THE EFFECT OF ISLANDS ON SURFACE WAVES

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ROBERT S. ARTHUR

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ABSTRACT

THE INTERRUPTION of the progress of waves by an island produces a zone of "wave shadow" to the "wave lee" of the island. Observations have demonstrated that the shadow is never completely devoid of waves and that on occasion relatively high waves may occur in this lee region. An investigation is made of factors influencing the wave conditions in the shadow.

The characteristics and mean direction of approach of the incident waves are assumed to be known. The penetration of wave energy into the region to the lee of the island is determined by the following factors: (1) the effect of underwater topography off the island's shores in refracting wave energy into the lee, (2) the effect of currents near the island in refracting energy, (3) the diffraction effect resulting when a barrier interrupts wave fronts, and (4) the effect of variability in direction of wave travel in limiting the extent of the shadow. The quantitative results indicate that the important effects in the penetration of wave energy into the lee are generally the result of refraction by underwater topography and variability in direction.

The factors discussed are adequate for explaining quantitatively the wave conditions observed in the lee of the Island of Sicily, San Clemente Island, and Santa Catalina Island. The methods may be of general interest in the prediction of wave heights near the lee shores of islands or on coastal shores which are in the shadow of offshore islands.

INTRODUCTION

MARINERS, particularly in coastal regions, have long used islands as barriers against heavy seas. When a train of waves is interrupted by an island there is generally a zone of "wave shadow" to the "wave lee" of the island. Contrary to what might be expected, this shadow zone does not usually extend any great distance beyond the island, and high waves occasionally break on the lee shore itself. The purpose of this paper is to evaluate the factors which determine the penetration of wave energy into the wave lee of islands.

This investigation was begun after the results of wave forecasting for the invasion of Sicily (Chief of Naval Operations, 1944) in World War II were published. Although the waves in the fetch extending to the northwest of the island were adequately forecast on the basis of the method of Sverdrup and Munk (1947), the breakers on the beaches of the supposedly protected lee shore from Licata to Scoglitti (fig. 11) were much higher than anticipated. Again, during World War II, the wave forecasting for the amphibious operation on the coast of Burma (Bates, 1949) involved consideration of the effect of the Andaman and Nicobar islands. And further, unpublished observations of breaker heights along the southern California coast, which were made or supervised by W. H. Munk, C. J. Burke, and M. A. Traylor in 1944, demonstrated that, for westerly waves, San Clemente and Santa Catalina islands do not produce a pronounced wave shadow.

The factors which influence the penetration of wave energy into the wave lee of an island are refraction, diffraction, and variability in direction of wave travel. In this paper each of these factors is considered separately. Although they are not independent, it is believed that a reliable indication of the effect of islands on the wave pattern can be obtained in this manner.

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EARLY OBSERVATIONS OF WAVE PATTERNS AROUND ISLANDS

The early Micronesians and Polynesians were among the keenest observers of the interaction of islands and waves. Their knowledge of the swell patterns around islands was used as an aid in navigating from island to island over distances of as much as 120 nautical miles and perhaps even more. They have left no written explanation of their methods, but through interviews of native chiefs and sea pilots in the latter part of the nineteenth century Winkler (1901) and others have established and recorded some of their techniques. Their ability to read the "signs of the swell and currents" has lately been supplanted by the use of the compass and charts of western civilization. In museums can be found examples of their early charts, of which Krieger (1943, p. 24) states:

> Every chief and sea pilot possessed elaborate charts based upon his own experience and on knowledge handed down or gained from others. These sea charts are made with thin strips of the midrib of the leaflets of the coconut, arranged in a frame usually rectangular in shape. The knowledge they record is indicated by the arrangement of the leaf strips relative to one another and by forms given to them by bending and crossing. Curved strips indicate the altered direction taken by ocean swells when deflected by the presence of an island; their intersections are nodes where these meet and tend to produce a confused sea. These charts are never made to scale and are, in fact, little more than mnemonic devices for the use of the owner.

Since the regular wave pattern of a long swell is disturbed even at great distances beyond islands, the early navigators were able to use the phenomenon as a guide to new islands (Bigelow and Edmondson, 1947).

REFRACTION OF WAVES BY UNDERWATER TOPOGRAPHY

As waves approach a shore, their velocity begins to diminish significantly at a depth of about one-half the wave length. When the crests reach the beach, they tend to parallel the shoreline regardless of their direction of approach from deeper water, for as Harrison (1848, p. 343) has put it, "the velocity of the part which first reaches the shallow being lessened, the whole wave wheels round, and breaks nearly at right angles on the beach." According to D. W. Johnson (1919), this process was termed "wave refraction" by William Morris Davis. The wave height is influenced by refraction since changes in wave-crest orientation are associated with convergence and divergence of energy along the crest.

The quantitative effect is obtained from a refraction diagram, which shows lines representing the orientation of the wave crests. On such diagrams it is possible to construct a network of lines which are referred to as orthogonals since they are everywhere perpendicular to the wave crests. If the wave energy between two orthogonals is considered to remain constant, the convergence and divergence of energy along the crests is determined by the convergence and divergence of orthog-

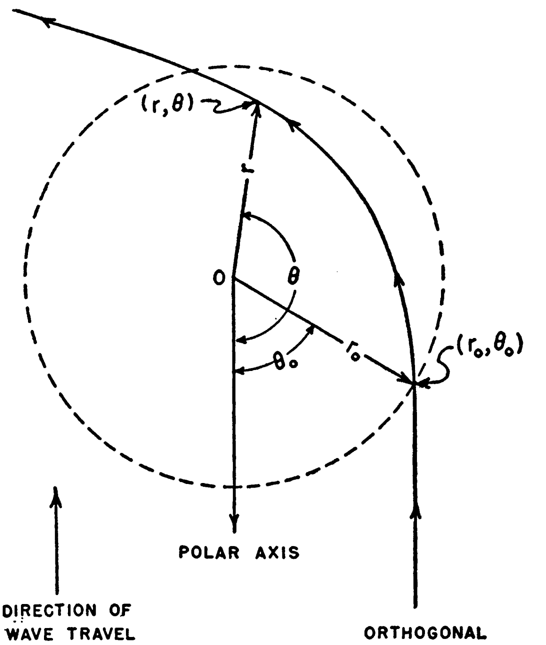

Fig. 1. Polar coördinate representation of an orthogonal associated with wave crests deformed by concentric circular bottom contours.

onals. This assumption of no energy transport across orthogonals can be only approximately valid, but the approximation may be expected to be close when the convergence or divergence of orthogonals is relatively small. Some regions in the refraction examples worked out in this section have large convergence or divergence of orthogonals, and diffraction losses may occur.

The construction of refraction diagrams by graphical methods on the basis of Snell's Law is discussed by O'Brien and Mason (1940) and others (Hydrographic Office, 1944; Johnson, O'Brien, and Isaacs, 1948). Such methods can be applied to determine the refraction of waves by an island, but the procedure is time consuming and the precision is limited in practice. This lack of precision is particularly disadvantageous in determining the refraction of energy into the wave lee of an island.

As an alternative to the graphical method, analytical solutions for the orthogonals are obtainable for "idealized" islands, the variation in depth being expressed as an analytic function. Precise determinations of the refraction effects are possible, and an insight into the effects of the critical parameters is readily gained. Solutions based on the application of Fermat's principle of least time to refraction by concentric circular bottom contours have been discussed elsewhere by the writer (1946); parts of the development are reviewed here because they apply to the present subject.

According to Fermat's principle, light travels in a minimal time path. The principle is equivalent to Snell's law of refraction. The paths of the light rays correspond to orthogonals in a refraction diagram which are thus to be regarded as paths of least time. The travel time along a path Γ is the integral

$$I' = \int_{\Gamma} \frac{ds}{c}, \tag{1}$$

where c is the variable wave velocity along the path, and ds is the arc length along the path. An orthogonal is a path which renders the integral of (1) a minimum. We introduce polar coördinates (fig. 1) and an initial point (r_0, θ_0) where the depth is infinite, so that the wave velocity has the value c_0 appropriate to the wave length L_0 in infinitely deep water. We assume that the depth remains infinite for $r > r_0$, and the orthogonals are, therefore, straight lines in this region. For practical purposes, the orthogonals may be extended as straight lines to a depth of 0.5 L_0, since even at that depth $c = 0.996\, c_0$. Multiplying (1) by c_0/r_0, we have as the equivalent integral to be minimized between the points (r_0, θ_0) and (r, θ)

$$I = \int_{\theta_0}^{\theta} \frac{\sqrt{R^2 + (R')^2}}{C}\, d\theta\,, \tag{2}$$

where $R' = dR/d\theta$, and where $R = r/r_0$ and $C = c/c_0$ are convenient nondimensional quantities.

