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Truth & Knowledge in Logic & Mathematics

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Author

Sher, Gila

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Gila Sher

Abstract

In this paper I develop an account of truth and knowledge for logic and mathematic. The underlying methodology is a synthesis of holistic and foundational principles, logical and mathematical truth are based on indirect correspondence, and logical and mathematical knowledge is quasi-apriori (as opposed to being either empirical or purely-apriori).

Keywords: truth, knowledge, logic, mathematics, composite reference, composite correspondence, quasi-apriori, Benacerraf

Logic and mathematics are abstract disciplines par excellence. What is the nature of truth and knowledge in these disciplines? In this paper I investigate the possibility of a new approach to this question. The underlying idea is that knowledge qua knowledge, including logical and mathematical knowledge, has a dual grounding in mind and reality, and the standard of truth applicable to all knowledge is a correspondence standard. This applies to logic and mathematics as much as to other disciplines; i.e., logical and mathematical truth are based on correspondence. But the view that logical and mathematical truth are (i) based on correspondence and (ii) require a grounding in reality demands a change in the common conception of both correspondence and epistemic grounding.

Before turning to this task, however, I have to address the questions (a) Do logic and mathematics, as highly abstract disciplines, require a grounding in reality (or do humans need a logic and a mathematics that are grounded in reality)?, and (b) Is there is anything in reality for logic and mathematics to correspond to? After giving positive answers to these questions, I will turn to the traditional methodology of grounding branches of knowledge in reality ("foundationalism") and the traditional theory of truth associated with correspondence ("copy", "mirror", or "isomorphism" theory), and I will propose an alternative. A foundational methodology for logic and mathematics, I will argue, ought to be holistic rather than foundationalist; but being holistic does not mean being coherentist. On the contrary. Grounding highly abstract disciplines in reality requires a large array of interconnected cognitive resources and a wide network of interconnected routes from mind to reality, and a holistic conception of cognition is better suited to explaining how these requirements are satisfied. I will call the use of holistic methods to pursue the foundational project "foundational holism".

As for correspondence, traditionally correspondence is viewed as based on a single and simple principle, one that assumes the same form in all fields, be they largely observational or highly abstract. However, given the large array of fields and the substantial differences between them (for example, differences in the aspects of reality they study), this view is quite unreasonable. Likewise, the idea that the correspondence between true statements (true theories) and reality is always simple or direct is unreasonable. Instead, I will suggest that mathematical correspondence is "composite" (indirect) and that logical correspondence is closely related to it. I will describe a template of composite correspondence that can be used for mathematics and an associated template for logic, and I will show how these and other elements of the present account equip it to solve some outstanding problems in the philosophy of logic and mathematics. Due to limitations of space I will be able to offer only a general outline and a few examples.

1 Truth in mathematics

Our first question is: If truth in mathematics is based on correspondence, what is mathematical correspondence correspondence with? My proposed answer is that mathematical correspondence is correspondence with the formal layer (aspect, dimension, structure) of reality, the layer of formal features, or formal "behavior", of objects. This answer, however, raises two new questions, the critical question "Does reality have a formal layer or structure?" and the clarificatory question "What do we mean by 'formal'"? Starting with a somewhat

