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Author
Kim, K.-J.

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Lawrence Berkeley Laboratory

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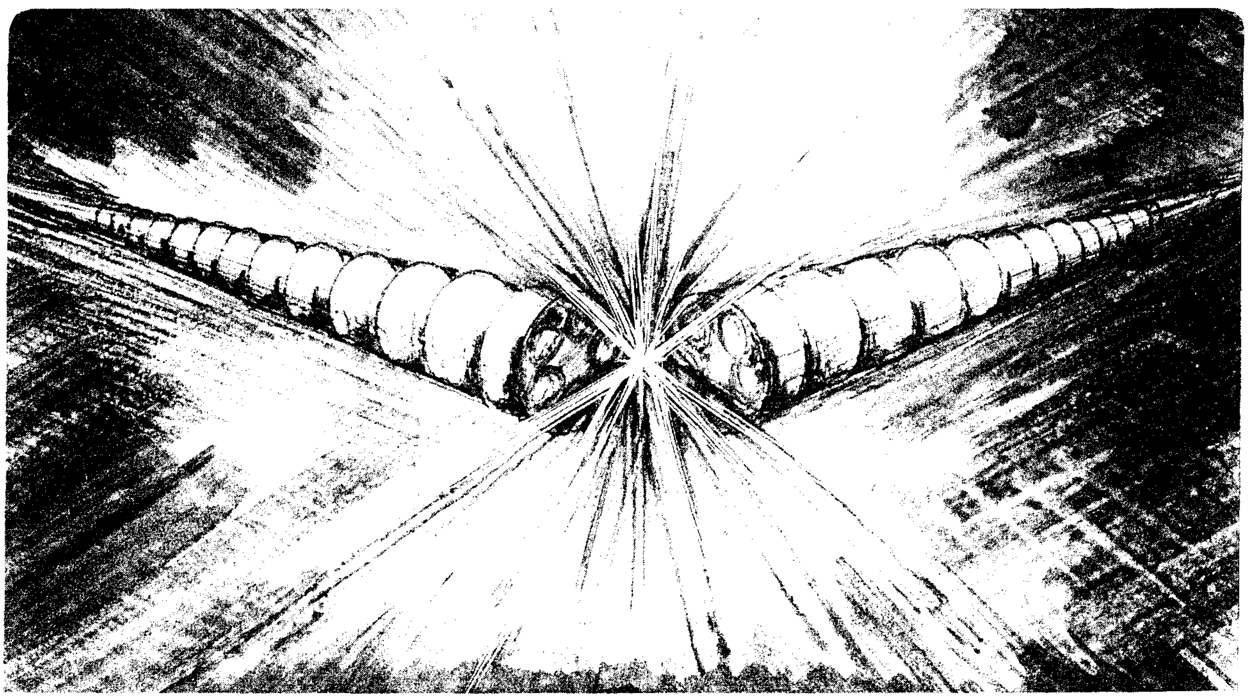

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Photon Storage Cavities

Kwang-Je Kim and Andrew M. Sessler*

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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PHOTON STORAGE CAVITIES

Kwang-Je Kim and Andrew M. Sessler

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Abstract

A general analysis is presented of a photon storage cavity, coupled to a free-electron laser (FEL) cavity. It is shown that if the coupling between the FEL cavity and the storage cavity is uni-directional (for example, a ring resonator storage cavity) then storage is possible, but that if the coupling is bi-directional then storage is not possible. Parameters are presented for an infra-red FEL storage cavity giving an order of magnitude increase in the instantaneous photon power within the storage cavity.

I. Introduction

Saturation of a free-electron laser (FEL) occurs at a field level which depends upon many things such as the wiggler length and strength, the electron beam characteristics (for example, emittance and current), the wavelength of the light, and the reflectance of the mirrors forming the optical cavity. Luis Elias [1] has asked if one can design a storage cavity, coupled to the FEL cavity, so as to remove the saturation limit of an FEL. In this note we answer this question by analyzing storage cavities, and show that they can be effective if the coupling from the FEL cavity to the storage cavity is uni-directional, but not if the coupling is symmetric. The general analysis is given in Sections II and III.

A reflecting plate with holes that allow some of the radiation to pass through (usually only a few percent is needed) can provide coupling in the infra-red. We consider this case in Section IV, and give parameters leading to an order of magnitude increase in photon power. Numerical studies are presented in Section V supporting the analytic work of the previous sections.

