A Coasian Model of International Production Chains

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Abstract

International supply chains require coordination of numerous activities across multiple countries and firms. We adapt a model of supply chains and apply it to an international trade setting. In each chain, the measure of tasks completed within a firm is determined by a tradeoff between transaction costs and diseconomies of scope linked to management of a larger measure of tasks within the firm. The structural parameters that determine firm scope explain variation in supply-chain length and gross-output-to-value-added ratios, and determine countries' comparative advantage along and across supply chains. We calibrate the model to match key observables in East Asia, and evaluate implications of changes in model parameters for trade, welfare, the length of supply chains and countries' relative position within them.

Keywords: Fragmentation of production, Transaction costs, Trade in intermediate goods, Boundary of the firm
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1 Introduction

The nature of international trade changed dramatically in recent decades, as vertically integrated production processes spread across international borders, increasing trade in parts and components along the way.[1] The sequential arrangement of production activities across multiple countries and firms is an important empirical reality, but one that can be difficult to model analytically.[2] Firms' outsourcing and offshoring decisions must be linked, even as the entire international production chain is arranged to successfully compete with other possible chains. Incorporating such complexity in a parsimonious way that allows transparent links to data poses additional modeling challenges.

In this paper we develop a model of vertically integrated international supply chains. A continuum of sequentially arranged tasks are accomplished by perfectly competitive firms that are themselves arranged sequentially in equilibrium. The length of supply chains – indicated by the number of firms engaged in sequential production – is endogenous, and driven by optimal scope decisions made by every firm participating in the chain. The two parameters that govern firms' scope decisions vary across countries, and one of these parameters is shown to fully determine comparative advantage within an optimized international supply chain. We develop links between the model's structural parameters and two data elements: the gross-output-to-value-added (GO/VA) ratio and countries' relative position within an optimized chain. We embed the partial equilibrium supply chain model in a two-country general equilibrium, and derive relationships between trade costs, the trade elasticity and welfare. We then calibrate a multi-country version of the general equilibrium model. We use the calibrated model to evaluate key model predictions, and to illustrate quantitative implications of model shocks for trade, welfare, and the relative position of countries within international supply chains. We also compare our welfare results to those from more familiar models of international trade.

The model is an extension of the single-country partial equilibrium model of supply chains proposed by Kikuchi et al. (forthcoming). Motivated by Coase (1937), firms in the model face a tradeoff between a) increasing costs of expanding firm scope and b) the costs of outsourcing tasks to upstream firms. Because inter-firm transactions costs are *ad valorem*, their effects are larger downstream, giving downstream firms a stronger incentive to produce a greater range of tasks.[3] We extend the Kikuchi et al. model in a number of ways. First, we develop a continuous firm representation of the model, which facilitates tractable analytics as well as calibration of the general equilibrium version of the model we develop later.[4] Second, we show that the model can be formalized as a Lagrangian optimization problem, with the Lagrange multiplier clearing a shadow market for tasks. This generalizes an insight Kikuchi et al. offer about the relationship between neighboring firms, and provides intuition about

[1]Baldwin (2012) surveys these developments and provides insights into how they should affect our thinking about the economics of international trade and trade policy.

[2]Despite the challenges, a number of authors contribute theories along these lines, including Antràs and Chor (2013), Antràs and de Gortari (2017), Johnson and Moxnes (2016), and Yi (2010).

[3]Firms are identical *ex ante*, but make different optimal scope decisions in equilibrium, depending on their position in the chain.

[4]The continuous firm extension of Kikuchi et al. (forthcoming) relies on a functional form proposed in Chaney and Ossa (2013).

the forces that organize the chain both within and across countries. Third, we move the model to an international setting and develop implications for within-chain comparative advantage. We also show strong links between our model's structural parameters and a key data element that can be retrieved from international data, the GO/VA ratio.

The structure of our multi-country model is quite similar to that in Costinot et al. (2013), with a continuum of sequential tasks produced in countries that are themselves sequentially ordered in equilibrium. In the Costinot, et al. model, a single parameter determines both comparative advantage within a production chain and the cross-country distribution of wages. Relative to this framework our model introduces a distinction between firms and tasks. Key outcomes are driven by two parameters: a firm-to-firm transaction cost parameter that operates much like the key parameter in Costinot, et al., and a parameter that governs within-firm diseconomies of scope. In our model the scope parameter fully determines within-chain comparative advantage; the transaction cost parameter plays no role in determining comparative advantage within an optimized chain.[5]

Our model offers a theoretical framework for interpreting recently developed quantitative measures of supply chain length. Antràs et al. (2012) propose an 'upstreamness' index that represents a weighted-average number of firms that operate between a given industry and consumption of the final good.[6] In addition to the upstreamness index, Fally (2012) proposes a measure of the number of stages embodied in an industry's production. Both of these papers show how input-output tables can be used to calculate the indices. The structural parameters of our model predict both the relative position of a country within a production chain and the GO/VA ratio. We use regional input-output data from Asia to calculate the two indices as well as related aggregates that are useful for fitting the quantitative trade model. Consistent with our model's predictions, we show a positive relationship between a country-industry pair's upstreamness measure and its GO/VA ratio. This correlation holds unconditionally, and conditional on industry fixed effects and a range of country characteristics. We also show negative correlations – both unconditional and conditional – between the GO/VA ratio and the costs of enforcing contracts, validating yet another model prediction.

We also contribute to the literature on trade costs, trade elasticities and economic welfare. Arkolakis et al. (2012) demonstrate that a variety of single-factor trade models link trade cost changes to economic welfare through a simple formula involving a constant global trade elasticity. (Henceforth, we refer to this formula as the ACR formula.) A number of authors have drawn out changing implications when input-output relationships are involved.[7] In a simple two-country version of our general equilibrium model, we show that the relatively downstream country has gains from trade that are relatively larger than predicted by the ACR formula, while the gains from trade in the upstream

[5]We show that, within a chain, the transaction cost parameter matters for absolute advantage, not comparative advantage. In the general equilibrium model with multiple chains, the transaction costs parameter plays an indirect role in comparative advantage because it can affect a country's ability to participate in a given chain.

[6]Ours is a perfectly competitive model, so we elide the distinction between plants and firms. Input-output tables are sometimes built from plant-to-plant flows (e.g. US tables), while in the model we formalize firm-to-firm transactions. In our framework there is no distinction between plants and firms, though we can also interpret the solution to our model as the allocation of tasks across plants by a single planner in a multinational firm.

[7]See Costinot and Rodriguez-Clare (2014) and Caliendo and Parro (2015), for example.

country are relatively smaller than in the ACR benchmark. We also show that the trade elasticity is magnified in our framework, relative to the benchmark model.[8] Analytical predictions from the two-country model hold up in the calibrated multi-country version of the model.[9]

The structure of the paper is as follows. Section 2 provides the mathematical framework and defines the relevant partial and general equilibrium concepts. Section 3 solves the model equilibrium for domestic and international chains. Solution of the international model nests solution of the within-country model, and we illustrate the nested solution method. We also derive theoretical propositions about within-chain comparative advantage, as well as several model implications linked to changing trade costs. In section 4 we develop the two-country general equilibrium, and derive additional implications of changing trade costs. Section 5 calibrates the multi-country quantitative trade model and applies it. We describe the data and use it to calculate the relative position of countries and industries within chains. We show that hypothesized correlations between data elements are consistent with model predictions. Finally, we conduct counterfactual analysis to quantify model responses to trade cost and other shocks. Section 6 concludes.

2 Model setup

We develop a model where the production of each variety of final good requires a continuum of tasks to be organized across firms and countries. We describe, in turn, consumers' preferences in final goods, tasks and firms involved in the production of each good, the forces shaping firm scope and firm entry along the chain, differences between varieties and the labor market.

Preferences: Consumers have identical Cobb-Douglas preferences over varieties of final goods indexed by ω:

$$U_i = \int_{\omega \in \Omega_i} \alpha_i(\omega) \log y_i(\omega)\, d\omega \tag{1}$$

where Ω_i denotes the (fixed) set of varieties available to consumers in i, $y_i(\omega)$ denotes quantities of final goods and $\alpha_i(\omega)$ is a constant term such that $\int_{\omega \in \Omega_i} \alpha_i(\omega) d\omega = 1$. We obtain that expenditures in each variety ω equals:

$$p_i^Y(\omega) y_i(\omega) = \alpha_i(\omega) L_i w_i \tag{2}$$

where $L_i w_i$ refers to income and p_i^Y to final good prices.

As a large part of the theory focuses on the organization of supply chains for a given product variety, we will omit the ω indexation for most of the paper for the sake of exposition.

Tasks and firms along the chain: In order to produce the final good of variety ω, a range $[0, 1]$ of tasks must be performed sequentially. These tasks may be performed across different firms and different countries.

[8] In a somewhat different framework, Johnson and Moxnes (2016) illustrate the magnification of the trade elasticity in the presence of sequential production, and demonstrate it in a calibrated version of their trade model. Yi (2010) and Hillberry and Hummels (2002) also show that trade responses are magnified when input-output relationships are present.

[9] As in Johnson and Moxnes (2016), the magnification of the trade elasticity is rather modest in our calibrated exercise.

Firms are arranged sequentially along the chain to produce each good. A chain is specific to each variety ω of the final good and the location of final producers. On each chain, we assume that there is a continuum of firms indexed by f. Firms may be located in different countries. For each chain, we rank countries along the chain and index by $i(n)$ the n^{th} country, with $i(1)$ indicating the most downstream country and $i(N)$ the most upstream country along the chain.

We denote by F_n the range of firms involved in the chain in the n^{th} country $i(n)$. An elementary firm df performs a range $s_n(f)$ of tasks. Both the range of firms F_n and firm scope $s_n(f)$ are endogenous, but the range of tasks performed across all firms must sum up to one to produce a final good:

$$\sum_n \int_{f=0}^{F_n} s_n(f)df = 1 \tag{3}$$

Denoting $S_n = \int_{f=0}^{F_n} s_n(f)df$ the total range of tasks to be performed in country n, the last constraint can be rewritten:

$$\sum_n S_n = 1$$

for each chain.

Diseconomies of scope: There are costs and benefits to fragmenting production across firms and countries. Fragmentation across firms reduces total costs because of diseconomies of scope. As firms must manage employees across different tasks and perform tasks that are away from their core competencies, unit costs increase with the scope of the firm.

Formally, we assume that an elementary firm df in country i requires one unit of intermediate goods and $c_i(s)df$ units of labor which is a function of firm scope s. The cost of labor is w_i in country i and labor is the only production input besides intermediate goods. We assume that c_i is convex in firm scope s, thus generating gains from fragmentation across firms.

In particular, we specify the following labor requirements:

$$c_i(s) = a_i \frac{s^{\theta_i+1}}{\theta_i + 1}. \tag{4}$$

where a_i and θ_i are parameters that are specific to each country i (and variety ω), but are constant along the chain within a given country. The marginal cost $c_i'(s) = a_i s^{\theta_i}$ of performing additional tasks within the firm increases with s. This follows recent work on the division of labor (the specification is similar to Chaney and Ossa, 2013), and in this context can represent the productivity loss associated with movement away from the firm's core competencies. θ_i parameterizes "diseconomies of scope" and governs the convexity of the cost function. The higher is θ_i, the greater the increase in costs when firms need to manage a larger range of tasks.[10] Accounting for the unit cost of labor w_i in country i,

[10]Note that we assume diseconomies of scope but constant returns to scale in production. This differs from Chaney and Ossa (2013) and more closely follows Kikuchi et al (forthcoming). In keeping with Kikuchi et al (forthcoming), this framework allows us to examine patterns of fragmentation across firms while keeping a perfectly-competitive framework where the competitive allocation of tasks across firms is optimal.

the cost function for value added by an elementary firm df is $w_i c_i(s) df$.

Transaction costs: Fragmenting production across firms incurs transaction costs. We model transaction costs like iceberg transport costs in standard trade models. Let $q_i(f)$ be the quantity of an input produced by firm f in country i (for a given variety ω of the final good). A transaction in country i with an elementary firm df involves losing a fraction $\gamma_i df$ of the good when upstream firm $f + df$ sells to firm f.

$$q_i(f+df) \;=\; (1+\gamma_i\, df)\, q_i(f) \tag{5}$$

Within each country, quantities thus follow a simple evolution that depends on transaction costs γ_i and the position on the chain f. This can be rewritten as a simple ordinary differential equation:

$$\frac{dq_i}{df} \;=\; \gamma_i\, q_i(f) \tag{6}$$

This implies that, as we go upstream, quantities increase exponentially with the measure of firms f participating in the chain:

$$q_i(f) = \mathrm{e}^{\gamma_i f} q_i(0) \tag{7}$$

Since part of the production is lost when transactions occur, upstream firms must produce larger quantities. The necessary increase in quantities is starker when transaction costs are high and when the chain is more fragmented.

In a similar fashion, a *cross-border* transaction between two consecutive countries $i = i(n)$ and $j = i(n+1)$ along the chain involves an iceberg trade cost $\tau > 1$ such that:

$$q_j(0) \;=\; \tau\, q_i(F_i) \tag{8}$$

where $q_j(0)$ denotes the quantities produced by the most downstream firm in the upstream country j and $q_i(F)$ denotes quantities produced by the next firm, i.e. the most upstream firm $f = F_i$ in the next country i along the chain, going downstream. For simplicity, we assume away geographical elements other than borders and impose a common border cost.[11] Cross-border trade costs τ also apply to trade in the final good, between the most downstream firm and final consumers if those are located in different countries.

Market structure: We assume perfect competition. Since we have constant returns to scale in quantities,[12] the price of each variety in each location equals its unit cost of production. Consistent with the perfect competition assumption, we impose free entry and zero profits. Imposing the zero profit condition everywhere along the chain, and for the chain as a whole, implies that the least cost solution to the problem is consistent with perfect competition as we will show in Lemma 1. The zero

[11] In our setting, there is a continuum of firms but only a discrete number of countries involved sequentially. When crossing a border, the unitary transaction cost is $\tau + \gamma_j df$ but $\gamma_j df$ is infinitesimally small relative to τ with a continuum of firms.

[12] There are decreasing returns to scope in tasks, s, but constant returns in terms of quantities, q.

profit constraint will hold at optimized values of $s_i(f)$, which will also be incentive compatible in equilibrium for every firm in the chain.[13]

Prices along the chain: The price of intermediate goods at each step along the chain is equal to their unit cost of production. Here, this cost accounts for all transaction costs and labor costs incurred by all upstream firms. Within country borders, the price of intermediate goods satisfies the following differential equation which describes its evolution along the chain:

$$p_i(f) = w_i c_i(s_i(f))df + (1 + \gamma_i\, df)\, p_i(f{+}df) \tag{9}$$

where $c_i(s_i(f))$ denotes the cost of performing a range $s_i(f)$ of tasks at stage f in country i as specified above. Similar to quantities, this can be rewritten as a differential equation in $s_i(f)$ and $p_i(f)$:

$$\frac{dp_i}{df} = -w_i c_i(s_i(f)) - \gamma_i\; p_i(f) \tag{10}$$

This equation is similar to its counterpart in Costinot, et al. (2013) and also features increasing intermediate goods prices as we go downstream. A key difference, however, is that the labor share is endogenous since $s_n(f)$ is endogenous and thus not simply driven by differences in input prices along the chain. In particular, the cost of inputs per unit of labor is no longer necessarily larger for downstream firms. Many of the results in Costinot et al. (2013) are driven by this feature and thus no longer hold in our framework.

Across borders, the price is simply multiplied by the international trade cost τ:

$$p_j(F) = \tau\, p_i(0) \tag{11}$$

for cross-border transactions from the most downstream firm in j to the most upstream firm in i. This arbitrage condition also applies to final goods.

Industry heterogeneity: While the previous assumptions are sufficient to generate interesting patterns of specialization along a particular chain, we still need to specify how chains vary across varieties. Following Eaton and Kortum (2002), we assume that labor efficiency is a random variable drawn independently across varieties and countries. Specifically, we assume that the labor cost parameter a_i for each variety ω is drawn from a Weibull distribution as in Eaton and Kortum (2002). For each

[13] In the competitive equilibrium, tasks can be allocated across firms through a series of contracts. For instance, one can have a recursive contracting process: each firm f takes as given the measure of tasks $\bar{S}_f(f)$ that must be completed before selling the good to the next firm, but chooses the measure of tasks to complete in-house $s_i(f)df$ and the measure of completed tasks to be required of the subsequent upstream firm, $\bar{S}_i(f+df)$, such that $\bar{S}_i(f) = \bar{S}_i(f+df) + s_i(f)df$. Conversely, one can think of a forward contracting process: firms take as given the range of tasks $\bar{S}_i(f+df)$ performed by upstream firms and chooses the range of tasks $\bar{S}_i(f) = \bar{S}_i(f+df) + s_i(f)df$ to be completed before selling to the next downstream firm. Both approaches lead to the same outcome. We develop this point in more length in the Appendix (proof of Lemma 1).

country i, the cumulative distribution function for a_i is:

$$Proba(a_i < a) = 1 - \mathrm{e}^{-T_i a^{\xi}} \tag{12}$$

where T_i parameterizes the country average productivity and where ξ is inversely related to productivity dispersion. Note that a_i is constant along the chain for a specific country and variety ω. Unlike Yi (2003, 2010), Rodriguez-Clare (2010) and Johnson and Moxnes (2013), our framework does not require a_i to differ across tasks along the chain to generate trade in intermediate goods. Another component of the cost function is θ_i. We will explore different settings. In section 4.1, we do not impose any restriction on θ_i. In section 4.2 we only consider two countries U and D: one where θ_U is constant across all varieties, and another country with $\theta_D < \theta_U$ also constant across all varieties. In section 5 (the calibration exercise), we allow θ_i to vary across countries and varieties ω, assuming that θ_i is log-normally distributed with a country-level shifter $\bar{\theta}_i$ and a common standard deviation $\bar{\sigma}$.

