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Network Coding Does Not Change The Multicast Throughput Order of Wireless Ad Hoc Networks

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Abstract—We demonstrate that the gain attained by network coding (NC) on the multicast capacity of random wireless ad hoc networks is bounded by a constant factor. We consider a network with n nodes distributed uniformly in a unit square, with each node acting as a source for independent information to be sent to a multicast group consisting of m randomly chosen destinations. We show that, under the protocol model, the per-session capacity in the presence of arbitrary NC has a tight bound of $\Theta\left(\frac{1}{\sqrt{mn\log(n)}}\right)$ when $m = O\left(\frac{n}{\log(n)}\right)$ and $\Theta\left(\frac{1}{n}\right)$ when $m = \Omega\left(\frac{n}{\log(n)}\right)$. Our result follows from the fact that prior work has shown that the same order bounds are achievable with pure routing based only on traditional store-and-forward methods.

I. INTRODUCTION

The concept of network coding was first explored by Yeung et. al. [1] and subsequently generalized by Ahlswede et. al. [2] for a single source multicast in arbitrary directed graphs. Since then, the interest in network coding has increased rapidly. A large number of studies have investigated the utility of network coding (NC) for wireless networks, and widely cited experiments [3], [4] have been reported in which NC has been used successfully in combination with other mechanisms to attain large throughput gains compared to approaches based on conventional protocol stacks. These results have led many to believe that a combination of NC with wireless broadcasting can lead to significant improvements in the order throughput of wireless networks. Understandably, there is significant interest in identifying the true impact of NC on the throughput order of wireless networks. However, the exact characterization of network capacity with NC in the presence of multiple access interference is a very hard problem, even for simple networks, and limited results have been reported to date on the subject.

Recent work [5]–[7] has shown that the throughput gain due to the use of NC in a wireless network is bounded by a constant when the traffic in the network consists of multiple unicast sessions. However, the motivation for the original work by Ahlswede et. al [2] was improving network performance for multicasting, not unicasting. Furthermore, many commercial and defense applications, such as conferencing, require multicasting of large amounts of information, and the study of the multicast capacity of wireless ad hoc networks is an important research topic in its own right.

Several works [8]–[11] have studied the multicast and broadcast capacity of wireless networks under conventional routing, and these results show consistently that broadcasting and multicasting significantly alter the throughput order of wireless networks. In light of these findings, the importance of multicasting and broadcasting, and recent practical results on NC, it is natural to inquire whether the introduction of NC can improve the throughput capacity of multi-pair multicasting. In this work, we undertake the characterization of the multicast and broadcast throughput order of wireless ad-hoc networks in presence of network coding, which had remained an open problem for the past 10 years.

We consider a network consisting of n nodes distributed randomly in the network space, with each node acting as source for m randomly chosen nodes in the network. We make two key contributions. First, we show that in the presence of arbitrary NC, the per-session multicast capacity of random wireless ad hoc network under the protocol model has a tight bound of $\Theta\left(\frac{1}{\sqrt{mn\log(n)}}\right)$ when $m = O\left(\frac{n}{\log(n)}\right)$ and $\Theta\left(\frac{1}{n}\right)$ when $m = \Omega\left(\frac{n}{\log(n)}\right)$. Second, we show that, in the presence of arbitrary NC, the per-session multicast capacity of random wireless ad hoc network under the physical model has a tight bound of $\Theta\left(\frac{1}{\sqrt{mn}}\right)$ when $m = O\left(\frac{n}{\log(n)^2}\right)$, and $\Theta\left(\frac{1}{n}\right)$ when $m = \Omega\left(\frac{n}{\log(n)^2}\right)$.

It has already been established in the literature that the above bounds are also achievable on the basis of traditional store-and-forward routing methods. Consequently, our analysis demonstrates that the throughput gain due to NC for multicast as well as broadcast is bounded by a constant factor!

The remainder of this paper is organized as follows. Section II surveys relevant prior work. Section III describes the network models and other concepts used proofs. Section IV deduces the capacity results under the protocol model. Section V summarizes our conclusions.

II. LITERATURE REVIEW

Gupta and Kumar's original work focused on the unicast capacity of wireless networks [12], and many subsequent contributions have been made on the capacity of wireless networks subject to unicast traffic. However, the focus of this paper, and

therefore this section, is on the capacity wireless networks under broadcast and multicast traffic.

