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A Closer Look at the Physical and Protocol Models for Wireless Ad Hoc Networks with Multi-Packet Reception

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Abstract—In this paper we introduce the throughput capacity of wireless ad hoc network utilizing Multi-Packet Reception (MPR) such that every node in a network has a capability of decoding packets simultaneously transmitted from the nodes inside the receiver range $R(n)$. It was shown in [1] that per source-destination throughput capacity under the protocol model with MPR has a tight upper and lower bounds of $\Theta(R(n))$. On the other hand, the throughput capacity under the physical model using successive interference cancellation (SIC) [2] has a tight upper and lower bounds of $\Theta(R(n)^{(1-2/\alpha)}/n^{\frac{1}{\alpha}})$ where α is the path loss parameter. Motivated by the discrepancy between the two capacity results, we investigated the throughput capacity with MPR utilizing Maximum Likelihood Decoding(MLD). To estimate the throughput capacity under the physical model with MLD capability of nodes in the network we assumed a random wireless ad hoc networks with n nodes uniformly and randomly distributed in a unit square area. Using this optimum decoding strategy, we demonstrate for the first time, that physical and protocol models render the same capacity when we utilize MLD in the physical model. The per source-destination throughput capacity is under the physical model similar to the results in protocol model [1]. More specifically, we demonstrate that a tight upper and lower bounds of $\Theta(R(n))$can be achieved where $R(n)$ is the receiver range in MPR model. It is also proved that this capacity is achieved when $R(n) \geq \Theta\left(\sqrt{\log n/n}\right)$.

I. INTRODUCTION

The seminal work by Gupta and Kumar [3] showed that the amount of information between each source-destination pair with single-user decoding and forwarding of packets is not scalable with increasing number of nodes in the wireless ad hoc networks. Particularly in their physical model based on SINR(Signal to Interference plus Noise Ratio), it is assumed that the successful communication between the specific source-destination pair is mainly restricted by the signals received from other nodes. Since the communication between nearest neighbor maximizes the number of simultaneous transmissions, the optimal strategy is to utilize the multi-hop routing. Thus there has been a lot of research to increase the throughput capacity on a wireless ad hoc networks. In [4] it is shown that per-user throughput capacity can be increased dramatically when nodes are endowed with mobility. Another line of research demonstrate that per node throughput capacity

is increased by spatially reusing wireless channel [5], [6]. In addition to this increasing the available bandwidth [7], [8] and various kind of cooperation technique [9] were researched to increase the throughput capacity.

In this paper we demonstrate the throughput capacity utilizing Multi-Packet Reception(MPR). The first framework on many-to-one communication was introduced by Ghez et al. [10], [11] and Tong et al. [12]. In extended research on MPR there have been efforts to increase the throughput capacity by allowing multiple nodes cooperate to transmit their packets simultaneously to the same node based on directional antennas, multiuser detection (MUD), multiple input multiple output (MIMO) techniques [5], [6], [13] and successive interference cancelation (SIC) to decode multiple packets [14]. Recently in [1] it was shown that the throughput capacity with MPR is tightly bounded by $\Theta(R(n))$ under the protocol model. This represents a minimum gain of $\log n$ compared to the capacity bounds obtained by Gupta and Kumar for point-to-point communication under the protocol model. Furthermore it was also shown in [2] that with the MPR based on SIC throughput capacity was tightly bounded by $\Theta(R(n)^{(1-2/\alpha)}/n^{\frac{1}{\alpha}})$ under the physical model. The main contribution of this paper is that we introduced the throughput capacity with MPR utilizing Maximum Likelihood Decoding under the physical model. Hence we find out the closer relationship between two different capacity result on MPR under physical and protocol model. In Section II we summarized the prior work on the capacity of wireless ad hoc network. Section III presents the network model we use to obtain the upper and lower bounds on the throughput capacity of wireless networks with MPR, which are derived in Section IV. We show that $\Theta(R(n))$ bits per second constitutes a tight bound for the throughput capacity per node in random wireless ad hoc networks, where $R(n)$ and α are the MPR receiver range and channel path loss parameter, respectively. When $R(n) \geq \Theta\left(\sqrt{\log n/n}\right)$, the throughput capacity is tightly bounded by $\Theta\left(\sqrt{\log n/n}\right)$. This is a gain of $\log n$ compared to the bound $\Theta(1/\sqrt{n})$ in [15] and [3]. The assumptions we use to obtain these results are similar to

those made by Gupta and Kumar [3], except that each node is equipped with MPR capabilities.

II. RELATED WORK

The seminal work by Gupta and Kumar [3] on the capacity of wireless ad-hoc networks introduced capacity result in the network where n static nodes are arbitrary and randomly distributed under the protocol model and physical model based on unicast routing scheme. In the extended research by Franceschetti et al. [15], they proved that the gap between lower and upper bound in the random wireless network can be closed under the physical model utilizing percolation theory. Thus they have the same throughput order $\Theta\left(1/\sqrt{n}\right)$.