Labor supply: Finally, to close the model, we assume that workers are homogeneous and perfectly mobile within each country, with an inelastic supply of labor L_i in country i.

Labor demand corresponds to unit labor requirement at each stage, multiplied by output $q_{i,f}$, summing across all varieties and all stages performed in the country. Factor market clearance sets labor demand equal to the value of fixed labor supply.[14]

$$\int_{\omega} \int_{f} q_i(f)\, c_i(s_i(f)) \;=\; L_i \tag{13}$$

By Walras' law, trade is balanced.

Equilibrium can then be characterized as:

Definition 1 *For each variety of good ω, a partial equilibrium is a competitive equilibrium taking wages w_i and final consumption y_i given, defined as an allocation of tasks $s_i(f)$ to firms $f \in [0, F_i]$ and countries i to rank positions $n = 1, ...N$, a set of quantities $q_i(f)$ and intermediate prices $p_i(f)$, such that: only the lowest-price chain produces (with free entry of chains and firms); prices equal marginal costs all along the chain; all tasks are performed (3); prices and costs satisfy (4), (9) and (11), and quantities satisfy (7) and (8).*

Definition 2 *General equilibrium is defined as a set of wages w_i to satisfy the labor market clearing condition (13), a set of final demands y_i as in (2) and production chains in competitive equilibrium as described in Definition 1.*

[14] Here we sum across final goods varieties ω, keeping in mind that q_i, c_i and s_i vary across varieties ω because of different productivity draws.

3 Partial equilibrium: optimal organization of chains

In this subsection, we take wages w_i as given and focus on the optimal fragmentation and location of production for a specific chain corresponding to a final good variety ω.[15] For a given chain, we can reformulate the equilibrium as the solution to a social planner's problem.[16] Given our assumption of perfect competition and constant returns to scale, prices equal unit costs and the competitive equilibrium corresponds to the social optimum. This result is standard with a discrete number of firms and it holds here with a continuum of tasks and firms along each chain. In particular, we show in the appendix that the first-order conditions associated with the social planner's problem correspond to the free entry conditions and firm scope choices in the competitive equilibrium.[17]

Lemma 1 *Taking wages as given (Definition 1), a competitive equilibrium is unique and corresponds to the social planner's solution, i.e. it minimizes the cost of producing final goods subject to the full range of tasks to be performed sequentially along the chain.*

Let us denote by $i(n)$ the ranking of countries along the chain, with $i(1)$ being the most downstream country and $i(N)$ the most upstream country, assuming that N countries are involved in the chain. One should keep in mind that the ranking of countries is an equilibrium outcome that we will characterize subsequently.

As expressed in Lemma 1, equilibrium can be summarized by the following optimization problem:

$$\min P_1 \tag{14}$$

$$\begin{aligned}
\textit{over:} \quad & i(n)\ ,\ s_n(f)\ ,\ F_n\ ,\ S_n\ ,\ P_n \\
\textit{under the constraints:} \quad & P_n = \left[\int_{f=0}^{F_n} \mathrm{e}^{\gamma_{i(n)} f} c_{i(n)}(s_n(f)) df \ + \ \mathrm{e}^{\gamma_{i(n)} F_n} \tau P_{n+1} \right] \\
& S_n = \int_{f=0}^{F_n} s_n(f) df \\
& \sum_{i=1}^{N} S_n = 1
\end{aligned}$$

where N is the optimal number of countries involved in the chain and $P_n \equiv p_{0,n}$ denotes the price at the most downstream stage in country $i(n)$ at the n^{th} position. Recall that exponential terms $\mathrm{e}^{\gamma_{i(n)} f}$ reflect the evolution of quantity requirements along the chain as described in equation (7). The transaction cost parameter $\gamma_{i(n)}$ and the cost function $c_{i(n)}(s)$ are indexed by $i(n)$ because they depend

[15]As noted above, we drop the index ω for the sake of presentation. The reader should keep in mind, however, that the optimal fragmentation and allocation of value across firms, as well as costs parameters a_i and θ_i, are all specific to each variety of final good ω.

[16]One could also view the partial equilibrium as the solution to a cost minimization problem for a large integrated multinational corporation allocating tasks across plants. The firm's price and quantity choices in final goods markets would not affect the organization of the optimal chain.

[17]A similar result is obtained in the assignment literature with a continuum of agents on each side (Galichon, 2016).

on which country $i(n)$ is at the n^{th} position upstream. As an abuse of notation, P_{N+1} refers to the price of the most upstream good and is set to zero.[18] The solution to the model in autarky occurs when each country i produces all tasks $S_i = 1$ purchases inputs at price $P_{i+1} = 0$.

The optimization problem described in (14) can be formulated as a nested optimization problem. In the inner nest, firms in a specific country $i(n)$ are organized to minimize the price P_n of goods exported by this country, conditional on a given measure of tasks S_n to be performed and the price P_{n+1} of the imported intermediate good. The outer nest allocates measures of tasks to be completed to each country, and determines the import and export prices of participating countries, conditional on a sequential ranking of the countries. The solution must also provide a mapping of countries i to their rank order position $n = 1, ..., N$ and determine the number of countries N that participate in each chain. Our solution method is to first solve the within-country problem; then solve the global problem for any given ranking of countries. The optimal rank order and the choice of N are determined by comparing minimized prices of each chain.

3.1 Fragmentation of production within countries

Before turning to the cross-border organization of chains, we focus on the within-country problem. In this problem we allocate tasks $s_i(f)$ across firms $f \in [0, F_i]$ to minimize country i's last-stage (export) price P_i, given a measure of tasks to be completed and an import price.[19] P_i can be expressed as the solution of the following optimization:

$$\widetilde{P}_i(S_i, P_i^M) = \min_{s_i(f), F_i} \left[\int_{f=0}^{F_i} \mathrm{e}^{\gamma_i f} \, w_i c_i(s_i(f)) df \; + \; \mathrm{e}^{\gamma_i F_i} P_i^M \right] \tag{15}$$

under the constraint:

$$\int_{f=0}^{F_i} s_i(f) df = S_i \tag{16}$$

To examine the optimal allocation of tasks across firms and the optimal range of firms, it is useful to introduce the Lagrange multiplier λ_i associated with the constraint $\int_0^{F_i} s_i(f)df = S_i$.

The first-order conditions of this planning program are:

$$\text{For } s_i(f): \quad e^{\gamma_i f} w_i c_i'(s_i(f)) \; = \; \lambda_i \tag{17}$$

$$\text{For } F_i: \quad e^{\gamma_i F_i} w_i c_i(s_i(F_i)) \; + \; e^{\gamma_i F_i} P_i^M \gamma_i \; = \; s_i(F_i) \, \lambda_i \tag{18}$$

These conditions help us solve for firm scope $s_i(f)$ and the number of firms involved in the chain F_i. Both $s_i(f)$ and F_i depend on λ_i, the shadow cost of a task.

Equation (17) defines a shadow market for tasks. All firms in the chain provide a measure of tasks $s_i(f)$ such that their marginal cost of tasks equals the shadow price of a task, λ_i. In this way, the

[18] Alternatively, we could set an exogenous price $P_{N+1} = \bar{p}$ of the most upstream good reflecting the price of primary commodity such as oil and minerals available from an outside economy that trades for final goods.

[19] The autarky solution of the model is the solution to the following problem for $S_i = 1$ and $P_i^M = 0$.

conditions that determine the scope of individual firms also define the allocation of tasks across firms that minimizes the cost of producing a measure of tasks S_i in country i.[20] From a broader perspective, λ_i also links firm scope decisions across countries, a relationship we develop further in the following section of the paper. For those relationships it is helpful to recognize that $\lambda_i = \frac{\partial \widetilde{P}_i}{\partial S_i}$.

Condition (17) offers an additional insight about the relationship between firm heterogeneity and relative position along the chain. A move upstream (i.e. towards higher index f) increases required quantities $e^{\gamma_i f}$, which must be balanced by a reduction in the marginal cost $c_i'(s)$. Hence, with convex costs, condition (17) implies that more upstream firms have smaller firm scope $s_i(f)$ and provide less value added. We can be more explicit about this using our parameterization: $c_i'(s) = a_i s^{\theta_i}$, which implies that firm scope is log-linear in upstreamness f:

$$\frac{\partial \log s_i}{\partial f} = -\frac{\gamma_i}{\theta_i} < 0 \tag{19}$$

In an appendix we solve for $s_i(f)$ and F_i as a function of λ_i. We apply these in turn to the constraint $\int_0^{F_i} s_i(f)df = S_i$ and derive an explicit solution for the shadow cost of fragmentation.

$$\lambda_i = w_i a_i \left[\frac{\gamma_i S_i}{\theta_i} + \left(\frac{(\theta_i\!+\!1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i} \right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i} \tag{20}$$

λ_i increases with all cost parameters a_i, θ_i and γ_i, with the price of intermediate goods P_i^M and with the range of tasks to be performed S_i. Having solved for the shadow cost of fragmentation, we can now solve for the price of the last-stage goods P_i, the extent of fragmentation F_i in country i and firm scope $s_i(f)$ across all firms f within the country. We also examine the (endogenous) intermediate goods intensity at each stage.

Firm scope: The model is tractable enough to solve for firm scope $s_i(f)$ all along the chain. Firm scope $s_i(F_i)$ for the most upstream and downstream firms are respectively:

$$s_i(F_i) = \left[\frac{(\theta_i\!+\!1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i} \right]^{\frac{1}{\theta_i+1}} ; \qquad s_i(0) = \frac{\gamma_i S_i}{\theta_i} + s_i(F_i) \tag{21}$$

Using expression (19), scope at intermediate positions corresponds to: $\log s_i(f) = -\frac{\gamma_i}{\theta_i} f + \log s_i(0)$. Note again that firms are *ex ante* homogeneous but end up with different firm scope due to their position on the chain. The difference $\frac{\gamma_i}{\theta_i} S_i$ between the scope of the most downstream and upstream firms in country i is illustrative of this within-country heterogeneity in firm scope. Heterogeneity is rising in S_i because more tasks produced in country i implies more firms, and thus more room for heterogeneity, conditional on θ_i and γ_i.

Of further interest is the relationship between firm scope and the price of intermediate goods relative to labor costs $\frac{P_i^M}{a_i w_i}$. The scope of both the most upstream and downstream firms are rising

[20] Our Lagrangian formulation in (17) generalizes the condition $\delta c'(s(f+1)) = c'(s(f))$ in Kikuchi et al (forthcoming) that links the marginal costs of tasks between (discrete) firms f that neighbor one another in the chain.

in this ratio. The intuition is that when the price of intermediates is relatively high, the cost of outsourcing is relatively higher and firms will choose to add more value in-house. Conversely, when labor costs are high, firms will produce relatively few stages before outsourcing to upstream firms.

Length of the chain: The number (mass) of firms involved sequentially in production is a key measure of fragmentation of the chain. Here, since the range of tasks performed by each firm is endogenous, the length of the chain also becomes endogenous and is no longer proportional to S_i. For a given price P_i^M of imported intermediate goods and range S_i of tasks to be performed, the mass of sequential suppliers is:

$$F_i = \frac{\theta_i}{\gamma_i} \log \left[1 + \frac{S_i}{\theta_i + 1} \left(\frac{A_i w_i}{P_i^M} \right)^{\frac{1}{\theta_i+1}} \right] \tag{22}$$

The mass of suppliers depends negatively on the price of intermediate goods because more expensive components make transactions more costly, which leads to less fragmentation. The number of suppliers also depends negatively on transaction costs and positively on θ_i, the parameter for diseconomies of scope.

Aggregate price: After solving for firm scope s_{if} and the number of firms F_i, we find that the price of the most downstream good in country i, i.e. the solution of the minimization program (15), is:

$$P_i = \widetilde{P}_i(S_i, P_i^M) = \left[\frac{S_i}{\theta_i + 1} (A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i+1} \tag{23}$$

expressed as a function of the synthetic parameter A_i:

$$A_i = a_i \left(\gamma_i \, \tfrac{\theta_i + 1}{\theta_i} \right)^{\theta_i} \tag{24}$$

This A_i depends on exogenous country-specific parameters θ_i, a_i and γ_i, and reflects the effective labor productivity in country i. Note that, conditional on A_i, prices no longer depend on transaction costs γ_i. The price mimics a CES cost function with two inputs: imported intermediate goods and labor, where the weight for labor depends on the range of tasks, productivity, transaction costs and diseconomies of scope. The apparent elasticity of substitution is $\theta_i + 1$. When diseconomies of scope θ_i are stronger, production has to be more fragmented and there is a larger amount of production lost in transaction costs. These costs are larger when the price of intermediate goods P_i^M is high.

Labor vs. imported intermediate goods demand: Each unit of the final-stage good produced in country i also generates a demand $e^{\gamma_i F_i}$ for the most upstream intermediate goods, i.e. intermediate goods imported from the next country in the chain. In terms of value rather than quantities, by applying Shephard's Lemma we obtain that the share of imported inputs in the total cost of production in country i is:

$$\frac{P_i^M q_i(F_i)}{P_i q_i(0)} = \frac{\partial \log \widetilde{P}_i}{\partial \log P_i^M} = \frac{\left(P_i^M\right)^{\frac{1}{\theta_i+1}}}{\frac{S_i}{\theta_i+1} (A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}}} \tag{25}$$

Using this expression, we can retrieve the demand for local labor in country i. The share of local demand in the production of country i has a simple interpretation: it corresponds to the value-added content of exports for country i in that chain. As with the price of the produced good, this expression mimics a CES cost function. The share of labor (one minus the above expression) depends positively on the range of tasks to be performed as well as the price of intermediate goods. The elasticity of substitution between imported inputs and local labor is in turn endogenously determined by diseconomies of scope at the firm level.

Gross-output-to-value-added ratio: The ratio of these two variables informs on the extent of fragmentation and has a useful empirical counterpart since it is readily available in typical input-output tables provided by statistical agencies. The gross-output-to-value-added ratio also maps well into our framework with a continuum of firms if we adopt an appropriate measure of gross-output. Over a continuous segment of firms, say $[0, F_i]$, we define gross output as the integral sum of transactions along that segment: $GO_i = \int_0^{F_i} p_i(f)q_i(f)df$. With this definition, the scaling is such that gross output over a segment of measure one corresponds to the average sales of a measure one of firms. Symmetrically, value added over a segment $[0, F_i]$ corresponds to: $VA_i = \int_0^{F_i} w_i c_i(s_i(f))q_i(f)df$.[21]

We find that the GO/VA ratio in equilibrium for country i is equal to:

$$\frac{GO_i}{VA_i} = \frac{\theta_i}{\gamma_i} \tag{26}$$

Strikingly, this result also holds at the firm level. To be more precise, the ratio of the price to the unit factor cost $w_i c_i$ at each stage f is constant and equal to:

$$\frac{p_i(f)}{w_i c_i(s_i(f))} = \frac{\theta_i}{\gamma_i} \tag{27}$$

We can interpret this GO/VA ratio as an index of fragmentation across firms. Here, in particular, it reflects the two key forces that are present in our model: stronger diseconomies of scope θ_i lead to more fragmentation while larger transaction costs γ_i lead to less fragmentation.[22] As seen in equations (19) and (21), this ratio also dictates the difference in scope between upstream and downstream firms.

The relationship between the structural parameters and summary measures of fragmentation are summarized in the following lemma:

Lemma 2 *Production fragmentation within countries – captured either by the GO/VA ratio or by the range F_i of firms involved in the chain – increases with diseconomies of scope θ_i and decreases with transaction costs γ_i. Specifically, the GO/VA ratio equals $\frac{\theta_i}{\gamma_i}$.*

[21] These two indexes in a continuum provide natural extensions of GO and VA with discrete firms. With discrete firms, note that GO is the sum of upstream VA. Here with a continuum, GO is the integral sum of upstream VA.

[22] Every firm within a given chain and country shares the same GO/VA ratio. This is a strong prediction that could be tested with firm-to-firm transaction data linked to firm level GO/VA ratios, which we lack, unfortunately. It is very likely that this strong empirical prediction would fail, but the manner in which it failed would be highly informative.

