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The mathematics connection : a curriculum promoting mathematical application through the home-school connection

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The Mathematics Connection:

A Curriculum Promoting Mathematical Application Through the Home-School Connection

A thesis submitted in partial satisfaction of the requirements of the degree of Master of Arts

in

Teaching and Learning (Curriculum Design)

by

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The thesis of Kristin Mie Komatsubara is approved and it is acceptable in quality and form for publication on microfilm and electronically:

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Chair

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2008

DEDICATION

This work is dedicated to my parents, who were my first and most influential teachers.

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I would like to acknowledge my family, colleagues, professors, and members of my cohort who supported my efforts in completing this project. I owe special thanks to my professor, Claire Ramsey for her honest feedback and to my advisor, Caren Holtzman for her endless support. I also owe great thanks to my father, Kerry Komatsubara who offered thoughtful insight where mine was lacking. Finally, the students and families of my seventh grade class deserve my gratitude for willingly participating in this project.

ABSTRACT OF THE THESIS

The Mathematics Connection: A Curriculum Promoting Mathematical Application

Through the Home-School Connection

by

Kristin Mie Komatsubara

Master of Arts in Teaching and Learning (Curriculum Design)

University of California, San Diego, 2008

Caren Holtzman, Chair

The Mathematics Connection aims to increase student mathematical application and self-evaluation through collaborative home math activities. The curriculum views students as individuals, who interact with members of their school, home, and community to access and construct knowledge. This approach is based on research

suggesting analysis of multiple strategies support student computational fluency. The curriculum is viewed through the lens of Lave's situated learning theory, which implies learning must occur in an authentic context through social interaction.

The Mathematics Connection was implemented in one seventh grade Pre-Algebra class during the 2007-2008 school year at a charter middle school. Outside of the classroom, students collaborated with a mentor to tackle rigorous math challenges. Mentors included parents, elder siblings, coaches, and tutors. Students presented and evaluated multiple strategies to recognize the strengths, weaknesses, and effectiveness of each strategy. Changes in student reflection and mathematical application were measured by evaluating student written reflections and solutions on a standards-based rubric, examining student interviews, organizing family responses to an online survey and interpreting classroom interactions. Through collaborative home experiences, student presentations, writing opportunities, and teacher scaffolding, students improved their mathematical reasoning, mathematical modeling, mathematical flexibility, and ability to self evaluate. Specifically, students developed multiple approaches toward problem solving, which assisted them in deepening their understanding and identifying the correct solution. While the general trend showed improvement in mathematical application, individual student progress and response to The Mathematics Connection varied.

I: BUILDING MATHEMATICAL APPLICATION THROUGH FAMILY INVOLVEMENT

> "It is a paradoxical that many educators and parents still differentiate between a time for learning and a time for play without seeing the vital connection between them." ~Leo F. Buscaglia

Family involvement and its relationship to student achievement, motivation, and behavior has been highly debated in education. Educators acknowledge a relationship between families and students but disagree about the nature of the relationship. Researchers have yet to fully explore the influence of multiple home-school factors including the type of family involvement, amount of family involvement, learning activities, and home-school communication on student achievement. While family involvement has often been associated with elementary grades, educators have taken little action to involve families at the middle and high schools. This may be a result of students' developing independence and responsibility in combination with a lack of teacher preparedness to incorporate family activities. It is a constant challenge to fuse the boundaries between home and school.

Recently, I transferred teaching positions and started working in a charter middle school. Since the school serves a small population of a little over 300 students, teachers and students form close relationships. While I appreciated working closely with students, I was most interested in the interactions I had with their parents. As a middle school teacher, I manage multiple classes and finding time to meet with each family individually is challenging. However, the limited conversations I had with parents were very insightful and informative. Parents helped me to see each student in a new light as they shared information about their children's personal experiences, interests, and academic challenges. Although I had expertise in teaching, my parents were experts about their children. It occurred to me that families were untapped funds of knowledge that could support student learning outside of the classroom. My idea seemed fundamental but foreign because I had never looked beyond the school environment. I sought to build communication with families and view them as education partners. The Mathematics Connection is my attempt at creating curriculum that bridges the home and school gap.

In my mathematics classroom, I emphasize collaborative learning where students actively co-construct knowledge. Visitors are often surprised when observing my classroom because they expect to see a teacher-centered lecture with students passively taking notes. In contrast, my classroom is a dynamic community with students using language to exchange, internalize, and create knowledge. As a teacher, I provide feedback and guide student learning. I teach students to develop mathematical application, which consists of mathematical reasoning, modeling, and flexibility. Students learn math content through authentic contexts. In the end, I wanted students to recognize their math skills as valuable and applicable to their surrounding community.

To extend learning beyond the classroom, I invited families to participate in The Mathematics Connection. I created a series of home activities to involve families in mathematical puzzles that would promote learning math but still be enjoyable and interesting. To ensure that learning at home was valued, I provided opportunities for students to present their solutions to home math activities in class. The Mathematics Connection provided a rich learning experience that allowed students to develop and transfer mathematical knowledge between school and home. This approach is student-centered and allows families access content from multiple perspectives. As a result, The Mathematics Connection forms meaningful relationships with families as partners in improving student mathematical achievement.

II. ESTABLISHING A NEED FOR MATHEMATICAL APPLICATION IN HOME MATH ACTIVITIES

Middle school is a transition period into adolescence. It can be a challenging time for many students because they are no longer learning in a self-contained classroom. Instead, middle school pushes students to develop an independence and responsibility of managing multiple assignments, courses, and teacher expectations. Without clear lines of communication between parents and teachers, most of the responsibility of school falls upon a student. This transition between elementary and middle school allows many students to fall into academic and personal holes such as a drop in student grades, decreased motivation, and low self-efficacy.

Mathematics Achievement in the United States

With the implementation of No Child Left Behind, the United States public schools have seen a push for improved test scores particularly in mathematics. The United States is one of the most powerful and wealthiest countries in the world but our mathematics performance and education system do not reflect this. The Trends in International Mathematics and Science Study (TIMSS) is a study dedicated to comparing the mathematical and scientific understanding of students in various countries. In 2003, 46 countries participated in the TIMSS, at the fourth grade, eighth grade, or both grade levels. The fourth grade study evaluates students' understanding of numbers, geometric shapes, measurement and data display. By the eighth grade, a second study reevaluates students on all fourth grade categories plus algebraic operations. The TIMSS reveals that by middle school, American students understand and know less mathematics content than their counterparts in many other Asian and European countries (NCES, 2004). Out of 46 countries that participated in the TIMSS study, the United States ranked 15th and scored over 100 points below the top ranked country, Singapore (NCES, 2004).

A second international study highlights the United States' need for improvement and change in mathematics teaching and curriculum. The Program for International Student Assessment (PISA) tests 15 year-old students' ability to apply mathematical skills and reasoning. Out of 57 international educational systems participating, 30 participants were Organization for Economic Cooperation and Development (OECD) countries and 27 participants were subnational education systems. On average, U.S. students scored 474 points on the mathematics literacy scale; 24 points lower than the OECD average (NCES, 2007b). Overall, U.S. students ranked 35th out of 57 participating countries and education systems (NCES, 2007b). These international studies indicate that U.S. students struggle to master mathematical content and apply their mathematics knowledge. This information is an alarm for educators and government to adequately prepare our students for a globally competitive society.

In 1999, the PISA revealed that about 23 percent of 13 year old American students were at or above proficiency in moderately complex procedures and reasoning. Only 0.9 percent of American students were at or above proficiency in multiple step problem solving and algebra. However, the majority of students had mastered numerical operations and beginning problem solving (NCES, 2004).

Math Achievement in California

The National Assessment of Educational Progress (NAEP) assesses student achievement in United States public schools in all subject areas. In mathematics, NAEP tests students at the fourth and eighth grade in five content areas: number properties and operations, data analysis and probability, measurement and geometry, and algebra. The NAEP mathematics scales range from 0 to 500. In 2007, California eighth graders attained an average scale score of 270 points, which was 14 points higher than the average scale score in 1999 (NCES, 2007a). Although students displayed progress in math achievement, out of the 52 states and other jurisdictions that participated in the 2007 eighth-grade assessment, California was only higher than four jurisdictions and performed around the same level as four other jurisdictions (NCES, 2007a). This left 43 jurisdictions performing above California's average scale score.

While California mathematics scores are slowly rising, the achievement gap between selected groups is still present (NCES, 2007a). In 2007, NAEP assessment results of eighth grade students in mathematics show the average score for Hispanic students was 31 points lower than the average score of White students. In 1990, the average score for Hispanic students was lower than that of White students by 34 points. The NAEP data show similar results for economically disadvantaged students. Students, who were eligible for free/reduced lunch prices, a factor of poverty, scored 26 points lower than students, who were not eligible for free/reduced lunch prices in 2007. The gap between group performances has slightly decreased but still remains close to the 30 point difference in 1996. NAEP results also reveal that the range of student scores between the 75^{th} percentile and 25^{th} percentile has increased from 1990 to 2007 (see Figure 1). In 1990 the score gap between the 75^{th} percentile and 25^{th} percentile was 51 points but by 2007 the gap had stretched to 54 points (NCES, 2007a).

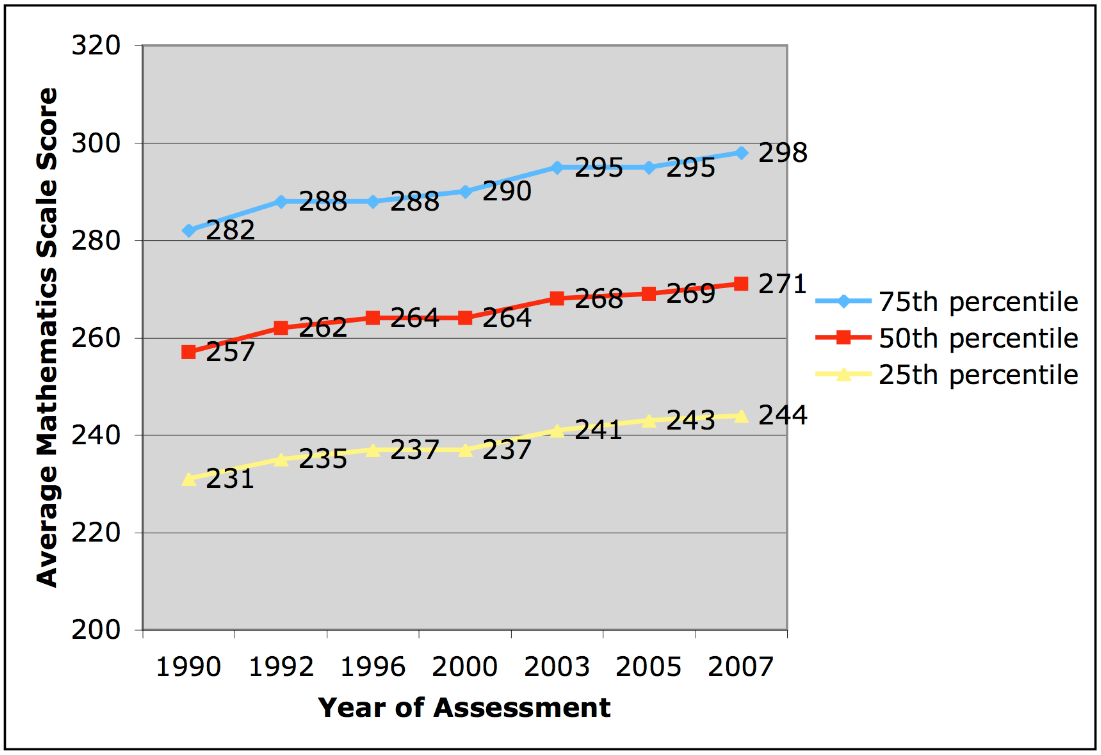


Figure 1:
California Mathematics Scores at Selected Percentiles

While NAEP data suggests general math achievement of all California students is gradually increasing, score gaps between selected groups have alarming results. Not only are the gaps between student groups still apparent, but underperforming students of color, socio-economic status, and language compose a large portion of California's student population. In 2007, California public schools enrolled over six million students with 48.5 percent of students eligible for free/reduced lunch prices, 24.9 percent enrolled in limited-English proficiency programs, and 48.5 percent identified as Hispanic (NCES, 2007a). These categories do not include all California's underperforming minority student groups such as Black and American Indian/Alaskan Native students. The presence of minority student groups makes it clear that California public schools must provide equal access to curriculum and resources in order to raise student math achievement.

Student Transitions from Elementary to Middle School

In addition to content-based assessment, student surveys indicate that as students progress through school from first to twelfth grade, there is steady decline in student academic self-perception (Jacobs, Lanza, Osgood, Eccles & Wigfield, 2002). The study used Hierarchical Linear Modeling to track changes in beliefs of students from childhood to adolescence in mathematics, language arts, and sports. Student transitions from elementary to middle school show that there is a decline in student motivation and self-efficacy (Jacobs et al., 2002). The link between decrease in student motivation and decline in student achievement occurs across all subjects including mathematics. Research suggests that this downward trend in student-belief occurs because "they are reality based and inevitable in any skill-based domain as children become aware of others' levels of competence and where they fall in the 'pecking order'" (Jacobs & Eccles, 2000). As students progress through school, they enter and encounter a larger student population and become aware of potential competitors. They vie for a limited number of spots that become more competitive as the population increases. In mathematics, students recognize competence and prestige through tracking. For all but a limited number of students entering advanced math courses, student self-efficacy decreases. Jacobs et al. (2002) emphasize that as children make school transitions, the researcher has little control over the courses students will enroll in, teaching methods, family support, and school culture. Therefore, the declining trajectory presented in this study does not take into account common factors experienced inside and outside of the classroom (Jacobs et al., 2002). These common factors may have influenced the data collected.

Students Entering STEM Careers

The number of students currently entering and completing fields of science, technology, engineering, and mathematics (STEM) in the United States is not promising. The National Center on Educational Statistics (NCES) reveals the number of bachelor's degrees conferred in the combined field of engineering and engineering technologies decreased by 1.5 percent between 1990-1991 and 2004-2005 (2006). The number of mathematics and statistics degrees awarded decreased by 0.3 percent between 1990-1991 and 2004-2005 (NCES, 2006). These decreases are dismal when compared to the total number of bachelor degrees awarded. From 1990-1991 to 2004-2005, the overall number of bachelor degrees increased by 31.9 percent (NCES, 2006). Other fields of study such as business, visual and performing arts, philosophy, and journalism have drastically increased the number of bachelor degrees awarded. The NCES (2006) reports from 1997-1998 to 2004-2005, universities awarded 34.3 percent more business bachelor degrees and 55.5 percent more visual and performing art degrees.

Challenges of Family Support

The literature investigating parent involvement and student outcome has been extensive (DePlanty, Coulter-Kern, & Duchane, 2007; Sheldon & Epstein, 2005; Xu & Corno, 2003; Okpala & Smith, 2001; Balli, Demo & Wedman, 1998). Yet, multiple factors including but not limited to socioeconomic status, ethnic background, parent educational level, and parent-teacher communication influence research findings.

Therefore, parent involvement is an educational issue that researchers continue to explore from various perspectives.

The amount of parent involvement, particularly in the school setting decreases from elementary to middle school (NCES, 2005; Paulson & Sputa, 1996). However, there has not been significantly publicized data and large studies conducted on the amount of parent involvement that occurs in any level of education. Researchers conducted numerous studies on the effects of parent involvement, reasons for parent involvement, and ways to involve families in curriculum (Green, Walker, Hoover-Dempsey & Sandler, 2007; Epstein & Sanders, 2006; Sheldon & Epstein, 2005; Anderson, 1997). Most research linked with family involvement lacks extensive numbers of schools, students, and resources. Few studies related to parent involvement observe the long-term effects on student outcomes. While previous study findings provide a window of understanding into family involvement, educators cannot use these studies to draw overall or definitive conclusions.

A limited number of parent involvement studies examine parent-student relationships outside the school setting, where students spend most of their time. This lack of research is likely because it is difficult to observe and document interactions that occur in outside of the controlled school setting. It is also unclear how researchers are defining family and parent involvement. Many of today's families are no longer nuclear with a father, mother, and children. The composition, beliefs, experiences, and identity of each family are unique and teachers cannot assume that students come from "traditional" stereotypical family backgrounds.

Parent Influence on Student Outcome

Although school provides a structured learning environment, children encounter many informal learning experiences outside of the school setting. Children spend the majority of their time with their families and surrounding communities. While teachers strive to improve their classroom curriculum, they should recognize the potential influence of parents, families, and communities outside of the classroom. The majority of curriculum reform concentrates on instruction within the classroom and practitioners have neglected the developing connection between home and school in academic reform (Sheldon & Epstein, 2005). For example, researchers Okpala and Smith (2001) conducted an investigation to examine how parent involvement, socioeconomic status, and access to instructional resources affected student mathematics achievement. The study included fourth graders and their families of a low-income neighborhood in North Carolina. The study compared the three factors and concluded that socioeconomic status most strongly influenced the child's academic performance. While socioeconomic status may be linked to student performance, Okpala and Smith's (2001) study focused solely on parent involvement in the classroom and neglected to study the effects of home interactions. The study indicated there was a low correlation between the amount of parental volunteer hours and student mathematic achievement. These findings are reasonable when considering the challenges families of lower socioeconomic status must overcome. However, Okpala and Smith's data does not provide strong evidence about the influence of the home-school connection on academic performance.

Alternative studies indicate that at-home parent involvement with school-related activities has positive influences on academic achievement. Desimone (1999) analyzed parent and student surveys from the National Education Longitudinal Study (NELS), which included survey data and standardized test scores for eighth graders across the nation. In an effort to categorize types of parent involvement, Desimone applied Epstein's parent-involvement framework. Epstein's (2002) framework split parent involvement into six categories: (1) parenting practices at home; (2) home-school communication; (3) volunteering in the school setting; (4) learning at home; (5) decision making on school policies; and (6) collaborating in community. Since achievement is a general outcome, Desimone (1999) measured achievement through student grade point average, standardized math test scores, and cognitive reading test scores. Research results vary in relation to a student's family income and ethnic background but suggest that of Epstein's (2002) six types of parent involvement, learning at home through parent-student interaction is one of the strongest influences on student achievement (Desimone, 1999). Future models can expand on Desimone's (1999) findings by investigating the specific types of parent-student interactions and their effect on student motivation, self-efficacy, and homework habits.

In a separate study conducted by Sheldon and Epstein (2005), content-focused, learning-at-home activities consistently and positively increased percentages of students proficient in standardized tests. Focus on Results in Math was a project designed to study the measureable effects of school, family, and communities partnerships on students in elementary through high school grades. Sheldon and Epstein (2005) attempted to link the effects of family involvement on student outcomes such as behavior, mathematics achievement, and attendance. Participating schools reported an increase of 6 percent more students scoring at or above satisfactory levels in math.

However there was a wide range of performances from a decrease in 18 percent to an increase in 27 percent. While results suggested that home-school connections supported student learning, data analysis indicates that the quality of implementation was more significant than the use of an activity (Sheldon & Epstein, 2005).

Parent Involvement in Middle School

Recent trends indicate that parent involvement in education has increased. Parent involvement measured by attendance at a meeting, school event, or volunteering on a committee, rose between 1999 and 2003 (NCES, 2005). In 2003, 88 percent of students in kindergarten through twelfth grade had parents who attended a school meeting compared to 78 percent in 1999 (NCES, 2005). While overall parent involvement appears to be increasing, there are still discrepancies between ethnicity, parent education, parent language, socioeconomic status, and grade levels.

As students grow and enter middle school, there is a decline in the amount of parent participation in their child's education. Parents are more likely to volunteer and attend school events when children are in elementary school (NCES, 2005). In 2003, the NCES (2005) reported over 90 percent of parents of kindergarten through fifth grade students and 75 percent of middle school students' parents attended a meeting with teachers. By high school, the percent of parents attending parent-teacher meetings continues to drop to 59 percent for ninth through tenth graders and 53 percent for eleventh through twelfth graders (NCES, 2005).

Exploring the Home-School Connection

Learning at home has positive outcomes on student achievement (Desimone, 1999), but this type of learning requires a commitment of time and effort from both families and teachers. Homework can cause tension and frustration between teachers, parents, and students, if not properly implemented. Teachers often report a lack of homework completion and students report time invested in homework takes away from extra-curricular activities (Hoover-Dempsey, Walker & Reed, 2002). Parents tend to focus on the stress homework causes between family members (Hoover-Dempsey et al., 2002). However, the effort of involving parents in home activities may be well merited. Although most studies support and recognize the need for parent involvement (DePlanty et al., 2007; Sheldon & Epstein, 2005; Xu & Conner, 2003; Desimone, 1999), many teachers lack substantial training to provide home curriculum for parents. Research indicates teachers need in-service programs such as Hoover-Dempsey, Walker, and Reed's (2002) Teachers Involving Parents (TIP) program in order to improve their sense of efficacy and their belief about parent's efficacy towards student achievement. Hoover-Dempsey et al. (2002) highlights the potential gains from effective family-school communication: (1) increased student achievement, (2) improved student behavior, (3) increased parent satisfaction with the quality of student education, and (4) sustained parent support for student educational goals. The TIP program provides beneficial forms of family-school communication that are typically implemented to involve families (Hoover-Dempsey et al, 2002). The TIP program (Hoover-Dempsey et al, 2002) and Epstein's (2002) parent involvement framework can provide guidelines for creating a successful home-school connection.

Looking to Research

Standardized assessment data such as the TIMSS, PISA, and NAEP suggest that there are fundamental problems with mathematics instruction in the United States, and particularly California. Results from the NAEP suggest that California students, especially minority students are performing noticeably lower than the rest of their U.S. peers. The underachieving performance of minority students is a red flag for educators that teaching practices must change in order to provide equal access to curriculum.

Data collected from the NCES (2007b) reveals that fewer U.S. students are entering STEM careers today than 15 years ago. The NCES (2007b) results show that even though the total number of bachelor degrees continues to increase, the number of degrees awarded in STEM fields decreases. These trends suggest a lack of preparation, interest, and success in earlier student science and math education.

NCES (2005) data exhibits that parent participation decreases as students transition from elementary to secondary school settings. This decline in parent involvement has implications on mathematics achievement since numerous studies (Sheldon & Epstein, 2005; Pezdek et al, 2002; Desimone, 1999) have shown a strong relationship between parent involvement and student achievement. In the chapter that follows, I explore the research on how the home-school connection can influence and improve student mathematical achievement. Research assists educators in making informed decisions about curriculum development and implementation.

III. REVIEW OF RELEVANT LITERATURE

This chapter examines research supporting families' positive influence on developing students' mathematical knowledge. The chapter begins by outlining three sociocultural constructs of situated learning, semiotic mediation, and pedagogical content knowledge. The sociocultural constructs lay the foundation of ideal conditions for learning in general. Next, I explore the literature highlighting effective methods of strengthening students' mathematical reasoning, mathematical modeling, and mathematical flexibility. Finally, I conclude with research advocating for family involvement in learning math.

A Sociocultural Approach to Learning

Vygotsky's Contribution to Sociocultural Learning

There are multiple learning theories that support mathematical development through active participation inside the classroom, at home, and within the community. Sociocultural theorist Vygotsky first suggested that learners co-construct knowledge. In referring to co-construction, Vygotsky states that learners wield language to interact with other members of a community and create a shared understanding (Wink & Putney, 2002). Language is a powerful tool, which forms and expresses a learner's level of cognition (Wink & Putney, 2002).

Recognizing the importance of a sociocltural context, Vygotsky maintains that students learn by extending their understanding beyond the knowledge and skills they possess independently. Vygotsky's zone of proximal development refers to the distance between a child's actual developmental level and the potential developmental level that can be reached through problem solving and expert guidance (Wink & Putney, 2002). Vygotsky observed that even if a child was not developmentally ready for a concept, the child could still achieve and practice the concept through an academic experience with a more advanced learner (Wink & Putney, 2002). Eventually the learner will be able to perform the same academic activity independently. The intellectual skills children acquire impact how they interact with experts in specific problem-solving opportunities. Research suggests that assisted learning should be completed at the highest level of the zone of proximal development (Wells, 2007). While teachers often assume the role of an advanced learner, all individuals can provide contributions to group learning.

Situated Learning Theory

Students often perceive learning mathematics as a series of abstract concepts removed from an accessible context. Sociocultural theorists Lave and Wenger (1991) refute the over simplified perception that a learner simply receives factual knowledge. Repetition of the same information, same activity, and same environment does not spontaneously produce learning. Situated learning theory builds from Vygotsky's study of sociocultural learning and suggests that in addition to using language as a tool for learning, all learning is embedded within a social activity or context (Lave & Wenger, 1991). Lave and Wenger (1991) encourage educators to view learning as a "cognitive apprenticeship." A cognitive apprenticeship guides students to interact with more advanced members of their community using language as a tool to mediate and exchange ideas. Students gain skills, experience, and knowledge from their interactions within the social community. They participate in community work until they can become a fully contributing member.

In addition to viewing learning as changes in social identity through cognitive apprenticeship, Williams, Linchevski, and Kutscher (2008) expand the concept of situated learning. A learner participates in multiple, dynamic communities and develops a unique identity within each community (Williams et al., 2008). Applying this idea to education, students actively participate with members of their schools, families, and surrounding communities. They take on a different role in each community and move freely between communities.

Semiotic Mediation

As suggested from the work of sociocultural theorists, language plays a central role in learning and understanding (Wells, 2007; Wink & Putney, 2002; Lave & Wenger, 1991). The situated learning theory maintains that in order for communication to occur, students must encounter an authentic challenge. A challenge, which sparks interest, encourages students to apply their skills in meaningful ways. As students communicate their thoughts, they use of signs and symbols to negotiate meaning. This negotiation is a struggle as learners respond and attempt to internalize and express their understanding with other members of a community. Socioculturalists refer to this struggle with language as semiotic mediation (Williams et al, 2008; Wells, 2007). Semiotic mediation can take many forms such as reflective writing, drawing, and oral discussion.