Assuming the storage cavity can be Q-switched, operation of an FEL with a storage cavity can lead to a significant increase of instantaneous photon power, which for certain experiments would be advantageous.

The use of optical elements, in order to improve performance, is a well-known technique in conventional laser technology [2]. In this note we show how some of these techniques are useful for an FEL.

II. General Analysis of Coupled Cavities

The system we analyze is shown in Fig. 1. The quantities g_1 and g_2 are the round trip factors

$$\begin{aligned} g_1 &= r r_1 e^{i\theta_1} g_F , \\ g_2 &= r r_2 e^{i\theta_2} , \end{aligned} \tag{2.1}$$

where g_F is the FEL amplitude gain factor and θ_i is the round-trip phase factor in each cavity.

Assuming that the round trip times for cavity 1 and cavity 2 are the same, we can derive the matrix recursion relations for the cavity fields at nth pass as follows:

$$\begin{pmatrix} E_1^- \\ E_2^+ \end{pmatrix}_n = \begin{pmatrix} g_1 & \left(\frac{\tau}{r}\right) g_2 \\ \left(\frac{\tau}{r}\right) g_1 & g_2 \end{pmatrix} \begin{pmatrix} E_1^- \\ E_2^+ \end{pmatrix}_{n-1} . \tag{2.2}$$

For purpose of seeing the nature of storage it is adequate to ignore saturation; i.e., to have g_1 and g_2 independent of field strength. In that case, we can simply find the eigenvalues, λ_1 and λ_2, of the matrix. We find:

$$\lambda_{\frac{1}{2}} = \frac{(g_1 + g_2) \pm \sqrt{(g_1 - g_2)^2 + 4\left(\frac{\tau}{r}\right)^2 g_1 g_2}}{2} . \tag{2.3}$$

For boundary conditions we can consider no field in the storage cavity $E_2^+(0) = 0$ while $E_1^-(0) = E_0$. The fields at nth pass are then given by

$$E_1^-(n) = \frac{E_0}{(\lambda_2 - \lambda_1)} \left[(\lambda_2 - g_1) \lambda_1^n + (g_1 - \lambda_1) \lambda_2^n \right] ,$$

$$E_2^+(n) = \frac{(\tau/r)_L\, g_1\, E_0}{(\lambda_2 - \lambda_1)} \left[- \lambda_1^n + \lambda_2^n \right] . \tag{2.4}$$

From the general expression for the eigenvalues, eq. (2.3), roughly (rigorous proof is given in ref. [3]) there are two cases (or orderings). The first is when

$$\left| 4\left(\frac{\tau}{r}\right)^2 g_1 g_2 \right| >> \left| (g_1 - g_2)^2 \right| \tag{2.5}$$

and the second is the reverse. It is clear that we want to consider the situation where g_1 and g_2 are very near unity. We may set

$$g_1 = 1 + \varepsilon \, ,$$
$$g_2 = 1 - \delta \, , \tag{2.6}$$

knowing that the FEL will make the real part of ε positive while losses in the storage cavity will make the real part of δ positive.

In the first case it is easy to show that

$$\lambda_{\frac{1}{2}} = 1 \pm \frac{\tau}{r} + \frac{\varepsilon - \delta}{2} + \ldots . \tag{2.7}$$

If τ/r has a non-zero real part then λ_1 dominates at long time and

$$\left| \frac{E_2 (n)}{E_1 (n)} \right| \rightarrow 1 \, . \tag{2.8}$$

In this case the storage cavity is not effective. If (τ/r) is pure imaginary then there is oscillation of energy between the first and second cavities. The field builds up, with n, but not at a greater rate than if there were a single FEL cavity; i.e., there is no storage.

In the second case we have

$$\lambda_1 = 1 + \varepsilon + \frac{\left(\frac{\tau}{r}\right)^2}{\varepsilon + \delta} \, ,$$
$$\lambda_2 = 1 - \delta - \frac{\left(\frac{\tau}{r}\right)^2}{\varepsilon + \delta} \, , \tag{2.9}$$

and since λ_1 dominates at long time

$$\frac{E_2(n)}{E_1(n)} \rightarrow \frac{(\tau/r)}{\varepsilon + \delta} . \tag{2.10}$$

But the ordering in the present case implies that

$$|2(\tau/r)| << \varepsilon + \delta ; \tag{2.11}$$

i.e., that $\frac{E_2(n)}{E_1(n)}$ is less than unity and there is no storage, either, in this case.

