Title
AN APPROXIMATE RELATION BETWEEN RESERVE MARGIN, GENERATING CAPACITY, AND LOSS OF LOAD PROBABILITY

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Probabilistic methods have been used in assessing power system reliability for some time (reference 1). Increasing sophistication of these methods has led to complicated computer simulation studies. The number of variables considered relevant has become quite large (reference 2). Because the present calculation procedure has become so complex, there is some interest in developing techniques that are approximate, but which allow easy qualitative and quantitative evaluation of changes in crucial variables (reference 3). It is in this spirit that the following work is offered.

We present a simple model of a large interconnected power system. We derive an analytic relation between reserve requirements, total generating capacity, and loss of load probability within the context of the model and in the limit of large total capacity. Among other things, this relation shows that if the LOLP is required to be fixed at some arbitrary value between zero and one, then in the limit of large total capacity, the following relation between reserve margin (R_m) and total capacity (T) is satisfied.

$$R_m(T) = A + \frac{B}{\sqrt{T}} + 0\left(\frac{1}{T}\right) , \qquad (1)$$

where $0\left(\frac{1}{T}\right)$ denotes a function which goes to zero as fast or faster than $\frac{1}{T}$ for T large. A and B are positive constants independent of T. This relation supports the "rule of thumb" in power engineering circles that reserve margin must remain finite even as capacity gets very large. In addition to Eq. 1, an expression for LOLP is derived in the limit T large and R_m fixed. An elegant relationship is presented which relates changes in B to changes in LOLP. This will be a useful tool to examine potential changes in reliability requirements, which is a subject of recent interest (reference 4).

Although the model presented is an oversimplification of a real-life power system, it contains all of the essential ingredients of a probabilistic interpretation of power system reliability.

Any large system can be thought of as built up by joining together many smaller systems. These smaller systems might be viewed as the building blocks of the large power pool. We can imagine dividing up a large power system into relatively small subsystems each with its own generation capacity and load. If the power system is large enough, then it will be possible to divide it into small subsystems each of which has roughly the same load and the same capacity.

With this division in mind, let us consider identical power systems each with generating capacity α and load P. Let the loss of load probability of each of these n systems be denoted by L_o.

If a snapshot is taken of the power system, then L_o represents the probability that some of the customers are losing service in that snapshot due to insufficient generating capacity.

Imagine that these n systems are originally unconnected. The reserve margin for each of the n systems is

$$R_m = \frac{\alpha - P}{P} = \frac{R}{1-R} \quad , \tag{2}$$

where R is defined as

$$R = \frac{\alpha - P}{\alpha} = \frac{R_m}{1 + R_m} \tag{3}$$

By multiplying the numerator and denominator of expression (2) by n we see that

$$R_m = \frac{n\alpha - nP}{nP} = \frac{\text{Total capacity - total load}}{\text{total load}} \tag{4}$$

Thus R_m is the usual reserve margin for this sytem; however in what follows, it will be convenient to work with R instead of R_m.

We will assume in the following that when a loss of load occurs in one of the n systems, then all of the power of that system is lost. This really assumes that these systems are actually quite small, consisting of only one or at most a few generators. This is clearly an oversimplification, since if some of our n systems consist of several generators, then even if one of these generators is out, the others may still operate and a complete loss of load will not occur. We cannot choose each of the n systems to be a single generator since this would contradict our previous postulate of equal capacities for each system. Despite the oversimplified nature of this postulate, we believe the model is reasonable enough to suggest some interesting relations.

Next we imagine that the n systems are connected together by an ideal transmission network. By an ideal network, we mean a network which is perfectly reliable and which can carry any load which it may be called upon to carry. This is clearly only an approximation. Any real-life transmission system has non-zero unreliability as a result of faulty switching, short circuits, lightning, etc. Transmission systems can and often are designed to be quite reliable by introducing redundant paths for power routing.

Finally, we assume that after the n systems have been connected together the probability for the outage of any one system remains L_o. We also assume that these systems remain statistically independent. This postulate of independence is once again an approximation. This is because if two turbine generators are connected by a transmission system, then they are electromechanically coupled. Thus an outage in one generator may cause transients to occur and phase slippage of the second generator may become so severe that it might have to shut off also, even if the transmission system is ideal or very good. We are currently investigating the precise implications of ignoring this.

