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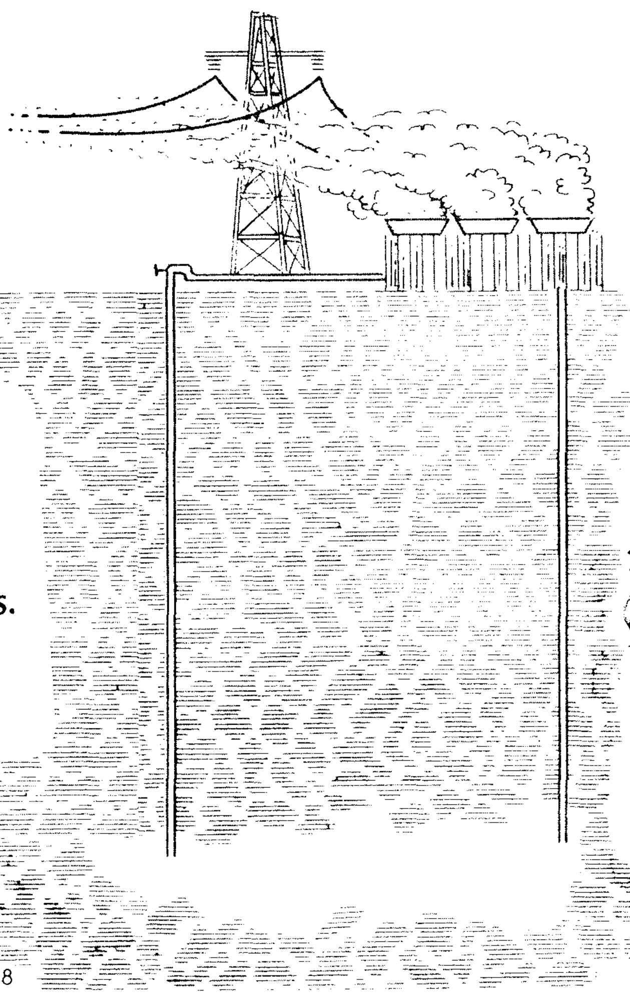
Geothermal Reservoir Engineering Management Program



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ANALYSIS OF PRODUCTION DECLINE IN GEOTHERMAL RESERVOIRS

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I. INTRODUCTION AND CONCLUSIONS

Objectives and Rationale

The major objectives of the Decline Curve project were to

1. Test the decline analysis methods used in the petroleum industry on geothermal production data,

2. Examine and/or develop new analysis methods,

3. Develop a standard operating procedure for analyzing geothermal production data.

Various analysis methods have long been available but they have not been tested on geothermal data because of the lack of publicly available data. The recent release to publication of substantial data sets from Wairakei, New Zealand, Cerro Prieto, Mexico and The Geysers, U.S.A. has made this study possible. Geothermal reservoirs are quite different from petroleum reservoirs in many ways so the analysis methods must be tested using geothermal data.

Data and Analysis Methods

Data and analysis methods were gathered from the petroleum, geothermal, and hydrological literature. The data sets examined include

1. Wairakei, New Zealand - 141 wells
2. Cerro Prieto, Mexico - 18 wells
3. The Geysers, U.S.A. - 27 wells
4. Larderello, Italy - 9 wells and groups
5. Matsukawa and Otake, Japan - 8 wells
6. Olkaria, Kenya - 1 well

The analysis methods tested were

1. Arps's equations
2. Fetkovich type curves
3. Slider's method for Arps
4. Gentry's method for Arps
5. Gentry's & McCray's method
6. Other type curves
7. P/z vs. Q method
8. Coats' influence function method
9. Bodvarsson's Linearized Free Surface Green's Function method

Conclusions

The conclusions are

1. The exponential equation fit is satisfactory for geothermal data.
2. The hyperbolic equation should be used only if the data fit well on a hyperbolic type curve.
3. The type curve methods are useful if the data are not too scattered. They work well for vapor dominated systems and poorly for liquid dominated systems.
4. Coats' influence function method can be used even with very scattered data.
5. Bodvarsson's method is still experimental but it shows much promise as a useful tool.

II. THEORY OF RESERVOIR DECLINE MODELS*

(1) Decline mechanism

A geothermal reservoir has essentially three capacitances, (1) fluid/rock compressibility, (2) free liquid surface mobility and (3) reservoir liquid vaporization. In essence, item (3) is also a compressibility effect similar to (1). In this section, we will very briefly review in a semi-quantitative manner, the relative magnitudes of the effects listed above.

Consider a reservoir consisting of a slab of thickness H and of a large horizontal extent. The porosity/permeability can be of the fracture or intergranular type but is assumed to be sufficiently homogeneous that an average porosity ϕ and capacitivity s (storage coefficient) can be defined.

On these premises, we find that lowering the pressure by Δp in a vertical column of unit area, releases because of compressibility a total liquid mass of

$$\Delta q_c = \rho s H \Delta p \tag{1}$$

where ρ is the density of the liquid. We can then define a specific release per unit area of

$$dq_e/dp = \rho s H. \tag{2}$$

Let g be the acceleration of gravity. Lowering the pressure by Δp corresponds to a lowering of the free liquid surface by $\Delta p/\rho g$. Hence, for the same Δp, the free surface releases a total of

$$\Delta q_f = \rho \phi \Delta p/\rho g = \phi \Delta p/g, \tag{3}$$

and then the specific release

$$dq_f/dp = \phi/g\,. \tag{4}$$

Finally, we consider the effect of intergranular vaporization. Let ρ_s be the density of the vapor, L the latent heat of vaporization of the liquid and T the temperature in kelvins. The Clausius-Clapeyron equation for the liquid is then approximately

$$(dp_s/dT)_v = \rho_s L/T \tag{5}$$

where p_s is the vapor pressure along the saturation line that is denoted by the subscript v. Hence, assuming saturation conditions, the lowering of the pressure by Δp lowers T by

$$\Delta T = T\Delta p/\rho_s L \tag{6}$$

and the release of heat per unit volume of the wet formation is

$$\Delta h = \rho_r C T \Delta p/\rho_s L \tag{7}$$

where ρ_r is the density and C is the heat capacitivity of the wet formation. The release of vapor is then

$$\Delta q_v = \Delta h/L = \rho_r C T \Delta p/\rho_s L^2 \quad , \tag{8}$$

and we can thus define a specific rate per unit area of a slab of thickness H

$$dq_f/dp = \rho_r C T H/\rho_s L^2. \tag{9}$$

The ratio of free surface to compressibility effect follows from (2) and (4)

$$(dq_r/dp)/(dq_c/dp) = \phi/g\rho s H \tag{10}$$

Considering porosities in the range $\phi = 0.01$ to 0.2, a thickness of $H = 10^3$m and taking that $s = 2 \times 10^{-11}\text{Pa}^{-1}$, we find that the ratio given in (10) varies from 50 to 10^3. Thus, at normal reservoir conditions the free surface lowering releases a much larger amount of reservoir liquid mass per unit pressure decline than the compressibility.

Along similar lines we obtain the ratio of the vaporization to the compressibility effect on the basis of (2) and (9)

$$(dq_v/dp)/(dq_c/dp) = \rho_r C T/\rho s \rho_s L^2 \,. \tag{11}$$

Considering the case of T = 200°C = 473 K and using standard values $\rho_r = 2500$ kg/m^3, $C = 10^3$ J/kg·K, $s = 2 \times 10^{-11}$, $\rho_s = 7$ kg/m^3 and $L = 2 \times 10^6$ J/kg, we find a ratio of about 2×10^3. Since ρ_s is the main variable in (11) this ratio will decrease with increasing temperature.

Summing up the results of the present section, we conclude that in the case of liquid dominated reservoirs with common porosities and where no vaporization takes place, the free surface effect is larger than the compressibility effect by a factor of 10^2-10^3. In such cases, the reservoir response to long-term production will be dominated by the free surface effect.

The situation is more complex when vaporization takes place. Theoretically, this effect can release approximately as much fluid mass as the free surface effect. However, in most practical cases where production is initiated at liquid dominated conditions, the vaporization is more or less confined to the local volumes around the boreholes and the ratio in (11) has then to be reduced by a volume factor that may very roughly be of the order of 0.1 or less. The free surface effect would also then dominate the global reservoir response to long term production.

Vapor dominated reservoirs have, as a matter of course, different characteristics. There is no near-surface free liquid surface and ρs in equation (2) has then to be replaced by the product $\phi\gamma$ where γ is the steam compressibility. Usually, there is a vaporization at a deep liquid surface and this effect dominates the long term reservoir behavior.

*Chapter II was written by Gunnar Bodvarsson, Geophysics Group, School of Oceanography, Oregon State University, Corvallis, Oregon.

(2) Pressure-flow fields in slightly compressible formations with Darcy type flow

(2.i) Diffusion equation. Let $p(t,P)$ be the pressure field at time t and at the point P in a Darcy type domain B with the stationary boundary surface Σ. Consider a general setting where the permeability k is a linear matrix operator and the kinematic viscosity of the fluid ν is also taken to be variable. It is convenient to introduce the fluid conductivity operator $c = k/\nu$ and express Darcy's law

$$\vec{q} = -c\nabla p \qquad (12)$$

where $\vec{q}$ is the mass flow density. Moreover, let ρ be the fluid density, s the capacitivity or storage coefficient of the formation and f be a source density. Combining (12) with the equation for the conservation of mass,

$$\nabla\cdot\vec{q} = -\rho s\partial_t p + f \qquad (13)$$

we obtain the diffusion equation for the pressure field

$$\rho s\partial_t p + \Pi(c)p = f \qquad (14)$$

where $\Pi(c) = -\nabla(c\nabla)$ is the generalized Laplacian operator. Appropriate boundary conditions that may be of the Dirichlet, Neumann, mixed or more complex convolution type, have to be ajoined to equation (14). The case of a homogeneous/isotropic/isothermal formation results in the simplication $\Pi(c) = c\Pi = -c\nabla^2$ where c is a constant. Moreover, stationary pressure fields satisfy the potential equation

$$\Pi(c)p = f. \qquad (15)$$

(2.ii) Eigenfunctions of the Laplacian. The eigenfunctions $u_n(P)$ of $\Pi(c)$ in B associated with (14) satisfy the equations

$$\Pi(c)u_n = \lambda_n u_n,\ n = 1,\ 2,\ .\ .\ . \qquad (16)$$

where the constants λ are the eigenvalues and the boundary conditions on Σ are homogeneous of the same type as those satisfied by $p(t,P)$ in (14) and (15.

(2.iii) Types of solutions. The key to solving equation (14) is the causal impulse response or Green's function $G(P,Q,t)$ which represents the pressure response of the causal system to an instantaneous injection of an unit mass of fluid at $t = 0+$ at the source point Q. This function satisfies the same boundary conditions as the eigenfunctions $u_n(P)$. Solutions to (14) in the case of a general source density $f(t,P)$, non-causal initial values and general boundary conditions can then be expressed in terms of integrals over the Green's function (Duff and Naylor, 1966).

Two fundamental types of expressions for the Green's function are available. First, in the case of simple layered domains B with a boundary Σ composed of a few plane faces, $G(P,Q,t)$ can be expressed as a sum (or integral) over the fundamental whole space source function

$$G_o(P,Q,t) = (8\rho s)^{-1}(\pi a t)^{-3/2} \exp(-r^2_{PQ}/4at)U_+(t) \qquad (17)$$

and its images. The symbol $U_+(t)$ is the causal unit step function, $a = c/\rho s$ the diffusivity, and r_{PQ} is the distance from Q to P. Whenever applicable, sums of this type represent the most elementary local and/or global expressions for $G(P,Q,t)$.

Second, the Green's function can be expanded in a series or integral over the eigenfunctions of $\Pi(c)$. If ρ and s are constants, then

$$G(P,Q,t) = (1/\rho s)\sum_n u_n(P)u_n(Q)\exp(-\lambda_n t/\rho s). \qquad (18)$$

The series expansion (18) is of a more general applicability than solutions of the type based on the fundamental source function (17).

The formal link between the two types (17) and (18) is provided by the Poisson summation formula (Stakgold, 1967). It is important to underline that all solutions of the type (17) can be expressed in the form (18).

From the numerical point of view, the form given by (17) is more convenient for the computation of relatively short term field responses, in particular, in the case of layered half-spaces. However, long-term responses in bounded domains are more effectively computed on the basis of (18). This expression is a sum over exponentials where the convergence improves with time.

A different type of solution of (14) that is of interest in the present context can be obtained by operational methods. Limiting ourselves to the pure initial value problem with $p(0,P) = p_o(P)$ in the case of an infinite domain, we can, since ρ, s and $\Pi(c)$ are independent of t, formally express the solution of the homogeneous form of (14) as

$$p = \exp[-t\Pi(c)/\rho s]p_o \qquad (19)$$

where the exponential operator is to be interpreted as a Taylor series in the operator $\Pi(c)$

$$\exp[-t\Pi(c)/\rho s] = 1-[t\Pi(c)/\rho s]+(\tfrac{1}{2})[t\Pi(c)/\rho s]^2 .. \qquad (20)$$

The series represents an iteration process where the convergence is limited to (properly defined) small values of t. The practical applicability is therefore fundamentally different from (18). Moreover, it is of considerable interest that rather general situations with regard to $\Pi(c)$ can be admitted in (19) and (20).

A number of other analytical and/or numerical techniques are available for solving (14). These include the path-integral technique of the Feynman-Kac type (Simon, 1979), compartmentalization or lumping and, as a matter of course, a series of numerical techniques.

(3) Nonstationary boundaries: effects of a free liquid surface

The presence of a free liquid surface in a reservoir requires the introduction of a rather complex non-stationary surface boundary condition. Let Σ now represent the free liquid surface at equilibrium and Ω be the free surface in a perturbed state. The boundary Ω is a surface of constant pressure which without loss of generality can be taken to vanish. The free surface condition (Lamb, 1932) is then expressed

$$Dp/Dt\big|_{p=o} = 0 \tag{21}$$

where D/Dt is the material derivative. This is an essentially non-linear condition which leads to a much more complex problem setting. Losing the principle of superposition the construction of solutions to the forward problem becomes a difficult task.

Bodvarsson (1977) has shown that when Ω deviates only little from Σ_1 (21) can be simplified and linearized. For this purpose, we place a rectangular coordinate system with the z-axis vertically down such that the (x,y) plane coincides with Σ. Moreover, let the amplitude of Ω relative to Σ be u and the scale of the undulation of Ω be L. Then provided $|u/L| << 1$, the condition (21) can be replaced by the approximation

$$(1/w)\partial_t p - \partial_z p = 0 \tag{22}$$

where $w = cg/\phi$ is a new parameter, namely, the free sinking velocity of the pore liquid under gravity (g = acceleration of gravity). Under these circumstances, the solution of the forward problem is obtained by constructing a solution to (14) which satisfies (22) at the free surface and appropriate conditions at other sections of the reservoir boundary.

The presence of a first order derivative with respect to time in the free-surface condition (21) obviously leads to an additional relaxation process analog to the purely diffusive phenomena associated with the first order time derivative in the basic equation (14). As we shall conclude below, the individual time scales of the two phenomena are, however, different.

For the sake of brevity, we shall limit the present discussion to the simplest but practically quite relevant case of the semi-infinite liquid saturated homogeneous, isotropic and isothermal half-space. To consider the pure free-surface related phenomena, we eliminate pressure field diffusion by neglecting the compressibility of the liquid/rock system. As shown in section (I) above, the long term dynamics of liquid reservoirs is dominated by the free surface phenomena. In this setting we can combine the potential equation (15) and the surface condition (22) in one single equation confined to the Σ plane (Bodvarsson, 1978a), which expressed in terms of the fluid surface amplitude $u(t,x,y) = p/\rho g$ takes the form

$$(1/w)\partial_t u + \Pi_2^{\frac{1}{2}} u = f/\rho gc \tag{23}$$

where $\Pi_2^{\frac{1}{2}} = (-\partial_{xx}-\partial_{yy})^{\frac{1}{2}}$ is th square root of the two-dimensional Laplacian and f is an appropriately defined surface source density. To obtain the pressure field in the space z>0, the boundary values derived from (23) have to be continued into the lower half-space on the basis of standard potential theoretical methods. The fractional order of the Laplacian in (23) is quite unusual, but the operator is well defined and poses no mathematical problems.

Some solutions of equations (23) of practical interest have been obtained by Bodvarsson (1977). Confining ourselves first to the simple semi-infinite half-space, some important results are given below.

(3.i) The source-free case. In a source-free case where f = 0, the homogeneous equation (23) is most easily solved by solving

$$-\nabla^2 p = 0, \qquad z \geq 0 \tag{24}$$

with the boundary condition (22) combined with a given initial condition which takes the form

$$p = \rho g h_o, \quad t = 0, \quad z = 0 \tag{25}$$

where $h_o(S)$ is a given initial free-surface amplitude.

This solution is obtained immediately by observing that a pressure function of the form

$$p = p(x,y,z + wt) \tag{26}$$

satisfies the boundary condition (22) at all times. Consequently, introducing the Dirichlet type Green's function for the half-space $z \geq 0$ (Duff and Naylor, 1966, page 276) which gives the pressure p(P) in z > 0 for a pressure $p_o(S)$ on Σ

$$p(P) = (z/2\pi)\int_\Sigma (1/r_{PU}^3)p_o(U)da_U, \quad z \geq 0 \tag{27}$$

where $U = (x',y')$, $da_U = dx'dy'$ and

$$r_{PU} = [(x-x')^2 + (y-y')^2 + z^2]^{\frac{1}{2}} \tag{28}$$

the solution to the present problem is

$$p(P,t) = [\rho g(z+wt)/2\pi]\int_\Sigma (1/r_{PUt}^3)h_o(U)da_U, \quad t \geq 0, \quad z \geq 0, \tag{29}$$

where

$$r_{PUt} = [(x-x')^2 + (y-y')^2 + (z + wt)^2]^{\frac{1}{2}} \tag{30}$$

The motion of the fluid surface is obtained by letting z = 0 in (13) and hence,

$$h(S,t) = (wt/2\pi)\int_\Sigma (1/r_{SUt}^3)h_o(U)da_U, \quad t > 0 \tag{31}$$

where now

$$r_{SUt} = [(x-x')^2+(y-y')^2+(wt)^2]^{\frac{1}{2}}. \tag{32}$$

(3.ii) Flow fields with sources. To select a relevant and important case of flow fields with sources, we will consider the following situation. Let the fluid at t = 0 be in static equilibrium and the fluid surface at t = 0 therefore coincide with Σ. Consider a concentrated sink of strength unity at the point Q = (0,0,d) which at t = 0+ starts withdrawing fluid mass at a constant rate equal to unity. In this case we have to solve

$$\Pi p = -\nabla^2 p = (-1/c)\delta(P-Q)U_+(t) \qquad (33)$$

where $U_+(t)$ is the causal unit step function for which $U_+(0) = 0$. The boundary condition on Σ is again given by (22) and the initial condition is p = 0 at t = 0.

A simple method of solving this problem has been given by Bodvarsson (1977). In the present context, it is of some interest to present a different approach via the combined Laplace-Hankel transform method.

Let $\hat{p}(P,s)$ be the Laplace-transform of p(P,t). The transform of (33) and (22) are then

$$\Pi\hat{p} = (-1/(cs))\delta(P-Q) \qquad (34)$$

and

$$s\hat{p} - w\partial_z\hat{p} = 0 , \quad z = 0 \qquad (35)$$

Moreover, let $\tilde{p}(k,z,s)$ be the (two-dimensional) Hankel-transform of $\hat{p}(P,s)$ and D = d/dz. The transform of (34) is then

$$k^2\tilde{p} - D^2\tilde{p} = (-1/2\pi cs)\delta(z-d) \qquad (36)$$

and (35) takes the form

$$s\tilde{p} - wD\tilde{p} = 0, \quad z = 0 \qquad (37)$$

The solutions of (36) for $z \gtrless z'$ are of the form $\exp(\pm kz)$ and we thus obtain

$$\tilde{p} = A\exp(kz) + B\exp(-kz), \; 0<z<z', \qquad (38)$$

and

$$\tilde{p} = C\exp(-kz), \quad z>z', \qquad (39)$$

where A, B and C are integration constants (with respect to z). From (37) we obtain the relation

$$A(s - wk) + B(s + wk) = 0, \qquad (40)$$

and our solution has to be continuous at z = d such that

$$A\exp(kd) + B\exp(-kd) = C\exp(-kd), \qquad (41)$$

Finally, integrating (36) with respect to z from d- to d+, we obtain the necessary third condition

$$D\tilde{p}\Big|_{d-}^{d+} = -(1/2\pi cs) \qquad (42)$$

which yields the relation

$$-kC\exp(-kd)-k[A\exp(kd)-B\exp(-kd)]=-(1/2\pi cs) \qquad (43)$$

Solving (40), (41) and (43) for A, B and C and inserting in (38) leads to

$$p = (-1/4\pi cks)\exp[-k(d-z)]-[(s-wk)/(s+wk)] \exp[-k(z+d)] \qquad (44)$$

which holds for z>0. Using the identity

$$(s-wk)/s(s+wk)=[2/(s+wk)]-(1/s), \qquad (45)$$

(44) is easily Hankel-Laplace inverted into (P,t) space (tables in Duff and Naylor, 1966) and the result is

$$p(P,t) = (-1/4\pi c)[1/r_{PQ} +(1/r_{PQ'})-(2/r_{PQ't})] \qquad (46)$$

where

$$r_{PQ} = [(x-x')^2 + (y-y')^2 + (z-d)^2]^{\frac{1}{2}}, \qquad (47)$$

$$r_{PQ'} = [(x-x')^2 + (y-y')^2 + (z+d)^2]^{\frac{1}{2}} \qquad (48)$$

$$r_{PQ't} = [(x-x')^2 + (y-y')^2 + (z+wt+d)^2]^{\frac{1}{2}} \qquad (49)$$

The surface elevation $h = p/\rho g$ is

$$h(S,t) = (-1/2\pi\rho gc)[(1/r_{SQ})-(1/r_{SQ't})] \qquad (50)$$

where S = (x,y) and

$$r_{SQ} = [(x-x')^2 + (y-y')^2 + d^2]^{\frac{1}{2}} \qquad (51)$$

$$r_{SQ't} = [(x-x')^2 + (y-y')^2 + (wt+d)^2]^{\frac{1}{2}} \qquad (52)$$

It is of a particular interest to note that the Hankel-inversion leading to the last term in (46) follows upon a Laplace-inversion on the basis of the Sommerfeld integral

$$\int_0^\infty \exp[-(z+wt+d)k]J_0(rk)dk = 1/r_{PQ't}, \qquad (53)$$

which we rewrite

$$\int_0^\infty \exp(-wkt)E(r,z,k)dk = 1/r_{PQ't}, \qquad (54)$$

where

$$E(r,z,k) = \exp[-(z+d)k]J_0(r,k), \qquad (55)$$

Equation (46) reveals that the effect of the free fluid surface on the pressure drawdown due to the concentrated sink of strength unity starting at time t = 0 can be represented by the pressure field due to a stationary image sink of strength unity located at Q' = (x',y',-d) and a moving image source of strength 2 located at Q't = [x',y',-(wt+d)]. At time t = 0+ the image sink and 1/2 of the image source cancel resulting in an initial pressure field of

$$p(P,0+) = -[1/(4\pi c)][1/r_{PQ}] - (1/r_{PQ'})]. \qquad (56)$$

At very large times, that is at t>>d/w, when the image source has retreated far into the negative half space, the third term in (46) becomes negligible and the pressure field reaches its stationary value p_S given by

$$p(P)\big|_{t\to\infty} = -[1/(4\pi c)][1/r_{PQ}) + (1/r_{PQ'})], \qquad (57)$$

The source-sink situation is illustrated in Figure 1.

It is appropriate to reiterate that the above free surface results have been obtained by neglecting the rock/liquid compressibility.

(3.iii) <u>Flow in slab with a free surface</u>. The results for the half-space set forth in the previous section are easily generalized to the model of a slab of thickness H and of infinite horizontal extent. As given in equation (46) and shown in Figure 1, the free surface dynamics reduces at any fixed time to a source-sink situation. Applying well known results of elementary potential theory, we can extend equation (46) to the case of the slab by adding an infinite sequence of source-sink images that is obtained by reflecting the source and the two sinks in Fig. 1 at the bottom and the equilibrium surface boundaries. Appropriate reflection coefficients have to be applied in this process. We will refraim from entering into details of the procedure. The practically most important case is obtained when the basement is impermeable and the reflection coefficient at the boundary is equal to unity. As shown above, the equilibrium surface has also a reflection coefficient of unity but on any reflection, we have to observe the splitting of an image source into a stationary image and a double moving image with an opposite sign. The picture is therefore a little more complex than in the usual cases involving single images.

(3.iv) <u>Discussion</u>. Equations (18) and (53) above show that both compressibility and free surface effects lead to decline functions that are sums or integrals over exponentials of negative time. In essence, therefore, the decline processes are governed by very simple functional relationships. Moreover, the analysis in section (I) indicates that from the quantitative point of view, the free surface effect dominates in all liquid reservoirs.

The decline or relaxation time is another parameter of major interest. By definition this is the time t_r during which the amplitude of a stationary wave of wavelength L decreases to (1/e) of its initial value. Inserting a waveform $\exp[-(t/t_r) + ikx]$ where $k = (2\pi/L)$ is the wavenumber, into equation (14) gives for compressibility the time $t_r = (1/ak^2)$. Similarly, we find on the basis of (23) for the free surface a value $t_r = (1/wk)$. At the same L, the ratio of the free surface to compressibility time is $(ak/w)=(\phi k/\rho sg)$. Inserting values of interest for long-term reservoir behavior such as, for example, $\phi = 0.1$, L = 6 km, $s = 3x10^{-11}Pa^{-1}$ we find values of this ratio of about 300. This indicates quite clearly that the compressibility phenomena are on a much shorter time scale and smaller magnitude than the free surface phenomena. Our approach of neglecting compressibility in the above analysis is, therefore, well justified.

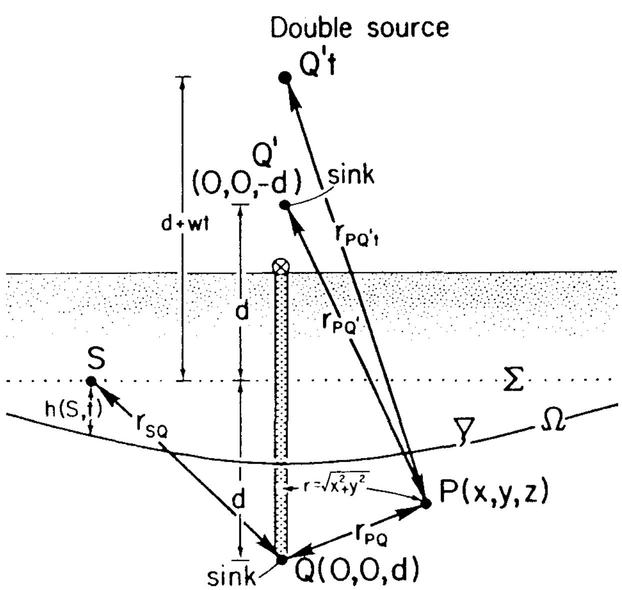


FIG. 1a. Infinite half space of linearized free surface method.

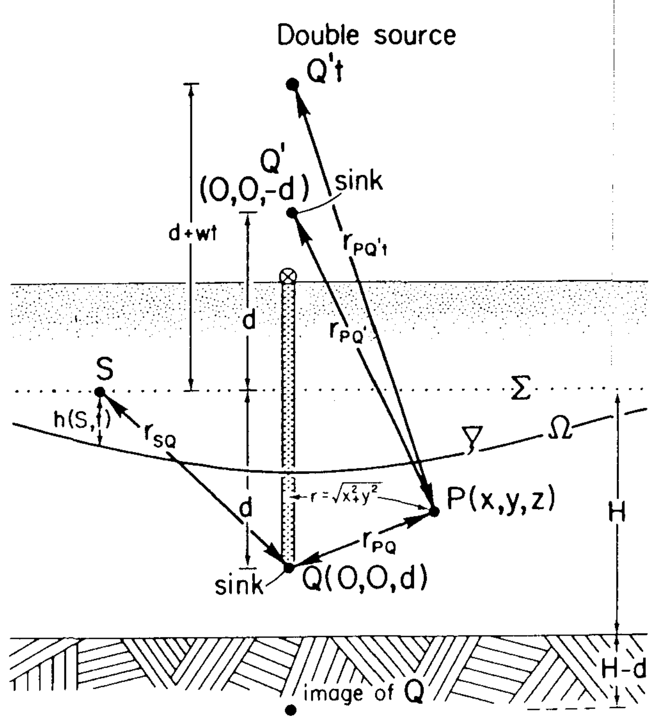


FIG. 1b. Reservoir half space for linearized free surface method with bottom layer.

(4) Reservoir simulation by lumping

The fact that the principal decline functions for liquid reservoirs are of the negative time exponential type suggests the use of lumping as a method of reservoir simulation. Below, we will briefly look onto this possibility.

Consider a liquid geothermal reservoir that is producing a constant mass flow q from a number of wells. We assume that the reservoir pressure is being monitored at a fixed point where a decreasing reference pressure function p(t) is being observed. Moreover, it is being assumed that production, started at time t = 0 from equilibrium conditions where we can take that the reference pressure p(0) = 0. The producing holes have a bottom-hole pressure of $p_w(t)$ that is also taken from an appropriate reference point as p(t) and therefore $p_w(0) = 0$.

The simplest lumped model to simulate this system is shown in Fig. 2 below.

The model consists of a liquid capacitor or container (I) with vertical walls having an area A. The production q is being extracted from this capacitor over a conductor that has a conductance c_1. This element represents the contact resistance of the producing holes. Recharge to the container is obtained from a capacitor (II) of an infinite area over another conductor that has a conductance c_2. Reference pressure in the large capacitor is taken to constant and equal to zero. The liquid level in (I) is measured by the pressure $p_1(t)$. In accordance with Darcy type flow conditions we assume that both conductors are linear and hence that the following system equations hold

$$q = c_1(p_1-p_w) \tag{58}$$

$$(A/g)Dp_1 = c_2(0-p_1)-q \tag{59}$$

where g is the acceleration of gravity and D = d/dt. Since we don't observe p_w, equation (58) is irrelevant and does not enter into the discussion below. The principal parameters of the simulation system are thus the capacitor area A and the conductance c_2. Given q(t) and $p_1(t)$ for some fixed time interval starting at t = 0, we are now interested in deriving values of A and c_2 such that the model simulates the given reservoir in the optimal way during at least a part of the production time. A convenient way of obtaining these values in the following.

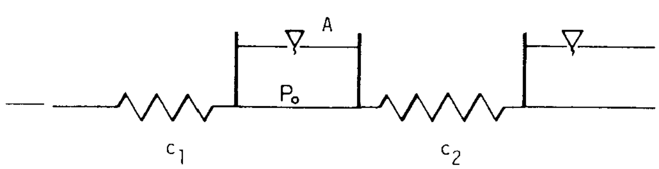


FIG. 2. Lumped parameter model of simulated reservoir.

Since we have assumed that q is constant, the present decline function $p_1(t)$ is characterized by a smooth negative time exponential behavior. We can then expand the known $p_1(t)$ into a Taylor series in t starting at t = 0 and that is truncated at the second order term,

$$p_1(t) = p_1(0)+tDp_1(0)+(t^2/2)D^2p_1(0), \tag{60}$$

where we have abbreviated $Dp_1|_{t=0}=Dp_1(0)$ and $D^2p_1|_{t=0} = D^2p_1(0)$.

Since $p_1(0) = 0$ this series reduces to

$$p_1(t) = tDp_1(0) + (t^2/2)D^2p_1(0), \tag{61}$$

Inserting this expression in equation (59) results in

$$(A/g)[Dp_1(0)+tD^2p_1(0)] = c_2[-tDp_1(0)- (t^2/2)D^2p_1(0)]-q, \tag{62}$$

On a second order appproximation, we obtain from the terms in t^o and t the parameter relations

$$A = -g[q/Dp_1(0)] \tag{63}$$

and

$$c_2 = -(A/g)D^2p_1(0)/Dp_1(0) = (A^2/qg^2)D^2p_1(0). \tag{64}$$

Since $p_1(t)$ is a known function, the derivatives at t = 0 are also known and we can thus derive A and c_2 from (63) and (64) above.

On the basis of the known parameters, we can then solve equation (59) for a given variable input q(t) and obtain

$$p_1(t) = (g/A)\int_0^t \exp[-gc_2(t-\tau)/A]q(\tau)dt \tag{65}$$

as a procedure to predict or extend $p_1(t)$ in time.

An analysis of the above type can be carried out on any field decline functions that have been obtained with sufficient accuracy to derive the derivatives. In most practical cases the mass production function will be a variable q(t) and the input function $p_1(t)$ for the above analysis will then have to be obtained by a deconvolution, that is by solving an equation

$$p_f(t) = \int_0^t p_1(t-\tau)Dq(\tau)d\tau, \tag{66}$$

where $p_f(t)$ is the reference pressure that is the output to q(t). There are no problems in solving (66).