The intermediate situation doesn't work either, as is shown rigorously in reference [3].

III. Photon Storage Cavity

For an effective storage, it is necessary to decouple the storage cavity from the FEL cavity. One way to achieve the unidirectional coupling is with a ring resonator as shown in Fig. 2. This consists of a resonant ring cavity and a prism so arranged that there is no coupling from the storage cavity to the FEL cavity.

Referring to Fig. 2, we have

$$E_2 = \tau E' + rE_3 , \tag{3.1}$$

where τ and r characterize the right hand surface of the prism. Clearly

$$E_3 = g_2 E_2 . \tag{3.2}$$

From these, it follows

$$E_2 = \frac{\tau}{1 - rg_2} E' . \tag{3.3}$$

By making the reflectivity r close to unity, and reducing the losses in the storage cavity so that g_2 is also close to unity, we can achieve $E_2 >> E'$. The goal of the storage cavity is to have $E_2 >> E$. In general E' is determined by the requirement that the reflectivity in the FEL cavity is adequate for the FEL operation. In practice, this implies that $E' < E$ ($E' \sim 0.1$ E), and hence rg_2 must be made very close to unity ($rg_2 >> 0.9$).

For the prism, as indicated in Fig. 2, with an index of refraction n, and with no plate coupler, we have

$$\frac{E'}{E} = \frac{2}{1+n}, \tag{3.4}$$

$$\frac{E_2}{E_1'} = \frac{2n \cos\theta \sin\theta}{n \sin\theta \cos\theta\,(1+g_2) + \left[1 - n^2\sin^2\theta\right]^{1/2} \sin\theta\,(1-g_2)} \tag{3.5}$$

where θ is the prism angle.

An alternative photon storage cavity could be a Fabry-Perrot resonator coupled with a FEL cavity. In this case, the requisite decoupling can be provided by a polarization rotator.

IV. Plate Coupling

The coupling between two cavities can be readily achieved by a plate with holes. Such a plate can be augmented with a prism, as shown in Fig. 2, for uni-directional coupling. For a plate the reflection coefficient is given by [4]

$$r = \frac{iB}{2-iB}, \tag{4.1}$$

where the real quantity $B > 0$ is given by

$$B = \frac{3a^2 Z_0}{4\omega r_0^3\, \mu_0}. \tag{4.2}$$

In this formula Z_0 is the impedance of free space, ω is the angular frequency of the light, and a is the spacing between holes of radius r_0.

A convenient representation is given by writing

$$B = 2 \cot \eta \ , \tag{4.3}$$

where η is a real quantity. Thus

$$r = - \cos \eta \ e^{-i\eta} \ , \ \tau = i \sin \eta e^{-i\eta} \ . \tag{4.4}$$

Note that, for a plate, $\tau = 1 + r$. It is easy to see that, as it should be, no energy is lost in the plate; i.e.,

$$|r|^2 + |\tau|^2 = 1 \tag{4.5}$$

V. Numerical Examples

A computer iteration program, employing the relation given for the fields in cavity 1 and cavity 2 on the n^{th} pass in terms of its value on the previous pass, eq. (2.2), was developed. For this purpose we specialized to plate coupling and therefore employed the notation of Section IV. We, furthermore, assumed plates at locations 1 and 2 in Fig. 1 and characterized by η_1 and η_2. Thus the central plate and the two reflecting plates are characterized by

$$\frac{\tau}{r} = - i \tan \eta \ , \ \frac{\tau_j}{r_j} = - i \tan \eta_j \ , \tag{5.1}$$

where j = 1 or 2.

For the round trip factors g_1 and g_2 we employed

$$g_2 = \cos\eta \cos\eta_2 \ ,$$

$$g_1 = \cos\eta \cos\eta_1 \left[1 + \frac{\Delta g}{1 + k\,|E_1|^2} \right] \ . \tag{5.2}$$

Thus g_2 describes the loss from the two plates of cavity 2. In cavity 1 we have the same thing, but in addition an FEL whose performance is assumed to saturate with increasing field in the FEL cavity.

We took for initial conditions $E_1 = 1$, $E_2 = 0$. We took the saturation parameter $k = 0.001$ and we assume that the cavity lengths can be adjusted so that g_1 and g_2 are real. The result is shown in Fig. 3. It is seen that, consistent with the discussion in Section II, there is no storage, but the field oscillates between the two cavities.