Thought and language are intertwined in semiotic mediation with one informing the other. Sometimes learners have thoughts but cannot express them in words. At other times, learners recognize words but fail to define them. Learners need both words and thoughts to use language to create complex thoughts. Wink and Putney clarify this idea of the relationship between thought and language.

> In other words, we use language, in our action of speaking, as a tool for developing thought, and, at the same time, we develop language through thought. This reciprocal relationship would then allow us to believe that the social action of using language could lead cognitive development (p. 89).

The situated learning theory believes children are social creatures, who want to use language to solve problems. Parents, teachers, and classmates can act as collaborators and engage in semiotic mediation with learners to develop mathematical concepts and skills. Parents are ideal partners for mathematical mediation because most children feel comfortable discussing ideas and receiving feedback from their parents. There is an established relationship of trust and respect that can be used when collaborating on homework assignments or math projects. However, researchers warn that this approach to education is unfamiliar to most practitioners (Wells, 2007). While semiotic mediation may hold promising results for learning, it is an area of study that has yet to be developed in diverse school settings.

Pedagogical Content Knowledge

Pedagogical Content Knowledge (PCK) refers to specialized skills and knowledge related to a subject area that expert teachers possess (Rowan, Schilling, Ball, & Miller, 2001). PCK combines effective pedagogical strategies with subject matter knowledge. Although educators generally associate PCK with teacher development, students can also use PCK to gain a deeper understanding of mathematical concepts. PCK encourages students to develop cognitive characteristics of expert learners. Students use PCK to chunk meaning into patterns, organize information, recognize relationships, and retrieve relevant information (Bransford, Brown, & Cocking, 2004). Ultimately, PCK provides a foundation for students to apply their knowledge and make their ideas accessible to others.

Self-Evaluation Through Reflection

Effective teachers provide opportunities for students to reflect and develop self-evaluation skills. Through consistent feedback from teachers, peers, and family members, students refine their self-evaluation skills by learning to assess their own work and the work of their peers (Bransford et al., 2000). Self-evaluation promotes better student understanding and reasoning using a sociocultural approach to learning. The process requires learners to use language to interpret, analyze, reflect, and respond to their work (Bransford et al., 2000).

A study conducted with fifth and sixth grade students evaluated the effects of self-evaluation training on mathematics achievement. Researchers engaged students in four types of self-evaluation training. Students defined evaluation criteria, applied evaluation criteria, received feedback on their self-evaluation, and created a learning action plan based on their self-evaluation. At the end of implementation, students who received self-evaluation training outperformed their peers who did not have training (Ross, Hogaboam-Gray, Rolheiser, 2002). Researchers assessed student achievement using a performance test. Teachers evaluated the performance test on a three-fold rubric, which judged the strategy generated, accuracy of concepts, and communication of solution. However, implementation only lasted eight weeks and teachers reported that students, even in the treatment group, were reluctant to self-evaluate. Teachers noted that many students lacked the vocabulary necessary to reflect and analyze their work (Ross et al., 2002). These findings indicate that self-evaluation may be even more effective at promoting mathematical achievement but it may require long-term training.

Mathematical Knowledge and Application

Even before children enter school, they are natural and curious learners exploring the world around them. Young children observe, question, and make connections between their previous experiences and current investigations. They engage in authentic problems and attempt to understand the world around them through modeling and discussion with a more advanced learner. This advanced learner is usually a parent, relative, or elder sibling within the family. However, as students enter and continue through the schooling system, some students struggle to obtain abstract mathematics concepts. This decline in mathematics performance is evident particularly in advanced courses as students enter middle and high school (Jacobs et al., 2002). This trend in mathematics is troublesome for the future of our education system and increasingly global and democratic society. Research reveals that a strong and stable mathematics experience is needed to successfully enter college and the workforce (Schmidt, Wang, & McKnight, 2005; Schmidt, McKnight, & Raizen 1996).

Building Mathematical Reasoning

Most teachers agree that understanding is vital to learning. However, teachers disagree on how to define understanding and the teaching practices that foster understanding. Results from the PISA suggest that although many U.S. students are able to perform mathematics skills, they are not able to apply their skills to solve authentic problems (NCES, 2007b). This data reveals that current teaching and learning practices are ineffective in promoting mathematical understanding. Not surprisingly, research supports this claim that traditional, direct instruction is ineffective in developing true mathematical reasoning and mastery of mathematical concepts (Schmidt et al., 2005).

The National Council of Teachers of Mathematics (NCTM) dedicates two standards solely to mathematical reasoning. All students from kindergarten through twelfth graders are encouraged to develop and evaluate mathematical proofs, evaluate mathematical strategies, and use language to express mathematical ideas (NCTM, 2004a). Even while mathematical reasoning and communication are national math standards, U.S. students failed to fully develop their reasoning skills and achieve these standards.

An examination of the six highest achieving countries in mathematics and science from the TIMSS revealed teaching trends and content standards that did not align with U.S. trends and standards (Schmidt et al., 2005). In higher achieving countries, mathematics teachers encouraged students to focus on connections between concepts and solutions. These math teachers pressed students to provide justification for their solutions through questioning and feedback. In high achieving classrooms, Schmidt et al. (2005) observed students identifying their misunderstandings, clarifying their reasoning, and justifying their solutions in peer and small group discussions. In general, classrooms of the six highest performing countries on the TIMSS dedicated a greater amount of instructional time to fewer math concepts than U.S. classrooms, which covered more concepts repeatedly throughout grade levels (Schmidt et al., 2005). Similar studies have observed instructors of undergraduate math classes hastily introduce pre-packaged formulas and theorems, before allowing students to understand how these math rules developed (Harel & Sowder, 2005). In an attempt to promote learning environments similar to the highest achieving countries in mathematics, teachers provide students with complex puzzles. Effective and engaging math problems should be multifaceted and allow for multiple routes to a solution, since students learn in different ways and at different rates (Myers, 2007). It seems reasonable that promoting discussion and mathematical reasoning rather than practicing a teacher rule would be beneficial to learning, especially when the discussion leads to mathematical rules.

This understanding of mathematical reasoning, learning, and teaching can be extended beyond classroom practice into homework assignments. Students can use resources such as community members and family members to further engage in discussion and questioning of mathematical application. Working outside of the class environment provides students with opportunities to view mathematics application in authentic contexts. As educators, we must facilitate this home learning process by creating home assignments that reflect classroom practices of mathematical reasoning.

Mathematical Modeling

Mathematical modeling tasks are encouraged to help students develop authentic application and mathematical concepts. The NCTM (2004a) highlights mathematical representation as a national standard for all students from kindergarten through twelfth grade. The national standard states the students will use representations to model and communicate mathematical ideas (NCTM, 2004a).

Mathematical modeling allows students to reveal the structure of a mathematical situation. Students attempt to internalize and organize information in representations that convey mathematical meaning. Modeling can take many forms. Models include tables, graphs, pictures, equations, and other representations. An observation study conducted by Zbiek and Conner (2006) revealed that not only did mathematical modeling help students clarify and illustrate their mathematical ideas but mathematical models also generated discussion between teacher and students, as well as the students themselves. In the study, students attempted an authentic problem of determining a location for a hospital between three cities. Students engaged in small groups discussions and acknowledged the mathematical properties and principles linked to the situation. Zbiek and Conner (2006) observed students tackle concepts of triangles, intersections, angles, and perpendicular bisectors through the models they created. The teacher questioned students further by asking about the relevance of population density.

Zbiek and Conner (2006) concluded that modeling served as a venue for dissecting students' understandings. The process of modeling generated discussion between learners and required students to rethink and revise their models. Zbiek and Conner's findings fall in line with previous research that describes modeling as an upward cyclic process to a shared solution (Lesh & Harel, 2003; Lesh & Lehrer, 2003). While mathematical modeling has potential to deepen student understanding, observations of researchers indicate that most classrooms incorporate mathematical modeling as an engaging alternative to learning math in a more traditional manner (Zbiek and Conner, 2006).

Mathematical Flexibility: Multiple Strategies in Mathematical Learning

Most mathematics problems can be solved using multiple strategies. The NCTM requires all students from prekindergarten through twelfth grade to have experiences applying multiple strategies in math. The national standard states that students must "understand how mathematical ideas interconnect and build on one another to produce a coherent whole" (NCTM, 2004). Teaching through multiple strategies allows students to combine information into more complex understandings and avoid misinterpretations of a single strategy (Ainsworth, 2006). Several studies have revealed that multiple strategies can foster mathematics learning in specific contexts (Gross & Renkl, 2006; Ainsworth, 2006; Harel & Sowder, 2005).

Grosse and Renkl (2006) conducted two studies with student teachers in Germany. The first study tested the effects of single versus multiple strategies of learning a probability concept. Student teachers participated by studying single and various worked-out examples. Participants completed pre and post tests to evaluate their procedural skills and conceptual knowledge before and after studying the worked-out examples. The results indicated that multiple strategies fostered mathematical learning but instructional support had no positive effect (Grosse and Renkl, 2006). The second study repeated the steps of the first study but in addition to pre and post tests, required students to solve and present a self-explanation of their work. Researchers were not able to duplicate the results of the first study. Strangely, researchers failed to identify any effects of using multiple strategies. Grosse and Renkl (2006) even suggested that multiple strategies reduced some of the spontaneous learning that may have occurred. Differences in the two studies may have occurred because of cognitive overload in the second study. It is taxing for students to connect and apply multiple strategies while verbally explaining their thought process behind the work. The first study may have had positive results with multiple strategies because students focused solely on studying student work and their connections between solutions.

Teaching multiple strategies to solve problems may be more beneficial to developing students' mathematical concepts than their procedural skills. These findings agree with previous research related to multiple strategies and cognitive development (Harel & Sowder, 2005; Brenner et al. 1997). Multiple strategies can provide an opportunity for student to identify and analyze interrelations between different representations. Yet, students require substantial support in order to comprehend interrelations. Interrelations are complex concepts that instructors should directly teach and not leave to student discovery (Grosse and Renkl, 2006). Instructors build interrelations by prompting self-explanations and encouraging students to compare different solutions. Students learn to value all solutions but begin to understand why certain ways of reasoning are more valuable or applicable in specific contexts. While it is clear that multiple strategies promote higher levels of understanding math concepts, the process requires extensive time and effort from teachers and students (Harel & Sowder, 2005; Brenner et al., 1997). Researchers warn that teachers should be careful of cognitive overload when presenting multiple strategies.

Exploring the Home-School Connection

Homework and Student Achievement

Most teachers at all grade levels assign homework to improve student understanding and retention of material covered in class. However, educators disagree about the amount and type of homework that should be assigned to students at various grade levels.

Cooper, Lindsay, Nye, and Greathouse (1998) investigated the effects of homework on student achievement with students of various ages. Students, parents, and teachers completed questionnaires about their attitudes toward the amount of homework assigned. In addition, researchers collected student achievement measures in the form of standardized test scores and teacher-assigned grades. While research revealed there was a weak correlation between homework assigned and student achievement across grade levels, students in grades 6 to 12 showed a positive relation between homework completed and student achievement (Cooper et al., 1998).

Recent research findings built on Cooper, Lindsay, Nye, and Greathouse's (1998) study of homework and achievement. Trautwein (2007) compared the affects of time invested on homework and homework effort on student mathematical achievement such as standardized test scores and student grades. The study involved 500 German eighth graders. Trautwein (2007) measured student homework efforts with surveys monitoring their homework compliance. Students also reported the average time they spent on homework per night. Results indicated that homework effort was positively related to mathematical achievement in both student grades and mathematics test scores (Trautwein, 2007). There was no positive correlation between time spent on homework and mathematical achievement or time spent on homework and homework effort (Trautwein, 2007). These findings align with previous research suggesting a greater effort on homework results in higher academic performance (Trautwein & Koller, 2003; Cooper, Nye, & Linsday, 2000).

Recognizing that homework has potential to increase student mathematical understanding does not come as a surprise to educators. However, research confirms that homework must align with classroom practices in order for students to see homework as valuable. With this knowledge, educators must seek to provide home assignments that promote student interests, motivation, and cognitive strategies. There are many factors related to homework and student outcomes that have yet to be explored, including parent involvement with home assignments.

Types of Parent/Family Involvement

Epstein et al. (2002) outlines six types of parent involvement: (1) parenting practices at home; (2) home-school communication; (3) volunteering in the school setting; (4) learning at home; (5) decision making on school policies; and (6) collaborating in community. According to Epstein et al. (2002), a comprehensive program attempts to build a home-school connection through the six types of parent involvement. It is widely accepted that parent involvement and children's home environment influence children's performance at school (Epstein & Sanders, 2006; Van Voorhis, 2001; Desimone, 1999).

The national Parent Teacher Association (PTA) advocates for parent, student, and teacher collaboration. In 1997, the PTA collaborated with parents, researchers, and educational leaders to create parent involvement standards. Numerous organizations, state departments of education, school districts, and teacher education programs endorse the standards outlined by the PTA. Although the PTA (1997) uses the term "parent involvement," they refer to the broad category of adults who play an important role in the child's family life. Research has found parent involvement to have benefits for all parties involved (PTA, 1997). Most notably, students with involved parents showed included higher attendance, graduation rates, grades, and self-confidence (PTA, 1997).

While extensive reasons for involving parents exist, there are multiple challenges and factors that teachers must consider. Middle and high school students do not always welcome or appreciate parent participation in the school setting. As students exit elementary grades, they strive to develop their own identity and separate their home and school relations. On the same note, parents are often extremely busy. Many parents work multiple jobs or come from single-family households and do not have the time to volunteer inside of the school (NCES, 2005). Therefore, it becomes the schools responsibility to create a home-school connection through at-home curriculum.

Each type of Epstein's (2002) parent involvement is important to promoting a complete home-school curriculum. Yet, each type of parent involvement has different results. The following sections focus on research studying the influence of parent involvement on student learning at home.

Parent Involvement Through Home Activities

Interactive homework, which requires students to show and discuss mathematics skills with a family member has been shown to increase home-school communication, increase family attitudes toward involvement, and improve student achievement (Van Voorhis, 2001; Balli, Demo, & Wedman, 1998).

In a study conducted by Sheldon and Epstein (2005) with fourth grade students, an analysis of multiple types of parent involvement indicated that at-home activities "consistently related to improvement in students' performance on mathematics achievement tests" (Sheldon & Epstein, 2005, p. 203). Participants included students and their parents from eighteen highly diverse schools. Before and after implementation, researchers measured student math performance on standardized achievement tests. In addition to analyzing standardized scores, researchers notes the types of parent involvement practices, schools implemented. Parent involvement practices included but were not limited to conducting parent workshops, providing families with information about contacting teachers, sending grade reports to families, and assigning math homework that required a family member to participate.

The study accounted for students' previous math achievement and found that schools which effectively assigned homework requiring parent-child interactions performed significantly higher than schools that did not include parent-involvement homework (Sheldon & Epstein, 2005). While this data shows a strong link supporting parent-involvement through home activities, it does not specify the type of home activities, instructions and materials provided for families, and the type of parent-teacher communication that occurred. Researchers also noted a wide range of student performance varying from an 18 percent decline to a 27 percent increase in different schools (Sheldon & Epstein, 2005). Again, the variation in school performance can be credited to the types of parent involvement and the methods and materials schools used to promote involvement.

Invitations for Family Involvement

Evidence suggesting family involvement as a positive influence on student outcomes abounds (Epstein & Sanders, 2006; Sheldon & Epstein, 2005; Epstein et al. 2002; Van Voorhis, 2001). However, educators are unclear about effective methods to encourage parent involvement.

In a small study conducted with middle school students, families received three prompts as invitations for becoming involved with home activities (Balli et al., 1998). The three home activities included a homework assignment without further instruction, a homework assignment with an invitation by the students only, and a homework assignment with an invitation for help by both the teacher and the student. The first group of students received home assignments that did not contain written or verbal instructions about inviting a family member to participate or help. The second set of students received homework assignments which provided suggestions to students about how to involve parents or family members with the home activity. The last group received homework assignments that provided suggestions to students and a direct request from the teacher for a parent signature. This last activity allowed a space for parents to provide feedback and suggestions about the home activity. After 20 home assignments, researchers surveyed students about the amount of parent support they received. Data indicated that parents were highly likely to offer support and services when both teachers and students request the help of parents with homework (Balli et al., 1998). Families of Group three students participated in 90 percent of the home activities while only 51 percent of families participated in activities with Group two students. Across the surveys, 87 percent of students who received family support reported that they had more than one family member helping them with their assignments at various times (Balli et al., 1998). Although research indicates that families are willing to support students with the proper invitations, the study does not provide any insight about the quality of family support provided (Balli et al., 1998).

Overall, studies suggest parents must be give clear instruction and invitations about how educators want them to participate (Epstein et al., 2002; Balli et al., 1998).

Challenges of Parent/Family Involvement

Thus far, research has established that targeted homework and parent participation in homework has positive effects on student achievement (Trautwein, 2007; Sheldon & Epstein, 2005; Balli et al., 1998). As students transition from elementary to middle school, there is a noticeable drop in the amount of parent involvement in the school and home environment (NCES, 2005). These findings raise two important questions for teachers. First, why is there a decline in parent involvement from elementary to secondary levels? Second, what are some challenges educators must overcome to effectively incorporate parents into the home-school curriculum?

There are numerous challenges associated with involving parents in school curricula (Gal & Stoudt, 1995). Parents often cite the increasingly challenging mathematic content, new methods of mathematics teaching, or student independence as reasons for decreased involvement (Gal & Stoudt, 1995). A survey conducted with middle school parents documented a decline in parent involvement as children aged (Constantino, 2007). Parents expressed varied concerns associated with participating in their child's education such as the increasing difficulty of homework assignments. Many parents no longer felt equipped to support their students with middle school and high school mathematics content (Constantino, 2007; Gal & Stoudt, 1995). Some parents lacked the educational experience to assist their children with homework while other parents had forgotten the skills required to teach their child at home (Constantino, 2007). Many parents felt intimidated by the academic environment and are uncertain of the best method of supporting their child at home (Constantino, 2007). Surveys indicate that parents have a wide range of concerns associated with providing student support (Constantino, 2007; Gal & Stoudt, 1995). This range of concerns suggest that teachers have numerous obstacles to address when considering effective ways of bridging home and school curricula.

Conclusion

Traditional mathematical teaching methods of lecture and independent practice have been linked with the decline in student motivation and academic performance from elementary to middle school (Roeser, Eccles, & Sameroff, 2000). However, current and innovative methods of problem-solving, small group discussion, and teachers as facilitators require parents to support children in new ways at home. In order for homework to be seen as valuable, it must promote the same type of sociocultural learning that occurs in classrooms. Changing mathematics curriculum to incorporate parents is not a quick fix. There are many challenges that must be addressed and the challenges require teachers to carefully and creatively seek solutions. It becomes the teacher's responsibility to educate parents about the benefits of modern mathematical methods and provide explicit instructions on how parents can support students outside the classroom. Together, sociocultural constructs and mathematical research support a need for change. In the following chapter, I review current mathematical curriculum available to building a home-school connection.

IV. REVIEW OF EXISTING HOME MATH CURRICULUM

Involving families in mathematical learning is not a novel idea. However, the number of quality resources promoting family involvement is limited. Creating new curriculum that encourages home-school connections requires an analysis of the curriculum that is currently available. I evaluated curricula based on mathematical content, organization, flexibility, and accessibility. An effective home-school curriculum should encourage families to build on children's previous knowledge, recognize mathematical application within the community, excite students and families about math, and reinforce multiple strategies of solving mathematical problems. I end the chapter with suggestions for exploration beyond the current existing designs.

Holt Pre-Algebra Series

The Holt, Rinehart, and Winston series (Bennett et al. 2004) is a California standards-based mathematics curriculum. The pre-algebra series includes teacher and student textbooks, worksheet generators, test and quiz generators, strategic problem solving handbooks, varied homework practices, and intervention activities. Compared to other traditional math textbooks, the Holt series is student-friendly, easy to follow, and organized. Large graphics generate interest and encourage the student to visualize the mathematical concept. The textbook provides a large bank of problems students may practice in a guided and independent section.

As a teacher, I appreciate the varied degree of difficulty for homework assignments. The textbook attempts to make connections between mathematics application and science, history, and art content. However, the major drawback with textbooks is a rigid structure that does not allow students to develop their own methods of problem solving. Simply offering word problems within a lesson does not necessarily provided opportunities to think critically and apply mathematical knowledge. Instead, many of the textbook problems are drills that repeat one isolated skill such as adding fractions, multiplying decimals, or balancing equations. There is little room for students to make mistakes, identify, analyze, and correct their errors. This is because every example and instruction in the math text is correct and leads students to only one line of thinking. Single strategy instruction limits instructional conversation and mathematical flexibility.

Naturally, students have varied methods of solving the same problem and educators should view each method as a valid strategy. Multiple strategies of solving problems can help students tackle problems from different perspectives. Each strategy builds a new level of mathematical understanding. When students encounter a problem with only one strategy and are not immediately successful, they are likely to quit. Homework that does not allow students the freedom to problem-solve forces them to perform math calculations without a deep understanding of the concept. As a result, students are less likely to later apply the math skills they gained.

The second concern with the Holt math curriculum is the lack of family involvement activities. Although home assignments are available in the Holt textbook, the assignments do not explicitly encourage students to seek the support of parents or families. This is important because home activities can be an opportunity for students to discuss their mathematical challenges and improve their reasoning. When children attempt to verbalize their thoughts, they share interpersonal understanding and internalize meaning through intrapersonal analysis. When families are involved, a stronger home-school connection is established. This connection supports the student from multiple communities and the family is more likely to view schools as partners and time working with the school as an important investment for their child.

While Holt's home assignments vary based on skill level, the assignments neglect to explicitly define the connection between activities and their application to an authentic context. This presentation of mathematics leads students to view problems as isolated skills. For example, when presented with the problem $6 + x = 15$, students understand that they must solve for the value of x. Some students are successful in determining the value of x, but most do not have a conceptual understanding of equations and variables. Students do not use authentic examples to represent the mathematical skills. However, students can easily understand that if they weigh two pumpkins that have a combined weight of 15 pounds and they know one pumpkin is 6 pounds, they can solve for the weight of the second pumpkin. When putting math into contexts, students see the purpose of using equations to solve for missing values. The idea of a variable (x) is no longer daunting and meaningless.

Online Resources

Explore Learning (2008) is an online site that offers interactive simulations called "gizmos" for math and science concepts. The gizmos are engaging tools for many students because they allow students to manipulate models to discover relationships and patterns. Modeling and manipulatives are powerful tools for allowing kinesthetic and visual learners an opportunity to connect abstract algebraic concepts with physical meaning (Zbiek & Conner, 2006; Lesh & Harel, 2003; Lesh & Lehrer, 2003). The steps of the gizmo are scaffolded so mathematical understanding is built and reinforced. While gizmos are effective for modeling mathematical concepts, they do not allow students to engage in discussion about patterns they observe. From teaching experience, the gizmo instructions are lengthy, academic and younger children often struggle to interpret the directions. When students do not understand the student exploration guide, many students face a roadblock. Without a teacher's assistance or parents equipped with mathematical vocabulary and training, students cannot continue their learning with gizmos. Gizmos provide an alternative support inside the classroom, yet many students are not able to access the gizmos from their homes because of technology deficiencies. With student diverse populations, schools face the challenge of balancing assignments that require home technology, which is unequally distributed among households. In summary, gizmos are effective tools for mathematical learning but should be limited to classroom use because they require extensive support from teachers.

The National Council for Teachers of Mathematics (NCTM, 2008) provides online "illumination" activities and lessons that are challenging and encourage complex and critical thinking. Similar to gizmos provided by Explore Learning (2008), the online illumination activities allow students to manipulate simulations to explore relationships and patterns within math concepts. As a teacher, I find the database of illumination lessons to be applicable in the classroom. Many of the illumination lessons engage students in authentic situations and motivate students to apply math in meaningful ways. The illumination lessons allow for variation in student skill level and multiple paths to achieving a solution. Students apply numerous mathematical skills in order to solve a complex problem. Scaffolded steps allow students to work at individual paces but still push students to work rigorously by building content and complexity at each step of the problem. Since these types of activities are challenging, they promote discussion between students, teacher and student, and possibly parents and students. Through this discussion, students clarify their understanding of the problem, justify their mathematical solutions, and recognize new solutions from their peers. These activities are applicable to the classroom but difficult to assign as homework because of specific mathematical questions involved. Many of the problems demand that students make algebraic connections from the situation presented. While a few students may be able to complete these activities without assistance, many students need the guidance of a mathematics teacher to draw these algebraic connections. Most parents will not have the training or mathematical background to help their child with traditional algebra but most parents will be able to engage in discussion with their child about possible solutions.

Marilyn Burns' Math Solutions

Burns is a veteran mathematics teacher, professional development specialist, and creator of numerous math curricula. Although Burns (2007) offers several books related to mathematical development, I focus on the *About Teaching Mathematics* resource. Burns provides engaging, open-ended math activities and lessons for teachers to implement in classrooms. While the activities encourage mathematical application and reasoning, the activities and lessons lack structure. Each activity cannot stand alone as a complete curriculum. Instead, Burns (2007) provides suggestions about how teachers should implement each activity in the classroom setting. Most of Burns' problems promote student discussion and allow students the freedom to tackle problems with multiple strategies. Burns (2007) argues that direct instruction of algebraic formulas should be presented only after students have gained a conceptual understanding of their purpose. A teacher should be responsible for guiding discussion and allowing opportunities for students to present their solutions in front of their peers.

Burns' activities and lessons are appealing because they are teacher and student friendly. The curriculum is flexible and Burns provides both suggestions for implementation and cautions about common student misunderstandings. Lessons can be skipped, reordered, and taught, as teachers deem most effective. Activities are appropriate for a range of learners because they offer a variety of access points and challenges. Students appreciate the activities because they are presented as a puzzle or game. Students enjoy working with manipulatives and tackling problems that are phrased with clarity.

