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$74.47 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 4.0235604664480361169$$

$t_f := 4.02\text{mm}$ $\quad t_f = 0.158\text{ in}$

Seismic Design Using Mander *et al*'s Strain Model and ACI 440 Strength Model

Assume given ductility desire is 8:

$$\mu_{\Delta} := 8$$

$$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Double Bending

$$L_p := 223.52\text{mm}$$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$$\mu_{\Phi} = 15.01$$

$$\varepsilon_{cc} := \mu_{\Phi} \cdot \Phi_y \cdot c_u \qquad \varepsilon_{cc} = 0.0137$$ Confined Concrete Strain

Mander *et al*'s Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc}}{f'_c} - 1\right)\right]$$

$$0.0137 = 0.002 \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc1}}{34.45} - 1\right)\right] \text{solve}, f'_{cc1} \rightarrow 74.756500000000000001$$

$$f'_{cc} := 74.76\text{MPa}$$ Required Confined Strength of Concrete

$$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f) \qquad \varepsilon_{fd} = 4 \times 10^{-3}$$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$74.76 \cdot \text{MPa} = 34.45 \cdot \text{MPa} \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}}} - 2 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}} - 1.25\right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 5.5589056513633647879 \cdot 10^{-3} \cdot \text{m} \\ .1336840837158304 3054 \cdot \text{m} \end{pmatrix}$$

$t_f := 0.134m$ $\qquad t_f = 134\,mm$

or

$t_f := 0.00556m$ $\qquad t_f = 5.56\,mm$ $\qquad t_f = 0.219\,in$

Assume given ductility desire is 10:

$\mu_\Delta := 10$

$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Double Bending

$L_p := 223.52mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$\mu_\Phi = 19.012$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\qquad \varepsilon_{cc} = 0.0173$ Confined Concrete Strain

Mander *et al*'s Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc}}{f'_c} - 1\right)\right]$$

$$0.0173 = 0.002 \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc1}}{34.45} - 1\right)\right] \text{ solve}, f'_{cc1} \rightarrow 87.158500000000000001$$

$f'_{cc} := 87.16MPa$ Required Confined Strength of Concrete

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\qquad \varepsilon_{fd} = 4 \times 10^{-3}$

$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2}$ $\qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$87.16 \cdot MPa = 34.45 \cdot MPa \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m} - 1.25 \right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 8.3855620402334051991 \cdot 10^{-3} \cdot m \\ .12323857156547771486 \cdot m \end{pmatrix}$$

$t_f := 0.123m$ $\quad t_f = 123\,mm$

or

$t_f := 0.00839m$ $\quad t_f = 8.39\,mm$ $\quad t_f = 0.33\,in$

Seismic Design Using Mander *et al.*'s Strain Model and Proposed Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 74.76MPa$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469 \right]$$

$$74.76 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45} \right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 3.8667147622115146809 \\ 40.459774470332154601 \end{pmatrix}$$

$t_f := 40.46mm$

or

$t_f := 3.87mm$ $\quad t_f = 0.152\,in$

Assume given ductility desire is 10:

$f'_{cc} := 87.16MPa$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469 \right]$$

$$87.16 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45} \right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 5.1712833737519678581 \\ 39.155205858791701423 \end{pmatrix}$$

$t_f := 39.16mm$

or

$t_f := 5.17mm$ $t_f = 0.204in$

Seismic Design Using Mander *et al.*'s Strain Model and Girgin's Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 74.76MPa$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$74.76 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 4.0564513506650747369$$

$t_f := 4.06mm$ $t_f = 0.16in$

Assume given ductility desire is 10:

$f'_{cc} := 87.16MPa$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$87.16 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 5.5017081161071624947$$

$t_f := 5.50mm$ $t_f = 0.217in$

Flexural Strengthening Using Proposed Strength Model

Assume Desired Ductility is 8

$$\mu_\Delta := 8$$

$$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Double Bending

$$L_p := 223.52\text{mm}$$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$$\mu_\Phi = 15.01$$

$$\varepsilon_{cu} := \mu_\Phi \cdot \Phi_y \cdot c_u$$

$$\varepsilon_{cu} = 0.0137$$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$$\phi_f := 0.9$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0137 - 0.004) \cdot \left[34.45 \cdot \left[\left[-0.503 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right) + 0.9469\right]\right]\right]}{0.9 \cdot 1310 \cdot 0.01} \ \text{solve}, t_{c11} \rightarrow \begin{pmatrix} -40.834808423821461515 \\ 2.9657951383028113994 \end{pmatrix}$$

$t_{c1} := 2.97mm$ $\quad t_{c1} = 0.117\,in$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 1.485\,mm$ $\quad t_{c2} = 0.058\,in$

Assume Desired Ductility if 10

$$\mu_{\Delta} := 10$$

$$L_p = 0.08 \cdot \frac{H}{2} + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Double Bending

$$L_p := 223.52mm$$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{\frac{H}{2}}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{\frac{H}{2}}\right)\right]}$$

$$\mu_{\Phi} = 19.012$$

$$\varepsilon_{cu} := \mu_{\Phi} \cdot \Phi_y \cdot c_u$$

$$\varepsilon_{cu} = 0.0173$$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$$\phi_f := 0.9$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0173 - 0.004) \cdot \left[34.45 \cdot \left[\left[-0.503 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right) + 0.9469\right]\right]\right]}{0.9 \cdot 1310 \cdot 0.01} \text{ solve}, t_{c11} \rightarrow \begin{pmatrix} -21.305062049173954847 \\ 5.6844554602736768984 \end{pmatrix}$$

