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### Authors

Guo, Xichen  
Jackson, David R  
Chen, Ji  
[et al.](#)

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# Leaky-Wave Analysis of Fabry-Pérot Resonant Cavity Antennas

Xichen Guo,<sup>#1</sup>, David R. Jackson<sup>#2</sup>, Ji Chen<sup>#3</sup>, Filippo Capolino<sup>\*4</sup>, and Donald R. Wilton<sup>#5</sup>

<sup>#</sup>Department of ECE, University of Houston, Houston, TX 77204-4005, USA

<sup>1</sup> xichenguo@gmail.com

<sup>2</sup> djackson@uh.edu

<sup>3</sup> jchen18@uh.edu

<sup>5</sup> wilton@uh.edu

<sup>\*</sup>Department of EECS, University of California, Irvine, Irvine, CA 92697-2625, USA

<sup>4</sup> f.capolino@uci.edu

**Abstract**—Fabry-Pérot resonant cavity antennas are investigated from a leaky-wave point of view, and the physics of the leaky waves responsible for the directive beams is explored.

## I. INTRODUCTION

Fabry-Pérot resonant-cavity antennas are structures that can be used to obtain directive beams, usually over a narrow bandwidth. The structure was originally introduced by von Trentini in 1956 [1], and has recently become popular, often in the context of a “metasurface” structure. The structure can produce narrow pencil beams at broadside and is simple to fabricate, being planar. The structure typically consists of a grounded substrate with a partially reflecting surface (PRS) on top, forming a resonant cavity region between the ground plane and the PRS. The PRS can consist of one or more dielectric layers, but is often instead composed of a periodic structure, such as a periodic array of metal patches or slots in a metal plane. The structure is usually excited with a simple source inside the cavity, such as a horizontal dipole in the middle of the cavity or a slot on the ground plane at the bottom of the cavity.

In this presentation the focus will be on examining the fundamental physics of the leaky waves that propagate on the structure, and establishing that the principle of operation is indeed that of a leaky-wave antenna.

## II. BACKGROUND

The Fabry-Pérot resonant-cavity antenna using a single high-permittivity superstrate as the PRS [2] or a stack of layers that alternate between low and high permittivities as the PRS [3] has been examined in the past. The fact that the basic principle of operation is from radiation of leaky waves has already been established for the single high-permittivity superstrate layer [4] and the stack of layers that alternate between low and high permittivities [5]. An example of such a structure is shown in Fig. 1, showing the case of a stack of low/high permittivity layers used as the PRS, with an  $x$ -directed horizontal electric dipole excitation.

It was established that such a structure operates as a “two-dimensional” leaky-wave antenna, in which the source excites one or more radially-propagating cylindrical leaky waves that propagate outward from the source location [6].

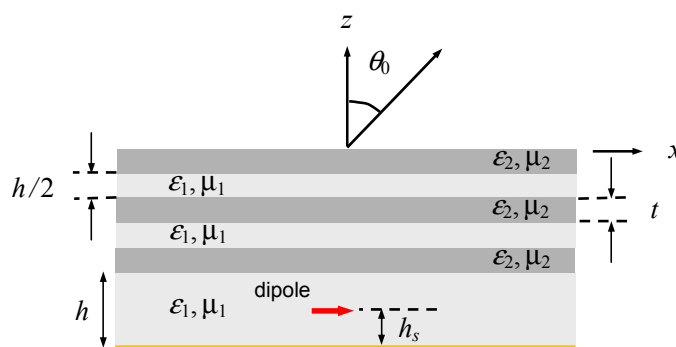


Fig. 1. Fabry-Pérot resonant-cavity antenna using a stack of low/high permittivity dielectric layers as the PRS structure. The substrate thickness is  $h$ .

For the structure of Fig. 1, with homogeneous dielectric layers acting as the PRS, there will be two leaky waves that can propagate on the layered structure, a  $TM_z$  leaky wave and a  $TE_z$  leaky wave. The  $TM_z$  leaky wave controls the pattern in the E-plane ( $\phi = 0^\circ$ ) while the  $TE_z$  leaky wave controls the pattern in the H plane ( $\phi = 90^\circ$ ). The two leaky waves have approximately the same complex wavenumber when the directivity of the antenna is high. When the two leaky waves are excited together with the optimum ratio of equal amplitudes, they radiate to form a pencil beam at broadside [6]. This happens automatically when the resonance gain condition is satisfied [1], [2]. For the layered PRS structure, this occurs when the substrate thickness  $h$  is a half-wavelength in the substrate dielectric. Results have shown that the field on the aperture just above the PRS is dominated by the leaky-wave fields, and hence the leaky-wave aperture field is a good predictor of the far-field radiation pattern.

Previous results have also established that the power density radiated at broadside from the source dipole is maximum when the frequency (or substrate thickness) is chosen so that  $\beta = \alpha$  for the  $TM_z$  and  $TE_z$  leaky modes [7]. This occurs when the electrical thickness of the substrate is one half of a wavelength in the dielectric. The directivity is

maximized when the frequency is slightly lower, so that  $\beta = 0.518 \alpha$  [7].