To improve the throughput capacity Grossglauser and Tse [4] examined the per-session throughput for applications with loose delay constraints allowing mobility in the wireless networks. They proved that under this assumption a non vanishing capacity $\Theta\left(1\right)$ can be achieved by exploiting node mobility. Negi and Rajeswaran [7] showed that the capacity of wireless network can be enhanced by assuming large bandwidth such as ultra wide-band (UWB) technology, power and rate adaptation. Thus they proved that the per node throughput capacity is upper and lower bounded by $O\left((n\log n)^{(\alpha-1)/2}\right)$ and $\Omega\left(\frac{n^{(\alpha-1)/2}}{(\log n)^{(\alpha+1)/2}}\right)$ respectively. Zhang et al. [8] improved the capacity bounds employing unlimited bandwidth resources and closed the gap in [7]

In other lines of research Ozgur et al. [9] proposed hierarchical cooperation and virtual MIMO(distributed multiple-input multiple-output) to achieve the linear capacity in an order of n. Collaboration-driven approach among nodes [16] also showed that the throughput capacity can be enhanced at a cost of increased processing complexity in the nodes. In recent research result by [1] and [2], it was also proved that with MPR based throughput capacity under the protocol model and physical model with SIC can be increased by $\Theta\left(\sqrt{\log n}\right)$ and $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$.

III. NETWORK MODEL

We consider a random wireless dense network which consists of n nodes distributed randomly and uniformly over a unit square area with intensity n. In this network model, the node density in the network goes to infinity as the number of nodes increase. Since we assume AWGN channel and path loss channel model, the received signal at the receiver node j is expressed as

$$Y_j = \sum_{i \neq j} X_i \cdot \sqrt{g_{ij}} + n_j, \qquad (1)$$

,where X_i, Y_j and n_j represent the transmitted signal from node i, received signal at node j and thermal noise at the receiver node j respectively, and $g_{ij} = \frac{1}{r_{ij}^\alpha}$ is the path loss channel gain between a pair of transmitting node i and receiving node j. Thus the transmitted signal decays with distance r_{ij} and path loss parameter $\alpha(\alpha \geq 2)$ as it propagates through the channel. We further assume that the $\{X_i\}_{i=1}^m$ and

n_j are independent, and that $E[n_j] = 0$, $E[n_j^2] = N$. Now we have the well-known m input Gaussian multiple access channel model.

To analyze the throughput capacity in out network model, we employ the physical model introduced by Gupta and Kumar [3] as a successful communication condition.

Definition 3.1: Physical Model with Point to Point Communication:
In a dense random wireless network based on plain routing, the successful communication between a pair of transmitting node i and receiving node j occurs when the following $SINR$ condition is satisfied based on the physical model.

$$SINR_{i \to j} = \frac{Pg_{ij}}{N + \sum_{k \neq i, k=1}^n Pg_{kj}} \geq \beta, \qquad (2)$$

,where N is the ambient noise power at the receiver side and concurrently transmitted signal from X_k $(i \neq k)$ is simply regarded as interference. Let all nodes transmit to their destination at a common transmission power level P and signal power decay based on the path loss channel model. Then we have $C_{ij} = B \log\left(1 + SINR_{i \to j}\right)$ bits/sec as capacity between transmit node i and receive node j. According to the physical model introduced by Gupta and Kumar [3], if the $SINR_{i \to j} \geq \beta$ at the receiver side is satisfied, the constant data rate W bits/second between the transmitter-receiver pair is achieved for C_{ij}.

On the other hand, in the physical model of MPR, we allow each receiving node decode concurrently transmitted signal from all the nodes within the common transmission range $R(n)$ while signal from nodes outside the $R(n)$ is considered interference. In the previous research [2], it was proved that based on SIC MPR can increase the throughput capacity of the network although throughput capacity of the network allowing SIC under the physical model does not correspond to the capacity result with MPR under the protocol model [1].

In this paper we applied ML optimum decoding scheme to analyze the throughput capacity in MPR to find out the relationship between two different throughput capacity result under the physical and protocol model respectively.

Note that it is unmanageably hard to implement this optimum decoding scheme in practice. First of all, every receiver should have the massive storage device that store the precise amplitudes of all received symbols and decoding table proportional to the length of the code and the number of active users. Second, ML decoding requires huge amount of computation proportional to the number of users and code length.

Although this optimum decoding scheme seems to be impractical compared to the SIC, we expect that MPR with MLD can reveal the closer relationship between protocol model and physical model based on MPR.

The following proposition describes the encoding and decoding procedure for MPR allowing MLD at the receiver node.

Proposition 3.2: We consider a set of m transmitters in the receiver range $R(n)$ and m code books for each transmitters. If the ith transmitter sends data to the receiver node at a rate of R_i, code book for ith user has 2^{nR_i} codewords of power

P. Each of the m transmitters chooses an arbitrary codeword from its own codebook and send these vectors simultaneously. At the receiver end these codewords are added together with the Gaussian noise N and interference.