Several recent efforts have addressed the multicast and broadcast capacity of wireless networks, primarily under the protocol model. The work by Keshavarz et al. [8] addresses the broadcast capacity of a wireless network for any number of sources in the network. Jacquet and Rodolakis, [9] proved that the scaling of multicast capacity is decreased by a factor of $O(\sqrt{m})$ compared to the unicast capacity result by Gupta and Kumar [12]. This result implies that multicasting gain, over transmitting the information from each source as m unicasts, is at least $\Theta(\sqrt{m})$. Li et al. [10] compute the capacity of wireless ad hoc networks for unicast, multicast, and broadcast applications. Zheng et. al. [11] independently generalized this work and introduced (n, m, k) -casting as a framework for the characterization of all types of information dissemination in wireless networks. This prior work has only addressed conventional store-and forward routing for multicast and broadcast traffic.

Since Ahlswede et. al.'s [2] seminal work, most of the theoretical research on NC has focused on directed networks, where each communication link is point to point and has a fixed direction. However, a wireless network is more appropriately modeled by bi-directional links. Li et. al [13], [14] have studied the benefits of NC in undirected networks. The result shows that, for a single unicast or broadcast session, there is no throughput improvement due to NC. In the case of a single multicast session, such an improvement is bounded by a factor of two. Nevertheless, the work by Li et. al does not account for multiple access interference, and hence cannot be directly applied to wireless networks.

Liu et. al. [5], [6] have shown that the NC for unicast traffic in a random network (i.e. a network in which the nodes are distributed randomly in a Euclidean space and the sources and destinations are also placed randomly) is bounded by a constant factor. Keshavarz et. al. [7] extended these conclusions to arbitrary networks and an arbitrary unicast traffic pattern. Furthermore, they also showed that the NC gain for even a single source multicast is bounded by a constant factor in any arbitrary network.

From the above, it is apparent that prior work has not determined whether NC by itself can provide any gains on the multicast order throughput in wireless networks, which is the subject of this paper.

III. PRELIMINARIES

For a continuous region A , we use $|A|$ to denote its area. We denote the cardinality of a set by $|\mathbb{S}|$, and by $\|X_i - X_j\|$ the distance between nodes i and j . Whenever convenient, we utilize the indicator function $1_{\{P\}}$, which is equal to one if P is true and zero if P is false. $Pr(E)$ represents the probability of event E . We say that an event E occurs with high probability (w.h.p.) if $Pr(E) > (1 - (1/n))$ as $n \rightarrow \infty$. We employ the standard order notations O, Ω , and Θ .

We assume that the topology of a network is described by a uniformly random distribution of n nodes in a unit square.

Let $V = 1, \dots, n$ represent the node-set and let X_i be the location of node $i \in V$. To avoid boundary effects, it is typical to assume that the network surface is placed upon a toroid or sphere. However, for mathematical convenience, in this work we ignore edge effects and thus assume that the network is placed in a 2-D plane. Further, in our model, as n goes to infinity, the density of the network also goes to infinity. Therefore, our analysis is applicable only to dense networks. We do not consider mobility of nodes and assume a static stationary distribution of nodes. Our capacity analysis is based on the protocol model introduced by Gupta and Kumar [12].

Definition 3.1: The Protocol Model

We assume that all nodes use an identical transmission range $r(n)$ for all their communication. Node i can successfully transmit to node j if for any node $k \neq i$, that transmits at the same time as i it is true that

$$\begin{aligned} \|X_i - X_j\| &\leq r(n) \\ \|X_k - X_j\| &\geq (1 + \Delta)r(n). \end{aligned} \quad (1)$$

We shall utilize the following well known property [15] in our analysis

Lemma 3.2: Connectivity Criteria

For a random distribution of n nodes in a unit-square, the network connectivity under the protocol model can be guaranteed w.h.p if and only if (iff)

$$r(n) \geq r_c(n) = \sqrt{\frac{3 \log(n)}{n}}. \quad (2)$$

We focus on the traffic scenario in which each node of the wireless network acts as a multicast source for a randomly chosen set of m distinct destinations.