$t_{c1} := 5.69\text{mm}$ $\quad\quad t_{c1} = 0.224\text{in}$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 2.845\text{mm}$ $\quad\quad t_{c2} = 0.112\text{in}$

Flexural Strengthening Using Girgin's Strength Model

Assume Desired Ductility is 8

$\varepsilon_{cu} := 0.0137$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0137 - 0.004) \cdot \left(\frac{2 \cdot t_{c11} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{c11} \cdot 1310}{610}}\right)}{0.9 \cdot 1310 \cdot 0.01} \text{ solve}, t_{c11} \rightarrow 2.9080238444453194364$$

$t_{c1} := 2.91\text{mm}$ $\quad\quad t_{c1} = 0.115\text{in}$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 1.455\text{mm}$ $\quad\quad t_{c2} = 0.057\text{in}$

Assume Desired Ductility if 10

$\varepsilon_{cu} := 0.0173$ From above calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0173 - 0.004) \cdot \left(\frac{2 \cdot t_{c11} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{c11} \cdot 1310}{610}}\right)}{0.9 \cdot 1310 \cdot 0.01} \text{ solve}, t_{c11} \rightarrow 5.2854171416123375589$$

$t_{c1} := 5.28\text{mm}$ $t_{c1} = 0.208\,\text{in}$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 2.64\,\text{mm}$ $t_{c2} = 0.104\,\text{in}$

Flexural Design Example

Column Section Properties:

$H := 3658mm$	Column Height
$D := 610mm$	Column Diameter
$cc := 19.05mm$	Concrete Cover
$f'_c := 34.45MPa$	Unconfined Concrete Strength
$E_c := 27580MPa$	Modulus of Concrete

Longitudinal Reinforcement Properties (Grade 40):

$d_b := 19mm$	Bar Diameter, 26 in total
$A_s := 284mm^2$	Total Bar Area
$f_y := 303.4MPa$	Yield Strength

Transverse Reinforcement Properties (Grade 40):

$d_h := 6.35mm$	Bar Diameter
$A_h := 31.7mm^2$	Bar Area
$s := 127mm$	Spacing

Column Load Properties:

$P := 1800kN$	Axial Load
$M_{yi} := 518.6kN \cdot m$	Yield Moment
$\Phi_y := 0.008196 \cdot \frac{1}{m}$	Yield Curvature
$c_u := 136.4mm$	Neutral Axis

Note: Moment, curvature and neutral axis were obtained through a moment curvature software program.

Jacket Material Properties:

$E_f := 124.1GPa$	Jacket Modulus	
$f_f := 1310MPa$	Jacket Ultimate Strength	
$\varepsilon_f := \frac{f_f}{E_f}$	$\varepsilon_f = 0.011$	Jacket Ultimate Strain

ACI 440.2R-02 (2002)

Axial Capacity Design

$k_e := 0.85$ For spiral reinforced columns

$\phi := 0.75$

$\Psi_f := 0.95$ For wrapped column jackets

$A_g := \frac{\pi \cdot D^2}{4}$ $A_g = 0.292\,m^2$

$\phi \cdot P_n = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot \left(A_g - A_s\right) + f_y \cdot A_s\right]$

Want ϕPn to equal axial load (591.61kN)

$P = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot \left(A_g - A_s\right) + f_y \cdot A_s\right]$

$$f'_{cc} := \frac{\frac{P}{k_e \cdot \phi} - f_y \cdot A_s}{0.85 \cdot \Psi_f \cdot \left(A_g - A_s\right)}$$

$f'_{cc} = 11.611\,MPa$ Required confining strength to achieve axial load given

$k_s := 1.0$ For circular columns

$\rho_f = \frac{4 \cdot t_f}{D}$ Volumetric ratio of FRP

$\varepsilon_{fd} := \min\left(0.004, 0.75\varepsilon_f\right)$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2}$ $f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$ Confining Pressure

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$11.611 \cdot MPa = 34.45 \cdot MPa \cdot \left[-1.25 + 2.25 \cdot \left[\sqrt{1 + 7.9 \cdot \frac{2 \cdot t_f \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_f \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}\right]\right] \text{solve}, t_f \rightarrow \begin{bmatrix} -\left(1.9524265035734277446 \cdot 10^{-3}\right) \cdot r \\ 2.8826967364438321645 \cdot 10^{-2} \cdot m \end{bmatrix}$$

Thickness is negative; therefore, no FRP required for axial capacity. $t_f := 0$

Seismic Design (

Assume given ductility desire is 8

$\mu_\Delta := 8$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Single Bending

$L_p := 419.5mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$\mu_\Phi = 22.584$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\quad \varepsilon_{cc} = 0.025$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$f'_{cc} = 109.002\,MPa$