To illustrate the above procedure we will carry out the lumping of the free surface dynamics model leading to equation (46). We obtain then the time derivatives $Dp_1(0)$ and $D^2p_1(0)$ from equation (46) and use (63) and (64) to derive the lumped system parameters. To simplfy the procedure, we consider only the case q = constant = unity. Omitting elementary details and irrelevant factors, the method, in essence, consists in approximating the function

$$f_1(t) = -[1-(R_o/R_1)] \qquad (67)$$

by the function

$$f_2(t) = -[(z+d)^2/R_2^2][1-\exp[-wtR_2^2/R_o^2(z+d))], \qquad (68)$$

where

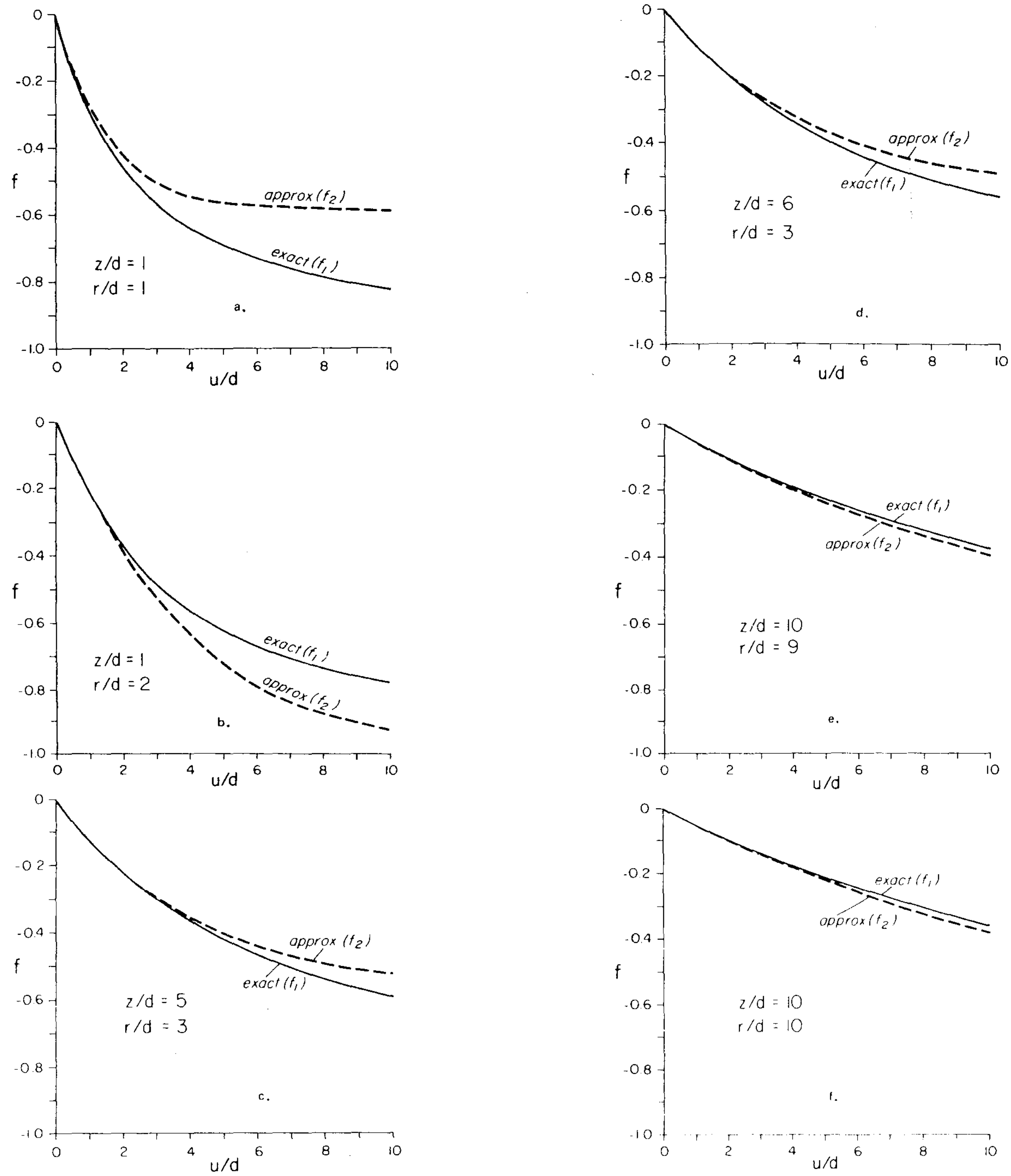
$$R_o = [r^2+(z+d)^2]^{\frac{1}{2}} \qquad (69)$$

$$R_1 = [r^2+(z+wt+d)^2]^{\frac{1}{2}} \qquad (70)$$

$$R_2 = [2(z+d)^2-r^2]^{\frac{1}{2}} \qquad (71)$$

with $r^2 < 2(z+d)^2$

The results of a numerical evaluation are illustrated in Fig. 3. It is evident that the lumped approximation holds quite well until the factor wt is of the order of a few depths d. Quite often w is of the order of 10^{-5} m/s and d about 10^3m. In this case the lumped approximation will give good results for a period of a few years.



FIGS. 3a-3f. Comparisons between lumped parameter approximation and exact solution.

III. REVIEW OF METHODS

Petroleum Reservoirs

Production decline methods are probably the most commonly used tool of the reservoir engineer because production data are always recorded and filed whereas temperature and pressure records are far less common. The uses of these methods are at least two fold. First, they are used to predict future production and second, they can provide insight into reservoir mechanisms and geology.

Production data for fields and individual wells are usually plotted on a monthly basis so a year's worth of data might be enough to use with the standard methods. When fields have been produced for a number of years, e.g., 10, production data are plotted on an annual basis and fitted. In the petroleum industry great care must be taken in trying to extrapolate past trends because conditions can change. For example, the reservoir pressure might pass through a bubble point causing dissolved gas to outgas thereby drastically changing flow conditions. The best discussion of and warning about decline methods is in Brons (1963).

Reserve estimates are calculated from predicted future production. It the predictions are bad, the estimates are bad. Brons shows an example using production from two wells, each with constant but different percentage decline rates. When their productions are added together and fitted with a hyperbolic $_{\text{eqn}}$ (the best fit) we get a very different reserve estimate from the one obtained by looking at each well separately. As always, the reservoir analyst must supply a great deal of insight.

Decline methods are not directly applicable to new fields except that if the new field appears to be similar to a previously studied field we might make some intelligent guesses about its production characteristics.

Decline methods are used to determine when additional wells should be drilled and when wells should be worked over. Production in individual wells can decrease in a steady regular manner from sand plugging the formation. This can be seen on a production vs. time graph.

Geothermal Reservoir

The decline methods developed for analyzing oil and gas wells can be used for geothermal wells but we must recognize that petroleum and geothermal reservoirs are very different from each other. These differences can cause production mechanisms to be drastically different in the two cases. Some of the more important differences and their consequences are as follows:

a. Petroleum reservoirs are usually sedimentary formations. Geothermal reservoirs are usually fractured igneous or metamorphic formations. Darcy flow holds in the first case and fracture flow in the second.

b. Temperature is relatively unimportant in petroleum production. It is critical in geothermal production. High temperatures stress tubing and cement in the wellbore.

c. Geothermal well flow volumes are often 1 to 2 orders of magnitude greater then petroleum volumes.

d. Precipitation is much more serious in geothermal wells than in petroleum wells.

e. Petroleum is a complex mixture with volatile components. Geothermal water is essentially one species.

Fracture size, quantity and distribution are drastically affected by precipitation, changes in temperature and seismic activity. Geothermal reservoirs seen to be much more complex than petroleum reservoirs so methods taken into geothermal work must be examined carefully. We have done this with the data and methods available but more work must be done as we produce more geothermal fields over time.

1. Arps

Arps's (1945, 1956) work forms the basis for all the decline curve methods currently in use. He brought together and codified work on oil reserve estimation that had been done as early as 1908. The commonest methods were graphical in which production q or cumulative production Q was plotted vs. time t. See Fig. 4 from Arps (1956). Examinations of production data showed that data with constant first differences fit an exponential equation while data with constant second differences fit a hyperbolic or harmonic equation. All three equations can be expressed as

$$a = Kq^b = \frac{-dq/dt}{q} \tag{72}$$

where a = fractional decline
some authors use D = fractional decline
q = production rate of time t
K = constant
b = constant

The solutions to equation (72) are shown in Table 1.

Guerrero (1961) gives a good "cookbook" approach to analyzing data using these methods. See Table 2 for problems worked out by Guerrero. Arps's equations were considered to be strictly empirical until 1973 when Fetkovich proposed some theoretical basis for the exponential equation (see below). The hyperbolic equation is still considered to be empirical.

2. Fetkovich

Fetkovich (1973) showed that log-log type curves can be used to analyze production data in an analogous manner to analyzing pressure data. He presented log-log plots of dimensionless flow rate, $q_{Dd} = q(t)/q_i$, vs. dimensionless time, $t_{Dd} = D_i t$, for $0 \le b \le 1$ and $D_i = 1$ (see Fig. 5). b = 0 is the exponential solution while b = 1 is the harmonic solution.

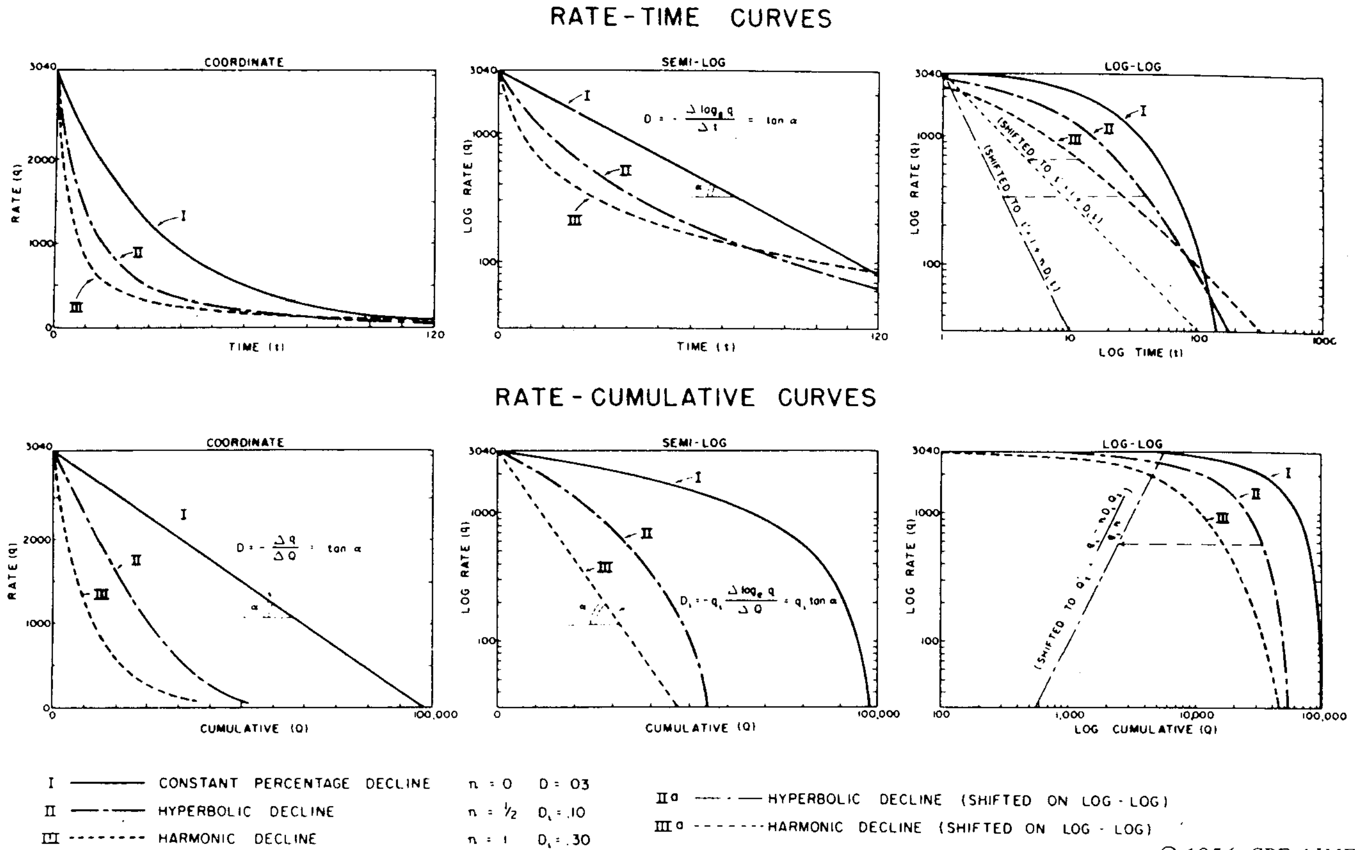


FIG. 4. Three types of production decline curves on coordinate, semi-log, and log-log graph paper (from Arps, 1956).

The exponential curve is given by

$$q_{Dd} = \exp(-D_i t), \quad D_i = 1 \tag{73}$$

while the hyperbolic curves are given by

$$q_{Dd} = (1+bD_i t)^{-1/b} \quad \text{for } 0<b\leq 1 . \tag{74}$$

Using an overlay technique as shown very clearly in Earlougher (1977), (see Fig. 6), production data can be plotted over the curves and a decline exponent can be picked. For $t_{Dd}<0.3$ all the curves are coincident.

Fetkovich showed that the exponential decline has a fundamental base by deriving it as a solution to the constant well pressure case. The equation for dimensionless flow rate is

$$q_{Dd} = q(t)/q_i = \exp[-(q_i)_{max} t/N_{pi}] \tag{75}$$

This equation can be related to (73) by setting

$$D_i = (q_i)_{max}/N_{pi} \tag{76}$$

then

$$t_{Dd} = \frac{(q_i)_{max} t}{N_{pi}} \tag{77}$$

We define

$$N_{pi} = \frac{\Pi(r_e^2 - r_w^2)\phi c_t h p_i}{5.615B} \tag{78}$$

$$(q_i)_{max} = \frac{khp_i}{141.3\ \mu B[\ln(r_e/r_w)-\frac{1}{2}]} \tag{79}$$

Finally we get

$$q_{Dd} = \frac{q(t)}{\dfrac{kh(p_i-p_{wf})}{141.3\ \mu B[\ln(\frac{r_e}{r_w} - \frac{1}{2})]}} \tag{80}$$

Fetkovich showed that production decline curve data could be used to derive values for permeability thickness kh which is usually obtained from pressure data. (see Fig. 7a and 7b). Compare kh calculations from rate-time data and pressure time data.

3. Slider's Method

Slider (1968) proposed a simple method of curve matching to obtain the hyperbolic exponent

TABLE 1. CLASSIFICATION OF PRODUCTION DECLINE CURVES

DECLINE TYPE	I. CONSTANT-PERCENTAGE DECLINE	II. HYPERBOLIC DECLINE	III. HARMONIC DECLINE
BASIC CHARACTERISTIC	DECLINE IS CONSTANT $n = 0$	DECLINE IS PROPORTIONAL TO A FRACTIONAL POWER (n) OF THE PRODUCTION RATE $0 < n < 1$	DECLINE IS PROPORTIONAL TO PRODUCTION RATE $n = 1$
	$D = K \cdot q^0 = -\frac{dq/dt}{q}$ $\int_0^t D\,dt = -\int_{q_i}^{q_t} \frac{dq}{q}$ $-Dt = \log_e \frac{q_t}{q_i}$	$D = K \cdot q^n = -\frac{dq/dt}{q}$ FOR INITIAL CONDITIONS: $K = \frac{D_i}{q_i^n}$ $\int_0^t \frac{D_i}{q_i^n} \cdot dt = -\int_{q_i}^{q_t} \frac{dq}{q^{n+1}}$ $\frac{n D_i t}{q_i^n} = q_t^{-n} - q_i^{-n}$	$D = K \cdot q^1 = -\frac{dq/dt}{q}$ FOR INITIAL CONDITIONS: $K = \frac{D_i}{q_i}$ $\int_0^t \frac{D_i}{q_i} \cdot dt = -\int_{q_i}^{q_t} \frac{dq}{q^2}$ $\frac{D_i t}{q_i} = \frac{1}{q_i} - \frac{1}{q_t}$
RATE - TIME RELATIONSHIP	$q_t = q_i \cdot e^{-Dt}$	$q_t = q_i (1 + n D_i t)^{-\frac{1}{n}}$	$q_t = q_i (1 + D_i t)^{-1}$
	$Q_t = \int_0^t q_t \cdot dt = \int_0^t q_i \cdot e^{-Dt} \cdot dt$ $Q_t = \frac{q_i - q_i \cdot e^{-Dt}}{D}$ Substitute From Rate-time Equation: $q_i \cdot e^{-Dt} = q_t$ To Find:	$Q_t = \int_0^t q_t \cdot dt = \int_0^t (1 + n D_i t)^{-\frac{1}{n}} \cdot dt$ $Q_t = \frac{q_i}{(n-1) D_i} \left[(1 + n D_i t)^{\frac{n-1}{n}} - 1 \right]$ Substitute From Rate-time Equation: $(1 + n D_i t) = \left(\frac{q_i}{q_t}\right)^n$ To Find:	$Q_t = \int_0^t q_t \cdot dt = \int_0^t q_i (1 + D_i t)^{-1} \cdot dt$ $Q_t = \frac{q_i}{D_i} \left[\log_e (1 + D_i t) \right]$ Substitute From Rate-time Equation: $(1 + D_i t) = \frac{q_i}{q_t}$ To Find:
RATE - CUMULATIVE RELATIONSHIP	$Q_t = \frac{q_i - q_t}{D}$	$Q_t = \frac{q_i^n}{(1-n) D_i} (q_i^{1-n} - q_t^{1-n})$	$Q_t = \frac{q_i}{D_i} \log_e \frac{q_i}{q_t}$

D = Decline as a fraction of production rate

D_i = Initial decline

q_i = Initial production rate

t = Time

q_t = Production rate at time t

Q_t = Cumulative oil production at time t

K = Constant

n = Exponent

TABLE 2. EXAMPLE OF USE OF EXPONENTIAL EQUATION

(1) Year	(2) Time, years Total	(3) Time, years Ave.	(4) q_o= Oil prod. rate bbl./yr.	(5) Δq_o $(4)_{n-1}-(4)_n$	(6) $q_{oav.} = \frac{(4)_{n-1}+(4)_n}{2}$	(7) $D = \frac{\Delta q_o/\Delta t}{q_{oav.}}$ (5):(6)	(8) N_p=Cum. oil recovery, bbl.	(9) $N_p - \frac{\Delta N_p}{2}$ $(8) - \frac{(4)}{2}$
1947 . . .	1	0.5	99,200	. . .	. . .	. . .	99,200	49,600
1948 . . .	2	1.5	88,210	10,990	93,705	0.117	187,410	143,305
1949 . . .	3	2.5	73,240	14,970	80,725	0.185	260,650	224,030
1950 . . .	4	3.5	63,990	9,250	68,615	0.135	324,640	292,645
1951 . . .	5	4.5	54,910	9,080	59,450	0.153	379,550	352,095
1952 . . .	6	5.5	47,400	7,510	51,155	0.147	426,950	403,250
1953 . . .	7	6.5	41,580	5,820	44,490	0.131	468,530	447,740
						0.868		
					$D_{av.}$=0.868:6=0.145			
Future Performance								
1954 . . .	8	...	35,960	5,620	38,770	0.145	504,490	.....
1955 . . .	9	...	31,099	4,861	33,530	0.145	535,589	.....
1956 . . .	10	...	26,895	4,204	28,997	0.145	562,484	.....
1957 . . .	11	...	23,260	3,635	25,078	0.145	585,744	.....
1958 . . .	12	...	20,116	3,144	21,688	0.145	605,860	.....
1959 . . .	13	...	17,397	2,719	18,757	0.145	623,257	.....
1960 . . .	14	...	15,045	2,352	16,221	0.145	638,302	.....
1961 . . .	15	...	13,011	2,034	14,028	0.145	651,313	.....
1962 . . .	16	...	11,252	1,759	12,132	0.145	662,565	.....
1963 . . .	17	...	9,731	1,521	10,492	0.145	672,296	.....
1964 . . .	18	...	8,416	1,315	9,074	0.145	680,712	.....

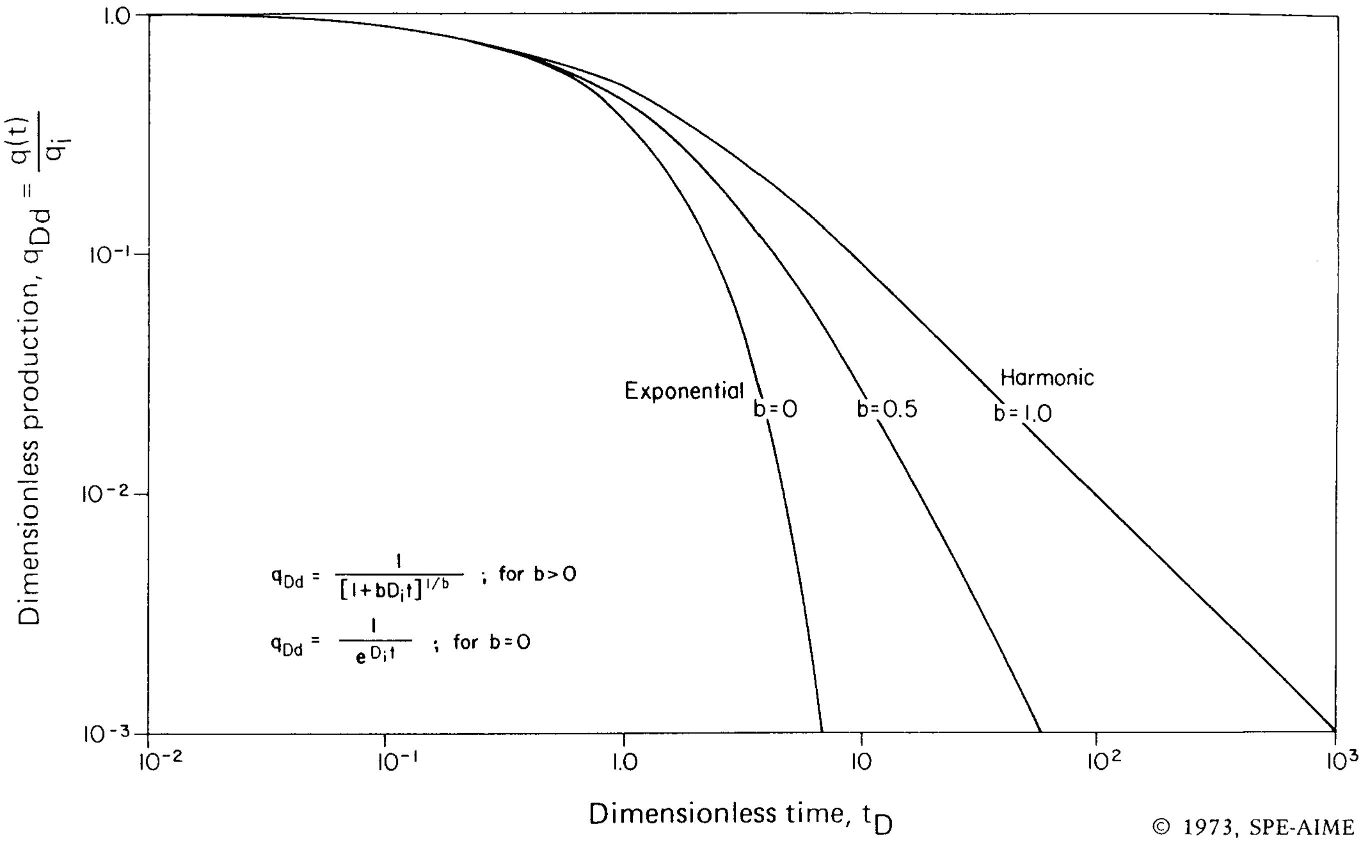


FIG. 5. Log-log type-curve of dimensionless flow rate vs dimensionless time (after Fetkovich, 1973).

b and the initial decline rate q_i. To use the method one needs to construct a set of curves of q/q_i vs. log time for various values of a_i and b using Arps's hyperbolic equation. Production data can then be plotted on the curves by using a transparent overlay. The overlay can be moved around until the best fit is found thus giving b and a_i. From equations or from a second set of curves, future production rates q and future cumulative production Q can easily be estimated. This method is easy to apply but it requires a separate set of curves for each possible value of b. Later methods eliminate this shortcoming.

4. Gentry's Method

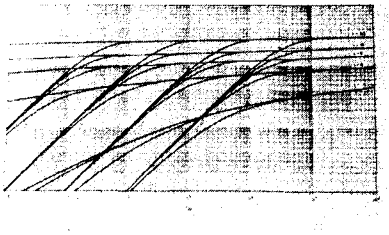
Gentry (1972) developed curves which are much easier to use then Slider's because only one set is needed for all values of b between 0 and 1. (see Figs. 8a and 8b) We can find b from a plot of Q/tq_i vs. log q_i/q. With this b we go to a plot of q_it vs. log q_i/q and find a_i. This gives us all the factors we need for a reserve analysis.

5. Gentry and McCray

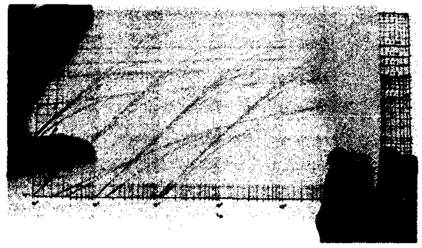
Reservoir analysts have usually assumed that $0 \leq b \leq 1$ in the solution of Arps's equations. See Higgins and Lechtenberg (1970) for exceptions. There is no mathematical basis for this restriction. b = 0 and b = 1 are special cases, the exponential and harmonic, respectively, but this does not restrict b from being larger than 1. Gentry and McCray (1978) investigated decline curve methods using semi-log plots of q_i/q vs. Q/q_it, cartesian plots of q/q_i vs. Q, and semi-log plots of q_i/q vs. a_it. See Figures 9a, 9b, and 9c. Some of their conclusions are (N_p = Q)

1. The dimensionless curves N_p/q_it vs. q_i/q a_it vs. q_i/q for a particular fluid-permeability system are not affected by the absolute permeability or size of the reservoir. The behavior of these plots is determined by (1) the characteristics of the contained fluid, (2) the relative permeability characteristics of the reservoir rock, (3) the reservoir drive mechanism, (4) reservoir heterogeneity, and (5) manual manipulation of production.

2. Reservoir heterogeneity tends to increase the magnitude of b as the degree of heterogeneity is increased. It is also apparent that b for a heterogeneous system

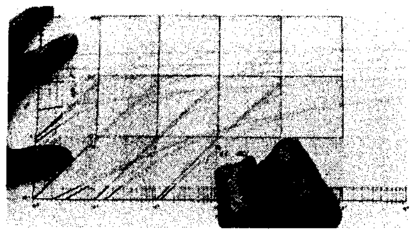


(a) Choose a type curve.



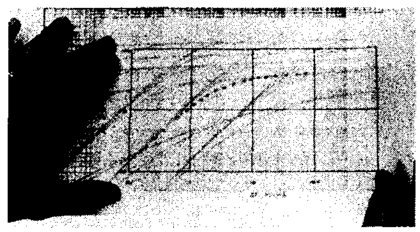
(b) Overlay with tracing paper.

(c) Trace major grid lines.

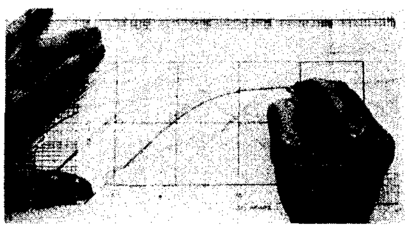


(d) Label axes.

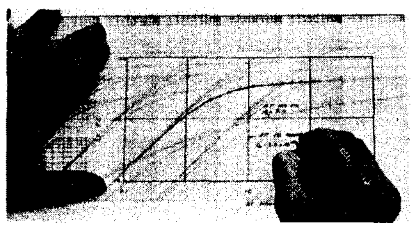
(e) Plot observed data using type-curve grid.



(f) Slide tracing paper to match a type curve.

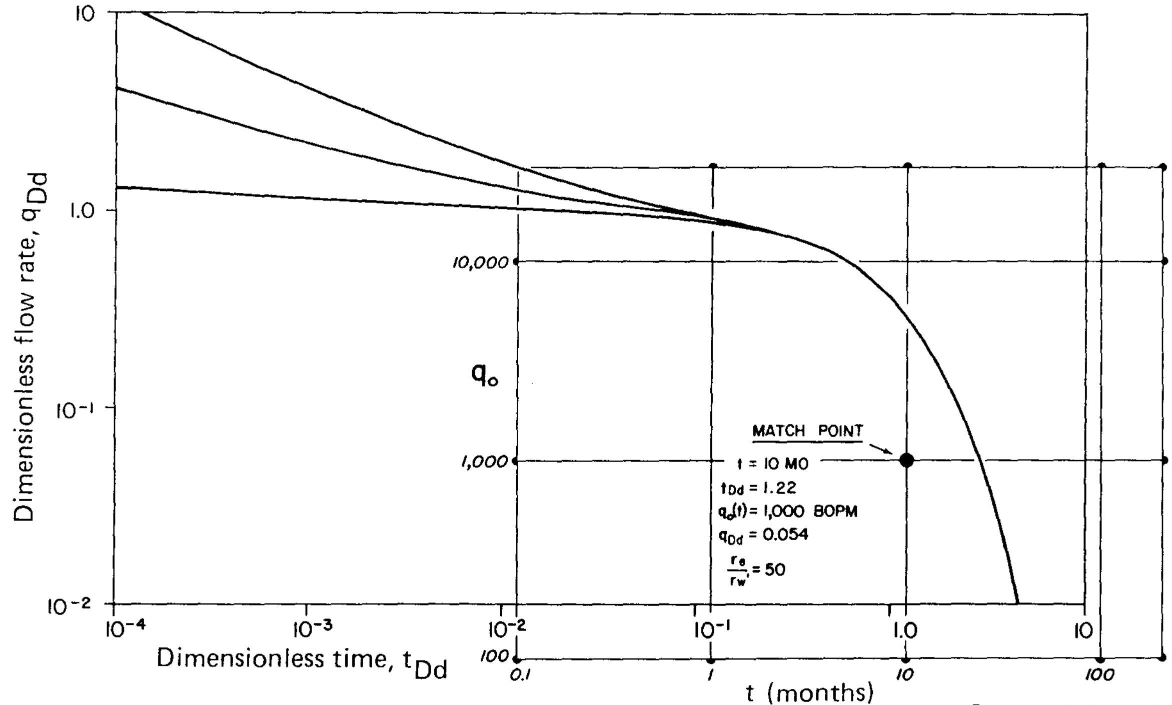


(g) Trace the matched curve.



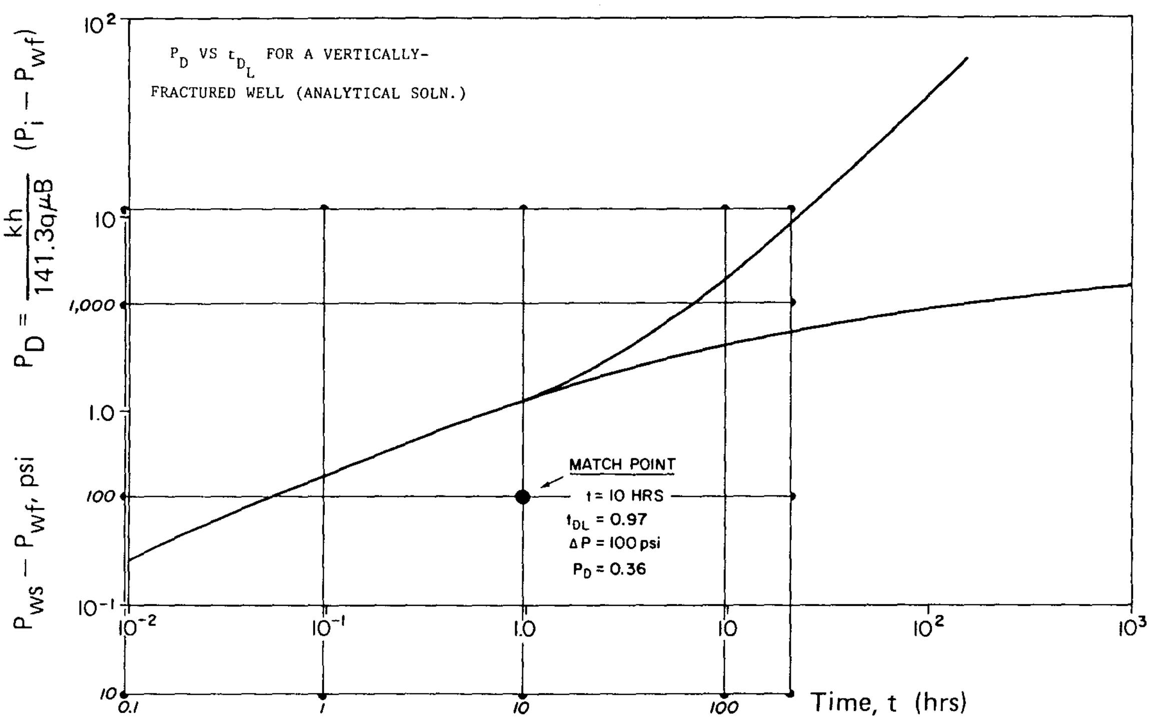
(h) Pick a match point

FIG. 6. Steps in type-curve matching (from Earlougher, 1977).



$$kh = \frac{q(t)\ 141.3\ (\ln r_e/r_w - .5)\mu B}{q_{Dd}\ (P_i - P_w)} = \frac{1000\ \text{BOPM}\ 141.3(\ln 50 - .5)}{30.4\ \text{D/M}\ (.054)\ (7259)} = 40.5$$

FIG. 7a. Type-curve matching example for calculating Kh using decline curve data (after Fetkovich, 1973).



$$P_D = 0.36 = \frac{(kh)(100\text{PSI})}{141.4(145)(0.47)(1.37)} \qquad t_{DL} = \frac{0.00634\ kt}{\phi\mu c_t L^2} \qquad kh = 47.5 \text{ md-ft}$$

FIG. 7b. Type-curve matching example for calculating Kh from pressure buildup (after Fetkovich, 1973).

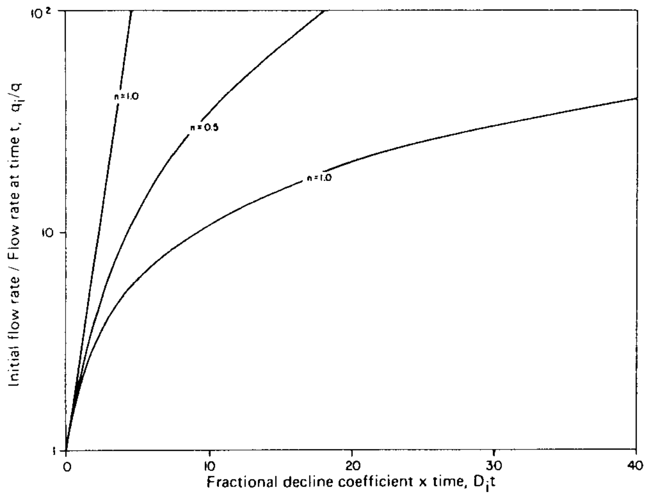


FIG. 8a. Decline curve analysis chart relating production rate to time (after Gentry, 1972).

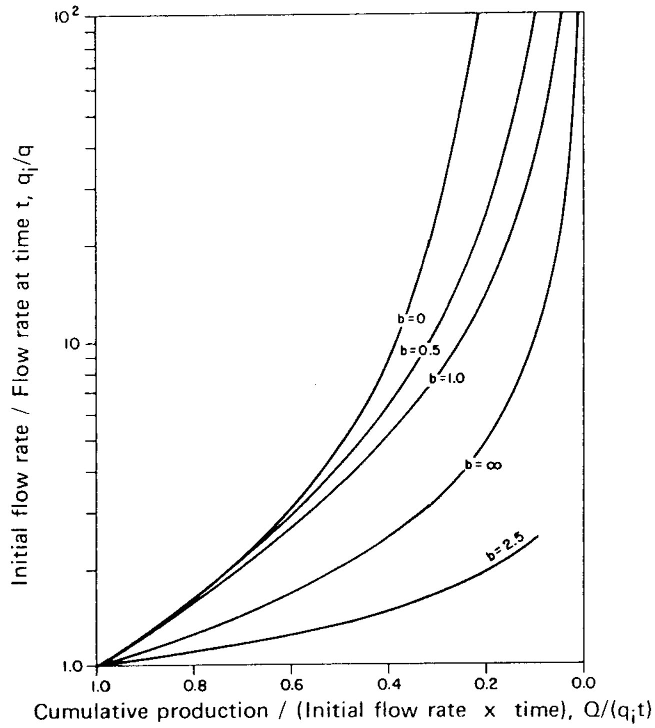


FIG. 9a. $Q/(q_i t)$ vs q_i/q (after Gentry & McCray, 1978).