Definition 3.3: Feasible rate and Throughput Order

Our definitions of feasible rate and throughput order are similar to those defined in [12].

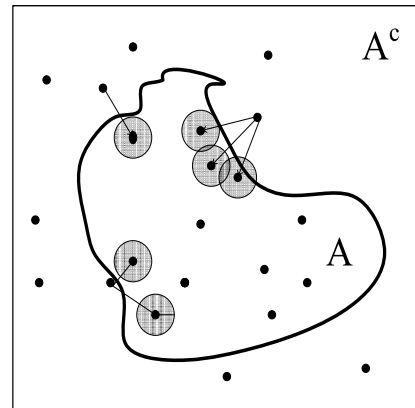


Fig. 1. Generalized Sparsity Cut

Definition 3.4: Cut

Given a node set V , a cut is the separation of the vertex set V into two disjoint and exhaustive subsets (S, S^c) . Here, a vertex partition can be completely described by partitioning the network-area into two regions (A, A^c) as shown in Fig. 1, thus

we also refer to a closed region A as a cut. The cut-capacity $C(A)$ is defined as the maximum number of simultaneous transmissions that can take place from A^c to A .

Definition 3.5: Multicast Cut-Demand

Given a cut A , a source node in A^c is said to have demand across the cut iff at least one of its destination lies in A . The multicast demand $D(A)$ across the cut is defined as the total number of sources in A^c such that there is at least one destination in the multicast group across the cut.

Definition 3.6: Sparsest Cut

We define the sparsity Γ_A of cut A as the ratio

$$\Gamma_A = \frac{C(A)}{D(A)} \quad (3)$$

Hence, the sparsest cut is given by

$$A^* = \arg \min_A \Gamma_A \quad (4)$$

where A^* has the least possible sparsity, denoted as Γ_{A^*} .

The conventional definition of Sparsity cut [16] is applicable only to unicast traffic [6]. Our definition generalizes the conventional definition to multicast traffic.

Finally we state the well-known Chernoff Bounds [17], which shall be repeatedly used in the rest of this paper.

Lemma 3.7: Chernoff Bounds: Consider n i.i.d random variables $Y_i \in \{0, 1\}$ with $p = \Pr(Y_i = 1)$. Let $Y = \sum_{i=1}^n Y_i$. Then for any $1 \geq \delta \geq 0$ and $\delta_2 \geq 0$ we have

$$\Pr(Y \leq (1 - \delta_1)np) < 2e^{-\frac{\delta_1^2 np}{3}} \quad (5)$$

$$\Pr(Y \geq (1 + \delta_2)np) < 2e^{-\frac{\delta_2^2 np}{3}} \quad (6)$$

IV. BOUNDS FOR PROTOCOL MODEL

It is well-known that under its conventional definition, the sparsity cut can be used to obtain an upper bound on the unicast traffic flow in a wireless network [6], [16]. In a similar way, our generalized definition provides an upper bound for multicast flows.

Lemma 4.1: Let $C_m(n)$ be the maximum multicast flow rate in a network and let A^* be the sparsest cut with sparsity Γ_{A^*} , then we have

$$C_m(n) \leq \Gamma_{A^*} \quad (7)$$

Proof: Let f be the total maximum feasible average rate at which bits can be transmitted from A^c to A , where A is any arbitrary cut. Then by Def. 3.4 we have

$$f \leq C(A) \quad (8)$$

The total information flow across a cut has to be greater than or equal to the sum of the data rates associated with individual multicast sessions that communicate across the cut. Hence,

$$\begin{aligned} f &\geq \sum_{i=1}^n C_m(n) \mathbf{1}_{\{\text{source } i \text{ in } A^c \text{ has demand across cut } A\}} \\ &= C_m(n) \sum_{i=1}^n \mathbf{1}_{\{\text{source } i \text{ in } A^c \text{ has demand across cut } A\}} \\ &= C_m(n) D(A). \end{aligned} \quad (9)$$

Inserting the above equation in Eq.8, we have

$$C_m(n) \leq \frac{C(A)}{D(A)} = \Gamma_A \leq \Gamma_{A^*}. \quad (10)$$

■

In the study of network capacity, cut arguments are typically employed only for unicast traffic and hence the use of such arguments for multicast traffic may seem counter intuitive to some readers. Therefore, some additional comments are in order. In particular, we can replace the use of Lemma 4.1 and Definition 3.6 with the following alternative result.