Storage can be achieved, as was discussed in Section III, with a resonant ring. For example, we may consider storage, even without a coupling plate, by having the prism made of germanium whose index of refraction $n = 4$ in the infra-red. From eq. (3.4) $(E'/E) = 0.4$, implying the power gain of the FEL must be more than 16%. From eq. (3.5), and taking $\theta = 12.75\,°$, and $g_2 = -0.9$, we obtain $(E_2/E')^2 = 10$. The storage of power $(E_2/E)^2 = 16$ for this example.

The parameters are all most reasonable, although g_2 implies good reflecting mirrors in the storage cavity (in practice, over a range of wavelengths). Use of a coupling plate would improve performance or further relax parameters.

Acknowledgments

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[1] Luis Elias, private communication.

[2] Anthhony E. Siegman, Lasers (University Science Books, Mill Valley, CA, 1968).

[3] Kwang-Je Kim and A.M. Sessler, "Analysis of Photon Storage Cavities for a Free-Electron Laser", Lawrence Berkeley Laboratory Report (August 1991) (unpublished).

[4] Robert E. Collin, Foundations for Microwave Engineering (McGraw-Hill, New York, 1966), p. 340.

Figure Captions

Fig. 1. General configuration of coupled storage cavity and FEL cavity. The reflecting surfaces are characterized by reflectivity r and transmitivity τ.

Fig. 2. A ring resonator storage cavity coupled through a reflective prism to an FEL cavity.

Fig. 3. Fields in cavity 1 (FEL) (solid line) and in cavity 2 (dashed line) for two coupled cavities. For this case $\eta = 0.1974$, $\eta_1 = \eta_2 = 0.03439$ and $k = 0.001$.

APPENDIX A

Rigorous Proof

We analyze completely a bi-directional cavity, but under the special assumption that the coupling is by means of a plate and, hence, that $r = r_L = r_R$, $\tau = \tau_L = \tau_R$ and $\tau \approx 1 + r$. When g_F does not vary as n increases,

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix}_n = M^n \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}_0$$

$$M^n = \frac{1}{\lambda_1 - \lambda_2} \begin{bmatrix} (g_1 - \lambda_2)\lambda_1^n - (g_1 - \lambda_1)\lambda_2^n \ , & \frac{\tau}{r} g_2 (\lambda_1^n - \lambda_2^n) \\ \frac{\tau}{r} g_1 (\lambda_1^n - \lambda_2^n) \ , & (g_2 - \lambda_2)\lambda_1^n - (g_2 - \lambda_1)\lambda_2^n \end{bmatrix} .$$

Here λ_1 and λ_2 are two eigenvalues of M:

$$(\lambda_i - g_1)(\lambda_i - g_2) - \left(\frac{\tau}{r}\right)^2 g_1 g_2 = 0$$

i.e.,
$$\lambda_i^2 - (g_1 + g_2)\lambda_i + g_1 g_2 \frac{1}{|r|^2} = 0 \quad ,$$

where we used the fact that $1 - (\frac{\tau}{r})^2 = \frac{1}{|r|^2}$. λ_i also satisfies

$$\lambda_1 + \lambda_2 = g_1 + g_2$$

$$\lambda_1 \lambda_2 = g_1 g_2 \frac{1}{|r^2|} .$$

When the FEL gain $|g_F|$ is above a certain threshold value, at least one of $|\lambda_1|$ and $|\lambda_2|$ is larger than unity and the field strength in the cavity grows. For a sufficiently high field, the gain starts to decrease, and the system reaches an equilibrium where the FEL gain just balances the loss through coupling. At this "saturated" state, at least one of the eigenvalues, say λ_1, must have $|\lambda_1| = 1$, and the other $|\lambda_2| \leq 1$. We analyze the behavior near this saturated state, and prove that it is impossible for the system to reach a configuration where $|E_2| >> |E_1|$. We do this in the following steps:

i) First consider the system at saturation. We prove that, assuming $|\lambda_1| = 1$, the other eigenvalue has also a unit magnitude $|\lambda_2| = 1$. The field ratios for the eigenvector corresponding to the eigenvalue λ_i,

$$R(i) = \left.\frac{|E_1|}{|E_2|}\right|_{\lambda = \lambda_i}$$

are shown to satisfy

$$|R_1 R_2| = 1 \quad .$$

The eigenvector corresponding to λ_1 is the desired configuration if $|R_1| >> 1$.