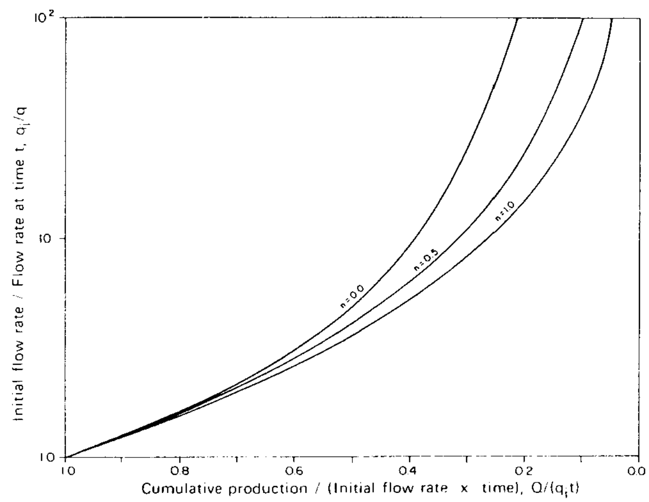


FIG. 8b. Decline curve analysis chart relating production rate to cumulative production (after Gentry, 1972).

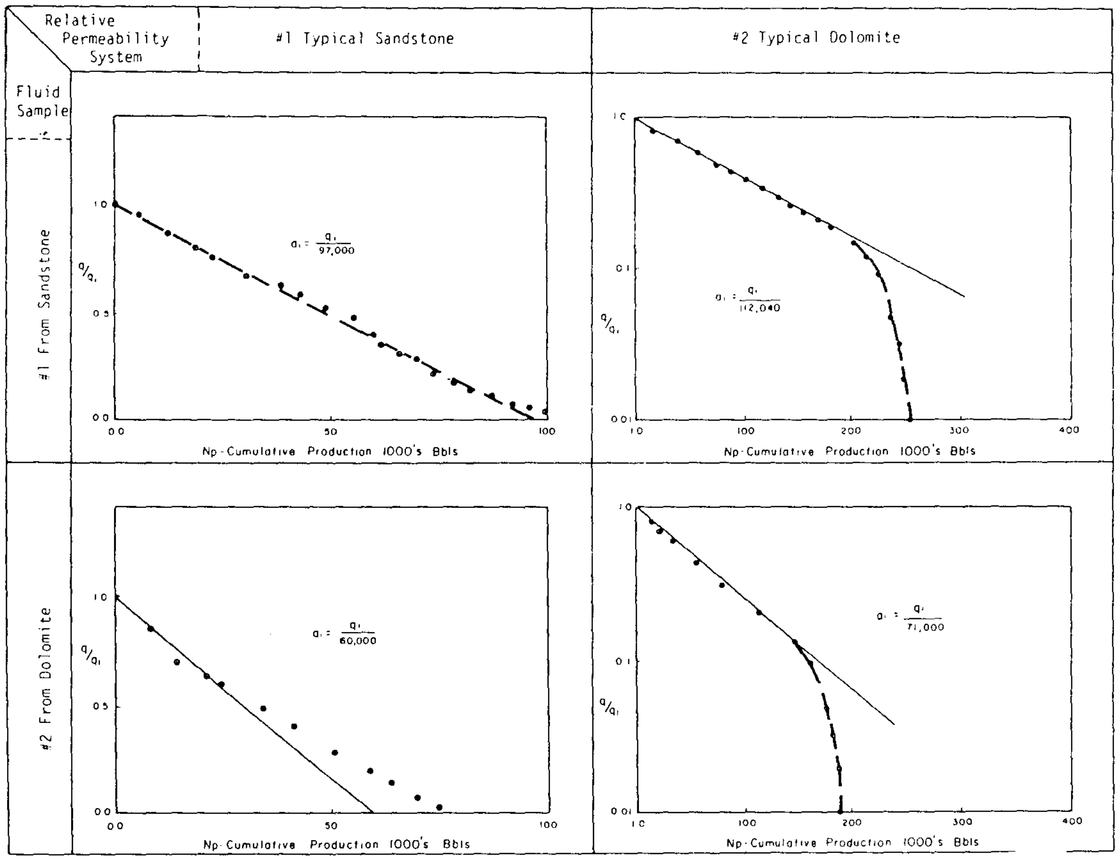


FIG. 9b. Plots of cumulative production vs q/q_i for four fluid-permeability systems (after Gentry & McCray, 1978). © 1978, SPE-AIME

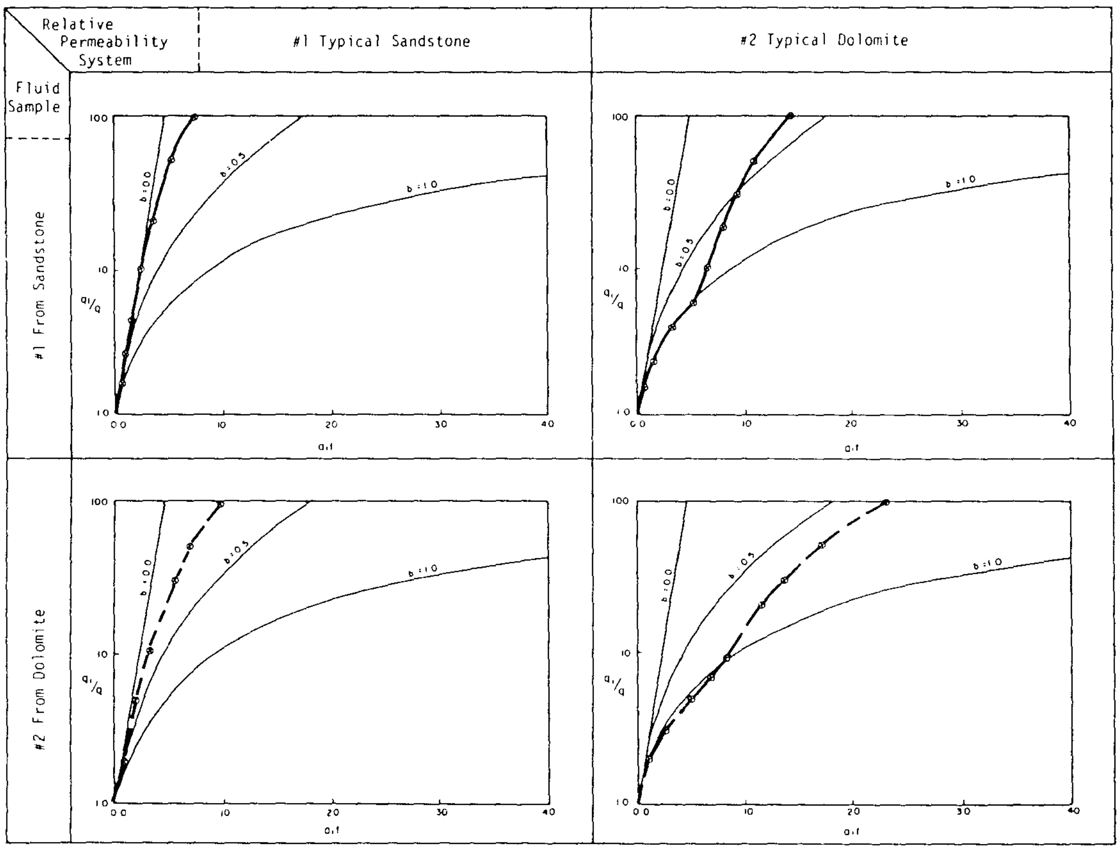


FIG. 9c. Dimensionless $a_i t$ curve histories for four fluid-permeability systems (after Gentry & McCray, 1978). © 1978, SPE-AIME

will increase to a maximum value and then as the ratio q_i/q becomes large, b will decrease and approach its homogeneous value.

3. Reservoir heterogeneity can and does cause b values to be greater than 1.0.

4. Manual manipulation of production can and does cause b values to be greater than 1.0.

5. The dimensionless plots for heterogeneous systems of 1 and 3 md, 3 and 9 md, and 5 and 15 md all plotted the same curve. This indicates that heterogeneous systems in the ratio of 1:3 will plot congruous dimensionless curves.

6. It appears that the relative-permeability characteristics of the reservoir have the greater effect on the decline exponent b, while the fluid characteristics have a greater influence on the constants a_i and q_i.

7. The equation

$$N_p/(q_i t) = (q/q_i)^{\alpha}$$

may better define certain decline curves than do the Arps equations.

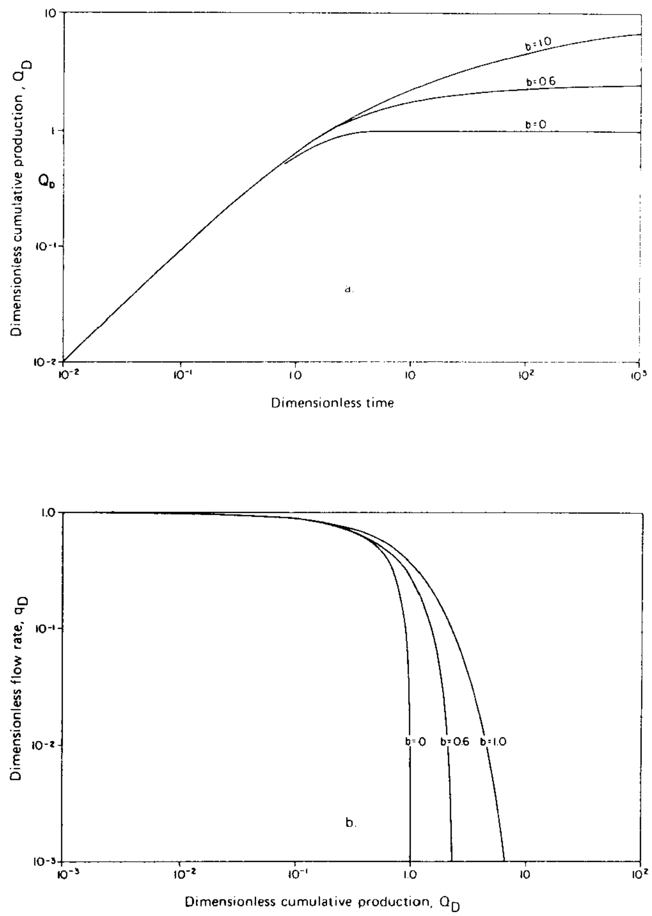
8. The plotting of production data on the $N_p/(q_i t)$ vs. q_i/q curve can be a helpful diagnostic tool for evaluating the production history of a well or lease."

6. Other Type Curves

Fetkovich developed log-log type curves using dimensionless production vs. dimensionless time, Q_D vs. t_D, but other variables can be used. We tried plots of dimensionless cumulative production vs. dimensionless time and dimensionless production vs. dimensionless cumulative production, q_D vs. Q_D. The plots were made by using the exponential equation and the hyperbolic equation for several values of b. See Figs. 10a and 10b. We had the same data scatter problem with these type curves as we did with Fetkovich's. A few data sets plotted very nicely on a particular curve, but most sets plotted very ambiguously.

7. P/z vs. Q

The natural gas industry has long used decline curves in which pressure divided by gas deviation factor, p/z, is plotted against cumulative production, Q (Katz, 1959). The straight line can be extrapolated to the economic limit of producing pressure quite easily. Brigham and Morrow (1974) have proposed adapting this method to steam fields. In plotting computer generated data they found that curve shape was strongly influence by porosity. Also, the presence of a boiling interface is critical. "If the wells are completed in the vapor zone it would be natural to graph p/z vs. production, as though this were a gas reservoir, and use an extrapolation of the best straight line as a predictive method to calculate reserves. The efficiency of this technique will be strongly dependent on the porosity if the actual reservoir contains boiling liquid." (see Fig. 11.)



FIGS. 10a-b. Log-log type-curve.

Pruess et al. (1979a, 1979b) have used the simulator SHAFT78 to test the use of p/z vs. Q plots for geothermal reservoirs. They conclude that ". . . the standard technique of estimating reserves by extrapolating a plot of p/z vs. cumulative production is not applicable to two-phase geothermal reservoirs." and ". . . in many cases pressure will be a linear function of cumulative production, with the slope allowing an estimate of reservoir volume. Reserve assessment requires knowledge of average porosity and vapor saturation, which cannot be obtained from pressure decline curves."

Brigham (1979) applied p/z techniques to a study of depletion in the Cabbro zone at Larderello, but he stated that the linearity of p/z with cumulative production doesn't hold for the entire life of a reservoir with a boiling interface. He claims that linearity is a good approximation for the first one-third to one-half of the reservoir's life.

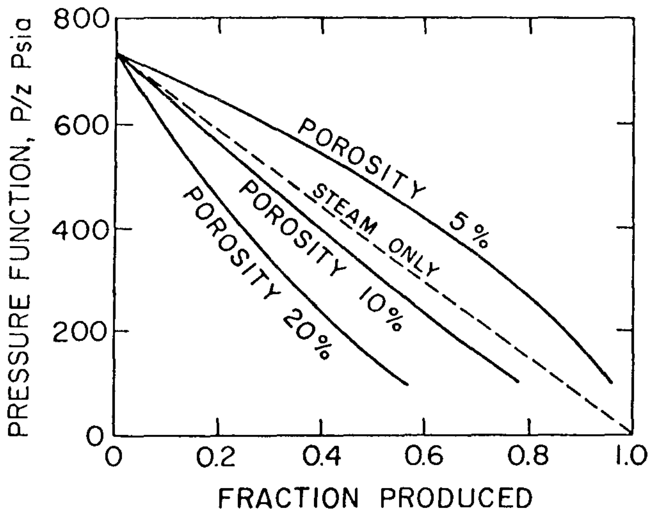


FIG. 11. Pressure depletion vs recovery, falling liquid level (from Brigham & Morrow, 1974).

8. Influence Functions

Unsteady state isothermal flow of slightly compressible liquid through a porous medium can be described using the diffusivity equation

$$\nabla\left(\frac{k\nabla p}{\mu}\right) = \phi c\frac{\partial p}{\partial t}$$

$$\frac{1\partial}{r\partial r}\left(\frac{rk\partial p}{\mu\partial r}\right) = \phi c\frac{\partial p}{\partial t} \qquad (81)$$

The equation can be solved using a Green's function approach to derive a "response", "resistance", "memory", or "influence" function. Katz and Coats (1969) in describing water movement in aquifers defined two influence functions: 1) P(t) = the "rate case" influence function which is defined as the pressure drop at the reservoir boundary (a function of time) corresponding to a unit rate (e.g. 1 cu. ft./day) of water influx." For a constant flow rate q we get $p_o-p(t) = qP(t)$, the constant terminal rate case equation. 2) Q(t) = the "pressure case" influence functions since a constant pressure p_b is specified at the outer boundary. The constant terminal pressure equation is $q(t) = (p_o-p_b)Q(t)$.

P(t) and Q(t) can be calculated either for idealized models or from field data. Let F(t) = Q(t) or P(t). For an idealized F(t) we must specify "1) model geometry, 2) exterior boundary conditions (e.g. infinite, closed or constant pressure), and 3) model parameters." The specification of reservoir parameters and geometry is particularly difficult in geothermal reservoirs so the calculation of F(t) from field data is more attractive and easier than trying to devise a thoroughly specified model. The advantages of the field method are "1) none of the above choices are required, and 2) an influence function which reflects unknown (and practically speaking, indeterminate) aquifer properties, as reflected by actual field performance, is determined. Disadvantages are 1) the resemblance of the backed out F(t) to the true function is proportional to the accuracy of field data, and 2) the influence function is obtained only up to the time of last available field data; extrapolation is required for purposes of predicting future water movement."

Coats et al. (1964) recommended the use of field influence functions for oil fields with adjacent aquifers. The method is directly applicable to geothermal fields. The influence function F can easily be generated as a function of pressure p and flowrate q using the following equations:

Integral form

$$\Delta p = \int_o^t \frac{dq(t-\tau)}{dt} F(\tau)d\tau = \int_o^t q(t)\frac{dF(t-\tau)}{d\tau}d\tau \qquad (82)$$

Discrete form

$$\Delta p_i = p_o - p_i = \sum_{j=1}^{i} (q_i-q_{i-1})F_{i-j+1}$$

Bodvarsson (1980, personal communication) has shown how the influence function problem can be formulated in a slightly different manner. The function F defined by Coats is a unit step response function. Instead of the unit step response, we can use the impulse response h, where h = dF/dt. The equation to be solved is then

$$\Delta p = \int_o^t \frac{dq(\tau)}{d\tau} F(t-\tau)d\tau = \int_o^t q(\tau)\frac{dF(t-\tau)d\tau}{dt} \qquad (83)$$

$$\text{Integral form } \Delta p = \int q(\tau)h(t-\tau)d\tau$$

$$\text{Discrete form } p_o-p_i = \sum_{j=1}^{i} q_j h_{i-j+1}$$

The first derivative of the curve from the F formulation should be identical to the curve derived from the h formulation. F can be calculated by hand (Jargon and van Poolen, 1965) and Hutchinson and Sikora, 1959) but we recommend against it. An F can be calculated which fits the data well, but which has no physical meaning.

Katz and Coats cite an example in Katz et al. (1963) in which an influence function is calculated by direct methods which exactly reproduces past performance but which cannot be extrapolated. The smoothness constraints below assure a physically meaningful solution and they can be arrived at both intuitively and analytically. From Katz and Coats "if water is injected into an aquifer at a constant rate through some fixed inner aquifer boundary (surface), then intuitively the pressure change at that boundary must always be positive. In addition, the pressure should always increase and the rate of increase should continually decrease with time." The analytical proof for the constraints is given in Coats et al.

Linear programming methods such as the package MPOS should be used with the smoothness constraints on F or h:

$$F > 0,\ t > 0 \qquad\qquad h > 0,\ t > 0$$

$$\frac{dF}{dt} \geq 0 \qquad\qquad \frac{dh}{dt} \leq 0$$

$$\frac{d^2F}{dt^2} \leq 0 \qquad\qquad \frac{d^2h}{dt^2} \geq 0$$

If the data are not "smooth and regular enough" the use of simple hand calculations can lead to the results shown in Fig. 12. The F function will reproduce the pressure very well, but the function cannot be extrapolated and is physically meaningless. The F calculated by the linear program is shown on the same figure for comparison.

Hutchinson and Sikora and Coats et al. discuss the effects of field geometry on the behavior of the pressure drop and the influence function. As production time increases, the rate of pressure change decreases. If the reservoir outcrops, both the pressure drop and the influence function become constant. This is an effect we will look for in geothermal areas which we know have fluid recharge. If the reservoir is infinite-acting or bounded the influence function and the pressure drop will increase monotonically for all time greater than 0.

Hutchinson and Sikora show how to extrapolate calculated influence functions. If a definite straight line has developed from the field data it may be extrapolated and the field assumed to be bounded. If no definite straight line has developed, the last 3 or 4 values of F should be examined. If the average ΔF for these times gives a good match to past performance the curve may be extrapolated using the slope of the average F. The extreme extrapolation assumes an infinite aquifer. In this case

$$\Delta F_{n+1} = \Delta F_n \frac{\log[(n+1)/n]}{\log[n/(n-1)]}\ .$$

All these extrapolations are included in our MPOS program.

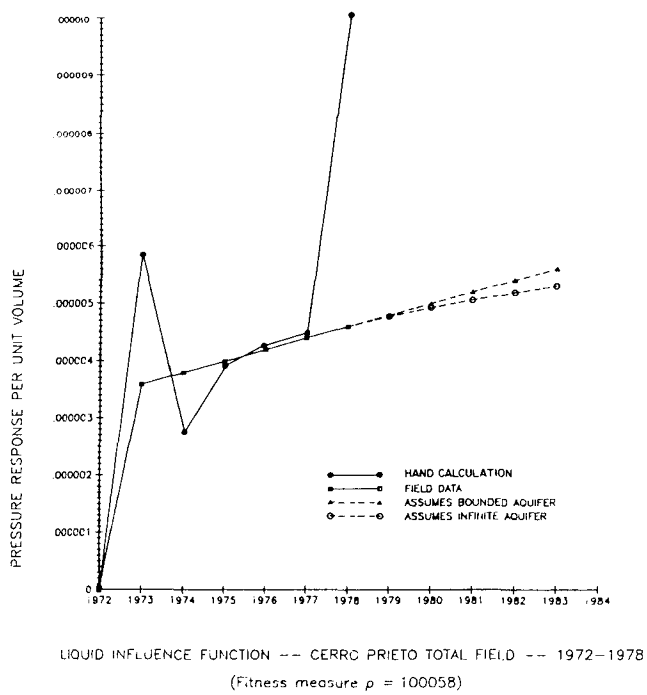


FIG. 12. Liquid influence function — Cerro Prieto total field.

9. Linearized Free Surface-Green's Function

One of the main vitures of the influence function method is described above is that it can be used to predict reservoir behavior without specifying a physical model for the reservoir. Long-time behavior of the influence function can tell something about the boundaries. If the reservoir has a free liquid surface and is assumed to be a porous half-space, Fig. 1a, a simple, distributed parameter model can be posited. Bodvarsson (1977) linearized the free surface condition and derived the following equations for pressure

$$p(P,t) = \frac{-q}{4\ c}\left(\frac{1}{r_{PQ}} + \frac{1}{r_{PQ'}} - \frac{2}{r_{PQ't}}\right) \qquad (84)$$

p = pressure, meters of head

q = flow rate, kg/s (constant)

$r_{PQ} = ((x-x')^2+(y-y')^2+(z-d)^2)^{\frac{1}{2}}$

$r_{PQ'} = ((x-x')^2+(y-y')^2+(z+d)^2)^{\frac{1}{2}}$

$r_{PQ't} = ((x-x')^2+(y-y')^2+(z+wt+d)^2)^{\frac{1}{2}}$

Bodvarsson (1978) also showed that the impulse response of a linearized free surface can be expressed as

$$G(t,S,Q) = \frac{-1}{2\pi\phi\rho}(wt+d)(x^2+y^2+(wt+d)^2)^{-3/2}U_+(t) \qquad (85)$$

where

d = depth from free surface to sink

G = Green's function

S = (x,y) a point on the free surface

ϕ = porosity, fraction

ρ = density, kg/m^3

$w = kg/(\nu\phi)$ = sinking velocity, m/s

ν = kinematic viscosity, m^2/s

g = 9.8 m/s^2

k = permeability, m^2

$U_+(t)$ = unit impulse function

$r = (x^2+y^2)^{\frac{1}{2}}$, radial distance from sink.

He obtained the following expression for drawdown in meters for P at a distance r from the sink (wellbore)

$$h(t) = \int_o^t G(t-\tau)q(\tau)d\tau \tag{86}$$

$$\approx \sum_{\tau=0.5}^{t-0.5} G(t-\tau)q(\tau)\Delta\tau$$

See Chpt. II for the derivation of these equations. The impulse response, h, is the drawdown in meters at the point, P, caused by the instantaneous withdrawal of one unit fluid mass at point Q. At a continuous withdrawal the total drawdown at P would be a summation over all fluid sinks. The equation is

$$h_{total}(P) = \sum_{n=1}^{N} G_n(t-\tau)q_n(\tau)d\tau \tag{87}$$

This equation for drawdown can be compared directly with the h function formulation of the influence function as described above.

If the reservoir has a relatively impermeable zone below the producing zone the half space assumption can be modified. An image source term or terms as necessary can be added to the equation for G. See Fig. 1b. With one image term the expression is

$$G = \frac{-1}{2\pi\phi\rho}\left[\frac{wt+d}{[x^2+y^2+(wt+d)^2]^{3/2}} + \frac{2H-d+wt}{[x^2+y^2+(2H-d+wt)^2]^{3/2}}\right] \tag{88}$$

More terms can be added as necessary.

The distributed model described above can be approximated by a lumped parameter model as described more fully in Chpt. II.

IV. DATA PROCESSING

Data Sets

The most complete data set is from Wairakei, New Zealand by Pritchett et al. (1978) published by Systems, Science and Software for Lawrence Berkeley Laboratory. Individual well monthly heat and mass flow rates are given from 1953 through 1976 for 141 wells. Furthermore, a fair amount of pressure and temperature data are presented.

The authors presented a substantial amount of data on the geology and subsidence problems at Wairakei. In addition to this report we received from Malcolm Grant, DSIR, a set of annotated individual well production graphs which indicated when wells were shut in and which steam lines the wells were connected to. See Fig. 13a for map.

The data set for Cerro Prieto by Bermejo et al. (1979) published by Lawrence Berkeley Laboratory included graphical production histories of most of the wells from 1973 to 1978. These graphs were digitized for analysis. The production was broken down into liquid and vapor production. In addition, we received data from Marcelo Lippman, LBL, which showed individual well total mass flow rates. The two data sets were treated separately and then compared. A theoretical pressure drawdown curve was taken from Sanchez and de la Palma (1979). See Fig. 13b for map.

The last large data set is from The Geysers, California, courtesy of the California Dept. of Conservation, Div. of Oil and Gas. The data include production injection and pressure data from 27 wells from March 1971 through December 1979. Additional pressure data are from Lipman, Strobel, and Gulati (1977). See Fig. 13c for map.

The Larderello data were taken from Sestini (1970). The sparse data for other fields were from various sources. See Fig. 13d for map.

Graphical Treatment of Data

The first step in the analysis was graphing all the available production data on cartesian paper using SPSS. These graphs allowed us to eliminate from further consideration wells with severely irregular production such as Bore 11.

Arps (1945, 1956) pointed out that the exponential equation would graph as a straight line on semilog paper. We tried plotting the data for several wells at Wairakei but found that production decline was insufficient to make the semilog plots look very different from the cartesian plots. The log-log plots, however, were significantly different from the cartesian plots so most of the data were plotted on log-log plots. We tried matching the log-log plots against Fetkovich's type curves. For the most part the data scatter rendered the method useless. We were also hindered by the fact that dimensionless time for almost all our wells was fairly short,

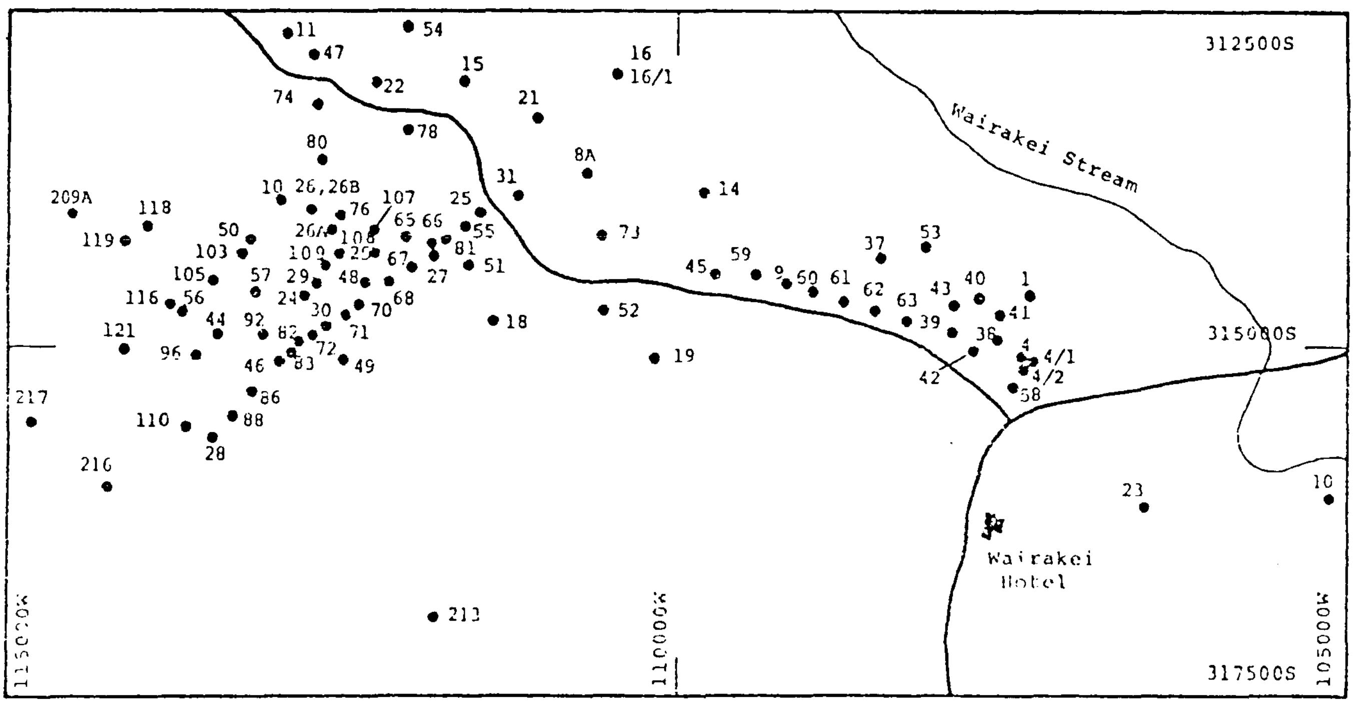


FIG. 13a. Bore locations in main bore field (from Pritchett et al., 1978).

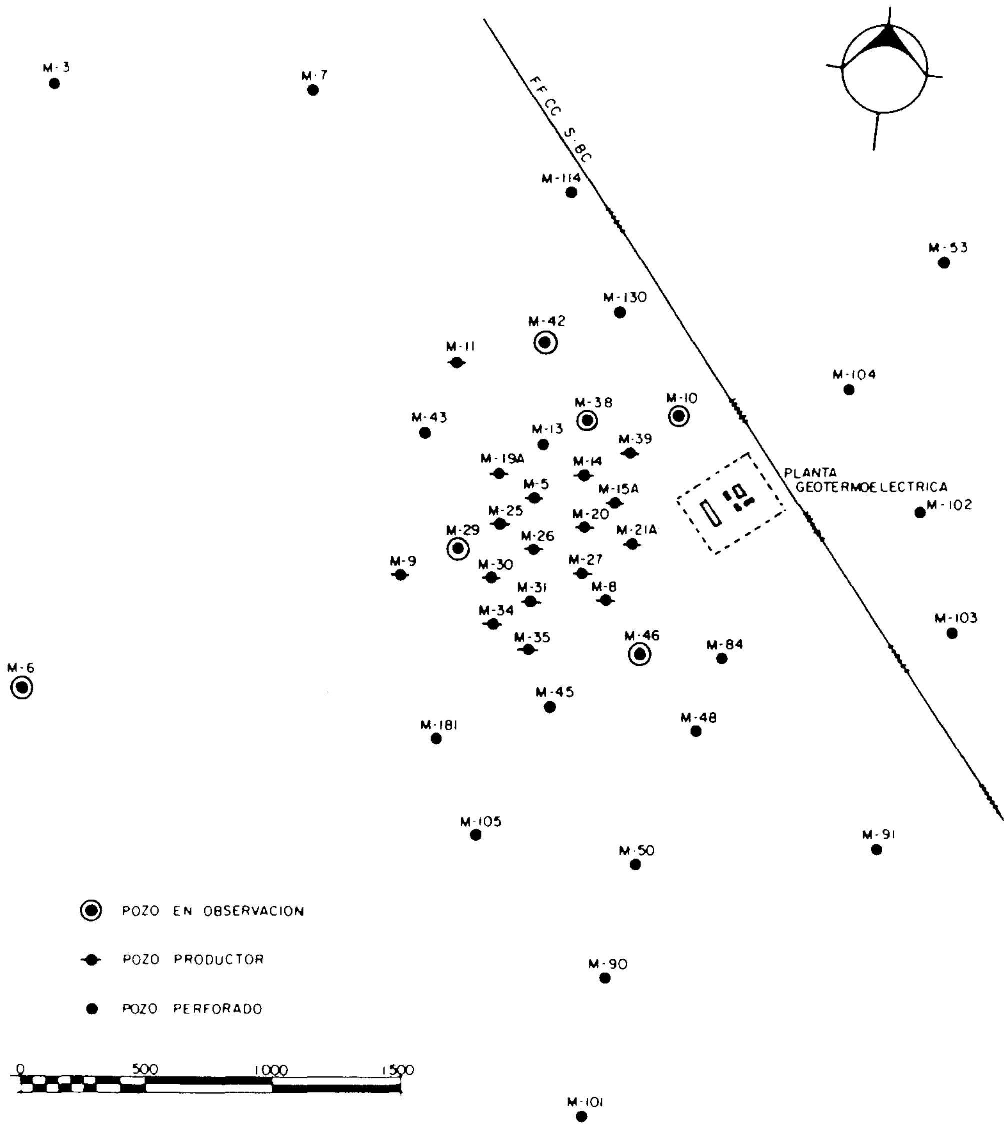


FIG. 13b. Location of wells in the Cerro Prieto Field.

about 1.0. The exponential and hyperbolic curves only start diverging at about $t_{Dd} = 0.2$, so with rough data we would like the last point to have $t_{Dd} = 2.0$, at least. We could not reproduce the fits reported by Rivera-R. (1977, 1978) using Cerro Prieto data. None of the data from liquid-dominated fields fit very well, but this is probably much more a function of data scatter than of the efficacy of the methods. See below for a discussion of data scatter. The only fair fits were for several wells from Larderello. Successful use of type curves with rough data may require a great deal of insight on the part of the analyst.

We tried two other kinds of type curves, Figs. 10a and 10b, with no more success than with Fetkovich's curves. Scatter and small dimensionless time caused problems again.

Gentry and McCray (1978) proposed the use of several different graphs, Figs. 9a, 9b, and 9c, for decline curve analysis. We had difficulty with the plots involving a_i because the data scatter gave a_i a very large uncertainty. We plotted $N_p/q_i\ t$ vs. q_i/q for several wells and got very peculiar results which were of no use. Again, the data are far more problematical than the models.

We tried plotting p/z vs. Q for The Geysers data using pressure from Cobb Mountain #1 well and yearly total production data from Finn (1975) and from the California Dept. of Oil and Gas (see Figs. 14a and b). Brigham (1979) analyzed some Larderello data using p/z vs. Q, but as mentioned above he cautions against expecting linearity after one-third to one-half of the fluid has been produced (see Figs. 15a and 15b for plots of the data).

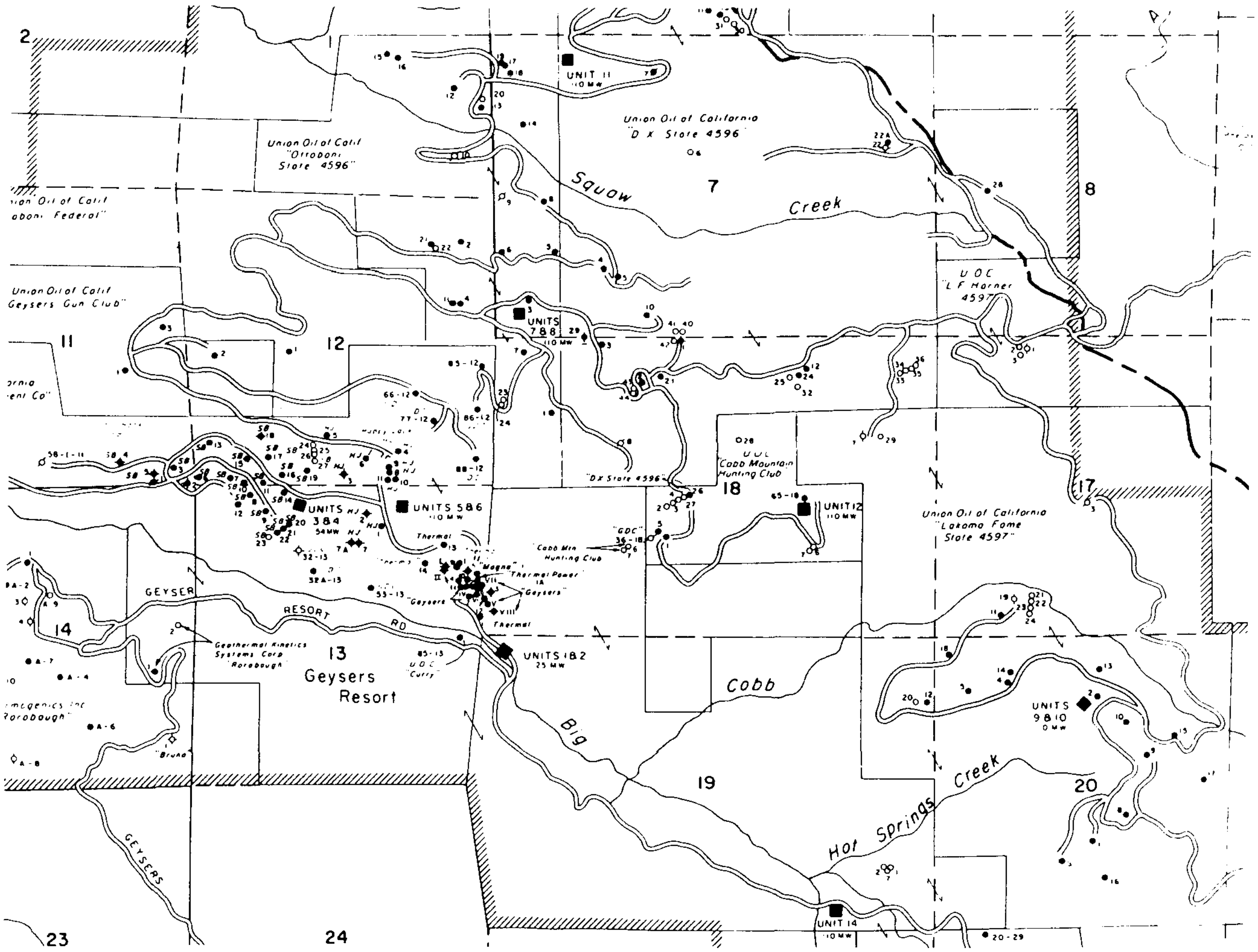


FIG. 13c. The Geysers, California (from California Dept. of Conservation, Division of Oil & Gas, 1978).