Lemma 4.2: Consider a network with n nodes $V = \{a_1, \dots, a_n\}$ and n multicast sessions. Each session consists of one of the n nodes acting as a source with an arbitrary finite subset of the set V acting as the set of destinations. Let s_i be the source of the i^{th} session and let $B_i = \{b_{i1}, \dots, b_{im_i}\}$ be the set of m_i destinations. Now, there exists a joint routing-coding-scheduling scheme that can realize a throughput of λ_i for the i^{th} session, i.e., $\lambda = [\lambda_1, \dots, \lambda_k]$ is a feasible rate vector. Then λ is also a feasible vector for any unicast routing problem in the same network, such that the traffic consists of k unicast sessions with s_i being the source of the i^{th} unicast session and the destination z_i is any arbitrary element of the set B_i .

The above lemma basically establishes that, if a multicast capacity from a source to multiple destinations is feasible, then clearly it is feasible to achieve the same capacity to any one arbitrarily chosen node from this set of destinations. We can thus deduce the bounds for the case of multi-source multicasting by reducing it to a suitable unicast routing problem. Under the reduction, an upper bound for the unicast problem also serves for the original multicast routing problem. Thus, in order to obtain an upper bound on the multicast capacity, we could construct a unicast problem by choosing destinations specifically from a suitably chosen region A .

To establish the relationship of the above argument to Lemma 4.1, recall the classical definition of sparsity used in [6], [16] for the analysis of unicast traffic.

Definition 4.3: Unicast Sparsity The (unicast) Sparsity of a cut A for a given set of sources $S = \{s_1, \dots, s_n\}$ and a chosen set of destinations $Z = \{z_1, \dots, z_n\}$ is defined as

$$\Upsilon_{B,A} = \frac{C(A)}{D_{unicast}(A)} \quad (11)$$

where $C(A)$ is the cut capacity and $D_{unicast}(A)$ is the unicast demand across cut A , i.e. the total number of unicast sources in A^c that have a destination in A .

Now let us consider a network with n nodes, a set of multicast sources $S = \{s_1, \dots, s_n\}$ and set of destination sets $\mathbf{F} = B_1, \dots, B_n$, such that B_i is the set destinations for s_i . Furthermore, let \mathbb{Z} be the set of all possible sets $Z = \{z_1, \dots, z_n\}$ such that $z_i \in B_i$ is a destination for source s_i . Let f_Z be a feasible per-session flow for a unicast scenario described by S as the set of sources and Z as the set of destinations. It is well known that $f_Z \leq \min_A \Upsilon_{Z,A}$ for all Z , where A can be any arbitrary cut. Hence, Lemma 4.2

basically states that f_B is a feasible flow rate for the multicast scenario iff

$$f_B \leq \min_{Z \in \mathbf{Z}} F_Z \leq \min_{Z \in \mathbf{Z}} \min_A \Upsilon_{Z,A} \quad (12)$$

Since $C(A)$ does not depend on our choice of Z , with the exchange of minima's we have

$$f_B \leq \min_A \frac{C(A)}{\max_{Z \in \mathbf{Z}} D_{unicast}(A)} = \min_A \frac{C(A)}{D(A)} = \Gamma_A^* \quad (13)$$

It should be highlighted that the above deductions imply that the maximum multicast flow-rate is less than the sparsity of any arbitrary cut. Thus, to obtain an upper bound on the network capacity, we are free to choose a region A of any arbitrary shape and size. In this work we shall utilize cuts of square shape as shown in Fig.2, with length $L_A = 4l_A$, i.e., each side of the square A has length l_A . The parameter l_A plays a crucial role in deducing the required upper bounds. In particular, we choose l_A so as to guarantee that the demand $D(A) = \Theta(n)$.

