Capacity of MIMO Mobile Wireless Ad Hoc Networks

Xiaohui Yu, Renato M. de Moraes, Hamid Sadjadpour
Department of Electrical Engineering
University of California at Santa Cruz (UCSC)
Santa Cruz, CA 95064, USA
Email: {xyu,renato,hamid}@soe.ucsc.edu

J.J. Garcia-Luna-Aceves
Department of Computer Engineering at UCSC and
Palo Alto Research Center
3333 Coyote Hill Road, Palo Alto, CA 94304, USA
Email: jj@soe.ucsc.edu

***Abstract*— We compute the capacity of wireless ad hoc networks when all the nodes in the network are endowed with M antennas. The derivation is based on a new communication scheme for wireless ad hoc networks utilizing the concept of cooperative many-to-many communications, as opposed to the traditional approach that emphasizes on one-to-one communications. We show that the upper bound average asymptotic capacity of each cell is $2\pi P_t M C_{cell}[1-\exp(-\frac{C_{cell}}{\theta})]$, for network parameters $C_{cell} \geq 1$, $0 < \theta < 1$, and transmit power P_t.**

I. Introduction

The capacity of fixed and mobile wireless ad hoc networks has been the subject of extensive study. Recently, Gupta and Kumar [1] showed that per source-destination throughput and delay of a stationary (fixed) wireless ad hoc network with n nodes scale as $\Theta(1/\sqrt{n\log(n)})$ [1] and $\Theta(\sqrt{n/\log(n)})$ [2], respectively. This is a disappointing result, because both the throughput and delay degrades as the number of nodes in the network increases. Grossglauser and Tse [3] demonstrated that per source-destination throughput and delay of wireless mobile ad hoc networks scale as $\Theta(1)$ and $\Theta(n)$ [2], respectively, by utilizing mobility and multi-user diversity scheme [4]. This significant improvement in throughput is achieved by a two-phase relaying strategy. Moraes et al [5] improved the delay behavior of the two-phase relaying strategy by utilizing a multi-copy forwarding strategy. The basic idea is to give a copy of the packet to multiple one-time relay nodes that are within the transmission range of the sender. The multi-copy forwarding strategy can achieve the capacity of the basic scheme in [3] but provides lower delay in the mobile wireless ad hoc networks. Another interesting study [6] shows that one can achieve significantly higher network throughput in fixed wireless ad hoc networks by changing the physical layer assumptions. Negi and Rajeswaran [6] proved that, in a power constrained ad hoc network, a design based on ultra wide band (UWB) communications can improve the throughput of each node as a function of n if we allow bandwidth expansion, i.e., $W \to \infty$ where W is the bandwidth requirement for each node.

We use a new scheme based on a collaboration-driven approach, called opportunistic cooperation [7]. In this new approach, multiple nodes simultaneously communicate with each other within a cell in the network. Sender-receiver nodes collaborate rather than compete with each other to access the channel. Each sender node either relays a message to all receiver nodes or to one of the receiver nodes as destination in a predefined cell. Within the context of a cell in the network, multiple sender nodes either relay or transmit data to a destination simultaneously. The multiple access scheme that is proposed in this paper is based on multiple-input multiple-output (MIMO) systems that does not require any bandwidth expansion. In this paper, we explore the capacity behavior of wireless ad hoc networks when every single node has M antennas. The neighbor discovery and scheduling techniques are required before transmission of packets in a cell. These subjects are beyond the scope of this paper and are described in [7]. For simplicity of the analysis, we assume that all the nodes in the network have the same number of antennas.

The capacity of MIMO systems has received considerable attention [8], [9], [10], [11], however, all these studies concentrate on the concept of one-to-one communication among nodes. Even the work in [11] studies the capacity of wireless ad hoc networks by regarding the whole network as a single MIMO system that some nodes are part of the transmitter and the remaining nodes in the network are part of the receiver,

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[1] Θ and o are the standard order bounds. $\log(\cdot)$ is the natural logarithm.

where all the nodes have only one antenna. A random line is used to cut the network into two parts for senders and receivers. While the work in [11] was the first attempt to compute the capacity of networks based on MIMO systems, the results are rather optimistic by assuming all the receiving nodes in the network are capable of cooperating with each other.