$\varepsilon_{fd} := \min\left(0.004, 0.75\varepsilon_f\right)$ $\quad \varepsilon_{fd} = 4 \times 10^{-3}$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$109 \cdot MPa = 34.45 \cdot MPa \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m} - 1.25\right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} 1.5768619047451032985 \cdot 10^{-2} \cdot m \\ .10243649763642285904 \cdot m \end{pmatrix}$$

$t_f := 0.102m$ $\quad$ $t_f = 102\,mm$

or

$t_f := 0.01577m$ $\quad$ $t_f = 15.77\,mm$ $\quad$ $t_f = 0.621\,in$

Assume given ductility desire is 10:

$\mu_\Delta := 10$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ $\quad$ Plastic Hinge Length for Single Bending

$L_p = 419.5\,mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$\mu_\Phi = 28.751$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\quad$ $\varepsilon_{cc} = 0.032$ $\quad$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$f'_{cc} = 131.24\,MPa$

$\varepsilon_{fd} := \min\left(0.004, 0.75\varepsilon_f\right)$ $\quad$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$131.24 \cdot \text{MPa} = 34.45 \cdot \text{MPa} \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}}} - 2 \cdot \frac{2 \cdot t_{fl} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}} - 1.25\right] \text{solve}, t_{fl} \rightarrow \begin{pmatrix} 7.3051173410894017478 \cdot 10^{-2} \cdot \text{m} \\ 3.1489156810449656976 \cdot 10^{-2} \cdot \text{m} \end{pmatrix}$$

$t_f := 0.0731\text{m}$ $\qquad t_f = 73.1\ \text{mm}$

or

$t_f := 0.03149\text{m}$ $\qquad t_f = 31.49\text{mm}$ $\qquad t_f = 1.24\ \text{in}$

Caltrans Memo 20-4 (2000)

To achieve ductility of 6:

Inside Plastic Hinge:

$f_l := 2068 \cdot 10^3 \text{Pa}$ — Recommended Confining Pressure

$\varepsilon_{cc} := 0.004$ — Recommended Confined Strain of Concrete

$$t_f := \frac{f_l \cdot D}{2 \cdot 0.9 \cdot E_f \cdot \varepsilon_{cc}}$$

$t_f = 1.412\text{mm}$ $\qquad t_f = 0.056\text{in}$

Outside Plastic Hinge:

$f_l := 1034 \cdot 10^3 \text{Pa}$ — Recommended Confining Pressure

$\varepsilon_{cc} := 0.004$ — Recommended Confined Strain of Concrete

$$t_f := \frac{f_l \cdot D}{2 \cdot 0.9 \cdot E_f \cdot \varepsilon_{cc}}$$

$t_f = 0.706\text{mm}$ $\qquad t_f = 0.028\text{in}$

Concrete Society Technical Report No. 55 (2004)

$f_l := 0.183 \frac{mm^2}{N}$ Recommended Confining Pressure

$t_f := \frac{f_l \cdot D \cdot f'_c{}^2}{2 \cdot E_f}$

$t_f = 0.534\,mm$ $t_f = 0.021\,in$

ISIS Canada Network of Centers of Excellence (2001)

$\alpha_{pc} := 1.0$ For circular columns

$f_{lmax} := \frac{0.29 f'_c}{\alpha_{pc}}$ Recommended Maximum Confining Pressure

$f_{lmax} = 9.991\,MPa$

$\phi_{frp} := 0.75$

$f_l = \frac{2 \cdot \phi_{frp} \cdot E_f \cdot \varepsilon_f \cdot t_f}{D}$

$t_f := \frac{f_{lmax} \cdot D}{2 \cdot \phi_{frp} \cdot E_f \cdot \varepsilon_f}$

$t_f = 3.101\,mm$ $t_f = 0.122\,in$

Associated Confined Strength of Concrete, f'cc:

$\phi_c := 0.6$

$w_w := \frac{2 \cdot f_{lmax}}{\phi_c \cdot f'_c}$ $w_w = 0.967$

$f'_{cc} := f'_c \cdot \left(1 + \alpha_{pc} \cdot w_w\right)$

$f'_{cc} = 67.752\,MPa$

Seible *et al.* (1997)

Shear Strengthening

$V_o := \frac{1.5 \cdot M_{yi}}{H}$ Shear Demand for Single Bending

$V_o = 212.657\,\text{kN}$

$V_c := 0\text{kN}$ Ignore Shear Contribution from Concrete, since concrete is damaged

$\theta := 45\text{deg}$ Assumed angle of shear crack, this is conservative

$D' := D - 2 \cdot cc - d_h$ Effective Column Diameter

$V_s := \frac{\pi}{2} \cdot \frac{A_h \cdot f_y \cdot D'}{s} \cdot \cot(\theta)$ Shear Contribution from Steel Reinforcement

$V_s = 67.276\,\text{kN}$

$V_p := \frac{P \cdot (D - c_u)}{H}$ Shear Contribution from Axial Load

$V_p = 233.045\,\text{kN}$

$\phi_v := 0.85$

$$t_v := \frac{\frac{V_o}{\phi_v} - (V_c + V_s + V_p)}{\frac{\pi}{2} \cdot 0.004 \cdot E_f \cdot D}$$