A path which makes the integral of (2) a minimum is found by the methods of the calculus of variations (Joos, 1934). Such a path must satisfy the Euler–Lagrange condition

$$\frac{d}{d\theta}\,(\partial F/\partial R') - (\partial F/\partial R) = 0\,, \tag{3}$$

where $F = \sqrt{R^2 + (R')^2}/C$. The condition is necessary and proves sufficient in some problems, but the sufficiency is not investigated here.

The assumption of concentric circular bottom contours requires that C be a function of R only, and that F be a function of R and R' only. Under these circumstances a solution of (3) is

$$F - R'(\partial F/\partial R') = a\,. \tag{4}$$

If the constant of integration, a, be evaluated in terms of the initial point (r_0, θ_0) and in terms of the initial direction of the orthogonal, e.g., parallel to the polar axis, we have

$$\frac{dR}{R\sqrt{\left(\dfrac{R}{C\sin\theta_0}\right)^2 - 1}} = \pm d\theta\,. \tag{5}$$

Since the variables in (5) are separated, the path of the orthogonal can always be produced, although for some forms of the function $C = C(R)$, it may be necessary to resort to numerical integration.

The depth, h, is related to the wave velocity by the familiar equation $c^2 = (gL/2\pi) \tanh (2\pi h/L)$. The form of this equation is conveniently altered (Hydrographic Office, 1944) by dividing both sides by c_0^2 and using the relationships $c_0^2 = (gL_0/2\pi)$ and $c/c_0 = L/L_0$. The resulting equation which relates the relative depth h/L_0 to $C = c/c_0$ is

$$C = \tanh\left(\frac{2\pi}{C} \cdot \frac{h}{L_0}\right). \tag{6}$$

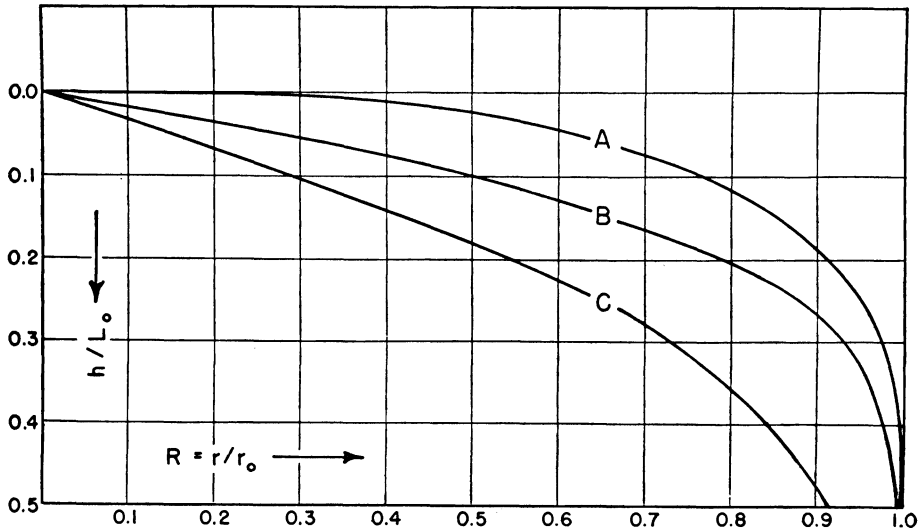

Fig. 2. Cross-section of three bottom slopes.

After a particular form of the function $C = C(R)$ is selected, numerical values of the function $R = f(h/L_0)$ are obtained from (6) and $C = C(R)$.

Three different types of bottom slopes are illustrated (fig. 2). Type A represents an island whose radius is one-fifth of the distance from the center to the circle $R = r/r_0 = 1$, and types B and C represent "point islands" whose central points only are at zero depth. For all three types, the island rises abruptly from infinite depth at $R = 1$ and reaches the depth at which refraction becomes effective, $h = 0.5\ L_0$, before $R = 0.9$. The functions $C = C(R)$ which have been utilized for the three types are $C = 5(R - 0.2)/4$, $C = \sqrt{R}$, and $C = \sqrt{R(2 - R)}$ respectively. These particular functions are chosen because they lead to interesting types of bottom slope when used in conjunction with (6) and because they are examples of forms which permit easy integration of (5).

Orthogonals appropriate to each of the three types are determined from (5). Values of the "refraction factor" (Munk and Traylor, 1947), K, are determined for the island shore for type A (fig. 3) and for the circle $R = r/r_0 = 5$ for types B and C (figs. 4 and 5). K is the factor by which deep-water wave height must be multiplied to determine the shallow-water wave height resulting from refraction alone.

In each type certain orthogonals reach the shores of the island or the very shallow water near shore, and the section of the incoming crests associated with this band of orthogonals produces breakers. Consequently, the island effectively absorbs the wave energy from this section. The width in deep water of this band of orthogonals which are associated with breakers we will call the "absorption length" of the island. In type A, any orthogonal which crosses the circle $R = r/r_0 = 1$ eventually spirals

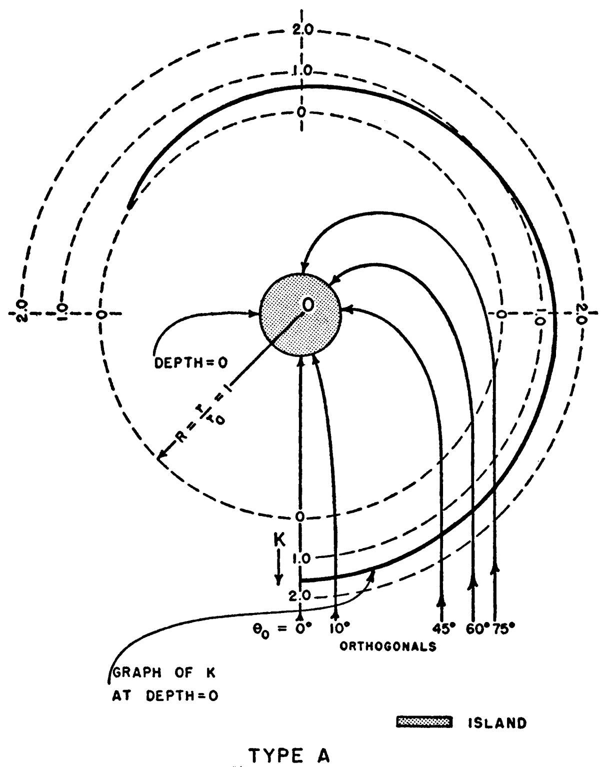

Fig. 3. Orthogonals and K-variation at shore for a circular island off which the slope changes from very gentle to very steep, type A.

in to the shore, and the absorption length of the island is, therefore, $2r_0$. For the point islands, type B and type C, each orthogonal reaches a minimum depth at some minimum distance, R_m, from the center of the island. The value of R_m is determined by setting $dR/d\theta = 0$ in (5), with the final result that R_m is a root of

$$R = C(R) \cdot \sin \theta_0 . \tag{5a}$$

Although the critical depth for breaking depends upon the wave height and, therefore, on the refraction factor, a comparison can be made between the absorption lengths of the two point islands by taking a typical value of the relative depth at breaking, for example, $h_b/L_0 = 0.02$. By using (5a), (6), and $C = C(R)$ it is possible to find the critical value $(\sin \theta_0)_b$ such that the associated orthogonal reaches the relative depth h_b/L_0 when $R = R_m$. The absorption length of the island is then $2r_0(\sin \theta_0)_b$.

The refraction results for the three types are summarized in table 1.