Statistical Treatment of Data

Most of the data sets had so much scatter that statistical treatment was the only reasonable approach. We used SPSS (Statistical Package for the Social Sciences) to reduce the data. See Appendix for discussion of SPSS and for the programs we used. SPSS is available at many computer centers and it requires a minimum of data handling.

We used SPSSPLOT to generate cartesian plots of the q vs. t data for all the wells. From the plots we chose wells to analyze further. Some of the wells had drastic rate changes in their histories so only selected parts of their histories were analyzed. We used a non-linear least squares regression subroutine to analyze the exponential equation

$$q(t) = q_i e^{-at}$$

The program requires initial estimates of q_i and a, and it returns $\hat{q}(t)$, the predicted value of q, and best estimates $\hat{a}$ and $\hat{q}_i$ for the fractional decline and the initial flow rate. A fit to the linear equation

$$q(t) = q_i + kt$$

is also generated. The primary statistic generated is R^2 defined as

$$R^2 = \frac{SSR}{SSTO} = \frac{\text{regression sum of squares}}{\text{total sum of squares}} =$$

$$1 - \frac{SSE}{SSTO} = 1 - \frac{\text{residual sum of squares}}{\text{total sum of squares}}$$

$$SSTO = \sum_{i=1}^{n} (q_i - \bar{q})^2 \quad \text{where } \bar{q} = \frac{\Sigma q_i}{n}$$

$$SSR = \sum_{i=1}^{n} (\bar{q} - \hat{q}_i)^2$$

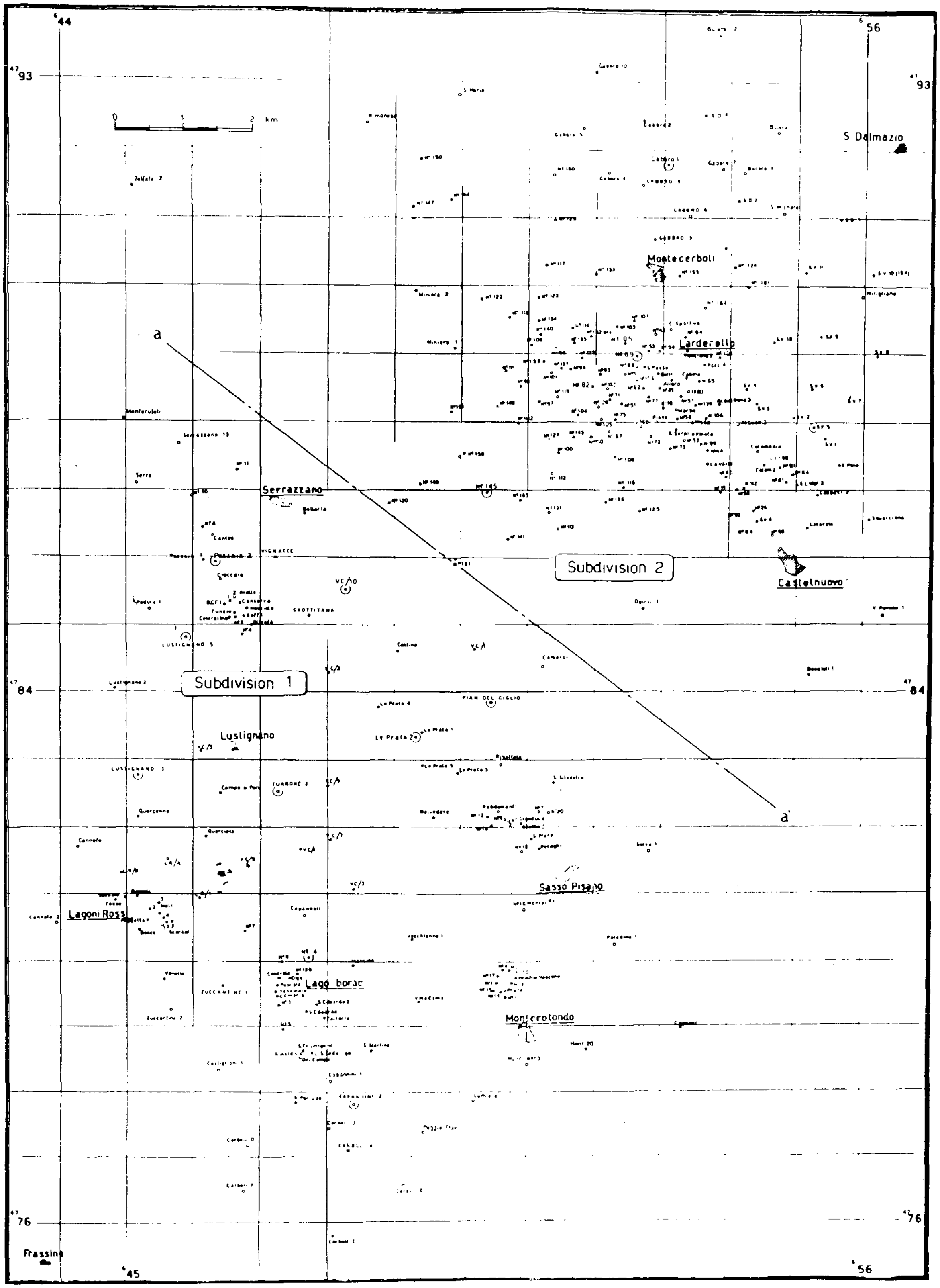


FIG. 13d. Larderello geothermal area (from Sestini, 1970).

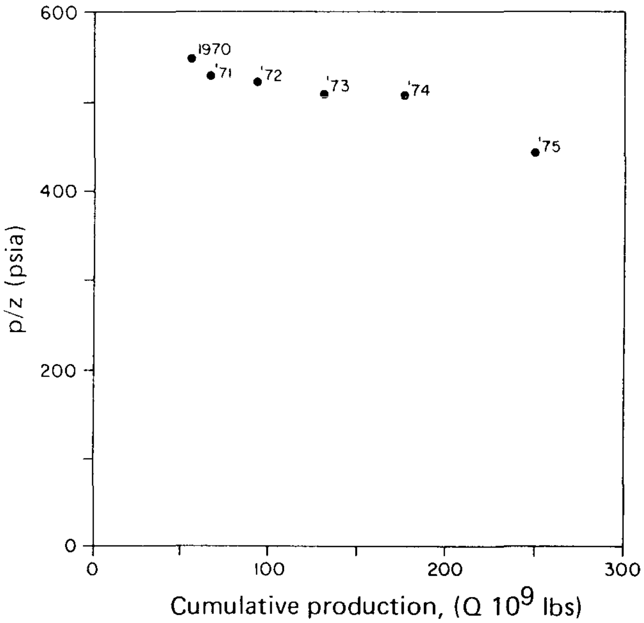


FIG. 14a. The Geysers p/z vs cumulative production, total field (pressure from Cobb Mt. No. 1 and Curry 85, ENEL Proceedings 1977, Strobel et al; production data from Finn, 1975).

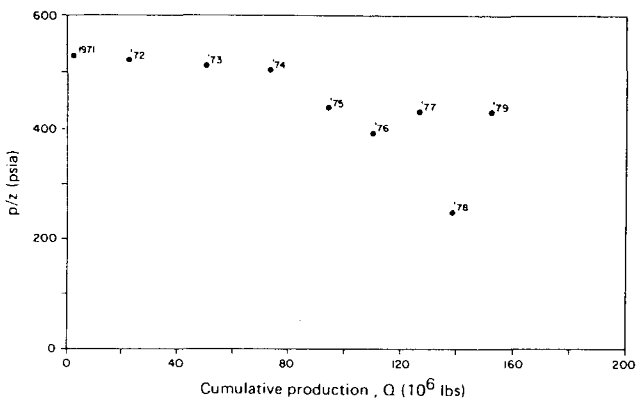


FIG. 14b. The Geysers p/z vs cumulative production, 20–30 wells. (pressure data from ENEL Proceedings 1977; production data from California Division of Oil & Gas, 1971-1979 data on 20-30 selected wells).

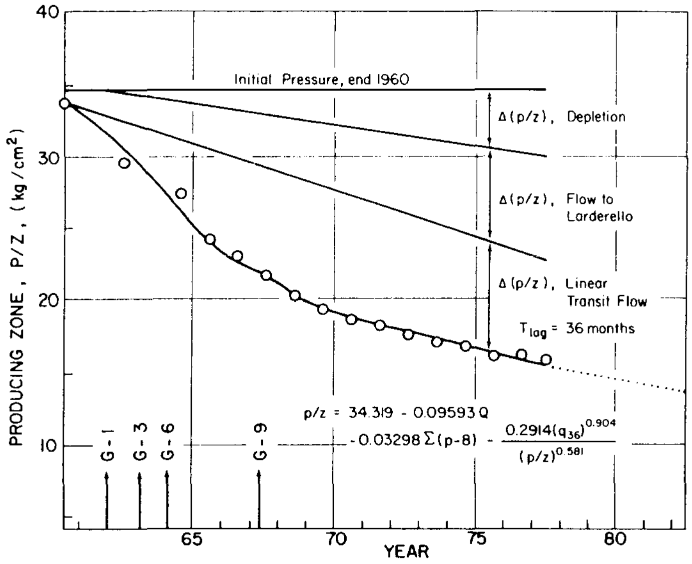


FIG. 15a. Gabbro Zone pressure–production history match, lag time=36 months (from Brigham, 1979).

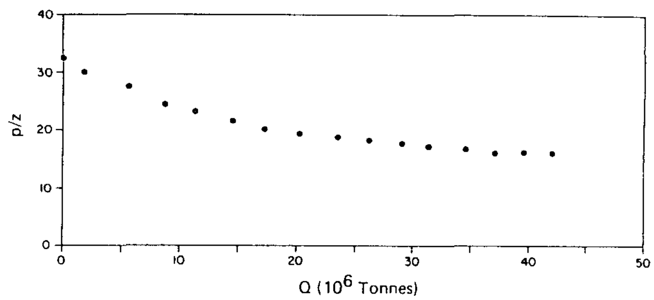


FIG. 15b. p/z vs Q–Larderello data (from Brigham & Neri, 1979).

SSE = SSTO - SSR

Values of R^2 greater than 0.65 indicate a good fit which can be extrapolated with some confidence. The value 0.65 is arbitrary, but is generally considered to be a good fit for raw data.

For the influence function method, we developed a fitness measure, ρ, which is the average fractional deviation of computed pressure differences, $\hat{\Delta p}$, from observed pressure difference, $\Delta p = p_i - p(t)$. For example, if $\rho = 0.1$ and $\hat{\Delta p} = 100$, the true value is between 90.91 and 111.11 because $\Delta p = \hat{\Delta p}/(1 \pm \rho)$.

Discussion of Data Scatter

Field data often have a great deal of scatter in them which can cause difficulties in analyzing them. The scatter can be of two general types, reservoir related and operations related. Reservoir related scatter can be caused by

1) rainfall
2) recharge
3) earthquakes
4) subsidence.

Production related scatter can be caused by

1) changes in production schedules
2) bad well completions
3) workovers
4) poor calibration techniques
5) poor data gathering techniques

Little can be done to prevent reservoir related scatter, but operations related scatter can always be reduced. Methods for reducing the chance of scatter are discussed in the Standard Operating Procedure section. Scattered data can be analyzed with the following techniques:

1) averaging the data
2) least squares fitting
3) subtracting our known effects and trends
4) using insight and experience.

We tried averaging data from several Wairakei and Cerro Prieto wells to see whether we could use Guerrero's method for Arps's equations. We could not get reasonable values for the decline exponent. See Fig. 16 for a graph of six month average production vs. time for Bore 18.

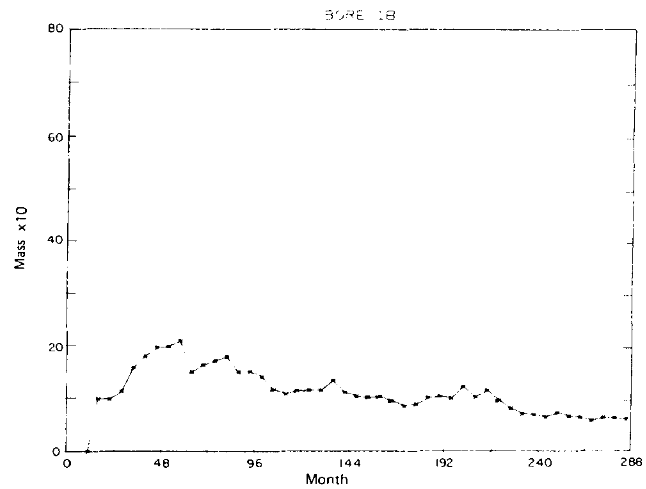


FIG. 16. Six month average production, Bore 18, Wairakei, New Zealand.

The easiest known effects to take into account are periodic shut downs. The annotated production graphs from Wairakei showed that many of the wells were shut in for periods of about 1 month every 1-2 years. The monthly production during these shut-in months is obviously much lower than the preceding and following month's production. If the other data are on a smooth trend, the low values can effectively be ignored in fitting an equation to the trend line. Since these points represent production, however, they should be included in any calculation involving the cumulative production, Q.

V. RESULTS

Arps's Equations

We tested Arps's exponential equation (73) on all individual well data, total field data and on several groups of wells. The results are summarized in Table 3 with complete results in the Appendix. $\hat{D}$ is the average calculated monthly fractional decline. $\hat{D}$ based on total field production from Wairakei, The Geysers, and Cerro Prieto ranges from 0.003 for Wairakei to 0.0115 for The Geysers. This converts to yearly declines of 3.6% and 13.8%, respectively. R^2 for individual wells ranges from 0.0004 for a well at Otake to 0.9712 for a well at Larderello. Eight of the ten wells and groups at Larderello had R^2's greater than 0.87, indicating a very good fit to the equation. Also, all three wells at Matsukawa had R^2's greater than 0.76. The wells from The Geysers did not fit as well as the wells from Larderello and Matsukawa, so we cannot draw definite conclusions about vapor-dominated fields and the exponential equation.

Cerro Prieto and Wairakei are both liquid-dominated fields, and their data did not fit the exponential equation quite as well as the vapor-dominated fields. However, for all the fields the equation fit at least several of the well's data quite well. See Figures 17a-g for a fit of the exponential equation to total Wairakei production and to several individual wells.

We only tested a few wells using the hyperbolic equation and got R^2's greater than 0.989 in all cases. This indicates that the equation is either nearly perfect or that it is a very poor model and will fit virtually any data set. We hold the latter view. Because the equation has no physical basis, we recommend against using it. However, if a particular data set fits a hyperbolic type curve well over a long stretch of dimensionless time, the curve can be used to extrapolate production.

Type-Curve Methods

None of the data sets fit any of the type curves well. The scatter is so high that no value of b can be picked with confidence. Some of the Larderello data fit Fetkovich's exponential curve for up to 80 months but then develop constant production which takes them off the curve. See Figure 18a. Figure 18b shows typical Cerro Prieto data plotted on the same curve. No value of b ca reasonably chosen.

Coats' Influence Function Method

Coats' method can be used with any but the most bizarre data because the derivative constraints imposed on the solution method guarantee that either a meaningful solution or no solution is generated. The fitness measure tells how useful

TABLE 3. SUMMARY RESULTS FOR FITS TO EXPONENTIAL EQUATION

Field	D	R^2	# of Wells	Range on Individual R^2s	# of Wells at R^2>0.65
Wairakei, NZ total prod.	0.0030	0.7847	36	0.0016-0.9049	11
Cerro Prieto B.C., Mexico					
Liquid prod.			17	0.0066-0.8524	6
Total prod.			19	0.0405-0.9409	8
The Geysers, CA., USA	0.01151	0.8103	26	0.0126-0.8127	6
Larderello, Italy			10	0.0416-0.9712	8
Matsukawa, Japan			3	0.7609-0.8633	3
Otake, Japan					
Liquid			4	0.1528-0.7043	1
Vapor			4	0.0004-0.7981	1

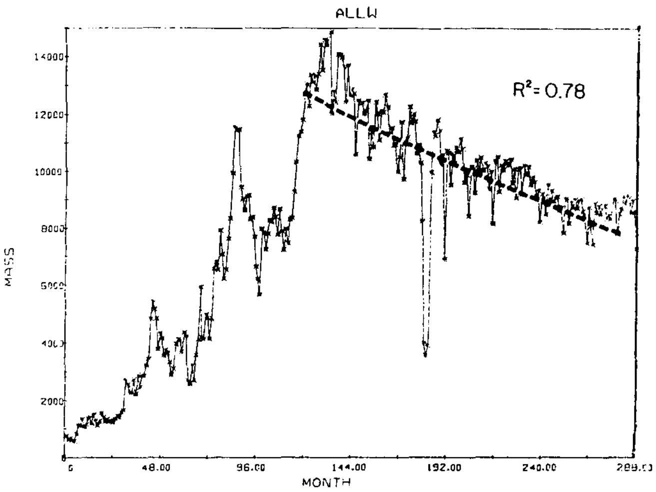


FIG. 17a. Wairakei total production, 1953–1976, with exponential fit 1964–1976.

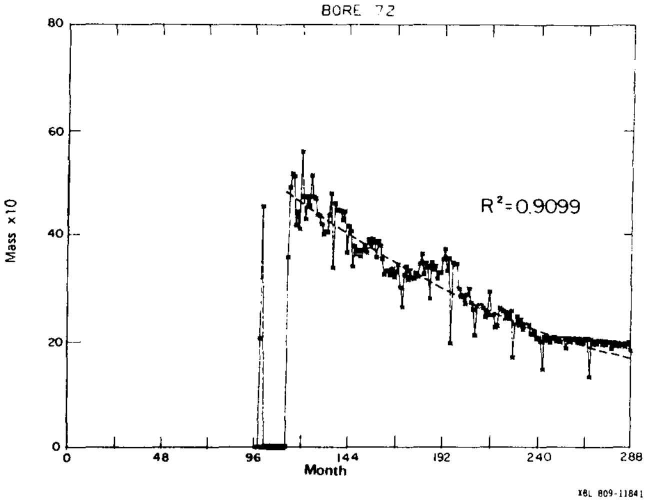


FIG. 17d. Bore 72, Wairakei, New Zealand.

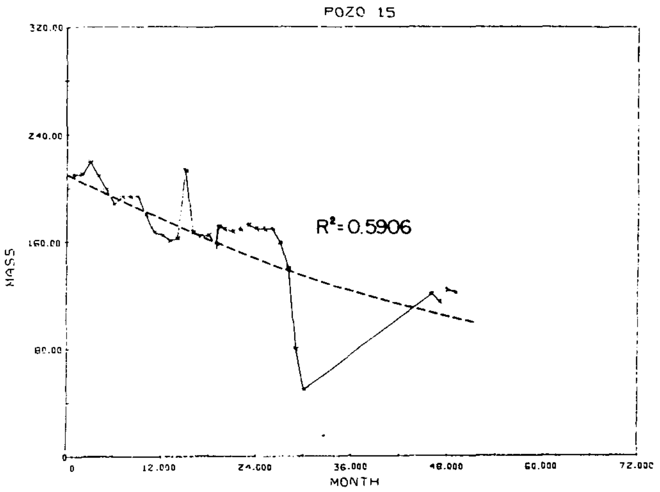


FIG. 17b. Pozo 15, Cerro Prieto, Mexico.

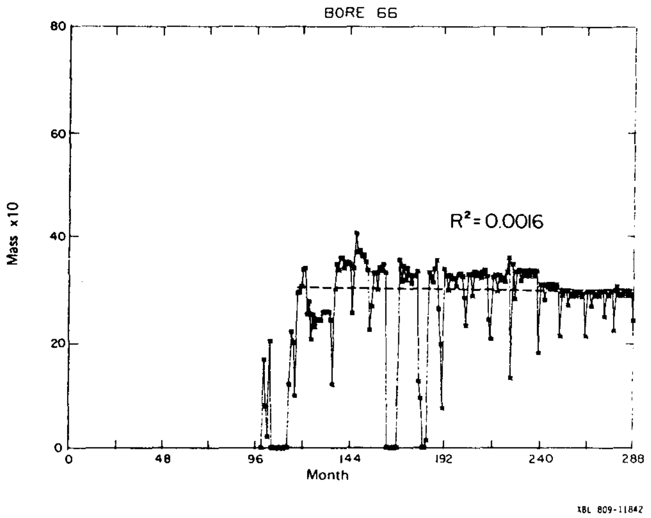


FIG. 17e. Bore 66, Wairakei, New Zealand.

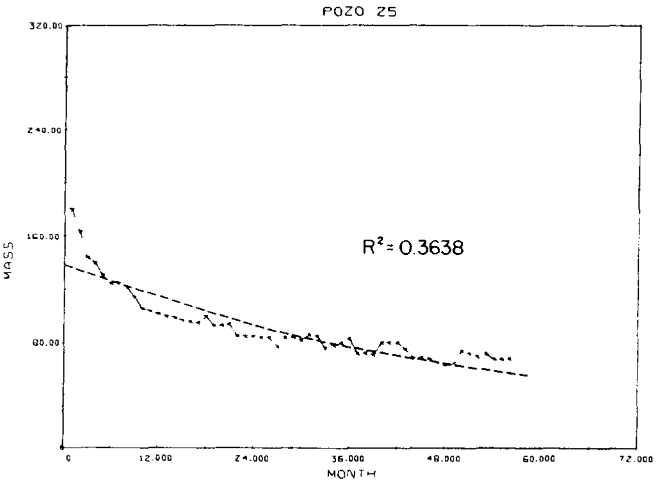


FIG. 17c. Pozo 25, Cerro Prieto, Mexico.

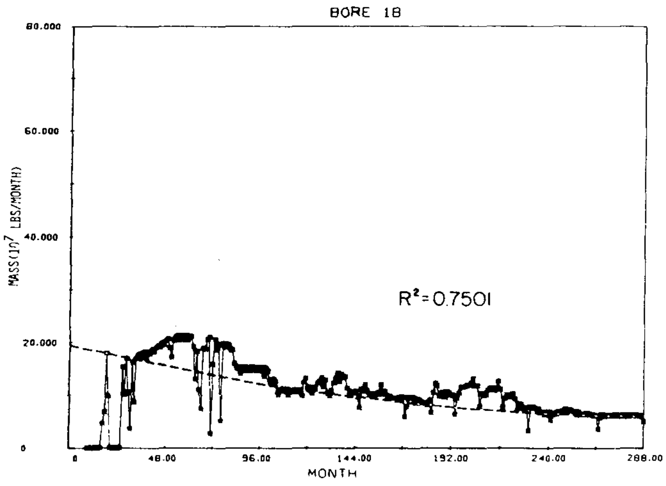


FIG. 17f. Bore 18, Wairakei, New Zealand.

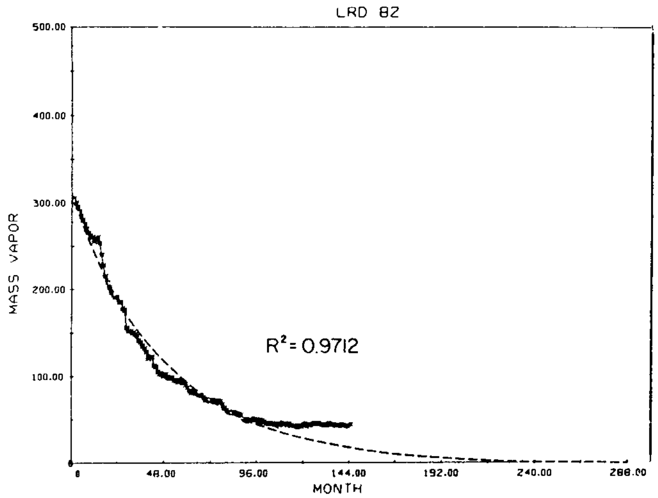


FIG. 17g. Larderello, Italy—Nella Sasso Rosso Nr. 82.

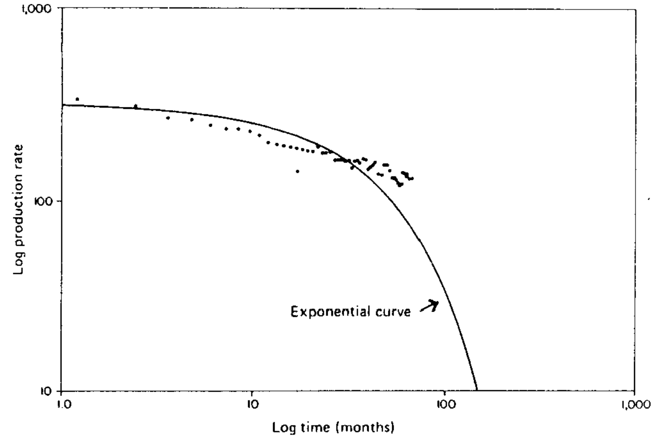


FIG. 18b. Pozo 25, Cerro Prieto, fit to Fetkovich type-curve.

the solution is. We tried the method on Wairakei and Cerro Prieto total production and on Travale 22 from Larderello. The fitness measure, ρ for Travale 22 was 0.038 indicating a very good fit (see Fig. 19). ρ was 0.1001 and 0.3366 for Cerro Prieto liquid production and Wairakei total production, respectively. We tested the predictive value of the method by fitting Wairakei data from 1955-62 and then extrapolating. The pressures obtained using both the infinite and bounded aquifer approximations are shown in Table 4 along with the actual pressures. The pressure drop is calculated as

$$\Delta p = \frac{QF}{t}$$

Where Q is cumulative production, F is the influence function and t is time. Figure 20 shows the calculated influence functions for Wairakei 1955-1962. The high fitness measure indicates very rough data. The observed pressures fall within the fitness measure for the infinite aquifer case. In an infinite aquifer the rate of pressure decline decreases with time as is the case for Wairakei.

Bodvarsson's Linearized Free Surface Method

We had enough data to try the linearized free surface method only with Wairakei. We divided the field into six regions and then assumed that the total production from each region was coming from a virtual well in the "center" of the region. The production depth for each virtual well was the production weighted average center of open zone for the wells in the region. A centroid was chosen for the entire field and then the pressure drawdown at the centroid was calculated for each virtual well and summed to get total drawdown. The drawdown curve obtained is shown in Figure 21 as Curve #2 with the actual drawdown as Curve #1 for comparison. By adding the term for a bottom as described in Chapter III and by adjusting the porosity, ϕ, permeability, k, and depth, H, we obtained Curve #3,

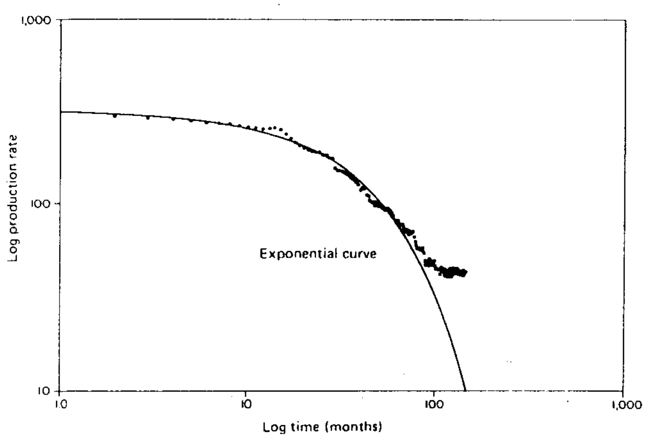


FIG. 18a. Larderello 82 (Nella Sasso Rosso Nr. 82) fit to Fetkovich type-curve.

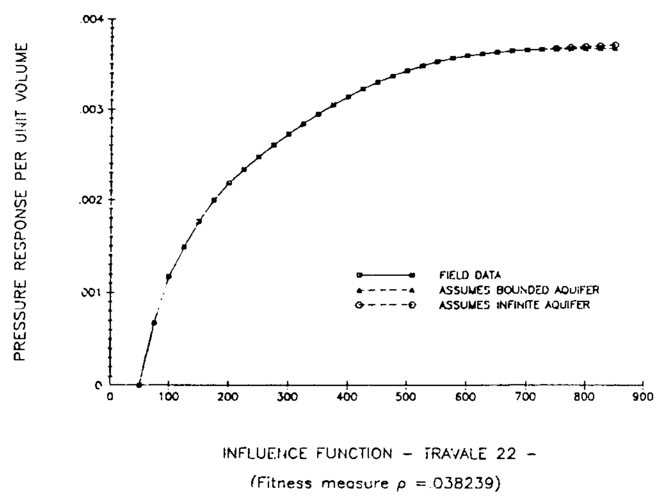


FIG. 19. Influence function—Travale 22.

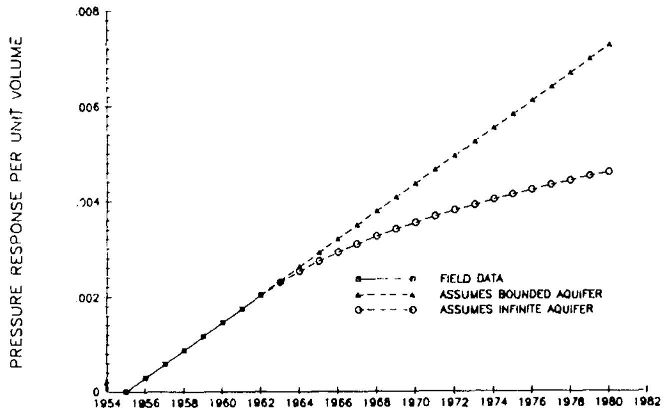


FIG. 20. Influence function—Wairakei total field 1955-1962.

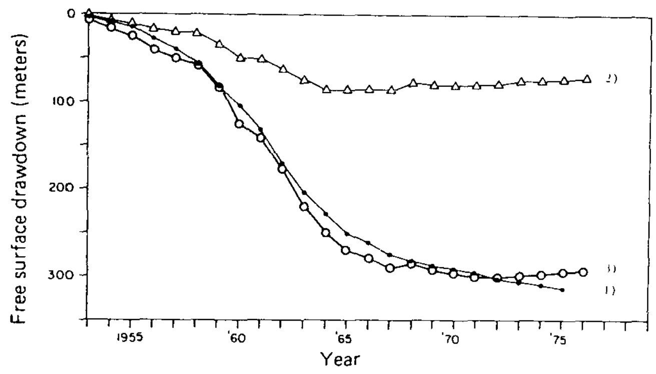


FIG. 21. Linearized free surface fits to Wairakei data.
Curve 1, ●—●: observed pressure drawdown
Curve 2, △—△: LFS fit, 1 term
Curve 3, ○—○: LFS fit, 2 terms

a plausible fit. This method is difficult to use because the necessary geologic and production data are usually lacking or sparse at best.

TABLE 4. CHECK OF EXTRAPOLABILITY OF COATS' METHOD USING WAIRAKEI DATA

(Values of the influence function are from Fig. 20)

Year	Cumulative Prod., Q	Producing Time, t, yrs.	F_∞	Δp	p	F_{bound}	Δp	p	p_{obs}
1963	726276	8	.0023	209	542	.002334	212	539	543
1965	10283401	10	.0027	278	473	.002917	300	451	491
1970	1644585	15	.0035	384	367	.004376	480	271	427
1976	2295755	21	.0042	459	292	.006126	670	81	405

VI. STANDARD OPERATING PROCEDURE FOR DATA GATHERING AND ANALYSIS

Data Gathering

The most important step for the analysis of data is the proper collection of it. The data must be as complete and clear as possible so that "bad data" can be eliminated as a possible cause of unusual results in the analysis. Some steps for ensuring good data gathering are

1) Set up regular testing schedules and stick to them.
2) Set up calibration schedules for all instruments used such as pressure gauges and temperature bombs.
3) Keep an updated calibration log for each instrument.
4) Use clear standard forms for recording data.

A data chart for routine measurements should include at least the following information

1) Well name and location
2) Date and time
3) Pressure-well head, tubing, bottom hole, meter run, etc. gauge or absolute
4) Temperature
5) Flow rate
6) Location of test points
7) Units for all measured quantities
8) Well status
9) Type of test being conducted - buildup, interference, etc.
10) Zone being tested
11) Instrument numbers
12) Name of tester

Data Analysis

The data can be analyzed by wells, by groups of wells and by fields.

Graphs

Graphing the data provides an easy way of examining the data for unusual behavior such as occasional high, low or erratic production. Such data sets can be flagged for special attention. The data should be plotted and analyzed according to Arps (1945, 1956) using cartesian semi-log, and log-log plots of production vs. time. However, this provides only a "quick look" and further analysis should be done. If the data are smooth enough, log-log type curves and Gentry's and McCray's curves can be used to fit current data and to extrapolate to future behavior. If the field is vapor dominated, p/z vs. Q plots can be used but only with great caution.

Least Squares Fits to Arps's Equations

Production data (q vs. t) should be fit to Arps's exponential equation using a non-linear least squares program. The program should calculate R^2 to indicate goodness of fit. A reasonably high value of R^2, e.g., greater than 0.65, allows extrapolation with some degree of confidence.

We recommend against using the computer to fit data to Arps's hyperbolic equation for the reasons described in Chapter V. However, if the data fall very well on a particular type curve then one may reasonably predict future production using that curve.