We propose that a more appropriate strategy for communications among nodes in wireless ad hoc networks with shared channel is a new approach based on cooperative many-to-many communication [7]. In this new paradigm, multiple nodes that are close to each other attempt to communicate concurrently. The nodes are divided into sender and receiver nodes. At each time, a node is selected as a sender or receiver node. If the node is a sender node, it sends packets from only one of its antennas while the receiver nodes receive and decode packets from multiple nodes simultaneously using all of their antennas. Therefore, each MIMO system in this scheme consists of multiple transmitting nodes and a single receiver node in a cell. Hence, this approach does not require any cooperation among receiving nodes for decoding the received packets. Such a new viewpoint allows us to achieve higher performance in the system in terms of capacity and delay. In this paper, we only explore the capacity of such systems. We also assume that these antennas create statistically independent channels.

The remaining of the paper is organized as follows. Section II summarizes the network model. In Section III, the design of parameters in the network is discussed. Section IV is dedicated to the capacity analysis. We conclude the paper in section V.

II. Network Parameters and Assumptions

A. Network Model

For simplicity of the analysis, the network is modeled as a unit square area with n nodes moving inside this area. The nodes are uniformly distributed in the network. The communication between senders and receivers is time-slotted. A global scheduler designates at each time slot the senders and receivers and makes sure the number of senders is $n_S = \theta n$, where $0 < \theta < 1$. The network is split into smaller square cells with area $a(n)$. Hence, there are $1/a(n)$ cells in the entire network.

B. Mobility Model

The nodes' movements are modeled as uniform mobility model [12]. In this model, the nodes are initially uniformly distributed and move at a constant speed $v(n)$ and the directions of motions are independent and identically distributed (iid) with uniform distribution in the range of $[0, 2\pi)$. As time passes, each node chooses a direction uniformly from $[0, 2\pi)$ and moves in that direction, at speed $v(n)$, for a distance d, where d is an exponential random variable with mean u. After the node reaches the destination, the process repeats. This model satisfies the following properties [12]:

- At any time, the positions of the nodes are independent of each other.
- The steady-state distribution of the mobile nodes is uniform.
- The direction of the node movement is uniformly distributed in $[0, 2\pi)$, conditional on the position of the node.

With this uniform mobility model, the distribution of the number of nodes in a cell is binomial [13]. Let the random variable Z denotes the number of nodes in a cell. The probability of $Z = l$ is

$$\mathbb{P}\{Z=l\} = \binom{n}{l} [a(n)]^l \, [1 - a(n)]^{n-l} \,. \tag{1}$$

C. Communication Model

We assume every node has M antennas where M is a constant that is greater than or equal to 2. The communication between senders and receivers is time-slotted and fully synchronized. We assume the communication in all the cells is operated on different frequencies. This assumption allows us to simplify the analysis and assume that there is no interference from adjacent cells. In practical applications, this assumption is equivalent to allow adjacent cells to use different frequencies, similar to the frequency re-use concept in cellular systems, and assume that interfering cells using the same frequency are so far that their interference is negligible compared to the thermal noise, i.e., the additive white Gaussian noise (AWGN) power is much stronger than the interference. Hence, nodes only communicate with other nodes in the same cell and there is no interference from other cells in the network.

The transmit power of each sender node is equal to P_t. The distance between a sender i and a receiver j is denoted as r_{ij}. Assuming no fading , the received signal power at node j from node i is

$$P_{ij} = \frac{P_t}{(1 + r_{ij})^{\alpha}}, \tag{2}$$

where α is the path loss parameter and assumed to be greater than 2. We use the path loss channel model described in [11] as opposed to the more common approach of $1/r_{ij}^{\alpha}$ [1], [3], [5].