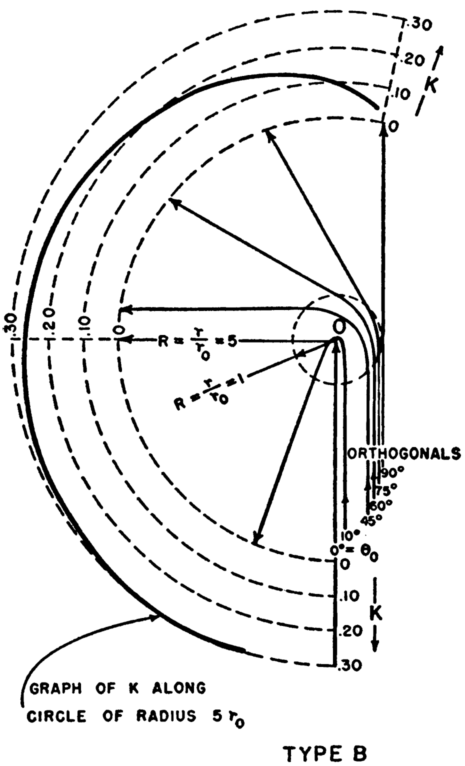

Fig. 4. Orthogonals and K-variation (at $R = 5$) for a "point island" off which slope changes from gentle to steep, type B.

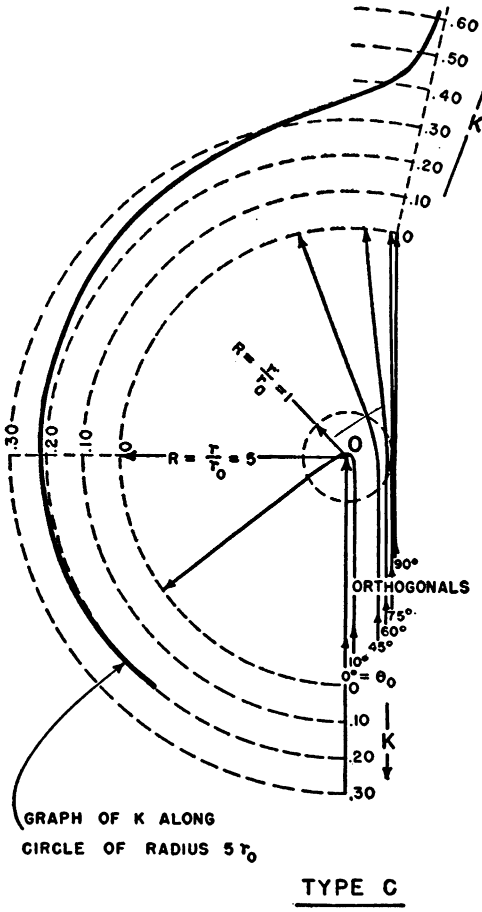

Fig. 5. Orthogonals and K-variation (at $R = 5$) for a "point island" off which the slope is moderate, type C.

These analytic results serve to illustrate the wide range of effects of wave refraction around islands. Although, in general, shelter may be expected along the lee shore of most islands, a large amount of wave energy may be turned onto the lee shore by refraction, as indicated by type A. Bigelow and Edmondson (1947, p. 173) mention that "the swell heaves right around Nukuoro Atoll, in the Carolines, during the winter season when the Northeast Trade Winds are at their height, there being no shelter anywhere, except within its entrance, for it is nearly round and only between 3 and 4 miles in diameter." The available soundings off Nukuoro Atoll are not sufficient to identify it further with type A. However, this type of bottom slope is characteristic of coral islands and volcanic islands with a shelf cut by wave erosion.

Although the "point island" cases are highly idealized, they indicate that the distribution of refracted wave energy in the wave lee of an island is critically dependent upon bottom slope. Thus, along a coastline which receives protection from incident waves as a result of the wave energy absorbed by offshore islands, the height

TABLE 1

SUMMARY OF REFRACTION RESULTS FOR THREE TYPES OF CIRCULAR ISLANDS

BOTTOM SLOPE

Type	Analytic function	Near $h = 0.5\,L_0$	In shallow water	Figure
A	$C = 5(R - 0.2)/4$	Very steep	Very gentle	2, A
B	$C = \sqrt{R}$	Steep	Gentle	2, B
C	$C = \sqrt{R(2 - R)}$	Moderate	Moderate	2, C

EFFECT OF REFRACTION

Type	Absorption length	Remarks	Figure
A	$2r_0$	No refracted waves in lee of island beyond contour $h = 0.5\,L_0$	3
B	$0.70\,r_0$*	Height of refracted waves lower at edge of lee but increasing toward center of lee. Waves refracted through greater angles higher.	4
C	$0.36\,r_0$*	Height of refracted waves rapidly decreasing from incident wave height at edge of lee to lower values toward center of lee. Waves refracted through greater angles lower.	5

* Assuming breaking at $h_b/L_0 = 0.02$.

of refracted waves may either increase or decrease from the edge of the shadow zone depending on whether the bottom slope at the edges of the islands is similar to type B or type C. The amount of wave energy dissipated by breakers on the shores becomes particularly significant for dense networks of islands such as those found in the western Pacific. The northeast trade-wind swell meets the Marshall Islands over a total lateral extent of 530 nautical miles from Bikini Atoll southeast to Mille Atoll. If it is assumed that the energy of waves reaching the island shores and connecting reefs in the atolls of the group is completely dissipated in breaking, the absorption occurs over a lateral extent of approximately 400 nautical miles. The "absorption coefficient" for northeast swell for the Marshalls is then 400/530 or 0.75. The coefficient for the Gilbert Islands is 180/360 or 0.50.

With regard to the protection afforded a coast by an offshore island, it is useful to consider a schematic example of an elongate island (fig. 6) typical of those off the coast of southern California. If it is assumed that the absorption length of the island is 18 miles and that all the wave energy passing across stretches of shallow water one mile wide at the extremities of the island spreads out by refraction over stretches ten miles wide along the opposite coastline, the average refraction factor for breakers (Munk and Traylor, 1947) on the sheltered coast is $K_b = \sqrt[4]{1/10} = 0.46$ for the dimensions indicated. This average value is maximal since some of the energy may be expected to be refracted beyond the ten-mile-wide stretch. Thus, the refracted breakers along the sheltered coastline may be expected to have an average height of less than 46 per cent of the incident, undisturbed breaker height.

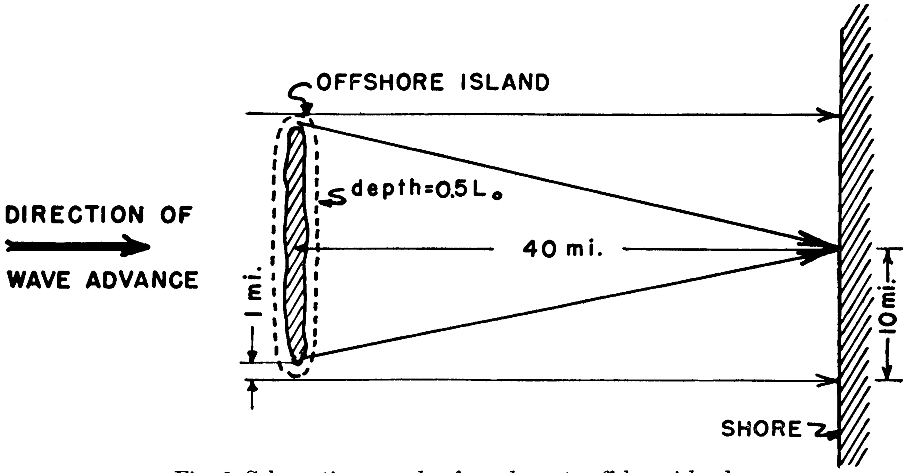

Fig. 6. Schematic example of an elongate offshore island.

EFFECT ON WAVES OF CURRENTS AROUND ISLANDS

The steepness and direction of waves which enter a region of currents from still water are altered. The steepness of waves meeting an opposing current increases, and if the increase is great enough the waves break; when waves meet a current at an angle, directional changes occur as a result of refraction. Some general estimates have been made regarding the effect on the wave pattern of a current pattern around islands.

The example of waves moving from still, deep water into a deep sound where a current runs directly with or against the advancing waves is treated by Unna (1942) and Sverdrup (1944). If a steady state is established, the wave period remains constant during the process. This requires that the wave length, L, decrease if the waves enter an opposing current. On the other hand, continuity of the supply of energy causes the energy to crowd up and the wave height, H, to increase. The wave steepness, H/L, in an opposing current therefore increases, and the result obtained by Sverdrup is

$$H/L = (H_0/L_0)\sqrt{\frac{2}{a\,(1+a)}}\left(\frac{2}{1+a}\right)^2 \tag{7}$$

where $a = \sqrt{1 + 4u/c_0}$, the initial steepness in still, deep water being H_0/L_0 and the ratio of current velocity to initial wave velocity being u/c_0, with u taken as negative in an opposing current. Sverdrup assumes that waves break when the steepness H/L reaches the critical value 1/7. The magnitude of the opposing current necessary to cause breaking of waves is computed from (7) for several sets of values of period T_0 and initial height H_0 (table 2). The heights of the breaking waves for the sets of values shown are from about 1.5 to 3 times the initial heights. Equation (7) also applies to waves entering a following current ($u > 0$), but the results are of much less interest in the present discussion.