vague characterization of the formal as sensitive to the *patterns* of objects having properties and standing in relations but not to the identity or type of objects involved (a more precise characterization will be given shortly), let us turn to the first, critical, question. To avoid a conflict with nominalists right at the beginning, let us think of the ontology of individuals as limited to observable individuals. The question, then, is: Do objects in the world have formal properties? My answer is positive: Individuals (0-level objects) have the formal property of self-identity; properties of individuals (1st-level objects) have cardinality properties, which are formal; relations of individuals (1st-level objects) have formal properties like reflexivity, symmetry, and transitively, and so on. As for a precise characterization of formality, I propose *invariance under isomorphisms* (Lindström, 1966) as capturing the informal idea with which we have started. To see what is meant by "invariance under isomorphisms", let us define for each property a class of argument-structures, i.e., a class of pairs consisting of a universe and an argument of the given property in that universe. An argument-structure of self-identity, for example, is a pair $\langle A, a\rangle$ where A is a non-empty set of individuals and a is a member of A; an argument-structure of a cardinality property is a pair, $\langle A, B\rangle$, where A is as above and B is a subset of A, and an argument-structure of a property like "is symmetrical" is a pair $\langle A, R\rangle$, where R is a binary relation on A. These properties are invariant under isomorphisms in the sense that they are preserved under isomorphisms of their argument-structures: (i) a is self-identical, $\langle A, a\rangle \cong \langle A', a'\rangle \Longrightarrow a'$ is self-identical, (ii) Exactly one individual is B (or all but two individuals are B), $\langle A, B\rangle \cong \langle A', B'\rangle \Longrightarrow$ Exactly one individual is B' (all but two individuals are B'), and (iii) R is symmetric, $\langle A, R\rangle \cong \langle A', R'\rangle \Longrightarrow R'$ is symmetric.

Now, if formal properties are real (in the sense that objects in the world—actual and potential—have, or could have, such properties), then it is reasonable to surmise that such properties are governed by laws. The question arises: Which discipline, if any, studies these laws? Our answer is that mathematics does (or some of its parts do). To see the force of this answer, suppose mathematics does not study the laws governing properties of objects in the world. This would be quite strange. It would be quite strange if arithmetic and set theory studied the laws governing *imaginary* cardinalities but *real* cardinalities were governed by altogether different laws. This does not mean that the only thing that mathematics does is study the formal laws governing the behavior of objects and structures of objects in the world, but that one important thing it does is study these laws.

Here, however, we come upon a puzzle: It appears that the formal features of objects in the world are for the most part of a relatively high level (2nd-level and above), but mathematical theories are for the most part 1st-order theories. Why do arithmetic and set theory study cardinalities as *individuals* (the numbers zero, one, two, ...) if they are 2nd-level properties (ZERO, ONE, TWO, ...)?

My answer is that there could be many reasons why people prefer to think of cardinalities as individuals and mathematicians prefer to study them by 1st-order theories. For example, it is quite possible that we, humans, work better with individuals and their properties than with properties and their properties. It is easier for us to figure out, and present in a systematic manner, the laws governing cardinalities when we think of cardinalities as individuals than as 2nd-level properties. The main issue is: Is it possible to account for 2nd-level phenomena *accurately* and *systematically* by 1st-order theories?

The answer to this question is quite clearly positive. Mathematics may study cardinalities indirectly yet accurately and systematically. Before we further elaborate on this answer, let us reflect, more generally, on some of the factors involved in humans' ability (or inability) to reach the world cognitively and develop a standard of truth for theories (statements, thoughts, beliefs) about it. Four such factors are:

(a) The complexity of the world.

(b) Our desire to know and understand it in all its complexity.

(c) The mind's cognitive limitations.

(d) Its intricate cognitive capacities.

These factors introduce some tensions into the two projects of theorizing (thinking) about the world and constructing a truth standard for our theories (thoughts) about it. But they also point to a solution. In particular, they suggest that we approach these projects with the following principles as guidelines:

(a) Seek a fruitful balance between unity and diversity. (Dyson, 1988, p. 47)

(b) Access reality holistically.