3.2 Cross-border fragmentation

Now that we have described the allocation of tasks along the chain within borders, we turn to the optimal allocation of tasks and firms across borders. In particular, we need to characterize the ordering of countries $i(n)$ on the chain, with $i(1)$ being the most downstream and $i(N)$ the most upstream country.[23]

Given the optimal fragmentation of production across firms in each country $i = i(n)$, summarized by the price function from equation (23), $\widetilde{P}_i(S, P^M)$, the optimal global value chain corresponds to the following minimization program:

$$\min_{\{S_n, P_n\}} P_1 \tag{28}$$

under the constraints:

$$P_n = \widetilde{P}_{i(n)}(S_n, \tau P_{n+1}) \tag{29}$$

and

$$\sum_{i=n}^{N} S_n = 1 \tag{30}$$

where the function $\widetilde{P}_i(S, P^M)$ is the solution of the optimization described in equation (23) in the previous section.

For a given sequence of countries $i(n)$, we can go quite far in characterizing prices, ranges of tasks completed and labor demand along the chain. First, it is useful to explicitly express the Lagrangian:

$$\mathcal{L} = P_1 - \sum_{n=1}^{N} q_n \left[P_n - \widetilde{P}_{i(n)}(S_n, \tau P_{n+1}) \right] - \lambda_G \Big[\sum_{n=1}^{N} S_n - 1 \Big] \tag{31}$$

The Lagrange multipliers associated with price equations correspond to quantities required for each unit of final good. To be more precise, q_n correspond to quantities $q_{i(n)}(0)/q_{i(1)}(0)$ required at the most downstream task performed in the n^{th} country $i(n)$ per unit of final good $q_{i(1)}(0)$. The first-order condition $\frac{\partial \mathcal{L}}{\partial P_{n+1}} = 0$ is equivalent to imposing $q_{n+1} = \tau q_n e^{\gamma_{i(n)} F_n}$ (using the price derivative described in equation 25).

The marginal cost of tasks $\frac{\partial \widetilde{P}_{i(n)}}{\partial S_n}$ in each country is tightly linked to the shadow cost of stages across markets, which must be equalized across all countries in the chain: $\lambda_G = q_{i(n)} \frac{\partial \widetilde{P}_{i(n)}}{\partial S_n} = q_n \lambda_n$. Since a move upstream along the chain increases quantities (because of transaction costs and cross-border trade costs), the shadow cost $\lambda_n > \lambda_{n+1}$ must decrease. Concretely, a first implication is that firm scope tends to decrease as we go upstream, not just within countries but also across countries. The F.O.C. in S_i implies the following expression which generalizes equation (17) across countries along the chain:

$$q_n e^{\gamma_{i(n)} f} w_{i(n)} c'_{i(n)}(s_n(f)) = \lambda_G \tag{32}$$

where $q_n e^{\gamma_{i(n)} f}$ corresponds to the quantities of intermediate goods required for each unit of final

[23]Recall that we drop for now the variety subscript ω while most parameters vary across varieties.

good. Since the latter increases with upstreamness, we obtain that firm scope $s_n(f)$ would be smaller if a country $i = i(n)$ specializes upstream than if it specializes downstream. Therefore, a country with stronger within-firm diseconomies of scope would have a relatively larger cost downstream than upstream compared to a country with weaker diseconomies of scope.

This feature has important implications for the sorting of countries along the chain. Countries with relatively high values of θ will have firms that manage fewer tasks, which means that they will be located upstream. Low-θ countries will host firms with larger scope, and be located downstream. Formally, we can confirm this insight by examining second-order conditions of the optimization problem described in Equation (28), which yields the following Proposition:

Proposition 1 *Let us denote by $i(n)$ the ranking of countries involved in the same production chain, $i(1)$ being the most downstream and $i(N)$ the most upstream country. In equilibrium, the relative position of countries along the chain is fully determined by diseconomies of scope θ_i; countries with weaker diseconomies of scope specialize downstream:*

$$\theta_{i(1)} < \theta_{i(2)} < ... < \theta_{i(N)}$$

Proposition 1 describes comparative advantage within a supply chain, conditional on a country's participation in the chain.[24] Two implications are of primary interest here: the central role of θ_i in determining within-chain comparative advantage, but also the absence of a role for the transaction cost parameter γ_i. The lack of a role for γ_i would seem to run counter to Costinot et al. (2013), where cross-country differences in the rates of mistakes in production drive comparative advantage within the chain. The closest counterpart in our model to the mistakes in Costinot et al. (2013) is the γ_i parameter.[25]

In both models cross-country sorting of sequential activities mitigates the effects of firm-to-firm transaction costs on the price of the completed good. In Costinot et al. (2013) countries with low transaction costs produce downstream in equilibrium because they impose the least "melt" on goods that are nearing completion. In our model firms can offset transaction costs by expanding firm scope. Because offsetting such costs is more valuable downstream, the countries in which firms can most easily expand firm scope, the low-θ countries, locate downstream.

Another way to see this is to exploit the insights in Costinot (2009), who links comparative advantage to the mathematics of log-supermodularity. The accumulation of value added along the chain insures that the cost of intermediate goods is rising along the chain. This means that if production costs are log-supermodular in input prices and a parameter, then countries that have low values of that parameter will locate downstream. With frictions along supply chains, the average cost of performing tasks increases along the chain with the cost of inputs $\left.\frac{\partial \tilde{P}_i}{\partial S_i}\right|_{S_i=0} = (A_i w_i)^{\frac{1}{\theta_i+1}} \left(\tau P_i^M\right)^{\frac{\theta_i}{\theta_i+1}}$, and the

[24]Countries with large values γ_i, w_i, a_i and/or θ_i may not participate at all in an equilibrium chain. Proposition 1 describes the sorting of countries that do participate in an equilibrium chain.

[25]Costinot et al. (2013) offer cross-country differences in contract enforcement as a rationale for differences in the rates of mistakes. Here, the parameter most closely related to contract enforcement is clearly γ_i.

effect of the input price is larger for larger θ_i. In fact, we can show that the effect of input prices (on the cost of a marginal task) is equal to the share of transaction costs, which is itself determined by θ_i when firm scope is endogenous:[26]

$$\frac{\partial \log\left(\frac{\partial \widetilde{P}_{i(n)}}{\partial S_n}\right)}{d\log P_{n+1}} = \frac{\gamma_i P_i^M}{w_i c_i + \gamma_i P_i^M} = \frac{\theta_i}{1+\theta_i} \tag{33}$$

In Costinot et al. (2013), $s_i(f)$ is fixed, so the average cost of performing tasks is log supermodular in input prices and γ_i. The middle term $\frac{\gamma_i P_i^M}{w_i c_i+\gamma_i P_i^M}$ would be increasing in γ_i if firm scope s_i and thus costs c_i are fixed. Hence, transaction costs γ_i have the largest impact on average cost when P_i^M is high (i.e. downstream), so that lower transaction costs create a comparative advantage in downstream tasks.

In contrast to Costinot et al. (2013), γ_i does not appear in the last term of (33) in our model where firm scope is endogenous. In countries with larger transaction costs, firms will endogenously increase firm scope to mitigate the role of higher transaction costs. These endogenous responses nullify the role that transaction cost would otherwise play if firm scope were exogenous. Instead, θ_i now plays a singular role in determining countries' positions within the chain. As shown in equation (33), countries with stronger diseconomies of scope θ_i should specialize upstream to mitigate the effect of input prices on value added.[27] A related implication is that there will be no international fragmentation without cross-country variation in θ_i. Proposition 1 also implies that there is no back-and-forth trade along a specific chain in equilibrium, except when a final good is shipped back to be consumed in an upstream country.

Equilibrium allocation of tasks across countries: Given the ranking of countries described in Proposition 1, we now describe the range of tasks performed by each one. Using marginal conditions imposed by the optimization problem, we can also determine prices and firm scope along the chain as functions of wages and relative productivity. Specifically, the first-order conditions determine the c.i.f. price between consecutive countries $i(n)$ and $i(n+1)$. First-order conditions between three consecutive countries $i(n-1)$, $i(n)$ and $i(n+1)$ then yield the range of tasks performed in $i(n)$. Denoting A_n, w_n and θ_n the productivity, wages and the scope parameter in the n^{th} country $i(n)$ along the chain, we

[26]To show equality between the last two terms, notice that $\frac{P_i^M}{w_i c_i}$ corresponds to GO/VA at the firm level and use Lemma 2. This equality holds for the imported good price if $\frac{\partial \widetilde{P}_i}{\partial S_i}$ is evaluated at $S_i = 0$, and it holds more generally as a function of the intermediate good price at any stage.

[27]Our specific choice of functional form removes γ_i altogether from the determination of within-chain comparative advantage. It is difficult to derive analytical solutions for other functional forms, and the effects of γ_i on within-chain comparative advantage may not be completely offset or may be more-than-offset by endogenous responses to θ for other functional forms of $c(s,\theta)$. The intuition nonetheless goes through for cost functions where the convexity is governed by parameter θ. Countries with low values of θ are better able to reduce inter-firm transaction costs through expansion and thus tend to locate downstream. Computational experiments with alternative functional forms (e.g. $a\left(e^{\theta s}-1\right)$ as in Kikuchi et al. forthcoming) confirm that θ is the main determinant of comparative advantage within the chain.

obtain:

$$\begin{cases} \tau P_{n+1} &= (A_n w_n)^{\frac{\theta_{n+1}+1}{\theta_{n+1}-\theta_n}} (\tau A_{n+1} w_{n+1})^{-\frac{\theta_n+1}{\theta_{n+1}-\theta_n}} \\ \frac{S_n}{\theta_n+1} &= \left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \end{cases} \tag{34}$$

where τ is the trade cost between any two countries.

Conditional on the set of countries participating (with θ_n increasing with n along the chain), we can obtain a simple expression for the price of final goods (i.e. price of downstream goods in country 1) as a function of costs parameters A_n, θ_n and wages w_n. Conditional on the set of countries, we can also derive simple expressions for the share of labor costs from a specific country.

Lemma 3 *Conditional on the set of countries $n = 1, 2, etc.$ participating (with θ_n increasing with n along the chain), the price of the final good is:*

$$P_1 \ = \ \frac{A_1 w_1}{(\theta_1+1)^{\theta_1+1}} \ \Theta\big(\mathbf{wA}, \tau\big) \tag{35}$$

where $\Theta\big(\boldsymbol{wA}, \tau\big) < 1$ captures the gains from fragmentation for the chain:

$$\Theta\big(\mathbf{wA}, \tau\big) = \left[1 \ - \sum_{n=1}^{N-1} (\theta_{n+1} - \theta_n) \left(\frac{w_n A_n}{\tau w_{n+1} A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \right]^{\theta_1+1}$$

In the expression for the final good price above, the first term $\frac{A_1 w_1}{(\theta_1+1)^{\theta_1+1}}$ is the cost of production in country 1 if there is no possibility to fragment production across countries, while the second term $\Theta\big(\mathbf{wA}, \tau\big)$ is the price reduction obtained from fragmenting production across countries. We can verify that this term increases with trade costs. It also increases with labor requirements A in each upstream country. Note also that the price of the final good is itself tightly linked to the shadow cost of fragmentation λ_G. The Θ term drives both the shadow cost of fragmentation and the final goods price:

$$\lambda_G \ = \ \frac{A_1 w_1}{(\theta_1+1)^{\theta_1}} \ \Theta\big(\mathbf{wA}, \tau\big)^{\frac{\theta_1}{\theta_1+1}}. \tag{36}$$

We now turn to implications of a decrease in trade costs for two key outcomes that reflect the extent of fragmentation across and within countries: the value-added contribution from each country and firm scope. Applying Shephard's Lemma, we can use Lemma 3 to obtain country $i(n)$'s labor contribution to each dollar of final good:

$$\frac{l_n w_n}{P_1} = \frac{d \log P_1}{d \log w_n} = \frac{d \log \Theta}{d \log w_n} = \frac{\left(\frac{w_{n-1}A_{n-1}}{\tau w_n A_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{w_n A_n}{\tau w_{n+1} A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}}{\left(\frac{P_1}{A_1 w_1}\right)^{\frac{1}{\theta_1+1}}} \tag{37}$$

Because ours is a single-factor model, labor shares also correspond to the value added by country $i(n)$. This expression can be used to compute various indicators of involvement in supply chains, such as the share of foreign vs. domestic value added embodied in exports, a key statistic for economic policy.[28] Intuitively, the share of local value added in exports is higher when the relative labor cost is lower, as lower labor costs allow the country to serve as the low-cost location for a larger measure of stages.

A change in trade costs and wages along the chain has implications for firm scope everywhere on the chain. Using equation (32), the marginal cost of increasing firm scope in the most downstream firm in the most downstream country, $w_1 c'(s_1(0))$ is equal to λ_G. This implies:

$$s_1(0) = \frac{\gamma_1}{\theta_1} \Theta\big(\mathbf{wA}, \tau\big)^{\frac{1}{\theta_1+1}} \tag{38}$$

which formalizes how a change in fragmentation and trade costs (changes in Θ) reduces firm scope for the last firm in the chain, the one that produces the finished good.

In upstream countries $n > 1$, the reduction in iceberg trade costs implies smaller quantities of upstream goods required for each unit of final goods, reflected in lower values of q_n in (32), which also implies that $s_n(f)$ can rise even as λ_G falls. Moving from the first order condition to the analytical solution combining the two effects, we find that the scope of firms within an upstream country increases as trade costs decrease:

$$s_n(0) = \frac{(\theta_n+1)\gamma_n}{\theta_n} \left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \qquad \text{and:} \qquad s_{n,F_n} = \frac{(\theta_n+1)\gamma_n}{\theta_n} \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \tag{39}$$

Trade cost reductions thus imply a reorganization all along the chain, with tasks shifting away from the most downstream firms, and towards firms in upstream countries.[29]

Proposition 2 below summarizes the effect of trade costs on a chain in partial equilibrium (exogenous wages) for a given set of countries involved in the chain:

Proposition 2 *Holding wages constant, a decrease in cross-border trade costs leads to:*

i) a decrease in the price of the final good;

ii) an increase in the share of imported inputs;

iii) an increase in the range of tasks being offshored;

iv) a decrease in firm scope $s_n(f)$ in the most downstream country $n = 1$

v) an increase in firm scope in upstream countries $n > 1$.

While the ranking of countries along the chain (from downstream to upstream stages) is dictated by the ranking in θ_i (Proposition 1), it is more difficult to characterize the participation of a specific country in the chain. Moreover, the reader should keep in mind that we have dropped the variety subscript ω to simplify the notation, but the costs parameters A_i and θ_i are assumed to be specific to a particular variety of final good ω. Hence, the organization of the chains across firms and countries is specific to each variety and country of final destination.

[28] For example Koopman et al. (2012) investigate the share of domestic value added in China's exports.

[29] A graphical representation of the effects of reduced trade costs on firm scope appears in Figure 3 of the appendix.

In the next section, we address this problem in a two-country case with trade costs and heterogeneous chains.[30] In Section 5, we examine numerically a ten-country case calibrated using input-output data. Using expression (35) from Lemma 3 and expression (37) on value-added shares, we can dramatically reduce the complexity of the numerical problem and reformulate it into a simpler linear programming problem that allows us to solve for large economies with a large number of final goods.

4 General equilibrium

The discussion so far has described the production structure of the equilibrium chain. As described in Definition 2, general equilibrium requires labor markets and the final goods market to clear.

Using value-added shares derived from Lemma 3 (equation 37) for each individual chain and our expression for labor demand per unit of final output, it is now easier to compute aggregate labor market demand. As in equation (13), factor market clearance sets labor demand (summing across varieties and chains indexed by ω) equal to fixed labor supply:

$$\sum_{j,n} \int_{\omega \in \Omega_j} \alpha_j(\omega) L_j w_j \frac{l_{i(n)}(\omega)}{P_{i(n)}(\omega)} \, d\omega \;=\; L_i \tag{40}$$

where $l_{i(n)}$ and $P_{i(n)}$ are the values of unit labor demands and export prices that apply to region i when it takes the n^{th} position in an equilibrium supply chain for variety ω that is completed in country j. $\alpha_j(\omega) L_j w_j$ corresponds to expenditures on the final good (equation 2).

Two-country case: trade elasticity and welfare

A central question in the trade literature is the link between trade and welfare. Arkolakis et al. (2012) show that several theoretical models summarize the trade welfare link with a simple formula requiring only the home trade share and a trade elasticity parameter. Our model of sequential production does not provide such a simple summary of trade and welfare links, but we are able to make a direct analytical comparison of our model's framework, relative to the Arkolakis et al. (2012) benchmark, in a two-country setting.

We consider country D and country U with homogeneous θ equal to θ_D and θ_U respectively. To justify these country names, we assume that $\theta_U > \theta_D$ which implies that country U is always upstream and country D is downstream when there is production sharing.

The first relationship we study is the response of trade to trade costs. While the previous sections examine the fraction of value-added by a country along a given chain (an intensive margin), here we examine how trade costs affect the fraction of varieties that a country sources from another country (an extensive margin) as a function of trade costs.