Proposition 3.3: The received signal at node j from all the nodes inside receiver range $R(n)$ comprises m symbols and has $(g_{1j}, g_{2j}, g_{3j}...g_{mj})$ for the actual channels for m symbols. If this channel vector is available to the decoder at the receiver side, then the joint ML decoding scheme can be executed by comparing the Euclidean distance between the received codeword and the all possible codewords for the m transmitters in the receiver range. The codewords that make the vector sum closest to the received vector in Euclidean distance is chosen to be the received codewords. With the capability of the optimum ML decoding scheme, it is obvious from chapter 15 [17] that for a gaussian multiple access system with m sources with power $\{Pg_{1j}, Pg_{2j}, ..., Pg_{mj}\}$ and ambient noise of power N, the total rate of information flow $\sum_{i=1}^{m} R_i$ from m sources to the receiver node j at the center of the receiver range $R(n)$ is.

$$\sum_{i=1}^{m} R_i \leq \frac{1}{2} \log_2 \left(1 + \sum_{i=1}^{m} \frac{Pg_{ij}}{N+I} \right), \qquad (3)$$

,where I denotes interference signal power received from outside of the receiver range $R(n)$ and N is the Gaussian noise. In Eq. (3), $\sum_{i=1}^{m} Pg_{ij}/(N+I)$ implies the summation of the individual $SINR_{ij}$ between a pair of transmitting node i and receiving node j.

In contrast to the transmission range $r(n)$ defined in point-to-point communication [3], the receiver range $R(n)$ in the MPR model defines the area where the receiver is capable of decoding. We also assume half-duplex communication which means that nodes in the MPR model can not transmit and receive at the same time. The following definition shows the SINR in MPR model with ML decoding capability at the receiver node.

Definition 3.4: Physical Model with Multi-packet Reception:
In the physical model of random wireless dense networks [3], the transmissions from all the nodes inside the receiver range $R(n)$ around a receiver j act as constructive signal while other transmissions outside $R(n)$ considered as interference. Hence in general the total $SINR_{i \in I(R(n)) \rightarrow j}$ at the receiver node j is

$$SINR_{i \in I(R(n)) \rightarrow j} = \frac{\sum_{i \in I(R(n))} Pg_{ij}}{N + \sum_{k \notin I(R(n))} Pg_{kj}}, \qquad (4)$$

where $I(R(n))$ is the set of nodes inside the receiver range $R(n)$.

Therefore by applying the $SINR_{i \in I(R(n)) \rightarrow j}$ to the Eq. (3) instead of $\sum_{i=1}^{m} Pg_{ij}/(N+I)$, the following condition can be achieved.

$$\sum_{i \in I(R(n))} R_i \leq B \log \left(1 + SINR_{i \in I(R(n)) \rightarrow j} \right)$$

$$= B \log \left(1 + \frac{\sum_{i \in I(R(n))} Pg_{ij}}{N_0 + \sum_{k \notin I(R(n))} Pg_{kj}} \right) (5)$$

,where R_i can be W or zero bits per second according to physical model constraint [3].

Hence as long as the Eq. (5) is satisfied, having a total throughput capacity $\sum_{i \in I(R(n))} R_i$ is feasible in a given receiver range $R(n)$.

We present the following definition from Gupta and Kumar's work [3] for completeness of the presentation.

Definition 3.5: Feasible throughput capacity of unicast:
" A throughput of $\lambda(n)$ bits per second for each node is feasible if there is a spatial and temporal scheme for scheduling transmissions, such that by operating the network in a multihop fashion and buffering at intermediate nodes when awaiting transmission, every node can send $\lambda(n)$ bits per second on average to its chosen destination nodes. That is, there is a $T < \infty$ such that in every time interval $[(i-1)T, iT]$ every node can send $T\lambda(n)$ bits to its corresponding destination node."

Definition 3.6: Order of throughput capacity: $\lambda(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim_{n \to \infty} \text{Prob} \left(\lambda(n) = cf(n) \text{ is feasible} \right) = 1 \\ \lim_{n \to \infty} \text{Prob} \left(\lambda(n) = c'f(n) \text{ is feasible} \right) < 1. \end{cases} \qquad (6)$$

The distribution of nodes in random networks is uniform. Therefore, if there are n nodes in a unit square, then the density of nodes equals n. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S) = n|S|$. Let N_j be a random variable defining the number of nodes in S_j. Then, for the family of variables N_j, we have the following standard results known as the Chernoff bound [18]:

Lemma 3.7: Chernoff bound
- For any $\delta > 0$, $P[N_j > (1+\delta)n|S_j|] < \left(\frac{e^\delta}{(1+\delta)^{1+\delta}} \right)^{n|S_j|}$
- For any $0 < \delta < 1$, $P[N_j < (1-\delta)n|S_j|] < e^{-\frac{1}{2}n|S_j|\delta^2}$

Combining these two inequalities we have, for any $0 < \delta < 1$:

$$P[|N_j - n|S_j|| > \delta n|S_j|] < e^{-\theta n|S_j|}, \qquad (7)$$

where $\theta = (1+\delta)\ln(1+\delta) - \delta$ in the case of the first bound, and $\theta = \frac{1}{2}\delta^2$ in the case of the second bound.

Therefore, for any $\theta > 0$, there exist constants such that deviations from the mean by more than these constants occur with probability approaching zero as $n \to \infty$. An event occurs with high probability (w.h.p.) if its probability tends to one as $n \to \infty$. It follows that, w.h.p., we can get a very sharp concentration on the number of nodes in an area, so we can find the achievable lower bound w.h.p., provided that the upper bound (mean) is given. In the next section, we first derive the

upper bound, and then use the Chernoff bound to prove the achievable lower bound w.h.p..