COMPLETE RESULTS FOR FITS TO EXPONENTIAL EQUATION

Field Well #	Start Date	End Date	Calculated Fractional Decline, $Dx10^3$	R^2
Wairakei - Total Production				
18	1-56	12-76	4.86	0.75
20	4-59	12-76	6.00	0.32
22	12-59	12-76	12.47	0.79
24	1-60	12-66	7.52	0.60
24	5-68	12-76	3.26	0.46
26A	1-63	12-76	5.38	0.52
26B	10-62	12-76	5.29	0.68
27	8-58	12-76	2.05	0.20
28	1-64	12-76	1.74	0.19
30	3-57	12-76	4.90	0.72
39	1-64	12-76	3.69	0.53
41	1-59	12-66	7.02	0.43
42	1-60	12-66	17.48	0.80
43	12-58	12-66	5.04	0.40
44	7-62	12-76	5.10	0.78
46	12-58	12-76	4.86	0.74
47	3-59	12-76	1.34	0.14
47	1-63	11-67	7.87	0.40
47	6-68	12-76	0.47	0.01
48	5-62, excl 5-68-3-69	12-76	11.22	0.80
55	5-62	12-76	3.05	0.52
56	8-62	12-76	9.81	0.86
57	9-62	12-76	6.01	0.69
58	8-61	12-76	2.84	0.45
66	5-64	12-76	1.03	0.06
67	8-60	12-76	4.67	0.61
70	1-65	12-76	1.94	0.38
71	5-63	12-76	1.77	0.20
72	7-62	12-76	5.97	0.90
74	12-63	4-66	2.50	0.27
76	12-62	12-76	5.72	0.59
80	1-63	12-76	6.81	0.72
81	12-60	5-62	3.99	0.59
82	1-66	12-76	3.40	0.33
83	9-63	12-76	2.54	0.38
88	1-64	12-76	0.45	0.01
108	8-64	12-76	2.87	0.51
ALL WELLS	1-64	12-76	3.00	0.78

COMPLETE RESULTS FOR FITS TO EXPONENTIAL EQUATION (Continued)

Field Well #	Start Date	End Date	Calculated Fractional Decline, $Dx10^3$	R^2
Cerro Prieto - Liquid Production				
5	3-73	7-78	4.72	0.55
8	6-73	7-78	14.36	0.64
9	3-73	12-77	10.26	0.13
11	3-73	7-78	24.22	0.73
14	8-76	8-78	4.65	0.09
15	8-74	8-78	14.55	0.59
19	2-75	7-78	-13.79	0.54
20	8-73	7-78	24.71	0.79
21	9-74	7-78	-15.02	0.62
25	12-73	7-78	16.08	0.36
26	8-73	7-78	-20.87	0.76
27	8-76	7-78	- 2.06	0.01
30	12-73	7-78	7.80	0.47
31	8-73	7-78	4.08	0.58
34	7-73	9-75	22.67	0.84
35	3-74	7-78	6.82	0.84
Cerro Prieto - Total Production				
5	1-73	12-79	8.01	0.81
8	1-73	12-79	18.12	0.81
9	1-73	12-79	99.09	0.15
11	1-73	12-79	19.87	0.72
14	1-73	12-79	9.74	0.52
15	1-73	12-79	17.31	0.73
19	1-73	12-79	- 1.40	0.06
20	1-73	12-79	25.31	0.83
21	1-73	12-79	4.60	0.19
25	1-73	12-79	9.52	0.58
26	1-73	12-79	- 8.36	0.31
27	1-73	12-79	2.76	0.04
29	1-73	12-79	7.55	0.22
30	1-73	12-79	6.93	0.68
31	1-73	12-79	5.84	0.67
34	1-73	12-79	27.20	0.58
35	1-73	12-79	8.07	0.94
39	1-73	12-79	33.26	0.63
42	1-73	12-79	7.18	0.32

Field Well #	Start Date	End Date	Calculated Fractional Decline, $Dx10^3$	R^2
Geysers - Total Production				
DX-2	3-74	11-79	7.98	0.44
DX-3	11-72	11-79	8.10	0.54
DX-4	8-72	11-79	10.86	0.69
DX-5	8-71	11-79	1.16	0.01
DX-10	8-71	11-79	5.00	0.16
DX-10	3-74	2-78	9.80	0.35
SUFBK3	8-71	11-79	11.59	0.49
GDC-32A	12-72	11-79	9.94	0.54
GDC-53	3-72	11-79	19.01	0.32
GDC-66	4-73	11-79	9.56	0.54
GDC-77	5-72	11-79	9.85	0.31
GDC-85	5-72	11-79	7.17	0.38
GDC-85	5-73	12-74	37.98	0.82
GDC-85	2-75	11-76	24.34	0.89
GDC-86	5-73	11-79	7.69	0.42
GDC-88	3-72	11-79	11.76	0.61
HAPYJK1	11-71	11-79	16.10	0.61
HAPYJK2	11-71	11-79	13.17	0.60
HAPYJK7	11-71	11-79	2.45	0.15
HAPYJK8	11-71	11-79	10.13	0.48
HAPYJK9	11-72	11-79	6.18	0.22
OS-1	6-73	11-79	9.66	0.59
OS-2	9-72	11-79	6.03	0.23
OS-3	9-72	11-79	13.70	0.46
OS-3	6-75	9-77	25.36	0.89
OS-4	11-72	11-79	11.64	0.54
OS-5	7-74	11-79	23.99	0.63
OS-6	8-72	11-79	9.58	0.59
OS-7	11-72	11-79	11.58	0.69
OS-8	8-72	11-79	10.80	0.81
Geysers Total Field				
	11-72	11-79	11.51	0.81
Olkaria	--	--	10.84	0.85

Influence Functions

If adequate production and pressure data are available, they should be analyzed using Coats's influence function method and a computer program with the constraints described in Chapter III. Data preparation is straightforeward and data handling is minimal. The first half of a data set can be modeled and extrapolated in several different ways. Comparing the extrapolation with the second half of the data can give insight to the placement of reservoir boundaries such as faults or outcrops.

Bodvarsson's linearized free surface method should be tried if the reservoir has a free liquid surface, and if enough data are available to estimate a sinking velocity.

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APPENDIX

Statistical Package for the Social Sciences SPSS

SPSS is a set of programs developed for general statistical analysis. We have used the two subprograms PLOT (Tuccy, 1977) and NON-LINEAR (Robinson, 1977) quite extensively. The listings for our SPSS main programs which used these subprograms are given below. SPSS2 will plot a set of data. SPSS4 will do a nonlinear least squares fit using the exponential equation. B(1) and B(]) are initial guesses for initial production, q_o, and monthly fractional decline, D. The other program names are self-explanatory. A complete description of SPSS is given in Nie, 1975.

Multiple Purpose Optimization System MPOS

MPOS is a linear programming package designed to solve a wide variety of linear programming problems. Coats' influence method can be formulated as a linear programming problem as follows.

$$\sum_{j=i}^{i} q_{i-j}X_j + u_i - v_i = b \qquad (1) \quad i = 1,2, \ldots n$$

$$X_j = F_j - F_{j-1}$$

$$X_i \geq 0 \qquad (2a) \quad i = 1,2, \ldots n$$

$$X_{n-1} - X_n \geq 0 \qquad (2b) \quad i = 1,2, \ldots n$$

$$X_{i+1} - 2X_i + X_{i-1} \geq 0 \qquad (2c) \quad i = 1,2, \ldots n$$

Objective function $\sum_{i}^{n}(u_i + v_i)$ = minimum where u_i and v_i are slack variables, b_i is the observed pressure change Δp, and $q_{i-j}X_j$ is the calculated pressure change. MPOS generates a tableau for the calculation which looks like the following for n = 4.

	X1	X2	X3	X4	u1	u2	u3	u4	v1	v2	v3	v4	b
Objective function					1	1	1	1	1	1	1	1	
2 a)	q_1				1				-1				$= b_1$
	q_2	q_1				1				-1			$= b_2$
	q_3	q_2	q_1				1				-1		$= b_3$
	q_4	q_3	q_2	q_1				1				-1	$= b_4$
2 c)			1	-1									≥ 0
2 d)	1	-2	1										≥ 0
		1	-2	1									≥ 0

Values for u_i and v_i are given in the output so that the fitness measures, ρ, can be calculated directly as

$$\rho = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{b_i} (u_i + v_i).$$

The listing for program INFUNC is given below.

COMPUTER LISTINGS AND EXAMPLES

SPSS PROGRAMS

```
     ===== SPSS2 =====
:::
PAGESIZE          NOEJECT
RUN NAME          PLOT ONLY -- POZO 5
VARIABLE LIST     MONTH53,MASS
INPUT FORMAT      FIXED (24X,F3.0,27X,F7.0)
N OF CASES        UNKNOWN
READ INPUT DATA
PLOT              PLOTS=MASS(0,320) WITH MONTH53(0,72)/
                  TITLE=POZO 5/
                  TITLEX=MONTH/
                  TITLEY=MASS/
                  SIZE=10.5,8.0/
                  XDIV=12/
                  YDIV=8/
                  SYMBOLS=-7/
OPTIONS           1,10
FINISH
```

```
     ===== SPSS4 =====
:::
PAGESIZE          NOEJECT
RUN NAME          NONLINEAR REGRESSION -- BORE 108
VARIABLE LIST     MONTH53,AMONTH,YEAR,MASS
INPUT FORMAT      FIXED (9X,F5.0,5X,2F5.0,F20.2)
N OF CASES        159
COMPUTE           PMASS=MASS
MISSING VALUES    MASS(0)
IF                (YEAR LT 1964)MASS=0
IF                (YEAR EQ 1964 AND AMONTH LT 8)MASS=0
COMPUTE           MONTHR=MONTH53-140
READ INPUT DATA
*REJECT IF        (MASS EQ 0)
NONLINEAR         VARIABLES=MASS WITH MONTHR,NB=2
MODEL             YHAT=1000000*B(1)*EXP(B(2)*.00001*MONTHR)
DERIVATIVES       G(1)=EXP(B(2)*.00001*MONTHR)*1000000
                  G(2)=10*MONTHR*B(1)*EXP(B(2)*.00001*MONTHR)
PARAMETERS        B(1)=435.299562
                  B(2)=-392.55
REGRESSION        VARIABLES=MASS,MONTHR/
                  REGRESSION=MASS WITH MONTHR/
PLOT              PLOTS=PMASS(0,800000000) WITH MONTH53(0,288)/
                  TITLE=BORE 108/
                  TITLEX=MONTH/
                  TITLEY=MASS/
                  SIZE=10.5,8.0/
                  XDIV=12/
                  YDIV=8/
                  SYMBOLS=-7/
OPTIONS           1,10
FINISH
```

```
...
     ===== SPSS5 =====
:::
PAGESIZE           NOEJECT
RUN NAME           SEMILOG PLOT ONLY -- BORE 22
VARIABLE LIST      MONTH53,AMONTH,YEAR,MASS
INPUT FORMAT       FIXED (9X,F5.0,5X,2F5.0,F20.2)
N OF CASES         UNKNOWN
COMPUTE            LOGMASS=LG10(MASS+1)
READ INPUT DATA
PLOT               PLOTS=LOGMASS(0,9) WITH MONTH53(0,288)/
                   TITLE=BORE 22/
                   TITLEX=MONTH/
                   TITLEY=LOG MASS/
                   SIZE=10.5,8.0/
                   XDIV=12/
                   YDIV=9/
                   SYMBOLS=-7/
OPTIONS            1,10
FINISH

     ===== SPSS6 =====
:::
PAGESIZE           NOEJECT
RUN NAME           LOG-LOG PLOT ONLY -- BORE 72
VARIABLE LIST      MONTH53,AMONTH,YEAR,MASS
INPUT FORMAT       FIXED (9X,F5.0,5X,2F5.0,F20.2)
N OF CASES         UNKNOWN
COMPUTE            LOGMASS=LG10(MASS+1)
COMPUTE            LOGT=LG10(MONTH53)
READ INPUT DATA
PLOT               PLOTS=LOGMASS(4,9) WITH LOGT(0,3)/
                   TITLE=BORE 72/
                   TITLEX=LOG TIME/
                   TITLEY=LOG MASS/
                   SIZE=-9.0,-15.0/
                   XDIV=12/
                   YDIV=20/
                   SYMBOLS=-7/
OPTIONS            1,10
FINISH

     ===== SPSSGEN =====
:::
PAGESIZE           NOEJECT
RUN NAME           GENTRY SEMILOG PLOT - - BORE 22
VARIABLE LIST      XTM,YPRD,XCM
INPUT FORMAT       FIXED(10X,F3.0,5X,F20.0,5X,F20.0)
N OF CASES         UNKNOWN
COMPUTE            QINIT=354000000
MISSING VALUES     YPRD(0)
COMPUTE            QIDQ=LG10(QINIT/YPRD)
ASSIGN MISSING     QIDQ(0.1)
COMPUTE            RQCDQIT=1-XCM/(QINIT*XTM)
READ INPUT DATA
CONDESCRIPTIVE     QIDQ,RQCDQIT
STATISTICS         ALL
PLOT               PLOTS=QIDQ(0,2) WITH RQCDQIT(0,1)/
                   TITLE=BORE 22/
                   TITLEX=BORE22 CUM. PROD. OVER INITIAL PROD.*TIME/
                   TITLEY=LOG(INITIAL PROD. OVER CURRENT PROD.)/
                   SIZE=-15,-10/
                   XDIV=20/
                   YDIV=16/
                   SYMBOLS=-7/
OPTIONS            10
FINISH
...
```

===== SPGEN2 =====

```
:::
PAGESIZE           NOEJECT
RUN NAME           GENTRY CARTESIAN FOR A-INITIAL -BORE 20
VARIABLE LIST      XTM,YPRD,XCM
INPUT FORMAT       FIXED(10X,F3.0,5X,F20.0,5X,F20.0)
N OF CASES         UNKNOWN
COMPUTE            QINIT=489000000
MISSING VALUES     YPRD(0)
COMPUTE            QDQI=YPRD/QINIT
ASSIGN MISSING     QDQI(0.01)
READ INPUT DATA
CONDESCRIPTIVE     QDQI
STATISTICS         ALL
PLOT               PLOTS=QDQI(0,1.5) WITH XCM(0,100000000000)/
                   TITLE=BORE 20/
                   TITLEX=BORE 20 PROD RATIO VS CUM PROD/
                   SIZE=15,10/
                   XDIV=20/
                   YDIV=12/
                   SYMBOLS=-9/
OPTIONS            10
FINISH
```

===== SPSSCUM =====

```
:::
PAGESIZE           NOEJECT
RUN NAME           CUMULATIVE VS CURRENT PRODUCTION, BORE 24
VARIABLE LIST      XTM,YPRD
INPUT FORMAT       FIXED(14X,F5.0,10X,F20.2)
N OF CASES         UNKNOWN
COMPUTE            XCM=ACCUM(YPRD)
WRITE CASES        (10X,F3.0,5X,F20.0,5X,F20.0)
                   XTM,YPRD,XCM
READ INPUT DATA
PLOT               PLOTS=YPRD(0,600000000) WITH XCM(0,100000000000)/
                   TITLE=BORE 24/
                   TITLEX=CUMULATIVE PRODUCTION/
                   TITLEY=CURRENT PRODUCTION/
                   SIZE=10.5,8.0/
                   XDIV=10/
                   YDIV=8/
                   SYMBOLS=-7/
PLOT               PLOTS=XCM(0,100000000000) WITH XTM(0,288)/
                   TITLE=BORE 24/
                   TITLEX=TIME IN MONTHS/
                   TITLEY=CUMULATIVE PRODUCTION/
                   XDIV=12/
                   YDIV=10/
                   SYMBOLS=-7/
FINISH
```

===== SPSSAVG =====

```
:::
PAGESIZE           NOEJECT
RUN NAME           OUTPUT SEMI-ANNUAL AVERAGES -- BORE 72
VARIABLE LIST      MONTH53,AMONTH,YEAR,MASS
INPUT FORMAT       FIXED(9X,F5.0,5X,2F5.0,F20.2)
N OF CASES         UNKNOWN
MISSING VALUES     MASS(0)
IF                 (AMONTH LE 6)HAFYEAR=1
IF                 (AMONTH GE 7)HAFYEAR=2
READ INPUT DATA
AGGREGATE          GROUPVARS=YEAR,HAFYEAR/
                   VARIABLES=MONTH53,MASS/
                   AGGSTATS=MEAN/
OPTIONS            3
STATISTICS         3
FINISH
```

```
                                                       79/07/25.    08.28.54.     PAGE 1
VOGELBACK COMPUTING CENTER
NORTHWESTERN UNIVERSITY

S P S S - - STATISTICAL PACKAGE FOR THE SOCIAL SCIENCES

VERSION 7.0 -- JUNE 27 1977


        PAGESIZE          NOEJECT
        RUN NAME          NONLINEAR REGRESSION -- BORE 18
        VARIABLE LIST     MONTH53,AMONTH,YEAR,MASS
        INPUT FORMAT      FIXED (9X,F5.0,5X,2F5.0,F20.2)

       ACCORDING TO YOUR INPUT FORMAT, VARIABLES ARE TO BE READ AS FOLLOWS

       VARIABLE  FORMAT  RECORD    COLUMNS

       MONTH53   F 5. 0       1    10-  14
       AMONTH    F 5. 0       1    20-  24
       YEAR      F 5. 0       1    25-  29
       MASS      F20. 2       1    30-  49

THE INPUT FORMAT PROVIDES FOR    4 VARIABLES.    4 WILL BE READ
IT PROVIDES FOR  1 RECORDS (*CARDS*) PER CASE.  A MAXIMUM OF    49 *COLUMNS* ARE USED ON A RECORD.


WARNING - A NUMERIC VARIABLE HAS A WIDTH GREATER THAN 14.  SMALL ROUNDING/TRUNCATION ERRORS MAY OCCUR.
        N OF CASES        280
        IF                (YEAR LT 1956)MASS=0
        COMPUTE           MONTHR=MONTH53-37
        ASSIGN MISSING    MONTHR(-1)
        REJECT IF         (MASS EQ 0)
        READ INPUT DATA

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

NONLINEAR REGRESSION -- BORE 18                        79/07/25.    08.28.54.     PAGE    2

        NONLINEAR         VARIABLES=MASS WITH MONTHR,NB=2
        MODEL             YHAT=1000000*B(1)*EXP(B(2)*.00001*MONTHR)
        DERIVATI          G(1)=EXP(B(2)*.00001*MONTHR)*1000000
                          G(2)=10*MONTHR*B(1)*EXP(B(2)*.00001*MONTHR)
        PARAMETERS        B(1)=185.615892
                          B(2)=-407.18
        STATISTICS        9
        FINISH

SPSS RELOADED

     00001400 CM NEEDED FOR NONLINEAR


     OPTION - 1
     IGNORE MISSING VALUE INDICATORS
```

```
NONLINEAR REGRESSION -- BORE 18                                79/07/25.     08.28.54.      PAGE    3

FILE    NONAME    (CREATION DATE =  79/07/25.)

N O N L I N E A R  -  P R O B L E M   S U M M A R Y

   252   CASES
     1   DEPENDENT VARIABLE(S)
     2   PARAMETERS
    50   ITERATION LIMIT
METHOD = MARQUARDT
TOL1 =   1.5000000E-08   REL. CHANGE IN A PARAMETER
TOL2 =           0       REL. CHANGE IN SUM OF SQUARES
TOL3 =           0       RATIO TO INITIAL SUM OF SQUARES
TOL4 =   1.0000000E-06   PIVOT TOLERANCE

PARAMETERS

 NO.        NAME           INITIAL VALUE

   1          B1           1.8561589E+02
   2          B2          -4.6718000E+02

              0 DERIVATIVE ERRORS WERE DETECTED

SETUP TIME =        .476   SECONDS


                                    T H E   L A S T   I T E R A T I O N

ITERATION NO.    5           BASE POINT         TEST POINT
SUM OF SQUARES              1.2714952E+17      1.2714952E+17
L                             0                  0
LAMBDA                              0                  0
GAMMA                       1.0000000E+00      1.0000000E+00
ANGLE IN DEGREES             8.7176             8.7257
MAX. PIVOT REDUCTION        3.9411999E-01      3.9411999E-01

PAR.     1      B1        S 1.9510662E+02    S 1.9510662E+02
         2      B2         -4.6583609E+02     -4.6583609E+02

CUMULATIVE NO. OF FUNCTION CALLS =      6
 ITERATION TIME =        .262  SECONDS
CUMULATIVE TIME =      1.[illegible]  SECONDS
```

ITERATION TERMINATES
MAX. RELATIVE CHANGE IN A PARAMETER .LT. TOL(1) = 1.5000000E-08

- -

NONLINEAR REGRESSION -- BORE 18 79/07/25. 08.28.54. PAGE 4

FILE NONAME (CREATION DATE = 79/07/25.)

F I N A L   P A R A M E T E R   V A L U E S

FINAL VALUE SUM OF SQUARES = 1.2714952E+17

$R^2 = .7501$

PAR.	1	B1	1.9510662E+02
	2	B2	-4.8583609E+02

- -

NONLINEAR REGRESSION -- BORE 18 79/07/25. 08.28.54. PAGE 5

FILE NONAME (CREATION DATE = 79/07/25.)

F I N A L   F U N C T I O N   V A L U E S   A N D   R E S I D U A L S

ROOT MEAN SQUARE RESIDUAL = 2.2552119E+07 D.F. = 250
THIS IS THE SCALE UNIT IN THE GRAPH OF THE RESIDUALS.

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL (-2 -1 0 +1 +2)
1	1	1.9510662E+08	1.8200000E+08	-1.3106622E+07	*******
2	1	1.9416102E+08	1.7100000E+08	-2.3161023E+07	***********
3	1	1.9322001E+08	1.8200000E+08	-1.1220006E+07	******
4	1	1.9228355E+08	1.7900000E+08	-1.3283550E+07	*******
5	1	1.9135163E+08	1.8700000E+08	-4.3516331E+06	***
6	1	1.9042423E+08	1.8300000E+08	-7.4242324E+06	****
7	1	1.8950133E+08	1.9200000E+08	2.4986735E+06	**
8	1	1.8858289E+08	1.9500000E+08	6.4171065E+06	****
9	1	1.8766891E+08	1.9100000E+08	3.3310882E+06	**
10	1	1.8675936E+08	2.0000000E+08	1.3240640E+07	*******
11	1	1.8585422E+08	1.9600000E+08	1.0145784E+07	*****
12	1	1.8495346E+08	2.0500000E+08	2.0046541E+07	**********
13	1	1.8405707E+08	2.0800000E+08	2.3942933E+07	************
14	1	1.8316502E+08	1.9000000E+08	6.8349795E+06	****
15	1	1.8227730E+08	1.7200000E+08	-1.0277297E+07	******
16	1	1.8139388E+08	2.0500000E+08	2.3606124E+07	***********
17	1	1.8051474E+08	2.1200000E+08	3.1485264E+07	***************
18	1	1.7963986E+08	2.0500000E+08	2.5360143E+07	************
19	1	1.7876922E+08	2.1200000E+08	3.3230781E+07	****************
20	1	1.7790280E+08	2.1200000E+08	3.4097200E+07	****************
21	1	1.7704058E+08	2.0500000E+08	2.7959420E+07	*************
22	1	1.7618254E+08	2.1200000E+08	3.5817461E+07	*****************
23	1	1.7532866E+08	2.0500000E+08	2.9671344E+07	**************

24	1	1.7447891E+08	2.1200000E+08	3.7521088E+07	*****************
25	1	1.7363329E+08	2.1200000E+08	3.8366713E+07	*****************
26	1	1.7279176E+08	1.9200000E+08	1.9208241E+07	**********
27	1	1.7195431E+08	1.3000000E+08	-4.1954311E+07	*******************
28	1	1.7112092E+08	1.8400000E+08	1.2879079E+07	*******
29	1	1.7029157E+08	1.1000000E+08	-6.0291570E+07	**************************
30	1	1.6946624E+08	7.3000000E+07	-9.6466238E+07	**************************
31	1	1.6864491E+08	1.9000000E+08	2.1355093E+07	**********
32	1	1.6782756E+08	1.8800000E+08	2.0172444E+07	**********
33	1	1.6701417E+08	2.0700000E+08	3.9985833E+07	******************
34	1	1.6620472E+08	2.1100000E+08	4.4795280E+07	********************
35	1	1.6539920E+08	2.7000000E+07	-1.3839920E+08	**************************
36	1	1.6459758E+08	1.6000000E+08	-4.5975754E+06	***
37	1	1.6379984E+08	2.0700000E+08	4.3200160E+07	*******************
38	1	1.6300597E+08	1.8600000E+06	2.2994028E+07	***********
39	1	1.6221595E+08	1.9700000E+08	3.4784050E+07	***************
40	1	1.6142976E+08	5.1000000E+07	-1.1042976E+08	**************************
41	1	1.6064738E+08	1.9900000E+08	3.8352624E+07	*****************
42	1	1.5986879E+08	1.9100000E+08	3.1131214E+07	**************
43	1	1.5909397E+08	1.9600000E+08	3.6906030E+07	****************
44	1	1.5832291E+08	1.9500000E+08	3.6677092E+07	****************
45	1	1.5755558E+08	1.8500000E+08	2.7444416E+07	*************
46	1	1.5679198E+08	1.8700000E+08	3.0208022E+07	**************

NONLINEAR REGRESSION -- BORE 13 79/07/25. 08.23.54. PAGE 6

FILE NONAME (CREATION DATE = 79/07/25.)

F I N A L F U N C T I O N V A L U E S A N D R E S I D U A L S

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL -2 -1 0 +1 +2
47	1	1.5663207E+08	1.6100000E+08	4.3679262E+06	***
48	1	1.5527585E+08	1.5300000E+08	-2.2758522E+06	**
49	1	1.5452330E+08	1.5300000E+08	-1.5232956E+06	**
50	1	1.5377439E+08	1.4300000E+08	-1.0774386E+07	******
51	1	1.5302911E+08	1.5300000E+08	-2.9106781E+04	*
52	1	1.5228744E+08	1.4800000E+08	-4.2874393E+06	***
53	1	1.5154937E+08	1.5300000E+08	1.4506337E+06	**
54	1	1.5081487E+08	1.4800000E+08	-2.8148704E+06	**
55	1	1.5008393E+08	1.5300000E+08	2.9160656E+06	**
56	1	1.4935654E+08	1.5300000E+08	3.6434592E+06	***
57	1	1.4863267E+08	1.4800000E+08	-6.3267269E+05	*
58	1	1.4791231E+08	1.5300000E+08	5.0876872E+06	***
59	1	1.4719544E+08	1.4800000E+08	8.0455577E+05	*
60	1	1.4648205E+08	1.5300000E+08	6.5179500E+06	****
61	1	1.4577211E+08	1.5200000E+08	6.2278867E+06	****
62	1	1.4506562E+08	1.3700000E+08	-8.0656174E+06	*****
63	1	1.4436255E+08	1.5200000E+08	7.6374545E+06	****
64	1	1.4366288E+08	1.4700000E+08	3.3371189E+06	**
65	1	1.4296661E+08	1.2800000E+08	-1.4966608E+07	********
66	1	1.4227371E+08	1.2700000E+08	-1.5273709E+07	********
67	1	1.4158417E+08	1.3200000E+08	-9.5841683E+06	*****
68	1	1.4089797E+08	1.2700000E+08	-1.3897970E+07	*******
69	1	1.4021510E+08	1.0100000E+08	-3.9215096E+07	******************
70	1	1.39[illegible]53E+08	1.1300000E+08	-2.6535[illegible]3E+07	*************
71	1	1.3885926E+08	1.0900000E+08	-2.9859263E+07	**************
72	1	1.3818627E+08	1.1200000E+08	-2.6186271E+07	*************

73	1	1.37. ,54E+08	1.1200000E+08	-2.5516 JE+07	************
74	1	1.3685006E+08	1.0100000E+08	-3.5850056E+07	*****************
75	1	1.3618680E+08	1.1100000E+08	-2.5186801E+07	************
76	1	1.3552676E+08	1.0700000E+08	-2.8526761E+07	**************
77	1	1.3486992E+08	1.1000000E+08	-2.486S920E+07	************
78	1	1.3421626E+08	1.0600000E+08	-2.8216263E+07	**************
79	1	1.3356577E+08	1.0900000E+08	-2.4565773E+07	************
80	1	1.3291844E+08	1.0900000E+08	-2.3918436E+07	************
81	1	1.3227424E+08	9.9000000E+07	-3.3274236E+07	****************
82	1	1.3163316E+08	1.2600000E+08	-5.6331590E+06	***
83	1	1.3099519E+08	1.3400000E+08	3.0048114E+06	**
84	1	1.3036031E+08	1.1200000E+08	-1.8360310E+07	*********
85	1	1.2972851E+08	1.1600000E+08	-1.3728509E+07	*******
86	1	1.2909977E+08	1.0300000E+08	-2.6099769E+07	*************
87	1	1.2847408E+08	1.1400000E+08	-1.4474077E+07	*******
88	1	1.2785142E+08	1.1200000E+08	-1.5851418E+07	********
89	1	1.2723178E+08	1.2400000E+08	-3.2317757E+06	**
90	1	1.2661514E+08	1.2800000E+08	1.3848631E+06	**
91	1	1.2600149E+08	1.3500000E+08	8.9985132E+06	*****
92	1	1.2539081E+08	1.1600000E+08	-9.3908107E+06	*****
93	1	1.2478309E+08	1.3000000E+08	5.2169057E+06	***
94	1	1.2417832E+08	1.0300000E+08	-2.1178323E+07	**********
95	1	1.2357648E+08	9.9000000E+07	-2.4576483E+07	************
96	1	1.2297756E+08	1.0900000E+08	-1.3977560E+07	*******

- -

F I N A L F U N C T I O N V A L U E S A N D R E S I D U A L S

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL (-2 -1 0 +1 +2)
97	1	1.2238154E+08	1.2400000E+08	1.6184601E+06	**
98	1	1.2178841E+08	1.3400000E+08	1.2211592E+07	******
99	1	1.2119815E+08	1.4400000E+08	2.2801849E+07	***********
100	1	1.2061075E+08	1.2600000E+08	5.3892451E+06	***
101	1	1.2002621E+08	1.4100000E+08	2.0973795E+07	**********
102	1	1.1944449E+08	1.3500000E+08	1.5555511E+07	********
103	1	1.1886559E+08	1.3700000E+08	1.8134408E+07	*********
104	1	1.1828950E+08	1.1400000E+08	-4.2895006E+06	***
105	1	1.1771620E+08	9.9000000E+07	-1.8716201E+07	*********
106	1	1.1714568E+08	1.0200000E+08	-1.5145681E+07	********
107	1	1.1657792E+08	1.0800000E+08	-8.5779248E+06	*****
108	1	1.1601292E+08	1.0600000E+08	-1.0012921E+07	*****
109	1	1.1545066E+08	9.6000000E+07	-1.9450655E+07	**********
110	1	1.1489111E+08	7.6000000E+07	-3.8891114E+07	******************
111	1	1.1433429E+08	1.1200000E+08	-2.3342857E+06	**
112	1	1.1378016E+08	1.0700000E+08	-6.7801556E+06	****
113	1	1.1322871E+08	1.2100000E+08	7.7712888E+06	****
114	1	1.1267994E+08	1.0900000E+08	-3.6799394E+06	***
115	1	1.1213383E+08	1.0100000E+08	-1.1133827E+07	******
116	1	1.1159036E+08	1.0100000E+08	-1.0590362E+07	******
117	1	1.1104953E+08	1.0100000E+08	-1.0049531E+07	*****
118	1	1.1051132E+08	1.0300000E+08	-7.5113203E+06	****
119	1	1.0997572E+08	1.0100000E+08	-8.9757185E+06	*****
120	1	1.0944271E+08	1.0500000E+08	-4.4427126E+06	***
121	1	1.0891229E+08	1.2100000E+08	1.2087710E+07	******

122	1	1.0838444E+08	1.0200000E+08	-6.3844380E+06	****
123	1	1.0785914E+08	1.0800000E+08	1.4085564E+05	*
124	1	1.0733640E+08	9.7000000E+07	-1.0336397E+07	******
125	1	1.0681618E+08	9.8000000E+07	-8.8161823E+06	*****
126	1	1.0629849E+08	9.4000000E+07	-1.2298489E+07	******
127	1	1.0578331E+08	9.4000000E+07	-1.1783305E+07	******
128	1	1.0527062E+08	9.3000000E+07	-1.2270618E+07	******
129	1	1.0476042E+08	9.3000000E+07	-1.1760416E+07	******
130	1	1.0425269E+08	9.7000000E+07	-7.2526865E+06	****
131	1	1.0374742E+08	9.2000000E+07	-1.1747418E+07	******
132	1	1.0324460E+08	9.7000000E+07	-6.2445978E+06	****
133	1	1.0274421E+08	5.9444285E+07	-4.3299930E+07	*******************
134	1	1.0224626E+08	7.8428571E+07	-2.3817686E+07	************
135	1	1.0175071E+08	9.5534285E+07	-6.2164274E+06	****
136	1	1.0125757E+08	9.6081427E+07	-5.1761426E+06	***
137	1	1.0076682E+08	9.5832855E+07	-4.9339619E+06	***
138	1	1.0027844E+08	8.5059998E+07	-1.5218445E+07	********
139	1	9.9792435E+07	9.3398569E+07	-6.3938663E+06	****
140	1	9.9308783E+07	9.0221427E+07	-9.0873565E+06	*****
141	1	9.8827476E+07	8.6818570E+07	-1.2008906E+07	******
142	1	9.8348501E+07	8.5194284E+07	-1.3154217E+07	*******
143	1	9.7871847E+07	8.4359998E+07	-1.3511849E+07	*******
144	1	9.7397503E+07	8.1637141E+07	-1.5760362E+07	********
145	1	9.6925459E+07	9.1932855E+07	-4.9926038E+06	***
146	1	9.6455702E+07	6.6801428E+07	-2.9654274E+07	**************

- -

F I N A L F U N C T I O N V A L U E S A N D R E S I D U A L S

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL -2 -1 0 +1 +2
147	1	9.5988222E+07	1.0540143E+08	9.4132042E+06	*****
148	1	9.5523007E+07	1.2439143E+08	2.8868421E+07	**************
149	1	9.5060048E+07	1.1844143E+08	2.3381378E+07	***********
150	1	9.4599332E+07	9.7515712E+07	2.9163803E+06	**
151	1	9.4140849E+07	1.0432143E+08	1.0180578E+07	******
152	1	9.3684588E+07	1.0649286E+08	1.2808268E+07	*******
153	1	9.3230538E+07	9.5879998E+07	2.6494600E+06	**
154	1	9.2778689E+07	1.0769143E+08	1.4912738E+07	********
155	1	9.2329030E+07	1.0516857E+08	1.2839540E+07	*******
156	1	9.1881550E+07	1.0072714E+08	8.8455911E+06	*****
157	1	9.1436239E+07	9.9011427E+07	7.5751882E+06	****
158	1	9.0993086E+07	6.4111428E+07	-2.6881658E+07	*************
159	1	9.0552081E+07	9.4319998E+07	3.7679172E+06	***
160	1	9.0113213E+07	1.0449428E+08	1.4381071E+07	*******
161	1	8.9676472E+07	1.1687571E+08	2.7199240E+07	*************
162	1	8.9241848E+07	1.1589000E+08	2.6648149E+07	*************
163	1	8.8809331E+07	1.1663428E+08	2.7824953E+07	*************
164	1	8.8378909E+07	1.1997000E+08	3.1591089E+07	***************
165	1	8.7950574E+07	1.1884714E+08	3.0896566E+07	***************
166	1	8.7524315E+07	1.2435000E+08	3.6825684E+07	*****************
167	1	8.7100121E+07	1.3309571E+08	4.5995591E+07	*********************
168	1	8.667[illegible]84E+07	1.1828000E+08	3.1602[illegible]E+07	******* [illegible] ******
169	1	8.6257892E+07	1.1866285E+08	3.2404962E+07	*******[illegible]*******
170	1	8.5839836E+07	7.8339999E+07	-7.4998375E+06	****