We use boldface capital letters to represent matrices and boldface lower case letters to denote vectors. Accordingly, the received signal vector for one receiver node is defined as

$\mathbf{y} = [y_1, y_2, ..., y_M]^t$, where the operator t is the transpose of a vector. The transmission signal is $\mathbf{x} = [x_1, x_2, ..., x_S]^t$, where S is the number of the senders in the cell, i.e., $S = \theta l$. This assumption implies that transmit nodes only use one antenna while receiving nodes utilize all their M antennas for communications. The received signal for each node is defined as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{n} = [n_1, n_2, ..., n_M]^t$ is a zero-mean complex AWGN. We assume $E[\mathbf{n}\,\mathbf{n}^*] = \mathbf{I}_M$, where the operator $*$ is defined as conjugate transpose and $\mathbf{I}_M$ is the $M \times M$ identity matrix. $\mathbf{H}$ is the $M \times S$ channel matrix with its elements defined as

$$h_{ms} = \frac{\eta_{ms}}{(1 + r_{sj})^\alpha}, \quad (3)$$

where $1 \leq m \leq M$, $1 \leq s \leq S$, and η_{ms} is a stationary and ergodic stochastic fading process that is independent for each sender and receiver antenna pair. The power of channel coefficients is normalized to one, i.e., $E[|\eta_{ms}|^2] = 1$. In (3), r_{sj}, the distance between receive node j and the sender node s is not a function of receive antennas, i.e., m. The reason is because the distances between the sender node s and all M antennas of the receiver j are assumed to be equal for practical considerations. Note that in this model, the packets are delivered using one time relaying, perhaps using multi-copy forwarding scheme [5] if there is enough receiver nodes in the cell.

In our analysis, we consider that channel state information (CSI) is only known at the receiver side. Furthermore, in every cell, each MIMO system consists of multiple transmitting nodes (the S senders) and a single receiver node (with M receiving antennas). As a cell may have more than one receiver node, the communication is S-to-many (opportunistic cooperation [7]).

III. Network Design

In a MIMO system, it is usually preferred that the number of transmit and receive antennas, to be defined within a range. Note that any arbitrary number of transmit and receive antennas may not be of practical interest and result in a poor performance for the MIMO system. The number of nodes within a single cell is a random number. Thus, it is desirable to set the network parameters such that the maximum number of nodes in a cell is bounded with a high probability as $n \to \infty$. For this reason, it is better that the number of nodes in the cell falls into a range, e.g., $[2, M/\theta)$. In this case, the total number of transmit antennas in each cell will not exceed M with high probability (*whp*). [2] Let's define the area of a cell in the network as

$$a(n) = \frac{C_{cell}}{n_S} = \frac{C_{cell}}{\theta n}, \quad (4)$$

where $C_{cell} > 1$ is a constant tuning parameter for the network. Accordingly, there are a total of $\frac{1}{a(n)} = \frac{\theta n}{C_{cell}}$ cells in the network.

If a cell has only one or no node, it is not possible to implement a MIMO system in that cell. For this reason, we make the probability of $\mathbb{P}\{Z = 0\} + \mathbb{P}\{Z = 1\}$ less than an arbitrary small positive number ϵ_1. That is, for n large

$$\begin{aligned} &\mathbb{P}\{Z = 0\} + \mathbb{P}\{Z = 1\} \\ &= \binom{n}{0}\left(\tfrac{C_{cell}}{\theta n}\right)^0\left(1 - \tfrac{C_{cell}}{\theta n}\right)^n + \binom{n}{1}\left(\tfrac{C_{cell}}{\theta n}\right)^1\left(1 - \tfrac{C_{cell}}{\theta n}\right)^{n-1} \\ &\leq \exp\left(-\frac{C_{cell}}{\theta}\right) + \left(\frac{C_{cell}}{\theta}\right)\exp\left(-\frac{C_{cell}}{\theta}\right) \\ &< \epsilon_1. \end{aligned} \quad (5)$$

Approximation in (5) will become equality when $n \to \infty$. We also used the facts that $\binom{n}{l} \leq \frac{n^l}{l!}$ for $n >> l$, and the limit $(1 - 1/x)^x \to e^{-1}$ as $x \to \infty$. Note that as the size of cell in the network increases, (5) goes to zero, i.e.,

$$\lim_{\frac{C_{cell}}{\theta} \to \infty} \left(1 + \frac{C_{cell}}{\theta}\right)\exp\left(-\frac{C_{cell}}{\theta}\right) = 0. \quad (6)$$

Now, let l_{max} represent the maximum number of nodes in a cell, and $\lceil x \rceil$ denote the ceil function (i.e., the smallest integer greater than or equal to x). The following lemma provides the relationship between l_{max} and n.