ii) To determine which of the eigenstate is the dominant one, we consider the situation before saturation where the FEL gain is $(1 + \varepsilon)$ times larger than that required for saturation. The eigenvalues for this case are

$$\lambda_1 = \lambda_1' = \lambda_1 + \delta\lambda_1 \ , \rightarrow \ \lambda_2' = \lambda_2 + \delta\lambda_2 \quad ,$$

and the new field ratios

$$R_1 \rightarrow R_1^{'} \quad , \quad R_2 \rightarrow R_2^{'} \quad .$$

We show that, if $R_1^{'} >> 1$, then $|\lambda_2| \approx (1 + \varepsilon)$, $|\lambda_1| \approx 1$, thus $|\lambda_2| > |\lambda_1|$. Furthermore $R_2^{'} << 1$. Thus the dominant eigenvalue is λ_2, the corresponding field configuration for which is such that the field in the storage cavity is much less than that in the FEL cavity. We now supply details of the proof:

Analysis of the Saturated State

Assume $|\lambda_1| = 1$. We show that $|g_1| = |g_2| = |r|$, and $|\lambda_2| = 1$. From the eigenvalue equation,

$$\lambda_1^2 - (g_1 + g_2)\,\lambda + g_1 g_2/|r|^2 = 0 \quad ,$$

it follows

$$g_1 = \frac{\lambda_1(\lambda_1 - g_2)}{\lambda_1 - g_2/|r|^2} \quad .$$

Let $\qquad \lambda_1 = e^{ix} \quad , \quad g_2 = |r|\, e^{i\psi} \quad .$

Then $\qquad g_1 = |r|\, e^{i(2x - \psi)} \dfrac{1 - |r|\, e^{i(\psi - x)}}{1 - |r|\, e^{-i(\psi - x)}} \quad .$

From this it follows that

$$|g_1| = |r \;\; r_1 \, g_F| = |r| \, , \; |r_1 \, g_F| = 1 \quad .$$

Thus both $|g_1|$ and $|g_2|$ are equal to $|r|$. From the relation $\lambda_1 \lambda_2 = g_1 g_2 / |r|^2$, it then follows that $|\lambda_1 \lambda_2| = 1$. Therefore $|\lambda_1| = 1$ and $|\lambda_2| = 1$.

The eigenvectors corresponding to λ_i satisfies

$$\begin{bmatrix} g_1 - \lambda_i \, , & \frac{\tau}{r} g_2 \\ \frac{\tau}{r} g_1 \, , & g_2 - \lambda_i \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = 0 \quad .$$

Thus

$$R_1 = \left|\frac{E_1}{E_2}\right|_{\lambda_1} = \left|\frac{\frac{\tau}{r} g_2}{\lambda_1 - g_1}\right| = \left|\frac{\lambda_1 - g_2}{\frac{\tau}{r} g_1}\right|$$

$$= \left|\frac{\tau}{\lambda_1 - g_1}\right| = \left|\frac{\lambda_1 - g_2}{\tau}\right| \quad ,$$

$$R_2 = \left|\frac{E_1}{E_2}\right|_{\lambda_2} = \left|\frac{\frac{\tau}{r} g_2}{\lambda_2 - g_1}\right| = \left|\frac{\lambda_2 - g_2}{\frac{\tau}{r} g_1}\right|$$

$$= \left|\frac{\tau}{\lambda_2 - g_1}\right| = \left|\frac{\lambda_2 - g_2}{\tau}\right| \quad .$$

Since $\lambda_1 - g_1 = -\lambda_2 + g_2$ and $\lambda_1 - g_2 = -\lambda_2 + g_1$, it follows from above that

$$R_1 R_2 = 1.$$

The desired configuration where $|E_2| >> |E_1|$ can be obtained for λ_1 eigenstate if $R_1 << 1$ which requires

$$|\lambda_1 - g_2| << \tau << |\lambda_1 - g_1| \quad .$$

This then implies $R_2 >> 1$. At this stage, we do not yet know which of the eigenvectors is the dominant one. If the eigenvector corresponding to λ_2 is dominant, then the field in the storage cavity is low. We show this is the case in the following:

Analysis of the Case Before Saturation

We consider now the case where FEL gain is larger than that required for saturation. Thus we write

$$g_F \rightarrow g_F' = g_F\,(1 + \varepsilon)\ ,\ g_1 \rightarrow g_1' = g_1\,(1 + \varepsilon)\ ,\ g_2 \rightarrow g_2'\ ;\ \varepsilon > 0 \quad .$$