171	1	8.54 07E+07	1.0365423E+08	1.8230 5E+07	*******
172	1	8.5009794E+07	9.9784285E+07	1.4774491E+07	********
173	1	8.4597737E+07	9.9985712E+07	1.5387925E+07	********
~~174~~	~~1~~	~~8.4107777E+07~~	~~1.1079714E+08~~	~~2.6689363E+07~~	*************
175	1	8.3779755E+07	1.1389714E+08	3.0117387E+07	**************
176	1	8.3373710E+07	1.1357429E+08	3.0200575E+07	**************
177	1	8.2969632E+07	1.0843714E+08	2.5467508E+07	************
178	1	8.2567514E+07	1.1157000E+08	2.9002485E+07	**************
179	1	8.2167344E+07	1.2813000E+08	4.5962653E+07	*********************
~~180~~	~~1~~	~~8.1769113E+07~~	~~1.1168236E+08~~	~~2.9913743E+07~~	**************
181	1	8.1372813E+07	7.7159999E+07	-4.2128139E+06	***
182	1	8.0978433E+07	9.4088570E+07	1.3110137E+07	*******
183	1	8.0585965E+07	1.0074571E+08	2.0159747E+07	**********
184	1	8.0195399E+07	9.6645712E+07	1.6450313E+07	********
185	1	7.9806725E+07	1.0002428E+08	2.0217558E+07	**********
~~186~~	~~1~~	~~7.9419936E+07~~	~~1.0315571E+08~~	~~2.3735776E+07~~	************
187	1	7.9035021E+07	9.3337142E+07	1.4302121E+07	*******
188	1	7.8651972E+07	7.1099999E+07	-7.5519726E+06	****
189	1	7.8270779E+07	6.4385713E+07	6.1149344E+06	****
190	1	7.7891433E+07	8.3351427E+07	5.4599938E+06	***
191	1	7.7513926E+07	7.5327142E+07	-2.1867843E+06	**
~~192~~	~~1~~	~~7.7138249E+07~~	~~7.1921428E+07~~	~~-5.2168210E+06~~	***
193	1	7.6764392E+07	7.7812856E+07	1.0484636E+06	*
194	1	7.6392348E+07	3.3102857E+07	-4.3289491E+07	********************
195	1	7.6022106E+07	7.8944284E+07	2.9221777E+06	**
196	1	7.5653659E+07	7.8382855E+07	2.7291958E+06	**

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~~FILE NONAME (CREATION DATE = 79/07/25.)~~

F I N A L   F U N C T I O N   V A L U E S   A N D   R E S I D U A L S

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL (-2 -1 0 +1 +2)
197	1	7.5286998E+07	7.5297142E+07	1.0144161E+04	*
198	1	7.4922114E+07	7.5715712E+07	7.9359848E+05	*
199	1	7.4558998E+07	7.4444284E+07	-1.1471364E+05	*
200	1	7.4197642E+07	6.7602855E+07	-6.5947866E+06	****
201	1	7.3838037E+07	6.8151427E+07	-5.6866100E+06	****
~~202~~	~~1~~	~~7.3480175E+07~~	~~6.3952856E+07~~	~~-9.5273191E+06~~	*****
203	1	7.3124048E+07	6.3441428E+07	-9.6826197E+06	*****
204	1	7.2769646E+07	6.9948570E+07	-2.8210763E+06	**
205	1	7.2416963E+07	5.2595714E+07	-1.9821249E+07	**********
206	1	7.2065988E+07	6.2952856E+07	-9.1131321E+06	*****
207	1	7.1716715E+07	6.7548570E+07	-4.1681446E+06	***
~~208~~	~~1~~	~~7.1369134E+07~~	~~6.4332857E+07~~	~~-7.0362774E+06~~	****
209	1	7.1023238E+07	6.9948570E+07	-1.0746679E+06	*
210	1	7.0679018E+07	6.7148570E+07	-3.5304482E+06	***
211	1	7.0336467E+07	6.9945713E+07	-3.9075380E+05	*
212	1	6.9995576E+07	7.4034284E+07	4.0387084E+06	***
213	1	6.9656337E+07	7.1838571E+07	2.1822344E+06	**
~~214~~	~~1~~	~~6.9318742E+07~~	~~7.2531428E+07~~	~~3.2126863E+06~~	**
215	1	6.8982783E+07	7.2859999E+07	3.8772160E+06	***
216	1	6.8648453E+07	6.9465713E+07	8.1726045E+05	*
217	1	6.8315742E+07	6.8988570E+07	6.7282754E+05	*
218	1	6.7984645E+07	6.2398571E+07	-5.5860744E+06	***
219	1	6.7655152E+07	6.9085714E+07	1.4305620E+06	**

220	1	6.7327256E+07	6.3991427E+07	-3.3358290E+06	**
221	1	6.7000949E+07	6.4138571E+07	-2.8623787E+06	**
222	1	6.6676224E+07	6.4272857E+07	-2.4033668E+06	**
~~223~~	~~1~~	~~6.6353072E+07~~	~~6.6754285E+07~~	~~4.0121219E+05~~	*
224	1	6.6031487E+07	6.4804285E+07	-1.2272025E+06	**
225	1	6.5711460E+07	6.2657143E+07	-3.0543177E+06	**
226	1	6.5392984E+07	6.4488571E+07	-9.0441347E+05	*
227	1	6.5076052E+07	6.0964286E+07	-4.1117668E+06	***
228	1	6.4760656E+07	6.2565713E+07	-2.1949431E+06	**
~~229~~	~~1~~	~~6.4446788E+07~~	~~3.5972857E+07~~	~~-2.8473932E+07~~	**************
230	1	6.4134442E+07	5.7982857E+07	-6.1515851E+06	****
231	1	6.3823610E+07	6.4855713E+07	1.0321035E+06	*
232	1	6.3514283E+07	6.1041428E+07	-2.4728559E+06	**
233	1	6.3206456E+07	6.2651428E+07	-5.5502846E+05	*
234	1	6.2900121E+07	6.0900000E+07	-2.0001219E+06	**
~~235~~	~~1~~	~~6.2595271E+07~~	~~6.2775714E+07~~	~~1.8044243E+05~~	*
236	1	6.2291898E+07	6.3324285E+07	1.0323868E+06	*
237	1	6.1989996E+07	6.2302856E+07	3.1286036E+05	*
238	1	6.1689556E+07	6.4124285E+07	2.4347287E+06	**
239	1	6.1390573E+07	6.1105714E+07	-2.8485902E+05	*
240	1	6.1093039E+07	6.3297142E+07	2.2041027E+06	**
~~241~~	~~1~~	~~6.0796947E+07~~	~~6.3249999E+07~~	~~2.4530519E+06~~	**
242	1	6.0502289E+07	5.9364285E+07	-1.1380045E+06	**
243	1	6.0209060E+07	6.3074285E+07	2.8652241E+06	**
244	1	5.9917252E+07	6.1322856E+07	1.4056030E+06	**
245	1	5.9626859E+07	6.3275714E+07	3.6488547E+06	***
246	1	5.9337873E+07	6.0945714E+07	1.6078409E+06	**

- -

FILE NONAME (CREATION DATE = 79/07/25.)

F I N A L   F U N C T I O N   V A L U E S   A N D   R E S I D U A L S

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL -2 -1 0 +1 +2
247	1	5.9050267E+07	6.3087143E+07	4.0368555E+06	***
248	1	5.8764095E+07	6.3228570E+07	4.4644749E+06	***
249	1	5.8479290E+07	6.0767142E+07	2.2878517E+06	**
250	1	5.8195866E+07	6.1883571E+07	3.6927051E+06	***
~~251~~	~~1~~	~~5.7913815E+07~~	~~6.0577142E+07~~	~~2.6633269E+06~~	**
252	1	5.7633131E+07	5.0198572E+07	-7.4345597E+06	****

TIME SINCE END OF THE LAST ITERATION = 1.533 SECONDS
TOTAL TIME = 3.368 SECONDS

- -

RUN COMPLETED

NUMBER OF CONTROL CARDS READ 13

ESTIMATING FIELD INFLUENCE FUNCTIONS -- PROGRAM USEAGE

The linear programming technique of Coats, Rapoport, McCord and Drews (JPT, Dec.,1964), is used to determine the aquifer influence functions from field data. Several programs have been written to take raw data and convert it into a tableau used by the M.P.O.S linear programming package, then extract the results and reformat it. The procedure is as follows:

1. INPUT DATA

The input data must follow the following arrangement:

Card No.	Information
1	Header Card
2	Input Format. Fields are integer-real-real for time-pressure-volume
3	N = number of data cards to follow
4	First Data Card (contains data for time, pressure, and volume)
N+4	Last Data Card
	repeat for more data sets

2. GENERATING THE INFLUENCE FUNCTION

The procedure to do this is simply:

```
GET,INFUNC.  or GET,INFUNC/UN=BKOQ5C.
INFUNC,IN.
```

where IN is the file containing the input data as described above.

The results will be found in file OUT and are arranged as follows:

1) header card
2) card containing number of data points and fitness value RHO in (I5,5X,F17.0) field
3) N data cards in the format (I5,5X,4F17.7) in order of time, F, delta-F, P, Q.
4) E-O-F mark
5) repetition of 1) to 3) for each data set

These results should be saved for future plotting. There is also a file named RESULTS generated which contains output from the M.P.O.S. package which should be sent to the line printer <u>with carriage control.</u>

3. PLOTTING THE INFLUENCE FUNCTION

To plot the influence functions generated above, it is necessary to use the INFPLOT program. The necessary libraries and object decks will have been gotten by INFUNC. All that is necessary is to enter

INFPLOT,OUT

where OUT is the file generated in step 2) above. The program should be run from a graphics terminal. Output can be sent to the GERBER plotter or plotted on a 4662 plotter. The program will prompt the user for the necessary information.

March 13, 1980

Influence Function Programs

```
.PROC,INFUNC,IN,OUT,RESULTS.
.* --------------------------------------------------------------------
.*   THIS PROCEDURE WILL TAKE RAW TIME/PRESSURE/VOLUME DATA AND
.*   CALL THE PROPER PROGRAMS TO PRODUCE AN INFLUENCE FUNCTION USING
.*   THE LINEAR PROGRAMMING ALGORITHM OF COATS, ET AL (1964).
.*
.*   CALLING SEQUENCE:  BEGIN,INFUNC,JPROC,IN,OUT,RESULTS.
.*
.*   WHERE:
.*
.*     IN      FILE WITH DATA SETS IN PROPER FORMAT.
.*     OUT     FILE WHICH WILL CONTAIN TIME, INFLUENCE FUNCTION, DELTA-F,
.*             PRESSURE, AND VOLUME IN (I5,5X,4F17.7) FORMAT.
.*     RESULT  FILE WHICH WILL CONTAIN OUTPUT FROM M.P.O.S. LINEAR PROGRAMMING
.*             PACKAGE.
.* --------------------------------------------------------------------
.*COPY,INF4,OUTPUT.
.*REVERT.
IF(FILE(JLIB,.NOT.AS))GET,JLIB/UN=AET 15C.
LIBRARY=OSULIB/JLIB/COMPLOT.
BEGIN,GETPROG,P,JPROGS,TABGEN,INFPLOT.
IF(FILE(MPOS,.NOT.AS))ATTACH,MPOS/UN=LIBRARY.
REWIND,IN.
SETTL,100.
RETURN,ZTAB.
TABGEN(IN,ZTAB)
COPY,INF1.
MPOS(ZTAB,RESULTS)
COPY,INF2.
BEGIN,REFORM,JPROC,RESULTS,IN,OUT.
IF(FILE(Z123Z,AS))REVERT.  ERROR ENCOUNTERED.
COPY,INF3.
RETURN,INF1,INF2,INF3.
REVERT.  PROC/INFUNC.
.DATA,INF1
L.P. TABLEAU GENERATED ΞIN LOCAL FILE <ZTAB>
ENTERING M.P.O.S.
.DATA,INF2.
M.P.O.S. RUN COMPLETE.  ΞRESULTS ARE ΞIN LOCAL FILE <RESULTS>
BEGIN DATA REFORMATTING.
.DATA,INF3
REFORMAT COMPLETE.  INFLUENCE FUNCTION PLUS ORIGINAL DATA ARE NOW
ΞIN LOCAL FILE <OUT>.  DATA SETS ARE SEPARATED BY E-O-F MARKS.
USE <APPEND,JINFUNC,OUT> TO ADD TO PREVIOUS ΞRESULTS,
OTHERWISE <SAVE,OUT=NEWNAME> TO SAVE THE OUTPUT.
.DATA,INF4
PROGRAMS ARE NOT WORKING TEMPORARILY.  CALL JEFF AT 754-4515 OR -4599
```

```
*TABGEN
      PROGRAM TABGEN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
      DIMENSION P(300), Q(300), BUFF(8), FORMI(8)
      DIMENSION HDR(8)
      DATA ZERO/0/, ONE/1./, ONEG/-1./, TWOM/-2./, X/1HX/, V/1HV/
      DATA U/1HU/, EQ/1H0/, DASH/1H-/
C
C    THIS PROGRAM READS IN PRESSURE AND PRODUCTION DATA AND
C    CREATES INPUT FOR USE IN LINEAR PROGRAMMING ROUTINE M.P.O.S.
C
C
C
C    READ HEADER AND INPUT AND OUTPUT FORMATS
C
    1 READ(5,101) HDR
      IF(EOF(5))99,2
    2 READ(5,101) FORMI
  101 FORMAT(8A10)
C
C    READ NUMBER OF DATA POINTS
C    READ DUMMY VARIABLE IN TIME FIELD, READ PRESSURE AND PRODUCTION DATA
C
      READ(5,*) N
      DO 5 I=1,N
      READ(5,FORMI) IDUM, P(I), Q(I)
    5 CONTINUE
C
C    SETUP INDEXES
C
      N1 = N-1
      N2 = N1*2
      N3 = N1+2
      N4 = 2*N1-1
C
C    COMPUTE DELTA-P
C
      DO 10 I=2,N
   10 P(I) = ABS(P(1) - P(I))
C
C    PRINT M.P.O.S. CONTROL CARDS
C
      WRITE(6,201)
  201 FORMAT(≠TITLE≠)
      WRITE(6,101) HDR
      WRITE(6,202)
  202 FORMAT(≠REVISED≠/≠VARIABLES≠)
      PRINT (6,*)≠X1 TO X≠,N1
      PRINT (6,*)≠U1 TO U≠,N1
      PRINT (6,*)≠V1 TO V≠,N1
      WRITE(6,203) 2*N1-1
  203 FORMAT(≠PACKED≠/≠MINIMIZE≠/≠CONSTRAINTS≠,I5)
      WRITE(6,103) (EQ,I=1,N1), (DASH,I=1,N3)
  103 FORMAT(80A1)
      WRITE(6,204) FORMO
  204 FORMAT(≠FORMAT≠/≠(2I5,F20.5)≠/≠READ≠)
C
C
C         REWRITE DATA AS INPUT TO M.P.O.S. IN TABLEAU FORMAT
C
C
C
C    WRITE OBJECTIVE FUNCTION
C
  111 FORMAT(2I5,F20.5)
      WRITE(6,111) (ZERO,I+N1,ONE,I=1,N2)
C
C    WRITE SECOND ORDER CONSTRAINTS
C
      DO 20 J=1,N1
      WRITE(6,111) (J,K,Q(J-K+2), K=1, J)
      WRITE(6,111) J,N1+J,ONE
      WRITE(6,111) J,N2+J,ONEG
      WRITE(6,111) J,ZERO,P(J+1)
   20 CONTINUE
C
C    WRITE THIRD ORDER CONSTRAINTS
C
      WRITE(6,111) N1+1, N1-1, ONE
      WRITE(6,111) N1+1, N1,   ONEG
      DO 22 J=N3,N4
      K = J-N3
      WRITE(6,111) J, K+1, ONE
      WRITE(6,111) J, K+2, TWOM
      WRITE(6,111) J, K+3, ONE
   22 CONTINUE
```

```
C
      WRITE(6,111) ZERO,ZERO,ZERO
      WRITE(6,205)
  205 FORMAT(*OPTIMIZE*)
      GO TO 1
   99 REWIND 6
      STOP
      END
```

```
*INFPLO
      PROGRAM INFPLOT(OUT,EXTRAP,INPUT,OUTPUT,TAPE10=0,TAPE5=INPUT,
     *  TAPE8=OUT,TAPE7=EXTRAP,TAPE6=OUTPUT)
C
C
C ----------------------------------------------------------------
C          PLOTS THE INFLUENCE FUNCTION FOR GEOTHERMAL FIELD
C
C          TIME AND VALUE OF THE FUNCTION AS GENERATED BY PROGRAM <LPM> ARE READ
C          AND PLOTTED ON TEKTRONIX 4662 PLOTTER
C
C
C   LOGICAL UNITS FOR INPUT/OUTPUT:
C
C       LUN 8    READS INFLUENCE FUNCTION AS GENERATED BY PROCEDURE <REFORM>
C                FORMAT IS ONE HEADER CARD FOLLOWED BY TIME AND FUNCTION VALUE CARDS
C                IN (F5.0,5X,F17.6) FORMAT.  SEPARATE DECKS SEPARATED BY --EOF--.
C                DEFAULT DATA FILE NAME IS <OUT>.
C       LUN 6    WRITES MESSAGES TO OPERATOR ON FILE <OUTPUT>
C       LUN 7    WRITES COMPUTED EXTRAPOLATIONS ON DEFAULT FILE <EXTRAP>
C       LUN 5    ASSIGNED TO FILE <INPUT> FOR RECEIVING OPERATOR INSTRUCTIONS
C
C ----------------------------------------------------------------
      INTEGER EXLAB1,EXLAB2(3),EXLAB3(3),EXLAB4(4)
      DIMENSION HDR(8), T(301), F(301), YLAB(8), BUFF(2), FOOT(8)
      DATA YES /≠YES≠/
      DATA MARK/0/
      DATA YLAB/≠PRESSURE RESPONSE PER UNIT VOLUME        ≠/
      DATA EXLAB1 /≠FIELD DATA≠/
      DATA EXLAB2 /≠ASSUMES BOUNDED ACUIFER≠/
      DATA EXLAB3 /≠ASSUMES INFINITE AQUIFER≠/
      DATA EXLAB4 /≠ASSUMES IMMEDIATELY BOUNDED AQUIFER≠/
      DATA FOOT /≠(F>ITNESS MEASURE ^Q<v = ≠/
C
C
C ----------------------------------------------------------------
C   INITIALIZATION -- DETERMINE WHICH, IF ANY, EXTRAPOLATIONS ARE REQUIRED
C ----------------------------------------------------------------
C
      CALL ALPHAS
      PRINT *,≠DO YOU WISH TO PLOT ON THE GERBER+≠
      LTYP=4
      IF(NOYES(5).EQ.0) LTYP=1
      PRINT *,≠ENTER NUMBER OF TIME PERIODS FOR EXTRAPOLATION≠
      READ (5,*) NEXT
      IF(EOF(5).EQ.1) NEXT=0
      IF(NEXT.EQ.0) GO TO 1
      WRITE(6,104) EXLAB2
  104 FORMAT(≠ENTER ≠≠YES≠≠ IF YOU DESIRE EXTRAPOLATION WHICH ≠,5A10)
      L2 = NOYES(5)
      WRITE(6,104) EXLAB3
      L3 = NOYES(5)
      WRITE(6,104) EXLAB4
      L4 = NOYES(5)
C
C ----------------------------------------------------------------
C   READ HEADER FOR X-LABEL
C ----------------------------------------------------------------
C
    1 READ (8,101) HDR
      IF(EOF(8)) 59,10
   10 N=0
      READ(8,107) RHO
  107 FORMAT(10X,F17.7)
      IF(MARK.EQ.0) GO TO 15
      CALL TEKPAUS
  101 FORMAT(8A10)
C
C ----------------------------------------------------------------
C   READ DATA
C ----------------------------------------------------------------
C
   15 N = N + 1
      READ (8,102) T(N), F(N)
  102 FORMAT(F5.0,5X,F17.6)
      IF(N.GE.301) GO TO 20
      IF(EOF(8)) 20,15
   20 N=N-1
C
C ----------------------------------------------------------------
C   PLOTTING SECTION
C ----------------------------------------------------------------
C
C   PHYSICAL PROPERTIES SETUP
```

```
      IF(MARK.EQ.1) GO TO 30
      MARK=1
      WIDTH=7.
      HEIGHT=5.
      CALL PLOTYPE(LTYP)
      CALL TKTYPE(4662)
      CALL BAUD(1200)
      CALL SIZE(WIDTH+1.0,HEIGHT+1.0)
C
C -----------------------------------------------------------------
C   RANGING
C -----------------------------------------------------------------
C
   30 NINTS=4
      FINCR=F(N)-F(N-1)
      TINCR = T(2) - T(1)
      CALL RANGE(0.,F(N)+NEXT*FINCR,NINTS,YMIN,YMAX,YTIC)
      CALL RANGE(T(1), T(N)+NEXT*TINCR, 8, XMIN, XMAX, XTIC)
      IF(XTIC.LT.1.0) XTIC=1.0
C
C -----------------------------------------------------------------
C   SCALING (INCHES/USER UNIT)
C -----------------------------------------------------------------
C
      XBIAS=2.
      YBIAS=1.5
      XDELT = XMAX - XMIN
      XFACT=(WIDTH-XBIAS)/XDELT
      YFACT=(HEIGHT-YBIAS)/(YMAX-YMIN)
      CALL SCALE(XFACT,YFACT,XBIAS,YBIAS,XMIN,YMIN)
C
C -----------------------------------------------------------------
C   GRID GENERATION AND DIMENSIONING
C -----------------------------------------------------------------
C
      CALL ERASE
CDEB  CALL ALPHAS
      CSIZE=.07
CDEB  PRINT(7,*)≠XMIN=≠,XMIN,≠ XMAX=≠,XMAX,≠ XTIC=≠,XTIC,≠ XFACT=≠,XFACT
CDEB  PRINT(7,*)≠YMIN=≠,YMIN,≠ YMAX=≠,YMAX,≠ YTIC=≠,YTIC,≠ YFACT=≠,YFACT
      CALL AXIS(XMIN,XMAX,XMIN,YMIN,YMAX,YMIN,XTIC,YTIC,0,9)
      CALL AXLABL
     * (1,XMAX,XMIN,XTIC,YMAX,YMIN,YTIC,XFACT,YFACT,CSIZE,1,1.,YDISP)
      CALL AXLABL
     * (2,XMAX,XMIN,XTIC,YMAX,YMIN,YTIC,XFACT,YFACT,CSIZE,1,1.,XDISP)
C
C -----------------------------------------------------------------
C   WRITE AXIS LABELS
C -----------------------------------------------------------------
C
      CSIZE = .1
      NCHARS = IGNORCB(1H ,HDR,1,80)
      X = XDELT - SYMWID(CSIZE,NCHARS,HDR)/XFACT
      X = XMIN + X/2.
      CALL SYMBEL(X,-.7/YFACT,0.,CSIZE,NCHARS,HDR)
C
      CALL FNUM(RHO,0.000001,BUFF,NCHAR)
      CALL MOVEST(BUFF,1,FOOT,25,NCHAR)
      CALL MOVEST(2H):,1,FOOT,25+NCHAR,2)
      X = XDELT - SYMWID(CSIZE,80,FOOT) / XFACT
      X = XMIN + X/2.
      CALL SYMBEL(X,-.98/YFACT,0.,CSIZE,80,FOOT)
C
      NCHARS=33
      Y = YMAX  - SYMWID(CSIZE,NCHARS,YLAB)/YFACT
      Y = Y/2.
      X = -.8/XFACT + XMIN
      CALL SYMBEL(X,Y,90.,CSIZE,NCHARS,YLAB)
C
C -----------------------------------------------------------------
C   PLOT THE VALUES
C -----------------------------------------------------------------
C
      CALL LINE(T,F,17,N)
C
C -----------------------------------------------------------------
C   CHECK IF EXTRAPOLATION WAS REQUESTED BY USER
C -----------------------------------------------------------------
C
      IF(NEXT.LE.0) GO TO 1
      IF(N.LT.5) GO TO 1
      CALL DASHSZ(.05,.03,.07,.05)
C
C -----------------------------------------------------------------
```

```
C     PERFORM FIRST EXTRAPOLATION -- ASSUME BOUNDED AQUIFER.
C     ALGORITHM: AVERAGE LAST THREE INTERVALS.
C  -----------------------------------------------------------------
C
         IF(L2.EQ.0) GO TO 42
         FDELT = (F(N)-F(N-3)) / (T(N)-T(N-3))
C
         CALL DASHES
         CALL PLOT(T(N),F(N),0,0)
         WRITE(7,101) HDR
         WRITE(7,103) NEXT,EXLAB2
     103 FORMAT(I3,≠ TIME PERIODS EXTRAPOLATION ≠,4A10)
C
         DO 40 I=1,NEXT
         Y = F(N) + FDELT*I
         X = T(N) + I * TINCR
         WRITE(7,102) X,Y
         CALL PLOT(X,Y,1,19)
      40 CONTINUE
C
C  -----------------------------------------------------------------
C     PERFORM SECOND EXTRAPOLATION -- ASSUME INFINITE AQUIFER
C     ALGORITHM:    HUTCHINSON ^ SIKORA, 1959, P.172, EQN. 16
C  -----------------------------------------------------------------
C
      42 IF(L3.EQ.0) GO TO 52
         CALL DASHES
         CALL PLOT(T(N),F(N),0,0)
         WRITE(7,103) NEXT,EXLAB3
         X0=T(1)
         Y = F(N)
         DELTY = FINCR
         DO 50 I=1,NEXT
         X = T(N) + I   * TINCR
         XX = X - X0
         D = (XX+1) / XX
         E = XX / (XX-1)
         DELTY = DELTY * ALOG(D) / ALOG(E)
         Y = Y + DELTY
         WRITE(7,102) X,Y
         CALL PLOT(X,Y,1,27)
      50 CONTINUE
C
C  -----------------------------------------------------------------
C     PERFORM THIRD EXTRAPOLATION -- ASSUME IMMEDIATELY BOUNDED AQUIFER
C     ALGORITHM:  USE LAST INTERVAL
C  -----------------------------------------------------------------
C
      52 IF(L4.EQ.0) GO TO 62
         FDELT = (F(N)-F(N-1)) / (T(N)-T(N-1))
C
         CALL DASHES
         CALL PLOT(T(N),F(N),0,0)
         WRITE(7,103) NEXT,EXLAB3
C
         DO 60 I=1,NEXT
         Y = F(N) + FDELT * I
         X = T(N) + I * TINCR
         WRITE(7,102) X,Y
         CALL PLOT(X,Y,1,21)
      60 CONTINUE
C
      62 CONTINUE
C
C  -----------------------------------------------------------------
C     WRITE LABELS
C  -----------------------------------------------------------------
C
         CALL SCALE(1.,1.,XBIAS,YBIAS,0.,0.)
         X = WIDTH/3.
         Y=HEIGHT/5.
         CALL VECTORS
         CSIZE=.07
         MM=2
         CALL WMARK(X,Y,CSIZE,17)
         CALL SYMBEL (X,Y,0,CSIZE,10,EXLAB1)
         CALL DASHES
         IF(L2.EQ.0) GO TO 71
         X = WIDTH/3.
         Y=HEIGHT/5 - CSIZE * MM
         MM = MM+2
         CALL WMARK(X,Y,CSIZE,19)
         CALL SYMBEL(X,Y,0,CSIZE,23,EXLAB2)
         CALL DASHES
```

```
   71 IF(L3.EQ.0) GO TO 72
      X = WIDTH/3.
      Y = HEIGHT/5. - CSIZE * MM
      MM = MM + 2
      CALL WMARK(X,Y,CSIZE,27)
      CALL SYMBEL(X,Y,0,CSIZE,24,EXLAB3)
      CALL DASHES
   72 IF(L4.EQ.0) GO TO 1
      X = WIDTH/3
      Y = HEIGHT/5 - CSIZE*MM
      CALL WMARK(X,Y,CSIZE,21)
      CALL SYMBEL(X,Y,0,CSIZE,35,EXLAB4)
      GO TO 1
C
C    END
C
   99 CALL PLOTEND
  999 STOP ≠END INFPLOT≠
      END
```

```
.PROC,REFORM,IN,DATA,OUT.
.*----------------------------------------------------------------
.*
.*   THIS PROCEDURE WILL REFORMAT THE OUTPUT FROM M.P.O.S. LINEAR PROGRAMMING
.*   AND MERGE IT WITH THE TIME VALUES FROM THE ORIGINAL DATA.  THE FINAL
.*   FORM IS DESIGNED TO BE USED WITH THE PLOTTING PROGRAM <INFPLOT>.
.*
.*   FILES:
.*
.*        IN   CONTAINS OUTPUT FROM M.P.O.S.
.*        DATA CONTAINS ORIGINAL DATA USED TO GENERATE M.P.O.S. TABLEAU.
.*        OUT  FILE WHICH WILL CONTAIN THE OUTPUT USED TO CALL <INFPLOT>
.*
.*----------------------------------------------------------------
RETURN,OUT,EDUMP,NUL,Z123Z.
NOEXIT.
XEDIT,IN,I=EDIT1,L=EDUMP,NH.
IFE,FILE(Z123Z,.NOT.AS),B.
ONEXIT.
XEDIT,NUL,I=EDIT2,L=EDUMP,NH.
IF(FILE(LPM,.NOT.AS))BEGIN,GETPROG,P,,LPM.
LPM(NUL,OUT,DATA)
RETURN,EDIT1,EDIT2,BADAT.
REWIND,OUT.
REVERT.   PROC/REFORM.   <OUT> CONVERTED.
ENDIF(B)
COPY,BADAT.
RETURN,BADAT,EDIT1,EDIT2.
REVERT.   PROC/REFORM.  ERROR FOUND.
.=DATA,EDIT1
NOBELLS
WM 81 84
Y;L/NO FEAS/;L/NO FEAS/0;COPY Z123Z 1;STOP
Y;L/Y OF R/;L/SUM/0;N-3;CSW//INF/;COPY NUL 1;N7;COPY NUL /SLACK/;L/OBJ...N/;-
STOP
.=DATA,EDIT2
NOBELLS
D/SLACK/*
N2
Y;L/ X1 /;L/ X1 /0;N-1;WEOF;N2;-
WM 78 84
Y;LW/---INF/;LW/---INF/0;D;-C
Q
.=DATA,BADAT
RUN ABORTED.
FILE <DATA> CONTAINS A =DATA SET FOR WHICH NO FEASIBLE SOLUTION
CAN BE FOUND BY THE LINEAR PROGRAMMING PACKAGE.
PLEASE EXAMINE FILE <IN> TO DETERMINE WHICH SET IS NOT FEASIBLE.
```

```
S
1     MPOS VERSION 3.2        NORTHWESTERN UNIVERSITY                                             80/03/13.   16.42.13.        PAGE    1
          ************************************
          *                                  *
          *             M P O S              *
          *                                  *
          *           VERSION 3.2            *
          *                                  *
          * MULTI-PURPOSE OPTIMIZATION SYSTEM *
          *                                  *
          ************************************

 ***** PROBLEM NUMBER  1 *****

         TITLE
         INFLUENCE FUNCTION -- BORE28 -- 1956 TO 1975
         REVISED
         VARIABLES
          X1 TO X19
          U1 TO U19
          V1 TO V19
         PACKED
         MINIMIZE
         CONSTRAINTS   37
         0000000000000000000--------------------
         FORMAT
         (2I5,F20.0)
         READ
         OPTIMIZE
1     MPOS VERSION 3.2        NORTHWESTERN UNIVERSITY                                             80/03/13.   16.42.13.        PAGE    2

 *********************
 * PROBLEM NUMBER  1 *
 *********************

 USING REVISED
 INFLUENCE FUNCTION -- BORE28 -- 1956 TO 1975

                                   PROBLEM INPUT SUMMARY

                  CONSTRAINTS   VARIABLES     NON-ZEROS        PARAMETERS        BOUNDS
                  -----------   -----------   -------------    -------------    --------------
                  EQS=   19     INT=     0    NUMBER=    281   TOL=  .100E-07
                  LES=    0     TOTAL=   57   PERCENT= 13.32   EPS=DEFAULT
                  GES=   18     NOUB=    0                     LIMIT=DEFAULT
                  TOTAL=   37   NOLB=    0                     RSCALE=   .100E+01

                                      VARIABLE TABLE

      1 - X1           2 - X2           3 - X3           4 - X4           5 - X5           6 - X6
      7 - X7           8 - X8           9 - X9          10 - X10         11 - X11         12 - X12
     13 - X13         14 - X14         15 - X15         16 - X16         17 - X17         18 - X18
     19 - X19         20 - U1          21 - U2          22 - U3          23 - U4          24 - U5
     25 - U6          26 - U7          27 - U8          28 - U9          29 - U10         30 - U11
     31 - U12         32 - U13         33 - U14         34 - U15         35 - U16         36 - U17
     37 - U18         38 - U19         39 - V1          40 - V2          41 - V3          42 - V4
     43 - V5          44 - V6          45 - V7          46 - V8          47 - V9          48 - V10
     49 - V11         50 - V12         51 - V13         52 - V14         53 - V15         54 - V16
     55 - V17         56 - V18         57 - V19

          INPUT TRANSLATION TIME =       .7570 SECONDS
1     MPOS VERSION 3.2        NORTHWESTERN UNIVERSITY                                             80/03/13.   16.42.13.        PAGE    3

 *********************
 * PROBLEM NUMBER  1 *
 *********************

 USING REVISED
 INFLUENCE FUNCTION -- BORE28 -- 1956 TO 1975

 ITNO.      1 IN VAR-   1   VAR-  96 TO LOWER BOUND   -WMIN=          -.128998
 ITNO.      2 IN VAR-   2   VAR-  97 TO LOWER BOUND   -WMIN=          -.128998
 ITNO.      3 IN VAR-  60   VAR-  76 TO LOWER BOUND   -WMIN=          -.114724
 ITNO.      4 IN VAR-   3   VAR-  98 TO LOWER BOUND   -WMIN=          -.114724
 ITNO.      5 IN VAR-   4   VAR-  99 TO LOWER BOUND   -WMIN=          -.114724
 ITNO.      6 IN VAR-  62   VAR-  60 TO LOWER BOUND   -WMIN=          -.106065
 ITNO.      7 IN VAR-   5   VAR- 100 TO LOWER BOUND   -WMIN=          -.106065
 ITNO.      8 IN VAR-   6   VAR- 101 TO LOWER BOUND   -WMIN=          -.106065
 ITNO.      9 IN VAR-  64   VAR-  62 TO LOWER BOUND   -WMIN=          -.098160
 ITNO.     10 IN VAR-   7   VAR- 102 TO LOWER BOUND   -WMIN=          -.098160
 ITNO.     11 IN VAR-   8   VAR- 103 TO LOWER BOUND   -WMIN=          -.098160
```