The first principle suggests that instead of either radical monism or radical pluralism with respect to truth, we adopt a family of diverse yet unified standards of truth. E.g., instead of either a single and simple correspondence standard of truth or radically divergent standards of truth for different fields (correspondence for physics, coherence for mathematics), we allow a *family* of correspondence standards that can take into account the special needs of different fields. The second principle says that in accessing reality we are free to use all the resources available to us (including those involving non-vicious circularity), and that using these resources we are free to forge multiple routes to reality, including multiple correspondence routes. Correspondence, on this conception, is not a "mirror" or a "copy" or even an "isomorphism" relation between language and reality, but rather a family of interrelated connections between the two. These connections enable us to say "of what is that it is and of what is not that it is not" in an accurate, if at times circuitous (indirect, composite, multi-staged), ways. As a standard, correspondence sets substantial yet flexible conditions on our discourse, requiring an appropriate truth-conferring connection between our discourse and reality, given our resources, reality, and what aspect(s) of reality we are speaking about. (The exact content of these conditions may change in the course of history, reflecting changes in our resources, our understanding of reality, and what aspects of reality we are interested in.)

Turning back to mathematics, our question was whether it is possible to accurately account for the formal aspect of reality, which is largely higher-level, by 1st-order mathematical theories. It should now be clear why a positive answer to this question is, in principle, justified. One way in which we can account for 2nd-level phenomena through 1st- and 0-level thoughts is by introducing an intermediate layer of posits into our picture of reality. For example, we can create a posited layer of mathematical individuals and their properties (levels 0 and 1) that systematically represents 2nd-level cardinality properties and their (3rd-level) properties. Mathematical correspondence will then be circuitous yet accurate. Starting with *reference*, we can display the difference between simple and "composite" reference as follows:

Simple Reference		
Lang.:	Ind. Consts^0	Predicates^1
	$\downarrow$	$\downarrow$
World:	Individuals^0	Properties^1

Composite Reference		
Lang.:	Ind. Consts^0	Predicates^1
	$\downarrow$	$\downarrow$
Posit:	Individuals^0	Properties^1
	$\downarrow$	$\downarrow$
World:	Properties^2	Properties^3

Turning to correspondence, consider the 1st-order arithmetic truths "$2 + 7 = 9$" and "$(\forall m)(\forall n)(m + n = n + m)$". We can describe their composite correspondence-conditions as follows:

Composite Mathematical Correspondence

1st-Order Language: "$2 + 7 = 9$" is true

iff

Posits: $+(2, 7) = 9$

iff

Reality: DISJOINT-UNION(TWO, SEVEN) = NINE

[*iff*

$(\forall P1)(\forall P2)((\text{TWO}(P1)\ \&\ \text{SEVEN}(P2)\ \&\ P1 \cap P2 = \emptyset] \supset \text{NINE}(P1 \cup P2))]$

1st-Order Language: "$(\forall m)(\forall n)(m + n = n + m)$" is true

iff

Posits: The operation of addition is symmetric

iff

Reality: DISJOINT-UNION IS SYMMETRIC.

Three advantages of the composite correspondence principle of mathematical truth together with the other alethic and epistemic principles delineated above are:

(a) They enable us to provide a substantive account of mathematical truth.

(b) They enable us to integrate this account in a unified correspondence account of truth.

(c) They lead to solutions to outstanding problems in the philosophy of mathematics (the large ontology problem, the identity problem, and the applications problem), as well as to progress toward solutions to other problems (the cognitive access problem and the "mathematics as algebra" problem). This will be discussed in Section 4 below.

2 Truth in logic

If truth is correspondence throughout, then logical truth, to the extent that it is a genuine type of truth, is also based on correspondence. Similarly, the other semantic properties (relations) associated with logical truth—logical consequence, logical consistency, etc.—are based on principles related to correspondence. This means that logic has as much to do with the world as it does with the mind (language, concepts, etc.), and the first question we are facing is, therefore: What does logic have to do with the world?