$t_v = -0.105\,\text{mm}$ $t_f = 0.122\,\text{in}$

Flexural Strengthening

Assume Desired Ductility is 8

$\mu_\Delta := 8$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Single Bending

$L_p = 419.5\,mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{H}\right)\right]}$$

$\mu_\Phi = 22.584$

$\varepsilon_{cu} := \mu_\Phi \cdot \Phi_y \cdot c_u$

$\varepsilon_{cu} = 0.025$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$\phi_f := 0.9$

$f'_{cc} := 1.5 \cdot f'_c$ Assumed Confined Strength of Concrete, conservative

$$t_{c1} := \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$t_{c1} = 5.113\,mm$ $t_{c1} = 0.201\,in$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 2.556\,mm$ $t_{c2} = 0.101\,in$

Assume Desired Ductility if 10

$\mu_{\Delta} := 10$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Single Bending

$L_p = 419.5\,\text{mm}$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{H}\right)\right]}$$

$\mu_{\Phi} = 28.751$

$\varepsilon_{cu} := \mu_{\Phi} \cdot \Phi_y \cdot c_u$

$\varepsilon_{cu} = 0.032$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$\phi_f := 0.9$

$f'_{cc} := 1.5 \cdot f'_c$ Assumed Confined Strength of Concrete, conservative

$$t_{c1} := \frac{0.09 \cdot D \cdot (\varepsilon_{cu} - 0.004) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$t_{c1} = 6.772\,\text{mm}$ $t_{c1} = 0.267\,\text{in}$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 3.386\,\text{mm}$ $t_{c2} = 0.133\,\text{in}$

Lap Splice Clamping

$L_s := 381\,mm$ Lap Splice Length

$p := 3.208 \cdot 10^3 mm$ Perimeter of Column

$n := 26$ Number of Longitudinal Bars

$A_s = 284\,mm^2$

$$f_l := \frac{A_s \cdot f_y}{\left[\frac{p}{2 \cdot n} + 2 \cdot \left(d_b + cc\right)\right] \cdot L_s}$$ Confining Pressure Required

$f_l = 1.641\,MPa$

$E_s := 29000ksi$ Modulus of Steel

$$f_h := \frac{0.002 \cdot A_h \cdot E_s}{D \cdot s}$$

$f_h = 0.164MPa$ Confining Pressure Provided by Spiral Hoops

$$t_s := \frac{500 \cdot D \cdot \left(f_l - f_h\right)}{E_f}$$

$t_s = 3.632mm$ $t_f = 0.122in$

Modifications using the top two confinement models described in chapter 3

Top Strength Models:

Proposed Model:

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

Girgin's Hoek-Brown Model:

$$f_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

Top Strain Models:

Proposed Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1.0427 \cdot \left(\frac{f'_{cc}}{f'_c}\right)^2 - 1.1181 \cdot \frac{f'_{cc}}{f'_c} + 6.1949\right]$$

Mander *et al.*'s Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc}}{f'_c} - 1\right)\right]$$

ACI 440.2R-02 (2002)

Axial Capacity Design Using Proposed Strength Model (

$k_e := 0.85$ For spiral reinforced columns

$\phi := 0.75$

$\Psi_f := 0.95$ For wrapped column jackets

$A_g := \frac{\pi \cdot D^2}{4}$ $A_g = 0.292\,m^2$

$$\phi \cdot P_n = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot \left(A_g - A_s\right) + f_y \cdot A_s\right]$$

Want ϕPn to equal axial load (591.61kN)

$$P = k_e \cdot \phi \cdot \left[0.85 \cdot \Psi_f \cdot f'_{cc} \cdot \left(A_g - A_s\right) + f_y \cdot A_s\right]$$

$$f'_{cc} := \frac{\frac{P}{k_e \cdot \phi} - f_y \cdot A_s}{0.85 \cdot \Psi_f \cdot (A_g - A_s)}$$

$f'_{cc} = 11.611\,\text{MPa}$ Required confining strength to achieve axial load given

$k_s := 1.0$ For circular columns

$\rho_f = \frac{4 \cdot t_f}{D}$ Volumetric ratio of FRP

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2}$ $f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$ Confining Pressure

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$11.611 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} -1.6948792185791849667 \\ 46.021368451122854248 \end{pmatrix}$$

Thickness is negative; therefore, no FRP required for axial capacity. $t_f := 0$

Axial Capacity Design Using Girgin's Strength Model (

Girgin Goek-Brown Model:

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$11.611 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow -1.6244706106470456516$$

Thickness is negative; therefore, no FRP required for axial capacity. $t_f := 0$

Seismic Design Using ACI 440 Strain Model and Proposed Strength Model

Assume given ductility desire is 8

$$\mu_{\Delta} := 8$$

$$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Single Bending

$$L_p = 419.5\,\text{mm}$$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$$\mu_{\Phi} = 22.584$$

$$\varepsilon_{cc} := \mu_{\Phi} \cdot \Phi_y \cdot c_u \qquad \varepsilon_{cc} = 0.025$$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$$f'_{cc} = 109.002\,\text{MPa}$$

$$\varepsilon_{fd} := \min\left(0.004, 0.75\varepsilon_f\right) \qquad \varepsilon_{fd} = 4 \times 10^{-3}$$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$109 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 7.7534031369975484005 \\ 36.573086095546120881 \end{pmatrix}$$