The quantities without prime are those for the saturated case. The matrix becomes

$$M \rightarrow M' = \begin{bmatrix} g_1\,(1 + \varepsilon) & , & \frac{\tau}{r}\,g_2 \\ \frac{\tau}{r}\,g_1\,(1 + \varepsilon)\ , & & g_2 \end{bmatrix} .$$

The new eigenvalues are

$$\lambda_1 \rightarrow \lambda_1' = \lambda_1 + \delta\lambda_1\ ,\ \lambda_2 \rightarrow \lambda_2' = \lambda_2 + \delta\lambda_2 \quad ,$$

and satisfy the equation

$$\left(\lambda_1 + \delta\lambda_i - g_1 (1 + \varepsilon)\right)\left(\lambda_i + \delta\lambda_i - g_2\right) - \left(\frac{\tau}{r}\right)^2 g_1\, g_2\, (1 + \varepsilon) = 0 \quad .$$

To first order in ε

$$\delta\lambda_i = \frac{\lambda_i\left[\lambda_i - g_2\right]}{2\lambda_i - g_1 - g_2}\,\varepsilon \quad .$$

Thus $$\delta\lambda_1 = \frac{\lambda_1\left(\lambda_1 - g_2\right)}{\lambda_1 - \lambda_2}\,\varepsilon \;,\quad \delta\lambda_2 = \frac{-\lambda_2\left(\lambda_2 - g_2\right)}{\lambda_1 - \lambda_2}\,\varepsilon \quad .$$

The new field ratios for eigenvectors λ_i are obtained from

$$\left(M' - \lambda_i\, I\right)\begin{pmatrix} E_1' \\ E_2' \end{pmatrix} = 0 \quad .$$

We find

$$R_1' = \left|\frac{E_1'}{E_2'}\right|_{\lambda_1} = R_1 \left|1 + \frac{\varepsilon\lambda_2}{\lambda_1 - \lambda_2}\right|$$

$$R_2' = \left|\frac{E_1'}{E_2'}\right|_{\lambda_2} = R_2 \cdot \left|1 - \frac{\lambda_1}{\lambda_1 - \lambda_2}\,\varepsilon\right|$$

Thus $$R_1'\, R_2' = R_1\, R_2\, (1 - \varepsilon) \quad .$$

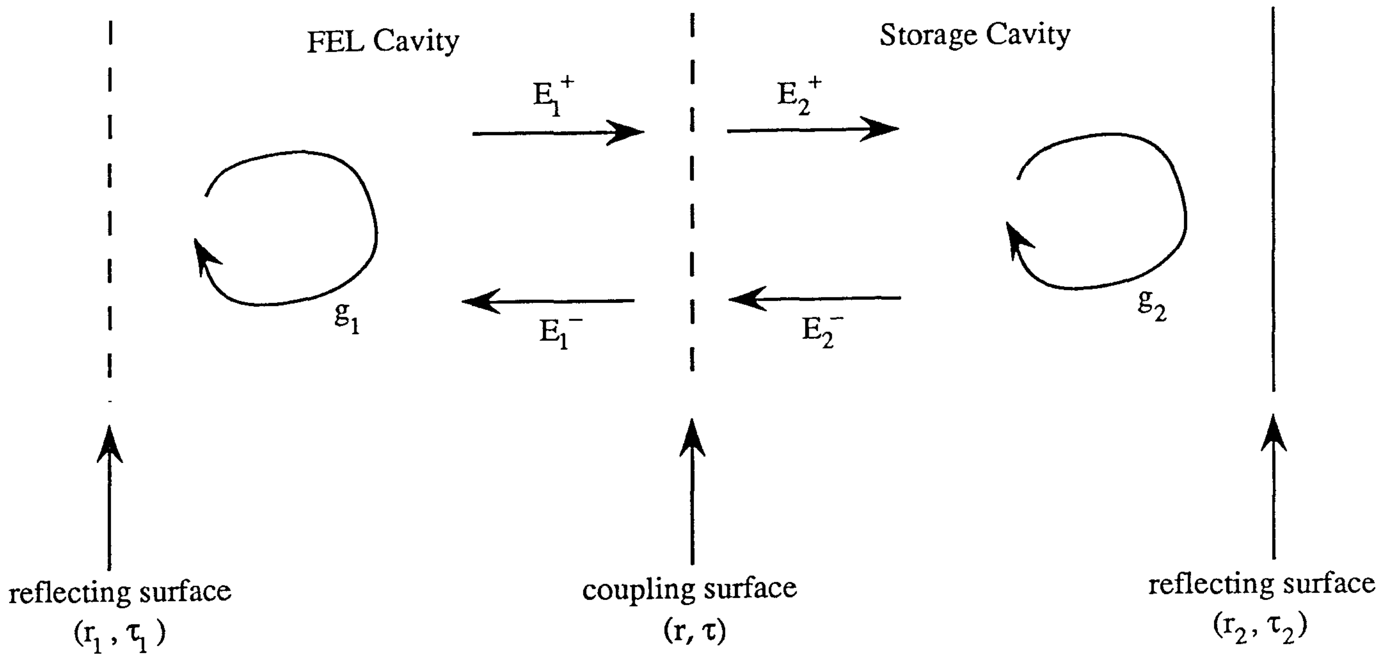
Assume that λ_1'-eigenstate is the desired configuration where $R_1' << 1$. Then $R_1 << 1$ leading to $|\lambda_1 - g_2| << \tau << |\lambda_1 - g_1| < 2$. (We assume $\varepsilon \sim 0(\tau)$, then the correction to R_1 is small.) Then