The probability for one of our n systems to be out and the other n-1 systems to be fully operative is

$$P_1 = nL_o(1-L_o)^{n-1} . \qquad (5)$$

The probability for two of these systems to be out and the other n-2 in is

$$P_2 = \frac{n(n-1)}{2!} L_o^2 (1-L_o)^{n-2}. \tag{6}$$

The probability for i of these systems to be out and the remaining n-i systems fully operative is

$$P_i = L_o^i (1-L_o)^{n-i} \frac{n!}{(n-i)!\,i!} \tag{7}$$

If i of our systems have failed but the remaining n-i systems are still operative then a loss of load will occur if the remaining capacity is not sufficient to meet the load. The total load of our connected system is np. Thus a loss of load will occur if

$$np > (n-i)\alpha, \quad \text{or} \quad i > (\alpha - p)\, n/\alpha \tag{8}$$

The total loss of load probability is the sum of probabilities for i systems to be out and the other n-i systems to be in, such that the inequality in Eq. (8) is satisfied. Namely,

$$\text{LOLP} = \sum_{i=Rn}^{n} (L_o)^i (1-L_o)^{n-i} \frac{n!}{(n-i)!\ i!} \tag{9}$$

where we recall that $R = \frac{\alpha - p}{\alpha}$.

Since the total capacity T of our system is related to n by the equation

$$T = n\alpha \tag{10}$$

we see that Eq. 10 is a functional relationship between LOLP, R, and T, with two constant parameters α and L_o. The summation of Eq. 10 is too complicated to do in closed form exactly. It could certainly be done on a computer, but for our purposes analytic methods will suffice. This is because we are interested in the limit n or T large, and in this case exact analytic results may be derived.

To see this, we begin by noting that for n large with R not equal to zero or one, the integers which explicitly appear in the factorials of the terms which contribute to the LOLP expression, namely n, i, and n-i, are very large. It can be shown that the few terms in the sum where the integers n-i are not large do not contribute to the sum in the large n limit. Thus we can use Stirling's approximation for the factorials in our expression for the LOLP, yielding

$$\text{LOLP} = \frac{1}{\sqrt{2\pi}} \sum_{i=Rn}^{n} L_o^i (1-L_o)^{n-i} \; n^{n+\frac{1}{2}} (n-i)^{-(n-i+\frac{1}{2})} \; i^{-(i+\frac{1}{2})} \tag{11}$$

In the limit of large n, this sum can be approximated by the following integral

$$\text{LOLP} \cong \frac{1}{\sqrt{2\pi}} \int_{Rn}^{n} di \; L_o^i (1-L_o)^{n-i} \; n^{n+\frac{1}{2}} (n-i)^{-(n-i+\frac{1}{2})} \; i^{-(i+\frac{1}{2})} . \tag{12}$$

Changing variables to y = i/n, this becomes

$$\text{LOLP} = \frac{\sqrt{n}}{\sqrt{2\pi}} \int_R^1 dy \frac{1}{\sqrt{y(1-y)}}$$

$$\times \exp\left[n\left(y \ln\left(\frac{L_o}{y}\right) + (1-y) \ln\left(\frac{1-L_o}{1-y}\right)\right)\right] \qquad (13)$$

Since the functional argument of the exponent dominates the behavior of the integral in the limit $n \to \infty$, we examine this function

$$f(y) = n \left\{ y \ln\left(\frac{L_o}{y}\right) + (1-y) \ln\left(\frac{1-L_o}{1-y}\right) \right\} \qquad (14)$$

Figure 1, shows the qualitative behavior of f(y). It is sharply peaked about (has a maximum at) the point $y = L_o$ where $f(L_o) = 0$, and becomes very negative on either side of this maximum. Because of the exponential damping, implied by this behavior of f(y), the integral only receives non-zero contributions (in the limit of large n) from the integration region around the point $y = L_o$. In this region, the function f(y) can be accurately represented by a Taylor series approximation,

$$f(y) = \frac{-n(y-L_o)^2}{2L_o(1-L_o)} \tag{15}$$

Using the above form for f(y) in the neighborhood of L_o, we have for the LOLP,

$$\text{LOLP} \approx \frac{\sqrt{n}}{\sqrt{2\pi}} \int_R^1 \frac{e^{\frac{-n(y-L_o)^2}{2L_o(1-L_o)}} dy}{\sqrt{L_o(1-L_o)}} \tag{16}$$