While Burn's activities effectively promote mathematical understanding and application, a teacher must be present to explicitly teach math concepts through scaffolding. Burn's curriculum also lacks defined home-school connections. Most activities and lessons are limited to classroom implementation.

Family Math

Family Math is an in-home family math program currently used in schools within the San Diego Unified School District. Family Math attempts to utilize families as funds of knowledge outside the classroom. The program is a series of fun and interactive math challenges that require families and children to engage in math "play" and talk (Thompson & Mayfield-Ingram, 1998). All activities are based on the National Council of Teachers of Mathematics Standards for Curriculum and Instruction. These standards outline the most appropriate methods of instruction at specific grade levels. The activities promote application of numerous, integrated math skills rather than skill repetition like most textbook homework assignments (Thompson & Mayfield-Ingram, 1998).

Educators encourage parents to attend Family Math meetings, which are conducted by a classroom teacher, administrator, parent, or community college instructor. Instructors group Family Math meetings by grade level and discuss topics ranging from arithmetic, geometry, probability, measurement, logical thinking, and future STEM careers. Instructors invite community experts into Family Math meetings to discuss how they use math within their careers. Incorporating experts into meetings is an essential component of the Family Math curriculum resource because it inspires students to imagine their future careers. Experts act as role models to help children and parents understand that math is a powerful tool used to access higher education and promote social mobility. Meetings help parents to recognize the value of incorporating math in the home. Parents also receive a brief overview of content covered within the classroom and learn basic mathematics skills their children will need to be successful.

While answering parent questions about mathematical content, instructors caution parents about common misunderstandings students have. This helps prepare parents to watch for mistakes their children may make and to guide their children to an accurate understanding of math concepts. Besides mathematical knowledge, parents are equipped with supplies and materials that students may use when solving family math activities. While instructors offer and encourage Family Math meetings, the curriculum recognizes that not all families may be able to attend the event. Therefore, attendance at meetings is optional and all activities are equipped with instructions for parents so that they can still be successful implementing Family Math without training.

An average Family Math activity includes the objective of the activity, materials needed to implement the activity, steps parents should follow to guide their students through the learning process, and common student misunderstandings to avoid. This information acts as a teacher's voice instructing parents on how to interact with their children when working at home.

While Family Math creates an engaging curriculum for parents and children, this curriculum resource is structured in traditional methods of teaching mathematics. While, the activities allow some room for children to develop their own understanding and solutions for problems, I anticipate that this type of curriculum would not promote a significant amount of discussion to occur between parent and child, particularly with the child as a leader or problem solver. In Family Math, the parent acts as a teacher in the home setting, guiding the student through the activity.

Building on Vygotsky's research of the zone of proximal development and intersubjectivity (Wink & Putney, 2002), it seems reasonable to modify Family Math to incorporate more parent-child discussion. Family Math should provide parents with probing questions that give students the opportunity to clarify, prove, and judge their mathematical reasoning. In order to facilitate discussion, activities should be problem-based with many possible solutions. These solutions are ways of drawing students back into dialogue and learning within the classroom. Students should have the opportunity to reconnect the learning done at home and the learning done in the classroom and vice versa. This strong connection between parent, child, school, and home reinforces the application of math within all learning environments. Family Math strengthens this relationship by giving parents the opportunity to provide suggestions and questions for teachers after each assignment. Building bridges between funds of knowledge also shows students the value of teamwork between school and home, and helps them appreciate it.

In most middle schools, the courses students select determine the high school courses they can enroll in. Even further down the line, middle school mathematics influence the courses taken in college and the type of career students select. Therefore, it is important that students receive opportunities to further their knowledge no matter what level of understanding they possess. Family Math promotes equity in schools and attempts to provide a range of opportunities that challenge each student. The curriculum resource provides activities with fun and deep understandings of the math content but fails to scaffold the steps for advanced students to move beyond the basic assignment.

Conclusion

Various curricula fall short of fully capturing the potential of developing mathematical application through the home-school connection. After an exploration of the existing curricula, it is apparent that educators need to build activities that are flexible while maintaining rigorous standards. Effective curriculum must seek ways to incorporate social learning outside the classroom. Teachers can accomplish these goals by engaging families and students in fun, collaborative math activities. It seems reasonable to encourage families to explore mathematical puzzles with their children while allowing teachers to draw connections between home strategies and mathematical concepts.

To address the shortcomings of the present home math curriculum, I developed and implemented the Mathematics Connection in my seventh grade classroom. The following chapter describes my approach to building a rich, social learning experience that utilizes the home-school connection.

V. THE MATHEMATICS CONNECTION: AN APPROACH TO LEARNING MATH

Mathematics is a set of skills, strategies, concepts, and knowledge that students must manipulate and apply in various contexts. However many math courses in the United States emphasize direct instruction and skill repetition. Mathematics instructors lecture students and introduce math procedures without substantial explanation of their value or connection to the community outside of the classroom. Many students have difficulty simply memorizing math procedures and fail to make connections between their previous knowledge and the current content. When teachers present math as an isolated set of skills, students lack the context needed to apply and create meaningful math understanding.

Goals

The Mathematics Connection is a series of class and home activities designed to involve families in supporting student development and application of mathematical skills. Mathematical application consists of mathematical reasoning, mathematical modeling, and mathematical flexibility. In addition to teaching math content, The Mathematics Connection attempts to strengthen students' ability to self-evaluate by involving students in discussion and reflection. The home-school connection reinforces the problem-solving and collaborative learning occurring in the classroom.

The Mathematics Connection addresses the questions posed earlier:

1. In what ways does family involvement support student math application?
2. How can discussion about engaging, content-rich math problems promote student reflection and self-evaluation?

One goal of the Mathematics Connection is to improve student mathematical application. I define mathematical application as a combination of mathematical reasoning, mathematical modeling, and mathematical flexibility (see Figure 2). All three skills encourage students to apply and refine their mathematical understanding to obtain the "right" answer.

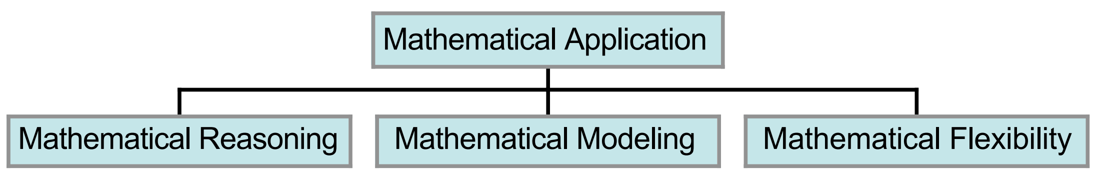


Figure 2:
Components of Mathematical Application

Mathematical reasoning requires students to analyze and explain their math procedures. Students use mathematical modeling to translate mathematical procedures into meaningful graphics such as pictures, tables, equations, and graphs. Mathematical modeling encourages students to develop standardized methods of algebraic expression. Finally, mathematical flexibility promotes students use of multiple strategies to solve and confirm their solution. Multiple strategies allow students to attack a problem from various angles and help students recognize the strengths and weaknesses of each strategy in specific contexts.

My second goal is to build student self-evaluation skills to monitor and reflect on their mathematical understanding and application. Students analyze their mathematical performance and strategies. Being able to identify their application of mathematical reasoning, modeling, and flexibility provides students with valuable reflective skills. Self-evaluation promotes elimination of previous misconceptions and future errors. Students also use self-evaluation to recognize the importance and limits of varied math strategies.

Goal three strives to build and strengthen a connection between home and school. Teachers view families and community members as important, untapped resources for student learning. Families and schools must communicate and work collaboratively to help students apply their math knowledge to authentic situations. Teachers urge students to extend their learning beyond the classroom into their local communities.

Instead of presenting many isolated mathematics concepts, the goals of The Mathematics Connection are highly integrated with the ultimate goal of mathematical application. Students tackle challenging, interesting, and authentic math problems with the support of classmates, teachers, and families. The curriculum makes connections between knowledge within the classroom and the home environment. I approached these goals using Lave and Wenger's (1991) situated learning theory for designing and implementing The Mathematics Connection.

Course Description

I designed The Mathematics Connection for students in grades six to eight but the curriculum can easily be adapted for students of all ages and levels. In the classroom, students engage in team discussions and attempt to tackle real-life, multi-layered problems. Students digest the problems and attempt their own strategies before the teacher introduces new math procedures. Students work collaboratively and share their understanding with classmates through modeling and formal presentation. Together, the teacher and students analyze the validity of student solutions. Teachers act as facilitators of student discussions, questioning students and attempting to eliminate misconceptions. Teachers do not push students toward one strategy but assist students in identifying the advantages and disadvantages of each strategy in a specific context. Each lesson presents students with an opportunity to deeply analyze and develop mathematical content and skills. The Mathematics Connection urges teachers to limit the number of concepts introduced and concentrate efforts on the depth of understanding.

Unlike traditional, independent, drill-based homework, The Mathematics Connection extends beyond the classroom and allows students to further their problem-solving skills with family and community members. Teachers create mathematical problems to target student interests and needs. Students recognize math problems as relevant and applicable to their environment. Outside the classroom, parents, older siblings, coaches or tutors act as facilitators of discussion, questioning students and pushing them to expand their reasoning. In The Mathematics Connection, students refer to discussion partners as "mentors". Mentors are not experts in mathematics content or teaching methods. Instead, mentors work as discussion partners who encourage children to develop their understanding of mathematical concepts and solutions. Since mentors are absent from class instruction, teachers encourage students to thoroughly explain the home math activity and any related math strategies presented in class. Students become responsible for carrying mathematical knowledge and strategies between their school and home environments. Students and mentors evaluate student performance based on a self-evaluation checklist, which requires students to monitor their mathematical reasoning, modeling, flexibility, and analysis. Teachers advise mentors to share any knowledge or experience they possess as this helps students recognize the personal value of mathematics application. In this mentor-student relationship, most students assume the role of an advanced student and have the opportunity to teach and apply their mathematical knowledge.

Components of The Mathematics Connection: An Approach to Learning Math

To explain and model mathematical content, students develop and use pedagogical content knowledge. Schulman defines pedagogical content knowledge as the ability or knowledge possessed by expert teachers (2001). Students assume the role of teacher during home math activities as they explain their strategies and solutions to their mentor. When presenting their predictions or solutions, students make knowledge accessible to their mentor and to peers by using mathematical reasoning, modeling, and flexible strategies. Acquiring pedagogical content knowledge is a highly demanding task and it is not enough to simply find a solution. Instead, pedagogical content knowledge requires students to use varied skills to instruct and debate with their mentor. Students begin by identifying and interpreting a math problem. They select an appropriate strategy and use reasoning to explain its purpose to their mentor. Students break down their calculations into sensible parts. They search for connections between a mathematical model and their selected strategy. As experts, students anticipate questions and misconceptions that may occur during their presentation and find alternative strategies that could help clarify their reasoning. Pedagogical content knowledge fosters a complex understanding of content and reduces students' previous misconceptions. Table 1 demonstrates how multiple activities within The Mathematics Connection require students to work collaboratively and communicate their mathematical understanding.

Table 1:
The Mathematics Connection Links between Goals, Activities, and Constructs

Goal	Activity	Pedagogical Content Knowledge	Situated Learning	Semiotic Mediation
Mathematical Application	Family Math Night		X	
	Home Math Activities	X	X	X
	Class Presentations & Discussions	X	X	X
	Quizzes & Tests	X		
Self Evaluation	Self-Evaluation Checklist	X	X	
	Class Reflection	X		X
	Mentor Discussion	X	X	X

A Math Celebration: Family Math Night

Many middle-school parents express desires to support the school and their children but feel uneasy about the increasingly difficult content, particularly in mathematics. The Mathematics Connection eases parents' fears by creating mathematical challenges that allow parents to engage in mathematics without needing a complete understanding of mathematical content and pedagogy. Through home activities, families support their children by being directly involved with their learning. Families gain a better understanding of the mathematical curriculum covered in the classroom and a better understanding of the classroom teaching methods. Through clear instructions from teachers and feedback from families, the home-school connection is strengthened. The Mathematics Connection strives to develop a shared understanding between families and teachers that they are both working together to further the mathematical achievement of the student. Families who feel valued and encouraged to participate in student learning are more likely to provide long-term support for their child.

Family Math Night is an event designed to invite families and students to learn about school math curricula, participate in engaging math exercises, and incorporate math into family activities. The event promotes the celebration and application of mathematics both in school and at home.

Family Math Night usually lasts between two to three hours. The first part of math night consists of a concise but formal presentation about the proposed curriculum. From the presentation, parents develop an appreciation of mathematical discovery and application. The presentation should be interactive and allow parents to challenge their own ideas about teaching and learning mathematics. Following the welcome introduction and curriculum information, students and families rotate to various math stations. The stations include interactive and interesting math games with manipulatives and illustrations (see Appendix). Teachers advise families to take home a Family Math Night Brochure with games they engaged in and tips about integrating math into their daily exercises. Teachers serve refreshments and the remaining portion of Family Math Night is dedicated to family questions and comments.

Home Math Activities

Each week students have a home math challenge which includes three mandatory sections to be completed with their mentor, one mandatory reflection to be completed in class, and one optional extension (see Table 2). Unlike traditional math homework assigned daily, students work on an independent schedule throughout the week. This demands that students budget their time and monitor benchmark deadlines.

Table 2:
Components of Weekly Home Math Activities

Home Math Activity Benchmarks	Mathematical Application	Self-Evaluation & Reflection
Part I: Interpreting and Predicting	X	
Part II: Searching for a Solution	X	
Part III: Self-Evaluation Checklist		X
Part IV: Reflection	X	X
Part V: Extension	X	

Students have varied needs and strengths, and work at different rates. Students approach problems with different perspectives and each student should have the opportunity to access their previous knowledge. Traditional homework assignments require all students regardless of academic ability, language, and learning styles to complete the same standard homework. Although some curricula allow for variation, they do not allow enough flexibility and challenge for many advanced students. Some students fail to complete homework assignments because they become frustrated by their inability to understand the content and lack the resources for support. The repetitive nature of assignments bores other students. The Mathematics Connection offers multiple levels of engagement and various strategy solutions. Meaningful, authentic challenges support engagement of all students regardless of ability, time availability, and parent educational background.

Many parents, particularly parents of lower socioeconomic status have previous commitments to work to provide for their households. Time constraints make it difficult and stressful for parents to discuss homework with their child. Often homework without purpose or clear direction is frustrating for both parent and child. Many parents feel they could spend family time happily engaged in other recreational activities. Some parents lack the education experience to adequately help their child with home assignments. Teachers may invite parents to be involved directly in the classroom and on field trips but most parents do not have the luxury of volunteering a workday dedicated to their child's school. Therefore, home math activities need to be flexible in time demands and mathematical understanding. The Mathematics Connection allows parents and children to decide the amount of time they can commit to an activity and how thoroughly they want to investigate the problem. Multiple layers of investigation build understanding and allow for student personalization.

Self-Evaluation Rubric

Part three of all home math activities requires students to evaluate their ability to apply mathematical skills and concepts (see Appendix). Students review their work with their mentors and attempt to tally the number of times they displayed mathematical reasoning, mathematical modeling, mathematical flexibility, and mathematical analysis. The checklist provides examples of student work for each category. This checklist requires students to analyze their reasoning and classify each step of their work as mathematical modeling, reasoning, flexibility, and analysis.

Classroom Presentations and Reflection

At the end of each week when students have completed their home math assignment, they have the opportunity to formally present their solutions and strategies to classmates. Students volunteer to approach the board, explain their reasoning, and model their solution. As volunteers presents their strategy, classmates in the audience observe the presenter, take notes for reflection, and ask questions for clarification. Presentation requires sustained critical thinking because students must explain their strategies in clear detail and must identify their own misconceptions, as well as classmates' misunderstandings.

After observing student presentations, all students have time to reflect on the advantages, disadvantages, strengths, and misconceptions of their solution. Observing multiple pathways of obtaining the correct answer aids students in recognizing the connection between their strategy and another student's strategy. It is the teacher's role to draw relationships between strategies and emphasize the value of each strategy presented. Teachers may also present strategies that students failed to include or utilize. Students use sentence starters and reflective vocabulary to self-evaluate their processes and reasoning. Looking back at their original predictions in part one of the home activity, students identify any changes in their thinking. They attempt to identify the causes for changes in their reasoning.

Quizzes and Tests

At the end of each week, students complete quizzes. Quizzes are benchmarks to evaluate student learning and mathematical application of the same content and math skills covered in the week's home math assignments and class lessons. Students receive feedback on their performance based on a math application rubric in the form of a checklist (see Appendix). Teachers grade students' ability to identify the correct answer, explain their solution, and model their strategy. Any additional strategies earn students extra credit. Tests have similar formats to quizzes but allow for student choice. Teachers assess student learning with tests at the end of a unit of study. Tests are cumulative in nature.

Mentor Communication

Opportunities that allow students to discuss their mathematical solutions with a mentor help to refine mathematical understanding and concepts. Teachers encourage mentors to ask questions that promote reflection and analysis, such as:

1. How did you solve the problem?
2. What is the relationship between this problem and the previous problem?
3. Why do you think this strategy was more effective than the first strategy you attempted?
4. Why did you change your prediction/answer?

In class, students develop pedagogical content knowledge through discussion with classmates, teachers, and tutors. When students offer a strategy or solution to their peers, they must defend their positions. As peers and teachers question them, students develop the ability to evaluate their thought processes. As engaged observers, students analyze and compare their thought process to other students' thought processes. Through home math activities students engage in multiple tasks to increase their mathematical application, communication skills, and self-evaluation.

Self-evaluation extends beyond the classroom into home activities in The Mathematics Connection. When students discuss weekly problems with their mentor, they interpret and describe the problem. It is through talking and writing that students assess their understanding, deepen their understanding through clarification, and make connections between different solutions and perspectives. When students engage in semantic mediation, they develop an understanding of their thought process and the misunderstandings they originally created. Homework completed in isolation does not support classroom methods and may even lead to negative outcomes. Students simply repeat rules "learned" in the classroom and miss the chance to consider how math rules can be applied in various situations.

VI. IMPLEMENTATION AND REVISION

This chapter outlines the implementation of The Mathematics Connection. The curriculum presented uses situated learning, pedagogical content knowledge, and semiotic mediation to promote math application.

The Setting

The implementation site was a public charter middle school in the San Diego Unified School District. Our school belongs to a larger group of technology-driven schools within a village. The schools promote project-based learning and students learn through collaboration and cooperation. Technology Middle does not track students but tailors curriculum to meet the interests, needs, and strengths of individuals. Educators allow students to learn content by performing real-life application and presenting student work to the public makes learning meaningful.

Technology Middle has an extensive list of students hopeful to enroll in the school. A lottery selects students by zip code and students with siblings already enrolled in one of the Technology village schools have preference over other candidates. This lottery system attempts to select a student population that represents the greater San Diego population. However, Technology Middle has not managed to enroll a student body representative of San Diego Unified School District (Great Schools, 2008). Table 3 displays the differences in student ethnicity between Technology Middle and the San Diego Unified School District.

Table 3:
Technology Middle Student Ethnicity compared to SDUSD

Ethnicity	Technology Middle	San Diego Unified School District
White, not Hispanic	45%	25%
Hispanic or Latino	26%	44%
African American, not Hispanic	12%	13%
Asian American, Filipino	9% 8%	9% 7%
Pacific Islander	1%	<1%
Multiple or No Response	N/A	1%

With a significantly smaller Hispanic and Latino population than San Diego Unified School District, Technology Middle also has a lower percentage of English Language Learners (see Table 4). The percent of Technology Middle students participating in the free or reduced-price lunch program is about 40 percent lower than the district (Great Schools, 2008). Student data reveals that Technology Middle serves a much more privileged student body than most San Diego Unified schools.

Table 4:
Technology Middle Student Subgroups

Student Subgroups	Technology Middle	San Diego Unified School District
Students participating in free or reduced-price lunch program	18%	57%
English Language Learners	4%	28%

Since the application and enrollment process require parents to submit personal information and attend meetings, parent involvement and investment in the school is generally very strong compared to other traditional public schools. The school welcomes parents and families to participate in parent-teacher conferences, exhibition of student work, student presentations of learning, field trips, and school fundraisers. Many parents have multiple college degrees and only three percent of parents did not receive their high school diploma (California Department of Education, 2008). Most of the student population is composed of White, native English speakers as seen in Table 3 and Table 4. The population of Technology Middle does not reflect most of the schools within San Diego Unified School District, which are predominantly composed of ELL and Hispanic students.

The average class size at Technology Middle is 28 students (Great Schools, 2008). The school does not pull out or separate IEP students into special needs classes. Teachers and supporting tutors address learning disabilities, English language learners, and differentiated abilities within the classroom. Therefore, 28 students is an accurate representation of the average classroom at Technology Middle. Traditional public schools in California report that the average class size is 27 students, however this mean has included smaller special education classes (Great Schools, 2008).

Table 5:
2007 CST Grade Level Student Math Scores

Student Subgroups	Technology Middle	San Diego Unified School District
Students Tested (current seventh graders)	103	9735
% Advanced	20%	14%
% Proficient	42%	29%
% Basic	26%	30%
% Below Basic	12%	21%
% Far Below Basic	0%	6%

Technology Middle caters to a population of students achieving at a higher level than most schools in the San Diego Unified School District (SDUSD). As shown in Table 5, the percent of Technology Middle students categorized as advanced and proficient learners is noticeably greater than the percent of SDUSD students (California

Department of Education, 2008). This reinforces the point that Technology Middle serves a privileged population that does not mirror the greater school district.

Technology Middle does not track students based on academic achievement, special disabilities, or language ability. All students work together in an attempt to appreciate and embrace their diversity. However, this structure challenges teachers to create differentiated lessons that target the special needs and interests of each student. Despite the interest in technology-driven, project-based learning, many classrooms within our school continue to teach traditional math through direct instruction. While independent practice and textbooks can reinforce mathematical skills, this type of curriculum often limits student development of mathematical application and exploration.

Specific Implementation Participants

My Classroom Compared to My Colleague's Classroom

I implemented The Mathematics Connection curriculum over a period of two months in my classroom, where students learn through collaborative discussion and inquiry. My colleague teaches two seventh grade math classes with traditional lecture and independent practice. At the beginning and end of implementation, I assessed students enrolled in traditional math and The Mathematics Connection. I evaluated students' ability to apply mathematical skills and knowledge of basic algebraic equations and functions with The Mathematics Connection checklist. Components of the checklist included mathematical reasoning, modeling, and flexibility. Students from both classes completed online surveys to determine the amount and type of support they received outside of the classroom.

Five Focus Student and Family Profiles of The Mathematics Connection

Currently, about half the student population at our school identifies themselves as white, non-Hispanic. Most students come from affluent families with college-educated parents. However, our school is beginning to observe changes in student population with an increase of minority and economically disadvantaged students. Our school is striving to build a student body, which reflects the composition of San Diego County. I selected a diverse handful of students to represent our class profile and critically observe their experience through the course of implementation. Student experiences reveal personal insight of how students viewed and responded to The Mathematics Connection. I describe the five students and their families I observed throughout implementation.

Jessica is an energetic and social seventh grader. Jessica is African American and qualifies for free or reduced lunch. She lives with her mother and three older sisters, who attend charter high schools within the village. Since Jessica's mother works several jobs, Jessica has a lot of responsibility outside of school. Immediately after school, Jessica catches a bus home and begins her daily chores around her apartment. Jessica has little time to do her homework and often does not have access to a computer because she must share the computer with her older sisters. Jessica is focused and observant during class but has difficulty finishing homework and larger projects. While Jessica is talkative outside of class, she is very restrained and shy during class discussion. On several occasions, Jessica expressed her discomfort presenting her answer if there is a possibility that she may be wrong. This may stem from the fact that Jessica struggles with her academic performance in all school subjects.

Michael is a lively and outgoing student. Michael is half Chinese and half French. He lives with his older sister, who also attends Technology Middle and his parents. He is often distracted in class but loves to participate in team or class discussions. He maintains a positive attitude and enjoys school. He has many friends and students appreciate his relaxed nature. He is not afraid to present his solution to his classmates and receive peer feedback. Michael has difficulty completing his class and home work with care and sometimes neglects to complete his work at all. Michael very rarely reviews his work and considers the reasoning behind his errors. Michael's parents support Michael and his teachers. They attend parent meetings, student presentations, and email teachers regularly. Michael's parents recognize and express that Michael needs support to improve his mathematical skills and application.

Alex is a quiet but bright student. He is Caucasian and an only child. He recently transferred to Technology Middle from a traditional public school, where his school placed him in an advanced math course. He rarely asks questions and participates in class discussion because he has adapted to working independently. Since Alex is proficient in most math concepts and skills, he performs mental calculations. He resists displaying his thinking process and argues that math's focus is finding the "right answer". Alex's parents are supportive of Alex but express concerns that he is bored and not challenged without new math content.

Sofia is a shy and respectful student. She is a second generation Mexican-American and both her parents are fluent in English and Spanish. She lives with her parents and younger brother. Sofia almost never participates in class discussions and rarely asks questions. She does not seek help outside of class from her teachers or parents but prefers to ask a peer for assistance. However, Sofia is diligent about completing her work on time and puts forth a consistent effort. Sofia's parents regularly attend school functions and are willing to help Sofia at home. Sofia's father expressed that he is concerned about Sofia's attitude towards math. He believes that math is a gatekeeper toward higher education and because of the cumulative nature of math, it is important that Sofia not fall behind in her studies.

Luis is a lively student and always participates during class discussions. Luis' father is Caucasian and his mother is Hispanic. He lives with his parents, his grandmother, and his older sister. Luis willingly shares his ideas and presents his solutions to his classmates. He seeks help outside of class from his teachers and tutors because he does not have assistance at home. His father and mother support Luis by expressing the importance of his education. Luis' mother only speaks Spanish and does not feel comfortable supporting Luis with his assignments. Both Luis' parents work during the evenings and do not consistently have time to monitor Luis at home. Luis receives weekly tutoring after school from high school mentors at Technology Middle.