Lemma 1 *For the uniform mobility model, the maximum number of nodes in any cell is given whp by*

$$l_{max} = \left\lceil \frac{3\log(n)}{\log(\log(n^{\theta/C_{cell}}))} \right\rceil. \quad (7)$$

Proof: see Appendix A.

This result demonstrate that as n increases in the network, the maximum number of nodes in a cell increases very slowly.

Accordingly, we can compute the probability that the nodes in a cell is greater than or equal to M/θ and attempt to limit this probability. Therefore,

$$\begin{aligned} \sum_{l=M/\theta}^{l_{max}} \mathbb{P}\{Z = l\} &= \sum_{l=M/\theta}^{l_{max}} \binom{n}{l}\left(\tfrac{C_{cell}}{\theta n}\right)^l\left(1 - \tfrac{C_{cell}}{\theta n}\right)^{n-l} \\ &\leq \sum_{l=M/\theta}^{l_{max}} \frac{1}{l!}\left(\frac{C_{cell}}{\theta}\right)^l \exp\left(\frac{C_{cell}}{\theta}\right) \\ &\leq \epsilon_2, \end{aligned} \quad (8)$$

[2] With high probability means with probability $\geq 1 - \frac{c}{n}$ [13], where c is a positive constant.

TABLE I

POSSIBLE NETWORK PARAMETERS AND CORRESPONDING PROBABILITIES.

θ	M	C_{cell}	Cell node range ($min, M/\theta$)	$\mathbb{P}\{2 \le Z < M/\theta\}$
1/2	4	2	(2,8)	0.7405
1/2	6	4	(2,12)	0.9224
1/2	6	4	(4,12)	0.8366
1/3	4	2	(2,12)	0.9292
1/3	4	2	(4,12)	0.7061
1/3	4	4	(2,12)	0.5754
1/3	6	4	(2,18)	0.9621
1/3	6	4	(4,18)	0.9550

where ϵ_2 is an arbitrarily small positive number.

The right term of inequality in (8) is the right-tail function of Poisson distribution with parameter $\lambda = C_{cell}/\theta$. This probability decreases with the increase in M/θ or decrease in λ. Using the three parameters, M, θ, C_{cell}, the inequalities in (5) and (8) can be satisfied. If M is fixed and cannot be changed, then we need to adjust the other two parameters to satisfy both (5) and (8) conditions.

In general, the network design becomes the task of carefully tuning the three parameters, (selected from a practical range) to make sure that the probability of the number of nodes in the cell, in the range of 2 and M/θ, is very high. Table I illustrates some examples for network parameters and their corresponding probabilities when the number of nodes in the cell is between 2 (or 4) and M/θ.

IV. CAPACITY ANALYSIS

In this paper, we compute the capacity for unit bandwidth also known as spectral efficiency. The communication bandwidth for the network configuration is a constant that depends on the transmission bit rate of the nodes.

Theorem 1 *The average capacity of each cell in the network is upper bounded by the following constant when $n \to \infty$*

$$2\pi P_t M C_{cell}\left[1 - \exp\left(-\frac{C_{cell}}{\theta}\right)\right]. \qquad (9)$$

Proof: The conditional mutual information between input sequence, $\mathbf{x}$, and the output data sequence, $\mathbf{y}$, is given by

$$I(\mathbf{x}, \mathbf{y}|\mathbf{H}) = E_{\mathbf{H}}[I(\mathbf{x}, \mathbf{y}|\mathbf{H} = H)], \qquad (10)$$

where H is a $M \times S$ channel matrix realization of $\mathbf{H}$, in which $S = \theta l$ is the number of senders in the cell.

With CSI known only at the receiver, maximizing mutual information is equivalent to maximize the following equation over $\mathbf{x}$ [8], [9], [10]

$$C = \max_{\mathbf{K_x} \ge 0,\, (\mathbf{K_x})_{ii} \le P_t} E_{\mathbf{H}}[\log\det(\mathbf{I} + \mathbf{H}\mathbf{K_x}\mathbf{H}^*)], \qquad (11)$$

where $\mathbf{K_x}$ is defined as $E(\mathbf{x}\,\mathbf{x}^*)$.