### III. PERIODIC PRS STRUCTURES

More recently, Fabry-Pérot resonant-cavity antennas have been designed with periodic PRS structures. An example is the metal-patch PRS structure shown in Fig. 2. The PRS consists of an infinite periodic array of metal patches of length  $L$  in the  $x$  direction and width  $w$  in the  $y$  direction, with a periodic spacing of  $a$  in the  $x$  direction and  $b$  in the  $y$  direction. If the patches are narrow ( $w \ll L$ ) as shown in the figure, the patch currents will be primarily in the  $x$  direction, and the leaky wave excited by an  $x$ -directed horizontal electric dipole will consist of a single  $TM_x$  leaky-wave mode when the substrate is air. This leaky mode will propagate outward radially from the dipole source, essentially acting as a leaky parallel-plate mode. The complex radial propagation wavenumber  $k_\rho$  will be somewhat dependent on the angle  $\phi$  in cylindrical coordinates on the PRS surface.

There is no simple analytic solution for the patch currents or the fields excited by the dipole source for the structure of Fig. 2, as there is for the dielectric-layer PRS structure, so a numerical solution must be sought. A direct integral equation approach can be used to analyze the Fabry-Pérot resonant cavity antenna, but this requires putting basis functions on each of the patches that make up the PRS. A more efficient method for obtaining the patch currents or the fields within the infinite periodic structure of Fig. 2, excited by the dipole source, is the array scanning method (ASM). The method requires only a single unit cell analysis, just as with most infinite periodic structure problems, but involves two spectral (wavenumber) integrations, which are necessary to synthesize the single source from an infinite periodic array of phased sources. The patch current or field in any unit cell of the PRS can be obtained directly by this method. This method is used to investigate the patch currents for the structure of Fig. 2, and it is shown how the patch currents are dominated by the currents of a  $TM_x$  leaky mode.

### IV. ARRAY SCANNING METHOD

In the array scanning method [8], the patch current at the center of the  $(m,n)$  patch due to the dipole source in Fig. 2 is expressed as

$$A_{mn} = \frac{ab}{(2\pi)^2} \int_{-\pi/b}^{\pi/b} \int_{-\pi/a}^{\pi/a} A_{00}^\infty(k_{x0}, k_{y0}) e^{-j(k_{x0}ma + k_{y0}nb)} dk_{x0} dk_{y0} \quad (1)$$

where  $A_{00}^\infty(k_{x0}, k_{y0})$  is the patch current at the center of the  $(0,0)$  patch when the infinite periodic structure is excited by an infinite periodic phased array of identical dipole sources with periods  $(a,b)$ , having wavenumber phasings of  $(k_{x0}, k_{y0})$ . This current is calculated by using a standard periodic spectral-domain moment-method analysis, in which full-domain sinusoidal basis functions are used to represent the  $x$ -directed patch current on the  $(0,0)$  unit-cell patch.

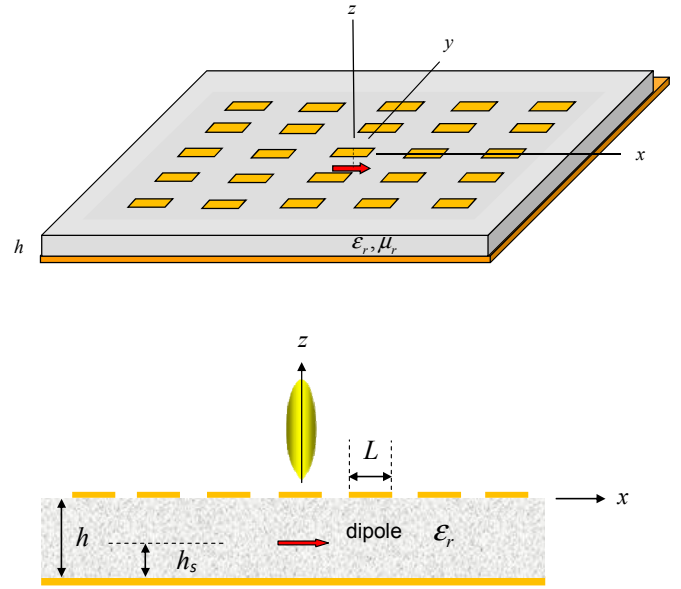


Fig. 2. Fabry-Pérot resonant-cavity antenna using a periodic metal patch array as the PRS structure. The top figure shows a three-dimensional view. The bottom figure shows a side view, with a sketch of the beam also shown

### V. LEAKY-MODE PROPAGATION

The complex wavenumber of the  $TM_x$  leaky mode is also studied by directly solving a unit-cell periodic spectral-domain determinantal problem, which enforces the electric field integral equation (EFIE) condition that the  $E_x$  component of the electric field vanishes on the unit-cell patch surface. It is important in this solution that the correct branch of the vertical wavenumber in the air region be chosen for each space harmonic (Floquet wave)  $(p,q)$ , since the leaky mode has a fundamental Floquet wave  $(p,q) = (0,0)$  that is improper (exponentially increasing) in the air region. Hence, for this Floquet wave the vertical wavenumber is chosen with a positive imaginary part. The  $TM_x$  leaky mode is also  $TM_z$  in the E plane ( $\phi = 0^\circ$ ) and  $TE_z$  in the H plane ( $\phi = 90^\circ$ ).