The following section describes how I implemented The Mathematics Connection over an eight-week period. Family Math Night, home math activities, student presentations, reflections, quizzes, and tests provided structure to promote student math application and self-evaluation. I discuss how I used semiotic mediation to build a connection between home and school activities.

Family Math Night

Family support is an important feature of The Mathematics Connection. In order for families to fully support students during home math activities, families needed an understanding about The Mathematics Connection approach and their responsibilities as a mentor. Family Math Night provided a casual and positive method of inviting families into the school to learn and enjoy mathematics. I sent invitations for Family Math Night home with students and through a mass parent email. I emphasized to students that Family Math Night was a stress-free opportunity to engage in fun math activities and enjoy refreshments. Students also earned extra credit for their attendance.

At Family Math Night, I enthusiastically welcomed families to express my gratitude for their support and participation. I presented information about the mathematics curriculum and provided families with a packet of suggestions with informal activities to engage in with their students. After the presentation, families played games at different stations in the room. The games included various math concepts such as probability, number sense, logic, and algebraic functions. From my observations, I noted that all families enjoyed the math stations and many reported that they felt very welcomed and valued. The night ended with a questions and discussion about the games played and support at home.

As I circulated around math stations to interact with families, I observed the connection between home and school mathematics. Jessica arrived at family math night with both her sisters and her mother. This was a significant event for her family because Jessica's mother often works at night. Jessica's mother commented that she and her family enjoyed the event and wanted to learn more ways to incorporate math activities into their home. Jessica commented that her sister had not been her "mentor" the previous week and therefore had no experience with the "PIN Problem". This is an example of the home-school connection drawing experience from home activities into the classroom and family math night. Not only did Jessica use The Mathematics Connection vocabulary but she also discussed the home activity and its relationship to the Family Math station challenge. Jessica is an example of how Family Math Night can incorporate mathematics content and learning into fun, collaborative family activities.

Home Math Activities

During the first week of implementation, students received instructions about home math activities. In class, I discussed the purpose of home math assignments and provided a brief summary of the home-school connection. Students selected a mentor to work with throughout the week. Each home math activity included at least five components. There were three required sections of the home math activity, one in class section, and one or two optional extensions. I assigned and distributed home math activities at the beginning of each week and collected the activities at the end of the week.

Students completed all sections of the week's assignment with a mentor, who signed at the end of each section to verify their presence and participation. The mentors could be parents, older siblings, tutors, teachers, or extended family. A mentor could change throughout the week depending on time and availability. For example, Jessica started the home math activities with her eldest sister but switched at the end of the week to work with her other sister. I did not give mentors direct instructions about how to solve the problem and provided no time limitations. I emphasized that mentors should not "teach" students procedures, although from my class observations of student discussion, some mentors provided students with content background.

Part One: Interpreting and Predicting

Part one of home math activities instructed students to interpret and predict the answer to the week's dilemma. I built student interest and excitement by creating problems that included student names and hobbies. I created open-ended problems that could be extended toward a more complex, rigorous challenge. For many of the home math activities, I modified problems from existing curricula to fit the structure of The Mathematics Connection.

In class, I introduced the week's dilemma by reading the problem aloud. Students then attempted to comprehend the problem as I posed the question to students, "What is this problem asking you to solve?" Students shared their understanding of the problem aloud and asked questions to clarify any misunderstandings. Outside of school, students explained the dilemma to their mentor and presented the reasoning behind their prediction. Students and mentors did not conduct formal math calculations during part one of the home math activity. However, students often had such a strong sense of how to approach the problem and what the solution would be that they began working continuously from part one to part two.

Part Two: Searching for a Solution

Part two instructed students to work with their mentor and attempt to find a solution to the dilemma. Unlike daily, independent homework, students had the power to select their mentor and determine the amount of time to spend on each section of the home math activity. During part two, students and mentors engaged in discussion about how to tackle the problem. Since the students had previous experience with math content and skills in the classroom, they often took the lead and provided suggestions about which mathematical strategy to apply. Mentors asked students to explain their reasoning and provide alternative suggestions. I stressed that mentors were not required to teach any mathematical content or skills. Instead, I encouraged mentors to allow students to identify patterns in their work. This made the patterns more meaningful, powerful, and personal.

Part Three: Self-Evaluation Checklist

Part three of the home math activity allowed students to analyze their thinking and solution to the dilemma with The Mathematics Connection checklist. With their mentor, students reviewed their thinking and work, tallying the number of times they displayed mathematical reasoning, mathematical modeling, and mathematical flexibility. In class, I pointed out examples of each category as students presented their work so they understood what they should look for when they were reviewing their thinking. For example, I clarified that tables, graphs, pictures, and equations with variables were all forms of mathematical models. Students practiced identifying categories of mathematical application by evaluating other students' work in small, collaborative groups. Students built familiarity with The Mathematics Connection vocabulary by using the terms in class discussions, on assessments, and during presentations. Although reflection and self-evaluation was an unfamiliar and strenuous process, many students recognized the relationship between the self-evaluation checklist and the quiz checklist.

The self-evaluation checklist (see Appendix) also provided a section for optional mentor feedback. Mentors were able to comment on the week's home math activity with suggestions, questions, frustrations, or insights they encountered with their student. While it was not required, I encouraged the students to share with their mentors solutions and alternative strategies presented in class. This ensured that the connection between home and school was strengthened and cyclical.

Part Four: Reflection

At the end of each home math activity, students had the opportunity to share their solutions with their classmates. Students, who did not volunteer to present their solution, were still required to engage in the presentation by observing, questioning, and commenting on the strategies applied. As students observed their classmates' solutions, they recorded alternative strategies that they did not apply during collaboration with their mentor in section four. At the end of the presentations, students reflected on the changes between their original prediction and their new understanding of the problem and content. I provided students with sentence starters to help them identify and organize their reflective thoughts and process. Optional sentence starters listed below, helped students to begin the reflective process.

1. My strategies was effective/ineffective because . . .
2. When I compare my strategy to __________'s strategy, I notice . . .
3. The advantage/disadvantage of this strategy is . . .
4. _________'s strategy and _________'s strategy are related because . . .
5. This strategy could be used to find . . .

Students also had the opportunity to read their reflection aloud to their classmates. This sharing of reflections emphasized the cyclic and situated learning process of mathematics application.

Part Five: Optional Extension(s)

Students who needed a challenge or had extended time to work with a mentor had the option of continuing with an extension activity. The extension is a similar problem to the home math activity but at an increased difficulty level. I awarded extra credit to students, who challenged themselves with extension activities.

Quizzes and Tests

Students completed quizzes at the end of each home math activity. Quizzes provided students with a consistent benchmark and immediate feedback on their performance. Each quiz consisted of only one open-ended problem related to the content or skills discussed in the week's home math activity. I evaluated quizzes with the mathematics application checklist as displayed in the appendix. The checklist evaluates student work in three categories; correct answer, mathematical reasoning or explanation, and mathematical modeling. Mathematical flexibility was optional and any alternative strategies earned bonus points.

Conclusion

Throughout the implementation, I observed a high level of student participation. There was a noticeable increase in work completion and student attentiveness. Students willingly volunteered to present their solutions before classmates and most students showed interest in different strategies. Students were alert and questioned the strategies of students presenting. While students recognized the connection between multiple strategies, students did not view all strategies as equally valuable. Instead, a few students evaluated strategies on strengths and weaknesses within a context and ranked strategies on a level of hierarchy. Since I graded mathematical flexibility as an optional bonus, students may not have viewed the ability to apply multiple strategies as significant as selecting the "best" strategy to find the solution.

Mentors were willing and supportive during family math activities but it was more challenging to balance the type to assignments than I originally anticipated. As I selected and created assignment dilemmas, I considered student interests, application to real-life problems, rigorous mathematical content, and accessibility to varied math content. Overall, students successfully improved their mathematical reasoning, mathematical modeling, and mathematical flexibility. These skills linked to mathematical application, ultimately helped students improve their ability to identify the correct solution. Most students reported an increase in their math motivation and confidence. Chapter VII that follows is a detailed account of the data collected from implementation and summaries I drew based on data analysis.

VII. AN EVALUATION OF THE MATHEMATICS CONNECTION

The evaluation component of this approach analyzed home math activities, pre and post tests, online surveys, field notes, and interviews from two groups of participants – my seventh grade math students and their selected mentors. Seventh grade math students from a colleague's traditional math course acted as a control group. Students participating in The Mathematics Connection and their mentors worked outside of the classroom according to their own schedule. This component of the project was challenging to evaluate because interactions between student and mentor took place at home. This ultimately required students to transport knowledge and experience between home and school.

Goals

The Mathematics Connection creates opportunities for students to develop mathematical application outside the school setting. Families act as funds of knowledge that students can access and interact with in order to develop mathematical application and communication. I attempted to shift math homework from an isolated activity to application of math skills situated in collaborative learning. Homework becomes beneficial when knowledge and skills are incorporated to classroom activities. The first goal of The Mathematics Connection was to improve student application of mathematical strategies. Mathematical application includes a combination of mathematical reasoning, mathematical modeling, and mathematical flexibility. The curriculum's second goal is to help students develop self-evaluation strategies to monitor and reflect on their mathematical understanding and application. Students engage in analytical discussion, utilize a self-evaluation checklist, and write reflective summaries. The third goal of The Mathematics Connection seeks to strengthen a connection between the home and school. Students aim to transfer and apply mathematical understanding between classroom lessons and authentic real world challenges.

Data Collection Strategies

I used a variety of strategies to collect and evaluate data from implementation. Table 6 displays the strategies used to evaluate the three goals of the Mathematics Connection. Data collected included field notes, pre and post assessments, pre and post online surveys, student work samples, and notes from informal student and mentor interviews. I compared my students' assessments performance and survey responses with students enrolled in a traditional mathematics course.

Table 6:
Evaluation Strategies with Corresponding Goals for The Mathematics Connection

Evaluation Strategies	Goal One: Mathematical Application	Goal Two: Self-Evaluation & Reflection	Goal Three: Home-School Connection
Pre Assessment Equations Test	X		
Post Assessment Equations Test	X		
Field Notes (Class Discussion)	X	X	X
Student Samples (Home Math Assignments)	X	X	X
Student Samples (Written Reflection)		X	
Pre Online Survey		X	X
Post Online Survey		X	X
Student Interview (Observation)	X	X	X
Mentor Interview (Observation)	X		X

Pre and Post Tests

Pre and post tests of an algebraic equations unit were the main source of data collected to monitor changes in student mathematical application. I evaluated mathematical application based on four categories; correct answer, mathematical reasoning, mathematical modeling, and mathematical flexibility. Students earned three points for the correct answer, three points for mathematical reasoning, and three points for mathematical modeling. Students tackled three problems on each test with a maximum score of 27 points. Multiple strategies were optional and awarded extra credit, which allowed some students to score over 27 possible points. To monitor the effects of The Mathematics Connection on mathematical application, I compared my class results with my colleague's results. My colleague teaches a traditional seventh grade math class through lecture, direct instruction, independent practice, and limited collaborative group work. Students in my colleague's traditional math course completed pre and post assessment tests. In addition to comparing test scores of the two classes, I noted the amount of time students in The Mathematics Connection spent on each assessment.

Field Notes

I recorded field notes during Family Math Night and class discussions at the end of each home math activity. Although I gathered much of the data from tests and home math activities, field notes allowed a qualitative analysis of experiences with the curriculum. I monitored student interactions, comments, motivation, and development throughout implementation. Actively monitoring and promoting discussion in class made it difficult to consistently collect detailed field notes. However, after each weekly class discussion I expanded my field notes focusing on specific events, such as student reactions to a classmate's strategy or confusion about a particular home math assignment. Field notes also helped me compare student comments and actions to the written work they produced on home math activities, tests, and quizzes. In addition to collecting student comments, I gathered mentor feedback from informal conversations and written responses on home math activities.

Student Samples

From early on in the project, I decided that I wanted to look at trends in class data but also concentrate on selected focus students. Analyzing individual student work allowed for closer observation of work quality, mathematical application, and self-evaluation. I selected student samples to reveal student responses to prompts and written reflections. To record changes over implementation, I analyzed student samples from the initial to final week of curricula implementation. I created a four-point rubric to categorize student reflections from basic to advanced levels of self-evaluation.

Pre and Post Surveys

In an effort to measure the effects of the curriculum on developing a home-school connection, students and mentors completed electronic surveys. Student and mentors completed the surveys before and after implementation. I designed the surveys to document and interpret the frequencies of responses to questions regarding time invested on home math activities, resources accessed outside of school, and attitudes toward math curricula. All online surveys were conducted anonymously to encourage students and parents to respond honestly without judgment. Besides analyzing changes in frequency of responses, I cross-referenced the frequency of responses between students and mentors.

Student and Parent Interviews

I selected and interviewed students and mentors after the curriculum implementation. Students responded to a series of questions about their experiences with home math activities, mentor interactions, family math night, tests, and quizzes. Students' responses were open ended and I probed them for clarification if their answers were vague. Some student interviews occurred in small groups of two to three students while I conducted other interviews individually. I encouraged all students to speak freely and honestly about their experience with The Mathematics Connection. I interviewed mentors through phone conversations and informal meetings. I conducted mentor interviews independently and asked mentors to respond to identical questions from my student interviews. I used audio recordings to extract specific quotes that highlighted mathematical understanding, attitudes toward the curricula, and situated learning between mentor and student.

Findings

Goal One: Improve Students' Mathematical Understanding and Application

Finding One: Improvement in Mathematical Application

Before implementation, students enrolled in The Mathematics Connection and in traditional math completed pre algebraic equations tests. Students selected and attempted three out of four possible problems. Each problem required students to create an algebraic equation with a variable, identify the solution to the problem, and explain their mathematical reasoning. Alternative strategies earned bonus points. Most students completed the test within 30 minutes and while a few students were able to identify an answer, most students could not explain how they verified the accuracy of their solution. Even fewer students had the ability to approach the problem from multiple perspectives. I focused on evaluating the effects of The Mathematics Connection on mathematical application and then compared my class results to my colleague's traditional math class.

Figure 3 displays the components of student scores before and after implementation of The Mathematics Connection.

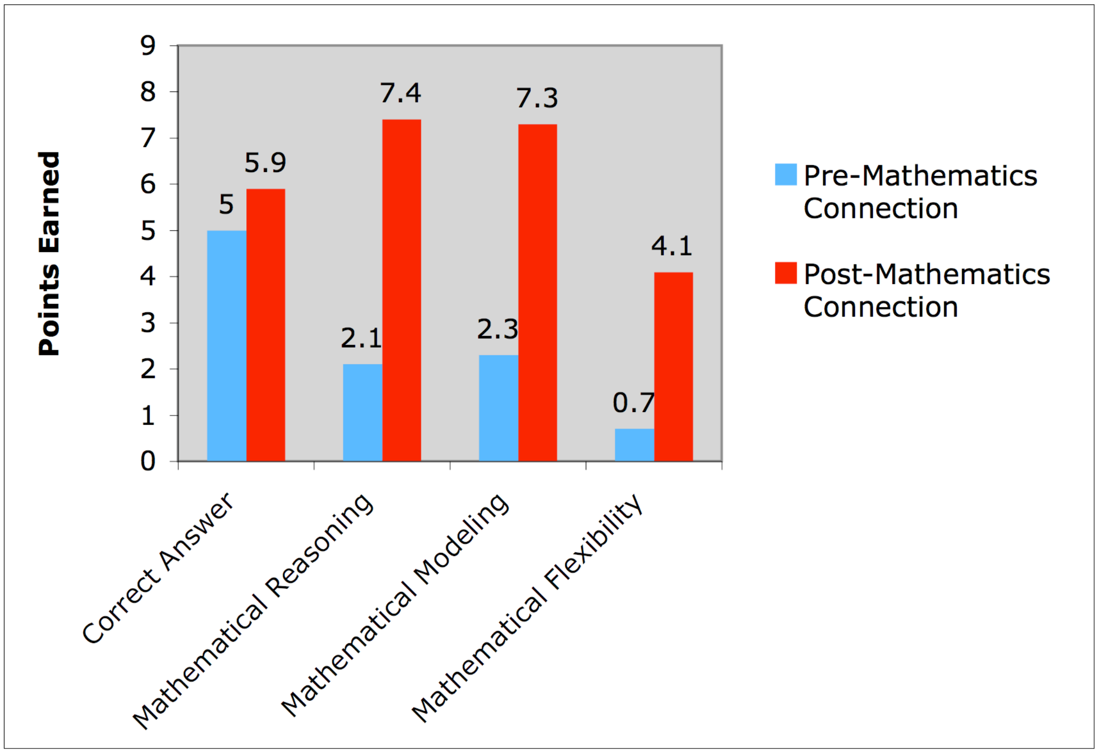


Figure 3:
Results from Algebraic Equations Pre and Post Tests

The Mathematical Connection shows noticeable improvement between students' pre and post performance. Most notably, students increased their mathematical reasoning, modeling, and flexibility. These skills supported students' overall ability to access the correct answer as shown in Figure 4.

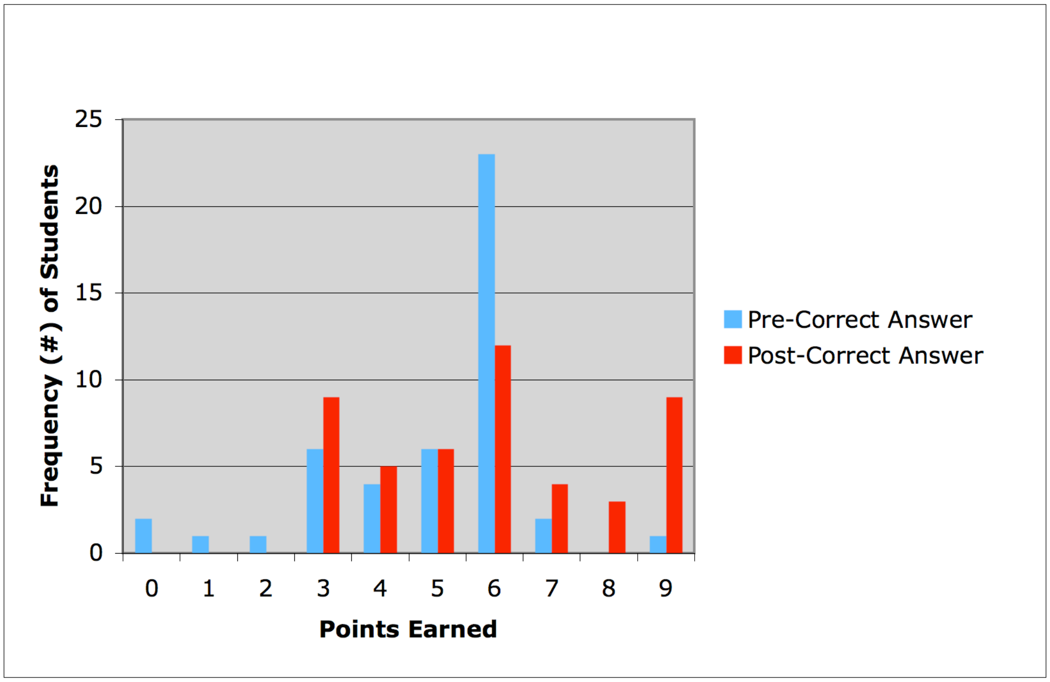


Figure 4:
Changes in Correct Answers from Pre and Post Tests

In addition to increasing their ability to identify an accurate answer, the majority of students in The Mathematics Connection increased their mathematical reasoning. Figure 5 shows student scores shifted up from pre to post implementation. This data suggest that students were able to find an accurate answer because they had a better understanding of the strategies they applied.

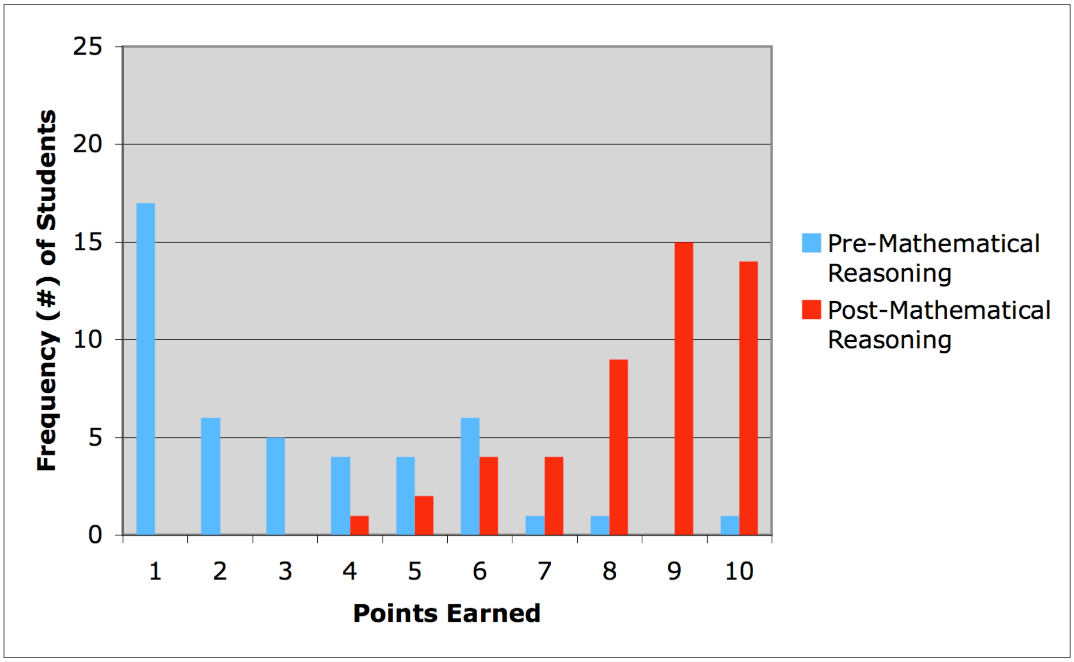


Figure 5:
Changes in Mathematical Reasoning from Pre and Post Tests

Like mathematical reasoning, mathematical modeling increased from pre to post test. Figure 6 shows the upward trend in mathematical modeling. I attributed the significant increase in mathematical modeling to the fact that many students had never used modeling in their math studies. In his student interview, Michael stated that modeling was a foreign concept before his experience with The Mathematics Connection, "it was neat to know that we could draw pictures of whatever model we wanted because then you can see it. They don't teach that stuff (modeling) in math".

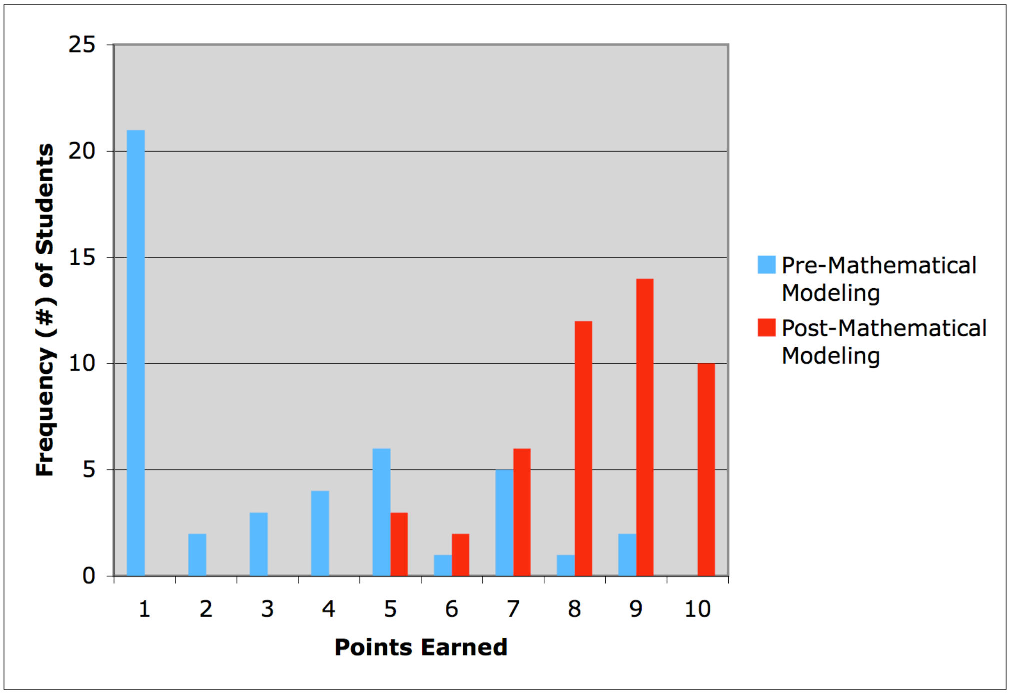


Figure 6:
Changes in Mathematical Modeling from Pre and Post Tests

Students enrolled in The Mathematics Connection displayed a noticeable increase in their overall mathematical application. Mean student scores rose 14.6 points from pre to post tests. This is a dramatic change compared to students enrolled in my colleague's traditional math course, who showed an increase of only 1.2 points (see Table 7). In addition to analyzing changes in mean test scores, I calculated the standard deviation of pre and post test scores. A small standard deviation in both The Mathematics Connection and my colleague's traditional math scores indicate there was a general consensus among student responses (see Table 7). While the standard deviation increased in both classes from pre to post test, I attribute this minor change to students' attempts at multiple strategies.

Table 7:
Comparing Traditional Mathematics and The Mathematics Connection Algebraic Equations Tests

	Pre Test (Class Mean)	Pre Test (StD)	Post Test (Class Mean)	Post Test (StD)
Traditional Math Overall Score	10.8	4.3	12.0	4.9
Mathematics Connection Overall Score	9.9	4.0	24.5	5.0

Figure 7 confirms students enrolled in The Mathematics Connection had greater gains in mean scores compared to students enrolled in the traditional math course. Differences in performance are likely due to different class environments, activities, and exposure to mathematical application.