Now, we define the following vector $\mathbf{r}$ with its elements representing the distance between different transmit nodes and the receive antennas of node j, i.e., $\mathbf{r} = [r_{11}, r_{21}, ..., r_{M1}, ..., r_{1S}, r_{2S}, ..., r_{MS}]$. Note that the elements of the vector $\mathbf{r}$ are random variables and every M values are equal, i.e., $r_{ij} = r_{ik}$ for all values of i, j, and k. Besides, when $n \to \infty$, then the area of each cell goes to zero that results in almost same value for all elements in $\mathbf{r}$.

Let R_j denote the capacity for the receiver j when there is no interference from adjacent cells. It is already shown in [11] that

$$\begin{aligned} R_j &\le \max_{\mathbf{K_x} \ge 0,\, (\mathbf{K_x})_{ii} \le P_t} E_{\mathbf{H}}[\log\det(\mathbf{I} + \mathbf{H}\mathbf{K_x}\mathbf{H}^*)] \\ &\le E_{\mathbf{H}}\left[\sum_{m=1}^{M} \log\left(1 + \sum_{s=1}^{S} P_t |h_{ms}|^2\right)\right] \\ &\le \sum_{m=1}^{M}\sum_{s=1}^{S} \frac{P_t}{(1 + r_{ms})^{2\alpha}}. \end{aligned} \qquad (12)$$

r_{ms} is a random variable that considers the uniform distribution of senders and receivers in the cell. We show in Appendix B that

$$E_r\left[\frac{P_t}{(1 + r_{ms})^{2\alpha}}\right] \le \frac{2\pi P_t}{\left(1 + \sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha - 1}}. \qquad (13)$$

From (12) and (13), and noting that $S = \theta l$, the upper bound unconditional capacity for the receiver node j is given by

$$\begin{aligned} R(l) = E_r[R_j] &\le E_r\left[\sum_{m=1}^{M}\sum_{s=1}^{S} \frac{P_t}{(1 + r_{ms})^{2\alpha}}\right] \\ &\le \sum_{m=1}^{M}\sum_{s=1}^{\theta l} E_r\left[\frac{P_t}{(1 + r_{ms})^{2\alpha}}\right] \\ &\le \frac{2\pi P_t M \theta l}{\left(1 + \sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha - 1}}. \end{aligned} \qquad (14)$$

Observe that this is also the total capacity of the cell because each receiver node in a cell receives all the transmitted packets in that cell (opportunistic cooperation [7]) as either destination, relay, or simply drops that packet because the destination is one of the receivers in the cell.

The number of nodes in the cell is a random variable with its probability given in (1). Hence, the average total capacity of the cell is upper bounded by

$$\begin{aligned} R_{cell} &= \sum_{l=2}^{l_{max}} \mathbb{P}\{Z=l\} R(l) \\ &\leq \sum_{l=2}^{l_{max}} \binom{n}{l} \left(\frac{C_{cell}}{\theta n}\right)^l \left(1-\frac{C_{cell}}{\theta n}\right)^{n-l} \frac{2\pi P_t M \theta l}{\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}} \\ &\leq \frac{2\pi P_t M\theta \exp\left(-\frac{C_{cell}}{\theta}\right)}{\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}} \sum_{l=2}^{\infty} \frac{1}{l!} \left(\frac{C_{cell}}{\theta}\right)^l l \qquad (15) \\ &= \frac{2\pi P_t M\theta \exp\left(-\frac{C_{cell}}{\theta}\right)}{\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}} \left[\left(\frac{C_{cell}}{\theta}\right) \exp\left(\frac{C_{cell}}{\theta}\right) - \frac{C_{cell}}{\theta}\right] \\ &= \frac{2\pi P_t M\, C_{cell}}{\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}} \left[1-\exp\left(-\frac{C_{cell}}{\theta}\right)\right], \qquad (16) \end{aligned}$$

where (15) is derived using similar technique as in (5). The final result for the theorem is achieved as $n \to \infty$. □

These results show that as the number of antennas for each node increases, the capacity of the cell increases linearly with that. However, in order to keep the number of cells in the network within an acceptable range so that we can perform space time signal processing, the size of the cell should be modified accordingly by adjusting the parameters θ and C_{cell}.