$t_f := 36.57mm$

or

$t_f := 7.75mm$ $t_f = 0.305in$

Assume given ductility desire is 10:

$\mu_\Delta := 10$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Single Bending

$L_p = 419.5mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$\mu_\Phi = 28.751$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\varepsilon_{cc} = 0.032$ Confined Concrete Strain

$$\varepsilon_{cc} = 1.71 \cdot \frac{5 \cdot f'_{cc} - 4 \cdot f'_c}{E_c}$$

$$f'_{cc} := \frac{\frac{\varepsilon_{cc} \cdot E_c}{1.71} + 4 \cdot f'_c}{5}$$ Required confined strength of concrete

$f'_{cc} = 131.24MPa$

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$131.24 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{ solve}, t_{f2} \rightarrow \begin{pmatrix} 10.979538813942047924 \\ 33.346950418601621358 \end{pmatrix}$$

$t_f := 32.35mm$

or

$t_f := 10.98mm$ $t_f = 0.432in$

Seismic Design Using ACI 440 Strain Model and Proposed Strength Model (

Assume Desired Ductility is 8

$f'_{cc} := 109MPa$ From above calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$109 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 8.2087905268312492042$$

$t_f := 8.21mm$ $t_f = 0.323in$

Assume Desired Ductility is 10

$f'_{cc} := 131.24MPa$ From above calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$131.24 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 11.138684786936818766$$

$t_f := 11.14mm$ $t_f = 0.439in$

Seismic Design Using Proposed Strain Model and ACI 440 Strength Model

Assume given ductility desire is 8:

$\mu_\Delta := 8$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Single Bending

$L_p = 419.5\text{mm}$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$\mu_\Phi = 22.584$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\varepsilon_{cc} = 0.0252$ Confined Concrete Strain

Proposed Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1.0427 \cdot \left(\frac{f'_{cc}}{f'_c}\right)^2 - 1.1181 \cdot \frac{f'_{cc}}{f'_c} + 6.1949\right]$$

$$0.0252 = 0.002 \cdot \left[6.1949 + 1.0427 \cdot \left(\frac{f'_{cc1}}{34.45}\right)^2 - 1.1181 \cdot \frac{f'_{cc1}}{34.45}\right] \text{solve}, f'_{cc1} \rightarrow \begin{pmatrix} -68.887623203981729130 \\ 105.82878077567061376 \end{pmatrix}$$

$f'_{cc} := 105.83\text{MPa}$ Required Confined Strength of Concrete

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$105.83 \cdot \mathrm{MPa} = 34.45 \cdot \mathrm{MPa} \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \mathrm{GPa}\right)}{34.45 \cdot \mathrm{MPa} \cdot 0.61 \cdot \mathrm{m}}} - 2 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \mathrm{GPa}\right)}{34.45 \cdot \mathrm{MPa} \cdot 0.61 \cdot \mathrm{m}} - 1.25\right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 1.442628918489225815 4 \cdot 10^{-2} \cdot \mathrm{m} \\ .10572655110897357585 \cdot \mathrm{m} \end{pmatrix}$$

$t_f := 0.106\mathrm{m}$ $\quad t_f = 106\,\mathrm{mm}$

or

$t_f := 0.01443\mathrm{m}$ $\quad t_f = 14.43\,\mathrm{mm}$ $\quad t_f = 0.568\,\mathrm{in}$

Assume given ductility desire is 10:

$$\mu_\Delta := 10$$

$$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Single Bending

$$L_p = 419.5\,\mathrm{mm}$$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$$\mu_\Phi = 28.751$$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\quad \varepsilon_{cc} = 0.0321$ Confined Concrete Strain

Proposed Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1.0427 \cdot \left(\frac{f'_{cc}}{f'_c}\right)^2 - 1.1181 \cdot \frac{f'_{cc}}{f'_c} + 6.1949\right]$$

$$0.0321 = 0.002 \cdot \left[6.1949 + 1.0427 \cdot \left(\frac{f'_{cc1}}{34.45}\right)^2 - 1.1181 \cdot \frac{f'_{cc1}}{34.45}\right] \text{solve}, f'_{cc1} \rightarrow \begin{pmatrix} -89.038722926851152547 \\ 125.97988049854003717 \end{pmatrix}$$

$f'_{cc} := 125.98\mathrm{MPa}$ Required Confined Strength of Concrete

$\varepsilon_{fd} := \min\left(0.004, 0.75\varepsilon_f\right)$ $\quad \varepsilon_{fd} = 4 \times 10^{-3}$

$f_l = \dfrac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2}$ $\quad f_l = \dfrac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$125.98 \cdot MPa = 34.45 \cdot MPa \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m} - 1.25\right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 2.6068116854416750882 \cdot 10^{-2} \cdot m \\ 8.1704082827039735821 \cdot 10^{-2} \cdot m \end{pmatrix}$$