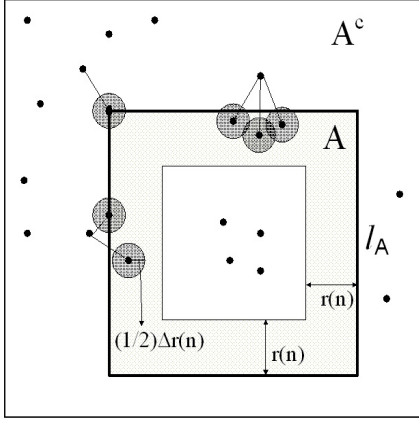


Fig. 2. Cut Capacity under Protocol Model

Lemma 4.4: In a random network with n nodes, each acting as source for m randomly chosen nodes, for every $\epsilon \geq 0$ if

$$l_A = \frac{1}{\sqrt{(1+\epsilon)m}} \text{ for } m \leq \frac{1}{4(1+\epsilon)r(n)^2} \quad (14)$$

$$l_A = 2r(n) \text{ for } m \geq \frac{1}{4(1+\epsilon)r(n)^2} \quad (15)$$

then for any $1 \geq \delta_1 \geq 0$ and n such that $\frac{n}{\log(2n)} \geq \frac{3}{\delta_1^2 c_1}$, w.h.p we have

$$D(A) \geq (1 - \delta_1)nc_1 \quad (16)$$

where $c_1 = \left(1 - \frac{1}{1+\epsilon}\right) \left(1 - \frac{1}{e^{1+\epsilon}}\right)$.

Proof: The proof is omitted due to space limitations. ■

A choice of $l_A = \frac{1}{\sqrt{(1+\epsilon_1)m}}$ can be used in the above lemma for all m , and such a condition would be sufficient to prove the required result that demand $D(A) \geq (1 - \delta_1)nc_1$ w.h.p. However, in the following analysis we require that $l_A \geq 2r(n)$. Therefore, we introduce the condition that $l_A = 2r(n)$ for

$m \geq \frac{1}{4(1+\epsilon_1)r(n)^2}$. Note that if $m \geq \frac{1}{4(1+\epsilon_1)r(n)^2}$, then $2r(n) \geq \frac{1}{\sqrt{(1+\epsilon_1)m}}$.

We invoke the following important observation to obtain an upperbound on the cut-capacity.

Remark 4.5: In [12], it was observed that a disk of radius $\frac{\Delta r(n)}{2}$ centered at each receiver in any time slot should be disjoint. However, this fact does not apply to the case in which nodes exploit broadcast transmissions, as is done when nodes are capable of employing NC. Indeed, as shown in Fig.2, the disks can overlap if the associated nodes are receiving identical information from a common transmitter. Nevertheless, as highlighted in [5], even under the NC assumption, the union of the disks centered at the receivers of one transmission should be disjoint from the union of the disks centered at the receivers of another transmission.

Lemma 4.6: If a square-shaped cut A has side-length $l_A \geq 2r(n)$, then the cut capacity satisfies

$$C(A) \leq \frac{16L_A}{\pi\Delta^2 r(n)} \quad (17)$$

under the protocol model.

Proof: In the protocol model, the distance between a transmitter and a receiver is bounded by $r(n)$. Hence, any node in A that receives a transmission from A^c should lie within a distance $r(n)$ from the boundary of the cut, i.e., all the receivers must be placed within an annular region of area

$$\begin{aligned} l_A^2 - (l_A - 2r(n))^2 &= 4l_A r(n) - 4r(n)^2 \\ &\leq 4l_A r(n) = L_A r(n) \end{aligned} \quad (18)$$

where length L_A of the cut is the perimeter of the region A .

We observe that each transmission across the cut will not allow any more transmission within an area of at least $\frac{\pi\Delta^2 r(n)^2}{4}$. Additionally, at least $\frac{1}{4}$ of this area has to fall within the annular region near the cut boundary. Therefore,

$$\begin{aligned} C(A) &= \text{max. no. of transmissions from } A^c \text{ to } A \\ &\leq \frac{\text{Area of annular region}}{\frac{\pi\Delta^2 r(n)^2}{4 \times 4}} = \frac{16L_A}{\pi\Delta^2 r(n)} \end{aligned}$$

Theorem 4.7: In a random geometric network with NC the multicast capacity under the protocol model has the following upper bound w.h.p.