Country D, in particular, relies on imports from U to produce some goods that it exports back to U. As in Yi (2010), this back-and-forth trade generates a higher trade elasticity. There are two

[30] In the working paper version (NBER working paper 21520), we also examine a fully tractable case as in Costinot et al (2012) with more than two countries, symmetric chains and frictionless trade.

reasons for that. When trade costs increase by 1%, the price of goods imported by U from D increases by more than 1% since the production of final goods in D relies itself on goods imported from U (a double penalty). The second reason is that, even if there were no double penalty of trade costs, labor costs in country D would need to be strictly more than 1% lower to offset a 1% increase in the price of its exports when its labor only contributes a fraction of the value of the good. Considering the extensive margin, this implies a larger decline in the fraction of goods sold by D when trade costs increase. Combining these two effects, we find that the trade elasticity for country U, on the extensive margin, is larger with than without fragmentation of production across countries:[31]

$$\frac{d \log \left(\frac{\pi_{ij}}{\pi_{ii}}\right)}{d \log \tau} \quad \leq \quad -\xi$$

where π_{ij} is the share of products from country j among final goods purchased by consumers in country i, and ξ is the Frechet parameter that defines the dispersion of productivity draws in each country. Moreover, as we describe in Appendix, lower trade costs generate more fragmentation of production and therefore increase the trade elasticity. Because lower trade costs lead to more fragmentation, the foreign labor content embodied in the marginal variety increases. When trade becomes frictionless, the foreign labor content for this marginal variety converges to unity and the trade elasticity can, in theory, go to infinity.

We also examine Johnson and Noguera (2012a)'s "Value-added-to-export" (VAX) ratio, which compares a country's value added embodied in its exports to its gross export value.[32] A decrease in the VAX ratio reflects an increase in fragmentation across borders, because embodied import value accounts for a larger share of gross export value (Johnson 2014). In our two-country model, the VAX ratio is below unity in both countries. The upstream country sells a combination of intermediate and final goods, while the downstream country adds value to the upstream country's intermediates. In the Appendix, we show that this back-and-forth trade grows faster than other trade flows as trade costs decrease, which implies that the VAX ratio for country U decreases as trade costs decrease. These results are both intuitive and supported by recent empirical evidence. In particular, Johnson and Noguera (2012b, 2017) use multi-country input-output tables to show that the VAX ratio has decreased over the past decades and that the bilateral VAX ratio depends positively on bilateral trade costs.

Next we turn to the distributional question of how trade affects welfare in upstream and downstream countries. To make this comparison we derive formal measures of the gains from trade in U and D, and compare them to the ACR formula developed by Arkolakis et al. (2012). Arkolakis et al. (2012) express the welfare gains from trade as $\Delta \log \left(\frac{w_i}{P_i}\right) = -\frac{1}{\xi} \log \pi_{ii}$, where π_{ii} represents the domestic share of consumption. This formula applies in a wide set of models, including Eaton and Kortum (2002, henceforth EK). In a simple EK model, each country benefits from trade in final goods because trade allows both countries to purchase rather than produce the varieties for which they have

[31]For country D, this elasticity remains unchanged as the goods it imports from U are entirely produced in U.

[32]The inverse VAX ratio for the world can also be interpreted as the embodied number of border crossings (Fally 2012)

comparative disadvantage. This same source of gains from trade applies in our model, but the introduction of international fragmentation also has asymmetric effects across countries. We can express gains from trade in our two-country model relative to the ACR formula and describe for each country whether it over- or understate the gains.

We first consider the situation in country D, for which the welfare gains are characterized by:

$$\Delta \log \left(\frac{w_D}{P_D} \right) \;=\; -\frac{1}{\xi} \log \pi_{DD} \;+\; \frac{1}{\xi} \int_{\omega_D^*}^{1} \frac{w_U \, l_U(\omega)}{P_D(\omega)} \frac{d\omega}{1-\omega}, \tag{41}$$

where ω indexes chains and final goods varieties ranked by productivity of D relative to U, and where $\pi_{DD} = 1 - \omega_D^*$ represents the share of the varieties with final goods produced in country D (see Appendix A for details). The first term on the right hand side represents the standard gains from trade, as calculated in the ACR formula. The second term on the right hand side, which is positive, involves the trade elasticity and the cost share of foreign labor embodied in D's production of final goods. This term captures D's additional gains from trade that arise because D obtains cheaper inputs when it offshores upstream tasks.

By contrast, for country U the inclusion of international fragmentation in a trade opening produces smaller gains from trade relative to the ACR benchmark. In appendix A we show that the gains from trade in country U can be expressed as

$$\Delta \log \left(\frac{w_U}{P_U} \right) \;=\; -\frac{1}{\xi} \log \pi_{UU} \;-\; \frac{1}{\xi} \int_{\omega_U^*}^{1} \frac{w_U \, l_U(\omega)}{P_D(\omega)} \frac{d\omega}{\omega}, \tag{42}$$

where π_{UU} is equal to ω_U^*, the cutoff variety as it applies to consumers in U. The first term once again corresponds to welfare gains in the ACR benchmark. The second term is again a function of the Frechet dispersion parameter ξ and U's cost share in D's production; it reflects the value-added contribution from U that is embodied in the final goods that it imports from D. However in U's case the adjustment term is negative. Relative to the benchmark model, U gains less from the varieties of final goods produced in D because it has itself contributed to a significant portion of D's final good production.

We summarize the results on the trade cost elasticity, VAX ratio and welfare in Proposition 3:

Proposition 3 *With two countries, the effect of trade costs on trade is such that:*
i) The elasticity of trade in final goods to trade costs is higher than without fragmentation;
ii) This elasticity is larger when trade costs are smaller;
iii) The value-added content of trade decreases as trade costs decrease.
iv) Welfare gains from trade are larger than $-\frac{1}{\xi} \log \pi_{ii}$ *for country* D *and smaller for country* U*.*

While these results are shown here only for a two-country case, our counterfactual simulations in Section 5 suggest that these insights hold more generally. In what follows, we calibrate our model and compute gains from trade, trade elasticities and VAX ratios by using input-output tables and information on domestic and foreign labor content which, as shown above, are crucial to obtain a more adequate measure of the gains from trade when production is fragmented across borders.

5 Quantitative analysis

5.1 Data

Our main sources of data are the Asian input-output tables developed by IDE-JETRO. These tables provide information on gross output, value-added, and (most importantly) input purchases by product, parent industry (downstream industry), source country and destination country. For instance, the data report the amount of metals purchased from China by the auto industry in Japan. These 4-dimensional input-output tables, are, as far as we know, the only tables that track international transactions directly, rather than imputing them from trade flows. This is an exceptional data set for investigating the organization and evolution of international production fragmentation in a region of the world where fragmentation is an important feature of international trading relationships.[33]

The dataset covers 9 Asian countries and the US.[34] Our analysis mostly focuses on the year 2000 (with most disaggregated product classification), but we also compare our results to IDE-JETRO data from 1975 and 1990. This period marks a time in which the region began to emerge as an important location for internationally fragmented production (see Baldwin and Lopez-Gonzalez (2015) for example).

Information on input purchases and production is disaggregated at the 76-sector level in 2000. For the sake of comparison to previous input-output tables (1975 and 1990), we also construct a more aggregated 46-sector classification to obtain harmonized product categories across years. The sector classification is far more detailed for manufacturing goods and commodities than services (among the 46 sectors, only 5 of them are service industries). We thus mostly restrict our attention to tradable goods: commodities and manufacturing goods.

The information provided in the IDE-JETRO tables goes beyond a simple aggregation of country-level input-output tables. Besides the harmonization of product categories, input flows by parent industry and source countries are estimated using supplementary surveys about firms' input choices. This supplementary information informs deviations from the proportionality assumption, which, according to Puzzello (2012), is rejected in these data.[35] This constitutes an important advantage

[33]Like other international IO tables such as the World Input-Output Database or Global Trade Analysis Project database, the Asian IO tables are constructed by merging harmonized national IO tables with international trade statistics. The Asian tables supplement these sources of information with a survey of input users, who report the specific country-of-origin of their inputs. See Meng et al. (2013) for details.

[34]The countries in the data base are the US, Japan, China, Taiwan, Korea, Singapore, Malaysia, Thailand, Indonesia and the Philippines.

[35]The proportionality assumption is made to construct input purchase by source country and parent industry when only partial information is provided. For instance, traditional country-level input-output tables describe how much steel is used by the auto industry in each country. Using trade flow data (which describe how much steel is imported from a particular country), previous international input-output tables have been constructed by allocating the use of input across source countries on a proportional basis.

of using the IDE-JETRO input-output compared to previous attempts at constructing input-output tables based on the proportionality assumption (as in Johnson and Noguera 2012, for example).[36]

5.2 Measuring the position along the chain: indexes N, U and UX

To better understand the degree of fragmentation in vertical production chains we adopt four indexes that generalize the two indexes proposed in Fally (2012) and Antras, et al. (2012) and applied there to US data. The indexes are designed to describe industries' position in vertical production chains by exploiting information about relationships in the input-output table. The 'U' index measures an industry's weighted average distance to final demand, or "upstreamness", as measured by the apparent number of firms visited by the industry's output before reaching consumers. The 'N' index calculates, for each industry, the number of stages that are embodied in each industry's production. These two calculations are distinct for each industry, and as in the US data there is only a weak correlation between them.

Distance to final demand or "upstreamness": We turn to a formal representation of the two indices. Consider a variable U_{ik}, which is intended to measure the distance of a product k from final demand. Some part of product k's sales will be intermediate trade purchased by downstream industries, so the industry in question's distance measure will depend upon which industries buy its output, and in turn how far those downstream industries are from final demand. Because an industry's sales go to several industries, which will vary in their respective measures of U, the industry measure must be weighted, and it must also be defined recursively. Let U_{ik} indicate the distance measure in region i for product k. We define U_{ik} as:

$$U_{ik} \;=\; 1 \;+\; \varphi_{ikik} U_{ik} \;+\; \sum_{(j,l)\neq(i,k)} \varphi_{ikjl} U_{jl} \tag{43}$$

where φ_{ikjl} denotes the share of output from sector k in country i that is used in sector l in country j. Taking φ_{ikjl} from the data, the entire system of equations that includes a U_{ik} for each industry and country can be solved to produce a measure for each sector-country pair.[37]

As shown in Antras, et al. (2012), this index can be interpreted as the average number of stages of production an industry's output passes through before reaching final consumers. Using the input-output matrix, we can decompose the different trajectories taken by the good across and within industries. Each trajectory is associated with a specific number of transactions across or within industries. Index U would then correspond to the average number of transactions weighted by the fraction of output corresponding to each trajectory. Notice that U_{ik} does not only rely on inter-industry

[36] Ideally, one would want to use firm-level data tracking chains across multiple countries in order to study why countries specialize at different positions along chains. While recent research has examined the structure of chains by matching suppliers and buyers within a country (e.g. Bernard et al. (2016)) or between two countries (e.g. Bernard et al. (forthcoming)), firm-level data of these kinds do not provide information that allow us to track chains on their entire length across multiple countries. The input-output tables allow us to do so, though at a more aggregate level.

[37] More details on the calculation of U_{ik} are available in Antràs et al. (2012). Our data do not track flows within Rest of World, so we impose the average U_{ik} among our countries for each industry there (robustness checks in Appendix B).

linkages but also depends on the extent of fragmentation within each industry. If an industry's production is partly used an input by other firms in the industry (e.g. electronic parts are used as inputs into other electronic parts within the same country), the coefficient φ_{ikik} would be strictly positive and would contribute to a higher index U_{ik} (since it would also correspond to a higher number of transactions).[38]

Embodied stages: The N_{ik} index captures a weighted average of the number of firms involved sequentially in the production of good k in country i. It is defined recursively by:

$$N_{ik} \;=\; 1 \;+\; \mu_{ikik} N_{ik} + \sum_{(j,l)\neq(i,k)} \mu_{ikjl} N_{jl} \tag{44}$$

where μ_{ikjl} denotes the amount of input l from country j used to produce one dollar of product k in country i. This is a single equation, but, as with the U index, the system of equations can be solved to produce a measure of N for each sector-country pair.[39] As shown in Fally (2012),[40] this index can also be expressed as a weighted average of the number of stages required to produce good k in country i, weighted by how much each stage of production contributes to the final value of that good.

Aggregation: Using the IDE-JETRO data, the U and N statistics are calculated at the level of country-industry (i, k) pairs. For the calibration exercise that follows a country-level statistic will be useful so as to better describe countries' average position in global supply chains. A weighted average across statistics is most suitable, although there are several options for defining weights, including value added- or export-weighting for U and output-weighting for N. As argued in Fally (2012), a natural weight for the upstreamness index U is value added, and final production by sector-country for index N.

In order to best capture the relative position of countries on international production chains, the aggregate we use in calibration is an export-weighted average of index U_{ik}. This export-weighted average is the statistic calculated in Antràs et al. (2012) to document countries' comparative advantage along production chains. Formally, we define UX_i by country with the following:

$$UX_i = \frac{\sum_k X_{ik} U_{ik}}{\sum_k X_{ik}} \tag{45}$$

where X_{ik} represents country i's exports of product k.

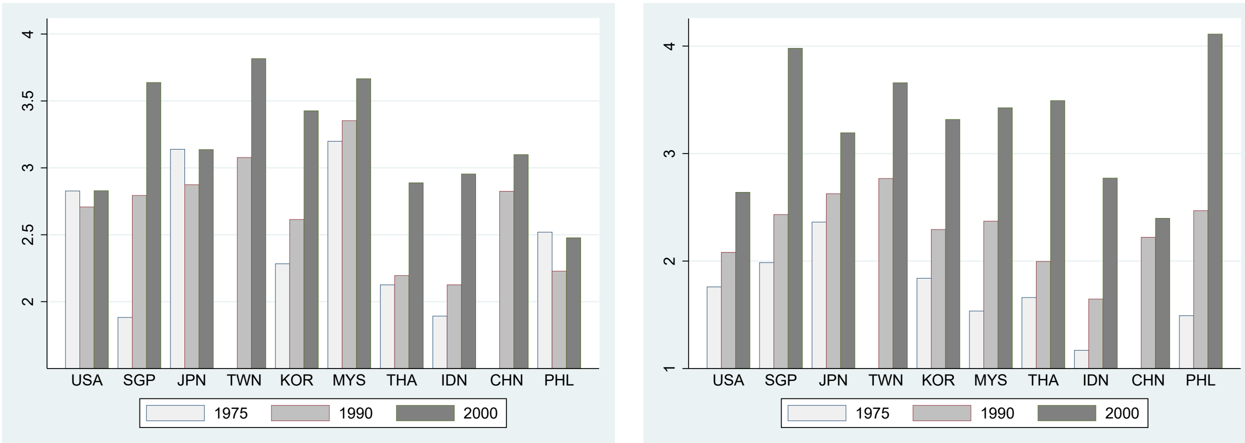
Descriptive statistics: We calculate these indices using the IDE-JETRO data from 1975, 1990 and 2000. First, we examine at upstreamness, weighted by value-added, across countries and years. Figure 1(a) exhibits the results for all tradable goods. There is some variation in the levels and trends

[38] One may argue that the industry classifications are too aggregated and create biases in computing U_{ik} and N_{ik} compared to what would be obtained with more precise data. Fally (2012) examines the aggregation properties of indexes U and N and shows that aggregating industries does not much affect the average of U and N across industries.

[39] See Fally (2012) for details on the calculation of these indices.

[40] See Proposition 1 in Fally (2012).

Figure 1: Average upstreamness index by country for 1975, 1990 and 2000



(a) Manufacturing (b) Electronics

of upstreamness index U across countries. This index increases over time for most countries (with Japan and the US as notable exceptions), which suggests that chains have become longer or that the countries in question have moved into upstream positions along production chains.

In this graph, countries are sorted by their per capita GDP's. One can see that there is no monotonic relationship between per capita GDP and average upstreamness. Countries at both end of the spectrum tend to be downstream while middle-income countries are relatively more upstream. This is not in line with the model developed by Costinot et al (2012) where more productive / higher income countries tend to be located downstream.

In Figure 1(b), we report results of our index for the electronics sector only, by country and by year (still accounting for input-output linkages across and within industries as described in equation 43). The electronics sector is particularly interesting over this period. Complex international production chains are, anecdotally, an important phenomenon in East Asian manufacturing. This is even more notably so within the electronics sector. Moreover, there has been important growth in the region's trade in electronics, which constituted only 8% of Asian exports in 1975, and 34% in 2000. For electronics, there has been a sharp upward movement in index U for most countries, which is consistent with increasing fragmentation of production chains in Asia in the electronic industry. Some countries such as China and the US remain downstream while other countries such as the Philippines have moved upstream. But again, there is no clear monotonic relationship between upstreamness and GDP per capita as predicted by Costinot et al (2013).

As one could expect, primary commodities such as ores and feeds are associated with higher upstreamness while finished goods tend to have lower upstreamness index values. In an appendix, we describe the variations in U and N across industries, showing that the two indexes are not strongly correlated. The two indexes thus provide a degree of independent evidence when we evaluate the calibrated model's fit to untargeted moments.