IV. THROUGHPUT CAPACITY WITH MPR

In this section we compute the upper and lower bounds of throughput capacity for MPR with capability of MLD at the receiver node. We will first introduce some definitions and preliminary results, of which some results are studied in the previous research result [1].

In a random wireless network on a unit square area, average distance between source and destination pair can be normalized such that the per node throughput capacity of the network is equivalent to transport capacity defined in [3]. Thus this paper only consider bits per second unit of throughput capacity.

To analyze the maximum throughput capacity, a cut Γ is introduced to partition nodes in the wireless networks into two sets. It is a well known fact that the maximum flow in a network is restricted by its bottleneck such that the amount of information packets flowing from one to the other cannot be greater than the weakest set of links among any two nodes connections. Similarly for the wireless networks, we can use the concept of *sparsity cut*, as defined by Liu et al. [19], considering the differences between wired and wireless links. Since we assumed nodes are uniformly and randomly deployed on a unit square area, the sparsity cut is induced in the middle of the network area such that it captures the traffic bottleneck of these network on average. The l_Γ shown in Fig. 1 is defined as the length of the sparsity cut Γ. The sparsity-cut capacity is upper bounded by deriving the maximum number of simultaneous transmissions across this cut.

By defining $R(n)$ as the radius of the receiver range A, i.e., $A = \pi R^2(n)$ and assuming that nodes are equipped with omni-antenna for broadcasting, receiver node can distinguish the decode-able transmitter nodes within $R(n)$ from the interference outside $R(n)$. In doing this, we can compute the maximum number of simultaneous transmissions across this cut. Note that each disk with radius $R(n)$ centered at any receiver should be disjoint from the other disk sets centered at the other receivers. If we allow overlapping of disks with radius $R(n)$, some nodes in the overlapping area can transmit data to more than one receiver node with W bits per second at the same time. This contradicts the fact that the unicast routing is assumed in this network. In addition to this, it will be shown later that this assumption is required to guarantee the physical model condition in Eq. (5) based on MPR and MLD.

A. Upper Bound

Lemma 4.1: The asymptotic throughput capacity of a sparsity cut Γ for an unit square region is upper bounded by $\frac{\pi l_\Gamma n W}{2} \frac{R^2(n)}{D(n)}$, where $R(n)$ and $D(n)$ denote the receiver range and division range in a dense random wireless network with MPR respectively. Fig. 2 shows the $R(n)$ and $D(n)$ with S_{xy} induced by a sparsity cut Γ.
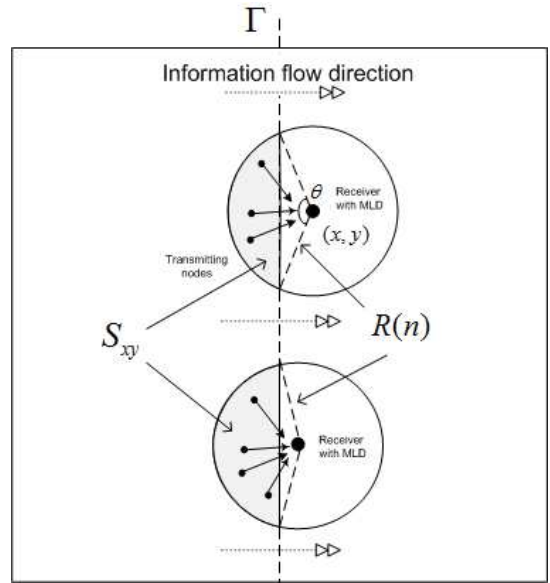

Fig. 1. For a receiver centered at (x, y), all the nodes in the shaded region S_{xy} can send a message successfully and simultaneously.

Proof: The definition of sparsity cut introduced in Section IV demonstrates that the cut capacity is upper bounded by deriving the maximum number of simultaneous transmissions across the cut. From Fig. 1 we can easily find out that all the nodes on the shaded region S_{xy} can transmit their packets to the receiver node centered at (x, y), such that simultaneous transmissions across the cut is maximized when all nodes lying on the left side of the cut Γ within an shaded area S_{xy} send their packets to the right side of the cut Γ.

Since we assume that simultaneously transmitted signal from nodes within $R(n)$ can be successfully decoded with MLD at the node located on (x, y), we can compute the total amount of edges crossing the cut Γ to find out the upper bound of throughput capacity of the sparsity cut. Assuming that the all transmitting nodes for the receiver node (x, y) are placed on the shaded region S_{xy} illustrated in Fig. 1, it is obvious that the average number of nodes on S_{xy} is $n \times S_{xy}$ considering the uniform distribution of the nodes. Then the average number of simultaneously transmitting nodes are upper bounded as a function of S_{xy}.