V. CONCLUSIONS

In this paper, we have shown that the capacity of the cells in the network can be upper bounded. This upper bound does not decrease with the increase in the number of nodes in the network. This result is rather optimistic assuming frequency reuse in the cells such that the interference is no longer the dominant factor. Another important aspect of these results is the use of a new communication scheme (called opportunistic cooperation [7]) that allows multiple nodes in a cell to communicate concurrently with other nodes in the same cell. For the future work, it is important to compute the delay of such networks.

APPENDIX

A. Proof of Lemma 1: maximum number of nodes in a cell

Let $\mathcal{E}_{k,l}$ denote the event that cell k contains at least l nodes. Let $z = l_{max} = \left\lceil \frac{3\log(n)}{\log(\log(n^{\theta/C_{cell}}))} \right\rceil$. The total number of cells in the network is (# of cells) $= 1/a(n) = \theta n/C_{cell}$. For any cell k, due to the uniform mobility model, the distribution of the nodes is Binomial [13]. Therefore, we have

$$\begin{aligned} \mathbb{P}\{\mathcal{E}_{k,z}\} &= \sum_{l=z}^{n} \binom{n}{l} \left(\frac{1}{\text{\# of cells}}\right)^l \left(1-\frac{1}{\text{\# of cells}}\right)^{n-l} \\ &= \sum_{l=z}^{n} \binom{n}{l} \left(\frac{C_{cell}}{\theta n}\right)^l \left(1-\frac{C_{cell}}{\theta n}\right)^{n-l} \\ &\leq \sum_{l=z}^{n} \binom{n}{l} \left(\frac{C_{cell}}{\theta n}\right)^l \\ &\leq \sum_{l=z}^{n} \left(\frac{ne}{l}\right)^l \left(\frac{C_{cell}}{\theta n}\right)^l \qquad (17) \\ &\leq \sum_{l=z}^{\infty} \left(\frac{e\,C_{cell}}{\theta l}\right)^l \\ &= \left(\frac{e\,C_{cell}}{\theta z}\right)^z \left(\frac{1}{1-\frac{e\,C_{cell}}{\theta z}}\right) \\ &\leq 2\left(\frac{e\,C_{cell}}{\theta z}\right)^z \qquad (18) \\ &\leq 2\left(\frac{e\,C_{cell}/\theta}{\frac{3\log(n)}{\log(\log(n^{\theta/C_{cell}}))}}\right)^z \\ &= 2\left(e^{1-\log(3)-\log(\log(n^{\frac{\theta}{C_{cell}}}))+\log(\log(\log(n^{\frac{\theta}{C_{cell}}})))}\right)^z \\ &\leq 2\left(e^{-\log(\log(n^{\theta/C_{cell}}))+\log(\log(\log(n^{\theta/C_{cell}})))}\right)^z \\ &\leq 2\,e^{-3\log(n)+\frac{\log(\log(\log(n^{\theta/C_{cell}})))}{\log(\log(n^{\theta/C_{cell}}))}3\log(n)} \\ &\leq 2\,e^{-2\log(n)} \qquad (19) \\ &= \frac{2}{n^2}, \qquad (20) \end{aligned}$$

where we used $\binom{n}{l} \leq \frac{n^l}{l!} \leq (\frac{ne}{l})^l$ to obtain (17), and for large values of n, we utilized $\left(\frac{1}{1-\frac{e\,C_{cell}}{\theta z}}\right) \leq 2$ for (18), as well as $\frac{\log(\log(\log(n^{\theta/C_{cell}})))}{\log(\log(n^{\theta/C_{cell}}))} < \frac{1}{3}$ for (19).

Now, since there are $\theta n/C_{cell}$ cells, the probability that any cell receives at least z nodes is bounded by

$$\mathbb{P}\left\{\bigcup_{k=1}^{\text{\# of cells}} \mathcal{E}_{k,z}\right\} = \mathbb{P}\left\{\bigcup_{k=1}^{\theta n/C_{cell}} \mathcal{E}_{k,z}\right\} \leq \sum_{k=1}^{\theta n/C_{cell}} \mathbb{P}\{\mathcal{E}_{k,z}\}. \qquad (21)$$

Let $\overline{\mathcal{E}}_z$ be the event that no cell has more than z nodes.