$t_f := 0.0817m$ $\quad$ $t_f = 81.7\,mm$

or

$t_f := 0.02611m$ $\quad$ $t_f = 26.11\,mm$ $\quad$ $t_f = 1.028\,in$

Seismic Design Using Proposed Strain Model and Proposed Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 105.83MPa$ $\quad$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$105.83 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 7.3506662446043623089 \\ 36.975822987939306973 \end{pmatrix}$$

$t_f := 36.98mm$

or

$t_f := 7.35mm$ $\quad$ $t_f = 0.289\,in$

Assume given ductility desire is 10:

$f'_{cc} := 125.98MPa$ $\quad$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$125.98 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{ solve}, t_{f2} \rightarrow \begin{pmatrix} 10.138122619476825347 \\ 34.188366613066843935 \end{pmatrix}$$

$t_f := 34.19\text{mm}$

or

$t_f := 10.14\text{mm}$ $\quad$ $t_f = 0.399\,\text{in}$

Seismic Design Using Proposed Strain Model and Girgin's Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 105.83\text{MPa}$ $\quad$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$105.83 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 7.8045027223731473711$$

$t_f := 7.80\text{mm}$ $\quad$ $t_f = 0.307\,\text{in}$

Assume given ductility desire is 10:

$f'_{cc} := 125.98\text{MPa}$ $\quad$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$125.98 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 10.431875607126321766$$

$t_f := 10.43\text{mm}$ $\quad$ $t_f = 0.411\,\text{in}$

Seismic Design Using Mander *et al*'s Strain Model and ACI 440 Strength Model

Assume given ductility desire is 8:

$$\mu_{\Delta} := 8$$

$$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Single Bending

$$L_p = 419.5\,\text{mm}$$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$$\mu_{\Phi} = 22.584$$

$$\varepsilon_{cc} := \mu_{\Phi} \cdot \Phi_y \cdot c_u \qquad \varepsilon_{cc} = 0.0252$$ Confined Concrete Strain

Mander *et al*'s Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc}}{f'_c} - 1\right)\right]$$

$$0.0252 = 0.002 \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc1}}{34.45} - 1\right)\right] \text{solve}, f'_{cc1} \rightarrow 114.37400000000000000$$

$$f'_{cc} := 114.37\text{MPa}$$ Required Confined Strength of Concrete

$$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f) \qquad \varepsilon_{fd} = 4 \times 10^{-3}$$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$114.37 \cdot \text{MPa} = 34.45 \cdot \text{MPa} \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}}} - 2 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot \text{GPa}\right)}{34.45 \cdot \text{MPa} \cdot 0.61 \cdot \text{m}} - 1.25\right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 1.835438347814283533 8 \cdot 10^{-2} \cdot \text{m} \\ 9.6551277121927672315 \cdot 10^{-2} \cdot \text{m} \end{pmatrix}$$

$t_f := 0.0966m$ $t_f = 96.6mm$

or

$t_f := 0.01835m$ $t_f = 18.35mm$ $t_f = 0.722in$

Assume given ductility desire is 10:

$\mu_\Delta := 10$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Single Bending

$L_p = 419.5mm$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5 \cdot \left(\frac{L_p}{H}\right)\right]}$$

$\mu_\Phi = 28.751$

$\varepsilon_{cc} := \mu_\Phi \cdot \Phi_y \cdot c_u$ $\varepsilon_{cc} = 0.0321$ Confined Concrete Strain

Mander *et al*'s Strain Model:

$$\varepsilon_{cc} = \varepsilon_{co} \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc}}{f'_c} - 1\right)\right]$$

$$0.0321 = 0.002 \cdot \left[1 + 5 \cdot \left(\frac{f'_{cc1}}{34.45} - 1\right)\right] \text{ solve}, f'_{cc1} \rightarrow 138.14450000000000000$$

$f'_{cc} := 138.14MPa$ Required Confined Strength of Concrete

$\varepsilon_{fd} := \min(0.004, 0.75\varepsilon_f)$ $\varepsilon_{fd} = 4 \times 10^{-3}$

$$f_l = \frac{k_s \cdot \rho_f \cdot \varepsilon_{fd} \cdot E_f}{2} \qquad f_l = \frac{2 \cdot k_s \cdot t_f \cdot \varepsilon_{fd} \cdot E_f}{D}$$

$$f'_{cc} = f'_c \cdot \left(2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{f_l}{f'_c}} - 2 \cdot \frac{f_l}{f'_c} - 1.25\right)$$

$$128.14 \cdot MPa = 34.45 \cdot MPa \cdot \left[2.25 \cdot \sqrt{1 + 7.9 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m}} - 2 \cdot \frac{2 \cdot t_{f1} \cdot \left(4 \cdot 10^{-3} \cdot 124.1 \cdot GPa\right)}{34.45 \cdot MPa \cdot 0.61 \cdot m} - 1.25\right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 2.8055017793795871406 \cdot 10^{-2} \cdot m \\ 7.8390026367918471864 \cdot 10^{-2} \cdot m \end{pmatrix}$$

$t_f := 0.07839m$ $\quad t_f = 78.39mm$

or

$t_f := 0.02806m$ $\quad t_f = 28.06mm$ $\quad t_f = 1.105in$

Seismic Design Using Mander *et al.*'s Strain Model and Proposed Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 114.37MPa$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$114.37 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 8.4626292383087867873 \\ 35.863859994234882494 \end{pmatrix}$$