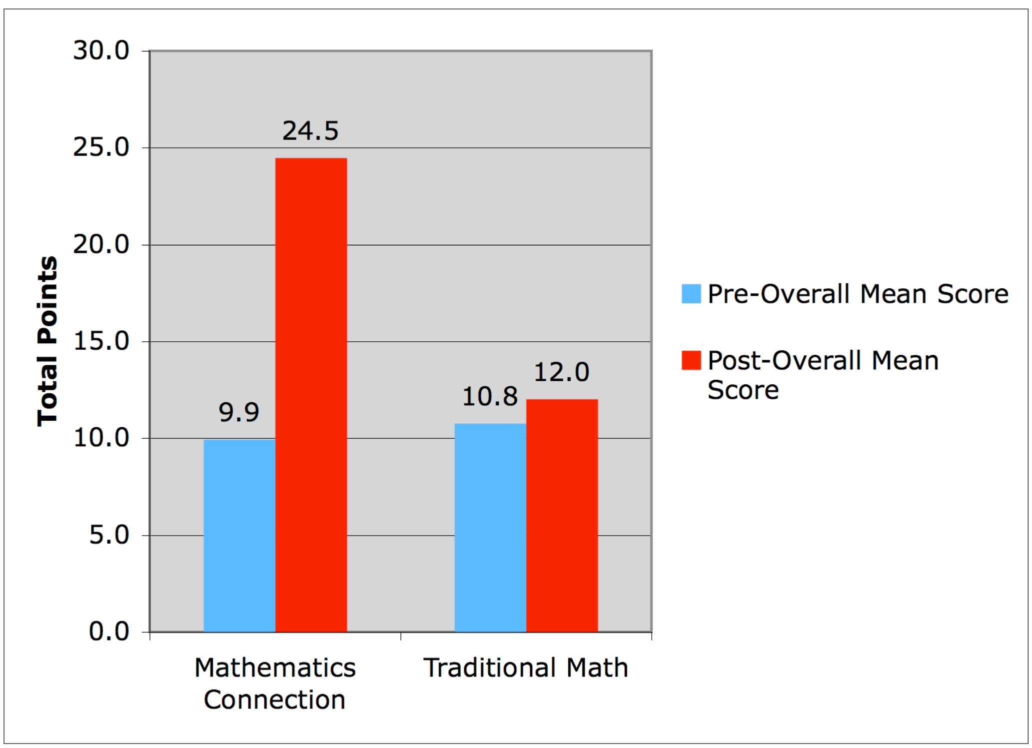


Figure 7:
Comparison of Student Math Application in The Mathematics Connection and Traditional Mathematics

Goal One: Improve Students' Mathematical Understanding and Application

Finding Two: Mathematical Flexibility Increased

Flexibility is a component of mathematical application and requires students to attempt multiple strategies to solve a problem. The ability to accurately apply various strategies requires a higher level of understanding than simply attempting strategies. For example, a student who attempts to write an equation describing a situation shows a lower level of comprehension than a student who writes an equation and uses the equation to verify his/her solution. I have considered this difference by awarding two points for an accurately applied strategy versus one point at an attempt to apply a second or third strategy. Although students may not be able to apply a strategy, the mere attempt is an indication that they possess the awareness that a second strategy exists. Although student data show that students in The Mathematics Connection improved their ability to identify multiple strategies, the data do not note the level of application or understanding of each strategy. Figure 8 suggests that before implementation of The Mathematics Connection, the majority of students lacked mathematical flexibility. After implementation, student flexibility increased. However, Figure 8 shows that there was a large range of abilities related to multiple strategies.

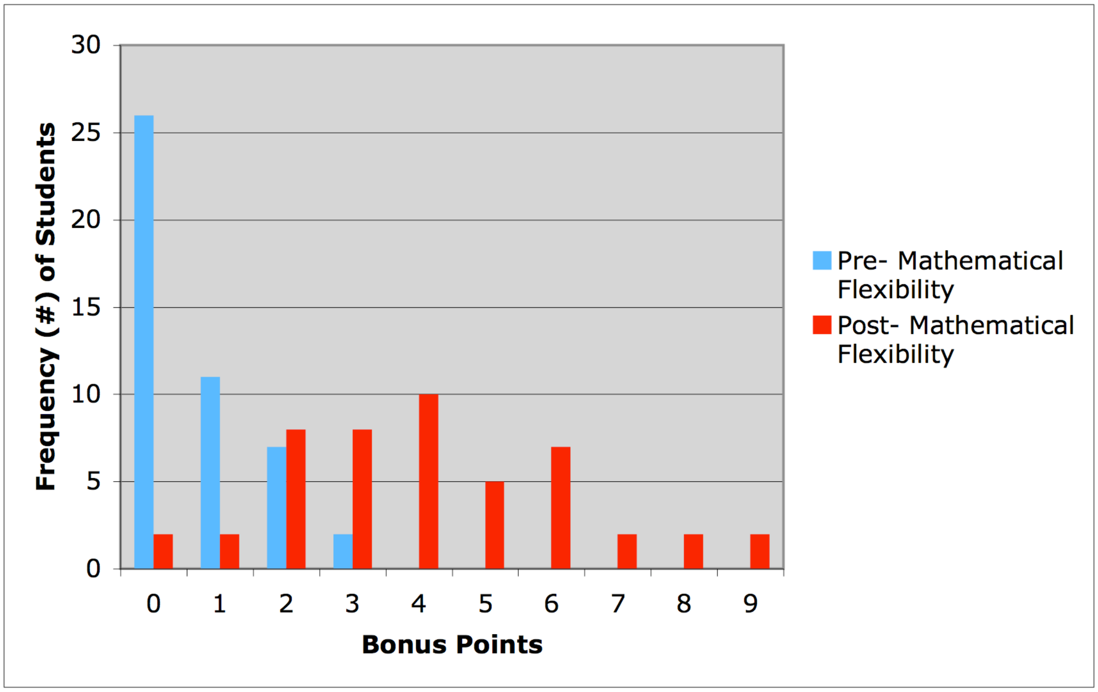
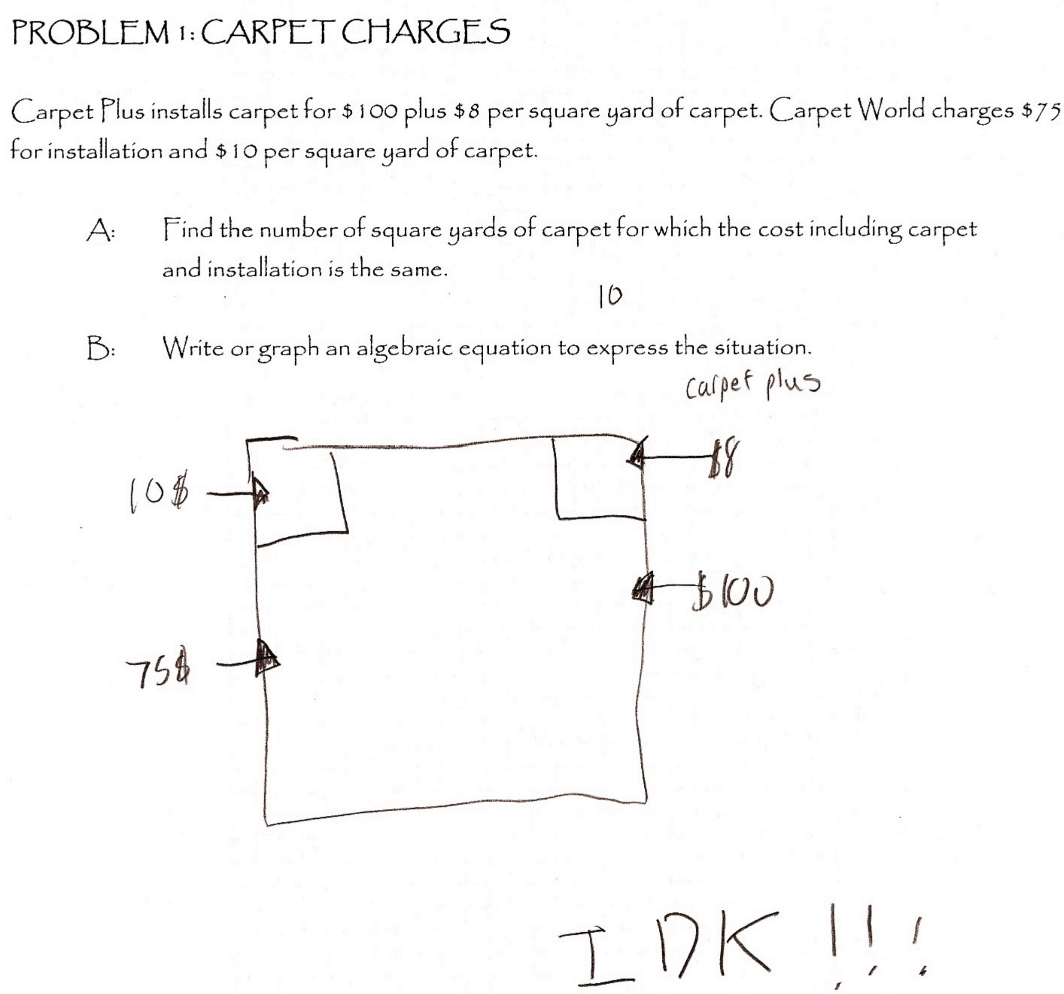


Figure 8:
Changes in Mathematical Flexibility from Pre and Post Tests

Max is an example of a student developing flexibility with multiple strategies. Max's pre test in Figure 9 shows that prior to implementation, Max was unable to solve or apply a strategy to the problem.



PROBLEM 1: CARPET CHARGES

Carpet Plus installs carpet for $ 100 plus $8 per square yard of carpet. Carpet World charges $75 for installation and $ 10 per square yard of carpet.

A: Find the number of square yards of carpet for which the cost including carpet and installation is the same.

10

B: Write or graph an algebraic equation to express the situation.

Figure 9:
Max's Pre Test

Through home math activities, class lessons, student presentations, and reflection, Max gained new strategies to effectively solve the problem. As shown in Figure 10, Max's post test suggests that Max interpreted the problem on multiple levels by creating a table to record the pattern of change in prices and tiles, as well as writing an equation to express the relationship between stores.

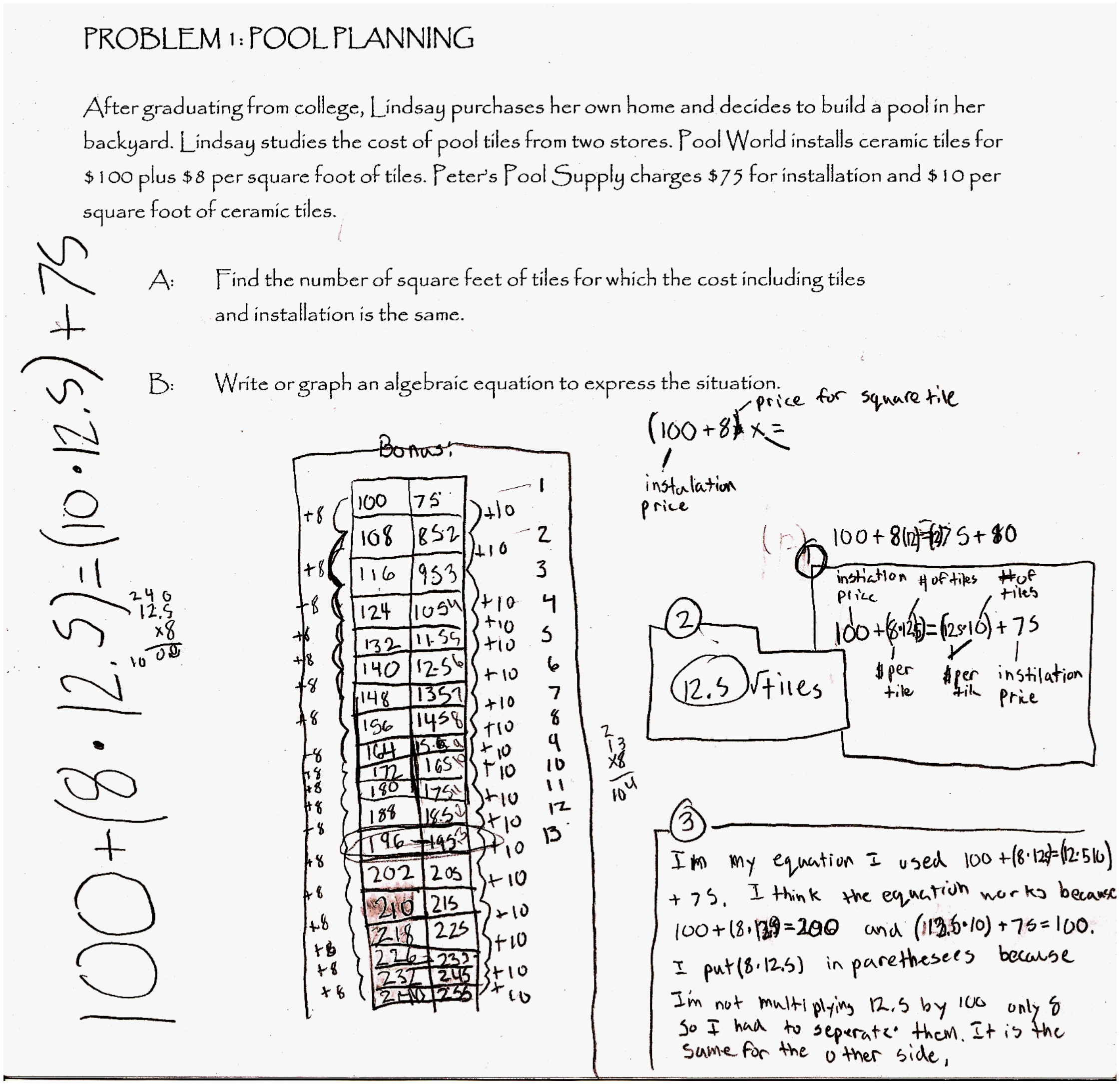
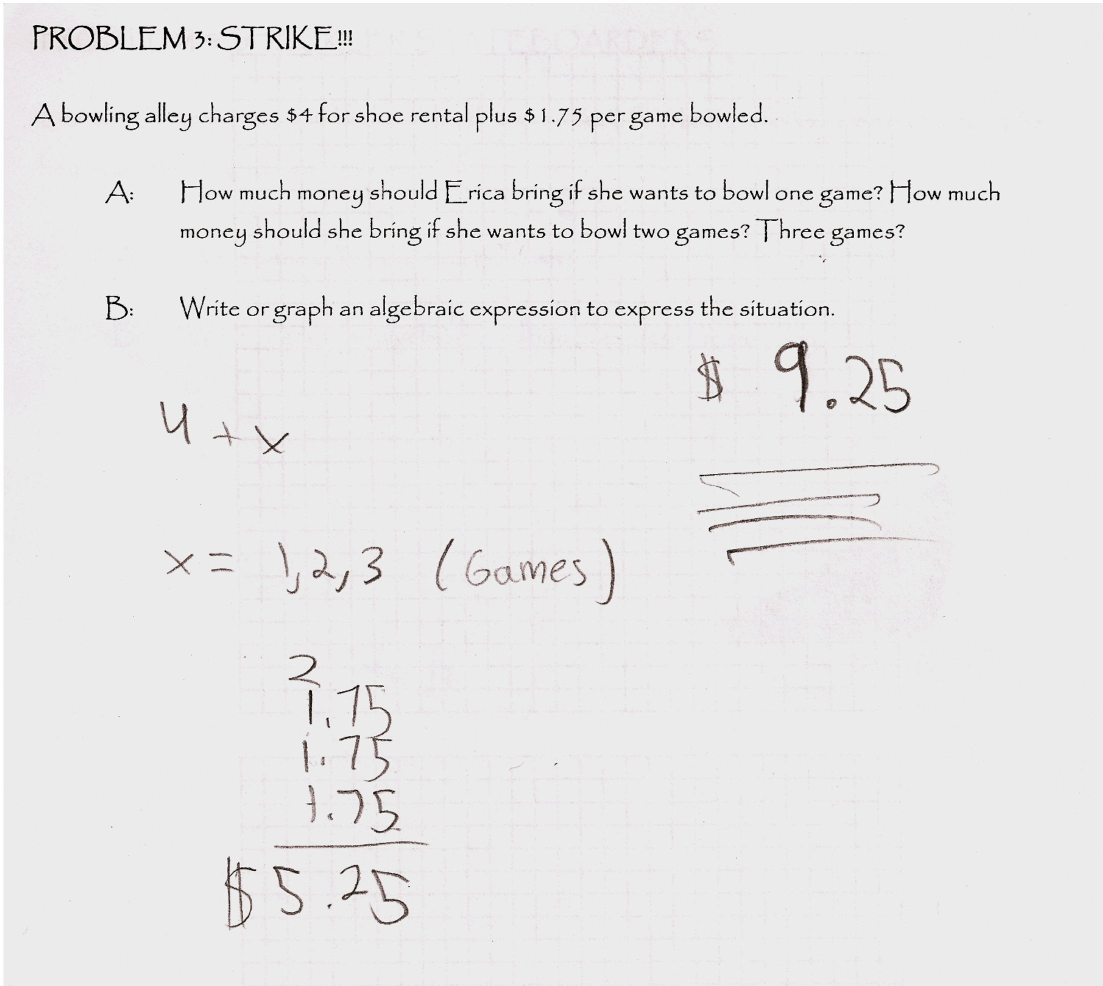


Figure 10:
Max's Post Test

Justin's pre and post tests in Figure 11 and 12 illustrate the new strategies students adopted to solve similar problems. Although Justin was able to find the correct answer in Figure 11, he did not provide any mathematical reasoning for his work.


PROBLEM 3: STRIKE!!!

A bowling alley charges $4 for shoe rental plus $1.75 per game bowled.

A: How much money should Erica bring if she wants to bowl one game? How much money should she bring if she wants to bowl two games? Three games?

B: Write or graph an algebraic expression to express the situation.

$9.25

4 + x

x = 1,2,3 (Games)

2
1.75
1.75
1.75
$5.25

Figure 11:
Justin's Pre Test

Through home math activities, class discussions, student presentations, and class lessons, Justin gained new strategies, which included creating a table, writing an algebraic equation, and graphing a linear equation. Justin's work shows that he was able to effectively use most new strategies and was able to explain the reasoning supporting his approach. However, it is interesting to note that Justin wrote an algebraic equation but did not solve the problem algebraically. Instead, he reverted back to his table and graph to verify his solution. Justin's post test suggests that while he recognized multiple strategies, he did not fully identify a relationship between each strategy.

PROBLEM 3: GO-CARTS

John goes to Boomers to race on go-carts. Boomers charges a \$7 entrance fee plus \$1.75 per go-cart ride.

A: John has \$15.75. How many times will he be able to ride the go-carts?

B: Write or graph an algebraic equation to express the situation.

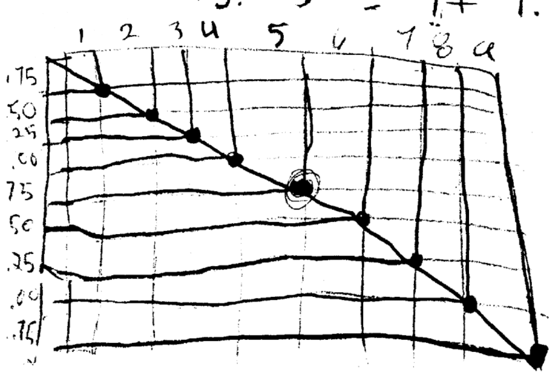
$\boxed{x = 5}$

$15.75 = 7 + 1.75x$

Johns total money — Entrance Fee — per go cart — How many times he can go on ride

$15.75 = 7 + 1.75x$

8.75
+7.00
$\boxed{15.75}$



1.75 per ride	How many Rides
1.75	1
2.50	2
5.25	3
7.00	4
8.75	5
10.50	6
12.25	7
14.00	8
15.75	9

1.75 × 3 = 5.25

1.75 × 4 = 7.00

1.75 × 6 = 10.50

10.50 + 1.75 = 12.25

12.25 + 1.75 = 14.00

14.00 + 1.75 = 15.75

I fand the answer by adding 1.75 to everytime u went on a go cart ide. Since he had 15.75 dollars, I added until I got to either the exact number or in between that # and another #. And I got 5 plus 7 dollar entrance fee

Figure 12:
Justin's Post Test

Goal One: Improve Students' Mathematical Understanding and Application

Discussion of Findings

Although student pre and post test scores on the algebraic equations tests indicate that students gained the ability to approach problems from different perspectives, the reasons for their multiple attempts are unknown. Since I awarded extra credit to multiple strategies, students may simply have been applying strategies to boost their grade. Nevertheless, students would not attempt multiple strategies if they did not know that the strategies existed and had some sense of how to use them.

Ethan's work is one example of a student's perspective toward mathematical flexibility. He is an advanced math student and often finished his work ahead of the class. He prefers approaches that are algebraically based and simple to complete. He has difficulty explaining his work and writes the minimal amount possible. While he recognizes that there are multiple methods of approach a problem, his comments in class suggested that some methods were more valuable than others. From my field notes of a class discussion, I recorded Ethan's comment about alternative strategies, "It's ok to do it that way. You could draw a picture and they got the same answer as using the equation." When prompted to explain why using a picture was "ok", Ethan responded, "Pictures work but they're for people who can't do the math". Ethan's statement suggested that he believed there was a hierarchy of strategies with some strategies being more mathematically appropriate than others. In order to recognize pictures as a mathematically significant strategy, I pointed out a previous project students had completed where all students had used models to find a solution. This example helped

Ethan reevaluate his attitude towards mathematical flexibility and acknowledge that even algebraic equations have limitations within specific contexts.

Goal Two: Increase Student's Self-evaluation Through Reflection

Finding One: Improvement in Quality of Student Reflection

As I evaluated student reflections from the first week of implementation to the final week, it was clear that there was an increase in both the quantity and quality of students' written reflections. I developed a four-point rubric to evaluate the quality of student reflections (see Table 8).

Table 8:
Self-Evaluation and Reflection Rubric

Rubric Score	Score Requirements
1	Student recognizes that strategy was effective or ineffective
2	Student identifies or explains the reason strategy was ineffective or effective Student recognizes multiple strategies
3	Student compares different strategy strengths and weaknesses
4	Student identifies the relationship between strategies and/or the extension potential of strategies in an appropriate context

Students who worked at the most basic level of reflection merely recognized a correct or incorrect answer. For example, a student who earned 1 point on the self-evaluation and reflection rubric wrote, "The probability that he wins is 50% because a coin has two sides." In contrast, students at the highest level of reflection summarized the relationship between strategies. A student who earned 4 points on the rubric wrote,

> I just did a tree diagram for this problem. Each person has 4 choices and I counted the bottom lines to show the combinations the girls could make. At first, I thought it was 16 (outcomes) because there was 16 on my tree – but then I wrote it out and got 24 different combinations. Writing the combinations takes a long time but you can see the pattern. The tree works too but you have to organize it because you might forget to remove 1 choice each time.

Since students enrolled in traditional mathematics did not receive instruction or opportunities related to self-evaluation, I only assessed the reflection skills of students participating in The Mathematics Connection. Data revealed that the class median was stable at 2.0 points before and after implementation. In contrast, the class mean increased from 1.6 points to 2.4 points. A change in class mean without a change in class median suggests the range of student reflection scores increased after implementation. To illustrate the general upward shift in student reflections, I created a histogram as seen in Figure 13. It is evident that the majority of students scored between one and two points on the Self-Evaluation and Reflection Rubric during the first week of implementation but gradually increased their scores to two and three points by the final week of implementation.