The area of S_{xy} is computed as

$$S_{xy} = \frac{1}{2} R^2(n)(\theta - \sin \theta). \qquad (8)$$

This area is maximized when $\theta = \pi$,

$$\max_{0 \le \theta \le \pi} [S_{xy}] = \frac{1}{2} \pi R^2(n). \qquad (9)$$

which means that the total information capacity C_j for one receiver j at the right side of the cut is

$$C_j \le \sum_{i=1}^{n\pi R^2(n)/2} R_i \qquad (10)$$

Note that the channel capacity R_i defined between a pair of transmitting node i and receiving node j is $B \log \left(1 + SINR_{ij} \right)$, and $B \log \left(1 + SINR_{ij} \right)$ becomes W bits per second if the $SINR_{ij} \geq \beta$ is satisfied otherwise zero.

Then by replacing R_i into $B \log \left(1 + SINR_{ij} \right)$ and considering that $SINR_{ij} \geq \beta$ as a successful communication condition for each source destination pair, the total information capacity C_j for a given receiver range $R(n)$ is upper bounded by

$$
\begin{aligned}
C_j &\leq \sum_{i=1}^{\frac{n\pi R^2(n)}{2}} R_i \leq \frac{1}{2}\pi R^2(n)nW \\
&= \sum_{i=1}^{\frac{n\pi R^2(n)}{2}} B \log \left(1 + \beta \right) \\
&\leq \sum_{i=1}^{\frac{n\pi R^2(n)}{2}} B \log \left(1 + SINR_{ij} \right) \quad (11)
\end{aligned}
$$

It is obvious from Eq. (5) that Eq. (11) should satisfy the Eq. (5) since we adopt ML decoding at the receiver side. Thus by combining Eq. (5) and Eq. (11), we have the following constraint for MPR with MLD.

$$
\sum_{i=1}^{\frac{n\pi R^2(n)}{2}} B \log \left(1 + \beta \right) \leq B \log \left(\frac{\sum_{i \in I(R(n))} P g_{ij}}{N_0 + \sum_{k \notin I(R(n))} P g_{kj}} \right)
$$
(12)

By replacing $(1+\beta)$ into β', Eq. (12) can be further reduced to.

$$
\beta'^{\frac{n\pi R^2(n)}{2}} \leq \frac{\sum_{i \in I(R(n))} P g_{ij}}{N_0 + \sum_{k \notin I(R(n))} P g_{kj}}, \quad (13)
$$

As long as Eq. (13) is satisfied in a given receiver range $R(n)$, $C_j = \frac{1}{2}\pi nWR^2(n)$ is achievable. In our MPR scheme we assumed unicast routing in the network. Thus the circles whose nodes are transmitting concurrently must be away from each other at least for $D(n) \geq 2R(n)$ as shown in Fig. 2, such that nodes can not transmit to multiple receivers in different receiver disks. Therefore, the total throughput capacity $C(n)$ across the sparsity cut is

$$
C(n) \leq \left(\left\lfloor \frac{l_\Gamma}{D(n)} \right\rfloor + 1 \right) C_j < \frac{\pi nWR^2(n)(l_\Gamma + D(n))}{2D(n)}.
$$
(14)

Since the $D(n)$ and $R(n)$ are decreasing functions of n which goes to zeros as $n \rightarrow \infty$, $\lim \left(l_\Gamma + D(n) \right) = l_\Gamma$ asymptotically. This proves the lemma. ∎

Lemma 4.2: The per source-destination throughput of MPR scheme in a 2-D random network is upper bounded by $O \left(\frac{R^2(n)}{D(n)} \right)$.

Proof: From lemma 4.1, there are $l_\Gamma / D(n)$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes on
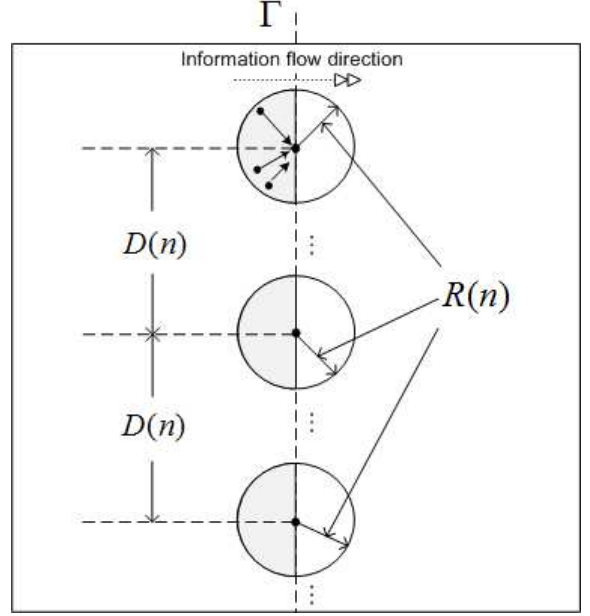

Fig. 2. Upper bound design of the network

average. Therefore, the average per node throughput capacity can be derived as

$$
\lambda(n) = \frac{C(n)}{n} = O \left(\frac{R^2(n)}{D(n)} \right). \quad (15)
$$
∎

To derive an upper bound for the throughput capacity, we need to obtain a minimum $D(n)$, such that it guarantees $C_j = \frac{1}{2}\pi nWR^2(n)$.

$$
\begin{aligned}
&\max_{SINR_{i \in I(R(n)) \rightarrow j} \geq \beta'^{\frac{n\pi R^2(n)}{2}}} \lambda(n) \\
&= \max_{SINR_{i \in I(R(n)) \rightarrow j} \geq \beta'^{\frac{n\pi R^2(n)}{2}}} \frac{R^2(n)}{D(n)} \quad (16)
\end{aligned}
$$

Note that the throughput capacity is maximized by minimizing $D(n)$, while if this value is too small, then Eq. (13) will not be satisfied. Our aim is to find the optimum value for $D(n)$ such that Eq. (13) is satisfied. The following theorem establishes the optimum value that will satisfy Eq. (13).