The breaking of waves in an opposing current requires relatively high current velocities according to table 2. Such velocities occur in certain straits and sounds around offshore islands, e.g., the Aleutian chain and the Orkneys and Shetlands

TABLE 2
VELOCITY OF OPPOSING CURRENT (KNOTS) NECESSARY TO CAUSE BREAKING
(Adapted from Sverdrup)

Initial height H_0 (in feet)	Period T_0 (in seconds)		
	5	7	9
5	3.1	5.1	6.7
10	2.0	4.3	6.3

north of Scotland. Stevenson (1866, pp. 65–66) notes that the rapid tideways, called *roosts* in the Orkneys and Shetlands, sometimes act as breakwaters.

It was proved by observations made especially for the purpose at Sumburgh Head in Shetland during a southwesterly storm, that so long as the Sumburgh Roost (one of the most formidable in those seas, and more than three miles in width) was cresting and breaking heavily outside there was comparatively little surf on the shore; but no sooner did the roost disappear towards higher water, than a heavy sea rolled toward the land, rising on the cliffs to a great height.

Regarding the Pentland Firth, which lies between the Orkneys and Scotland, Stevenson (1886, pp. 67–68) writes,

From careful inquiries, as well as from actual experience of such dangerous breaking waters as the Boar of Duncansby, and the Merry Men of Mey in the Pentland Firth, it appears that the true cause of these dangers [dangerous races or roosts] is the encounter between the *swell of the ocean and an opposing tidal current.* Two rapid tides may meet each other without any dangerous effects, if there be no ground-swell, yet, if they join together in a rough sea, as in coming around the islands of Stroma or Swona in the Pentland Firth, during ground swells, the effect of their union being to increase the current, highly dangerous waves will be produced. The meeting of the currents, therefore, though not the *cause* of the waves, is nevertheless sure to increase their height, and to make them break . . . *breaking waves are produced when the tide runs against a ground swell* . . . Thus, at the east end of the Pentland Firth the Boar of Duncansby is well known to rage with easterly swells and a flood tide; whereas, at the west end of the firth, the Merry Men of Mey are worst with ebb tide and westerly swell, at which time no boat can enter them without the greatest risk of being swamped.

Stevenson gives as the velocity at spring tide of the roost near Swona, Pentland Firth, 9 knots, and of Sumburgh Roost, Shetland, 7 knots. The results given in table 2 are, therefore, consistent with those observations.

Waves breaking in opposing currents are also noted in the Aleutian Island passages where "reports agree that the currents almost always flow into the Bering Sea" (Coast and Geodetic Survey, 1931, p. 14). *The Coast Pilot* (Coast and Geodetic

Survey, 1931) mentions that strong, dangerous "tide rips" are found in Unimak Pass when a strong wind opposes the current, which may run to about 2 knots. Dangerous, choppy seas occur at other passes when a strong wind or swell opposes currents that may reach 4 to 6 knots. Strong tidal influences on the generally northward-flowing current produce these higher velocities.

The breaking of waves in passes between islands means that a significant part of the wave energy will be prevented from traversing the opposing current zone, hence the "breakwater" effect mentioned by Stevenson. The effect on the wave pattern is generally not as spectacular as this, since the ordinary current velocities amount to but a fraction of a knot. The current pattern around islands may, however, cause

TABLE 3
VELOCITY OF OPPOSING CURRENT (KNOTS) NECESSARY TO CAUSE A CHANGE IN WAVE DIRECTION OF 10°
(Adapted from Johnson)

Angle, α, between crest and current discontinuity in still water	Period T_0 (in seconds)			
	5	7	9	11
45°	2.3	3.3	4.2	5.2
65°	0.9	1.2	1.6	1.9
85°	0.2	0.3	0.4	0.5

wave refraction; in fact, this will occur whenever there is a current velocity shear with a component along the wave crests. Johnson (1947) considers waves moving through still, deep water and encountering a uniform current moving at an angle with the wave direction, and the results serve as a guide for the magnitude of the effect in more complicated cases. Johnson assumes that the crest is continuous across the current discontinuity and that the wave in the current moves relative to the current with the velocity $c = \sqrt{gL/2\pi}$. If the acute angle between crest and discontinuity in still water is α and in the region of the current is β, the refraction law is

$$\sin \beta = c_0^2 (\sin \alpha)/(c_0 - u \sin \alpha)^2 , \tag{8}$$

where u is the current velocity and c_0 the still, deep water-wave velocity. Calculations of the refraction effect are made from (8) and given in table 3.

On the basis of the calculations, it can be stated that significant alterations of wave direction (say 10°) in the usual low-velocity currents take place only for shorter-period wind waves which have a still-water direction of travel very nearly opposite that of the current. Such a statement is very general but no more precision is warranted since the details, or even the general features, of current patterns around islands are not usually known. Steepness is also affected by refraction, but the effect is not significant for currents of low velocity.

DIFFRACTION OF WAVES

Wave energy is also propagated into the wave lee of an island barrier by diffraction. The phenomenon is analogous to optical diffraction; in fact, Penney and Price (1944) have shown that the classical Sommerfeld solution for optics is also a solution of the water-wave diffraction problem. The problem solved is that of diffraction by a semi-infinite plane barrier in water of uniform depth, h, with the assumption that the normal component of the fluid velocity be zero at the barrier. Putnam and Arthur (1948) introduce approximations to simplify application of the theoretical solution and compare the solution to results obtained in a laboratory investigation. The solution is reviewed below, and is employed to estimate the diffraction effect beyond an island barrier. The estimate may be rough, since an island cannot constitute a semi-infinite barrier, but the results should be useful, particularly in cases where the island interrupts the wave train over a distance of many wave lengths.

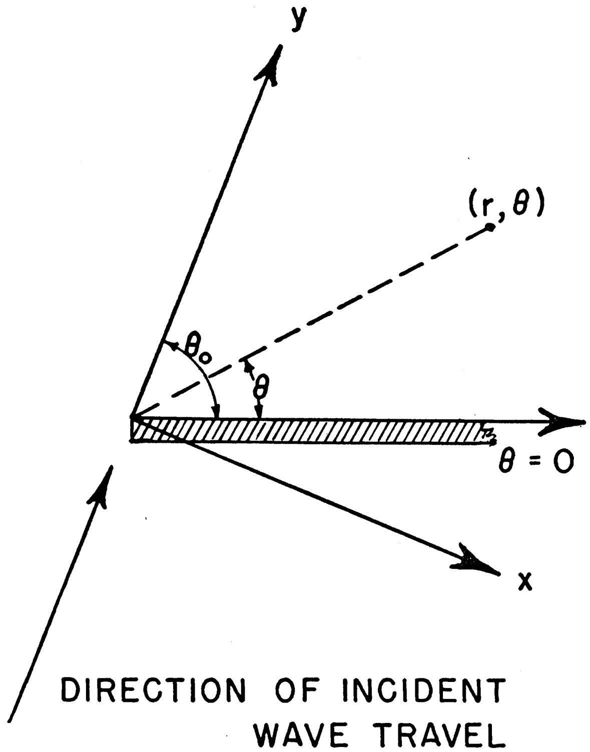

Fig. 7. Coördinate system for wave-diffraction analysis.