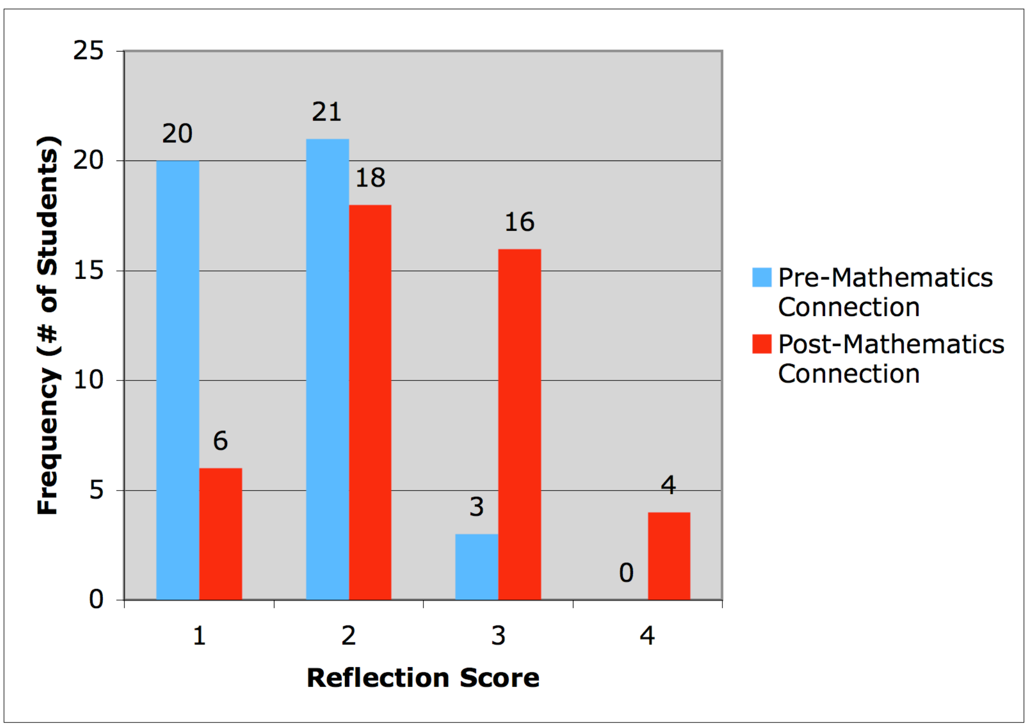


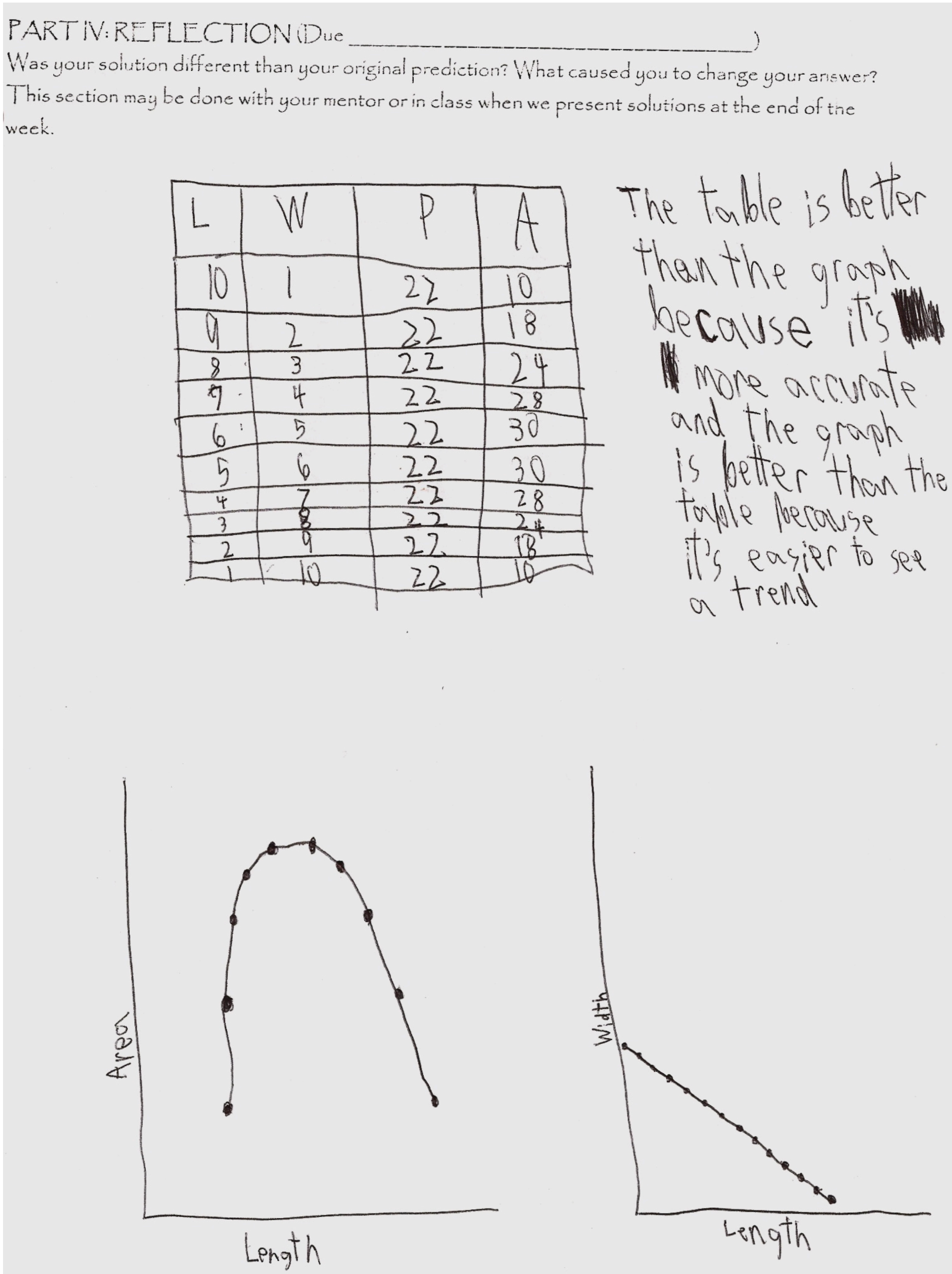
Figure 13:
Student Reflection Pre and Post Scores

In addition to observing trends in class data, I noted changes in individual student work from the first week to the final week of implementation. Alex is a focus student and an example of a student, who lacked experience reflecting but increased his skill through home math activities and student presentations. Alex is a high achieving math student but is not always able to articulate his reasoning about selecting a particular strategy or what the strategy represents. During the first week of implementing The Mathematics Connection, Alex commented that he could not complete a written reflection because he had identified the same answer as most of his classmates (see Figure 14). However, by the final week of implementation, Alex was able to identify the accuracy of his solution and compare the strengths and weaknesses of multiple strategies. Figure 15 reveals that through reflection Alex recognized the value of attempting the problem with a different strategy than his own.

PART IV: REFLECTION (Due Friday)
Was your solution different than your original prediction? What caused you to change your answer? This section may be done with your mentor or in class when we present solutions at the end of the week.

$$X = X \cdot [(x-1)(x-2)]$$

Figure 14:
Alex's Initial Reflection



PART IV: REFLECTION (Due ________________________________)

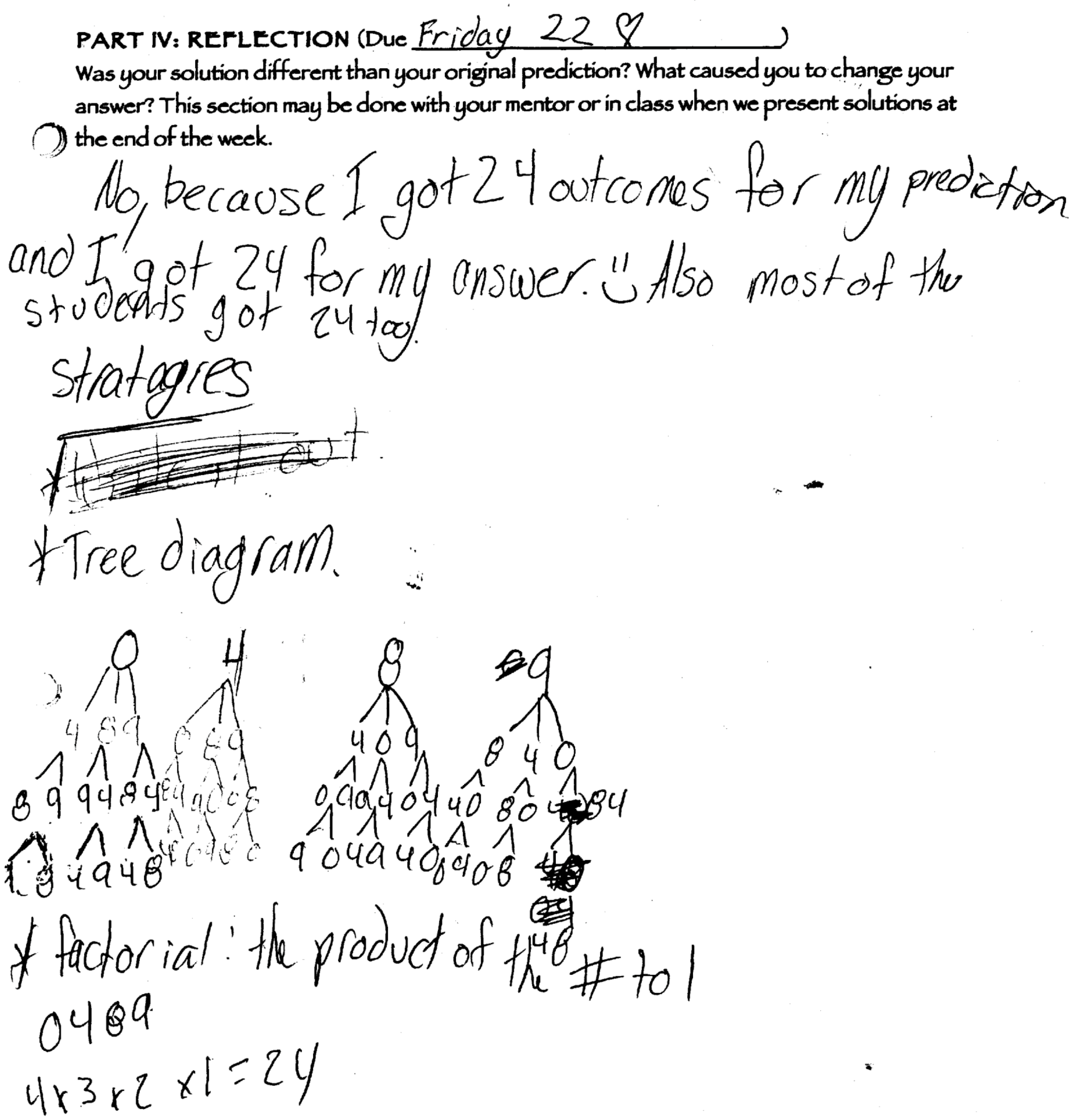
Was your solution different than your original prediction? What caused you to change your answer? This section may be done with your mentor or in class when we present solutions at the end of the week.

L	W	P	A
10	1	22	10
9	2	22	18
8	3	22	24
7	4	22	28
6	5	22	30
5	6	22	30
4	7	22	28
3	8	22	24
2	9	22	18
1	10	22	10

The table is better than the graph because it's [redacted] more accurate and the graph is better than the table because it's easier to see a trend

Figure 15:
Alex's Final Reflection

In most initial reflections, students simply noted if they had achieved the correct answer based on the solutions of the majority of students. During the first week of implementation, students observed their classmates' varied strategies of attempting a problem. Although multiple strategies intrigued students, the students failed to recognize relationships between the strategies. Many students could not support their solution with mathematical reasoning or mathematical modeling. For example, Ambar's initial reflection showed she relied on other classmates' answers to gage the accuracy of her solution (see Figure 16). Ambar recorded her classmates' strategies but did not explain why the strategies were effective or how they compared to her own strategy.


PART IV: REFLECTION (Due Friday 22)
Was your solution different than your original prediction? What caused you to change your answer? This section may be done with your mentor or in class when we present solutions at the end of the week.

No, because I got 24 outcomes for my prediction and I got 24 for my answer. Also most of the students got 24 too.

Stratagies

~~* test out~~

* Tree diagram.

* factorial: the product of the # to 1

0489

4x3x2 x1 = 24

Figure 16:
Ambar's Initial Reflection

By the final week of implementation, Ambar improved her reflection skills. Besides identifying the correct answer, Ambar explained new relationships she discovered from her classmates' presentations, when she wrote "the strategies today showed me that when you graph the length and width, it is a linear equation but the length and area is not linear" (see Figure 17). Although Ambar does not explicitly state the advantages and disadvantages of various strategies, comprehending her classmates' strategies allowed her to gain a new understanding of the problem that she had previously lacked.

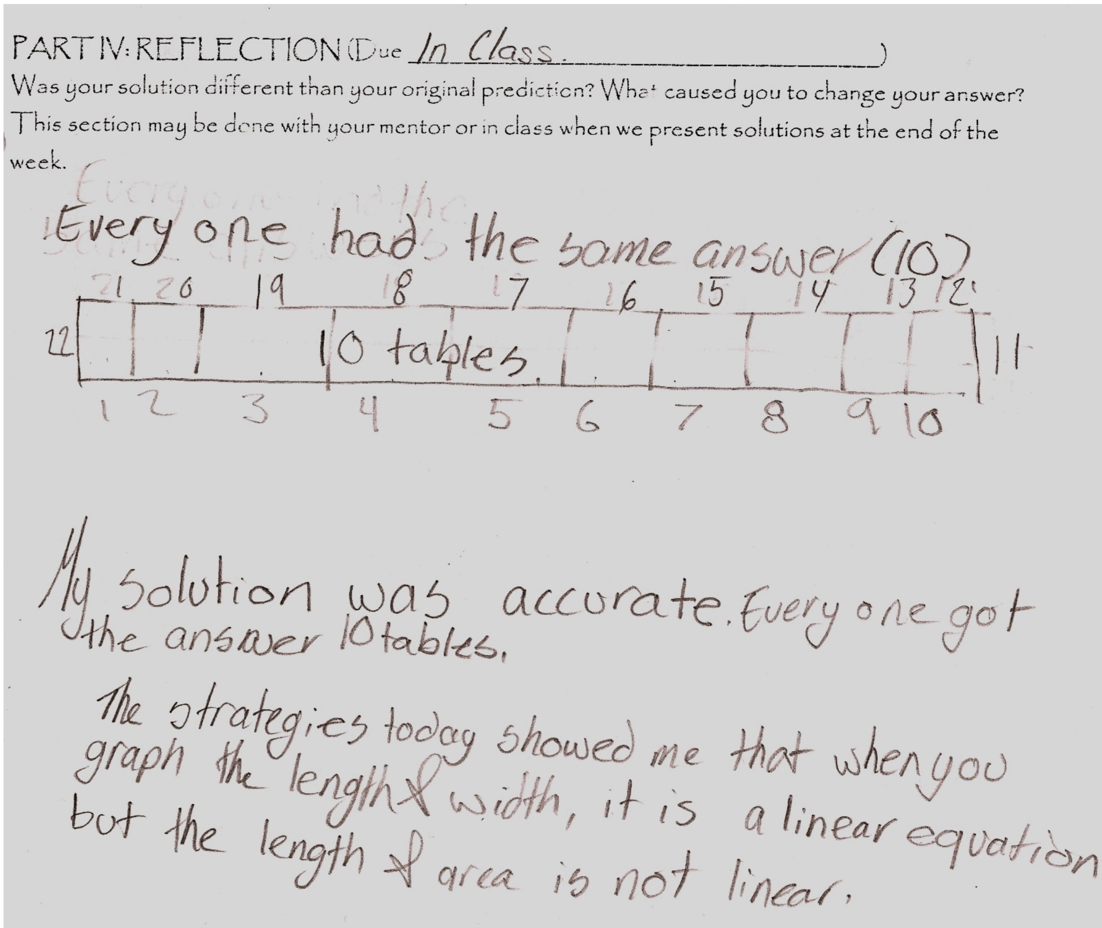


Figure 17:
Ambar's Final Reflection

Goal Two: Increase Student's Self-evaluation Through Reflection

Finding Two: Increase in Student Ability to Use Reflection to Self-evaluate

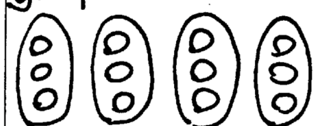
Reflection is a foreign skill to most seventh graders in mathematics. Student surveys completed before and after implementation of <u>The Mathematics Connection</u> revealed that few students check the reasonableness of their work after they have solved a problem. When students first engaged in home math activities, less than half of the students completed the self-evaluation checklist.

I recognized that students possessed the strategies to solve problems but they needed the skills to become aware of those strategies. During the first week of implementation, all students completed the home math activity with their mentors. As I observed the student presentations, students displayed a variety of strategies to the solve problem. Yet, students had difficulty explaining their strategies and claimed that they did not understand how to categorize their work into mathematical reasoning, mathematical modeling, mathematical flexibility and analysis. Students failed to differentiate between mathematical reasoning and writing. For example, Michael, a focus student of The Mathematics Connection presented his solution from the first home math activity and explained the process he used with his mentor. Next to his calculations, he wrote, "Yes, rolling the dice is a fair game." As he concluded his presentation, Michael pointed to his writing and referred to it as his "mathematical reasoning."

Through teacher modeling, consistent practice, and semiotic mediation, students used reflection to refine self-evaluation. Fred and Ben discussed how some work samples could display two or more components of mathematical application. Fred classified the student work as an example of mathematical reasoning because the student recorded the process they used to solve the problem. Ben responded that they should also categorize the work as an example of mathematical analysis because the students recognized they had made an error by counting the same values twice. The discussion between Ben and Fred demonstrates how students developed their reflection skills to improve their ability to self-evaluate. To strengthen student confidence in evaluating and categorizing mathematical application, I continued to label work in class activities throughout implementation of The Mathematics Connection. By the final week of implementation, all students who completed home math assignments also completed the self-evaluation checklist (Figure 18).

PART III: SELF-EVALUATION (Due ____________________________)

With your mentor, please monitor the number of times you find evidence of the actions below.

#	Action	Example
	MATHEMATICAL REASONING: Evidence of explanation or justification in writing or through discussion	Perfect squares have an odd number of factors because one factor repeats itself. 16: 1, 2, 4, 8, 16
	MATHEMATICAL MODELING: Evidence of using pictures, tables, or graphs to represent mathematical reasoning	$4 \times 3 = 12$ ← total; groups; oranges 
	MATHEMATICAL FLEXIBILITY: Evidence of more than one strategy to solve the problem	$3 \cdot \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$ ✓  $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$ ✓
	MATHEMATICAL ANALYSIS: Evidence of recognizing or correcting errors in reasoning	$\frac{1}{2} + \frac{1}{2} = \cancel{\frac{2}{4}}$ $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$ 

MENTOR FEEDBACK (Optional): Mentor's Initials: _______

Figure 18:
Self-Evaluation Checklist

While the checklist was provided to encourage students to monitor their mathematical application, it was not required that students complete a specific number of tallies in each category.

Goal Two: Increase Student's Self-evaluation Through Reflection

Discussion of Findings:

As the end of implementation neared, students became more comfortable with recognizing mathematical application categories in class discussions. For example, mathematical application vocabulary became more prevalent in student vocabulary as students became comfortable with the format of the home math activities, class discussions, and weekly assessments. During the final week of student presentations, multiple students referred to their "mentor" and "mathematical reasoning." On written reflections, students compared alternative "models" and "strategies" presented by their classmates. Some students extended their self-evaluation beyond written reflection by internalizing and applying new strategies toward problems on quizzes and tests.

Goal Three: Increase Family Involvement in Mathematics Curriculum

Finding One: Attendance and Response to Family Math Night

Over half of my 53 students attended a Family Math Night. While I documented that many of my students and their families attended Family Math Night, they cited many reasons for attending the event. I made an effort to interview my focus students about their response to Family Math Night. In her interview, Jessica mentioned that she wanted to earn credit for attendance. She expressed that because it seemed like a "fun" environment, she and her family were willing to participate. Scheduling conflicts were one reason for a lack of participation. In his student interview, Michael commented that he had wanted to attend Family Math Night when he saw most of his friends had signed up. Due to after school sports and previous commitments, he and his parents were not able to attend. Reviewing student and mentor comments, there were varied reasons for attending or not attending Family Math Night.

Of the families who attend Family Math Night, the majority of the feedback was positive. Jessica's mother stated that she enjoyed the event because she realized she could help Jessica without having to "know all the math." It was a revelation for many families to realize informal math challenges, puzzles, and games were available and could easily be implemented in their homes to build mathematical understanding. For example, Max and his mother attended a Family Math Night. Max and his mother enjoyed working together at the various math stations but found "The Card Problem" most engaging (see Appendix). Although Max and his mother were unsuccessful at completing the entire problem, they promised to continue solving the problem at home. The week after Family Math Night, Max's mother returned with Max and his younger sister to share their solution. Max's family experience reveals the interest and excitement Family Math Night generated in addition to the connection families created by sharing math activities with members of their extended family.

As expected, several parents were concerned about the lack of rigorous math content but as they participated in the math stations, they were able to recognize the connection between the math content and state standards.

Goal Three: Increase Family Involvement in Mathematics Curriculum

Finding Two: Recognizing and Strengthening the Home-School Connection

I collected anonymous responses from student and parent online surveys about their attitudes, ideas, and experiences with math homework. Before implementation of The Mathematics Connection, the majority of students enrolled in traditional mathematics and The Mathematics Connection agreed that homework was necessary in order to be a successful math student. The few students who viewed homework as insignificant believed that there was no connection between homework and class performance, "I think homework is not that important because you can have really good grades in tests and quizzes." Another student defined homework as repetition lacking critical reasoning, "it helps us memorize more about the lesson we learned." Many students described homework as "extra practice." After implementation, students in The Mathematics Connection completed a post online survey, identical to the pre online survey. The majority of students maintained their perception of homework as a useful tool for achieving mathematical success. I noted that while the attitude towards homework remained positive, students provided detailed comments that acknowledged the home-school connection. When asked if homework was necessary to become a successful math student, one student responded, "Yes, because it freshens your mind about the subject and it helps you for any upcoming quiz or test."

Mentors participating in The Mathematics Connection completed online surveys similar to student surveys before and after curriculum implementation. Prior to implementation, most parents strongly believed that homework was needed to achieve mathematical success. However, parents provided a wide range of reasons advocating for math homework. Some parents thought homework should provide repetition of class concepts and skills, "(homework) is very important for practice, practice makes perfect." Other parents expressed concerns related to rigorous content, authentic application, and providing home support:

> I think that homework is important to help support what the student is learning in class. I don't think that a ton of homework is necessary. It just becomes 'busy work'. In regards to textbooks, the only true advantage for me as a parent is that the beginning of a chapter will have the formulas and rules, as well as examples. It helps the parent learn, or remember how to do the work, then the parent can be more helpful. (Anonymous, Online Family Survey)

After implementation of The Mathematics Connection, parents maintained their support for math homework. Similar to the results of the pre implementation survey, parents' reasons for supporting math homework varied. Parents continued to request lessons, formulas, and samples to reference. Most parents still viewed homework as important extension of class lessons but parent responses revealed a new appreciation of mathematical application, which was absent in pre implementation surveys. For example, a parent stated that home math activities promoted meaningful application of concepts, "homework is a good practice for students as this will give them ample time and without constraints to analyze, understand, and find out how math relates to real life". Parents also recognized home math activities as informal progress reports that revealed their child's strengths, needs, and interests. One parent wrote, "home assignments also alert both the child and parents what areas the child excels in and what areas require improvement and/or better understanding of the concepts involved."

Online student surveys reported that students enrolled in The Mathematics Connection and traditional mathematics used a variety of resources to complete their math homework. Some students used more than one type of homework support. Figure 19 shows that after implementation of The Mathematics Connection, the percentage of students seeking help from a peer or mentor and referring to their class notes increased. It is not surprising that this data reveals all Mathematics Connection students worked collaboratively outside of the classroom since home math activities required students to work with a mentor. However, students voluntarily used their class notes to complete their homework and this implies that students recognized a connection between schoolwork and homework.

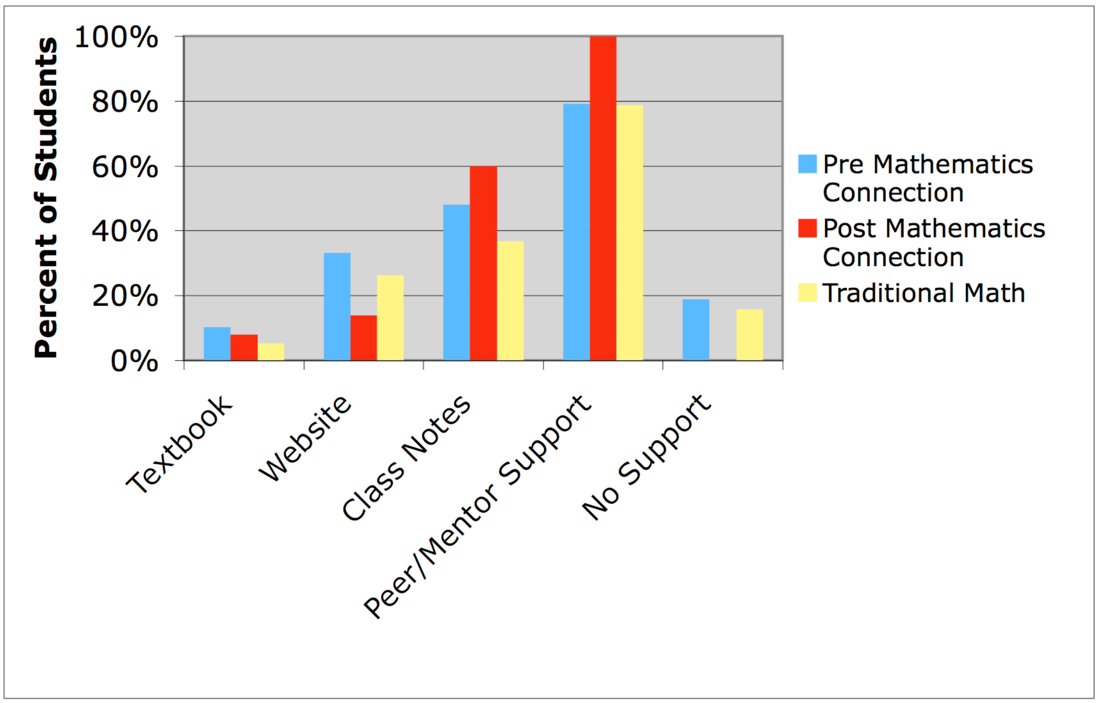


Figure 19:
Types of Homework Support Used by Students in Traditional Math and The Mathematics Connection

During his focus student interview, Luis identified the connection between homework and class performance, "I keep working until I can mark something in each box, you know, modeling and reasoning. It's because those are all the things I need to do on the quizzes so I try to do them all."

Goal Three: Increase Family Involvement in Mathematics Curriculum

Finding Three: Challenges of Mentor Collaboration

Parents have good intentions when attempting to advance their children's education. Yet, parents lack the professional training and experience of teachers. Therefore, parents need guidance about how to interact with their child to support learning. At Family Math Night and through informal meetings, I encouraged mentors to allow their student to lead problem-solving sessions. Mentors acted as advanced learners, who constantly questioned the student, clarified misunderstandings, and provided suggestions. Occasionally, mentors attempted to "teach" content to students without providing substantial scaffolding.

I observed Luis, a focus student and noticed how mentor-generated solutions often leave students without full mastery of the application. Luis receives weekly tutoring services from Technology Middle after school. He selected a high school tutor as his mentor since his father and mother work in the evenings. During a class presentation, Luis volunteered to display his solution and wrote an elaborate equation on the board to model the situation. He solved for the variable but faltered when answering questions about his reasoning for the equation. I questioned Luis, "Why did you choose to write this equation with variables on both sides?" Luis responded that his mentor had helped him but he believed that the equation should be split into two separate expressions. While Luis' equation was appropriate, he did not understand why he had applied the equation to the situation. Luis' equation displayed that mathematically advanced mentors can push their students toward algebraic skills without instructional support.