Theorem 4.3: The per source-destination throughput of MPR scheme in a 2-D random network is upper bounded by $O \left(R(n) \right)$.

Proof: First in order to find out the upper bound of the throughput capacity, we derive the upper bound of $SINR_{i \in I(R(n)) \rightarrow j}$ at the receiver node j. For the node that is in a circle close to the edge of the network, let all the interfering nodes placed at Euclidean distances of $(iD(n) + R(n))$ from the receiver node. Then the minimum interference signal

power at the receiver node is computed as

$$\sum_{k \notin I(R(n))} P g_{kj} \geq \sum_{i=1}^{l_\Gamma/D(n)} \frac{\frac{\pi}{2} n R^2(n) P}{(iD(n) + R(n))^\alpha}$$

$$\geq \frac{\pi n R^2(n) P}{2D(n)^\alpha} \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{\left(i + \frac{1}{2}\right)^\alpha}, \quad (17)$$

The second inequality of Eq. (17) stems from the fact that $\frac{R(n)}{D(n)} \leq \frac{1}{2}$. Since $l_\Gamma/D(n)$ goes to infinity as n increases, the summation $\sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i+\frac{1}{2})^\alpha}$ is bounded by two constant values c_1 and c_2 such that

$$c_1 \leq \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i + \frac{1}{2})^\alpha} \leq \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i)^\alpha} \leq c_2. \quad (18)$$

Consequently, we have the following minimum interference signal power at the receiver side.

$$I_{min} = \frac{\pi n R^2(n) P}{2D(n)^\alpha} c_1, \quad (19)$$

We next compute the received signal power from the nodes inside of the receiver range $R(n)$. It is known in [9] that for any positive value ϵ, the minimum distance between any two nodes in the network is larger than $\frac{1}{n^{1+\epsilon}}$ with high probability for large values of n. Equivalently, we say that there are no other nodes inside a circle of radius $\frac{1}{n^{1+\epsilon}}$. Thus we can conclude that nodes are uniformly distributed in the range of $[\frac{1}{n^{1+\epsilon}}, R(n)]$.

Next, based on the work by [20], the received signal power at node j from all nodes outside of circle of radius r_0 is given by

$$P_r(x_0, y_0, r_0) = \frac{2\pi \delta P n}{(\alpha - 2) r_0^{(\alpha-2)}} \left[1 - \frac{r_0^{(\alpha-2)}}{2\pi} C(x_0, y_0) \right], \quad (20)$$

,where $C(x_0, y_0)$ is the constant value related to the receiver location (x_0, y_0).

Then the received signal power from the shaded region in Fig. 2 is approximated as

$$\sum_{i \in I(R(n))} P g_{ij} = \frac{P_r(x_0, y_0, \frac{1}{n^{1+\epsilon}}) - P_r(x_0, y_0, R_0)}{2} \quad (21)$$

$$= \frac{\pi \delta P n}{(\alpha - 2)} \left(n^{(1+\epsilon)(\alpha-2)} - \frac{1}{R(n)^{(\alpha-2)}} \right),$$

Using Eq. (21) and Eq. (19), the upper bound of the $SINR_{i \in I(R(n)) \to j}$ is represented as

$$SINR_{i \in I(R(n)) \to j} \leq \frac{\sum_{i \in I(R(n))} P g_{ij}}{N_0 + \min \sum_{k \notin I(R(n))} P g_{kj}} \quad (22)$$

$$\leq \frac{\frac{\pi \delta P n}{(\alpha-2)} \left(n^{(1+\epsilon)(\alpha-2)} - \frac{1}{R(n)^{(\alpha-2)}} \right)}{N_0 + \frac{\pi n R^2(n) P}{2D(n)^\alpha} c_1}.$$

From the fact that the upper bound of the throughput capacity C_j can be achieved when the Eq. (13) is satisfied, we can set up the following constraint by combining Eq. (13) and Eq. (22).

$$\beta' \frac{n \pi R^2(n)}{2} \leq \frac{\frac{\pi \delta P n}{(\alpha-2)} \left(n^{(1+\epsilon)(\alpha-2)} - \frac{1}{R(n)^{(\alpha-2)}} \right)}{N_0 + \frac{\pi n R^2(n) P}{2D(n)^\alpha} c_1}, \quad (23)$$

In wireless ad hoc networks, interference is usually the dominant factor and if noise in negligible in the above equation, then it can be rewritten as

$$\left(\frac{(\alpha - 2)c_1}{2\delta} \frac{R(n)^\alpha \beta'^{\frac{n \pi R^2(n)}{2}}}{R(n)^{(\alpha-2)} n^{(1+\epsilon)(\alpha-2)} - 1} \right)^{\frac{1}{\alpha}} \leq D(n), \quad (24)$$