$t_f := 35.86mm$

or

$t_f := 8.46mm$ $\quad t_f = 0.333in$

Assume given ductility desire is 10:

$f'_{cc} := 138.14MPa$ From Above Calculations

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$138.14 = 34.45 \cdot \left[\left[0.9469 + -0.503 \cdot \left(\frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \frac{2 \cdot t_{f2} \cdot 1310}{610 \cdot 34.45}\right]\right] \text{solve}, t_{f2} \rightarrow \begin{pmatrix} 12.190365114857055531 \\ 32.136124117686613750 \end{pmatrix}$$

$t_f := 32.14mm$

or

$t_f := 12.19mm$ $\quad t_f = 0.48\,in$

Seismic Design Using Mander *et al.*'s Strain Model and Girgin's Strength Model

Assume given ductility desire is 8:

$f'_{cc} := 114.37MPa$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$114.37 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 8.9016585044928066555$$

$t_f := 8.90mm$ $\quad t_f = 0.35\,in$

Assume given ductility desire is 10:

$f'_{cc} := 138.14MPa$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$138.14 = \frac{2 \cdot t_{f3} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{f3} \cdot 1310}{610}} \text{ solve}, t_{f3} \rightarrow 12.077782744600068019$$

$t_f := 12.08mm$ $\quad t_f = 0.476in$

Flexural Strengthening Using Proposed Strength Model

Assume Desired Ductility is 8

$$\mu_{\Delta} := 8$$

$$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$$ Plastic Hinge Length for Single Bending

$$L_p = 419.5\,\text{mm}$$

$$\mu_{\Phi} := 1 + \frac{\mu_{\Delta} - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{H}\right)\right]}$$

$$\mu_{\Phi} = 22.584$$

$$\varepsilon_{cu} := \mu_{\Phi} \cdot \Phi_y \cdot c_u$$

$$\varepsilon_{cu} = 0.0252$$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$$\phi_f := 0.9$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0252 - 0.004) \cdot \left[34.45 \cdot \left[\left[-0.503 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right) + 0.9469\right]\right]\right]}{0.9 \cdot 1310 \cdot 0.01} \text{ solve}, t_{c11} \rightarrow \begin{pmatrix} -8.1470494902222521681 \\ 14.865219174407218708 \end{pmatrix}$$

$t_{c1} := 14.87mm$ $t_{c1} = 0.585in$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 7.435mm$ $t_{c2} = 0.293in$

Assume Desired Ductility if 10

$$\mu_\Delta := 10$$

$L_p = 0.08 \cdot H + 0.022 \cdot f_y \cdot d_b$ Plastic Hinge Length for Single Bending

$$L_p = 419.5mm$$

$$\mu_\Phi := 1 + \frac{\mu_\Delta - 1}{3 \cdot \left(\frac{L_p}{H}\right) \cdot \left[1 - 0.5\left(\frac{L_p}{H}\right)\right]}$$

$$\mu_\Phi = 28.751$$

$$\varepsilon_{cu} := \mu_\Phi \cdot \Phi_y \cdot c_u$$

$$\varepsilon_{cu} = 0.0321$$

$$\varepsilon_{cu} = 0.004 + \frac{2.8\rho_j \cdot f_f \cdot \varepsilon_f}{f'_{cc}}$$

$$\phi_f := 0.9$$

$$f'_{cc} = f'_c \cdot \left[-0.503 \cdot \left(\frac{f_l}{f'_c}\right)^2 + 2.7798 \cdot \frac{f_l}{f'_c} + 0.9469\right]$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0321 - 0.004) \cdot \left[34.45 \cdot \left[\left[-0.503 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 1310 \cdot t_{c11}}{610 \cdot 34.45}\right) + 0.9469\right]\right]\right]}{0.9 \cdot 1310 \cdot 0.01} \text{ solve}, t_{c11} \rightarrow \begin{pmatrix} -5.6151332247163695220 \\ 21.568086000847067340 \end{pmatrix}$$

$t_{c1} := 21.57mm$ $\qquad$ $t_{c1} = 0.849\,in$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 10.785\,mm$ $\qquad$ $t_{c2} = 0.425\,in$

Flexural Strengthening Using Girgin's Strength Model

Assume Desired Ductility is 8

$\varepsilon_{cu} := 0.0252$ $\qquad$ From Above Calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0252 - 0.004) \cdot \left(\frac{2 \cdot t_{c11} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{c11} \cdot 1310}{610}}\right)}{0.9 \cdot 1310 \cdot 0.01} \text{ solve}, t_{c11} \rightarrow 17.234214943810992391$$

$t_{c1} := 17.23mm$ $\qquad$ $t_{c1} = 0.678\,in$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 8.615\,mm$ $\qquad$ $t_{c2} = 0.339\,in$