Consider a cylindrical coördinate system with r and θ in the plane of the undisturbed water surface (fig. 7) and the z-axis vertically upward. Let ϕ denote the velocity potential and make the usual assumptions of the linear theory of surface waves (Lamb, 1945). An expression for ϕ which is periodic in the time t and which satisfies the boundary condition at the bottom $z = -h$ is

$$\phi = Ae^{ikct} \cosh k(z + h) \cdot F(r, \theta),$$

where $i = \sqrt{-1}$ (but only the real part is to be finally retained). The expression for ϕ satisfies Laplace's equation in cylindrical coördinates provided

$$(\partial^2 F/\partial r^2) + (1/r)\,(\partial F/\partial r) + (1/r^2)\,(\partial^2 F/\partial \theta^2) + k^2 F = 0\,. \tag{9}$$

The pressure condition at the surface leads to an expression for the surface elevation η of the form

$$\eta = (Aikc/g)\, e^{ikct} (\cosh kh)\, F(r, \theta)\ . \tag{10}$$

In the typical case of an uninterrupted train of waves traveling in the direction $\theta = \theta_0$,

$$F(r, \theta) = e^{-ikr\cos(\theta_0 - \theta)} \tag{11}$$

and the resulting waves have the following properties:

$$\begin{aligned}
&\text{Length} = L = 2\pi/k\ ,\\
&\text{Period} = T = 2\pi/kc\ ,\\
&\text{Amplitude} = a = (Akc/g)\cosh kh\ ,\\
&\text{Velocity} = c = \sqrt{(g/k)\tanh kh}\ .
\end{aligned}$$

The solution for (9) for which the normal component of the fluid velocity is zero along the barrier and which reduces to (11) for large values of r when $\theta =$ 180° is

$$\begin{aligned}
F(r, \theta) = {}& (1/\sqrt{2})\, e^{i[(\pi/4) - kr\cos(\theta_0 - \theta)]} \int_{-\infty}^{u_1} e^{-i\pi v^2/2}\, dv \\
&+ (1/\sqrt{2})\, e^{i[(\pi/4) - kr\cos(\theta_0 + \theta)]} \int_{-\infty}^{u_2} e^{-i\pi v^2/2}\, dv\ ,
\end{aligned} \tag{12}$$

where

$$u_1 = -\sqrt{4kr/\pi}\,\sin\,[(\theta_0 - \theta)/2] \quad \text{and} \quad u_2 = -\sqrt{4kr/\pi}\,\sin\,[(\theta_0 + \theta)/2]\ . \tag{13}$$

If a diffraction factor, K', represents the ratio of the heights of the diffracted wave and the incident wave, we have, using (10) and (11),

$$K' = |F(r, \theta)|\ , \tag{14}$$

where the symbol $|F(r, \theta)|$ denotes the *modulus* of the complex function $F(r, \theta)$. At sufficiently great distances beyond the barrier, the second term on the right-hand side of (12) can be neglected in comparison with the first term in computing $F(r, \theta)$. Then, after converting back to rectangular coördinates (fig. 7), we have

$$K' = |(1/\sqrt{2}) \int_{-\infty}^{u_1} e^{-i\pi v^2/2}\, dv| = |f(u_1)|\ , \tag{15}$$

where

$$u_1^2 = (4/L)\,(\sqrt{x^2 + y^2} - y)\ . \tag{16}$$

Values of K' in terms of u_1 are given graphically in figure 8.

The tabulated values of K' (table 4) indicate that relatively little wave energy is spread into the wave lee of a barrier through the process of diffraction.

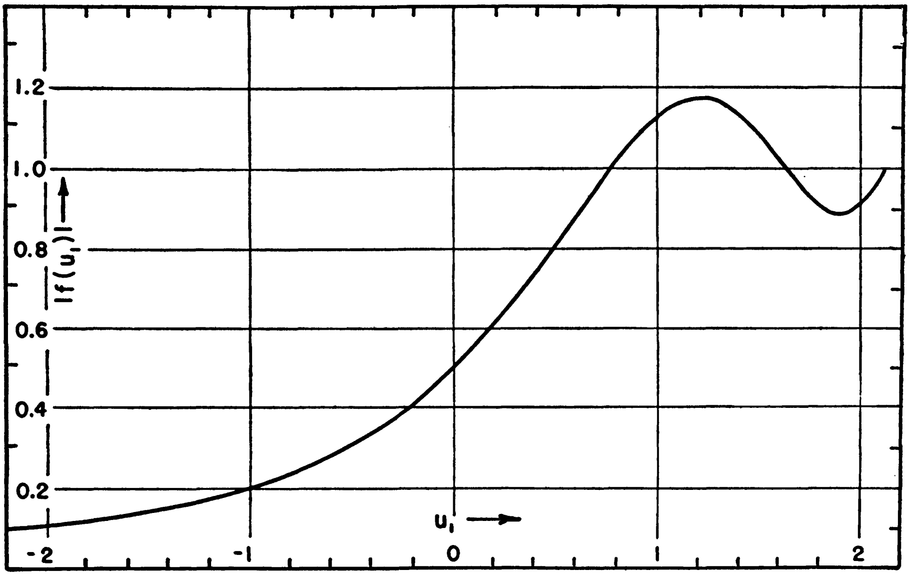

Fig. 8. Values of $K' = |f(u_1)|$ as a function of u_1.

TABLE 4

VALUES OF K' AT A DISTANCE OF 40 MILES TO THE LEE OF A SEMI-INFINITE BARRIER (I.E., y = 40 NAUTICAL MILES)

Distance, x, from edge of lee (in nautical miles)	Period T_0 (in seconds)			
	5	9	13	17
0	0.50	0.50	0.50	0.50
1	0.14	0.22	0.28	0.32
2	<0.10	0.13	0.18	0.22

VARIABILITY IN DIRECTION OF WAVE TRAVEL

The extent of the wave lee beyond an island depends largely upon the variability in direction of travel of the incident waves. The size of the sheltered region diminishes as the variability in wave direction increases. Such variability in wave direction can be traced ultimately to variability in the generating wind. Although it is true that the dominant waves run with the mean wind, careful observation of a generating area shows a complex pattern of short-crested waves with waves of appreciable height traveling not only in the mean wind direction, but also in other directions.

Stevenson (1886, pp. 32–33) noted the "partial action of the wind" and mentions that "during the continuance of a gale they [the waves] assume a very irregular appearance, and defy all attempts to trace any individual undulation for a long distance." Cornish (1912, pp. 132–133) writes on the irregularity of waves:

Wind is never really steady. Not only is it always more or less gusty, but it is always veering—i.e., changing its direction. Apart altogether from the progressive variation in the general direction of the wind which is characteristic of a cyclonic system, there is a rapid veering about a mean position, even in the Trades. The amount of this veering is sufficient to exercise an important effect upon the character of the waves and the appearance they present.

It results in the formation of waves running simultaneously in slightly different directions, and thus, even in the region of the Trade Winds, the open sea does not present a series of parallel ridges, each one of uniform height, with a lateral extension many times greater than the distance from crest to crest.

Quantitative estimates of the variability in wave direction can be made by generalizing the relationship for wave growth developed by Sverdrup and Munk (1947), if it is assumed that the "wavelets" which are first formed by the action of the wind exhibit a variability in direction of travel by virtue of irregularities in the wind. These small waves grow as a result of energy transferred from the wind by tangential stress and normal pressure even if the wind direction is inclined at an angle to the direction of advance of a particular crest. The variability in wave direction is retained as the waves leave the generating area and travel forth as swell.

TABLE 5

VALUES OF H_θ/H FOR $\theta = 30°$ AND $\theta = 45°$

Wind velocity (in knots)	Wind duration (in hours)					
	12		24		36	
	H_{30}/H	H_{45}/H	H_{30}/H	H_{45}/H	H_{30}/H	H_{45}/H
15	0.82	0.58	0.80	0.55	0.79	0.54
25	0.84	0.60	0.81	0.57	0.80	0.56
35	0.86	0.63	0.82	0.58	0.81	0.57

Relationships for wave growth have been developed by the writer (1949) following the method of Sverdrup and Munk, except that when tangential stress is exerted by wind on wave, the stress component in the direction of wave advance is introduced, and, with normal pressure, the wind component in the direction of wave advance is used. The components introduce no difficulty in solving the energy budget equations. The resulting solutions make it possible to obtain the wave height, H_θ, and wave period, T_θ, of waves moving at an angle, θ, to the wind direction if the wind duration and fetch are known. Practical considerations indicate that the results should not be used for values of θ greater than 45°. Results are conveniently presented in terms of the ratios H_θ/H and T_θ/T, where H and T are the height and period respectively of the waves traveling with the wind ($\theta = 0°$). Assuming that duration time is the critical parameter rather than fetch, values of H_θ/H are given for representative wind velocities and duration times and for $\theta = 30°$ and $\theta = 45°$ (table 5). The values of T_θ/T are generally 0.95 or greater for $\theta = 30°$ and 0.90 or greater for $\theta = 45°$.