Each student-mentor pair approached a problem with prior knowledge, varied levels of understanding, and a cultural context. The open nature of the home math activities allowed student-mentor pairs to approach the problem from multiple perspectives. Yet, a few student and mentor interviews revealed some issues of frustration with accessing mathematical concepts. Overly rigorous content seemed to be the dominant issue. Sofia is a focus student of The Mathematics Connection. She selected her father, Mr. Garcia, as her mentor for all her home math assignments. In his mentor interview, Mr. Garcia commented that he felt overwhelmed by a problem because he was not able to assist his daughter, "I didn't know where to begin and Sofia couldn't understand what she was supposed to use from class. It worried me to see (Sofia) upset because math is really important." Mr. Garcia's interview suggests that students need to be well prepared with mathematical strategies before attempting to solve home math activities with mentors. Teachers should provide explicit instruction about application of math strategies in the classroom. With substantial opportunities for exploration and application of math strategies, students will likely be more comfortable explaining and interacting with their mentors outside of the classroom.

Home math activities require a commitment of time, resources, and patience from both mentors and students. With busy schedules, it is a challenge to find time for mentors and students to work together. Bailey's parents are recently divorced and she often splits her school week between her father and mother's homes. During her interview, Bailey explained that she would work on the first section of the activity with her mother and the later parts of the activity with her father. Bailey found it difficult to have two mentors because they often disagreed about how to approach the problem. While having one mentor through the five parts of the home activity seems ideal, the benefits of working collaboratively outweigh the struggles of working independently. In his focus student interview, Michael expressed that he appreciated working with his father because his father was an immediate source of support. Before home math activities, Michael worked on his homework alone. When Michael encountered a challenging problem, he attempted to understand it, tried to apply a strategy, and ultimately disregarded the problem if he was not immediately successful. Michael's experience of working with his father showed that the time invested by mentors provides students with personalized attention and support for seeking alternative strategies.

Goal Three: Increase Family Involvement in Mathematics Curriculum

Discussion of Findings:

Teachers and families must share a dedication and vision in order to build a strong home-school connection, which allows students to transfer knowledge from one setting to another. This task is challenging because of limited time and resources of both families and schools. Yet, the home-school connection is a vital component of <u>The Mathematics Connection</u>. There are numerous pitfalls to implementing a curriculum involving families and their unique strengths, needs, and interests. However, providing open ended, flexible problems with proper scaffolding in class is critical to student success. A teacher should construct student-centered home math activities, which extend to mathematical content standards.

Summary and Discussion

I designed The Mathematics Connection to address three goals of improving mathematical application, developing student self-evaluation through reflection, and strengthening a home-school connection. Students worked collaboratively with mentors on weekly home math activities. Students and mentors used semiotic mediation to achieve intersubjectivity of mathematical concepts and mathematical application. Students acted as vehicles of transfer by applying their knowledge from class to home math activities. As students learned to reflect on and analyze multiple strategies, they improved their ability to self-evaluate. Ultimately, these skills assisted students in identifying an accurate solution to a math problem.

I assessed students' mathematical application with an array of evaluations to determine the overall effects of The Mathematics Connection. I found that students produced thoughtful work with evidence of mathematical reasoning, modeling, and flexibility. Rather than memorizing procedures, students focused on understanding, interpreting, and responding to a problem. They applied their prior knowledge to solve the problem.

There are some aspects of mathematical learning I was not able to address in this limited study. During implementation, students studied algebraic equations and probability. I am interested in extending The Mathematics Connection to teaching other math concepts such as geometry and proportions. Further research is also needed to examine the effects of motivation and self-efficacy related to The Mathematics Connection. As my students progress to the next grade, I am curious if they will continue to approach math problems with the application and self-evaluation skills they have developed. I am also interested to see if families will continue to support their children without direct instruction or teacher invitations. There is unlimited potential for future studies to determine the long-term effects of family involvement on student achievement in all subject areas.

VIII. CONCLUSION

Current educational reform often overlooks the home-school connection. My experience creating and implementing The Mathematics Connection has revealed that teachers limit student learning by focusing solely on mathematic concepts. Teachers must look beyond the classroom towards students' vast funds of knowledge. Students need opportunities situated in meaningful contexts to interact with their peers, mentors, and teachers to collaboratively construct knowledge. While this project has allowed me a glimpse at the possibilities of family involvement, I feel it has only skimmed the surface of analyzing the link between homes and schools.

The evidence I presented in Chapter VII suggests that students successfully increased their mathematical reasoning, modeling, and flexibility skills. In addition, students gained self-evaluation skills to identify their misunderstandings. While the overall findings suggest students increased their algebraic understanding, the curriculum fell short in terms of students acquiring math skills such as balancing an algebraic equation. This is understandable given the time constraints of the project. If I were to implement this curriculum again, I would be more explicit about connecting the strategies my students developed in home math activities to the "traditional" algebraic skills outlined in our textbook. Further, I would involve students in algebraic skill practice. Although I do not advocate introducing algebraic skills without an understanding of their application, I do think I made an oversight in not providing enough skill practice in class. I came to this realization when I observed students preparing for our annual state standardized test. I noticed students who had successfully solved problems in home math assignments struggled to perform the same skills on isolated test problems. Even though there was a slight gap between mathematical application and math skill performance, there were noticeable changes in students' learning process.

Although this change did not occur easily, I have faith that the curriculum can be implemented in other mathematics classrooms. I am fortunate to work with a population of parents willing and able to participate in their child's learning. Yet, even with dedicated parents, the changes I observed in student learning would not be possible without the curriculum. The Mathematics Connection served as a pathway for all families and students to have equal access to math achievement.

In schools with lower levels of family involvement, I suggest implementing The Mathematics Connection with a "graduated" approach. The curriculum should allow families to be involved with student learning at varied levels based on the time and resources they have available. In addition, the key to successfully implementing The Mathematics Connection with all families is to provide immediate feedback. Families react positively when they receive good news and evidence of how their efforts are helping their child's performance. I recognize providing feedback for all families and students can be time consuming for teachers but a brief e-mail can help to reinforce the home-school connection. Changing teaching and learning practices requires time and effort from all parties involved; teachers, students, and families. However, the benefits of the curriculum are worthwhile and have potential lasting effects.

A benefit of this curriculum is that it can easily be adapted to meet the math standards of different grade levels. Teachers have the freedom to evaluate and insert activities into the structure of home math assignments. The flexibility of this curriculum encourages both beginning and veteran teachers to apply this approach to learning mathematics. In addition, teachers can personalize activities with the names of their students. A highly critical component of this project is the communication between teachers and families. Inviting families to support learning has proven to be beneficial to students on multiple levels. Students gained multiple problem-solving strategies from their mentors and from their classmates. Students expressed interest in home math assignments and the rate of homework completion increased. In fact, many students reported that they spent less time on home assignments. While I can only speculate reasons for this drop of time invested in homework, I interpret it as students focusing on content while becoming familiar with the format of assignments. Although I set out to measure mathematical application and self-evaluation, I noted a general increase in student motivation and self-efficacy. From my field notes, I observed that students participated on a regular basis. Participation included the number of times students responded to prompts and the number of students participating in the discussion. I was most pleased to see students develop confidence and attempt their own strategies before seeking teacher feedback.

As an activist for the home-school connection, I want to ensure that families are welcomed into classrooms and seen as equal partners in education. To achieve this goal, I plan to disseminate my curriculum to colleagues at my school site and student teachers in credentialing programs. Ultimately, The Mathematics Connection utilizes the power of social, collaborative learning. My hope is that families will recognize the key role they play in shaping students' learning and development.

APPENDIX: <u>THE MATHEMATICS CONNECTION</u>

Table of Contents

Introduction: Letter to Potential Implementers

Dear Educator,

The Mathematics Connection addresses the need for student math application and self-evaluation through the home-school connection. Traditional math curriculum relies heavily on textbooks, worksheets, and teacher-centered lecture, which promote isolated repetition of math skills. This type of instruction fails to provide students with opportunities to apply math skills in authentic contexts. The Mathematics Connection is a series of fun and engaging activities that access families as resources for increasing student math understanding and application.

Goals

In designing and implementing The Mathematics Connection, I addressed the following questions:

1. In what ways does family involvement support student math application?
2. How can discussion about engaging, content-rich math problems promote student reflection and self-evaluation?

The goals of The Mathematics Connection are to increase student mathematical application, develop student self-evaluation, and strengthen the home-school connection.

Classroom Environment & Culture

Teachers should implement The Mathematics Connection in classrooms that emphasize critical analysis and collaborative teamwork when tackling open-ended math problems. As students apply multiple strategies for solving the same problem, the teacher should lead discussions about the advantages, disadvantages, and connections between each strategy. At least an hour daily should be dedicated to math instruction and exploration. Although subject integration is not necessary, it promotes students view of math as applicable to all aspects of life. The teacher is responsible for setting benchmarks in the class to check on and discuss student progress in home math activities.

Home math activities require the student to seek out a mentor. A mentor may be a parent, sibling, tutor, teacher, relative, coach or other community member. The purpose of the mentor is to question the student and help the student clarify his/her mathematical thinking. Collaboration and semiotic mediation is a key construct of The Mathematics Connection since language acts as a tool to build knowledge and achieve shared understanding between learners. Each week students and mentors receive a new challenge and students are responsible for connecting skills from class to home activities.

The Mathematics Connection Activities

The following materials can be modified to address the objectives of various math units. All activities should be situated learning opportunities, which allow students to access math content through an authentic context. Teachers should begin with Family Math Night as a celebration for math learning and introducing families to the curriculum. A home math activity should be scheduled each week with benchmark check-ins to monitor student progress. Quizzes and tests can be scheduled according to class and teacher needs.

Since the bulk of The Mathematics Connection curriculum focuses on the interactions between mentor and student, home math activities are the center of the curriculum. I organized home math activities in a minimum of five sections. Part one introduces the problem and requires students to interpret and predict. Part two is the sections where students should spend most of their time engaged in discussion with their mentor. Part three requires mentor and student to collaboratively assess their work based on four math application categories. Students completed the fourth section in class during student presentations at the end of the week. While the students have the chance to guide instruction, the teacher is still responsible for questioning and connecting student's mathematical strategy with mathematical content and skills. Extension sections may be added to challenge students who have more time with their mentor and a higher level of understanding.

Assessment Plans

After applying The Mathematics Connection, teachers may use quizzes and tests to formally assess student application. Teachers should not grade quizzes and tests solely on the correct answer. Using The Mathematics Connection checklist, teachers judge student work on displays of mathematical reasoning, mathematical modeling, and mathematical flexibility. This type of assessment reinforces the curriculum emphasis of mathematical application, multiple strategies, critical thinking, and self-evaluation. When assessment aligns with classroom and home math activities, students can transfer skills and knowledge.

I hope you will find The Mathematics Connection helpful in improving your students' mathematical application and self-evaluation. I encourage you build relationships with families of your students because they are funds of knowledge that can support student learning.

Family Math Night Materials: Family Math Night Flyer

The Komatsubara & Interlandi Team
invite you and your family to attend

Family Math Night

Come and enjoy a night of fun math games!
Learn about new math curriculum and how to help your child succeed in mathematics.

Refreshments Provided
No previous experience necessary

Tuesday, February 26th
OR
Thursday, February 28th
5:00 – 7:00 pm
Ms. Komatsubara's Room

Please R.S.V.P. by Friday, February 22nd
The portion below should be completed and returned to Ms. Komatsubara
~~~~~~

Student Name: ________________________________

________ Yes, I plan to attend family math night on **Tuesday, February 26th**
There are _______ people in my family (including student).

________ Yes, I plan to attend family math night on **Thursday, February 28th**
There are _______ people in my family (including student).

________ No, I will not be able to attend family math night
~~~~~~

Family Math Night Materials: Family Math Information Packet with Stations

Welcome to Family Math Night

Ms. Komatsubara, 2007 – 2008

Home math activities assess and encourage development of:

- ✓ Mathematical Reasoning
- ✓ Mathematical Modeling
- ✓ Mathematical Flexibility
- ✓ Mathematical Analysis & Reflection

How can families support student learning?

- ✓ Assume the role of "mentor"
- ✓ Ask questions that encourage children to analyze their math ideas
- ✓ Use mathematics connection rubric to evaluate student understanding

Suggestions for Family Math Fun

- ✓ play board and card games together
- ✓ check out Ms. K's DP for interactive, online math activities
- ✓ prepare recipes together
- ✓ allow students to calculate the cost of items at the supermarket or department store

Thank you for your continued support and involvement in family math!

The Card Problem

STATE STANDARDS:

Mathematical Reasoning

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

2.1 Use estimation to verify the reasonableness of calculated results.

MATERIALS:

❖ playing cards (ace to 10)

OBJECTIVE:

Figure out how to arrange the cards so that the following can happen: Turn over the top card, which should be an ace and place it face up on the table; move the next card to the bottom of the deck. Turn over the third card, which should be a 2, and place it face up on the table; move the next card to the bottom of the deck. Continue this way, turning over a card, placing it face up on the table, and moving the next card to the bottom of the deck. When you are done all the cards should be face up in order on the tabletop. (Burns, 2007, p. 140)

Birthdays

STATE STANDARDS:

Number Sense

1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

OBJECTIVE:

Follow these directions to find out on what day of the week you were born.

1. Write down the last two digits of the year in which you were born. Call this number A.

2. Divide the number (A) by four, and drop the remainder if there is one. This answer, without the remainder, is B.

3. Find the number of the month in which you were born from the Month Table below. This number is C.

4. On what day of the month were you born? This number is D.

5. Add the numbers from the first four steps: A + B + C + D.

6. Divide the sum you got in Step 5 by the number seven. What is the remainder from that division? (It should be a number from zero to six.) Find this remainder in the Day Table. That tells you what day of the week you were born. (This method works for any day in the twentieth century.)
(Burns, 2007, p. 166)

Month	#
January (leap year is 0)	1
February (leap year is 3)	4
March	4
April	0
May	2
June	5
July	0
August	3
September	6
October	1
November	4
December	5

Day	#
Sunday	1
Monday	2
Tuesday	3
Wednesday	4
Thursday	5
Friday	6
Saturday	0

(Burns, 2007, p. 166)

The Handshake Problem

<u>STATE STANDARDS:</u>

Algebra & Functions

1.1 Use variables and appropriate operations to write an expression, equation, inequality, or a system of equations or inequalities that represents a verbal description.

Mathematical Reasoning

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

<u>OBJECTIVE:</u>

With only one person in the room, there will be no handshake.

With two people, there will be one handshake.

How many handshakes will there be with three people? Four? Five?
(Burns, 2007, p. 156)

Foot Figuring

STATE STANDARDS:

Measurement & Geometry

1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems.

MATERIALS:

- pencil
- scissors
- construction paper
- ruler
- a partner

OBJECTIVE:

Trace your left foot (with your shoe off) on construction paper. Cut it out. Record your name, shoe size, and the length of your foot in centimeters. Compare your shoe size and foot length with those of others. Do longer feet always have larger shoe sizes?

Make a prediction. How many of your feet equal your height? How many of your feet equal your standing broad jump? Now find out. Record the information on the back of your cutout foot as show (Burns, 2007, p. 78).

Four in a Row

STATE STANDARDS:

Mathematical Reasoning

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

Mathematical Reasoning

2.4 Make and test conjectures by using both inductive and deductive reasoning.

MATERIALS:

- ❖ partner or a small group
- ❖ 1-inch grid paper
- ❖ color tiles (15 per participant)

OBJECTIVE:

This game can be played by two to four players. The grid paper is the playing board. Each player chooses a different color and uses the tiles of that color. On each turn, a player places a tile on a square.

After you have played the game, discuss a strategy for winning (Burns, 2007, p. 143).

The Game of Pig

STATE STANDARDS:

Number Sense

1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Mathematical Reasoning

2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

MATERIALS:

- ❖ pair of dice
- ❖ hundred chart (optional)
- ❖ partner or a small group

OBJECTIVE:

The goal of the game is to be the first to reach 100. On your turn, roll the dice as many times as you like, mentally keeping a running total of the sum. When you decide to stop rolling, record the total for that turn and add it to the total of the previous turns.

The catch: If a 1 comes up on one of the dice, the player's turn automatically ends and 0 is scored for that round. If 1 s come up on both dice, not only does the turn end, but the total accumulated so far returns to 0.

After becoming familiar with the game, discuss a strategy for winning (Burns, 2007, p. 99).

Home Math Activities: Part I: Interpreting & Predicting

PART I: INTERPRETING & PREDICTING (Due ________________)

Explain the problem to your parent/partner. Write or draw what you understand about the situation. Make a prediction about what you think the answer will be.

Mentor's Initials: ___________

Home Math Activities: Part II: Searching for a Solution

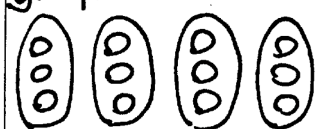
PART II: SEARCHING FOR A SOLUTION (Due _______________)
Discuss your solution with your parent/partner. Explain your answer with writing, drawing, or numbers.

Mentor's Initials: __________

Home Math Activities: Part III: Self-Evaluation Checklist

PART III: SELF-EVALUATION (Due ______________________________)

With your mentor, please monitor the number of times you find evidence of the actions below.

#	Action	Example
	MATHEMATICAL REASONING: Evidence of explanation or justification in writing or through discussion	Perfect squares have an odd number of factors because one factor repeats itself. 16: 1, 2, 4, 8, 16
	MATHEMATICAL MODELING: Evidence of using pictures, tables, or graphs to represent mathematical reasoning	$4 \times 3 = 12$ ← total groups oranges 
	MATHEMATICAL FLEXIBILITY: Evidence of more than one strategy to solve the problem	$3 \cdot \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$ ✓  $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$ ✓
	MATHEMATICAL ANALYSIS: Evidence of recognizing or correcting errors in reasoning	$\frac{1}{2} + \frac{1}{2} = \cancel{\frac{2}{4}}$ $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$ 

MENTOR FEEDBACK (Optional): Mentor's Initials: _______

Home Math Activities: Part IV: Reflection

PART IV: REFLECTION (Due ______________________________)

Was your solution different than your original prediction? What caused you to change your answer? This section may be done with your mentor or in class when we present solutions at the end of the week.

Home Math Activities: A Fair Game Problem & Extensions

A Fair Game Problem

Work with a mentor (preferably a family member) and take turns rolling the die two times each. Player A scores a point if the sum is even. Player B scores a point if the sum is odd. Is this game fair? If not, how could you make the game fair? Explain your reasoning (Burns, 2007, p. 102).

PART 1: INTERPRETING (Due ________________________)
Explain the problem to your parents/partner. Write or draw what you understand about the situation.

Mentor's Initials: _________

PART V: EXTENSION (Optional, Due ______________________)
Play the game again but this time use the product to determine the winner. Player A scores a point if the product is even. Player B scores a point if the product is odd. Is the game fair? If not, how could you make the game fair? Explain your reasoning (Burns, 2007, p. 102).

Mentor's Initials: __________

PART VI: EXTENSTION (Optional, Due ________________________)

Play the rock, paper, scissors game with three players. All players make a fist and on the count of three, each player shows either:

paper (by showing five fingers)
scissors (by showing two fingers)
rock (by showing a fist)

Play twenty times with the following rules:

Player A receives a point if all players show the same sign.
Player B receives a point if only two players show the same sign.
Player C receives a point if all players show different signs.

Tally the winning points:

Player	Tally	Total
A		
B		
C		

Is this a fair game? Which player would you rather be? How could you make the game fairer? (Burns, 2007, p. 102)

Mentor's Initials: ___________

Home Math Activities: The PIN Problem & Extensions

The PIN Problem

"I forgot the code! I'm ready to cry! How many arrangements do I need to try?" Ms. Morales has forgotten the code to access money from her bank account. It is a four-digit personal identification number (PIN) and luckily, she remembers that she used the numbers zero, four, eight, and nine. How many possible arrangements are there for Ms. Morales' PIN? (National Council of Teachers of Mathematics, 2004)

PART I: INTERPRETING & PREDICTING (Due ________________)
Explain the problem to your parent/partner. Write or draw what you understand about the situation. Make a prediction about what you think the answer will be.

Mentor's Initials: ________

PART V: EXTENSION (Optional, Due ______________________________)
Mr. Mike is not as fortunate as Ms. Morales. He has forgotten his PIN but he cannot remember any of the numbers he included. His PIN is also a four-digit code. How many possibilities are there for Mr. Mike's PIN? (National Council of Teachers of Mathematics, 2004)

Mentor's Initials: ____________

Home Math Activities: Splitting the Pot Problem & Extensions

Splitting the Pot Problem

Ms. Komatsubara and Mrs. Interlandi play a game of flipping a coin. If the coin lands on heads, Ms. K earns a point. If the coin lands on tails, Mrs. I earns a point. The first person to earn 10 points, wins the game. They each put $50 into the pot, making the total pot worth $100.

They are playing 'winner takes all'. However, a strange thing happens. Mrs. I is winning, 8 points to 7, when she receives an urgent message that Joey has pushed a pea into his nose. She rushes off leaving to rescue Joey. Based on probability, how should the $100 be divided?

PART I: INTERPRETING & PREDICTING (Due ____________________)
Explain the problem to your parents/partner. Write or draw what you understand about the situation. Without calculations, predict how Ms. Komatsubara will fairly divide the $100.

Mentor's Initials: ____________

Part V: EXTENSION (Optional, Due ______________________)
Suppose Mrs. Interlandi had been winning 9 to 6 when she left Ms. Komatsubara. How should the $100 be divided?

Mentor's Initials: __________

PART VI: EXTENSTION (Optional, Due ______________________________)

Suppose you had come upon them earlier and noticed that they were tied 5 to 5. You leave and come back 5 flips of the coin later. What's the probability of Mrs. Interlandi being ahead 9 to 6?

Mentor's Initials: ___________

Home Math Activities: A Mathematical Tug-of-War & Extensions

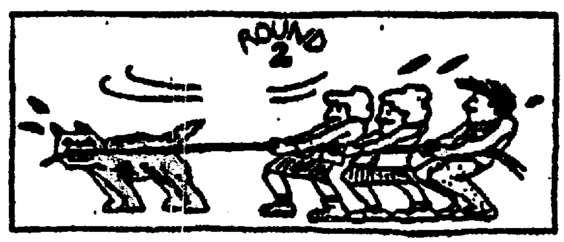
A Mathematical Tug-of-War

The First Round

On one side there are four acrobats who have come down to the ground during the off season for this special event. They have well-developed muscles because of all the swinging they do, and have proven themselves to be of equal strength. On the other side are five neighborhood grandmas, a tugging team for many, many years. They, too, are all equal in strength. In a content between these two teams, the result is dead even.

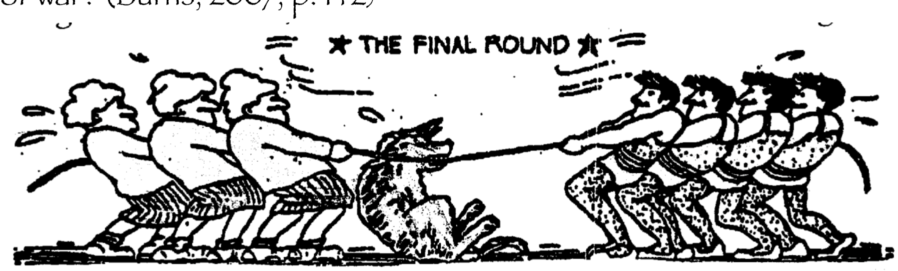
The Second Round

One team is Butchie, the specially trained dog that got his start as a pup when he was taken out for a walk by his owner. Butchie is pitted against a team of two grandmas and one acrobat. Again, it's a draw—an equal pull.



The Final Round

It's the final tug that you must figure out. It will be between these two teams: Butchie and three of the grandmas on one side, the four acrobats on the other. Who will win this tug of war? (Burns, 2007, p. 142)



PART V: EXTENSION (Optional, Due ____________________________)
A bonus round occurs. On one team there are five grandmas and two acrobats pitted against a team made of Butchie and three acrobats. Which team will win this tug-of-war round? (Burns, 2007, p. 142)

Mentor's Initials: ___________

Home Math Activities: Dragon Babies Problem & Extensions

Dragon Babies Problem

John and Beyra are purchasing paints to create their dragon baby. At Michael's a can of paint costs $17 and a set of brushes costs $30. At Lowe's a gallon of paint costs $20 and a set of brushes costs $16.50.

John and Beyra are not sure how much paint they will need to complete the project.

Task A: If they only need one gallon of paint and a set of brushes, which store is more affordable?

Task B: If they need six gallons of paint and a set of brushes, which store is more affordable?

Task C: Find the gallons of paint for which the cost of the paint and brushes is the same.

PART I: INTERPRETING & PREDICTING (Due ________________)

Explain the problem to your parent/partner. Write or draw what you understand about the situation. Make a prediction about what you think the answer will be.

Mentor's Initials: _________

PART V: EXTENSION (Optional, Due ________________________)
Christian and Ramon decide to go on the Washington D.C. trip as eighth graders. They walk to most of the museums but need to catch a taxi to the airport. The Yellow Taxi charges $5 for pickup and 70 cents per mile. The Checkered Taxi service has no pickup fee but charges $1.20 per mile.

Task A: If the airport is only one mile away, which taxi service is more affordable?

Task B: If the airport is miles away, which taxi service is more affordable?

Task C: Find the number of miles for which the cost of the taxi service is the same.