Since nodes are randomly distributed on the plain, $R(n)$ should be larger than $n^{-\frac{1}{2}}$ to maintain non zero nodes on average inside the circles, such that by neglecting minus one in the denominator Eq. (24) can be approximated into

$$\left(\frac{(\alpha - 2)c_1}{2\delta} \frac{R(n)^2 \beta'^{\frac{n \pi R^2(n)}{2}}}{n^{(1+\epsilon)(\alpha-2)}} \right)^{\frac{1}{\alpha}} \leq D(n), \quad (25)$$

The another inequality between $D(n)$ and $R(n)$ derived from Fig.2 is $2R(n) \leq D(n)$. However with respect to scaling law, constant gain does not change the result. Thus in general $[(1 + \Delta)R(n) + R(n)] \leq D(n)$, where Δ is constant greater than or equal to 1, is possible for another condition for $D(n)$. Therefore the minimum value of $D(n)$ is

$$\min D(n)$$
$$= \min \left[(2 + \Delta)R(n), \left(\frac{(\alpha - 2)c_1}{2\delta} \frac{R(n)^2 \beta'^{\frac{n \pi R^2(n)}{2}}}{n^{(1+\epsilon)(\alpha-2)}} \right)^{\frac{1}{\alpha}} \right]$$

Note that from Lemma 4.2 and the fact that we do not allow overlapping of any two communicating circles the maximum upper bound of the total throughput capacity through the cut is achieved by selecting minimum value of $D(n)$ which is $(1 + \Delta)R(n)$.

Thus in order to adopt $(2+\Delta)R(n)$ as a minimum separable distance between any two circles, we can set up the following inequality.

$$\left(\frac{(\alpha - 2)c_1}{2\delta} \frac{R(n)^2 (\beta')^{\frac{\pi}{2} R(n)^2 n}}{n^{(2+\delta)(\alpha-2)}} \right)^{\frac{1}{\alpha}} \leq (2 + \Delta)R(n) \leq D(n), \quad (26)$$

After simple manipulation of Eq. (26), it can be rewritten as

$$(\beta')^{\frac{\pi}{2\alpha} R(n)^2 n} \leq \left(\frac{2\delta}{(\alpha-2)c_1} \right)^{1/\alpha} n^{(1+\epsilon)(1-\frac{2}{\alpha})} (2+\Delta) R^{1-\frac{2}{\alpha}}(n), \quad (27)$$

Next by applying $\log_2$ operation to both sides of the equation, then we have

$$
\begin{aligned}
\frac{\pi}{2\alpha} R(n)^2 n \log_2 \beta' \;\leq\; & \frac{1}{\alpha} \log_2 \left(\frac{2\delta}{(\alpha-2)c_1} \right) \\
& + \; (1+\epsilon)(1 - \frac{2}{\alpha}) \log_2 n + \log_2 (2+\Delta) \\
& + \; \left(1 - \frac{2}{\alpha} \right) \log_2 R(n),
\end{aligned}
\tag{28}
$$

For the simplicity, Eq. (28) can be further reduced into

$$
R(n) \leq c_3 \cdot \sqrt{\frac{\log_2 n + c_4 + c_5 \log_2 R(n)}{n}}
\tag{29}
$$

,where c_3, c_4 and c_5 indicates $\sqrt{\frac{(1+\epsilon)(\alpha-2)}{\frac{\pi}{2} \cdot \log_2 \beta'}}$, $\frac{\alpha \log_2 (2+\Delta) + \log_2 [2\delta/\{(\alpha-2)c_1\}]}{(1+\epsilon)(\alpha-2)}$ and $\frac{1}{1+\epsilon}$ respectively.

As long as the receiver range $R(n)$ satisfy Eq. (28), the minimum $D(n)$ is

$$
minD(n) = (2+\Delta)R(n),
\tag{30}
$$

Therefore applying the minimum receiver range $D(n)$ lead us to the maximum per node throughput capacity which is

$$
\lambda(n) = O\left(R(n) \right),
\tag{31}
$$

∎

In the next section we will show that this upper bound capacity is also an achievable lower bound.

B. Lower Bound

In order to derive the lower bound, it is necessary to compute the number of nodes that transmit simultaneously from each communication circle.

For the purpose of finding the number of nodes in a communication circle, we will use the same approach based on Chernoff bound used in [2], such that we can prove that in a randomly distributed network the number of edges across the cut is sharply concentrated on its mean, and the actual number of edges across the sparsity cut is indeed $\Theta\left(R(n) \right)$ w.h.p..

The following theorem demonstrate that, when n nodes are distributed uniformly over a unit square area, there exit simultaneously at least $\frac{l_\Gamma}{D(n)}$ circular regions (see fig. 2), each one containing $\Theta(nR^2(n))$ nodes w.h.p...

Theorem 4.4: Each area A_j with circular shape of radius $R(n)$ contains $\Theta(nR^2(n))$ nodes w.h.p. and uniformly for all values of $j, 1 \leq j \leq \frac{l_\Gamma}{D(n)}$ under the condition that $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}} \right)$. Equivalently, this can be expressed as

$$
\lim_{n \to \infty} P \left[\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j) \right] = 1,
\tag{32}
$$

where δ is a positive arbitrarily small value close to zero.