The relationships for growth of waves moving at an angle to the wind are subject to the same limitations as is the basic Sverdrup-Munk theory, particularly in that the mechanism of growth requires physical explanation. The main justification for the quantitative results outlined above is that they appear to be useful in explaining certain wave observations which are referred to in the next section, and they are consistent with forecasting experience. Such experience (Hydrographic Office, 1943) has demonstrated the need for considering that waves decaying from a fetch where wind direction is uniform reach not only points in line with the wind (fig. 9,A), but also those at angles up to 30° (fig. 9,B). The same values for height and period have been used regardless of angle. This practice is in reasonable agreement with the variability results discussed above, since, if θ is less than 30°, the value of H_θ/H is generally greater than 0.80, and T_θ/T is greater than 0.95. The general accuracy in verifying wave forecasts cannot be considered to exceed 80 per cent.

The results discussed indicate that, within a fetch, waves may be expected to grow, even though they be moving at angles as great as 45° with reference to the mean wind direction. The height of such waves will be at least 50 per cent of the height of waves moving with the wind, although the waves moving with the wind appear to be dominant. Variability in direction of swell can be determined by con-

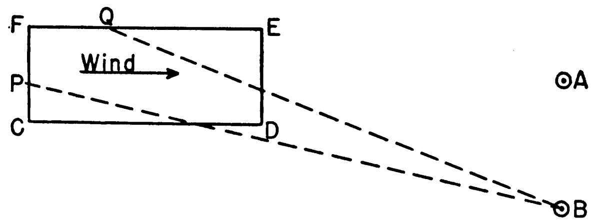

Fig. 9. Schematic representation of a fetch area, CDEF, from which waves arrive at points A and B.

sidering the different paths in the fetch which are directed toward the point where swell is to be considered. Lines PB and QB in figure 9 represent two such paths. Variability in direction of swell may be expected to decrease with increasing decay distance and decreasing width of fetch.

Short crestedness must be associated with variability in direction. The simplest patterns of short crestedness arise from interference between two wave trains with the same length and height but moving in different directions. The crest length, which is uniform in such a case, is $L/\sin \alpha$, where L is the wave length and α is the angular difference in the direction of travel. If waves are traveling simultaneously in several directions, the crest lengths are no longer uniform. However, it may be expected that the dominant crest length will be determined by the extremes in the directions of progress of waves whose heights do not differ excessively from those of the highest waves. Thus, an estimate of short crestedness may be obtained by computing the crest length which results from the interference of two trains moving in the extreme directions to be expected from the variability in direction as determined above. In table 6 estimates are made for typical values of wave length and variability in direction associated with wind waves and swell.

These results are in approximate agreement with visual estimates which the writer has made of crest length of wind waves and swell under various conditions. In particular, southerly swell of 18- to 20-second periods, breaking on the beaches of southern California, has been observed to have a crest length of more than one mile. The results also approach agreement with those noted by Bigelow and Edmondson (1947, pp. 46 and 66) who state:

> Also, it is only when the "seas" have been transformed into "swells" as described below that the individual crests extend far sidewise. In stormy weather, on the contrary, their lateral breadth

may not be more than three to five times as far as it is from one crest to the next, and sometimes no farther than it is from crest to crest, with their ends merging into the valleys in a wholly irregular pattern.

And with reference to swell:

Meantime, the individual wave crests, that are seldom more than a few times as broad transversely as the wave is long during windy weather, tend to expand farther and farther sidewise, while the narrowest of them seem also to be obliterated in some way, until finally a crest that was only 500 feet or so wide, while the gale was still blowing, may expand to a breadth of 1,500 to 2,000 feet or even more. We have ourselves observed swells that were well over one-half mile wide just before they broke upon the shore.

TABLE 6

SHORT CRESTEDNESS AS ESTIMATED FROM TYPICAL VALUES OF WAVE LENGTH AND VARIABILITY IN DIRECTION FOR WIND WAVES (PERIOD 5 SECONDS) AND SWELL (PERIODS 10, 15, 20 SECONDS)

Wave period, T (seconds)	Wave length, L (feet)	Variability in direction (degrees)	α (degrees)	Crest length $= L/\sin\alpha$ (feet)
5	130	±30	60	$1.2\,L = 160$
10	510	±15	30	$2.0\,L = 1000$
15	1200	±10	20	$2.9\,L = 3500$
20	2000	± 5	10	$5.8\,L = 12000$

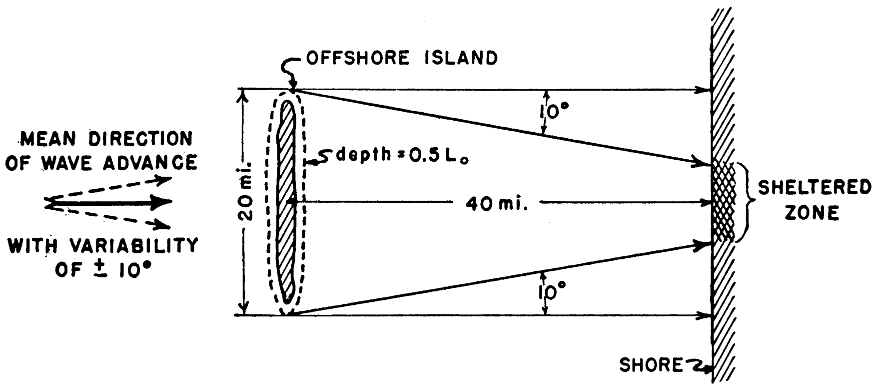

Fig. 10. Schematic example of effect of ±10° variability in direction on shadow zone beyond elongate offshore island.

The results on variability in direction are, therefore, consistent with the observed fact that the crest length increases greatly during the evolution of wind waves into swell.

Although waves moving at angles to the wind are lower, their effect is important when waves are interrupted by an island. If the variability in direction is ±10° for waves meeting an island 20 miles long and 40 miles offshore, the sheltered zone along the shore which results from the island barrier is significantly reduced, as shown in figure 10.

RECENT OBSERVATIONS IN THE LEE OF ISLANDS

Sicily and San Clemente

Wave observations made at Scoglitti, Sicily (fig. 11), and immediately south of Pyramid Head, San Clemente (fig. 12, station B) confirm the importance of variability in direction in determining the wave pattern in the lee of these two islands. The observations are discussed elsewhere (Arthur, 1949). It is sufficient to remark here that bottom refraction and diffraction, on the basis of results outlined above, cannot alone account for the wave heights observed in the lee of these islands, nor

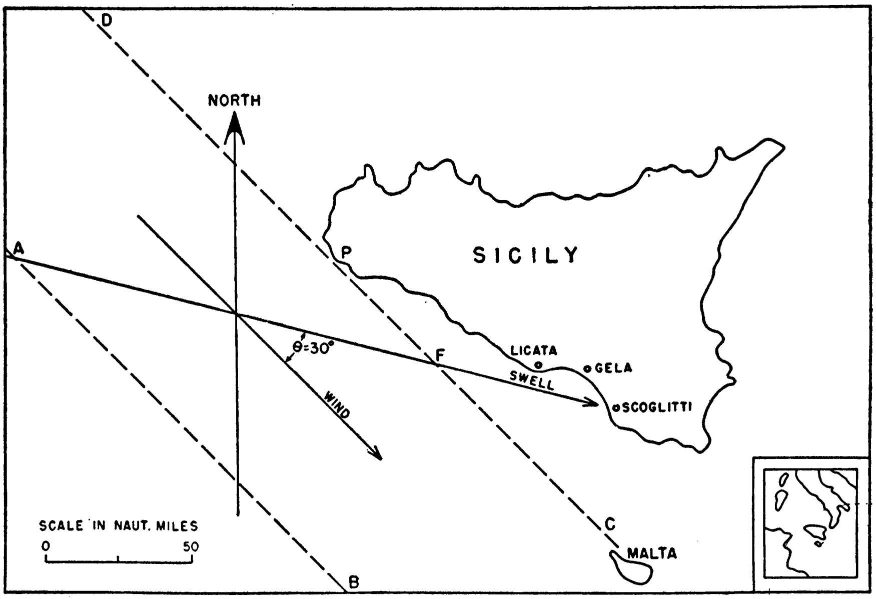

Fig. 11. Origin of swell at the Scoglitti beachhead during the invasion of Sicily.

does it appear possible to furnish an explanation solely on the basis of refraction by currents around the islands as they are now known.