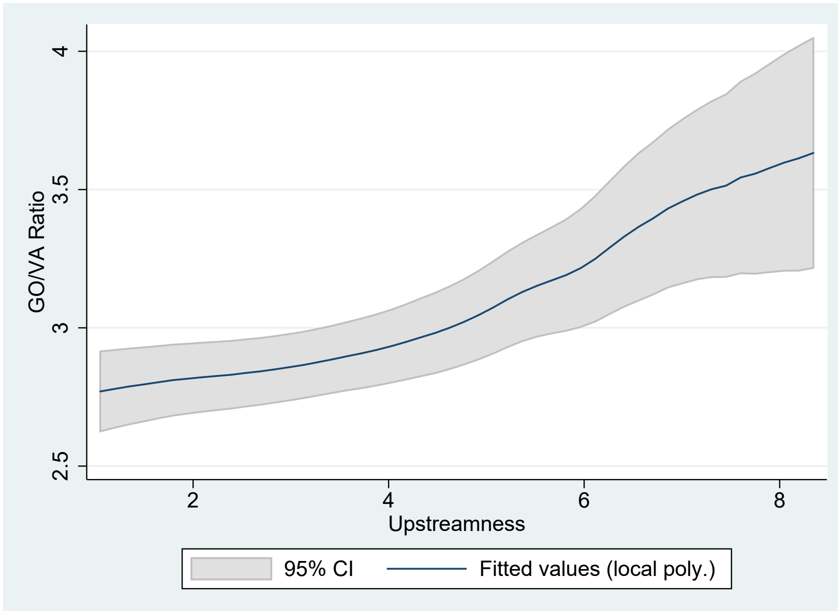


Figure 2: Gross-output-to-value-added ratio as a function of upstreamness U_{ik}

Correlation between upstreamness, value-added content and transaction costs: The model predicts that: i) countries with larger scope parameter θ should specialize upstream while countries with low θ should specialize downstream (Proposition 1); ii) the gross-output-to-value-added (GO/VA) ratio increases with diseconomies of scope θ (Lemma 2); iii) the GO/VA ratio decreases with transaction costs γ (Lemma 2).

By combining predictions i) and ii), we should observe a positive correlation between upstreamness and the GO/VA ratio. This correlation should hold after controlling for transaction costs γ_i. Moreover, we should observe a negative correlation between the GO/VA ratio and proxies for transaction costs γ_i. Here we use the "cost of enforcing contracts" from the Doing Business database (World Bank), but similar results obtain with measures from the Doing Business database on the "time to enforce contracts" or the recovery rate in insolvency proceedings.

Figure 2 shows a strong positive correlation between upstreamness and the GO/VA ratio. This finding supports the prediction that firm scope tends to be smaller upstream than downstream. If firm scope were fixed and held constant along production chains, the GO/VA ratio would be negatively correlated with upstreamness because a constant measure of value added per firm would accrue to increasing levels of gross output as production moved downstream. Our model with endogenous firm scope generates equilibria with relatively more value added by downstream firms, thereby allowing it to replicate the positive relationship observed in Figure 2.

In Table 1, we confirm the result from Figure 2 by regressing the gross-output-to-value-added ratio on upstreamness. We find a significant and positive correlation between the two, whether we include country fixed effects (column 2), industry fixed effects (column 3) or both (column 4). Consistent with the model, the correlation is the strongest when most of the variation is driven by cross-country

Table 1: GO/VA ratio and upstreamness U_{ik}

	(1)	(2)	(3)	(4)
Dependent var.:		GO/VA ratio		
Upstreamness	0.147	0.132	0.244	0.161
	[0.034]**	[0.031]**	[0.069]**	[0.060]**
Industry FE	No	No	Yes	Yes
Country FE	No	Yes	No	Yes
R2	0.05	0.16	0.49	0.59
N	344	344	344	344

Notes: OLS regression with robust s.e.; by country and sector in 2000, excluding services and trimming outliers with upstreamness above 10; ** significant at 1%.

variation (column 3), i.e. when we include industry fixed effects but no country fixed effects. Even assuming our model structure, the correlation might not be expected to emerge if the structural parameters γ_i and θ_i were strongly correlated across countries. Nevertheless, we do see the positive relationship.

As shown in Table 2, our results also corroborate another prediction of the model, that the GO/VA ratio decreases with transaction costs γ_i. This result is intuitive: higher transaction costs lead to fewer transactions and a larger range of activities performed internally, hence a higher value-added content and a lower GO/VA ratio. Note also that the correlation with upstreamness is robust to controlling for γ_i using proxies for transaction costs (columns 3 to 5). Table 2 also indicate that GDP per capita does not drive these results as it does not enter significantly in columns (4) or (5).

5.3 Calibration

Our calibration exercise focuses on the 10 countries that are covered by the IDE-JETRO input-output tables. The general equilibrium model described above is calibrated so as to reproduce key features of the data. The parsimony of the model allows us to consider only a small number of parameters to calibrate, those listed in the left column of Table 3.

Thanks to Lemma 2 and the analytical results described in section 3.2, we can reduce the optimization problem described in equation (14) to a linear programming problem for each chain. We consider chains of varying length, and identify the low cost chain completed in each destination country. The supplier of final goods to each market is the country with the lowest delivery cost, gross of trade costs.[41] Numerical simulations are performed in Matlab. We approximate a continuum of varieties by assuming 1,000,000 different final goods.

We now describe each calibrated parameter and its targeted moment, as described respectively in the left and right columns of Table 3.

[41]In general, if trade costs are sufficiently high, the final good associated with a single a variety may be produced for export by one country and for domestic consumption by one or more other countries.

Table 2: GO/VA ratio and contract enforcement

	(1)	(2)	(3)	(4)	(5)
Dependent var.:			GO/VA ratio		
Cost of enforcing contracts	-0.385 [0.136]**	-0.376 [0.083]**	-0.329 [0.082]**	-0.255 [0.094]**	-0.264 [0.097]**
Upstreamness			0.226 [0.069]**	0.213 [0.068]**	0.212 [0.066]**
GDP per capita				0.067 [0.048]	0.003 [0.049]
Population					-0.086 [0.036]*
Industry FE	No	Yes	Yes	Yes	Yes
R2	0.02	0.48	0.50	0.50	0.52
N	344	344	344	344	344

Notes: OLS regression with robust s.e.; by country and sector in 2000, excluding services; the cost of enforcing contracts is from the Doing Business Database; population and (output-based) per capita GDP are from the Penn World Table 9 (in log); * significant at 5%, ** significant at 1%.

Labor supply: Each country is endowed with an exogenous supply of factors. In the benchmark case, we consider only one factor of production: labor. For each country, we choose the labor force L_i to match aggregate value-added in tradeable goods sectors (i.e. excluding services) divided by the cost of labor (proxied by income per capita in our benchmark simulation).

Labor productivity: Labor productivity is calibrated such that labor demand equals labor supply, while wages are set equal to per capita income, which we obtained from the Penn World Tables for the year 2000.

As in Eaton and Kortum (2002), average labor productivity across all varieties is equal to $\bar{A}_i \equiv T_i^{-\frac{1}{\xi}}$ where T_i is the shift parameter for the Weibull distribution of $A_i(\omega)$ in country i (equation 12). The dispersion parameter $\xi = 5$ is calibrated based on recent estimates such as Simonovska and Waugh (2014). Conditional on wages and calibrated parameters, we can compute labor demand for each country. Equality between aggregate labor demand and labor supply is attained by adjusting labor productivity $\bar{A}_i$ (or, equivalently, by adjusting T_i). Note that there is a tight link between wages and implied labor productivity in country i. As shown in Table 3, there is a nearly log-linear (downward-sloping) relationship between wages w_i and $\bar{A}_i$ in our benchmark calibration.

Diseconomies of scope: As shown in Proposition 1, the scope parameter θ_i is a key determinant of the position of a country along the chain, downstream or upstream. A country tends to export final goods when diseconomies of scope are weak and export intermediate goods when they are strong. Since all countries export a mix of final and intermediate goods, we assume that θ_i is heterogeneous across varieties ω, as discussed in section 4.3. We assume that it is log-normally distributed. In calibration countries are allowed a different shift parameter $\bar{\theta}_i$ and a different standard deviation $\bar{\sigma}$. We use UX as the primary moment to calibrate $\bar{\theta}_i$, and calibrate $\bar{\sigma}$ to fit countries' intermediate share in total exports. While the correlation of the $\bar{\theta}_i$ and UX_i in Table 3 is weak (0.22), the prediction of the model is confirmed: countries that are more upstream have higher average values of $\bar{\theta}_i$. Most countries share similar values of UX_i, with China and, to a lesser degree, Thailand positioned relatively downstream, while Indonesia is relatively upstream.[42]

Transaction costs: Another key parameter of the model is γ_i, the cost of transactions between two firms. This cost is assumed to be positive even for transactions that occur within borders. Transaction costs are difficult to estimate in practice, but our model indicates that the gross-output-to-value-added ratio equals the ratio of diseconomies of scope and transaction costs parameters $\frac{\theta_i}{\gamma_i}$ and thus can be used to retrieve an estimate of γ_i once we know $\bar{\theta}_i$. Results are provided in Table 3. As a credibility check we compare our results to plausible real-world counterparts of the γ_i parameter using the Doing Business Database (World Bank). Reassuringly, we find expected correlations of our calibrated γ_i variables with the costs of enforcing a contract claim (0.69), the time to enforce contracts (0.29), and the recovery rate in insolvency proceedings (-0.39). We note that Singapore has unusually low transaction costs in the calibration, which is consistent with its high gross output to value added ratio. At the other end of the spectrum, Indonesia has the highest transaction costs, which is consistent with the Doing Business indicators and recent literature (Olken and Barron, 2009).

Trade costs: In addition to transaction costs that are also incurred within borders, cross-border transactions face an additional burden. The nature and size of the extra cost affecting international transactions is the matter of an extensive debate in the trade literature (transport costs, asymmetric information, marketing cost, technical barriers, or cultural differences are plausible sources of such costs). There is however a consensus that these costs are large and have a large effect on cross-border trade. While distance usually plays a key role in explaining the pattern of international trade (see Disdier and Head, 2008), distance is not as crucial for trade among IDE-JETRO countries.[43] We therefore assume that cross-border trade costs are uniform across all country pairs in our sample. We fit the trade cost parameter by asking the model to replicate the global ratio of trade to output.

Note that, as a consequence of trade costs, not all countries participate in a given production chain. In our fitted model, less than one percent of the chains involve more than three countries.

Simulation results on other moments: Before turning to the counterfactual results we briefly describe the benchmark equilibrium. By construction, our model is able to reproduce key indicators

[42]The average diseconomy of scope parameters for the United States and Japan are quite high in the calibration, even though these countries' upstreamness measures are in line with some other countries in the sample. Meanwhile Singapore has a low calibrated value of $\bar{\theta}$. *Ceteris paribus*, larger countries tend to be downstream – a mechanism highlighted in Antràs and de Gortari (2017) that also operates in our model. With this mechanism, our calibration procedure tends to assign higher θ to large countries to match observed upstreamness.

[43]In a standard gravity equation of trade using the IDE-JETRO data, the coefficient for distance is not significant while the estimated border effect is large and significant, economically and statistically. Of course, this is a small sample of only 10 countries.

Table 3: Parameter choice and moments to match

Parameters:			*Moments to match:*		
Average $\bar{A}_i = T^{-\frac{1}{\xi}}$	USA	1.000	GDP per capita	USA	35,080
by country	SGP	1.284	(PWT)	SGP	32,808
(relative to the US)	JPN	0.484		JPN	26,721
	TWN	1.463		TWN	21,891
	KOR	1.434		KOR	17,208
	MYS	2.644		MYS	7,917
	THA	2.847		THA	5,178
	IDN	3.182		IDN	2,549
	CHN	3.193		CHN	2,442
	PHL	3.680		PHL	2,210
Labor supply	USA	53,551	Total value-added	USA	1,878.6
in tradeable goods	JPN	41,665	in tradeable goods	JPN	1,113.3
(x1000 workers)	SGP	735	(in $M)	SGP	24.1
	TWN	3,889		TWN	85.1
	KOR	10,491		KOR	180.5
	MYS	5,637		MYS	44.6
	THA	10,410		THA	53.9
	IDN	36,585		IDN	93.3
	CHN	266,707		CHN	651.3
	PHL	13,618		PHL	30.1
Average scope param. $\bar{\theta}_i$	USA	0.875	UX Index	USA	2.923
by country	SGP	0.384	(Export weighted)	SGP	2.879
	JPN	0.760		JPN	2.663
	TWN	0.444		TWN	2.853
	KOR	0.560		KOR	2.892
	MYS	0.390		MYS	2.679
	THA	0.455		THA	2.439
	IDN	0.679		IDN	2.957
	CHN	0.530		CHN	2.009
	PHL	0.515		PHL	2.776
Transaction costs γ_i	USA	0.378	aggregate GO / VA ratio	USA	2.599
by country	SGP	0.209		SGP	3.842
	JPN	0.336		JPN	2.745
	TWN	0.216		TWN	3.865
	KOR	0.276		KOR	3.224
	THA	0.210		THA	3.718
	MYS	0.276		MYS	2.801
	IDN	0.460		IDN	2.161
	CHN	0.249		CHN	3.049
	PHL	0.359		PHL	2.434
Simple average border cost	All	15%	Trade/output ratio	All	26%

of fragmentation such as the gross-output-to-value-added ratio and values of UX. Alternatively, we can examine how the fitted model fares in terms of other indexes such as indexes U, N and the share of intermediate goods in exports, M-share.

Table 4 compares indexes from the model vs. data. In broad terms the magnitudes of U and N are consistent with the data, even though they are constructed in very different ways. These indexes from the data are computed at the industry level then averaged across industries. Index N in the benchmark calibration is computed for the most downstream firm in the chain while index U is a weighted average across firms weighted by value-added at each stage. The levels of U and N are approximately the same in model and data, and the cross-country correlations are high.

The calculated share of intermediates in exports is generally lower in the calibration than in the model. Recall that our model has no scope for back-and-forth trade in intermediates. Neither does our model have an explicit role for assembly nor multiple sources of inputs ("spiders"). Any of these features would lead real world data to report higher shares of intermediates in exports.[44] We nonetheless find M-share useful for model analytics in subsequent counterfactual analysis and so report it here for consistency.

Table 4: Fragmentation indexes: model vs. data

Index	U		N		M share	
	Data	Model	Data	Model	Data	Model
USA	2.829	2.666	3.397	2.871	0.649	0.365
SGP	3.638	3.842	3.833	3.976	0.690	0.240
JPN	3.137	2.735	3.152	3.053	0.596	0.301
TWN	3.817	3.805	3.691	3.997	0.711	0.238
KOR	3.426	3.312	3.565	3.472	0.676	0.280
MYS	3.666	3.654	3.453	3.897	0.640	0.216
THA	2.888	2.869	3.432	3.156	0.609	0.216
IDN	2.955	2.786	2.642	2.579	0.675	0.353
CHN	3.099	2.771	3.255	3.360	0.439	0.162
PHL	2.477	2.824	2.725	2.798	0.692	0.280
Correl. with data		0.882		0.843		0.506

5.4 Counterfactual simulations

East-Asian economies have been the setting for tremendous changes in recent decades. Arguably the most significant changes are the increased fragmentation of production, China's opening to international trade and its subsequent rapid economic growth. In our theory these phenomena can certainly be related, as China's opening to trade could have facilitated fragmentation along chains in which it is now involved. Rapid economic growth may be associated with trade-related increases in productivity, but multi-factor productivity growth not specifically related to trade might also have been important. With a calibrated model at hand, we can now examine various counterfactual simulations to study how structural changes would affect economic outcomes such as output, trade, welfare and the fragmentation of production.

We see at least four experiments that would provide interesting insight into the reorganization of supply chains in Asia:

- Counterfactual 1): Trade costs have fallen significantly over the past decades and their reduction is cited as the most likely source of the increased fragmentation of production in Asia. Trade costs

[44] An earlier draft of this paper showed that a capstone assembly sector can easily be added to our model, see NBER working paper version 21520.

may decline even further in the near future, as there is still room to improve trade agreements, especially on a multi-lateral basis (Baldwin, 2008). We model this structural change with a 10% reduction in cross-border trade costs.

- Counterfactual 2): Arguably the most dramatic economic change in Asia over the past two decades has been the very high rates of growth in the Chinese economy, along with its opening to trade. With GDP growth rates reaching 10%, the Chinese economy is inducing very large changes in how production in Asia is organized. To examine the role of China in light of our model, we shock the productivity parameter there. We first simulate a 10% productivity increase in China. Formally, this corresponds to a 10% increase in $T_{CHN}^{\frac{1}{\xi}}$.[45]

- Counterfactual 3): We simulate a reduction in the transaction costs γ_{CHN} for China. This scenario could be used to understand growing transparency in contractual disputes, for example. Reduced transaction costs should encourage relatively more domestic outsourcing in China, and raise the share of domestic value added in production/exports.[46]

- Counterfactual 4): Finally, we consider a bilateral trade cost reduction. This allows us to offer a local estimate of the trade elasticity. In particular we are interested in a quantitative evaluation of the claim in Proposition 3, that the elasticity of final goods trade to trade cost changes is larger in the presence of fragmentation. In order to do this, we reduce trade costs between China and the US.

5.4.1 Reduction in trade costs

Our first scenario is a 10% reduction in international trade costs τ. These results are reported in Table 4, which reports reductions in trade weighted upstreamness (UX_i), the intermediate share of trade (M-share) and the value added content of trade (VAX). Reductions in VAX indicate a lower value added content of trade, which occurs because a larger fraction of exported goods now use imported intermediate goods. This change is consistent with Hummels, Ishii and Yi (2001) and Johnson and Noguera (2012), who document a decrease in the VAX ratio over the decades.