Mentor's Initials: __________

Home Math Activities: Tech Deck Mania & Extensions

Tech Deck Mania

Jaylon and Mikito have the same number of tech decks in their collections. Jaylon has 6 complete sets and 2 individual tech decks, and Mikito has 3 complete sets plus 20 individual tech decks. How many tech decks are in a complete set? How many tech decks do Jaylon and Mikito have?
(Bennett et al., 2004, p. 510)

PART 1: INTERPRETING & PREDICTING (Due ________________)
Explain the problem to your parent/partner. Write or draw what you understand about the situation. Make a prediction about what you think the answer will be.

Mentor's Initials: __________

PART V: EXTENSION (Optional, Due ___________________________)

Two cars are traveling in the same direction toward Seattle. Derek is in the first car traveling at 45 miles per hour. Tim is in the second car traveling at 60 miles per hour. Derek leaves San Diego 2 hours before Tim leaves. How long would it take for Tim to catch up to Derek? (Bennett et al., 2004, p. 511)

Mentor's Initials: ___________

Home Math Activities: Animal Ages Problem & Extensions

The Animal Ages Problem

Kristin's father is 47 years old. Kristin's family adopts a new-born Irish Setter puppy. Below is a table comparing dog and human age rates. When will Kristin's father and the Irish setter be the same age?

Dog's Age in Years	0	1	2	3	4
Equivalent Human Age	0	12	19	26	

PART I: INTERPRETING & PREDICTING (Due ___________________)
Explain the problem to your parent/partner. Write or draw what you understand about the situation. Make a prediction about what you think the answer will be.

Mentor's Initials: ________

PART V: EXTENSION (Optional, Due ________________________)
LT and Fred build go-carts for a race. Fred starts 6 meters behind LT and his cart travels steadily at 5 meters every second. LT's cart starts slowly but increases exponentially. After the first second, LT has traveled one meter. After two seconds, LT has traveled 4 meters and after three seconds, LT has traveled 9 meters.

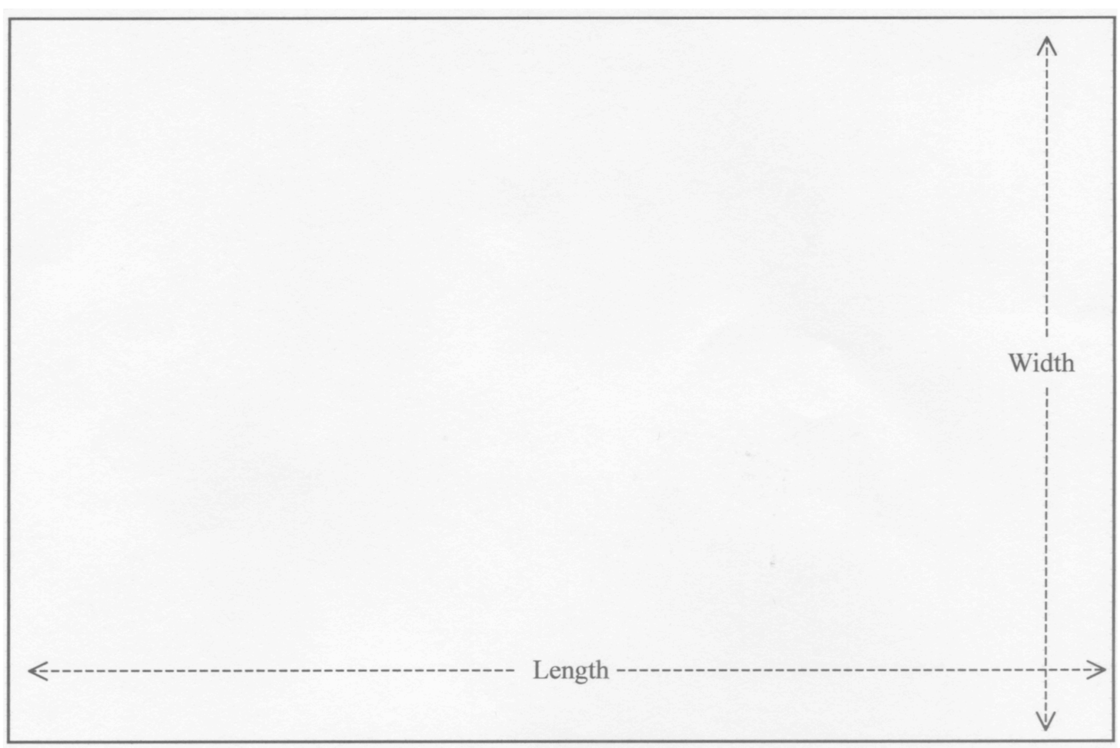
At what point will Fred catch up to LT? What happens during the race and who do you predict will win?

Mentor's Initials: __________

Home Math Activities: The Random Rectangles Problem & Extensions

The Random Rectangles Problem

Use a ruler to measure the dimensions of the rectangle in inches. Then, measure the dimensions in centimeters. Record your measurements in the chart on part II. Determine the length and width of the rectangle using four other units of measure and record them in the chart. Different units might include but are not limited to pennies, pencil erasers, lego blocks, and paper clips. When measuring with different units, do you think the relationship between the length and the width ($\frac{l}{w}$) will change?
(National Council of Teachers of Mathematics, 2008a)



PART I: INTERPRETING & PREDICTING (Due ________________)
Explain the problem to your parent/partner. Write or draw what you understand about the situation. Make a prediction about what you think the answer will be.

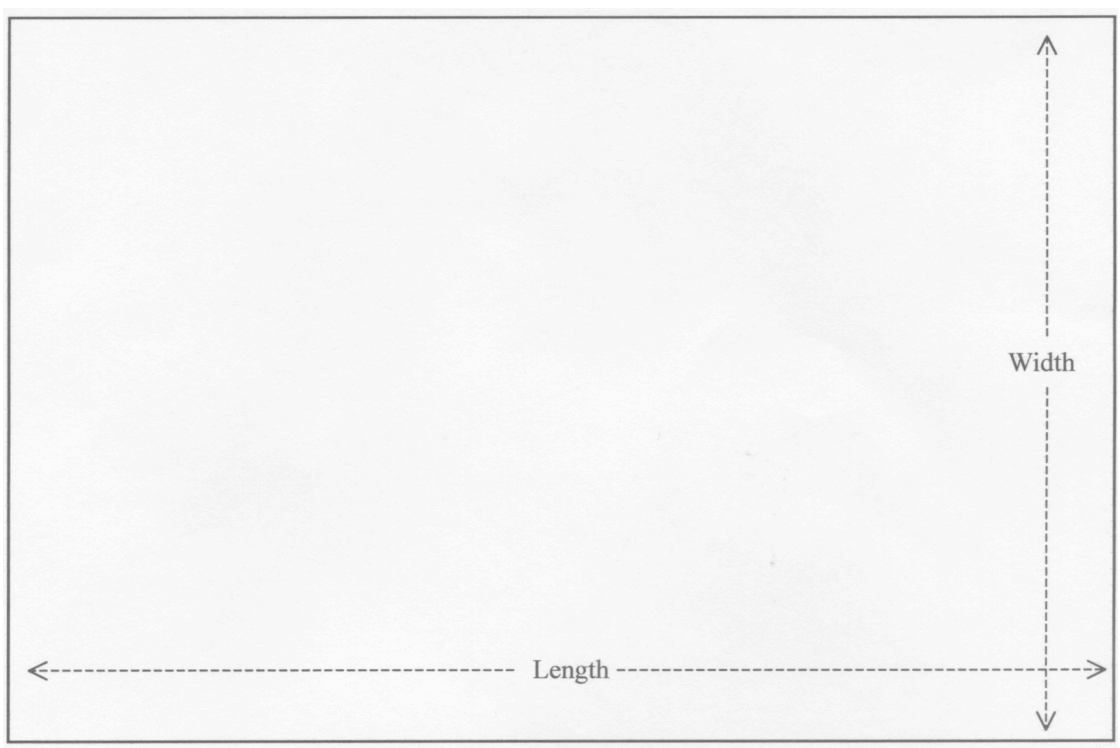
Mentor's Initials: __________

PART II: SEARCHING FOR A SOLUTION (Due ________________)

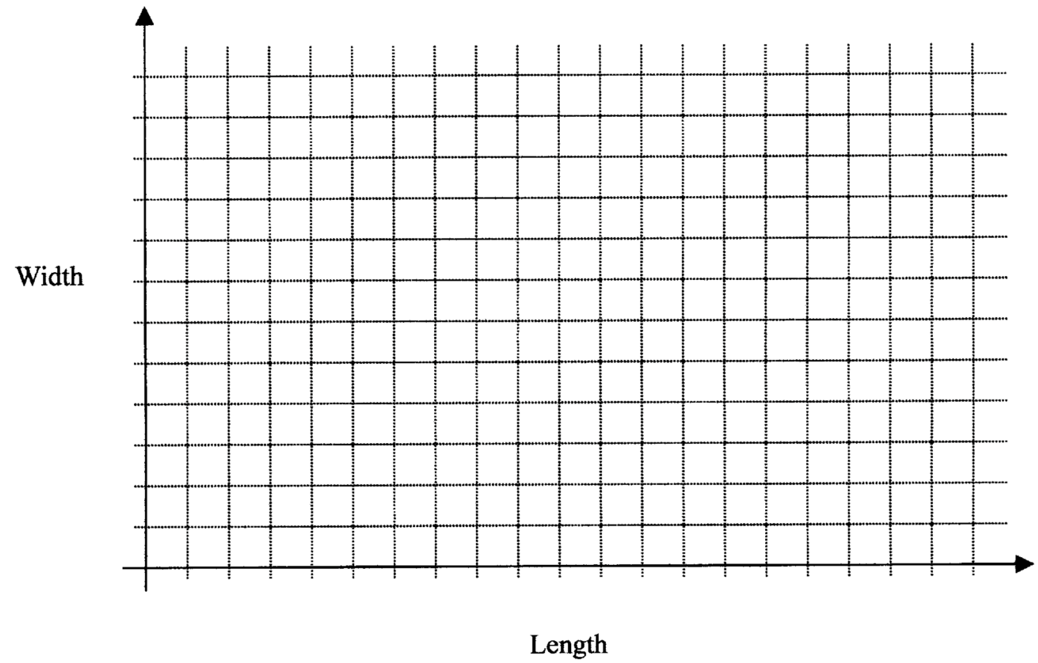
Discuss your solution with your parent/partner. Explain your answer with writing, drawing, or numbers.

Units	Length	Width
Inches		
Centimeters		

(National Council of Teachers of Mathematics, 2008a)



PART II: Continued . . .



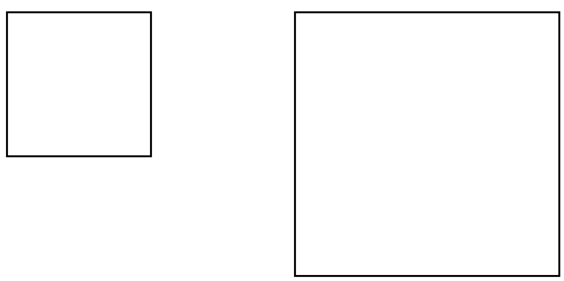
Mentor's Initials: __________

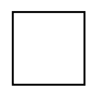
PART V: EXTENSION (Optional, Due ______________________________)

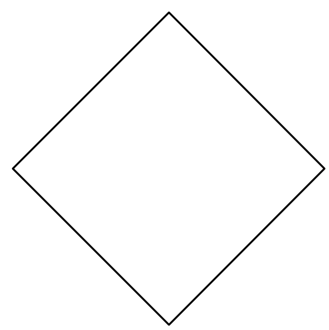
Use a ruler to measure the dimensions of the square A in centimeters. Measure the dimensions of the remaining squares. If the size of the square changed, do you think the relationship between the length and width will change?

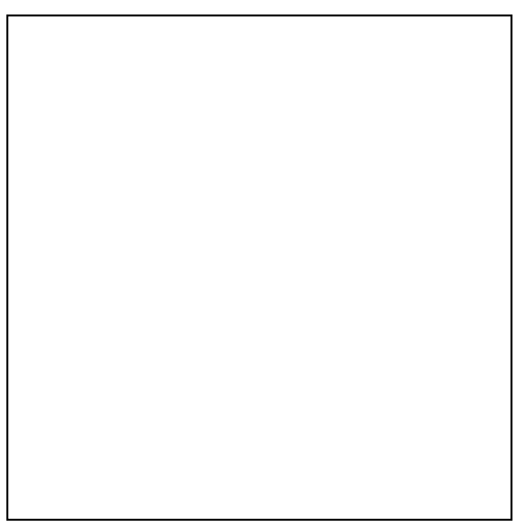
Square	Length	Width
A		
B		
C		
D		
E		

(National Council of Teachers of Mathematics, 2008a)









Mentor's Initials: __________

Home Math Activities: The Birthday Party Problem

The Birthday Party Problem

Erica and Rachel started their own summer business - putting on birthday parties for small children. Their neighbors agreed to loan them square card tables to seat the children for refreshments. However, when some of the neighbors were away on vacation, Erica and Rachel couldn't use their tables, and they really hated hauling the tables back and forth. Therefore, using as few tables as possible was important to them. Because all the children wanted to sit together, Erica and Rachel had to place the card tables together into rectangles. Only one child could sit on each side of a card table.

Their first party had twenty-two children. How many tables should they borrow? (National Council of Teachers of Mathematics, 2008b)

PART I: INTERPRETING & PREDICTING (Due ______________)
Explain the problem to your parent/partner. Write or draw what you understand about the situation. Make a prediction about what you think the answer will be.

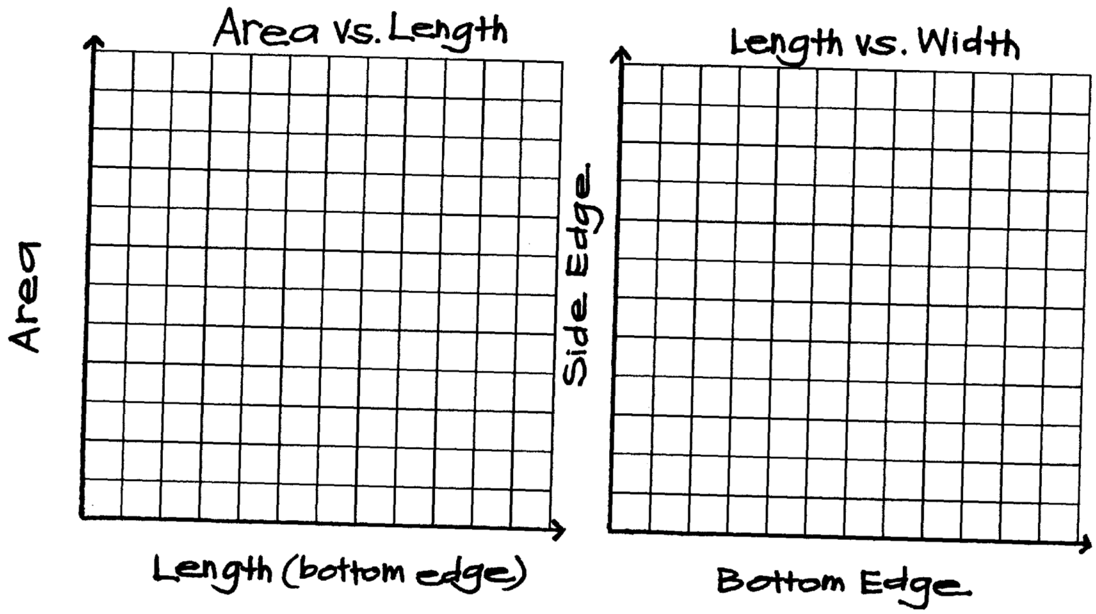
Mentor's Initials: __________

PART II: SEARCHING FOR A SOLUTION (Due ________________)
Discuss your solution with your parent/partner. Explain your answer with writing, drawing, or numbers. Record all of the arrangements in the table below.

Length (units)	Width (units)	Perimeter (units)	Area (square units)

(National Council of Teachers of Mathematics, 2008b)

Graph the length, width, and area of the possible birthday tables below.



What is the relationship between the area and length?

What is the relationship between length and width?

Will the line representing the relationship between table length and width ever cross the x-axis or y-axis? Explain your reasoning (National Council of Teachers of Mathematics, 2008b).

Mentor's Initials: ___________

PART V: EXTENSION (Optional, Due ________________________)
How many people can be seated if we have 24 square unit tables?
How is this problem different than a table with a fixed perimeter?

Length (units)	Width (units)	Perimeter (units)	Area (square units)
			24
			24
			24
			24
			24
			24
			24
			24
			24
			24

(National Council of Teachers of Mathematics, 2008b)

Mentor's Initials: __________

Building Self-Evaluation & Reflection: Sentence Starters for Reflection

1. My strategies was effective/ineffective because . . .

2. I know . . . because . . .

3. When I compare my strategy to ________'s strategy, I notice . . .

4. The advantage/disadvantage of this strategy is . . .

5. Today I changed the way I . . . because . . .

6. ________'s strategy and ________'s strategy are related because . . .

7. This strategy could be used to find . . .

8. At the end of today's lesson, I understand the section about... but need to clarify the bit about...

Building Self-Evaluation & Reflection: Critical Questions

1. How did you solve the problem?

2. What pattern(s) do you notice?

3. What evidence do you have that your solution is accurate?

4. What is the relationship between this problem and the previous problem?

5. Why do you think this strategy was more effective than the first strategy you attempted?

6. Why did you change your prediction/answer?

7. What knowledge have you gained from this problem? What concepts are you still unclear about?

Formal Assessments: Introduction to Probability Quiz

Introduction to Probability Quiz

LT and Lindsay play a game of "marble snap". There are four marbles in a bag, two red marbles, one green marble, and one yellow marble. Each player selects a marble, returns it to the bag, and then selects a second marble.

If the player selects two red marbles, LT earns a point. If they select any other combination, Lindsay earns a point. Is this a fair game? Explain your answer.

Formal Assessments: Outcomes Quiz

Outcomes Quiz

Rachel, Mikito, Justin, Ben, and Ines run a mile for the 7th grade fitness test. There are prizes for a first place, second place, and third place finisher. How many different arrangements are possible for first, second, and third prize?

Formal Assessments: Chairs in a Row Quiz

Chairs in a Row Quiz

Jessica and Kezia arrange chairs for a dance performance. If they arrange the chairs in 5 rows, they have 2 chairs left over. If they arrange the chairs in 3 rows, they have 14 chairs left over. How many chairs are in each row? (Bennett et al., 2004, p. 510)

1. Write an algebraic equation with a variable to express the problem.

2. Solve the problem using any strategy you prefer.

3. Model the problem with a picture, graph, or chart.

4. Explain how you solved the problem.

Formal Assessments: The Bicycle Ramp Quiz

The Bicycle Ramp Quiz

Derek builds a ramp for his bicycle. He makes the ramp 6 feet tall with a slope of $-\frac{2}{5}$. How long (feet) is the base of Derek's ramp?

1. Solve the problem using any strategy you prefer.
2. Model the problem with a picture or graph.
3. Explain how you solved the problem.
4. Use multiple strategies to earn extra credit.

Formal Assessments: Mini Horse Quiz

The Mini Horse Quiz

Max and Tim are cowboys. They need to build an arena for their mini horses to run around and exercise. They went out to the Home Depot and purchased 32 meters of fence. The boys want to build a rectangle that will give their mini horses the biggest area to run around and play. What dimensions (length and width) should they use?

Formal Assessments: Quiz Checklist

QUIZ CHECKLIST

Correct Answer:	________/3 pts
Mathematical Reasoning: (explanation/justification)	________/3 pts
Mathematical Modeling: (picture/graphs/tables)	________/3 pts
Mathematical Flexibility: (multiple strategies earn extra credit)	________/0 pts
TOTAL POINTS:	________/9 pts

Formal Assessments: Probability Test & Checklist

Probability Test

Choose 3 out of the 4 problems to solve.

PROBLEM 1: POTLUCK PROBLEM

Zoe, Kassidy, Ambar, and Kimberly decide to have a picnic to celebrate the end of POLs. They decide to have sandwiches, lemonade, apple slices, and cupcakes. They randomly choose to bring a dish to the picnic. How many different combinations are possible?

PROBLEM 2: IS IT FAIR?

Gabe and Daniel play a game of 1, 2, 3. Each player randomly holds up 1, 2, or 3 fingers. If they hold the same number of fingers, Gabe earns a point. If they hold up a different number of fingers, Daniel earns a point. Is this a fair game? Explain your answer (Burns, 2007, p. 102).

PROBLEM 3: TO THE MOVIES

Carly can invite two friends to the movies. She has a list of five friends, Rachel, Lexi, Wade, Trent, and Jaylon. If she randomly selects two friends, what is the probability that she will choose one boy and one girl?

PROBLEM 4: THE CHANCE OF WINNING

Ben and Ethan flip a coin to see who gets to use the last laptop in Mrs. Interlandi's cabinet. If the coin lands on tails, Ethan earns a point. If the coin lands on tails, Ben earns a point. The first person to reach 4 points wins. Ethan currently has 3 points while Ben has 2 points. What is the probability of Ben winning the game and using the laptop?

PROBABILITY TEST CHECKLIST

PROBLEM 1: POTLUCK PROBLEM

CORRECT ANSWER: ________/3 pts

MATHEMATICAL REASONING: ________/3 pts
(explanation/justification)

MATHEMATICAL MODELING: ________/3 pts
(picture/graph/table)

MATHEMATICAL FLEXIBILITY: ________/ 0 pts
(additional strategies earn extra-credit)

TOTAL: ________/ 9 pts

PROBLEM 2: IS IT FAIR?

CORRECT ANSWER: ________/3 pts

MATHEMATICAL REASONING: ________/3 pts
(explanation/justification)

MATHEMATICAL MODELING: ________/3 pts
(picture/graph/table)

MATHEMATICAL FLEXIBILITY: ________/ 0 pts
(additional strategies earn extra-credit)

TOTAL: ________/ 9 pts

PROBLEM 3: TO THE MOVIES

CORRECT ANSWER:	________/3 pts
MATHEMATICAL REASONING: (explanation/justification)	________/3 pts
MATHEMATICAL MODELING: (picture/graph/table)	________/3 pts
MATHEMATICAL FLEXIBILITY: (additional strategies earn extra-credit)	________/ 0 pts
TOTAL:	________/ 9 pts

PROBLEM 4: THE CHANCE OF WINNING

CORRECT ANSWER:	________/3 pts
MATHEMATICAL REASONING: (explanation/justification)	________/3 pts
MATHEMATICAL MODELING: (picture/graph/table)	________/3 pts
MATHEMATICAL FLEXIBILITY: (additional strategies earn extra-credit)	________/ 0 pts
TOTAL:	________/ 9 pts

OVERALL SCORE: __________/27 pts

GRADE: __________

Formal Assessments: Algebraic Equations Test & Checklist

Algebraic Equations Test

Choose **3** out of the 4 problems to solve. Besides finding the correct answer, you will be evaluated on your ability to demonstrate your mathematical reasoning, modeling, and flexibility. For each problem, you must write or graph an algebraic equation to express the situation.

PROBLEM 1: POOL PLANNING

After graduating from college, Lindsay purchases her own home and decides to build a pool in her backyard. Lindsay studies the cost of pool tiles from two stores. Pool World installs ceramic tiles for $100 plus $8 per square foot of tiles. Peter's Pool Supply charges $75 for installation and $10 per square foot of ceramic tiles.

A: Find the number of square feet of tiles for which the cost including tiles and installation is the same.

B: Write or graph an algebraic equation to express the situation.

PROBLEM 2: THE FAT CAT

Ines has two cats. The first cat is three times the weight of the second cat. The average weight of the two cats is 13 pounds.

A: What are the weights of the two cats?

B: Write or graph an algebraic equation to express the situation.

PROBLEM 3: GO-CARTS

John goes to Boomers to race on go-carts. Boomers charges a $7 entrance fee plus $1.75 per go-cart ride.

A: John has $15.75. How many times will he be able to ride the go-carts?

B: Write or graph an algebraic equation to express the situation.

PROBLEM 4: SHOW ME THE MONEY

Melissa is paid 1.5 times her normal hourly rate for each hour she works over 40 hours in a week. Last week she worked 48 hours and earned $624 (Bennett et al., 2004, p. 505).

A: What is her normal hourly rate?

B: Write or graph an algebraic equation to express the situation.

ALGEBRAIC EQUATIONS CHECKLIST

PROBLEM 1: POOL PLANNING

Correct Answer	______/3 pts
Mathematical Reasoning (explanation/justification)	______/3 pts
Mathematical Modeling (algebraic equation)	______/3 pts
Mathematical Flexibility (extra-credit for additional strategies)	______/0 pts
TOTAL	______/9 pts

PROBLEM 2: THE FAT CAT

Correct Answer	______/3 pts
Mathematical Reasoning (explanation/justification)	______/3 pts
Mathematical Modeling (algebraic equation)	______/3 pts
Mathematical Flexibility (extra-credit for additional strategies)	______/0 pts
TOTAL	______/9 pts

PROBLEM 3: GO-CARTS

Correct Answer	_______/3 pts
Mathematical Reasoning (explanation/justification)	_______/3 pts
Mathematical Modeling (algebraic equation)	_______/3 pts
Mathematical Flexibility (extra-credit for additional strategies)	_______/0 pts
TOTAL	_______/9 pts

PROBLEM 4: SHOW ME THE MONEY

Correct Answer	_______/3 pts
Mathematical Reasoning (explanation/justification)	_______/3 pts
Mathematical Modeling (algebraic equation)	_______/3 pts
Mathematical Flexibility (extra-credit for additional strategies)	_______/0 pts
TOTAL	_______/9 pts

OVERALL SCORE: _________/27 pts

GRADE: _________

Formal Assessments: Self-Evaluation Rubric

Rubric Score	Score Requirements
1	Student recognizes that strategy was effective or ineffective
2	Student identifies or explains the reason strategy was ineffective or effective Student recognizes multiple strategies
3	Student compares different strategy strengths and weaknesses
4	Student identifies the relationship between strategies and the extension potential of strategies in an appropriate context

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