My answer is:

(a) Logic has to "work" in the world. This point was already noted by Russell. Speaking about logical laws (the "*law of identity*", the "*law of contradiction*", the "*law of excluded middle*"), Russell says that "what is important [with respect to these laws] is not the fact that we think in accordance with [them], but the fact that **things behave in accordance with them**". It is this that is responsible for "the fact that when we think in accordance with them we think *truly*". (Russell, 1959, p. 34, my bolding)

(b) There have been cases of *factual* error in logical theories, and some say there still are. The most dramatic example of a discovery of a factual error in a logical theory is the discovery of an error in Frege's logic by Russell. Frege's logic, Russell discovered, affirms the existence of a set that does not, and cannot, exist. Moreover, advocates of so-called nonclassical logics (fuzzy logic, quantum logic, etc.) claim that classical logic fails to work in the world and this is naturally understood as due to errors in understanding the formal structure of reality.

(c) Logic is both constrained and enabled by reality through its inherent connection with truth. (This is a theoretical rendition and expansion of a point noted briefly in the citation from

Russell above.) Consider the case of logical consequence, $\Sigma \models \sigma$, where the truth-conditions of both σ and the sentences in Σ are based on correspondence principles. Say, σ is true iff the situation $\mathfrak{C}_2$ is the case, and the sentences in Σ are true iff $\mathfrak{C}_1$ is the case. For σ to be a logical consequence of Σ, the truth of the sentences in Σ (assuming they are all true) has to be transmitted to (or preserved by) σ with an especially strong modal force. But because the truth of σ and the sentences in Σ is a matter of whether $\mathfrak{C}_2$ and $\mathfrak{C}_1$ are the case, $\mathfrak{C}_1$ must be connected to $\mathfrak{C}_2$ with an especially strong modal force as well. The relation between logical consequence and reality is, thus, as in the following diagram:

$$\begin{array}{ll} \textit{Logic:} & \Sigma \models \sigma \\ & \Updownarrow \\ \textit{Truth:} & T(\Sigma) \Rightarrow T(\sigma) \\ & \Updownarrow \\ \textit{World:} & \mathfrak{C}_1 \rightarrow \mathfrak{C}_2, \end{array}$$

where $\Rightarrow$ and $\rightarrow$ have an especially strong modal force.

This relation means that the world both constrains and gives rise to logical consequences, or that the world can both falsify and justify logical-consequence claims. Starting with falsification, the point is that we cannot choose a logical theory without paying attention to the world. In particular, if our chosen logical theory says that $\Sigma \models \sigma$ but in the world one of the following three situations is the case:

(a) $\mathfrak{C}_1$ but *not* $\mathfrak{C}_2$,

(b) $\mathfrak{C}_1$ and $\mathfrak{C}_2$, but no law connects $\mathfrak{C}_1$ to $\mathfrak{C}_2$,

(c) $\mathfrak{C}_1$, $\mathfrak{C}_2$, and some law connects $\mathfrak{C}_1$ to $\mathfrak{C}_2$, but this law is not sufficiently strong for logical consequence,

then our chosen theory is wrong: σ is *not* a logical consequence of Σ.

One way in which the world can give rise to logical consequences or support (provide positive evidence) for logical-consequence claims is:

(d) $\mathfrak{C}_1$, $\mathfrak{C}_2$, and $\mathfrak{C}_1$ is connected to $\mathfrak{C}_2$ by a law whose modal force is sufficient for logical consequence.

The questions arise: What modal force is sufficient for logical consequence? Are there laws possessing such modal force? My proposal is that the modal force sufficient for logic is that of formal laws (laws that hold in all formally possible states of affairs), and that there are such laws, namely those studied (directly or indirectly) by mathematical theories like arithmetic and set theory. We have already seen that objects in the world have formal properties and that it is reasonable to surmise that these properties are governed by laws. We have also seen that arithmetic and set theory study these laws. An example of such a law and the logical claim it grounds is:

$$\begin{array}{ll} \textit{World:} & \text{UNIVERSALITY OF } \mathcal{A} \rightarrow \text{UNIVERSALITY OF } \mathcal{A} \cup \mathcal{B} \\ & \Downarrow \\ \textit{Truth:} & T[(\forall x)Ax] \Rightarrow T[(\forall x)(Ax \vee Bx)] \\ & \Downarrow \\ \textit{Logic:} & (\forall x)Ax \models (\forall x)(Ax \vee Bx), \end{array}$$

where $\mathcal{A}$ and $\mathcal{B}$ are the properties denoted by "A" and "B", respectively. The logical form of sentences standing in the relation of logical consequence is determines by their (linguistic) formal parameters ("$\forall$" and "$\vee$" in the example above), and these parameters are formal in virtue of representing the (objectual) formal parameters of the situations corresponding to these sentences (UNIVERSALITY and $\cup$).