Reductions in VAX do not imply, however, that trade is growing faster for upstream goods. Indeed, our simulations indicate that trade grows faster for downstream stages, as is evident in the reductions in both UX_i and M-share.[47] This finding illustrates that final goods trade is more sensitive than intermediate goods trade to reduced trade costs (see Proposition 3): trade flows increase relatively more for goods that embody small shares of domestic labor and larger shares of traded intermediate goods.

[45] Barrot and Sauvagnat (2016) study the effects of natural disasters operating through input suppliers on downstream firms. In our model such shocks would best be represented as negative productivity shocks on the countries affected by the disasters, and these effects could be traced through the entire chain as in Counterfactual 2.

[46] Kee and Tang (2013) and Li and Liu (forthcoming) provide evidence that the share of domestic value added in Chinese exports has been growing over time.

[47] Fally (2012, Figure 4) documents reductions in trade-weighted upstreamness over a period (1962-1996) when trade costs arguably fell substantially.

Table 4 also reports changes in welfare, given the trade cost shock. For comparison purposes, we also provide welfare results for two versions of an Eaton and Kortum model, a one-stage model without intermediate goods and an alternative model featuring intermediate production (EK-loop). In the EK-loop model, all goods are both final good and intermediate goods. Production costs are a Cobb-Douglas function of labor costs and good price indexes in each country. We calibrate the share of labor such that it equals the value-added-to-gross-output ratio in each country. Hence, all varieties are traded internationally multiple times and welfare gains from trade are magnified by a factor equal to the gross-output-to-value-added ratio.[48]

In level terms the calculated welfare gains in our model are much more similar to the standard EK model than to the EK-loop model. As illustrated in Proposition 3, our supply chain model does not necessarily yield larger welfare gains than one-stage models, especially for countries that tend to be upstream. On average our simulations yield relatively larger gains for downstream countries such as Japan and China, and lower gains for some upstream countries such as Singapore, compared to the one-stage model. In the EK-loop model, all of value added is exposed to trade costs, repeatedly, so the welfare costs of trade cost changes are significantly higher. The data have GO/VA ratios between two and three, so welfare gains in the EK-loop model are multiplied by more than a factor of two. This leads to systematically larger gains from trade then our sequential production chain model.

Table 5: Counterfactual 1): 10% decrease in border trade costs

(10 x change)	Fragmentation					Welfare		
Country	UX	U	N	M share	VAX	Model	EK 1-stage	EK loop
USA	-0.075	0.007	0.008	-0.002	-0.076	0.060	0.054	0.147
SGP	-0.441	-0.383	-0.193	-0.067	-0.099	0.209	0.240	0.537
JPN	-0.197	-0.046	0.019	-0.025	-0.077	0.094	0.069	0.209
TWN	-0.277	-0.197	-0.089	-0.031	-0.083	0.186	0.186	0.480
KOR	-0.238	-0.103	-0.023	-0.028	-0.080	0.170	0.156	0.398
MYS	-0.263	-0.217	-0.052	-0.028	-0.085	0.202	0.222	0.439
THA	-0.249	-0.199	0.004	-0.033	-0.092	0.196	0.198	0.408
IDN	-0.224	-0.022	0.061	-0.050	-0.078	0.189	0.180	0.325
CHN	-0.084	-0.112	0.006	-0.003	-0.084	0.117	0.085	0.288
PHL	-0.210	-0.112	0.024	-0.034	-0.087	0.197	0.202	0.379

5.4.2 Increasing labor productivity in China

Table 5 reports results from a 10% shock to labor productivity in China, $T_{CHN}^{\frac{1}{\xi}}$. This is a uniform shock that improves labor productivity at all points in the chain. Our interest is in seeing how such shocks affect China's relative position in chains, and the degree to which such shocks spill over into other countries.

The most notable effects of the Chinese productivity shock are the associated changes in relative position in supply chains. Changes in China's UX, and U indices indicate that the technology

[48]Costinot and Rodriguez-Clare (2014) model a similar loop with multiple industries.

shock moves Chinese production significantly closer to final demand, while the other countries move upstream. China's move downstream can also be seen in the fourth column (M share), which shows a reduction in the intermediate share of China's exports. This market size effect is similar to the overshooting effect in Baldwin and Venables (2012). With a larger fraction of world income spent by consumers in China and a larger fraction of tasks being performed in China, other vertically-related tasks are also more likely to be performed there to save on trade costs.

The welfare changes reported in column 6 show that the vast majority of the welfare gains accrue to China, which sees a 9.33% increase in welfare from a 10% labor productivity shock. The gains elsewhere are limited, and reasonably similar across countries. In order to put the spillovers in context we again consider shocks to the one-stage EK model and the EK model with input-output loops described above. We apply the same 10% labor productivity shock to the single-stage EK model, and find a somewhat smaller welfare gain in China. Spillovers are approximately the same magnitude – slightly lower in large countries US and Japan, and slightly higher in the other seven countries. Turning to the EK-loop model, the impact of a labor productivity increase in China is much smaller.[49] The benefits are diverted to other countries who benefit from lower intermediate goods prices. Once again the results from our model are closer to the one-stage model than the EK-loop model, even though the EK-loop model is calibrated to match the same GO/VA ratios.

Table 6: Counterfactual 2): 10% increase in labor productivity in China

(10 x change)	Fragmentation					Welfare		
Country	UX	U	N	M share	VAX	Model	EK 1-stage	EK loop
CHN	-0.322	-0.281	-0.171	-0.038	0.000	0.933	0.919	0.784
USA	0.182	0.075	0.011	0.038	0.008	0.022	0.018	0.046
SGP	0.177	0.159	0.109	0.022	0.006	0.031	0.051	0.084
JPN	0.200	0.104	0.024	0.039	0.010	0.021	0.018	0.054
TWN	0.185	0.172	0.096	0.023	0.007	0.036	0.046	0.088
KOR	0.234	0.195	0.063	0.036	0.012	0.026	0.033	0.079
MYS	0.198	0.185	0.053	0.027	0.012	0.028	0.045	0.073
THA	0.101	0.099	-0.003	0.014	0.007	0.036	0.049	0.074
IDN	0.262	0.237	0.051	0.045	0.017	0.029	0.041	0.048
PHL	0.224	0.199	0.028	0.035	0.014	0.037	0.047	0.060

5.4.3 Reduced transaction costs in China

When compared with the productivity shock, the welfare effects of a 10% reduction in transaction costs in China are smaller for China and relatively larger for other countries. In the case of a shock to internal transaction costs, China's production moves upstream, as indicated by the movements in UX, U and the share of exports in intermediate goods. Similarly, the increase in N reflects greater fragmentation of production within China. Other countries also move upstream because falling Chinese transaction costs lead to longer chains. This counterfactual also leads to a higher value-added export to gross

[49] The inclusion of intermediates in the EK loop model means that multiplying labor productivity by 1.1 is equivalent to multiplying TFP by 1.1 x β_{CHN}, where β_{CHN} is the VA/GO ratio in China.

export ratio (VAX) for China, and heterogeneous effect on other countries' VAX ratio. In broad terms, a move upstream by China is consistent with the evidence presented in Kee and Tang (2013), who find that Chinese exporters have been shifting their purchases of inputs from foreign to domestic sources. Relative to the productivity shock considered in the previous exercise, the transaction cost shock produces smaller welfare gains for China, but spillovers to other countries are roughly the same magnitude as in the earlier exercise.[50]

Table 7: Counterfactual 3): 10% decrease in Chinese transaction costs γ_i

(10 x change)	UX	U	N	M share	VAX	Welfare
CHN	1.418	4.646	3.832	0.095	0.090	0.667
USA	0.223	0.130	0.195	-0.003	0.015	0.029
SGP	0.062	0.106	0.155	-0.002	-0.014	0.032
JPN	0.139	0.114	0.190	-0.004	0.006	0.028
TWN	0.067	0.089	0.078	-0.005	-0.005	0.040
KOR	0.166	0.174	0.126	0.007	0.002	0.032
MYS	0.044	0.071	0.024	-0.008	-0.003	0.036
THA	0.081	0.096	0.081	0.003	-0.004	0.038
IDN	0.229	0.250	0.123	0.009	0.001	0.035
PHL	0.183	0.217	0.079	0.001	0.000	0.039

5.4.4 Reduced trade costs between China and the US

As noted above and in Yi (2010), international fragmentation raises the elasticity of trade to trade costs. In order to explore the quantitative magnitude of this effect we shock trade costs for a single country pair (US and China) and measure trade responses. We compare results in our model to those in the calibrated EK model described above (note that one-stage EK and EK-loop models yield the same elasticities). Recall that our two-country model showed a higher elasticity for final goods trade than in the standard EK model. We see that final goods trade is indeed more responsive to trade cost changes in our model, as is the elasticity of total trade.

Table 8: Counterfactual 4): 10% bilateral decrease in US-China trade costs

	Importer-exporter pair	With cross-border fragmentation		EK
		All trade	*Final goods*	*All trade*
$\Delta \log \pi_{ni}$	USA-CHN	0.063	0.065	0.050
	CHN-USA	0.059	0.057	0.043
Trade cost elasticity	USA-CHN	5.510	5.727	4.589
	CHN-USA	6.170	6.201	5.272

[50] Because the EK benchmarks contain no role for transaction costs, we are unable to offer a model comparison for this exercise.

6 Concluding remarks

International supply chains play a large and growing role in international trade. In this paper we attempt to link firm-level behavior within such chains to countries' relative position within the chain and to value added at each stage. We develop a continuous firm representation of the optimal organization of a multi-country supply chain, with an endogenous allocation of tasks across countries and firms. We derive formal and intuitive representations of the gains from fragmentation within a chain, and relate these to the implicit price of tasks and the price of the final good.

In the Coasian framework we adopt, we show that one of the two parameters that shape the boundaries of firms also determines comparative advantage within an international supply chain. Countries where diseconomies of scope are weaker host firms that produce a larger range of tasks. Downstream firms have greater scope in equilibrium, which means that countries with lower diseconomies of scope parameters are located downstream. Conditional on participation in a chain, transaction costs do not affect countries' position in the chain. Transaction costs can play an indirect role in comparative advantage when there are multiple chains, because high transaction costs and strong diseconomies of scope make participation very unlikely.

In order to link the model to the prominent literature on the welfare gains of trade, we use a conventional Ricardian framework to produce a general equilibrium model with multiple chains, and with exogenous productivity shocks *across* chains. In a two-country version of the model we derive implications for trade elasticities and welfare, relative to standard theoretical benchmarks (Arkolakis et al. 2012). We show that the elasticity of final goods trade to trade costs is larger in the presence of fragmentation. We also link a country's gains from trade to its position in the chain. We also compare welfare results in each country to those calculated with the formula in Arkolakis et al (2012).

A key finding in the paper is that the Coasian structural parameters determine the GO/VA ratio. We use this finding to motivate a model validation exercise. The model associates higher scope parameters with higher GO/VA ratios - conditional on transaction costs - and with higher levels of upstreamness. We verify that the implied positive correlation between upstreamness and the GO/VA ratio holds in the data, both unconditionally and conditional on measures of countries' transaction costs.

We then use the model's link to the GO/VA ratio to calibrate a 10-country version of the model to data from East Asia. We shock international trade costs, and illustrate the consequences for welfare and for countries' average position in chains, among other outcomes. We compare welfare predictions to two alternative benchmark models. We also find that shocks to Chinese productivity and to China's transaction costs generate different implications for welfare, for spillovers from China to other countries and for China's relative position within production chains. We also demonstrate that trade responses to trade costs shocks are magnified, relative to the benchmark model.

References

Antràs, Pol and Alonso de Gortari, "On the Geography of Global Value Chains," *Working paper*, 2017.

_ **and Davin Chor**, "Organizing the Global Value Chain," *Econometrica*, 2013, *81* (6), 2127–2204.

_ **,** _ **, Thibault Fally, and Russell Hillberry**, "Measuring the Upstreamness of Production and Trade Flows," *American Economic Review Papers and Proceedings*, 2012, *102* (3), 412–416.

Arkolakis, Costas, Arnaud Costinot, and Andres Rodriguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Baldwin, Richard, "Global Supply Chains: Why They Emerged, Why They Matter, and Where They Are Going," *C.E.P.R. Discussion Papers*, August 2012, *9103.*

_ **and Javier Lopez-Gonzalez**, "Supply-chain Trade: A Portrait of Global Patterns and Several Testable Hypotheses," *The World Economy*, 2015, *38* (11), 1682–1721.

Baldwin, Richard E., "Managing the Noodle Bowl: The Fragility of East Asian Regionalism," *The Singapore Economic Review (SER)*, 2008, *53* (03), 449–478.

Barrot, Jean-Noël and Julien Sauvagnat, "Input specificity and the propagation of idiosyncratic shocks in production networks," *The Quarterly Journal of Economics*, 2016, *131* (3), 1543–1592.

Bernard, Andrew B., Andreas Moxnes, and Karen Helene Ulltveit-Moe, "Two-Sided Heterogeneity and Trade," *Review of Economics and Statistics*, forthcoming.

_ **,** _ **, and Yukiko U. Saito**, "Production Networks, Geography and Firm Performance," *Dartmouth Tuck manuscript*, 2016.

Caliendo, Lorenzo and Fernando Parro, "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 2015, *82* (1), 1–44.

Chaney, Thomas and Ralph Ossa, "Market Size, Division of Labor, and Firm Productivity," *Journal of International Economics*, 2013, *90* (1), 177–180.

Coase, Ronald H., "The Nature of the Firm," *Economica*, 1937, *4* (16), 386–405.

Costinot, Arnaud, "An Elementary Theory of Comparative Advantage," *Econometrica*, 2009, *77* (4), 1165–1192.

_ **and Andres Rodriguez-Clare**, "Trade Theory with Numbers: Quantifying the Consequences of Globalization," *Handbook of International Economics*, 2014, *4*, 197–261.

_ **, Jonathan Vogel, and Su Wang**, "An Elementary Theory of Global Supply Chains," *Review of Economic Studies*, 2013, *80*, 109–144.

Disdier, Anne-Celia and Keith Head, "The Puzzling Persistence of the Distance Effect on Bilateral Trade," *The Review of Economics and Statistics*, 2008, *90* (1), 37–48.

Fally, Thibault, "Production Staging: Measurement and Evidence," *Working paper*, 2012.

Galichon, Alfred, *Optimal Transport Methods in Economics*, Princeton University Press, 2016.

Hillberry, Russell and David Hummels, "Explaining Home Bias in Consumption: The Role of Intermediate Input Trade," *NBER working paper no. 9020*, 2002, pp. 1–33.

Johnson, Robert and Andreas Moxnes, "Technology, Trade Costs, and the Pattern of Trade with Multi-Stage Production," *Working Paper*, 2016.

_ and Guillermo Noguera, "Accounting for Intermediates: Production Sharing and Trade in Value Added," *Journal of International Economics*, 2012, *86* (2), 224–236.

Johnson, Robert C and Guillermo Noguera, "Proximity and Production Fragmentation," *The American Economic Review*, 2012, *102* (3), 407–411.

_ and _ , "A Portrait of Trade in Value Added over Four Decades," *The Review of Economics and Statistics*, 2017, *99* (5), 896–911.

Kikuchi, Tomoo, Kazuo Nishimura, and John Stachurski, "Span of Control, Transaction Costs and the Structure of Production Chains," *Theoretical Economics*, forthcoming.

Koopman, Robert, Zhi Wang, and Shang-Jin Wei, "Estimating Domestic Content in Exports When Processing Trade is Pervasive," *Journal of Development Economics*, 2012, *99* (1), 178–189.

Li, Ben G. and Yibei Liu, "The Production Life Cycle," *Scandinavian Journal of Economics*, forthcoming.

Meng, Bo, Yaxiong Zhang, and Satoshi Inomata, "Applications of IDE-JETRO's International Input Output Tables," *Economic Systems Research*, 2013, *25* (1), 122–142.

Olken, Benjamin and Patrick Barron, "The Simple Economics of Extortion: Evidence from Trucking in Aceh," *Journal of Political Economy*, 2009, *117* (3), 417–452.

Puzzello, Laura, "A Proportionality Assumption and Measurement Biases in the Factor Content of Trade," *Journal of International Economics*, 2012, *87* (1), 105–111.

Rodriguez-Clare, Andres, "Offshoring in a Ricardian World," *American Economic Journal: Macroeconomics*, 2010, *2* (2), 227–258.

Simonovska, Ina and Michael E Waugh, "The Elasticity of Trade: Estimates and Evidence," *Journal of International Economics*, 2014, *92* (1), 34–50.

Yi, Kei-Mu, "Can Multistage Production Explain the Home Bias in Trade?," *American Economic Review*, 2010, *100* (1), 364–93.

Appendix for "A Coasian Model of International Production Chains"

Thibault Fally and Russell Hillberry

A) Mathematical Appendix

Proof of Lemma 1: First best

Sketch of proof. Given that we have constant returns to scale, prices equal unit costs and marginal costs in the competitive equilibrium. With free entry, firms in production chain have exactly zero profits all along the chain in equilibrium, and any chain associated with a lower final good price would have negative profits somewhere along the chain. As we will describe, the equilibrium chain corresponds to the chain that yields the lowest price of final goods under the constraints that: i) all firms along the chain choose their scope to maximize their profits; ii) profits are zero in equilibrium. What is not trivial is that firm scope in the social planner's solution maximizes profits of each firm along the chain. We need to show that, defining the price schedule as the cumulative cost along the chain, profit maximization for each firm along the chain leads to the same decision in firm scope as with the social planner's problem.