$$|\delta\lambda_1| << \left|\frac{\tau}{\lambda_1 - g_1}\right| \varepsilon << \varepsilon \ , \quad \delta\lambda_2 \approx \lambda_2 \varepsilon \quad .$$

Therefore

$$|\lambda_2'| = 1 + \varepsilon > |\lambda_1| \approx 1 \quad .$$

Therefore λ_2'-eigenstate is the dominant one, and the energy storage is not possible.



FEL Cavity
Storage Cavity
E_1^+
E_2^+
g_1
g_2
E_1^-
E_2^-
reflecting surface
(r_1, τ_1)
coupling surface
(r, τ)
reflecting surface
(r_2, τ_2)

Fig. 1

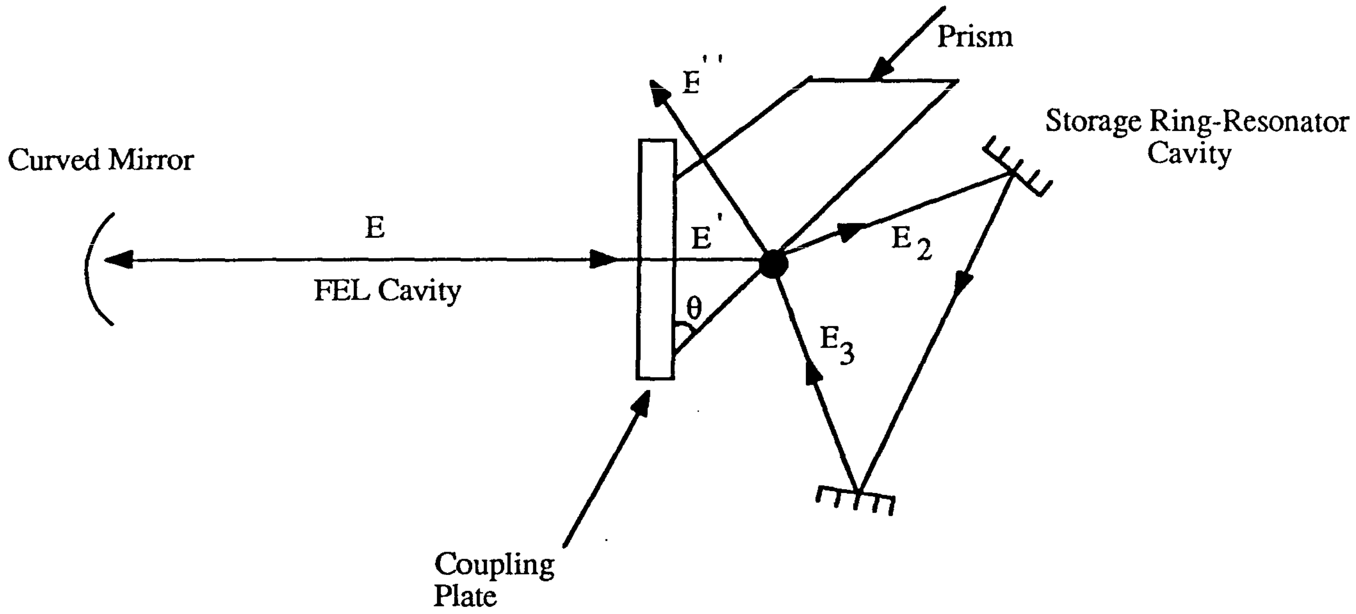

Prism
E''
Storage Ring-Resonator Cavity
Curved Mirror
E
E'
E2
FEL Cavity
θ
E3
Coupling Plate

Fig. 2.