From this explicit form, we see that the integral only has non-zero contributions, if the integration region includes at least some finite part of the interval $L_o - \frac{a_1}{\sqrt{n}} < y < L_o + \frac{a_2}{\sqrt{n}}$, where $0 \leq a_1$, $a_2 << \sqrt{n}$ and a_1 and a_2 are arbitrary constants that don't vary with n. (For example if $y = L_o + \frac{1}{2}$, the exponential in the integral becomes

$e^{\frac{-n}{8L_o(1-L_0)}}$ which goes to zero as $n \to \infty$; whereas if $y = L_o + \frac{a_2}{2\sqrt{n}}$

the exponential becomes $e^{\frac{-a_2^2}{8L_o(1-L_0)}}$ which is non-vanishing.)

Whether this interval is included in the region of integration for the LOLP or not is determined by the lower limit of integration R.

In considering the LOLP, it is useful to consider several different ranges of R separately.

<u>Case 1</u>: n large, $R < L_o - \frac{a_1}{\sqrt{n}}$, R fixed (i.e. not a function of n)

In this case, the entire neighborhood around the point $y = L_o$ which has non-zero contributions to the LOLP, is included in the integration region.

We use Eq. (16) for the LOLP with the change of integration variable $Z = \frac{(y - L_o)\sqrt{n}}{\sqrt{(L_o)(1-L_o)}}$

to obtain

$$\text{LOLP} = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\sqrt{n}\,(L_o-R)}{\sqrt{L_o(1-L_o)}}}^{\sqrt{n}\sqrt{\frac{(1-L_o)}{L_o}}} e^{-Z^2/2}\, dZ \qquad (17)$$

For large n, this becomes

$$\mathrm{LOLP} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{Z^2}{2}} dZ = 1 \qquad (18)$$

Thus in this case, the system becomes totally unreliable. Therefore if we are planning a large power system we must choose R to be bigger than a lower limit L_o. Next we consider the case $R > L_o + \frac{a_2}{n}$.

Case 2: n large, $R > L_o + \frac{a_2}{\sqrt{n}}$, R fixed (i.e. not a function of n).

In this case, the entire neighborhood around the point $y = L_o$, which has non-zero contributions to the LOLP is excluded from our region of integration, and thus the LOLP is zero. For the above integration region f(y) is negative and rapidly decreasing with increasing y. Thus, in the limit of large n, only the lower limit of integration contributes. We may therefore Taylor expand f(y) about the point y = R, without changing the aymptotic result. The square roots in the denominator of the integrand may also be replaced by their values at the lower limit. Thus we find

$$\mathrm{LOLP} = \frac{\sqrt{n}}{\sqrt{2\pi}} \int_{R}^{1} dy \; \exp\Big(f(R)\Big) \exp\left(\frac{df(R)}{dR}(y-R)\right) \frac{1}{\sqrt{R}\,\sqrt{1-R}} \quad . \qquad (19)$$

The upper limit of integration may be taken to infinity without affecting the leading term for large n. The final result is

$$\text{LOLP} = \frac{1}{\sqrt{2\pi}} \quad \frac{1}{\sqrt{n}} \quad \frac{1}{\sqrt{R}} \quad \frac{1}{\sqrt{1-R}} \left[\left| \ \ell n \left(\frac{L_o}{R} \right) - \ell n \left(\frac{1-L_o}{1-R} \right) \right| \right]^{-1}$$

$$\times \exp \left(-n \left| R \ \ell n \left(\frac{L_o}{R} \right) + (1-R) \ \ell n \left(\frac{1-L_o}{1-R} \right) \right| \right) \tag{20}$$

Thus an arbitrarily small LOLP can be achieved by choosing $R > L_o + \frac{a_2}{\sqrt{n}}$ and choosing n large by power pooling.