Assume Desired Ductility if 10

$\varepsilon_{cu} := 0.0321$ From above calculations

$$f'_{cc} = f_l + \sqrt{f'_c{}^2 + 3.5 \cdot f'_c \cdot f_l}$$

$$t_{c1} = \frac{0.09 \cdot D \cdot \left(\varepsilon_{cu} - 0.004\right) \cdot f'_{cc}}{\phi_f \cdot f_f \cdot 0.01}$$

$$t_{c11} = \frac{0.09 \cdot 610 \cdot (0.0321 - 0.004) \cdot \left(\frac{2 \cdot t_{c11} \cdot 1310}{610} + \sqrt{34.45^2 + 3.5 \cdot 34.45 \cdot \frac{2 \cdot t_{c11} \cdot 1310}{610}}\right)}{0.9 \cdot 1310 \cdot 0.01} \text{ solve}, t_{c11} \rightarrow 48.405970701396900267$$

$t_{c1} := 48.41\text{mm}$ $t_{c1} = 1.906\text{in}$

$$t_{c2} := \frac{t_{c1}}{2}$$

$t_{c2} = 24.205\text{ mm}$ $t_{c2} = 0.953\text{ in}$

Appendix B

Column Properties:

$D := 305mm$

$cc := 19mm$

$A_s := 16 \cdot 71mm^2$ $\qquad A_s = 1.136 \times 10^3\,mm^2$

$d_h := 4.877mm$ $\qquad d_h = 0.192\,in$

$d_s := D - 2 \cdot cc - 2 \cdot d_h$ $\qquad d_s = 257.246\,mm$

$f'_c := 34.48MPa$

$A_h := \frac{\pi \cdot (d_h)^2}{4}$ $\qquad A_h = 18.681\,mm^2$ $\qquad A_h = 0.029\,in^2$

$E_s := 29000ksi$ $\qquad E_s = 1.999 \times 10^5\,MPa$

$s := 32mm$

$f_{yh} := 413.685MPa$ $\qquad f_{yh} = 60\,ksi$

$f_{ys} := 468.43MPa$ $\qquad f_{ys} = 67.94\,ksi$

Composite Properties:

$f_f := 752MPa$ $\qquad f_f = 109.068\,ksi$

$E_f := 59360MPa$ $\qquad E_f = 8.609 \times 10^3\,ksi$

Confining Pressure Provided by Steel Hoops:

$f_h := \frac{2A_h \cdot f_{yh}}{d_s \cdot s}$

$f_h = 272.319\,psi$ $\qquad f_l := f_h$ $\qquad f_l = 1.878\,MPa$

Initial Axial Capacity:

$f'_{ccs} := f'_c + 4.1f_l$ $\qquad f'_{ccs} = 42.178\,MPa$

$A_g := \frac{\pi \cdot D^2}{4}$ $\qquad A_g = 7.306 \times 10^4\,mm^2$

$A_{cc} := \frac{(d_s)^2 \cdot \pi}{4}$ $\qquad A_{cc} = 5.197 \times 10^4\,mm^2$

$\phi_s := 0.75$ $\qquad$ for spiral hoops

$\phi P_{no} := 0.8 \cdot 0.85 \cdot \phi_s\left(f'_c \cdot A_g + f'_{ccs} \cdot A_{cc}\right) + 0.8 \cdot \phi_s \cdot A_s \cdot f_{ys}$

$\phi P_{no} = 2.722 \times 10^3\,kN$ $\qquad$ Initial Axial Capacity

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New column capacity:

$$\phi P_{new} := 174.9kN$$

Required strength from jacket:

$$f'_{cc} := \frac{\phi P_{no} - \phi P_{new}}{0.8 \cdot \phi_s \cdot 0.85 A_g}$$

$$f'_{cc} = 68.359\,MPa$$

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$$68.359 = 34.48 \cdot \left[-1.254 + (2.254) \cdot \sqrt{1 + 7.94 \cdot \frac{f_{l1}}{34.48}} - 2 \cdot \frac{f_{l1}}{34.48}\right] \text{solve}, f_{l1} \rightarrow \begin{pmatrix} 6.9979949684714119552 \\ 229.12982551632858804 \end{pmatrix}$$

$$f_l := 7.0MPa$$

$$\phi := 0.9$$

$$\varepsilon_{cc} := 0.004$$

$$t_f := \frac{f_l \cdot D}{2 \cdot \phi \cdot E_f \cdot \varepsilon_{cc}}$$

$t_f = 4.995\,mm$ $\quad$ $t_f = 0.197\,in$

Proposed Strength Model

$$68.359 = 34.48 \cdot \left[-0.503 \cdot \left(\frac{2 \cdot 752 \cdot t_{f1}}{305 \cdot 34.48}\right)^2 + 2.7798 \cdot \left(\frac{2 \cdot 752 \cdot t_{f1}}{305 \cdot 34.48}\right) + 0.9469\right] \text{solve}, t_{f1} \rightarrow \begin{pmatrix} 2.80935947915053 \\ 35.833105835345 \end{pmatrix}$$

$t_f := 2.81mm$ $\quad$ $t_f = 0.111\,in$

Girgin Strength Model

$$68.359 = \frac{2 \cdot 752 \cdot t_{f2}}{305 \cdot 34.48} + \sqrt{(34.48)^2 + 3.5 \cdot 34.48 \cdot \frac{2 \cdot 752 \cdot t_{f2}}{305}} \text{solve}, t_{f2} \rightarrow 5.6695295948074228016$$

$t_f := 5.67mm$ $\quad$ $t_f = 0.223\,in$