Our account unites logic with mathematics in a way that is in some respects similar to *logicism*. Like logicism, it unifies the two disciplines, thereby replacing two foundational tasks—the task of grounding logic and the task of grounding mathematics—by one. However, unlike logicism, it provides a grounding not just for one of these disciplines (logicism grounds mathematics in logic leaving logic itself ungrounded), but for both: both logic and mathematics are grounded in the formal. On this conception, the relationship between logic and mathematics is a back and forth relationship: we use our mathematical knowledge (knowledge of the formal) to construct our logical system, and we use our logical system as a framework for constructing mathematical theories. (Think of the mutual relationships between 1st-order logic and set theory, each having played an important role in the development of the other.)

Under this conception, logical constants are referring constants and logical truths are based on correspondence with formal laws governing reality. Logical reference and correspondence have two dimensions: a simple, 1-step dimension, in which they are directly connected to certain features of reality, and a composite, 3-step dimension, in which they are connected to the same features of reality indirectly through mathematical reference and correspondence. The former dimension is *ontological*, the latter—*epistemic*. Logical laws are grounded in largely *higher-level formal laws*, but these laws are known to us through *lower-level mathematical theories* and *lower-level posits*. An example of these two dimensions is:

Simple Logical Reference

Logical language: "$\exists$" (2nd-order predicate serving as 1st-order quantifier)

$\Downarrow$

World: (2nd-level property of) NON-EMPTINESS

Simple Logical Correspondence

Logical language: "$\sim(\exists x)(Px \;\&\; \sim Px)$"

$\Updownarrow$

World: EMPTY[$\mathcal{P} \cap$ COMPLEMENT($\mathcal{P}$) IN $\mathcal{A}$], where $\mathcal{A}$ is a given universe

But to *know* this and more complex formal laws governing the world, logic turns to mathematics, which establishes them, often indirectly, as 1st-level laws. Therefore, in the order of knowledge, logical reference and correspondence may very well be indirect:

Composite (3-layered) Logical Reference

Logical language: "$\exists$"

$\Downarrow$

Mathematical language: "> 0" / "$\neq \emptyset$"

$\Downarrow$

Posit: being larger than (the individual) 0 / being different from (the individual) $\emptyset$

$\Downarrow$

World: NON-EMPTINESS (of properties of individuals)

Composite (3-layered) Correspondence

Logical language: "$\sim(\exists x)(Px \;\&\; \sim Px)$"

$\Updownarrow$

Mathematical language: "$p \cap \text{complement}(p) = \emptyset$"

$\Updownarrow$

Posit: Empty[intersection(p, complement(p) in A)]

$\Updownarrow$

World: EMPTY[$\mathcal{P} \cap$ COMPLEMENT($\mathcal{P}$) in $\mathcal{A}$]

3 Knowledge in logic & mathematics

Logical and mathematical knowledge, on the present proposal, has several distinctive features:

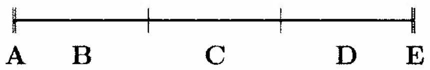
(a) It is grounded in reality, yet it is holistic.

(b) It is grounded in the same reality that scientific knowledge is grounded in, yet in different features of this reality. It follows that there are no two separate realities, a Platonic reality that grounds the abstract sciences (including logic and mathematics) and a mundane reality that grounds the empirical sciences, but there is a single reality with diverse features that grounds both.