ITNO.	12	IN VAR-	66	VAR-	64	TO LOWER BOUND	-WMIN= -.090962
ITNO.	13	IN VAR-	9	VAR-	104	TO LOWER BOUND	-WMIN= -.090962
ITNO.	14	IN VAR-	10	VAR-	105	TO LOWER BOUND	-WMIN= -.090962
ITNO.	15	IN VAR-	68	VAR-	66	TO LOWER BOUND	-WMIN= -.084425
ITNO.	16	IN VAR-	11	VAR-	106	TO LOWER BOUND	-WMIN= -.084425
ITNO.	17	IN VAR-	12	VAR-	107	TO LOWER BOUND	-WMIN= -.084425
ITNO.	18	IN VAR-	70	VAR-	68	TO LOWER BOUND	-WMIN= -.078604
ITNO.	19	IN VAR-	13	VAR-	108	TO LOWER BOUND	-WMIN= -.078604
ITNO.	20	IN VAR-	14	VAR-	109	TO LOWER BOUND	-WMIN= -.078604
ITNO.	21	IN VAR-	72	VAR-	70	TO LOWER BOUND	-WMIN= -.073566
ITNO.	22	IN VAR-	15	VAR-	110	TO LOWER BOUND	-WMIN= -.073566
ITNO.	23	IN VAR-	16	VAR-	111	TO LOWER BOUND	-WMIN= -.073566
ITNO.	24	IN VAR-	74	VAR-	72	TO LOWER BOUND	-WMIN= -.069265
ITNO.	25	IN VAR-	17	VAR-	112	TO LOWER BOUND	-WMIN= -.069265
ITNO.	26	IN VAR-	18	VAR-	95	TO LOWER BOUND	-WMIN= -.069265
ITNO.	27	IN VAR-	58	VAR-	74	TO LOWER BOUND	-WMIN= -.065605
ITNO.	28	IN VAR-	19	VAR-	58	TO LOWER BOUND	-WMIN= -.035495
ITNO.	29	IN VAR-	39	VAR-	94	TO LOWER BOUND	-WMIN= -.031840
ITNO.	30	IN VAR-	75	VAR-	77	TO LOWER BOUND	-WMIN= -.028161
ITNO.	31	IN VAR-	58	VAR-	75	TO LOWER BOUND	-WMIN= -.028131
ITNO.	32	IN VAR-	40	VAR-	78	TO LOWER BOUND	-WMIN= -.023459
ITNO.	33	IN VAR-	41	VAR-	79	TO LOWER BOUND	-WMIN= -.019915
ITNO.	34	IN VAR-	42	VAR-	93	TO LOWER BOUND	-WMIN= -.009621
ITNO.	35	IN VAR-	67	VAR-	92	TO LOWER BOUND	-WMIN= -.009395
ITNO.	36	IN VAR-	71	VAR-	80	TO LOWER BOUND	-WMIN= -.004666
ITNO.	37	IN VAR-	73	VAR-	58	TO LOWER BOUND	-WMIN= -.004294
ITNO.	38	IN VAR-	60	VAR-	65	TO LOWER BOUND	-WMIN= -.003394
ITNO.	39	IN VAR-	59	VAR-	60	TO LOWER BOUND	-WMIN= -.003148
ITNO.	40	IN VAR-	66	VAR-	67	TO LOWER BOUND	-WMIN= -.003135
ITNO.	41	IN VAR-	53	VAR-	73	TO LOWER BOUND	-WMIN= -.003099
ITNO.	42	IN VAR-	37	VAR-	91	TO LOWER BOUND	-WMIN= -.003072
ITNO.	43	IN VAR-	72	VAR-	58	TO LOWER BOUND	-WMIN= -.003039
ITNO.	44	IN VAR-	54	VAR-	71	TO LOWER BOUND	-WMIN= -.003020
ITNO.	45	IN VAR-	73	VAR-	72	TO LOWER BOUND	-WMIN= -.002957
ITNO.	46	IN VAR-	48	VAR-	90	TO LOWER BOUND	-WMIN= -.002016
ITNO.	47	IN VAR-	70	VAR-	86	TO LOWER BOUND	-WMIN= -.001703
ITNO.	48	IN VAR-	75	VAR-	70	TO LOWER BOUND	-WMIN= -.001490
ITNO.	49	IN VAR-	67	VAR-	83	TO LOWER BOUND	-WMIN= -.001333
ITNO.	50	IN VAR-	74	VAR-	73	TO LOWER BOUND	-WMIN= -.001321
ITNO.	51	IN VAR-	49	VAR-	67	TO LOWER BOUND	-WMIN= -.001308
ITNO.	52	IN VAR-	71	VAR-	75	TO LOWER BOUND	-WMIN= -.001276
ITNO.	53	IN VAR-	72	VAR-	37	TO LOWER BOUND	-WMIN= -.001174
ITNO.	54	IN VAR-	70	VAR-	84	TO LOWER BOUND	-WMIN= -.001093
ITNO.	55	IN VAR-	75	VAR-	74	TO LOWER BOUND	-WMIN= -.001093
ITNO.	56	IN VAR-	65	VAR-	66	TO LOWER BOUND	-WMIN= -.001076
ITNO.	57	IN VAR-	56	VAR-	75	TO LOWER BOUND	-WMIN= -.001075
ITNO.	58	IN VAR-	53	VAR-	70	TO LOWER BOUND	-WMIN= -.001050
ITNO.	59	IN VAR-	73	VAR-	58	TO LOWER BOUND	-WMIN= -.001047
ITNO.	60	IN VAR-	37	VAR-	71	TO LOWER BOUND	-WMIN= -.001026
ITNO.	61	IN VAR-	66	VAR-	65	TO LOWER BOUND	-WMIN= -.001023
ITNO.	62	IN VAR-	47	VAR-	73	TO LOWER BOUND	-WMIN= -.001013
ITNO.	63	IN VAR-	65	VAR-	66	TO LOWER BOUND	-WMIN= -.000992
ITNO.	64	IN VAR-	55	VAR-	59	TO LOWER BOUND	-WMIN= -.000977
ITNO.	65	IN VAR-	71	VAR-	37	TO LOWER BOUND	-WMIN= -.000975
ITNO.	66	IN VAR-	46	VAR-	55	TO LOWER BOUND	-WMIN= -.000877
ITNO.	67	IN VAR-	75	VAR-	71	TO LOWER BOUND	-WMIN= -.000812
ITNO.	68	IN VAR-	69	VAR-	75	TO LOWER BOUND	-WMIN= -.000721
ITNO.	69	IN VAR-	66	VAR-	81	TO LOWER BOUND	-WMIN= -.000706
ITNO.	70	IN VAR-	60	VAR-	66	TO LOWER BOUND	-WMIN= -.000705
ITNO.	71	IN VAR-	71	VAR-	82	TO LOWER BOUND	-WMIN= -.000699
ITNO.	72	IN VAR-	70	VAR-	69	TO LOWER BOUND	-WMIN= -.000691
ITNO.	73	IN VAR-	52	VAR-	70	TO LOWER BOUND	-WMIN= -.000636
ITNO.	74	IN VAR-	55	VAR-	71	TO LOWER BOUND	-WMIN= -.000624
ITNO.	75	IN VAR-	66	VAR-	65	TO LOWER BOUND	-WMIN= -.000600
ITNO.	76	IN VAR-	56	VAR-	68	TO LOWER BOUND	-WMIN= -.000406
ITNO.	77	IN VAR-	70	VAR-	66	TO LOWER BOUND	-WMIN= -.000443
ITNO.	78	IN VAR-	31	VAR-	87	TO LOWER BOUND	-WMIN= 0.000000

ENTERING PHASE II

ITNO.	79	IN VAR-	66	VAR-	70	TO LOWER BOUND	-ZMIN= -298.326532
ITNO.	80	IN VAR-	26	VAR-	60	TO LOWER BOUND	-ZMIN= -281.582360
ITNO.	81	IN VAR-	25	VAR-	46	TO LOWER BOUND	-ZMIN= -247.772863
ITNO.	82	IN VAR-	71	VAR-	66	TO LOWER BOUND	-ZMIN= -247.727265
ITNO.	83	IN VAR-	27	VAR-	47	TO LOWER BOUND	-ZMIN= -242.085940
ITNO.	84	IN VAR-	36	VAR-	71	TO LOWER BOUND	-ZMIN= -241.764187
ITNO.	85	IN VAR-	73	VAR-	72	TO LOWER BOUND	-ZMIN= -241.616360
ITNO.	86	IN VAR-	32	VAR-	49	TO LOWER BOUND	-ZMIN= -228.031608
ITNO.	87	IN VAR-	72	VAR-	52	TO LOWER BOUND	-ZMIN= -224.518162
ITNO.	88	IN VAR-	43	VAR-	59	TO LOWER BOUND	-ZMIN= -217.842603
ITNO.	89	IN VAR-	33	VAR-	56	TO LOWER BOUND	-ZMIN= -216.599545
ITNO.	90	IN VAR-	38	VAR-	73	TO LOWER BOUND	-ZMIN= -216.[illegible]
ITNO.	91	IN VAR-	37	VAR-	58	TO LOWER BOUND	-ZMIN= -216.[illegible]
ITNO.	92	IN VAR-	49	VAR-	37	TO LOWER BOUND	-ZMIN= -216.180349

ITNO. 93 IN VAR= 36 VAR= 33 TO LOWER BOUND -ZMIN= -216.007277

****
* PROBLEM NUMBER 1 *
****

USING REVISED
INFLUENCE FUNCTION -- BORE28 -- 1956 TO 1975

SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	LOWER BOUND	UPPER BOUND
1	X1	--	B	.0015534	0.0000000	0.0000	INF
2	X2	--	B	.0014565	0.0000000	0.0000	INF
3	X3	--	B	.0013546	0.0000000	0.0000	INF
4	X4	--	B	.0012526	0.0000000	0.0000	INF
5	X5	--	B	.0011509	0.0000000	0.0000	INF
6	X6	--	B	.0010490	0.0000000	0.0000	INF
7	X7	--	B	.0009471	0.0000000	0.0000	INF
8	X8	--	B	.0008453	0.0000000	0.0000	INF
9	X9	--	B	.0007434	0.0000000	0.0000	INF
10	X10	--	B	.0006415	0.0000000	0.0000	INF
11	X11	--	B	.0005396	0.0000000	0.0000	INF
12	X12	--	B	.0004378	0.0000000	0.0000	INF
13	X13	--	B	.0003359	0.0000000	0.0000	INF
14	X14	--	B	.0002340	0.0000000	0.0000	INF
15	X15	--	B	.0001321	0.0000000	0.0000	INF
16	X16	--	B	.0001321	0.0000000	0.0000	INF
17	X17	--	B	.0001321	0.0000000	0.0000	INF
18	X18	--	B	.0001321	0.0000000	0.0000	INF
19	X19	--	B	.0001321	0.0000000	0.0000	INF
20	U1	--	LB	0.0000000	-2.0000000	0.0000	INF
21	U2	--	LB	0.0000000	-2.0000000	0.0000	INF
22	U3	--	LB	0.0000000	-2.0000000	0.0000	INF
23	U4	--	LB	0.0000000	-2.0000000	0.0000	INF
24	U5	--	LB	0.0000000	-2.0000000	0.0000	INF
25	U6	--	B	15.8021086	0.0000000	0.0000	INF
26	U7	--	B	22.8909735	0.0000000	0.0000	INF
27	U8	--	B	15.3320343	0.0000000	0.0000	INF
28	U9	--	B	10.4803422	0.0000000	0.0000	INF
29	U10	--	LB	0.0000000	-2.0000000	0.0000	INF
30	U11	--	LB	0.0000000	-2.0000000	0.0000	INF
31	U12	--	B	9.7945394	0.0000000	0.0000	INF
32	U13	--	B	4.1359250	0.0000000	0.0000	INF
33	U14	--	LB	0.0000000	-.1833732	0.0000	INF
34	U15	--	LB	0.0000000	-2.0000000	0.0000	INF
35	U16	--	LB	0.0000000	-2.0000000	0.0000	INF
36	U17	--	LB	0.0000000	-2.0000000	0.0000	INF
37	U18	--	LB	0.0000000	-.9341121	0.0000	INF
38	U19	--	B	.1620919	0.0000000	0.0000	INF
39	V1	--	B	15.9862000	0.0000000	0.0000	INF
40	V2	--	B	28.8280040	0.0000000	0.0000	INF
41	V3	--	B	37.6508044	0.0000000	0.0000	INF
42	V4	--	B	41.4257540	0.0000000	0.0000	INF
43	V5	--	B	3.9929934	0.0000000	0.0000	INF
44	V6	--	LB	0.0000000	-2.0000000	0.0000	INF
45	V7	--	LB	0.0000000	-2.0000000	0.0000	INF
46	V8	--	LB	0.0000000	-2.0000000	0.0000	INF

****
* PROBLEM NUMBER 1 *
****

USING REVISED
INFLUENCE FUNCTION -- BORE28 -- 1956 TO 1975

SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	LOWER BOUND	UPPER BOUND
47	V9	--	LB	0.0000000	-2.0000000	0.0000	INF
48	V10	--	B	2.7927569	0.0000000	0.0000	INF
49	V11	--	B	1.9074293	0.0000000	0.0000	INF
50	V12	--	LB	0.0000000	-2.0000000	0.0000	INF
51	V13	--	LB	0.0000000	-2.0000000	0.0000	INF
52	V14	--	LB	0.0000000	-1.3166268	0.0000	INF
53	V15	--	B	1.2759498	0.0000000	0.0000	INF
54	V16	--	B	2.0046895	0.0000000	0.0000	INF
55	V17	--	B	1.6040808	0.0000000	0.0000	INF

56	V18	--	LB	0.0000000	-1.0158879	0.0000	INF
57	V19	--	LB	0.0000000	-2.0000000	0.0000	INF
58	--SLACK	20	LB	0.0000000	141242.5142500	0.0000	INF
59	--SLACK	21	LB	0.0000000	26189.4837854	0.0000	INF
60	--SLACK	22	LB	0.0000000	111623.0296808	0.0000	INF
61	--SLACK	23	LB	0.0000000	232157.5100201	0.0000	INF
62	--SLACK	24	LB	0.0000000	338441.2328062	0.0000	INF
63	--SLACK	25	LB	0.0000000	391371.6071770	0.0000	INF
64	--SLACK	26	LB	0.0000000	337264.5890740	0.0000	INF
65	--SLACK	27	LB	0.0000000	227548.0067963	0.0000	INF
66	--SLACK	28	LB	0.0000000	158433.9406710	0.0000	INF
67	--SLACK	29	LB	0.0000000	139572.1615504	0.0000	INF
68	--SLACK	30	LB	0.0000000	160051.3101301	0.0000	INF
69	--SLACK	31	LB	0.0000000	152555.6783389	0.0000	INF
70	--SLACK	32	LB	0.0000000	98770.6893817	0.0000	INF
71	--SLACK	33	LB	0.0000000	31459.5911939	0.0000	INF
72	--SLACK	34	B	.0001019	0.0000000	0.0000	INF
73	--SLACK	35	LB	0.0000000	32857.3415054	0.0000	INF
74	--SLACK	36	LB	0.0000000	81456.9988181	0.0000	INF
75	--SLACK	37	LB	0.0000000	122642.5142501	0.0000	INF
76	--ARTIF D-	1	LB	0.0000000	-1.0000000	0.0000	INF
77	--ARTIF D-	2	LB	0.0000000	-1.0000000	0.0000	INF
78	--ARTIF D-	3	LB	0.0000000	-1.0000000	0.0000	INF
79	--ARTIF D-	4	LB	0.0000000	-1.0000000	0.0000	INF
80	--ARTIF D-	5	LB	0.0000000	-1.0000000	0.0000	INF
81	--ARTIF D-	6	LB	0.0000000	1.0000000	0.0000	INF
82	--ARTIF D-	7	LB	0.0000000	1.0000000	0.0000	INF
83	--ARTIF D-	8	LB	0.0000000	1.0000000	0.0000	INF
84	--ARTIF D-	9	LB	0.0000000	1.0000000	0.0000	INF
85	--ARTIF D-	10	LB	0.0000000	-1.0000000	0.0000	INF
86	--ARTIF D-	11	LB	0.0000000	-1.0000000	0.0000	INF
87	--ARTIF D-	12	LB	0.0000000	1.0000000	0.0000	INF
88	--ARTIF D-	13	LB	0.0000000	1.0000000	0.0000	INF
89	--ARTIF D-	14	LB	0.0000000	.8166268	0.0000	INF
90	--ARTIF D-	15	LB	0.0000000	-1.0000000	0.0000	INF
91	--ARTIF D-	16	LB	0.0000000	-1.0000000	0.0000	INF
92	--ARTIF D-	17	LB	0.0000000	-1.0000000	0.0000	INF

****
* PROBLEM NUMBER 1 *
****

USING REVISED
INFLUENCE FUNCTION -- 309528 -- 1956 TO 1975

SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	LOWER BOUND	UPPER BOUND
93	--ARTIF D-	18	LB	0.0000000	.0158879	0.0000	INF
94	--ARTIF D-	19	LB	0.0000000	1.0000000	0.0000	INF
95	--ARTIF D-	20	LB	0.0000000	141242.5142500	0.0000	INF
96	--ARTIF D-	21	LB	0.0000000	26189.4837854	0.0000	INF
97	--ARTIF D-	22	LB	0.0000000	111623.0296808	0.0000	INF
98	--ARTIF D-	23	LB	0.0000000	232157.5100201	0.0000	INF
99	--ARTIF D-	24	LB	0.0000000	338441.2328062	0.0000	INF
100	--ARTIF D-	25	LB	0.0000000	391371.6071770	0.0000	INF
101	--ARTIF D-	26	LB	0.0000000	337264.5890740	0.0000	INF
102	--ARTIF D-	27	LB	0.0000000	227548.0067963	0.0000	INF
103	--ARTIF D-	28	LB	0.0000000	158433.9406710	0.0000	INF
104	--ARTIF D-	29	LB	0.0000000	139572.1615504	0.0000	INF
105	--ARTIF D-	30	LB	0.0000000	160051.3101301	0.0000	INF
106	--ARTIF D-	31	LB	0.0000000	152555.6783389	0.0000	INF
107	--ARTIF D-	32	LB	0.0000000	98770.6893817	0.0000	INF
108	--ARTIF D-	33	LB	0.0000000	31459.5911939	0.0000	INF
109	--ARTIF D-	34	LB	0.0000000	0.0000000	0.0000	INF
110	--ARTIF D-	35	LB	0.0000000	32857.3415054	0.0000	INF
111	--ARTIF D-	36	LB	0.0000000	81456.9988181	0.0000	INF
112	--ARTIF D-	37	LB	0.0000000	122642.5142501	0.0000	INF

MINIMUM VALUE OF THE OBJECTIVE FUNCTION = 216.067277

CALCULATION TIME WAS 4.1520 SECONDS FOR 93 ITERATIONS.

DATA STORAGE MEMORY = 004564(OCTAL) TOTAL MEMORY = 045000(OCTAL)

2TOTAL TIME FOR THIS PROBL WAS 5.667 SECONDS

Linearized Free Surface Programs

```
*LFS
      PROGRAM LFS(INPUT=65/150,OUTPUT=65/150,LFSDUM,TAPE5=INPUT,
     *   TAPE6=OUTPUT,TAPE7,TAPE8=LFSDUM)
C
C   ---------------------------------------------------------------
C
C   FIELD DRAWDOWN AND PRESSURE WITH A LINEARIZED FREE SURFACE CONDITION
C
C   ---------------------------------------------------------------
C
C   PROGRAM BY OREGON SYSTEMS ANALYSTS
C   6660 RESERVOIR ROAD
C   CORVALLIS, OREGON 97330
C   PHONE: 503/929-5622
C
C   COPYRIGHT ELLIOT ZAIS AND ASSOCIATES, 1980.
C
C   ---------------------------------------------------------------
C   THIS PROGRAM CALCULATES PRESSURE AT A FIELD POINT USING A DESCRETIZED
C   ESTIMATE OF THE GREENS FUNCTION.    THE FIELD POINT LIES ON THE
C   STATIC EQUILIBRIUM FREE SURFACE AT SOME RADIAL DISTANCE FROM
C   THE BOREHOLE.
C
C   THE PROGRAM READS IN ONE DATA SET AT A TIME AND WILL CALCULATE THE
C   FIELD PRESSURE FOR UP TO SIX RADIAL DISTANCES FROM THE BORE.
C   DATA SETS MAY BE SEPARATED BY AN E-O-R BUT NOT BY AN E-O-F.
C   A SINGLE DATA SET CONSISTS OF CONTROL CARDS, PARAMETERS, AND
C   TIME SERIES DATA FOR FLOW OF THE
C   REAL OR VIRTUAL WELL BEING ANALYZED.  THIS TIME-FLOW
C   DATA MAY BE READ EITHER IN FREE-FORMAT FROM THE SAME INPUT STREAM
C   AS THE CONTROL CARDS, OR IT MAY BE READ FROM A DESIGNATED
C   PERMANENT INDIRECT ACCESS FILE.
C
C
C   A SINGLE DATA SET CONSISTS OF CONTROL CARDS AND DATA AS DESCRIBED
C   BELOW.  ORDER OF INPUT IS UNIMPORTANT EXCEPT THAT THE <BEGIN>
C   CARD MUST COME FIRST, THE <END> CARD MUST COME LAST.
C
C     1)   A CARD WITH THE WORD <BEGIN> IN COLS 1-5.
C             FOLLOWED BY A TITLE CARD OF UP TO 80 CHARACTERS
C
C     2)   A CARD WITH THE WORD <PARAMETER> IN COLS 1-10 WITH THE
C             FOLLOWING PARAMETERS ENTERED IN FREE FORMAT (BLANK SEPARATED)
C             AND IN ORDER.
C             ONE OR MORE LINES (CARDS) MAY BE USED.
C
C         A)  THE TIME INCREMENT IN SECONDS (DTEE)
C         B)  THE WELL DEPTH IN METERS (D)
C         C)  THE POROSITY (PHI) WHICH IS DIMENSIONLESS
C         D)  THE DENSITY (RHO) IN KILOGRAMS/CUBIC METER
C         E)  THE PERMIABILITY (K) IN SQUARE METERS
C         F)  THE DYNAMIC VISCOSITY (MU) IN KILOGRAMS/METER-SECONDS
C         G)  THE P.Z. THICKNESS (H) IN METERS
C         H)  THE NUMBER OF RADII TO BE USED IN THE CALCULATIONS (MAXIMUM OF EIGHT)
C         I)  THE VALUES OF THE RADII (IN METERS)
C
C    **** NOTE ****
C             THE P.Z. THICKNESS, ITEM G, IS USED IN THE SECOND TERM OF THE EXPANSIOI
C             OF THE GREENS FUNCTIONS.  IF <H>=0, THE SECOND TERM IS NOT
C             INCLUDED IN THE GREENS FUNCTION CALCULATION.
C
C     3)  A <DATA> CARD WHICH SPECIFIES LOCATION OF THE TIME AND FLOW DATA
C             THERE ARE TWO FORMS OF THE DATA CARD DEPENDING ON HOW THE INPUT
C             IS TO BE READ.
C
C         A)  FREE FORMAT -- FIRST READ IN A <DATA N> CARD
C             WITH <N> EQUAL TO THE NUMBER OF DATA POINTS, E.G. <DATA 27>.
C             ON FOLLOWING LINE OR LINES ENTER <N> DATA POINTS FOR TIME,
C             FOLLOWED BY <N> DATA POINTS FOR FLOW.
C
C         B)  STORED ON FILE -- READ IN A <DATA N FILE FORM (S)> CARD WHERE
C             <FILE> IS THE FILE NAME, <FORM> IS THE INPUT FORMAT,
C             <S> IS AND AN OPTIONAL INTEGER FROM 1 TO 9 WHICH TELLS THE
C             PROGRAM TO SKIP THAT MANY CARDS BEFORE READING THE DATA.
C             THE FILE MUST BE AN INDIRECT ACCESS PERMANENT FILE.
C             ANY LOCAL FILE WITH THE SAME NAME WILL BE IRRETRIEVABLY LOST.
C             DATA IN THIS CASE IS ORGANIZED IN COLUMNS AS TIME-FLOW
C             AND MUST BE READ IN INTEGER-REAL FORMAT.  AN EXAMPLE IS:
C
C                  <DATA 27 LFSDAT (I10,15X,F15.5) 3 >
C
C             WHICH MEANS READ 27 OBSERVATIONS OFF THE PERMANENT FILE NAMED
C             NAMED <LFSDAT> USING THE SPECIFIED FORMAT AND SKIPPING THE
C             FIRST 3 LINES OF THE FILE./
```

```
C
C     4)  AN OPTIONAL CARD WHICH HAS THE WORD <GREEN> IN COLUMNS 1-5.
C             THIS WILL PRINT THE VALUES OF THE GREENS FUNCTION FOR
C             VALUES OF REAL TIME AND ALL RADII.
C
C     5)  AN OPTIONAL <LINES N> CARD WHERE N IS THE MAXIMUM NUMBER OF OUTPUT
C             LINES ON A PAGE.  THE DEFAULT IS 66 LINES.  IF <N> IS NOT SPECIFIED,
C             THE DEFAULT FOR <LINES> IS N=51.  N MUST HAVE THE RANGE 10-99.
C             THIS CARD NEED ONLY BE ENTERED ONCE PER RUN.
C
C     6)  AN OPTIONAL <MULT> CARD WHICH INDICATES THE PROGRAM IS TO
C               START SUMMING PRESSURE DROPS FOR A CENTRAL POINT WHICH
C               IS AT A DISTANCE EQUAL TO THE FIRST SPECIFIED RADIUS FOR
C               ALL WELLS AT EACH TIME PERIOD.  THIS SUMMATION WILL CONTINUE
C               UNTIL A <MULTEND> CARD IS FOUND IN A DATA SET OR
C               UNTIL THE END-OF-FILE IS REACHED.  ADDITIONAL <MULT> CARDS
C               ENCOUNTERED BETWEEN THE FIRST ONE AND THE <MULTEND> OR
C               E-O-F ARE EXTRANEOUS AND ARE IGNORED.
C
C     7)  AN OPTIONAL <CONVERT CODE> CARD WHICH TELLS THE PROGRAM
C               TO CONVERT THE FLOW DATA FROM THE UNITS GIVEN TO KG/SEC.
C               VALID ARGUMENTS FOR <CODE> ARE:
C
C                 CODE:             CONVERSION FROM:
C
C                   A                  TON/HOUR
C                   B                  10E6 LBS/YR
C                   C                  TONNE/HOUR
C                   D                  LBS/YR
C                   E                  LBS/HOUR
C                   F                  LBS/DAY
C                   G                  1000 LBS/MONTH
C                   Z                  EXTERNALLY SUPPLIED MULTIPLICATIVE
C                                      FACTOR GIVEN AS REAL NUMBER
C                                      FOLLOWING CODE LETTER <Z>.
C
C     8)        AN OPTIONAL <FIELD P> CARD WHICH SPECIFIES AN INITIAL
C               PRESSURE P IN POUNDS-PER-SQUARE-INCH (PSI).  IF THIS CARD
C               IS SUPPLIED THEN A TABLE GIVING FIELD PRESSURE OVER TIME
C               IS PRODUCED FROM THE CALCULATED DRAWDOWNS.  IF <P> IS
C               .LE. ZERO AN ERROR MESSAGE IS GIVEN AND THE
C               OPTION IS SKIPPED.
C
C     9)  AN END CARD WHICH HAS THE WORD <END> IN COLUMNS 1-3.
C
C
C   -------------------------------------------------------------------
C
C     INPUT AND OUTPUT LOGICAL UNITS:
C
C        UNIT 5:  DATA SET AND CONTROL CARD INPUT
C        UNIT 6:  OUTPUT
C        UNIT 7:  OPTIONAL UNIT ASSIGNED TO FILE SPECIFIED IN <FILE> CARD
C        UNIT 8:  TEMPORARY FILE <LFSDUM> ASSIGNED TO STORE HEADERS
C
C   -------------------------------------------------------------------
C
      REAL K, MU, CENTER(100), FIELD(8,100), FPHEAD(7), CONV(10)
      REAL FP(100), Q(100), GREENS(8,100), R(8), RSQ(8), PARAMS(16)
      INTEGER NAMES(2,10), ITEST(8), JCONV(2)
      INTEGER BUFF(8), IFORM(8), HDR(8), NTIME(100), FPDHED(7), GHEAD(7)
      INTEGER DASH(14), CHEAD(7), HBUF(13)
      COMMON /FPAR/ DTEE, D, PHI, RHO, K, MU, H, M, R
      EQUIVALENCE(DTEE,PARAMS)
      DATA ICONV/10HZABCDEFGHI/
      DATA CARDS/10HBEPDGLMCF /, IACTIV/0/
      DATA NAMES/≠EXTERNAL≠,≠ ≠,≠TONS/HOURS≠,≠ ≠,≠10E6 POUND≠,≠S/YEAR≠,
     * ≠TONNES/HOU≠,≠R ≠,≠POUNDS/YEA≠,≠R ≠,
     * ≠POUNDS/HOU≠,≠R≠,≠POUNDS/DAY≠,≠  ≠,≠1000 POUND≠,≠S/MONTH≠/
      DATA CONV/0.0,.2520669,.014377929,.2777777,1.4377928E-8,
     * 1.2603347E-4,5.2513947E-6,1.7274324E-04/
      DATA GRAVITY/9.801018/, TWOPI/6.283183/, DASH/14*10H----------/
      DATA LINES/66/, BLANK/10H          /, DOT/1H./, CF1/1.1560122/
      DATA FPHEAD
     */≠ FIELD PRESSURE AT SPECIFIED DISTANCE FROM BOREHOLE           ≠/
      DATA FPDHED
     */≠ FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE           ≠/
      DATA GHEAD
     */≠ VALUES OF GREENS FUNCTION AT SPECIFIED DISTANCE FROM BOREHOLE≠/
      DATA CHEAD
     */≠ FIELD DRAWDOWN AT CENTRAL POINT IN METERS OF HEAD            ≠/
      DATA FLO/≠FLOW≠/, NOLD/0/, IBAD/0/
      DATA TPH/≠  TPRIME≠/, IMULT/0/, CENTER/100*0.0/, NMULT/0/
      GFUNC(A,B,ASQ) = (A/( (B+ASQ) **1.5 ))
```

```
      REWIND 8
      CPU = SECOND(DUM) - .025
C ---------------------------------------------------------------------
C   READ A CONTROL CARD, DETERMINE WHICH TYPE IT IS AND GO TO PROPER SECTION
C ---------------------------------------------------------------------
    1 READ(5,101) ITEST
      IF(EOF(5).EQ.1) GO TO 990
      CALL FINDC(CARDS,9,ITEST,1,1,IFIND,ICF,IRET)
CD    PRINT *,≠IACTIV,IRET,IFIND,ICF: ≠,IACTIV,IRET,IFIND,ICF
CD    WRITE(6,101)ITEST
      IF(IRET.NE.1) GO TO 1
      IF(IACTIV.EQ.0.AND.ICF.GT.2) GO TO 1
      IBEG = LOCC(BLANK,ITEST,1,80)
      GO TO (801,802,803,804,805,806,807,808,809), ICF
C              B   E   P   D   G   L   M   C   F
C ---------------------------------------------------------------------
C   CHECK FOR INACTIVE STATUS, THEN
C   READ THE HEADER CARD WHICH CONTAINS AN IDENTIFIER FOR THIS DATA SET
C ---------------------------------------------------------------------
  801 IF(IACTIV.EQ.1) GO TO 997
      IACTIV = 1
      READ(5,101) HDR
  101 FORMAT(8A10)
      IF(EOF(5).EQ.1)  GO TO 995
      WRITE(6,138)
  138 FORMAT(1H1)
      CALL TIME(TIM)
      CALL DATE(DAT)
      PZERO = 0.0
      FACTOR = 0.0
      IGREEN=0
      ISIG = 0
      GO TO 1
C
C ---------------------------------------------------------------------
C   READ PARAMETERS IN FREE FORMAT
C ---------------------------------------------------------------------
  803 IEND=16
      KK=0
    3 KK=KK+1
      IF(KK.GT.IEND) GO TO 1
    4 PARAMS(KK) = XPAR(1,ITEST,IBEG,80,IRET)
CD    PRINT *,≠IBEG,IRET,KK,PARAMS(KK): ≠,IBEG,IRET,KK,PARAMS(KK)
      IBEG = IRET
      IF(IRET.GT.0) GO TO 5
C   READ ANOTHER CARD
      IBEG = 1
      READ(5,101) ITEST
      IF(EOF(5).EQ.1) GO TO 983
      GO TO 4
    5 IF(KK.NE.8) GO TO 3
C   ADJUST LOOP TERMINUS IF LESS THAN 8 RADIUS SPECIFICATIONS
      M = PARAMS(8)
      IX = M
      IF(M.GT.8)  M=8
      IEND = 8+M
      GO TO 3
C ---------------------------------------------------------------------
C   READ TIME AND FLOW DATA
C   IF <FILE> OPTION, READ FROM FILE, ELSE READ FROM <INPUT>
C ---------------------------------------------------------------------
  804 N = XPAR(2,ITEST,1,80,IRET)
      NN=N
      IF(N.LE.0.OR.N.GT.100)NN=0
      IF(IRET.LE.0) GO TO 996
      CALL FNDWRD(3,ITEST,1,80,NAME,LEN,IRET)
      IF(IRET.EQ.0) GO TO 10
C ---------------------------------------------------------------------
C   DATA IS LOCATED ON A SEPARATE FILE.
C   THE FORMAT IS INTEGER-REAL FOR TIME-FLOW COLUMNS
C   GET FORMAT FROM THE <DATA> CARD
C   CLOSE UNIT 7, ASSIGN FILE TO UNIT 7 AND READ DATA.
C ---------------------------------------------------------------------
      CALL FNDWRD(4,ITEST,1,80,IFORM,LEN,IRET)
      LENF = (LEN-1)/10 + 1
      IF(IRET.LE.0) GO TO 996
      NSKIP = XPAR(5,ITEST,1,80,IRET)
C
      ISIG=1
      CALL FCLOSE(7)
      CALL UGET(7,NAME)
C
      IF(NSKIP.LE.0) GO TO 16
      DO 15 I=1,NSKIP
```

```
      READ(7,101) DUMMY
   15 CONTINUE
   16 CONTINUE
C
      READ (7,IFORM) (NTIME(I), Q(I), I=1,N)
      IF(EOF(7).EQ.1) GO TO 992
      GO TO 1
C -----------------------------------------------------------------------
C   DATA IS LOCATED ON THE INPUT UNIT.
C   READ, IN FREE FORMAT, ALL DATA FOR TIME, FOLLOWED BY ALL DATA
C   FOR FLOW.
C -----------------------------------------------------------------------
   10 READ(5,*) (NTIME(I),I=1,N), (Q(I),I=1,N)
      IF(EOF(5).EQ.1) GO TO 995
      GO TO 1
C -----------------------------------------------------------------------
C   CHECK FOR FOLLOWING OPTIONAL CARDS:
C -----------------------------------------------------------------------
C
C -----------------------------------------------------------------------
C   <GREEN> CARD
C -----------------------------------------------------------------------
  805 IGREEN=1
      GO TO 1
C -----------------------------------------------------------------------
C   <LINES> CARD
C -----------------------------------------------------------------------
  806 LINES = XPAR(2,ITEST,1,80,IRET)
      IF(IRET.LE.0) LINES=51
      GO TO 1
C
C -----------------------------------------------------------------------
C   <MULT> AND <MULTEND> CARD
C -----------------------------------------------------------------------
  807 IMULT=1
      IF(ICOMPAR(7HMULTEND,1,ITEST,1,7).EQ.0) IMULT=2
      GO TO 1
C -----------------------------------------------------------------------
C   <CONVERT> CARD
C -----------------------------------------------------------------------
  808 CALL FNDWRD(2,ITEST,1,20,JCONV,LEN,IBEG)
      IF(IBEG.EQ.0.OR.LEN.GT.1) GO TO 998
      CALL FINDC(ICONV,10,JCONV,1,1,IFIND,NAMEF,IRET)
      IF(IRET.GT.1) GO TO 998
      FACTOR = CONV(NAMEF)
      IF(NAMEF.GT.1) GO TO 37
      FACTOR = XPAR(3,ITEST,1,80,IRET)
      IF(IRET.LE.0) GO TO 998
   37 DO 36 I=1,N
   36 Q(I) = Q(I) * FACTOR
      GO TO 1
C
C -----------------------------------------------------------------------
C   <FIELD> CARD
C -----------------------------------------------------------------------
  809 PZERO = XPAR(2,ITEST,1,80,IRET)
      IF(IRET.LE.0) GO TO 980
      IF(PZERO.LE.0) GO TO 980
      GO TO 1
C
C -----------------------------------------------------------------------
C   MANDATORY <END> CARD.  START ANALYSIS.
C -----------------------------------------------------------------------
  802 IF(IACTIV.EQ.0) GO TO 997
      IF(IACTIV.LT.0) GO TO 1
      IACTIV = 0
C -----------------------------------------------------------------------
C   CALCULATE SQUARE OF RADIUS
C -----------------------------------------------------------------------
      IF(NN.EQ.0) GO TO 390
      DO 22 IRAD=1,M
   22 RSQ(IRAD) = R(IRAD) * R(IRAD)
C -----------------------------------------------------------------------
C   CALCULATE SINKING VELOCITY, W.
C -----------------------------------------------------------------------
      IF(MU.EQ.0.OR.PHI.EQ.0) GO TO 994
      W = (K * GRAVITY * RHO) / (MU * PHI)
C -----------------------------------------------------------------------
C   CALCULATE GREENS FUNCTION FOR VALUES OF REAL TIME T AND ALL RADII.
C -----------------------------------------------------------------------
      IF(RHO.EQ.0) GO TO 994
      G1 = -1. / (TWOPI * PHI * RHO)
C ***********************************************************************
C   LOOP OVER ALL TIME PERIODS
```

```
C
      DO 39 I=1,N
      TPRIME = I - .5
      TAU = TPRIME * DTEE
      G2 = W * TAU + D
      G2SQ = G2 * G2
      IF(H.LE.0) GO TO 380
      G3 = W * TAU + 2*H - D
      G3SQ = G3 * G3
C >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C   INTERNAL LOOP OVER RADII
C
C   <GFUNC> IS A STATEMENT FUNCTION
C     GFUNC(A,B,ASQ) = (A/( (B+ASQ) **1.5 ))
C
  380 DO 38 IRAD = 1,M
      GRN = GFUNC(G2,RSQ(IRAD),G2SQ)
      IF(H.GT.0.0) GRN = GRN + GFUNC(G3,RSQ(IRAD),G3SQ)
      GREENS(IRAD,I) = G1 * GRN
   38    CONTINUE
C >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C
   39 CONTINUE
C ******************************************************************
C
C
C --------------------------------------------------------------------
C   PRINT INITIAL TIME AND HEADER
C --------------------------------------------------------------------
  390 CALL PEJECT(0)
      NEED=27
      WRITE(6,102) DASH(1), DASH(1), HDR
      WRITE(6,122) DAT,TIM
  102 FORMAT
     *(1X,A10,≠ FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE≠,
     *  ≠ CONDITION ≠,A10//1X,8A10)
      IF(ISIG.EQ.1) WRITE(6,104) NAME, (IFORM(I),I=1,LENF)
  104 FORMAT(//≠ TIME AND FLOW DATA READ FROM FILE ≠≠≠,
     *   A7,≠≠≠ WITH FORMAT:≠,2A10)
      IF(NSKIP.GT.0) PRINT *,≠FIRST ≠,NSKIP,≠ CARDS WERE SKIPPED≠
      IF(ISIG.EQ.0) WRITE(6,105)
  105 FORMAT(//≠ TIME AND FLOW DATA READ FROM INPUT STREAM≠)
      NAM = LOADCH(ICONV,NAMEF)
      IF(FACTOR.NE.0) WRITE(6,130) NAM, (NAMES(I,NAMEF),I=1,2)
     *    ,FACTOR
  130 FORMAT(//≠ FLOW DATA WERE CONVERTED FROM TYPE ≠,R1,2H (,2A10,1H),
     *   ≠ TO KG/SEC BY THE MULTIPLICATIVE FACTOR ≠,1PG20.8)
      IF(PZERO.GT.0.0) WRITE(6,131) PZERO
  131 FORMAT
     * (//≠ FIELD PRESSURE TABLE IS BASED ON AN INITIAL PRESSURE AT≠,
     * ≠ BOREHOLE AT TIME ZERO OF≠,F12.2, ≠ PSI≠)
      WRITE(6,103) N,DTEE,D,PHI,RHO,K,MU,H,W,IX,(R(I),I=1,M)
  103 FORMAT(//≠ INPUT PARAMETERS:≠/ 1P,
     *   /T5,≠NUMBER OF DATA POINTS (N):≠, T31, I10
     *   /T5,≠DELTA-TAU:≠, T30, E18.3,T50,≠SECONDS≠
     *   /T5,≠FLUID LEVEL (D):≠, T30, E18.3,T50,≠METERS≠
     *   /T5,≠POROSITY (PHI):≠, T30, E18.3
     *   /T5,≠DENSITY (RHO):≠, T30, E18.3,T50,≠KG/CU. METERS≠
     *   /T5,≠PERMEABILITY (K):≠, T30, E18.3,T50,≠SQ. METERS≠
     *   /T5,≠DYNAMIC VISCOSITY (MU):≠,T30,E18.3,T50,≠KG/METER-SECOND≠
     *   /T5,≠P.Z. THICKNESS (H):≠,T30, E18.3,T50,≠METERS≠
     *   /T5,≠SINKING VELOCITY (W):≠,T30,E18.3,T50,≠METER/SECOND≠
     *   /T4,I2,0P,≠ RADIUS SPECIFICATIONS:≠,2(T33,10F10.2))
      IF(IX.GT.8) PRINT *,
     *≠***** ONLY 8 RADIUS SPECIFICATIONS ALLOWED..  EXCESS IGNORED≠
      IF(NN.EQ.0) GO TO 985
C
C --------------------------------------------------------------------
C   WRITE GREENS FUNCTION RESULTS
C --------------------------------------------------------------------
      IF(IGREEN.EQ.0) GO TO 41
      WRITE(6,116)
      NEED = NEED + N + 7
      CALL PEJECT(NEED,LINES,6)
      WRITE(6,118) HDR
      WRITE(6,112) DASH
      WRITE(6,106) TPH,GHEAD
      WRITE(6,112) DASH
      WRITE(6,113) (R(IRAD),IRAD=1,M)
      WRITE(6,112) DASH
  116 FORMAT(/)
  118 FORMAT(1X,8A10)
C
      DO 42 I=1,N
      TPRIME = I - .5
```

```
      WRITE(6,115) NTIME(I), TPRIME, (GREENS(IRAD,I),IRAD=1,M)
  115 FORMAT(I10,F13.1,8(P1E13.3))
   42 CONTINUE
      WRITE(6,112) DASH
      WRITE(6,122) DAT,TIM
C
C --------------------------------------------------------------------
C   WRITE HEADER FOR TIME SERIES DATA
C --------------------------------------------------------------------
   41 WRITE(6,116)
      NEED = NEED + N + 11
      CALL PEJECT(NEED,LINES,6)
      WRITE(6,118) HDR
      WRITE(6,112) DASH
      WRITE(6,106) FLO,FPDHED
  106 FORMAT(T7,≠TIME≠,T19,A10,T35,7A10)
      WRITE(6,111)
      WRITE(6,112) DASH
      WRITE(6,113) (R(IRAD),IRAD=1,M)
  111 FORMAT(T17,≠(KG/SEC)≠,T50,≠(METERS OF HEAD)≠)
  112 FORMAT(1X,13A10,A1)
  113 FORMAT(≠ METERS:≠,T24,8F13.2)
      WRITE(6,112) DASH
C --------------------------------------------------------------------
C   CALCULATE TIME SERIES FIELD PRESSURE FOR ALL RADII
C --------------------------------------------------------------------
C ********************************************************************
C     LOOP OVER TIME PERIOD
C
      DO 45 I=1,N
      DO 40 IRAD=1,M
   40 FP(IRAD) = 0.0
C ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C        LOOP OVER GREENS FUNCTION VALUES UP TO CURRENT TIME PERIOD
C
         DO 44 J=1,I
         KK = I-J +1
C
C >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C             LOOP OVER RADII
C
              DO 43 IRAD=1,M
              FP(IRAD) = FP(IRAD) - GREENS(IRAD,KK) * Q(J) * DTEE
   43         CONTINUE
C >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C
   44    CONTINUE
C ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C
C --------------------------------------------------------------------
C   PRINT RESULTS FOR THIS TIME PERIOD
C --------------------------------------------------------------------
      WRITE(6,107) NTIME(I), Q(I), (FP(IRAD),IRAD=1,M)
      IF(IMULT.GT.0) CENTER(I) = CENTER(I) + FP(1)
  107 FORMAT(I10,9F13.3)
C
C   CALCULATE ACTUAL FIELD PRESSURE BASED ON DRAWDOWNS
      IF(PZERO.LE.0.0) GO TO 45
      DO 47 IRAD=1,M
   47 FIELD(IRAD,I) = PZERO - FP(IRAD) * CF1
   45 CONTINUE
      WRITE(6,112) DASH
      WRITE(6,122) DAT,TIM
C ********************************************************************
C
C --------------------------------------------------------------------
C   PRINT RESULTS FOR ACTUAL FIELD PRESSURE
C --------------------------------------------------------------------
      IF(PZERO.LE.0.0) GO TO 46
      WRITE(6,116)
      NEED = NEED + N + 11
      CALL PEJECT(NEED,LINES,6)
      WRITE(6,118) HDR
      WRITE(6,112) DASH
      WRITE(6,106) FLO,FPHEAD
      WRITE(6,136)
  136 FORMAT(T17,≠(KG/SEC)≠,T50,≠(POUNDS PER SQUARE INCH)≠)
      WRITE(6,112) DASH
      WRITE(6,113) (R(IRAD),IRAD=1,M)
      WRITE(6,112) DASH
      DO 60 I=1,N
      WRITE(6,107) NTIME(I),Q(I),(FIELD(IRAD,I),IRAD=1,M)
   60 CONTINUE
      WRITE(6,112) DASH
```

```
      WRITE(6,122) DAT, TIM
C
C ---------------------------------------------------------------------
C   PRINT RESULTS OF CENTRAL POINT DRAWDOWN IF IMULT > 1.
C       IMULT = 0 IMPLIES NO SUMMING GOING ON
C       IMULT = 1 IMPLIES SUMMATION OCCURRING
C       IMULT = 2 IMPLIES FINISH SUMMATION, PRINT RESULTS, AND RESET COUNTERS
C       IMULT = 3 IMPLIES E-O-F ENCOUNTERED.  PRINT RESULTS AND END.
C ---------------------------------------------------------------------
   46 IF(IMULT.EQ.0) GO TO 50
      NMULT = NMULT + 1
      WRITE(8,119) HOR, R(1)
  119 FORMAT(1X,8A10,1X,≠DISTANCE TO CENTRAL POINT =≠,F13.2)
      IF(NOLD.NE.N.AND.NMULT.GT.1) IBAD=1
      NOLD=N
      IF(IMULT.EQ.1) GO TO 50
   51 REWIND 8
      WRITE(6,116)
      NEED = NEED + N + NMULT + 8
      CALL PEJECT(NEED,LINES,6)
      IF(IBAD.EQ.1) PRINT *,
     * ≠***** ERROR: DATA SETS HAVE DIFFERENT NUMBER OF TIME STEPS≠
      IBAD=0
      DO 48 I=1,NMULT
      READ(8,120) HBUF
      WRITE(6,121) HBUF
  120 FORMAT(13A10)
  121 FORMAT(4X,13A10)
   48 CONTINUE
      WRITE(6,112) DASH
      WRITE(6,139) CHEAD
  139 FORMAT(T7,≠ TIME≠,T21,7A10)
      WRITE(6,112) DASH
      WRITE(6,123) (NTIME(I),CENTER(I),I=1,N)
  123 FORMAT(I10,10X,F13.3)
      WRITE(6,112) DASH
      WRITE(6,122) DAT,TIM
  122 FORMAT(≠ REFERENCE DATE AND TIME:≠,A10,3X,A10)
      IF(IMULT.EQ.3) GO  TO 991
      DO 49 KK=1,N
   49 CENTER(KK) = 0.0
      IMULT=0
      NMULT=0
      REWIND 8
C
C ---------------------------------------------------------------------
C   RETURN TO BEGINNING AND SEE IF ANOTHER DATA SET IS AVAILABLE
C ---------------------------------------------------------------------
   50 GO TO 1
C
C ---------------------------------------------------------------------
C   NORMAL EXIT
C ---------------------------------------------------------------------
  990 IF(IMULT.EQ.0) GO TO 991
      IMULT=3
      GO TO 51
  991 CPU = -CPU+SECOND(DUM)
      WRITE(6,109) CPU
  109 FORMAT(/////≠ L.F.S. PROGRAM UTILIZED≠,F6.3,≠ SECONDS CPU TIME≠)
      STOP ≠END LFS≠
C ---------------------------------------------------------------------
C   ERROR EXITS
C ---------------------------------------------------------------------
  992 PRINT *,≠***** PREMATURE END-OF-FILE ENCOUNTERED ON DATA FILE≠
      GO TO 1000
  993 PRINT *, ≠***** ≠≠END≠≠ CARD NOT FOUND.  RUN ABORTED≠
      GO TO 991
  994 PRINT *,≠***** VALUE FOR RHO, MU, OR PHI IS EQUAL TO ZERO.≠
      GO TO 1000
  995 PRINT *,≠***** PREMATURE END-OF-FILE ENCOUNTERED ON INPUT UNIT≠
      GO TO 999
  996 PRINT *, ≠***** ERROR IN ≠≠DATA≠≠ CARD.  ≠
      GO TO 1000
  997 PRINT *, ≠***** ERROR.  ≠≠BEGIN≠≠ AND ≠≠END≠≠ CARDS OUT OF ORDER≠
      GO TO 999
  998 PRINT *,≠***** ERROR IN ≠≠CONVERT≠≠ CARD.  ≠
      GO TO 1000
  980 PRINT *,≠***** INITIAL PRESSURE SPECIFICATION IS ABSENT, ZERO OR≠
      PRINT *,≠      AN INTEGER.  FIELD PRESSURES CANNOT BE CALCULATED.≠
      PZERO = 0.0
      GO TO 1
  981 PRINT *,≠***** ERROR IN ≠≠CONVERT≠≠ CARD.  NO DECIMAL POINT IN≠
      PRINT *,≠      FACTOR SPECIFICATION.  ≠
      GO TO 1000
```

```
 982 PRINT *,≠***** NO ≠≠BEGIN≠≠ CARD, OR OUT OF PLACE.≠
     GO TO 999
 983 PRINT *,≠***** ERROR IN PARAMETER LIST.≠
     GO TO 995
 984 PRINT *,≠***** ERROR IN CONTROL CARDS.  ≠
     GO TO 1000
 985 PRINT *,≠***** NUMBER OF DATA POINTS EXCEEDS MAXIMUM OF 100≠
     GO TO 1000
 999 PRINT *,≠***** RUN ABORTED≠
     GO TO 991
1000 PRINT *,≠***** ANALYSIS ON THIS DATA SET ABORTED.≠
     IACTIV=-1
     GO TO 1
     END
```