In the preceding two cases, a value of R was picked and fixed and two subsequent values of LOLP determined, namely 1 and 0, respectively. These represent the two extreme values for the LOLP, and are not of real practical interest. We will see that for values of R in the interval $\left(L_o - \frac{a_1}{n}, L_o + \frac{a_2}{n}\right)$ arbitrary values of the LOLP between 0 and 1 can be obtained.

<u>Case 3:</u> n large, $L_o - \frac{a_1}{\sqrt{n}} < R < L_o + \frac{a_2}{\sqrt{n}}$, R not fixed (i.e. a function of n.)

In this case, we take R to be a function of n with the form

$$R = L_o + \frac{B}{\sqrt{n}} , \tag{21}$$

where $-a_1 < B < a_2$ $(0 < a_1, a_2 \ll \sqrt{n})$. This form leads to an LOLP which doesn't depend on n, for large n. Using Eq. (16) and letting $Z = \frac{\sqrt{n}\,(Y-L_o)}{\sqrt{2L_o\,(1-L_o)}}$, we obtain for the LOLP,

$$\text{LOLP} = \frac{1}{\sqrt{\pi}} \int_{\frac{B}{\sqrt{2L_o(1-L_o)}}}^{\infty} e^{-Z^2} \, dZ \; ,$$

where the upper limit of integration has been taken to infinity since we are interested in the large n limit.

This can be written using one of the standard forms of the error function, as

$$\text{LOLP} = \frac{1}{2}\,\text{erfc}\left(\frac{B}{\sqrt{2L_o(1-L_o)}}\right) \tag{22}$$

where

$$\text{erfc}\,(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} \, dt \tag{23}$$

Now the function $\frac{1}{2}$ erfc(x) takes a continuous range of values between zero and one depending on B and L_o. For example

$$\text{LOLP}\Big|_{B = -\infty} = 1$$

$$\text{LOLP}\Big|_{B = 0} = \frac{1}{2} \tag{24}$$

$$\text{LOLP}\Big|_{B = +\infty} = 0$$

{In the cases where $B \to \pm\infty$, the approach to infinity must be slower than $\sqrt{n}$}.

Using Eqs. (2) (21) we can relate R to the reserve margin R_m in the large n limit (recall that $T = n\alpha$)

$$R_m = \frac{L_o}{1 - L_o} + \frac{\sqrt{\alpha}}{\sqrt{T}} \frac{B}{(1-L_o)^2} \tag{25}$$

We thus expect the final form for reserve margin as a function of total capacity T to be

$$R_m = A_m + \frac{B_m}{\sqrt{T}} \tag{26}$$

for constants A_m and B_n which are independent. If LOLP is to be smaller than .5 then we must have

$$B_m > 0 \quad , \tag{27}$$

and of course

$$0 < L_o < 1 \ . \tag{28}$$

We have used Eq. (26) for the reserve margin (R_m) to fit and extend data for a large power pool in the eastern United States (see Figure 2).[5] For this case Rm becomes

$$R_m = .0273 + \frac{.95}{\sqrt{T}} \tag{29}$$

Our analytic expression (22) for the LOLP provides a useful tool for analyzing various changes in the power system or its reliability requirements. As an example of this, we consider how a change in the LOLP requirement from 1 day in 10 years to 1 day in 4 years would effect the needed reserve margin. As a basis for this comparison we use formula (25) for the reserve margin

$$R_m = \frac{L_o}{1-L_o} + \frac{\sqrt{\alpha}}{\sqrt{T}} \frac{B}{(1-L_o)^2} = A_m + \frac{B_m}{\sqrt{T}} ,$$

where we will hold L_o, and α constant but allow B to change to accommodate to the shift in the LOLP. We recall that

$$\text{LOLP} = \frac{1}{2} \ \text{erfc} \left\{ \frac{B}{\sqrt{2L_o(1-L_o)}} \right\} .$$