(c) The grounding of logical and mathematical knowledge in reality can be either direct or indirect (with the possibility of various combinations of the two).

(d) Logical and mathematical knowledge is grounded not just in reality but also in the mind. Its grounding in the mind is both passive and active. On the one hand it is both constrained and enabled by our biological, psychological, cultural and other resources; on the other hand we are always free to devise new ways of expanding our knowledge and improving its quality: new tests, new forms of evidence, new concepts, etc.

(e) Logical and mathematical knowledge is neither purely apriori nor empirical in the sense of relying primarily on sensory perception. Instead, it is *quasi apriori*. We can represent the contrast between our conception and the traditional conception by a line with five regions:



A is the (almost infinitesimally) narrow space of absolutely sensory cognition, i.e., cognition for which not even the slightest influence of reason is permitted; **E** is the (almost infinitesimally) narrow space of absolutely reason-based cognition, i.e., cognition for which not even the slightest influence of sensory experience is permitted; and **B**–**D** are the large intervals of cognitions that can in principle be based both on sensory perception and on reason in various combinations and in a gradually increasing ratio of the latter to the former. In traditional epistemology the *apriori-aposteriori* division has on one side the (almost infinitesimally) narrow region **E** and on the other side essentially the whole line of knowledge, from **A** to (and including) **D**. This is a very uneven division and, moreover, it is a division with sharp boundaries. Logical and mathematical knowledge fall on the (almost infinitesimally) narrow side of this division—it is *purely apriori*, i.e., strictly limited to **E**. As such its resources are very limited in the sense that no combination of experience and reason is available to it. In contrast, aposteriori, or *empirical* knowledge has a wide array of resources, being entitled (in principle) to *all* combinations of sense-based and reason-based cognitive elements. Given this uneven distribution of cognitive resources between logical and mathematical knowledge on the one side and scientific knowledge on the other, it is not surprising that the task of explaining logical and mathematical knowledge within traditional epistemology is especially difficult and has led philosophers to resort to extreme measures. Two such measures are (i) postulating a separate abstract (Platonic) reality and (ii) viewing logical/mathematical knowledge as (purely) conventional (thereby giving up on genuine truth in these fields).

On the present (foundational holistic) account, regions **A** and **E** are eliminated as epistemically significant regions. Human knowledge consists of three continuous (i.e., not sharply divided) regions: the region of largely observational knowledge—**B**; the region of more theoretical scientific knowledge—**C**; and the region of highly abstract knowledge—**D**. Logical and mathematical knowledge resides in **D**. Due to the special nature of its subject-matter, the *formal*, such knowledge is *quasi-apriori*. I.e., its use of cognitive resources is characterized

by a relatively high ratio of reason-based to sensory resources. But we do not rule out cases in which empirical considerations make a significant contribution to changes in mathematics and logic. Among other things, we allows that occasionally empirical discoveries point beyond themselves to phenomena that are highly abstract in nature.

4 Solution to outstanding problems in the philosophy of logic and mathematics

Our account offers a solution to a number of outstanding problems in the philosophy of logic and mathematics, and makes significant steps toward a solution to others.

4.1 Problems in the philosophy of mathematics

4.1.1 The large ontology problem

The problem is to explain the large ontology of mathematics. This is thought to be especially difficult if mathematics is true about the world since the number of individuals in the world appears to be much smaller than that required by mathematics.

On our account, the large ontology of mathematical individuals is an ontology of posits and as such need not be limited to the ontology of real individuals (whatever this is). As for the question, "Why does a theory of the formal need a very large posited ontology?", our answer is: Laws in general are counterfactual in scope, and as such hold for, and may be best formulated in terms of, a counterfactual ontology that, being counterfactual, might be larger than the "actual" ontology. Due to the especially high degree of invariance of formal properties, formal laws have an especially large counterfactual scope; hence they require an especially large posited ontology. For example: to express the laws of finite cardinalities in complete generality, we need an ontology on the order of the denumerable ontology of the natural numbers, and to express the formal laws governing a denumerable ontology (laws like the law of power-set cardinality) we need an ontology on the order of the indenumerable ontology of ZFC.