Hence,

$$\begin{aligned}\mathbb{P}\left\{\overline{\mathcal{E}}_z\right\} &= 1-\mathbb{P}\left\{\bigcup_{k=1}^{\theta n/C_{cell}} \mathcal{E}_{k,z}\right\} \\ &\geq 1-\sum_{k=1}^{\theta n/C_{cell}} \mathbb{P}\left\{\mathcal{E}_{k,z}\right\} \\ &\geq 1-\frac{\theta n}{C_{cell}}\frac{2}{n^2} \\ &= 1-\frac{2\theta/C_{cell}}{n}. \qquad (22)\end{aligned}$$

Thus, no cell has more than $\left\lceil \frac{3\log(n)}{\log(\log(n^{\theta/C_{cell}}))}\right\rceil$ nodes *whp*. □

B. Computation of inequality (13)

The location of the considered receiver in polar coordinates is (r,γ), where r and γ represent radius and angle from the center of the cell, respectively. Accordingly,

$$\begin{aligned} E_r\left[\tfrac{P_t}{(1+r)^{2\alpha}}\right] &= \iint_{cell} \tfrac{P_t}{(1+r)^{2\alpha}}\tfrac{1}{a(n)} r\,dr\,d\gamma \\ &= \tfrac{P_t\theta n}{C_{cell}}\int_0^{2\pi}\int_0^{f(\gamma)\sqrt{\frac{C_{cell}}{\theta n}}} \tfrac{r\,dr}{(1+r)^{2\alpha}}d\gamma \\ &\leq \tfrac{P_t\theta n}{C_{cell}}\int_0^{2\pi}\int_0^{\sqrt{\frac{2C_{cell}}{\theta n}}} \tfrac{r\,dr}{(1+r)^{2\alpha}}d\gamma \qquad (23) \\ &= \tfrac{P_t\theta n}{C_{cell}}\int_0^{2\pi}\left[\tfrac{(2\alpha-2)-(2\alpha-1)(1+r)}{(2\alpha-2)(2\alpha-1)(1+r)^{2\alpha-1}}\right]_0^{\sqrt{\frac{2C_{cell}}{\theta n}}} d\gamma \\ &= \tfrac{P_t\theta n}{C_{cell}}\int_0^{2\pi}\left[\frac{\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}-1-\sqrt{\frac{2C_{cell}}{\theta n}}\,(2\alpha-1)}{(2\alpha-2)(2\alpha-1)\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}}\right]d\gamma \\ &= \tfrac{P_t\theta n}{C_{cell}}\int_0^{2\pi}\left[\frac{\frac{(2\alpha-2)(2\alpha-1)}{2}\frac{2C_{cell}}{\theta n}+o\left(\frac{2C_{cell}}{\theta n}\right)}{(2\alpha-2)(2\alpha-1)\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}}\right]d\gamma \qquad (24) \\ &= \tfrac{2\pi P_t\theta n}{C_{cell}}\frac{\frac{(2\alpha-2)(2\alpha-1)}{2}\frac{2C_{cell}}{\theta n}+o\left(\frac{2C_{cell}}{\theta n}\right)}{(2\alpha-2)(2\alpha-1)\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}} \\ &\approx \frac{2\pi P_t}{\left(1+\sqrt{\frac{2C_{cell}}{\theta n}}\right)^{2\alpha-1}}, \qquad (25)\end{aligned}$$

where we used for (23) that $\frac{r}{(1+r)^{2\alpha}} \geq 0$ if $r \geq 0$, and $f(\gamma)$ is a function related to the location of the receiver and the sender nodes in a square cell. It can be easily shown that $f(\gamma) < \sqrt{2}$ for any γ and any location of the nodes. To get (24), we use the Taylor expansion

$$\left(1+\sqrt{\tfrac{2C_{cell}}{\theta n}}\right)^{2\alpha-1} = 1+(2\alpha-1)\sqrt{\tfrac{2C_{cell}}{\theta n}}+\tfrac{(2\alpha-2)(2\alpha-1)}{2}\tfrac{2C_{cell}}{\theta n}+o\left(\tfrac{2C_{cell}}{\theta n}\right).$$

From (4), $a(n) = \frac{2C_{cell}}{\theta n}$ is very small for large n. Therefore, the additional terms of the Taylor series expansion are negligible. □

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