The results are explicable on the basis of variability in direction. In the example from Sicily, waves moving in the fetch at an angle of 30° to the mean wind would arrive at Scoglitti with a computed height of 11 feet. Observers recorded heights of between 10 and 12 feet. At San Clemente, waves moving from the fetch at an angle of 45° to the mean wind would arrive at the lee station with a computed height of 2.3 feet. This has been verified by heights observed at the station.

Southerly Swell from Port Hueneme to Oceanside

Wave observations made along the southern California coast indicate the degree of effectiveness of the offshore islands in sheltering the coast from waves. Saville (1948) has examined photographs of southerly swell from an aerial sortie made along the southern California coast from Port Hueneme to Oceanside on February 4, 1948. This entire section of coastline was photographed with the exception of the stretch from Malaga Point to Long Beach. Saville concludes from the photographs:

The resulting pictures show the arrival of a southerly swell (from about S by E) along parts of the coast line. From Port Hueneme ESE along the coast to Trancas Canyon, just west of Pt. Dume, the southerly swell is quite evident. East of Pt. Dume, however, the southerly swell is no longer evident, cut off presumably by Santa Catalina and San Clemente Island to the south. The swell reappears just SE of Santa Monica and disappears again south of Redondo Beach, in Malaga Cove, cut off by Palos Verdes Point. No pictures were taken between Malaga Cove and Long Beach, but the southerly swell is very obvious again south from Long Beach as far as Oceanside, where the sortie ended.

A study of the complete set of photographs indicates that Saville's conclusions are valid, with the possible exception of those for the section between Point Dume and Santa Monica. This section of the coast is either rocky or possesses beaches of

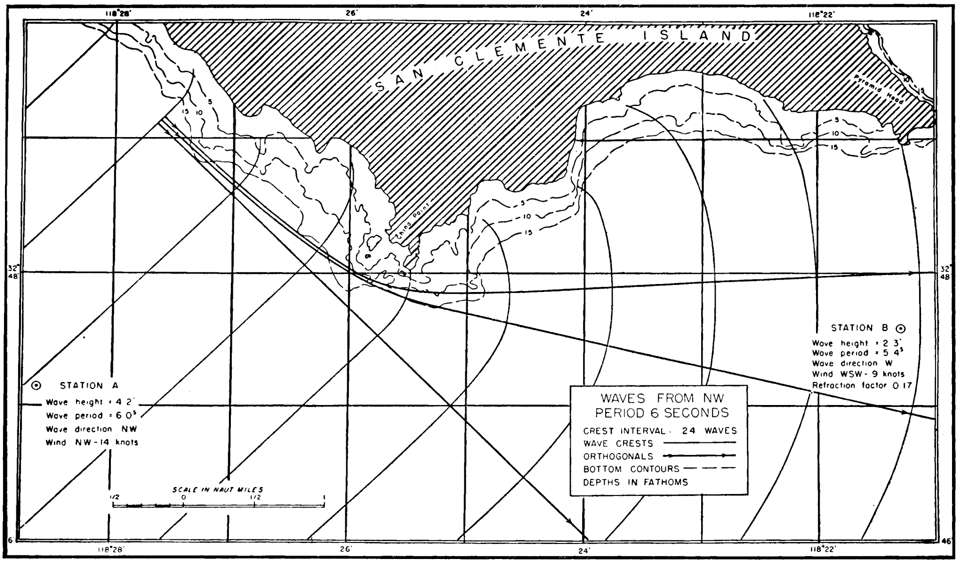

Fig. 12. Refraction diagram for China Point region, San Clemente Island, with wind-wave observations for both a sheltered and unsheltered station.

steep slope, and the southerly swell, therefore, breaks very near shore rather than at some distance seaward. The situation is different at several places, particularly Malibu Point where there is a strong convergence. However, the photograph of Malibu Point very definitely shows the southerly swell, and although other photographs along the steep beaches manifest only a shore break, this occurs simultaneously over a stretch of shore in a fashion that is characteristic only of the long-crested southerly swell. The conclusion is, then, that the islands of San Clemente and Santa Catalina do not prevent the appearance of the southerly swell along the photographed sections of the coast.

Southerly Swell from Oceanside to Point Dume

Observations of southerly swell on beaches between Oceanside and Point Dume were made during the period July 30–August 1, 1949, and on September 15, 1949. The observations (fig. 13) consisted of visual estimates of significant breaker heights and determinations of significant breaker period made by timing the interval be-

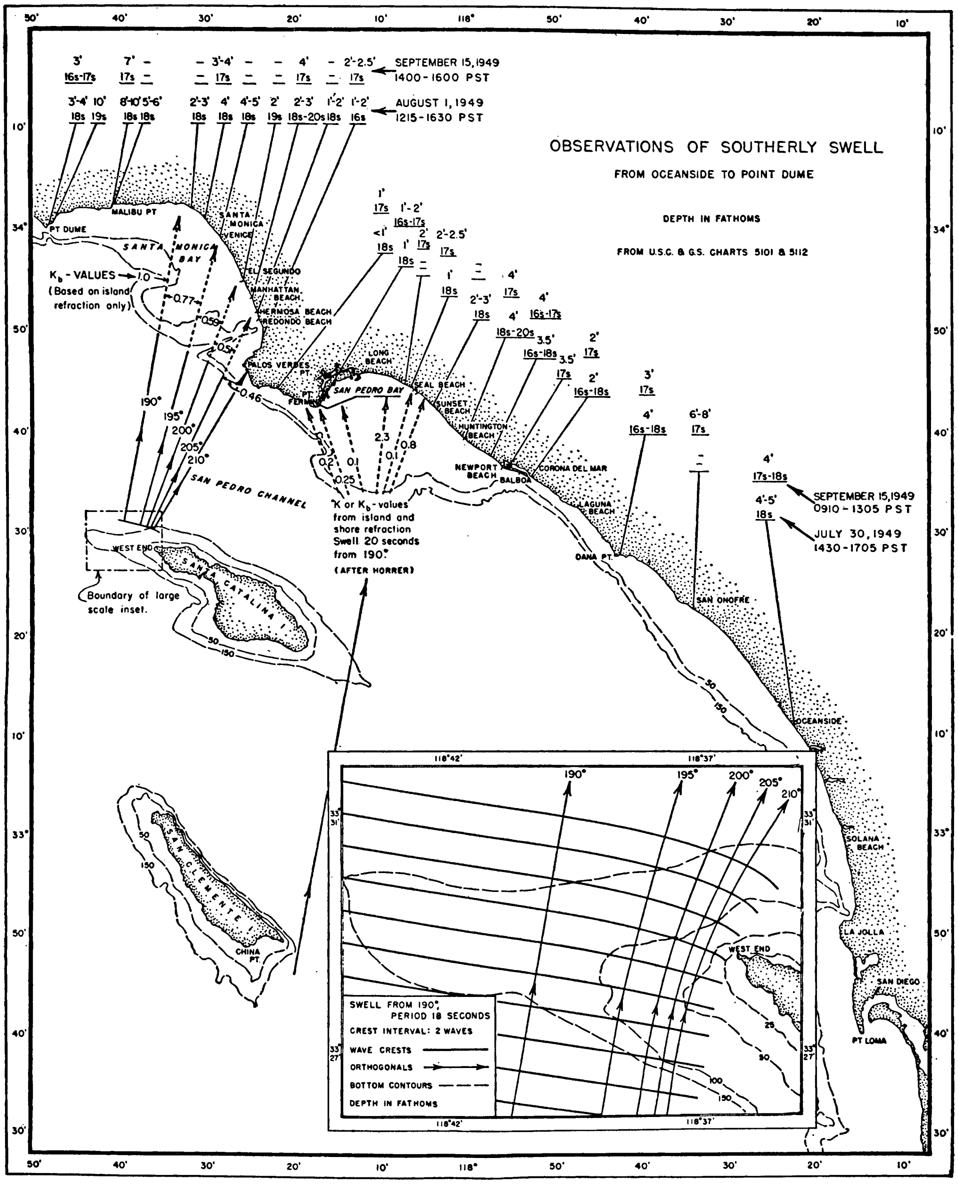

Fig. 13. Observations of southerly swell between Oceanside and Point Dume and refraction effect of San Clemente and Santa Catalina islands.

tween successive crests. The southerly swell, on the occasion of both sets of observations, was of greater height and longer period than experience has indicated to be normal. It was dominant over any shorter-period westerly waves on virtually all beaches. The southerly swell was characterized by long, even crests and an approach from south or near-south as indicated by the breaker angle (pl. 1). Groups of five to ten higher waves were separated by intervals of several minutes during which the wave heights were much lower.