Tables of the erfc may be found in "The Handbook of Mathematical Functions," ed. by M. Abramowitz and I. Stegen. For a loss of load of 1 day in 10 years we have

$$\text{LOLP}(1) = 1 \text{ day}/10 \text{ years} = .000274 \tag{30}$$

For a loss of load of 1 day in 4 years we have

$$\text{LOLP}(2) = 1 \text{ day}/4 \text{ years} = .000684 \tag{31}$$

Using the tables for erfc with Eq. 3 we find

$$\text{LOLP} = 1 \text{ day/10 yrs.} \rightarrow \frac{B}{\sqrt{2L_o(1-L_o)}} = 2.44 \qquad (32)$$

$$\text{LOLP} = 1 \text{ day/4 yrs.} \rightarrow \frac{B}{\sqrt{2L_o(1-L_o)}} = 2.26 \qquad (33)$$

If B_1 denotes the B for the first case and B_2 the second case then we have

$$\frac{B_2}{B_1} = \frac{2.26}{2.44} = .926 \,. \qquad (34)$$

The values for B_m will change by this same proportion.

$$\frac{B_{m_2}}{B_{m_1}} = .926 \qquad (35)$$

At an LOLP of 1 day/10 years we obtained a model fit to the data from a large eastern power pool.

$$R_m = .0273 + .95/\sqrt{T} \,. \qquad (36)$$

If an LOLP of 1 day/4 years were chosen then the model predicts that the required reserve margin would be for this case

$$\begin{aligned} R_m &= .0273 + .95(.926)/\sqrt{T} \\ &= .0273 + .8797/\sqrt{T} \,, \end{aligned} \qquad (37)$$

with T expressed in units of 1,000's of MW. The results of this function are tabulated in Table 1. This table shows the savings in reserve margin attainable by increasing LOLP to 1 day in 4 years and within the context of our model. For large capacity, these savings tend to be a smaller percentage than for small capacity. In the capacity range of 45,000 MW to 70,000 WM, the savings in reserve margin are about 1%.

Table 1: Reserve Margins for LOLP = 1 day/4 years

R_m	T(1000's of MW)	ΔR_m
.421	5	-.031
.306	10	-.022
.224	20	-.016
.188	30	-.013
.166	40	-.012
.158	45	-.011
.152	50	-.010
.146	55	-.009
.141	60	-.009
.136	65	-.009
.132	70	-.009
.129	75	-.008
.115	100	-.007
.090	200	-.004

Conclusion:

In this paper we have shown that for a particularly simple example of a power system an analytic investigation of loss of load probability, reserve margin, and capacity which greatly clarifies questions of reliability for large power systems is possible. In a subsequent publication we will show that these results can be generalized to arbitrary power systems so long as the number of generators is large.

The advantage of the analytic approach taken here over detailed computer simulations is that functional relationships between variables like reserve margin and capacity are explicitly exhibited rather than being buried on some magnetic tape containing output. We do not wish to suggest that computer simulations of reliability for large power systems be abandoned, but that they be complemented by the analytic results developed here and in our forthcoming publication. This will provide a check on the results of the computer models and will give a physical insight into the problem of reliability.

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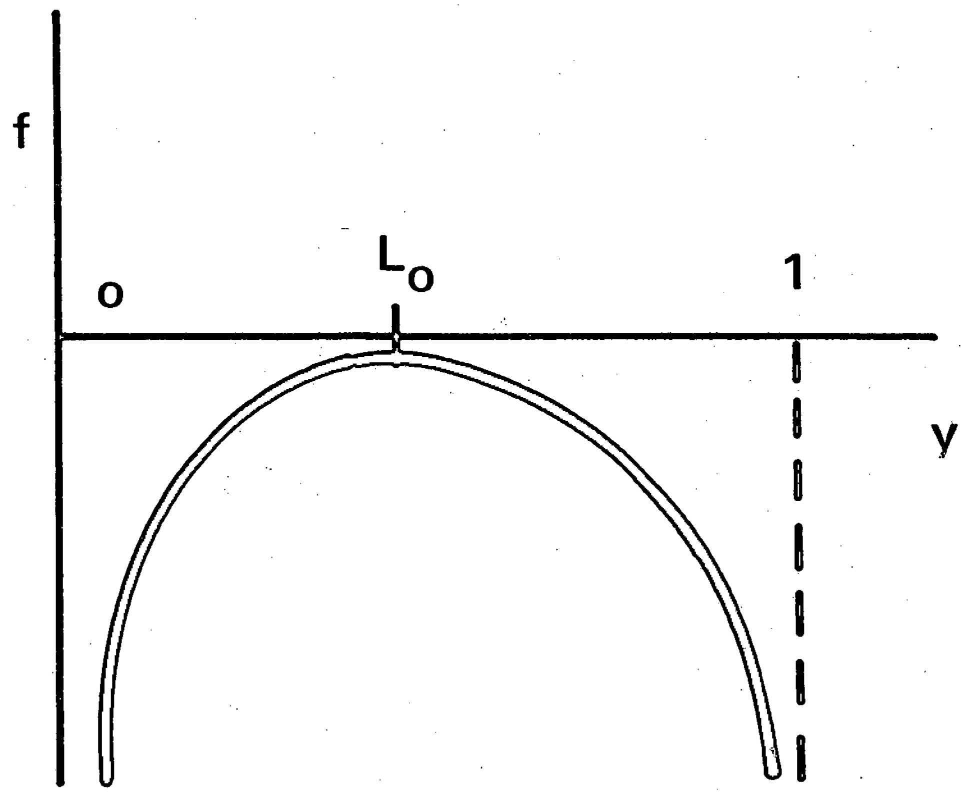