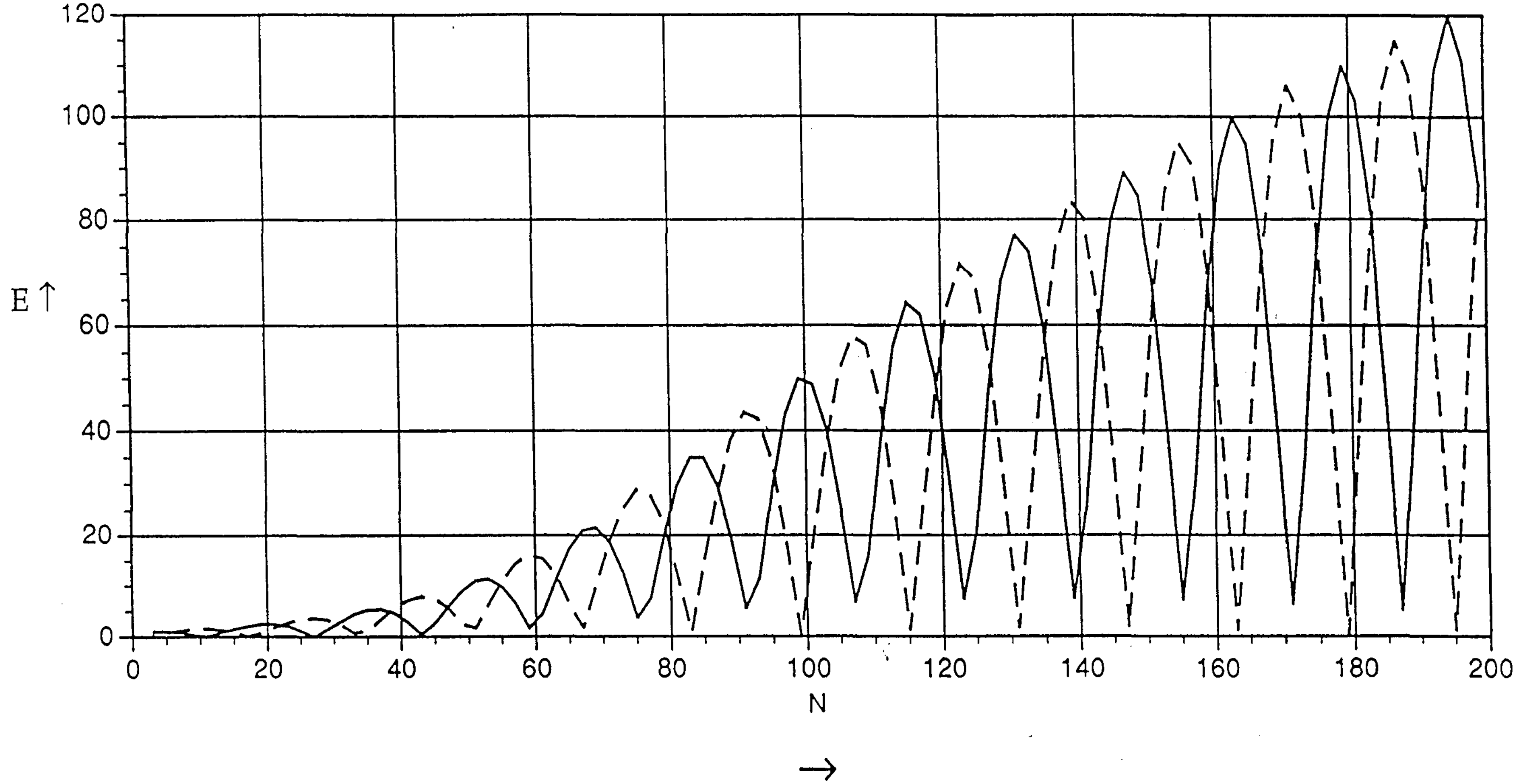


Fig. 3