This investigation allows for the leaky-mode propagation to be studied as a function of angle  $\phi$  on the PRS surface. Having knowledge of the leaky-wave propagation as a function of angle  $\phi$  allows for the development of a simple closed-form approximate expression for the patch currents of the  $TM_x$  leaky mode. A simple CAD formula for the patch currents of the leaky mode has been formulated, and it is given by

$$A_{mn} = A^{TM} \cos^2 \phi H_1^{(2)'}(k_\rho(\phi)\rho) + A^{TE} \sin^2 \phi H_1^{(2)'}(k_\rho(\phi)\rho), \quad (2)$$

where  $H_1^{(2)'}$  denotes the derivative of the Hankel function of the second kind, of order 1. The values of  $(\rho, \phi)$  are chosen to be the centers of the patches. This expression comes about from representing the field  $E_x$  of a radially propagating  $TM_x$  leaky mode, and then assuming that the patch currents of the leaky mode are proportional to the field  $E_x$  of the leaky mode. For this, the  $TM_x$  leaky mode is assumed to have a magnetic vector potential on the aperture that is of the form

$$A_x(\rho, \phi) = A_{x0} H_0^{(2)}(k_\rho \rho). \quad (3)$$

## VI. RESULTS

Results are obtained for the structure shown in Fig. 2, in which an air substrate is used ( $\epsilon_r = 1$ ) [9]. For this structure the following dimensions are chosen:  $h = 1.333$  cm,  $L = 1.25$  cm,  $w = 0.1$  cm,  $a = 1.35$  cm,  $b = 0.3$  cm,  $h_s = h/2$ . This structure produces a broadside beam at 12 GHz. Results show that at 12 GHz the complex wavenumber of the  $TM_x$  leaky mode in the E and H planes is approximately

$$\begin{aligned} k_\rho^{TM} / k_0 &\approx 0.11 - j0.12 \\ k_\rho^{TE} / k_0 &\approx 0.12 - j0.13. \end{aligned}$$

Figure 3 shows the normalized phase and attenuation constants vs. the propagation angle  $\phi$  measured from the  $x$  axis. It is seen that the propagation wavenumber is not independent of the angle  $\phi$ , though the variation is not extreme.

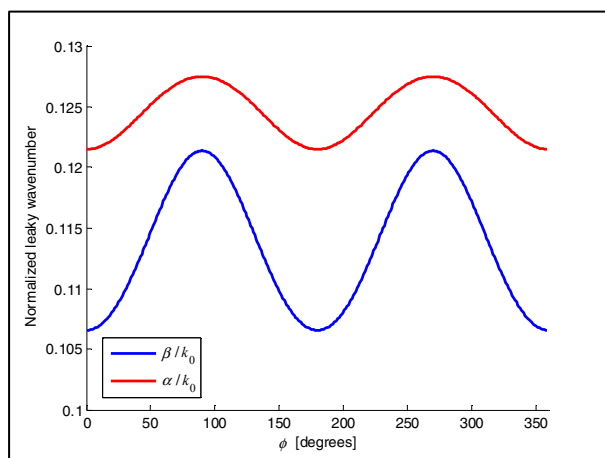


Fig. 3. Normalized wavenumber vs. propagation angle  $\phi$ .

Figure 4 shows the agreement between the exact patch currents as calculated by the ASM method and the leaky-mode CAD formula (2). The leaky-mode CAD formula curves has the constant  $A^{TM}$  in (2) chosen so that the two curves agree at patch  $(m, n) = (6, 0)$ . The agreement is seen to be excellent for moderate patch distances from the source dipole. For smaller distances the agreement worsens due to the fact that the leaky-mode is not yet dominant over the reactive

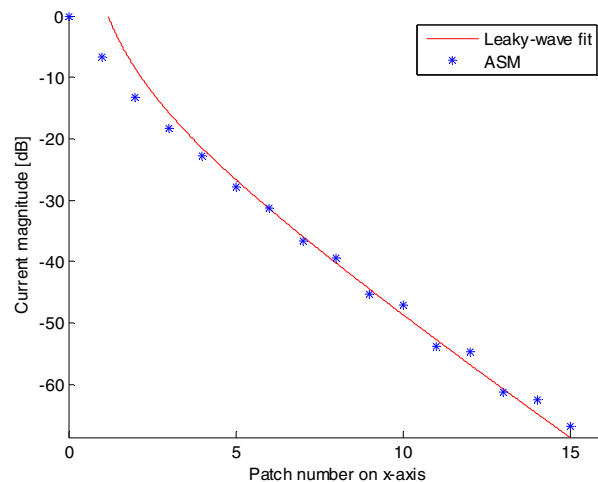


Fig. 4. Normalized wavenumber vs. patch number  $m$  in the E plane.

near field of the dipole source. For large distances the agreement also worsens due to the fact that the exponential decay of the leaky mode causes it to become small relative to the remaining part of the continuous spectrum that is excited by the dipole source.

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