For this Lemma, we introduce two pieces of notation:

- We denote by $S_i(f)$ the amount of tasks embodied at stage f in country i. The planning problem's constraint $\sum_i \int_{f=0}^{F} s_i(f)df = 1$ cannot be used directly in the competitive solution, because the requirement that the entire chain is completed cannot enter directly into an individual firm's problem. Instead, each firm takes as given its position on the chain and the range of tasks being performed by their suppliers.
- We denote by $p^C(S_i(f))$ the sequence of prices associated with the range of tasks $S_i(f)$ in the competitive equilibrium. This price is imposed upon each firm f by its downstream buyer. Similarly, we denote by $p^W(s_i(f))$ the sequence of prices associated with the range of tasks $S_i(f)$ in the social planner's solution.

We focus on a specific chain ω (hence we remove ω from the notation below for the sake of exposition).

Characterization of the competitive equilibrium. Given a sequence of prices $p^C(S_i(f))_{f=0}^{F_i}$ (and wages, w_i which we subsume in $c_i(s_i(f))$) and a required bundle of quantities $q_0 = \bar{q}_i(f)$ and embodied stages $\bar{S}_i(f)$ to be delivered to the next downstream firm, the problem facing each firm f in country i is to choose $q_i(f)$ and $s_i(f)$ to maximize profits $\pi_i(f)$ solving:

$$\pi_i(f) = \max_{q_i(f),s_i(f)} \quad q_i(f)\left[p^C(S_i(f)) - c(s_i(f))df\right] - p^C(S_i(f+df))q_i(f+df) \tag{1}$$

$$\text{s.t.} \quad q_i(f) = q_i(f-df)(1+\gamma_i df)$$

$$S_i(f+df) + s_i(f)df = S_i(f)$$

$$S_i(f) = \bar{S}_i(f)$$

$$q_i(f) = \bar{q}_i(f)$$

The first constraint represents the goods market clearance condition for the output of firm f. The second constraint insures that the firm's choice of $s_i(f)$ together with the stages embodied in its own inputs are sufficient to meet the input requirements of the downstream firm, which demands $S_i(f)$ embodied stages. The final two constraints define the contractual requirements for output and tasks to be performed.

We can simply rewrite (1) as:

$$\pi_i(f) = \max_s \quad q\left[p^C(S_i(f)) - c_i(s)df - (1+\gamma_i df)p^C(S_i(f) - sdf)\right] \tag{2}$$

Joint solution of the first order conditions (w.r.t. q and s) represent a solution to the competitive firm's problem. We link the solution to the firms' problem to the equilibrium price function in what follows.

First, the first-order condition in s yields:

$$c_i'(s)\ df = (1+\gamma_i df)\ \frac{dp_i^C}{dS}\ df \tag{3}$$

Taking the limit where df is infinitesimally small (i.e. ignoring second order terms in df^2), we simply obtain that the marginal cost of an additional task performed within the firm should be equal to the marginal price associated with this task along the chain for the equilibrium firm scope $s_i(f)$:

$$c_i'(s_i(f)) = \frac{dp_i^C}{dS} \tag{4}$$

Note that we would obtain exactly the same result if, instead of choosing their intermediate goods and how much to outsource, each firm were to choose how much to produce given the intermediate goods that they receive. The marginal cost would be $c_i'(s_i(f))$ while the marginal gains would be $\frac{dp_i^C}{dS}$

In turn, the zero-profit condition (i.e. the first-order condition in q) implies that the output price for each firm equals its average cost. This leads to:

$$p^C(S_i(f)) - c_i(s)df - (1+\gamma_i df)\ p_i^C(S_i(f) - sdf) = 0 \tag{5}$$

Rearranging, this can be written:

$$p^C(S_i(f)) - p^C(S_i(f) - sdf) = c_i(s)df + \gamma_i df p^C(S_i(f) - sdf)$$

Taking the limit where df is infinitesimally small (i.e. ignoring second-order terms in df^2), we obain:

$$\frac{dp^C}{dS} s_i(f) = c_i(s_i(f)) + \gamma_i p^C(S_i(f)) \tag{6}$$

for the competitive price schedule and equilibrium firm size $s_i(f)$. To summarize, the price schedule for the competitive equilibrium is characterized by the optimal firm scope in equation (4) and the free-entry condition (6).

Comparison with the social planner's solution. We have yet to show that the social planner's solution satisfies these two equations. To do so, we need to characterize the price in the social planner's solution associated with one unit of the intermediate good as a function of the range of tasks that has been completed. The first-best chain is the chain that minimize the price of the final good:

$$\min P_1 \tag{7}$$

$$
\begin{aligned}
\textit{over:} \quad & i(n)\ ,\ s_n(f)\ ,\ F_n\ ,\ S_n\ ,\ P_n \\
\textit{under the constraints:} \quad & P_n = \left[\int_{f=0}^{F_n} \mathrm{e}^{\gamma_{i(n)} f} c_{i(n)}(s_n(f)) df \ + \ \mathrm{e}^{\gamma_{i(n)} F_n} \tau P_{n+1} \right] \\
& S_n = \int_{f=0}^{F_n} s_n(f) df \\
& \sum_{i=1}^{N} S_n = 1
\end{aligned}
$$

First, our goal is to show that the allocation of tasks across firms within a given country satisfies the competitive market equilibrium conditions described above. Taking the sequence of countries $i(n)$ as given (we discuss below why the sequence of countries is identical to the one in the competitive equilibrium), we define $p^W(S)$ as the minimum unit cost to performed a range S of tasks, for each $S \in [S_{n+1}, S_n]$ and country $i = i(n)$ where S_n, P_{n+1} and $i(n)$ are the solution from above:

$$
p^W(S) = \min_{s_i(f),F} \left[\int_f^F e^{\gamma_i f} c_i(s_i(f)) df + e^{\gamma_i F} \tau P_{n+1} \right] \tag{8}
$$

$$
\text{s.t.} \quad S_{n+1} + \int_{f=0}^{F} s_i(f) df = S
$$

Notice that we can split the minimization problem in two parts. For any $S' \in (S_{n+1}, S)$, we obtain:

$$
\begin{aligned}
p^W(S) &= \min_{s_i(f),F} \left[\int_f^F e^{\gamma_i f} c_i(s_i(f)) df + e^{\gamma_i F} P_{n+1} \right] \qquad \text{s.t.} \ \int_{f=0}^{F} s_i(f) df = S \\
&= \min_{s_i(f),F,s_i'(f),F'} \left\{ \int_f^F e^{\gamma_i f} c_i(s_i(f)) df + e^{\gamma_i F'} \left[\int_{F'}^F e^{\gamma_i (f-F')} c_i(s_i'(f)) df + e^{\gamma_i (F-F')} P_{n+1} \right] \right\} \\
& \qquad \text{s.t.} \int_{f=0}^{F'} s_i(f) df = S - S' \qquad \text{and} \qquad \int_{f=F'}^{F} s_i'(f) df = S' \\
&= \min_{s_i(f),F'} \left\{ \int_{f=0}^{F'} e^{\gamma_i f} c_i(s_i(f)) df + e^{\gamma_i F'} p^W(S') \right\} \qquad \text{s.t.} \ \int_{f=0}^{F'} s_i(f) df = S - S'
\end{aligned} \tag{9}
$$

This implies that the optimal sequence of firm scope $s_i(f)$ is common across all price minimization $p^W(S)$ within a given country. In other words, the scope of firm after completing a range S of tasks is independent of what happens downstream (within a country).

Let us now examine optimal firm scope in the social planner's minimization problem. After completing a range S of tasks, the price minimization associated with $p^W(S)$ implies that the marginal cost $c'(s)$ equals the Lagrange multiplier associated with the constraint $\int_{f=0}^{F} s_i(f) df = S$. Since the range S of tasks appears only in this constraint, the Lagrange multiplier (shadow cost of completing a task) also equals the derivative $\frac{dp^W}{dS}$. Hence, we obtain the same condition as (4) in the competitive equilibrium:

$$
c_i'(s) = \lambda_i(S) = \frac{dp^W}{dS}
$$

Next, taking equation (9) with $S' = S - sdf$, where s is the optimal scope at this stage f, we obtain:

$$
p^W(S) = c_i(s_i(f)) df + (1 + \gamma_i df) p^W(S - sdf)
$$

Taking the limit where df is infinitesimally small (i.e. neglecting second order terms df^2) yields:

$$\frac{dp^W}{dS} = \frac{c_i(s_i(f)) + \gamma_i p^W(S)}{s_i(f)}$$

for the optimal firm scope s at this stage f. This is the same condition as the free-entry condition (6) in the competitive equilibrium within each country.

Finally, we argue that the allocation of tasks and firms across countries is identical. Obviously, the first-best solution corresponds to the sequence of countries and cross-country allocation of tasks S_n that yields the minimum final good price. The same applies to the competitive equilibrium. If a sequence of countries in a chain does not yield the lowest price, a lower-cost chain can enter (with a better sequence of countries) and capture all its consumers. The free-entry condition for chains is key to this argument.

Proofs for Section 3.1: Within-country fragmentation

FOCs: The first-order conditions of this planning program correspond to equations (17) and (18):

$$\begin{aligned} \text{For } s_i(f): &\quad e^{\gamma_i f} w_i c_i'(s_i(f)) \; = \; \lambda_i \\ \text{For } F_i: &\quad e^{\gamma_i F_i} w_i c_i(s_i(F_i)) \; + \; e^{\gamma_i F_i} P_i^M \gamma_i \; = \; s_i(F_i)\, \lambda_i \end{aligned}$$

Using our parameterization of the cost function, the first-order condition for $s_i(f)$ can be rewritten:

$$e^{\gamma_i f} a_i w_i s_i(f)^{\theta_i} = \lambda_i$$

which yields:

$$s_i(f) = \left(\frac{\lambda_i}{a_i w_i}\right)^{\frac{1}{\theta_i}} e^{-\frac{\gamma_i f}{\theta_i}}$$

By combining the first-order condition in F_i and the first-order condition in $s_i(f)$, we obtain:

$$e^{\gamma_i F_i} a_i w_i \frac{s_i(F_i)^{\theta_i+1}}{\theta_i + 1} \; + \; e^{\gamma_i F_i} P_i^M \gamma_i \; = \; s_i(F_i)\,.\, e^{\gamma_i F_i} a_i w_i s_i(F_i)^{\theta_i}$$

which can be simplified into:

$$\tfrac{w_i\, a_i\, \theta_i}{\theta_i+1}\, s_i(F_i)^{\theta_i+1} = \gamma_i P_i^M$$

and thus:

$$s_i(F_i) = \left[\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i}\right]^{\frac{1}{\theta_i+1}}$$

Lagrangian multiplier: It is the solution of:

$$\int_0^{F_i} s_i(f) df = S_i$$

where $s_i(f)$ and F_i functions of the Lagrangian multiplier as shown above. The left-hand side can be rewritten:

$$\int_0^{F_i} s_i(f)df = \left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \int_0^{F_i} e^{-\frac{\gamma_i f}{\theta_i}} df$$
$$= \frac{\theta_i}{\gamma_i}\left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \left[1 - e^{-\frac{\gamma_i F_i}{\theta_i}}\right]$$
$$= \frac{\theta_i}{\gamma_i}\left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \left[1 - \left(\frac{\lambda_i}{w_i a_i}\right)^{-\frac{1}{\theta_i}} s_i(F_i)\right]$$
$$= \frac{\theta_i}{\gamma_i}\left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} - \frac{\theta_i s_i(F_i)}{\gamma_i}$$

We obtain the following solution in λ_i such that the expression above equals S_i:

$$\lambda_i = w_i a_i \left[\frac{\gamma_i S_i}{\theta_i} + \left(\frac{(\theta_i + 1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i}\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i}$$

Final price: Expression for P_i:

$$P_i = \int_{f=0}^{F_i} e^{\gamma_i f} w_i c_i(s_i(f))df + e^{\gamma_i F_i} P_i^M$$
$$= \int_{f=0}^{F_i} e^{\gamma_i f} \frac{w_i a_i s_i(f)^{\theta_i + 1}}{\theta_i + 1} df + e^{\gamma_i F_i} P_i^M$$
$$= \frac{w_i a_i}{\theta_i + 1}\left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \int_{f=0}^{F_i} e^{-\frac{\gamma_i f}{\theta_i}} df + e^{\gamma_i F_i} P_i^M$$
$$= \frac{w_i a_i}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \left[1 - e^{-\frac{\gamma_i F_i}{\theta_i}}\right] + e^{\gamma_i F_i} P_i^M$$
$$= \frac{w_i a_i}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{w_i a_i}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} e^{-\frac{\gamma_i F_i}{\theta_i}} + \frac{w_i a_i}{\gamma_i}\left(\frac{\gamma_i P_i^M}{w_i a_i}\right) e^{\gamma_i F_i}$$
$$= \frac{w_i a_i}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{1}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \lambda_i s_i(F_i) + \frac{wa}{\gamma} \frac{\theta}{\theta + 1} s_i(F_i)^{\theta_i + 1} e^{\gamma_i F_i}$$
$$= \frac{w_i a_i}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{1}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \left[\lambda_i s_i(F_i) - w_i a_i s_i(F_i)^{\theta_i + 1} e^{\gamma_i F_i}\right]$$
$$= \frac{w_i a_i}{\gamma_i} \frac{\theta_i}{\theta_i + 1} \left(\frac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} + 0$$

Using the expression above for λ_i, we obtain equations (23) and (24) in the text:

$$P_i = \left[\frac{S_i}{\theta_i + 1}(A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i + 1}$$

with

$$A_i = a_i \left(\gamma_i \frac{\theta_i + 1}{\theta_i}\right)^{\theta_i}$$

It is also useful to note that:

$$\lambda_i = (w_i A_i)^{\frac{1}{\theta_i+1}} (P_i)^{\frac{\theta_i}{\theta_i+1}} \tag{10}$$

Labor demand: Each unit of last-stage good produced in country i generates the demand for labor $w_i L_i^D = \frac{\partial \log P_i}{\partial \log w_i}$ in country i. This yields:

$$w_i L_i^D = \frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} \left[\frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i} \tag{11}$$

Prices along the chain: To obtain a simple expression for the value-added-to-gross-output ratio $\frac{c_i(s_i(f))}{p_i(f)}$, the first step is to compute $p_i(f)$ is the price along the chain.

$$\begin{aligned}
p_i(f) &= \int_{f'=f}^{F_i} e^{\gamma_i(f'-f)} c(s_i(f'))df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} \int_{f'=f}^{F_i} e^{\gamma_i(f'-f)}\, s_i(f')^{\theta_i+1} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} \int_{f'=f}^{F_i} e^{-\gamma_i(F_i-f')}\, s_i(f')^{\theta_i+1} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} s_i(F_i)^{\theta_i+1} \int_{f'=f}^{F_i} e^{-\gamma_i(F_i-f')} e^{\gamma_i\left(\frac{\theta_i+1}{\theta_i}\right)(F_i-f')} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} s_i(F_i)^{\theta_i+1} \int_{f'=f}^{F_i} e^{\frac{\gamma_i(F-f')}{\theta_i}} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} e^{\gamma_i(F_i-f)} s_i(F_i)^{\theta+1} \left[e^{\frac{\gamma_i(F_i-f)}{\theta}} - 1\right] \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} s_i(f)^{\theta_i+1} \;-\; \frac{e^{\gamma_i(F_i-f)}}{\gamma_i}\left[\frac{w_i a_i \theta_i}{\theta_i+1} s_i(F_i)^{\theta_i+1} \;-\; \gamma_i P_i^M\right] \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} s_i(f)^{\theta_i+1} \;-\; 0 \\
&= \frac{\theta_i}{\gamma_i}\; w_i c_i(s_i(f))
\end{aligned}$$

Hence the gross-output-to-value-added ratio at the firm level is:

$$\frac{p_i(f)}{w_i c_i(s_i(f))} = \frac{\theta_i}{\gamma_i}$$

We also obtain the same expression for the aggregate gross-output-to-value-added ratio. If we define gross output as the total value of all transactions:

$$GO_i = \int_0^{F_i} q_i(f) p_i(f) df$$

we obtain:

$$\frac{GO_i}{VA_i} = \frac{\int_0^{F_i} q_i(f)p_i(f)df}{\int_0^{F_i} q_i(f) w_i c_i(s_i(f))df} = \frac{\int_0^{F_i} \frac{\theta_i}{\gamma_i} q_i(f) w_i c_i(s_i(f))df}{\int_0^{F_i} q_i(f) w_i c_i(s_i(f))df} = \frac{\theta_i}{\gamma_i}$$

Proof of Proposition 1

To simplify the exposition, we index countries by n, with $n = 1$ referring to the most downstream country and $n = N$ the most upstream country. The goal is to minimize:

$$\min P_1 \tag{12}$$

under the constraints:

$$P_{n+1} = \widetilde{P}_n(S_n, \tau P_{n+1}) \qquad \text{and} \qquad \sum_{n=n}^{N} S_n = 1$$

with:

$$\widetilde{P}_n(S, P^M) = \left[\frac{S}{\theta_n + 1} (A_n w_n)^{\frac{1}{\theta_n+1}} + \left(P^M\right)^{\frac{1}{\theta_n+1}} \right]^{\theta_n+1}$$

Under which condition can country n be downstream from country $n+1$? Let us take as given the price in country $n+2$ and consider the following function:

$$m(x)^{\theta_n+1} = \widetilde{P}_n(S_n - x, \tau \widetilde{P}_n(S_{n+1} + x, \tau P_{n+2}))$$

This function $m(x)$ indicates by how much the price of output in n will increase if we shift a measure x of tasks from country n to country $n+1$.

$$m(x) = \frac{(S_n - x)}{\theta_n + 1} (A_n w_n)^{\frac{1}{\theta_n+1}} + \left[\frac{(S_{n+1} + x)}{\theta_{n+1} + 1} (A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}} \right]^{\frac{\theta_{n+1}+1}{\theta_n+1}}$$

If we are at equilibrium, the function $m(x)$ must be at its minimum at $x = 0$. The first-order condition imply that $m'(x) = 0$. We obtain that:

$$m'(x) = -\frac{(A_n w_n)^{\frac{1}{\theta_n+1}}}{\theta_n + 1} + \frac{(A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n + 1} \left[\frac{(S_{n+1} + x)}{\theta_{n+1} + 1} (A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}} \right]^{\frac{\theta_{n+1}-\theta_n}{\theta_n+1}} \tag{13}$$

must equal zero at $x = 0$.