Proof: The proof follows the same procedure in [2]. Hence the key constraint of $R(n)$ to satisfy theorem is given as

$$
R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}} \right).
\tag{33}
$$

∎

This theorem shows that w.h.p., there are indeed $\Theta(nR^2(n))$ nodes in each communication region with the constraint in (33). The achievable capacity is only feasible when the receiver range of each node in MPR scheme is at least equal to the connectivity criterion of transmission range in point-to-point communication [3]. Combining the result of Eq. (31) in Theorem 4.3 and (33) in Theorem 4.4, we can state the following theorem for the lower bound of throughput capacity, which implies the lower bound order capacity achieves the upper bound.

Theorem 4.5: The per source-destination throughput capacity of MPR scheme in a 2-D static wireless ad hoc network is lower bounded by $\Omega\left(R(n) \right)$ provided that $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}} \right)$, which means the tight bound is at least $\Theta\left(\sqrt{\frac{\log n}{n}} \right)$ for $\alpha > 2$.

Proof: We first prove that Eq. (31) is an achievable bound and then by applying the minimum receiver range constraint in Eq. (33), we derive the lower bound for this theorem. In order to compute the achievable lower bound, we derive the following inequality

$$
\beta'^{\frac{n\pi R^2(n)}{2}} \leq \frac{\sum_{i \in I(R(n))} Pg_{ij}}{N_0 + \max \sum_{k \notin I(R(n))} Pg_{kj}},
\tag{34}
$$

The maximum interference is experienced by receiver node when the interfering nodes have the closest distance to the receiver Then the maximum interference is computed as

$$
\sum_{k \notin I(R(n))} Pg_{kj} \;\leq\; \sum_{i=1}^{l_\Gamma/D(n)} \frac{\frac{\pi}{2} nR^2(n)P}{(iD(n) - R(n))^\alpha}
\tag{35}
$$

$$
\leq \frac{\pi nR^2(n)P}{2D(n)^\alpha} \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{\left(i - \frac{1}{2}\right)^\alpha},
\tag{36}
$$

Now we can prove that $\sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i-\frac{1}{2})^\alpha}$ converge to the constant value.

$$
c_6 \;\leq\; \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i+\frac{1}{2})^\alpha} \leq \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i-\frac{1}{2})^\alpha}
\tag{37}
$$

$$
\leq \sum_{i=1}^{l_\Gamma/D(n)} \frac{i}{(i - \frac{1}{2})^\alpha} \leq c_7.
\tag{38}
$$

Hence the maximum interference signal power at the receiver side is

$$
I_{max} = \frac{\pi nR^2(n)P}{2D(n)^\alpha} c_7,
\tag{39}
$$

Combining all the results derived so far, the minimum SINR for MPR can be computed as

$$\frac{\sum_{i \in I(R(n))} P g_{ij}}{N_0 + \max \sum_{k \notin I(R(n))} P g_{kj}}$$

$$= \frac{\frac{\pi \delta P n}{(\alpha-2)} \left(n^{(1+\delta)(\alpha-2)} - \frac{1}{R(n)^{(\alpha-2)}} \right)}{N_0 + \frac{\pi R(n)^2 n P}{2D(n)^\alpha} \cdot c_7},$$

By applying the same condition for $D(n)$ and $R(n)$ computed in the upper bound analysis, we arrive at

$$\frac{c_1}{c_7}(\beta')^{\frac{\pi R(n)^2 n}{2}} \leq \frac{\sum_{i \in I(R(n))} P g_{ij}}{N_0 + \max \sum_{k \notin I(R(n))} P g_{kj}} \quad (40)$$

$$= \frac{\frac{\pi \delta P n}{(\alpha-2)} \left(n^{(1+\delta)(\alpha-2)} - \frac{1}{R(n)^{(\alpha-2)}} \right)}{N_0 + \frac{\pi R(n)^2 n P}{2D(n)^\alpha} \cdot c_7},$$

Using simple manipulations, we can compute the lower bound of throughput capacity as $\lambda(n) = \Omega\left(R(n)\right)$ ∎

The above theorem demonstrates that a gain of at least $\Theta\left((\log n)^{\frac{1}{2}}\right)$ can be achieved compared with the results by Gupta and Kumar [3] and Franceschetti et al. [15]. Combining Theorems 4.3 and 4.5, we arrive at our first major contribution of this paper.

Theorem 4.6: The per source-destination throughput capacity of MPR scheme in a 2-D static wireless ad hoc network is tight bounded as $\Theta\left(R(n)\right)$. The minimum receiver range is lower bounded as $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, which implies a lower tight bound of $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$.

Note that this result shows that we can close the gap in the physical model similar to the results derived by Franceschetti et al. [15] but achieving higher throughput capacity with MPR.

V. CONCLUSION

This paper shows that the use of MPR can close the gap for the transport (throughput) capacity in random wireless ad hoc networks under the physical model, while achieving much higher capacity gain than that of [15]. The tight bound is $\Theta\left(R(n)\right)$ where $R(n)$ is the receiver range in MPR model. For the minimum value of $R(n)$, a gain of $\Theta\left(\sqrt{\log n}\right)$ is achievable in MPR scheme.

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