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Fig. 1

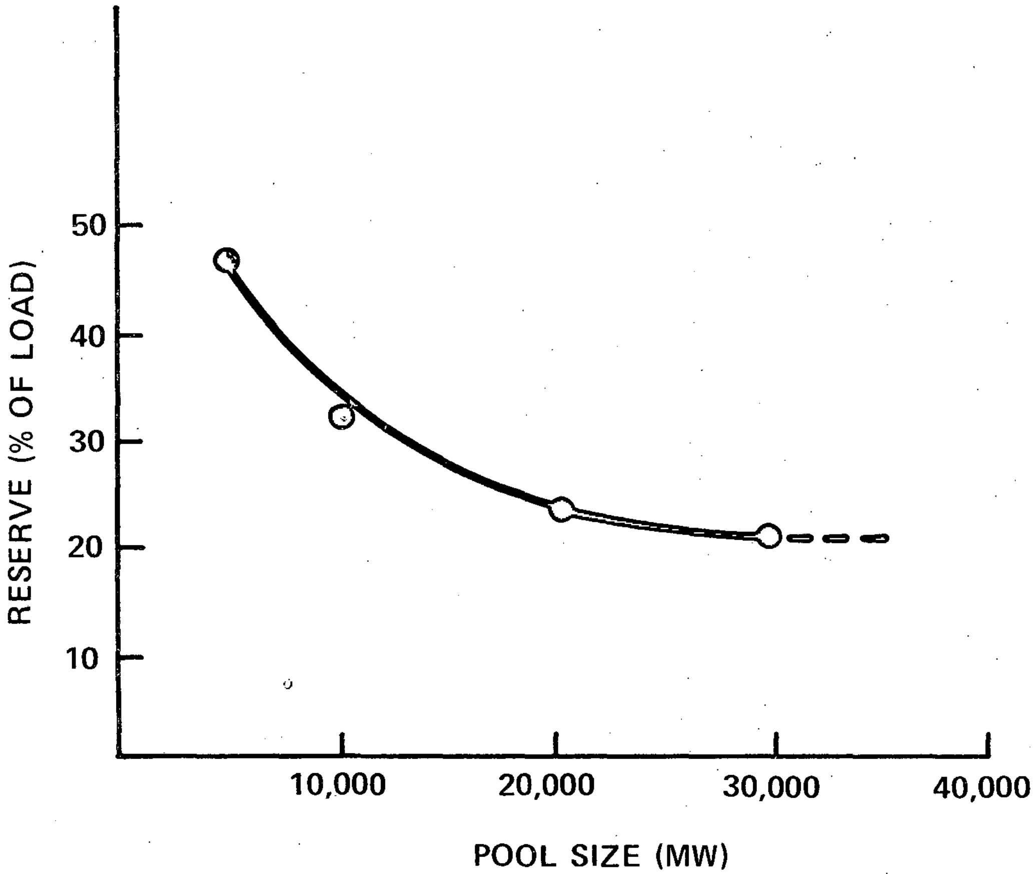

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Fig. 2

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