$$\begin{aligned} C_m(n) &= \frac{c_2}{\sqrt{3(1+\epsilon_1)mn\log(n)}} \text{ if } m \leq \frac{n(1+\epsilon_1)^{-1}}{12\log(n)} \\ C_m(n) &= \frac{2c_2}{n} \text{ if } m \geq \frac{n(1+\epsilon_1)^{-1}}{12\log(n)} \end{aligned} \quad (19)$$

when $\frac{n}{\log(2n)} \geq \frac{3}{\delta_1^2 c_1}$, where $c_2 = \frac{64(1+\epsilon_1)e^{\frac{1}{1+\epsilon_1}}}{\pi\Delta^2 \epsilon_1(1-\delta_1)(e^{\frac{1}{1+\epsilon_1}} - 1)}$ and $\delta_1, \epsilon_1 \geq 0$

Proof: On account of Lemma 4.1, we can obtain an upper bound on the network capacity by just providing a bound for

the sparsity Γ_A . Furthermore, note that $L_A = 4l_A$. Hence, due to Lemma 4.6 we can say that, for all $l_a \geq 2r(n)$, we have

$$C_m(n) \leq \frac{64l_A}{\pi\Delta^2 r(n)D(A)}. \quad (20)$$

Consider $m \geq \frac{1}{4(1+\epsilon_1)r(n)^2}$. If we choose $l_A = 2r(n)$, then from Lemma 4.4 we have that $D(A) \geq (1-\delta_1)nc_1$ w.h.p. for all n such that $\frac{n}{\log(2n)} \geq \frac{3}{\delta_1^2 c_1}$. Therefore,

$$C_m(n) \leq \frac{128}{\pi\Delta^2(1-\delta_1)nc_1} \quad (21)$$

Similarly, if we choose $l_A = \frac{1}{\sqrt{(1+\epsilon)m}}$ for all $m \leq \frac{1}{4(1+\epsilon_1)r(n)^2}$, we have

$$C_m(n) \leq \frac{64 \frac{1}{\sqrt{(1+\epsilon_1)m}}}{\pi\Delta^2(1-\delta_1)r(n)nc_1}. \quad (22)$$

Note that, for all $m \leq \frac{1}{4(1+\epsilon_1)r(n)^2}$, $C_m(n)$ is maximized by choosing the smallest possible value of $r(n)$. Nevertheless the connectivity criteria (Lemma 3.2) requires that $r(n) \geq \sqrt{\frac{3\log(n)}{n}}$. The final result is obtained by substituting the value of c_1 and $r(n) = \sqrt{\frac{3\log(n)}{n}}$ in Eqs. 21-22. ■

The multicast capacity under pure routing has been characterized in [10], [11].

Theorem 4.8: [10], [11] In a random geometric network with pure routing, the multicast capacity under the protocol model has a tight bound

$$C_m(n) = \Theta\left(\frac{1}{\sqrt{mn\log(n)}}\right) \text{ if } m = O\left(\frac{n}{\log(n)}\right) \quad (23)$$

$$C_m(n) = \Theta\left(\frac{1}{n}\right) \text{ if } m = \Omega\left(\frac{n}{\log(n)}\right) \quad (24)$$

NC is a generalization of routing and thus any capacity achieved by routing is necessarily achieved by NC. Hence,

Theorem 4.9: In a random geometric network with NC, the multicast capacity under the protocol model has a tight bound

$$C_m(n) = \Theta\left(\frac{1}{\sqrt{mn\log(n)}}\right) \text{ if } m = O\left(\frac{n}{\log(n)}\right) \quad (25)$$

$$C_m(n) = \Theta\left(\frac{1}{n}\right) \text{ if } m = \Omega\left(\frac{n}{\log(n)}\right) \quad (26)$$

Finally, we can arrive at the following conclusion.

Corollary 4.10: The multicast throughput order gain provided by network coding over pure routing in a random geometric network under the protocol model is $O(1)$

V. CONCLUSION

Network coding (NC) has received considerable attention, and recent results for specific instantiations of NC have led many to infer that NC could lead to order throughput gains for multicasting in wireless networks. In this work, we used the protocol model to show that the order throughput gain derived from NC for multicasting and broadcasting in wireless networks is bounded by a constant. That is, as the network size increases, NC renders the same order throughput as traditional

store-and-forward routing. A similar argument can be proved under the more realistic physical model assumption, and this is the subject of another paper.

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