4.1.2 The identity problem (Benacerraf, 1965)

The problem, as formulated by Benacerraf through an example, is: Zermelo's 2 is $\{\{\emptyset\}\}$, von Neumann's 2 is $\{\emptyset, \{\emptyset\}\}$; which is the real 2?

Our solution to this problem is straightforward: Since 2 is a posit representing the 2nd-level property TWO, it does not matter whether we construe it as $\{\{\emptyset\}\}$, using Zermelo's method, or as $\{\emptyset, \{\emptyset\}\}$, using von Neumann's method (so long as we are consistent). The two are different but equally good representational methods.

4.1.3 The application problem

The problem is how results concerning mathematical objects, which are highly abstract, can apply to physical objects, which are not abstract.

This question is especially difficult for radical Platonists, who believe in two disconnected realities, an abstract reality and a physical reality, and for radical conventionalists, who believe that mathematics is purely conventional and as such disconnected from reality. But on the present account this problem does not arise: physical objects have formal properties, and therefore laws governing these properties apply to them (directly or indirectly) in an unproblematic way.

4.1.4 The cognitive-access problem (Benacerraf, 1973)

Benacerraf's cognitive-access problem is the problem of how, given that our only access to reality is *causal*, we have access to mathematical objects, which do not stand in causal relations to us (or to anything else, for that matter).

Our response has two parts. First, we question the assumption that humans' *only* access to reality is causal; second, we point to the existence of a large network of routes from mind to reality (combining both causal and non-causal elements) that were not considered by Benacerraf. Both points are based on our foundational holistic methodology. The access question has so far been asked from two perspectives, a foundationalist perspective and an anti-foundational perspective, but not from the perspective of foundational holism. Foundational holism, with its rich and multi-layered conception of cognitive access to reality and its view of mathematics as *quasi-apriori*, questions the exclusiveness of causal access to reality, and suggests other ways of accessing it.

4.1.5 The "mathematics as algebra" problem

If mathematics is a theory of the formal, what is the status of algebraic theories, theories that seem to study structures for their own sake?

The present approach points to two ways in which this problem can be dissolved. First, mathematics is a broad and multi-faceted discipline with a variety of interests. One of its central interests is a theory (or theories) of the formal (or of various aspects thereof), but it has other interests as well. Second, algebraic theories offer potential *models* of phenomena in the world (as it is or as it could have been), i.e., models of some states of affairs, but not all. The difference between, say, arithmetic and group theory is, then, this, that while in all formally-possible states of affairs properties of individuals have cardinalities, not all formally-possible structures are group structures.

4.2 Problems in the philosophy of logic

The situation in the philosophy of logic is quite different from that in the philosophy of mathematics. While philosophers of mathematics have, for a long time, been aware of the need to establish the veridicality of mathematics, explain how mathematics works in the world, and clarify the nature of mathematics in a systematic and theoretical manner, philosophers of logic have often neglected these tasks with respect to logic. One exception is Russell who, as we have noted above, was fully aware of the importance of the questions of veridicality and "how it works in the world" for understanding logic. In addition, Russell was fully aware of the need for, and the difficulty of, explaining the nature of logic:

> The fundamental characteristic of logic, obviously, is that which is indicated when we say that logical propositions are true in virtue of their form. ... I confess, however, that I am unable to give any clear account of what is meant by saying that a proposition is "true in virtue of its form". (Russell, 1938, p. xii)

The present paper has offered a solution to these questions, if only in an outline form. (For discussion of related issues see Sher, 1991, 2008, in press.)

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Gila Sher
Department of Philosophy, University of California, San Diego
La Jolla, 92093-0119
e-mail: gsher@ucsd.edu