---------- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION ----------

L.F.S. FOR WAIRAKEI GROUP 1 , 1953 TO 1976
~~REFERENCE DATE AND TIME: 80/05/11. 17.16.36.~~

TIME AND FLOW DATA READ FROM FILE ≠WLFSG1 ≠ WITH FORMAT:(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 3 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24	
DELTA-TAU:	3.156E+07	SECONDS
FLUID LEVEL (D):	1.610E+02	METERS
POROSITY (PHI):	4.000E-01	
DENSITY (RHO):	8.140E+02	KG/CU. METERS
~~PERMEABILITY (K):~~	~~3.000E-14~~	~~SQ. METERS~~
DYNAMIC VISCOSITY (MU):	1.099E-04	KG/METER-SECOND
P.Z. THICKNESS (H):	9.200E+02	METERS
SINKING VELOCITY (W):	5.445E-06	METER/SECOND
7 RADIUS SPECIFICATIONS:	990.00 0.00 941.00 1692.00 1540.00 1588.00 1312.00	

L.F.S. FOR WAIRAKEI GROUP 1 , 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
METERS:		990.00	0.00	941.00	1692.00	1540.00	1588.00	1312.00
1953	36.779	.253	9.488	.277	.097	.115	.109	.153
1954	57.382	.692	18.189	.760	.260	.310	.293	.416
~~1955~~	~~76.396~~	~~1.303~~	~~26.744~~	~~1.428~~	~~.490~~	~~.585~~	~~.552~~	~~.784~~
1956	283.159	3.360	83.902	3.678	1.280	1.525	1.441	2.038
1957	201.320	5.092	84.089	5.567	1.938	2.310	2.182	3.090
1958	269.126	7.233	105.430	7.884	2.799	3.331	3.149	4.442
1959	461.877	10.655	164.705	11.588	4.197	4.984	4.715	6.614
1960	417.449	13.884	176.849	15.068	5.555	6.583	6.231	8.701
1961	374.660	16.680	175.005	18.045	6.822	8.061	7.639	10.595
1962	328.536	18.870	165.807	20.337	7.945	9.352	8.873	12.199
~~1963~~	~~373.984~~	~~21.028~~	~~176.248~~	~~22.582~~	~~9.129~~	~~10.698~~	~~10.165~~	~~13.838~~
1964	356.932	22.845	176.494	24.457	10.216	11.917	11.341	15.285
1965	311.598	24.132	166.341	25.755	11.119	12.909	12.304	16.414
1966	311.268	25.177	163.665	26.795	11.940	13.795	13.170	17.393
1967	280.095	25.833	154.889	27.419	12.603	14.491	13.856	18.117
1968	248.952	26.116	143.944	27.645	13.105	14.995	14.361	18.590
1969	246.311	26.243	139.157	27.714	13.521	15.399	14.770	18.938
1970	238.904	26.246	134.974	27.660	13.852	15.706	15.087	19.173
~~1971~~	~~258.140~~	~~26.346~~	~~138.007~~	~~27.725~~	~~14.179~~	~~16.014~~	~~15.402~~	~~19.428~~
1972	235.243	26.306	132.792	27.650	14.410	16.217	15.615	19.562
1973	209.163	26.081	124.225	27.376	14.538	16.304	15.716	19.559
1974	211.675	25.847	121.548	27.096	14.636	16.362	15.788	19.531
1975	208.465	25.597	119.179	26.806	14.700	16.386	15.826	19.472
1976	252.694	25.671	129.325	26.877	14.861	16.532	15.977	19.589

REFERENCE DATE AND TIME: 80/05/11. 17.16.36.

---------- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION ----------

L.F.S. FOR WAIRAKEI GROUP 2 , 1953 TO 1976
REFERENCE DATE AND TIME: 80/05/11. 17.16.38.

TIME AND FLOW DATA READ FROM FILE ≠WLFSG2 ≠ WITH FORMAT≠(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 8 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24	
DELTA-TAU:	3.156E+07	SECONDS
FLUID LEVEL (D):	2.670E+02	METERS
POROSITY (PHI):	4.000E-01	
DENSITY (RHO):	8.140E+02	KG/CU. METERS
PERMEABILITY (K):	3.000E-14	SQ. METERS
DYNAMIC VISCOSITY (MU):	1.099E-04	KG/METER-SECOND
P.Z. THICKNESS (H):	9.200E+02	METERS
SINKING VELOCITY (W):	5.445E-06	METER/SECOND

7 RADIUS SPECIFICATIONS: 211.00 0.00 941.00 1017.00 734.00 709.00 398.00

L.F.S. FOR WAIRAKEI GROUP 2 , 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
METERS:		211.00	0.00	941.00	1017.00	734.00	709.00	398.00
1953	63.637	5.332	8.238	.576	.499	.914	.975	2.630
1954	98.883	10.749	15.623	1.441	1.252	2.241	2.380	5.832
1955	[illegible]	15.613	21.873	2.476	2.162	3.763	3.980	9.020
1956	177.136	24.479	34.280	4.095	3.591	6.133	6.475	14.269
1957	102.730	23.806	31.159	5.011	4.427	7.286	7.654	15.275
1958	91.530	22.992	29.345	5.675	5.055	8.003	8.369	15.496
1959	325.358	42.437	59.212	8.325	7.404	11.897	12.479	25.389
1960	351.569	56.271	76.816	11.272	10.024	16.141	16.936	34.378
1961	313.209	61.974	81.964	13.785	12.293	19.511	20.431	39.640
1962	445.515	77.398	103.491	17.267	15.426	24.305	25.434	49.101
1963	460.913	88.614	117.004	20.731	18.565	28.941	30.248	57.218
1964	361.504	87.975	112.595	23.051	20.736	31.647	32.991	59.423
1965	315.826	84.906	106.646	24.596	22.246	33.134	34.444	59.255
1966	212.693	75.240	91.529	24.890	22.664	32.751	33.928	55.062
1967	287.113	76.269	94.532	25.574	23.397	33.200	34.338	55.093
1968	222.827	71.560	87.219	25.599	23.531	32.759	33.818	52.819
1969	258.010	71.883	88.490	25.854	23.840	32.818	33.850	52.637
1970	253.196	71.672	88.220	26.064	24.091	32.890	33.983	52.489
1971	248.284	71.193	87.494	26.230	24.292	32.940	33.938	52.268
1972	241.900	70.446	86.387	26.339	24.435	32.931	33.911	51.907
1973	233.597	69.375	84.835	26.371	24.504	32.827	33.786	51.351
1974	231.000	68.572	83.779	26.379	24.545	32.712	33.652	50.875
1975	230.724	68.097	83.210	26.390	24.584	32.627	33.553	50.556
1976	233.634	68.072	83.279	26.440	24.652	32.621	33.541	50.484

REFERENCE DATE AND TIME: 80/05/11. 17.16.38.

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---------- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION ----------

L.F.S. FOR WAIRAKEI GROUP 3 , 1953 TO 1976
REFERENCE DATE AND TIME: 80/05/11. 17.16.41.

TIME AND FLOW DATA READ FROM FILE ≠WLFSG3 ≠ WITH FORMAT:(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 6 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24	
DELTA-TAU:	3.156E+07	SECONDS
FLUID LEVEL (D):	5.270E+02	METERS
POROSITY (PHI):	4.000E-01	
DENSITY (RHO):	8.140E+02	KG/CU. METERS
PERMEABILITY (K):	3.000E-14	SQ. METERS
DYNAMIC VISCOSITY (MU):	1.099E-04	KG/METER-SECOND
P.Z. THICKNESS (H):	1.200E+03	METERS
SINKING VELOCITY (W):	5.445E-06	METER/SECOND
7 RADIUS SPECIFICATIONS:	827.00	0.00 1692.00 1017.00 360.00 533.00 727.00

L.F.S. FOR WAIRAKEI GROUP 3 , 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
METERS:		827.00	0.00	1692.00	1017.00	360.00	533.00	727.00
1953	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1954	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1955	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1956	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1957	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1958	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1959	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1961	4.426	.052	.200	.015	.037	.134	.094	.063
1962	0.000	.048	.126	.015	.036	.098	.077	.057
1963	16.463	.238	.830	.071	.173	.570	.412	.284
1964	12.365	.364	1.091	.115	.271	.793	.597	.429
1965	12.034	.470	1.270	.157	.357	.955	.740	.547
1966	13.515	.580	1.477	.203	.447	1.129	.888	.669
1967	11.613	.654	1.546	.241	.512	1.209	.970	.747
1968	11.212	.712	1.596	.276	.566	1.267	1.031	.807
1969	10.838	.759	1.629	.307	.610	1.308	1.076	.854
1970	9.753	.786	1.616	.332	.640	1.314	1.094	.879
1971	9.084	.803	1.590	.353	.660	1.306	1.097	.892
1972	8.884	.815	1.574	.371	.676	1.301	1.099	.901
1973	8.478	.821	1.551	.385	.686	1.289	1.096	.905
1974	8.277	.825	1.533	.397	.694	1.279	1.092	.907
1975	7.927	.826	1.509	.407	.698	1.264	1.083	.905
1976	7.233	.819	1.465	.414	.696	1.235	1.064	.894

REFERENCE DATE AND TIME: 80/05/11. 17.16.41.

----------- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION -----------

L.F.S. FOR WAIRAKEI GROUP 4 , 1953 TO 1976
REFERENCE DATE AND TIME: 80/05/11. 17.16.47.

TIME AND FLOW DATA READ FROM FILE *WLFSG4 * WITH FORMAT:(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 8 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24	
DELTA-TAU:	3.156E+07	SECONDS
FLUID LEVEL (D):	3.100E+02	METERS
POROSITY (PHI):	4.000E-01	
DENSITY (RHO):	8.140E+02	KG/CU. METERS
PERMEABILITY (K):	3.000E-14	SQ. METERS
DYNAMIC VISCOSITY (MU):	1.899E-04	KG/METER-SECOND
P.Z. THICKNESS (H):	9.200E+02	METERS
SINKING VELOCITY (W):	5.445E-06	METER/SECOND

7 RADIUS SPECIFICATIONS: 578.00 0.00 1540.00 734.00 360.00 194.00 388.00

L.F.S. FOR WAIRAKEI GROUP 4 , 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
METERS:		578.00	0.00	1540.00	734.00	360.00	194.00	388.00
1953	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1954	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1955	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1956	30.999	.703	3.234	.117	.465	1.406	2.388	1.280
1957	54.392	1.874	7.307	.331	1.273	3.501	5.595	3.220
1958	69.891	3.245	11.155	.616	2.266	5.739	8.762	5.320
1959	110.408	5.331	17.633	1.063	3.777	9.227	13.918	8.575
1960	102.342	6.948	20.437	1.493	5.044	11.451	16.551	10.719
1961	73.859	7.631	19.444	1.813	5.718	11.831	16.238	11.173
1962	101.034	8.755	22.248	2.213	6.648	13.412	18.459	12.674
1963	156.317	10.986	29.669	2.808	8.305	17.173	24.218	16.169
1964	232.333	14.681	41.674	3.687	10.987	23.457	33.683	22.017
1965	219.709	17.625	47.034	4.532	13.294	27.569	38.645	25.968
1966	21[illegible].918	20.053	50.664	5.361	15.308	30.631	42.098	28.951
1967	189.461	21.394	50.292	6.052	16.604	31.680	42.431	30.076
1968	142.395	21.397	45.929	6.518	16.958	30.476	39.512	29.093
1969	211.203	22.894	51.366	7.181	18.190	32.881	43.465	31.319
1970	198.020	23.989	52.844	7.762	19.158	34.214	44.937	32.621
1971	20[illegible].707	25.138	54.938	8.338	20.152	35.691	46.764	34.046
1972	196.194	25.907	55.270	8.840	20.890	36.411	47.299	34.783
1973	186.059	26.335	54.755	9.268	21.381	36.589	47.102	35.008
1974	189.273	26.780	55.111	9.670	21.848	36.974	47.455	35.482
1975	186.611	27.130	55.198	10.030	22.230	37.240	47.620	35.682
1976	182.956	27.375	55.038	10.347	22.522	37.357	47.583	35.821

REFERENCE DATE AND TIME: 80/05/11. 17.16.47.

---------- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION ----------

L.F.S. FOR WAIRAKEI GROUP 5 , 1953 TO 1976
REFERENCE DATE AND TIME: 80/05/11. 17.16.49.

TIME AND FLOW DATA READ FROM FILE ≠WLFSG5 ≠ WITH FORMAT:(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 3 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24	
DELTA-TAU:	3.156E+07	SECONDS
FLUID LEVEL (D):	3.300E+02	METERS
POROSITY (PHI):	4.000E-01	
DENSITY (RHO):	8.140E+02	KG/CU. METERS
PERMEABILITY (K):	3.000E-14	SQ. METERS
DYNAMIC VISCOSITY (MU):	1.099E-04	KG/METER-SECOND
P.Z. THICKNESS (H):	9.200E+02	METERS
SINKING VELOCITY (W):	5.445E-06	METER/SECOND
7 RADIUS SPECIFICATIONS:	602.00	0.00 1588.00 709.00 533.00 194.00 311.00

L.F.S. FOR WAIRAKEI GROUP 5 , 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
METERS:		602.00	0.00	1588.00	709.00	533.00	194.00	311.00
1953	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1954	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1955	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1956	37.771	.836	3.597	.136	.611	.979	2.731	1.946
1957	68.065	2.186	8.355	.393	1.688	2.614	6.548	4.850
1958	55.139	3.119	9.791	.617	2.473	3.653	7.998	6.211
1959	87.418	4.569	13.960	.960	3.663	5.313	11.385	8.876
1960	156.978	7.304	23.021	1.558	5.855	8.506	18.616	14.380
1961	98.000	8.445	22.503	1.961	6.987	9.676	18.982	15.238
1962	100.775	9.378	23.199	2.356	7.789	10.628	19.697	16.129
1963	220.385	12.694	36.322	3.163	10.456	14.507	29.100	23.069
1964	252.649	16.350	44.997	4.097	13.459	18.690	37.260	29.658
1965	267.156	19.807	52.259	5.086	16.388	22.550	43.681	35.145
1966	295.035	23.326	59.688	6.164	19.404	26.451	50.157	40.635
1967	297.996	26.378	64.811	7.232	22.094	29.763	54.883	44.885
1968	200.481	26.870	59.187	7.912	22.845	29.972	51.327	43.126
1969	243.564	28.067	61.258	8.672	24.029	31.174	52.958	44.474
1970	255.076	29.390	63.725	9.418	25.265	32.570	55.072	46.265
1971	248.775	30.443	64.799	10.091	26.280	33.646	56.188	47.392
1972	237.479	31.130	64.709	10.675	26.997	34.297	56.357	47.790
1973	206.392	31.071	62.066	11.098	27.121	34.073	54.496	46.651
1974	190.889	30.688	59.481	11.412	26.950	33.511	52.493	45.233
1975	166.862	29.872	55.925	11.591	26.405	32.473	49.675	43.143
1976	195.237	29.806	56.750	11.829	26.380	32.396	50.083	43.246

REFERENCE DATE AND TIME: 80/05/11. 17.16.49.

---------- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION ----------

L.F.S. FOR WAIRAKEI GROUP 6 , 1953 TO 1976
REFERENCE DATE AND TIME: 80/05/11. 17.16.51.

TIME AND FLOW DATA READ FROM FILE ≠WLFSG6 ≠ WITH FORMAT:(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE B (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24	
DELTA-TAU:	3.156E+07	SECONDS
FLUID LEVEL (D):	3.380E+02	METERS
POROSITY (PHI):	4.000E-01	
DENSITY (RHO):	6.141E+02	KG/CU. METERS
PERMEABILITY (K):	3.000E-14	SQ. METERS
DYNAMIC VISCOSITY (MU):	1.099E-04	KG/METER-SECOND
P.Z. THICKNESS (H):	9.200E+02	METERS
SINKING VELOCITY (W):	5.445E-06	METER/SECOND

7 RADIUS SPECIFICATIONS: 318.00 0.00 1312.00 398.00 727.00 388.00 311.00

L.F.S. FOR WAIRAKEI GROUP 6 , 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
METERS:		318.00	0.00	1312.00	398.00	727.00	388.00	311.00
1953	12.192	.606	1.121	.065	.474	.190	.488	.619
1954	40.690	2.444	4.332	.284	1.942	.816	1.999	2.493
1955	110.739	7.212	12.524	.886	5.777	2.498	6.940	7.351
1956	102.874	10.168	16.314	1.470	8.385	3.950	8.592	10.337
1957	190.134	16.648	26.867	2.512	13.742	6.581	14.077	16.925
1958	182.255	20.891	32.186	3.520	17.541	8.858	17.931	21.206
1959	194.634	24.644	36.964	4.565	20.931	11.034	21.367	24.992
1960	438.642	39.611	62.675	6.857	33.059	16.720	33.816	40.236
1961	375.163	47.110	71.094	8.837	39.926	21.013	40.766	47.784
1962	565.225	62.146	94.800	11.752	52.561	27.686	53.677	63.051
1963	642.978	86.873	134.281	16.098	73.125	38.040	74.721	88.175
1964	916.162	108.279	163.951	20.816	91.772	48.618	93.698	109.833
1965	880.202	122.163	179.607	25.235	104.708	57.427	106.762	123.814
1966	904.300	134.052	193.558	29.558	115.772	65.376	117.927	135.753
1967	820.991	139.218	196.181	33.176	121.394	70.933	123.506	140.868
1968	691.723	137.412	188.300	35.790	121.137	73.579	123.076	138.907
1969	790.872	142.005	195.150	38.583	125.326	77.181	127.306	143.545
1970	745.861	143.699	195.569	40.888	127.318	79.723	129.265	145.209
1971	754.140	145.917	197.752	43.600	129.574	82.089	131.516	147.424
1972	720.564	146.347	196.738	44.737	130.381	83.701	132.281	147.817
1973	688.694	145.555	194.128	46.125	130.105	84.642	131.946	146.976
1974	662.901	144.151	191.005	47.216	129.219	85.085	130.999	145.524
1975	666.569	143.713	190.097	48.192	128.984	85.564	130.738	145.067
1976	619.195	141.337	185.496	48.818	127.226	85.329	128.909	142.633

REFERENCE DATE AND TIME: 80/05/11. 17.16.51.

L.F.S. FOR WAIRAKEI GROUP 1 , 1953 TO 1976 DISTANCE TO CENTRAL POINT = 990.00
L.F.S. FOR WAIRAKEI GROUP 2 , 1953 TO 1976 DISTANCE TO CENTRAL POINT = 211.00
L.F.S. FOR WAIRAKEI GROUP 3 , 1953 TO 1976 DISTANCE TO CENTRAL POINT = 827.00
~~L.F.S. FOR WAIRAKEI GROUP 4 , 1953 TO 1976 DISTANCE TO CENTRAL POINT = 578.00~~
L.F.S. FOR WAIRAKEI GROUP 5 , 1953 TO 1976 DISTANCE TO CENTRAL POINT = 602.00
L.F.S. FOR WAIRAKEI GROUP 6 , 1953 TO 1976 DISTANCE TO CENTRAL POINT = 318.00

TIME	FIELD DRAWDOWN AT CENTRAL POINT IN METERS OF HEAD
1953	6.190
1954	13.885
1955	24.128
1956	39.516
1957	49.604
1958	57.481
1959	87.635
1960	124.019
1961	141.892
1962	176.596
1963	226.432
1964	250.494
1965	269.123
1966	278.427
1967	289.748
1968	284.067
1969	291.851
1970	295.782
1971	299.838
1972	300.950
1973	299.233
1974	296.863
1975	295.235
1976	293.079

REFERENCE DATE AND TIME: 80/05/11. 17.16.51.

L.F.S. PROGRAM UTILIZED 3.415 SECONDS CPU TIME