Since southerly swell is long crested and of long period, its identification is relatively simple, and the effects of the offshore islands of San Clemente and Santa Catalina on the wave train are ascertainable. The direction of wave approach is important in this respect and has been determined for the swell of August 1, 1949 from aerial photographs of the breakers at Camp Pendleton, Oceanside. The bottom contours off the beach in this area are approximately straight and parallel; measurement of the angle between a given crest and the bottom contour at a given depth allows the direction of approach in deep water to be determined (Hydrographic Office, 1944). A number of determinations of wave direction have been made from aerial photographs taken just north of the Santa Marguerita River and off Aliso Canyon where recent soundings were available. The results indicate that the waves approach from 190°. Individual determinations show deviations of as much as 11°, part of which may be real as a result of variability in direction, but part of which may arise from the uncertainty in the wave length. The over-all uncertainty and variation in direction is estimated to be $\pm$ 10°. However, the direction of 190° is adopted in order to provide a definite basis for comparison of observations with expected effects of the islands. No aerial photographs are available for indicating the conditions prevailing on September 15, 1949, but visual estimates of the breaker angles indicate that the direction of approach was slightly west of 190°.

With swell approaching from 190°, the islands cast a wave shadow on the coast from Santa Monica to Sunset Beach. The observations show that the shadow is not complete; swell is not entirely eliminated from this stretch of coast, but rather there is a general decrease of breaker height from either edge of the shadow toward the center, which lies between Palos Verdes Point and Point Fermin. This decrease is not altogether regular, there being important effects of the bottom topography immediately offshore, but the over-all tendency of the change in breaker height is toward a decrease. The changes in breaker height are explicable on the basis of refraction and variability in direction.

The effect of refraction around West End, Santa Catalina Island, is indicated by values of the refraction factor, K_b, along the shore extending from Santa Monica to Palos Verdes Point (fig. 13). These values do not include the refraction effect of the bottom topography immediately offshore. The effect of refraction around the southeast extremities of San Clemente and Santa Catalina islands is given by values of K_b applying to breakers along the shore from Sunset Beach to Point Fermin and by values of K applying to waves meeting the San Pedro Bay breakwater. This second set of values of the refraction factor, which includes the effect of the offshore bottom topography, is given by Horrer (1948), who reproduces the refraction diagrams on which they are based. Horrer has assumed a wave period of 20 seconds, but the pattern would be little altered by an alternative period of 18 seconds. The magnitude and trend of the refraction factors are in agreement with the observations of breaker heights.

Since swell from a very distant generating area has a relatively small variability in direction, the southerly swell may be expected to have the least directional variability of any swell reaching the southern California coast. Wiegel and Kimberley (1950) find, by using the observed decrease of period with time of trains of southerly swell at Camp Pendleton and the storm-tracking methods developed by Munk (1947), that the decay distance is approximately 5,000 nautical miles. This places the generating area latitudinally in the "Roaring Forties" and the "Furious Fifties" of the South Pacific Ocean, as anticipated. If the swell is assumed to travel a great-circle route, then a fetch width of 600 nautical miles, which is relatively large, is associated with a variability in direction of $\pm 5°$ after a decay of 5,000 nautical miles. The effect of such a variability on the wave pattern beyond San Clemente and Santa Catalina islands is not great, since Catalina Island is only 20 nautical miles from the coast in the direction of wave advance. The effect is roughly that of rotating the refraction patterns by 5° about the extremities of Santa Catalina Island, and the alteration is still in agreement with observations. There is, therefore, a region beyond the islands into which wave energy penetrates as a result of refraction. Because of the long period of the southerly swell and the gentle slopes at the extremities of Santa Catalina Island, refraction is a relatively effective process in causing the penetration.

SUMMARY AND CONCLUSIONS

1. The penetration of wave energy into the shadow zone beyond an island is determined by estimating separately the effects of: (*a*) refraction by underwater topography, (*b*) refraction by currents, (*c*) diffraction, and (*d*) variability in direction of wave travel.

2. Refraction by underwater topography is an important factor if there is a zone of shallow water at the edges of the island. The distribution of refracted wave energy in the lee is shown to be critically dependent upon the type of bottom slope in the shallow-water zone.

3. Refraction of swell by the usual semipermanent currents around islands is not significant because of the low current velocities involved. Tidal currents around certain islands may, however, be sufficiently strong to exercise important refraction effects.

4. The effect of diffraction is obtained from theoretical considerations wherein waves meet a semi-infinite plane barrier. On this basis, relatively little wave energy is spread into the shadow zone of an island through the process of diffraction.

5. Variability in direction of wave travel is an important factor in determining the extent of the wave lee beyond an island. Variability in wave direction decreases as the width of the fetch decreases and the decay distance increases.

6. The factors discussed are adequate for explaining quantitatively the wave conditions observed in the lee of the Island of Sicily, San Clemente Island, and Santa Catalina Island.

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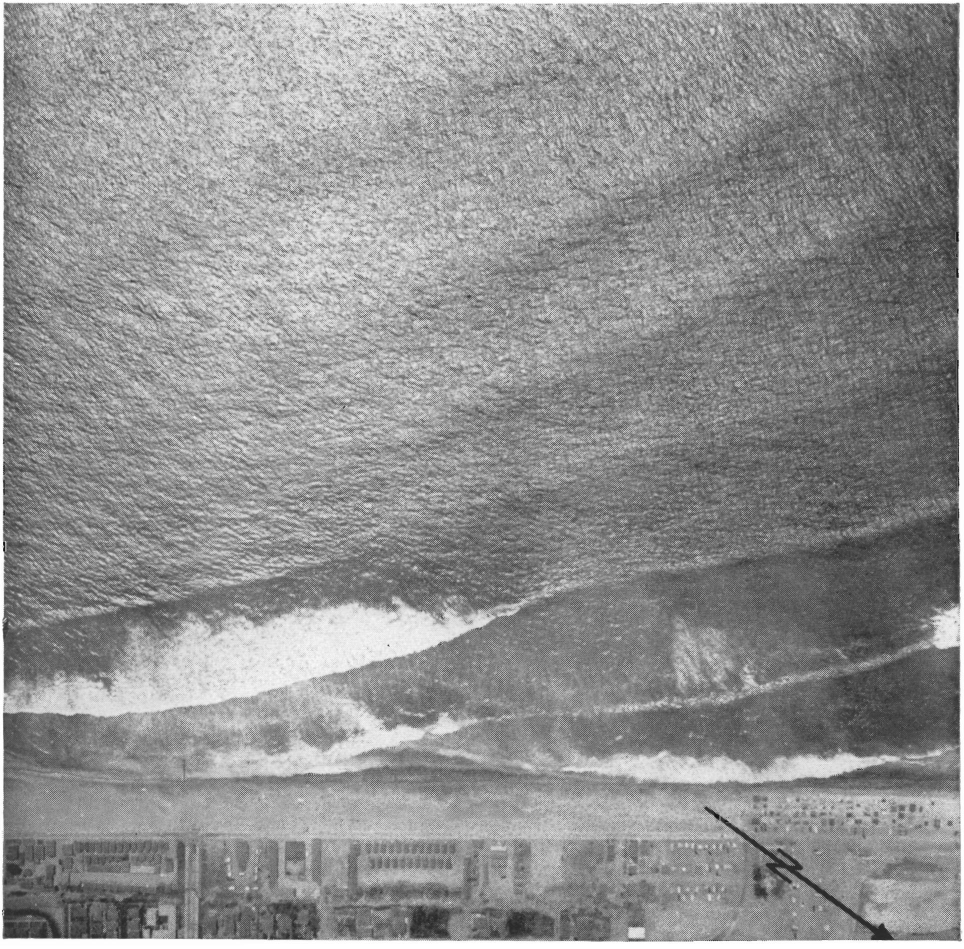

Aerial photograph of southerly swell on August 1, 1949, just north of Oceanside pier (U. S. Navy photo. Used through courtesy of the U. S. Marine Corps, Camp Pendleton, California, and the Department of Engineering, University of California, Berkeley).