More importantly, to prove Proposition 1, one needs to examine the second order condition, which imposes $m''(x) > 0$. If $m''(x)$ were negative, $x = 0$ would not be a local minimum and it would be more efficient to shift some tasks to either country n or $n+1$.

As one can see in equation (13), the right-hand-side term is increasing in x (i.e. $m''(x) > 0$) only if the exponent $\frac{\theta_{n+1}-\theta_n}{\theta_n+1}$ is positive. This proves that we must have $\theta_{n+1} > \theta_n$ at equilibrium.

Finally, it is not difficult to verify that two consecutive countries cannot have the same $\theta_n = \theta_{n+1}$ as long as we have non-zero trade costs $\tau - 1 > 0$.

Other proofs for Section 3.2

Prices along the chain: Using again appendix equation (13), the first-order condition $m'(0) = 0$ implies:

$$\begin{aligned}
\frac{(A_n w_n)^{\frac{1}{\theta_n+1}}}{\theta_n + 1} &= \frac{(A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n + 1} \left[\frac{S_{n+1}}{\theta_{n+1} + 1} (A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}} \right]^{\frac{\theta_{n+1}-\theta_n}{\theta_n+1}} \\
&= \frac{(A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n + 1} (P_{n+1})^{\frac{1}{\theta_n+1} \cdot \frac{\theta_{n+1}-\theta_n}{\theta_{n+1}+1}}
\end{aligned}$$

This yields expression (34) for the price of goods sold by country $n+1$:

$$\tau P_{n+1} = (A_n w_n)^{\frac{\theta_{n+1}+1}{\theta_{n+1}-\theta_n}} (\tau A_{n+1} w_{n+1})^{-\frac{\theta_n+1}{\theta_{n+1}-\theta_n}}$$

For country i, this gives:

$$P_n = (A_{n-1} w_{n-1}/\tau)^{\frac{\theta_n+1}{\theta_n-\theta_{n-1}}} (A_n w_n)^{-\frac{\theta_{n-1}+1}{\theta_n-\theta_{n-1}}}$$

Allocation of tasks across countries: The range of tasks performed by country i can then be obtained as:

$$\frac{S_n}{\theta_n+1} = (A_n w_n)^{-\frac{1}{\theta_n+1}} \left[P_n^{\frac{1}{\theta_n+1}} - (\tau P_{n+1})^{\frac{1}{\theta_n+1}} \right]$$
$$= (A_n w_n)^{-\frac{1}{\theta_n+1}} \left[(A_{n-1} w_{n-1}/\tau)^{\frac{1}{\theta_n-\theta_{n-1}}} (A_n w_n)^{-\frac{\theta_{n-1}+1}{(\theta_n+1)(\theta_n-\theta_{n-1})}} - (A_n w_n)^{\frac{\theta_{n+1}+1}{(\theta_n+1)(\theta_{n+1}-\theta_n)}} (\tau A_{n+1} w_{n+1})^{-\frac{1}{\theta_{n+1}-\theta_n}} \right]$$

which can be simplified into expression (34) given in the text:

$$\frac{S_n}{\theta_n+1} = \left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

For the last country N in the chain, we obtain:

$$\frac{S_N}{\theta_N+1} = \left(\frac{A_{N-1} w_{N-1}}{\tau A_N w_N} \right)^{\frac{1}{\theta_N-\theta_{N-1}}}$$

Finally, the range of tasks performed by the last country in the chain is:

$$S_1 = 1 - \sum_{n=2}^{N-1} S_n$$
$$= 1 - (\theta_N+1) \left(\frac{A_{N-1} w_{N-1}}{\tau A_N w_N} \right)^{\frac{1}{\theta_N-\theta_{N-1}}} - \sum_{n=2}^{N-1} (\theta_n+1) \left[\left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} \right]$$
$$= 1 - \sum_{n=2}^{N} (\theta_n+1) \left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} + \sum_{n=2}^{N-1} (\theta_n+1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$
$$= 1 - \sum_{n=1}^{N-1} (\theta_{n+1}+1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} + \sum_{n=2}^{N-1} (\theta_n+1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$
$$= 1 - (\theta_1+1) \left(\frac{A_1 w_1}{\tau A_2 w_2} \right)^{\frac{1}{\theta_2-\theta_1}} - \sum_{n=1}^{N-1} (\theta_{n+1}-\theta_n) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

Final good price: Using the above expressions for S_1 and P_2, we obtain the price of the final good:

$$\begin{aligned}
P_1 &= \left[\frac{S_1}{\theta_1+1}(A_1 w_1)^{\frac{1}{\theta_1+1}} + (\tau P_2)^{\frac{1}{\theta_1+1}}\right]^{\theta_1+1} \\
&= \left[\frac{S_1}{\theta_1+1}(A_1 w_1)^{\frac{1}{\theta_1+1}} + (A_1 w_1)^{\frac{\theta_2+1}{(\theta_1+1)(\theta_2-\theta_1)}}(\tau A_2 w_2)^{-\frac{1}{\theta_2-\theta_1}}\right]^{\theta_1+1} \\
&= \frac{A_1 w_1}{(\theta_1+1)^{\theta_1+1}}\left[S_1 + (\theta_1+1)\left(\frac{A_1 w_1}{\tau A_2 w_2}\right)^{\frac{1}{\theta_2-\theta_1}}\right]^{\theta_1+1} \\
&= \frac{A_1 w_1}{(\theta_1+1)^{\theta_1+1}}\left[1 - \sum_{n=1}^{N-1}(\theta_{n+1}-\theta_n)\left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1+1} \\
&= \frac{A_1 w_1}{(\theta_1+1)^{\theta_1+1}}\Theta\big(\mathbf{wA}, \tau\big)
\end{aligned}$$

This corresponds to expression (35) in the text with the term in Θ reflecting gains from fragmentation:

$$\Theta\big(\mathbf{wA}, \tau\big) = \left[1 - \sum_{n=1}^{N-1}(\theta_{n+1}-\theta_n)\left(\frac{w_n A_n}{\tau w_{n+1} A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1+1}$$

Demand for labor: By the envelope theorem, demand for labor in upstream countries can be obtained by:

$$\frac{l_n w_n}{P_1} = \frac{d\log P_1}{d\log w_n} = \frac{d\log\Theta}{d\log w_n}$$

This gives expression (37) in the text:

$$\frac{l_n w_n}{P_1} = \frac{\left(\frac{w_{n-1}A_{n-1}}{\tau w_n A_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{w_n A_n}{\tau w_{n+1} A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}}{\left(\frac{P_1}{A_1 w_1}\right)^{\frac{1}{\theta_1+1}}}$$

Lagrangian multiplier: The Lagrangian multiplier λ_G is equal to the Lagrangian multiplier λ_1 in the most downstream country (since $q_1 = 1$). Using (10), we obtain:

$$\lambda_G = (A_1 w_1)^{\frac{1}{\theta_1+1}} P_1^{\frac{\theta_1}{\theta_1+1}} = \frac{A_1 w_1}{(\theta_1+1)^{\theta_1}}\Theta\big(\mathbf{wA}, \tau\big)^{\frac{\theta_1}{\theta_1+1}}$$

Firm scope: For the most downstream firm in the most downstream country, equation (32) becomes:

$$w_1 c_1'(s_1(f=0)) = \lambda_G$$

This gives:

$$w_1 a_1 s_1(0)^{\theta_1} = \lambda_G$$

Using the expression above for λ_G, we obtain:

$$\begin{aligned} s_1(0) &= \left(\frac{\lambda_G}{w_1 a_1}\right)^{\frac{1}{\theta_1}} \\ &= \frac{\gamma_1(\theta_1+1)}{\theta_1}\left(\frac{\lambda_G}{w_1 A_1}\right)^{\frac{1}{\theta_1}} \\ &= \frac{\gamma_1}{\theta_1}\Theta\big(\mathbf{wA},\tau\big)^{\frac{1}{\theta_1+1}} \end{aligned}$$

To obtain downstream firm scope for other countries (expression 39), we use the first-order condition for firm scope for $f=0$:

$$w_n c_n'(s_n(0)) = \lambda_n$$

which gives:

$$w_n a_n s_n(0)^{\theta_n} = \lambda_n$$

Using $\lambda_n = (w_n A_n)^{\frac{1}{\theta_n+1}}(P_n)^{\frac{\theta_n}{\theta_n+1}}$ (expression 10) together with the expression for P_n, we obtain:

$$\begin{aligned} s_n(0) &= \left(\frac{\lambda_n}{w_n a_n}\right)^{\frac{1}{\theta_n}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{\lambda_n}{w_n A_n}\right)^{\frac{1}{\theta_n}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{P_n}{A_n w_n}\right)^{\frac{1}{\theta_n+1}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}(A_n w_n)^{-\frac{1}{\theta_n+1}}(A_{n-1}w_{n-1}/\tau)^{\frac{1}{\theta_n-\theta_{n-1}}}(A_n w_n)^{-\frac{\theta_{n-1}+1}{(\theta_n+1)(\theta_n-\theta_{n-1})}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \end{aligned}$$

We follow similar steps to find the scope of the most upstream firm in each country, $s_n(F_n)$:

$$s_n(F_n) = \frac{(\theta_n+1)\gamma_n}{\theta_n}\left(\frac{A_n w_n}{\tau A_{n+1}w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

Proof of Proposition 2

Proposition 2: Results in Proposition 2 are obtained simply by taking the derivative of the expressions above w.r.t trade costs τ. In particular, we find:

$$\frac{\partial\Theta\big(\mathbf{wA},\tau\big)}{\partial\tau} > 0$$

which implies that: i) the price in the final good and that the shadow cost λ_G decrease when trade costs decrease. Given equation (32)

$$q_n e^{\gamma_n f} w_n c_n'(s_n(f)) = \lambda_G,$$

we obtain that a decrease in λ_G affects firm scope everywhere along the chain and leads to a decrease in $s_n(f)$, conditional on the position on the chain, wages and the set of countries involved in the chain. As trade costs decrease, however, countries tend to move downstream. Since firms scope is larger downstream, moving up the chain implies larger average firm scope for each country $i > 1$ (except the most downstream one). This can be seen in expressions (39): firm scope at both end of the chain in country i is a decreasing function of τ (point v in Proposition 2). Finally, we can also see above that S_1 is an increasing function of trade costs (conditional on wages), which proves point iii).

Two-country case and proof of Proposition 3

Two-country setting: As stated in the text, we assume that $\theta_U > \theta_D$. As specified in equation (12) in section 2, labor efficiency $a_D(\omega)$ and $a_U(\omega)$ are distributed Weibull with coefficient T_D and T_U respectively for countries D and U. We make no assumption about the relative ranking of T_D and T_U. We also make no assumption about relative transaction costs γ_D and γ_U for countries D and U. Also, we normalize $w_D = 1$.

As shown in equation (24), it is useful to instead define an adjusted labor costs parameter $A_D(\omega) = a_D(\omega)\left(\gamma_D \frac{\theta_D+1}{\theta_D}\right)^{\theta_D}$ and $A_U(\omega) = a_U(\omega)\left(\gamma_U \frac{\theta_U+1}{\theta_U}\right)^{\theta_U}$. The effect of transaction costs is equivalent to a shift in labor productivity. The resulting $A_D(\omega)$ and $A_U(\omega)$ parameters also follow a Weibull distribution with adjusted shift parameters:[1]

$$\begin{aligned}\tilde{T}_D &= T_D\left(\gamma_D \frac{\theta_D+1}{\theta_D}\right)^{-\xi\theta_D}\\ \tilde{T}_U &= T_U\left(\gamma_U \frac{\theta_U+1}{\theta_U}\right)^{-\xi\theta_U}\end{aligned}$$

Following Dornbusch, Fisher and Samuelson (1977), we rank varieties ω between 0 and 1 and specify the following relative cost:

$$\frac{A_U(\omega)}{A_D(\omega)} = \left[\frac{\tilde{T}_D}{\tilde{T}_U}\left(\frac{\omega}{1-\omega}\right)\right]^{\frac{1}{\xi}} \equiv A(\omega) \tag{14}$$

where $A(\omega)$ is defined as the relative labor requirement in country U. This ordering implies that U has a comparative advantage in low-ω chains while country D has a comparative advantage in high-ω chains. For the sake of exposition, we normalize $A_D(\omega)$ to unity. It is otherwise equivalent to redefine all prices as relative to $A_D(\omega)$.

Sourcing patterns: As shown in Proposition 1, the ranking $\theta_D < \theta_U$ determines relative position on the chain. A chain that involves the two countries necessarily features country U specializing upstream and country D specializing downstream. Some chains may also involve country U only. However, when country D produces the final good, we find that country U is also involved in the chain, at least for some of the most upstream tasks.

When country D produces the final good (with country U involved in upstream tasks), the price of the final good in D is:

$$P_D(\omega) = \frac{1}{(\theta_D+1)^{\theta_D+1}}\left[1 - (\theta_U - \theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D+1} \tag{15}$$

[1] Our parameter ξ is the same as the dispersion parameter θ in Eaton and Kortum (2002).

Consumers in U can also import these goods at a price $\tau P_D(\omega)$. When country U produces the entire range of tasks, the price of final goods in U is:

$$P_U(\omega) = \frac{w_U A(\omega)}{(\theta_U+1)^{\theta_U+1}} \tag{16}$$

while consumers in country D can also import these goods for a price $\tau P_U(\omega)$.

Given the patterns of labor costs across varieties, the ratio of prices $\frac{P_D(\omega)}{P_U(\omega)}$ strictly increases with ω. For each final destination $X \in \{D, U\}$, there is a unique threshold ω_X^* for which the two prices are equal. These thresholds ω_D^* and ω_U^* are implicitly defined by:

$$P_D(\omega_D^*) = \tau P_U(\omega_D^*) \tag{17}$$

$$\tau P_D(\omega_U^*) = P_U(\omega_U^*) \tag{18}$$

As in Dornbusch, Fisher and Samuelson (1977), these cutoffs ω_D^* and ω_U^* correspond to the goods for which consumers (resp. in D and U) are indifferent between purchasing locally or importing. There is no analytical solution for ω_U^* but it is easy to check the following solution in ω_D^*:

$$\omega_D^* = \frac{\tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D+1)^{-(\theta_U-\theta_D)\xi}}{\tilde{T}_D + \tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D+1)^{-(\theta_U-\theta_D)\xi}}$$

General equilibrium and wages: While production in U only relies on local labor, production in D relies on country U to perform upstream tasks. Using the results from Lemma 2, the demand for labor in U for each dollar of final goods produced in D (at a price P_D) equals:

$$\frac{w_U\, l_U(\omega)}{P_D(\omega)} = \frac{(\theta_D+1)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}{1-(\theta_U-\theta_D)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}} \tag{19}$$

Trade balance imposes:

$$w_U L_U(1-\omega_U^*) \;=\; L_D \omega_D^* \;+\; L_D \int_{\omega=\omega_D^*}^{1} \frac{w_U\, l_U(\omega)}{P_D(\omega)} d\omega \;+\; w_U L_U \int_{\omega=\omega_U^*}^{1} \frac{w_U\, l_U(\omega)}{P_D(\omega)} d\omega \tag{20}$$

where the left-hand side correspond to exports of final goods by D and the right-hand side corresponds to exports of final and intermediate goods by U. Note that the marginal variety ω_D^* is such that: $\frac{w_U\, l_U(\omega_D^*)}{P_D(\omega_D^*)} = 1$. Hence the trade balance above is equivalent to a trade balance in value-added content:

$$w_U L_U \int_{\omega=\omega_U^*}^{1} \left(1 - \frac{w_U\, l_U(\omega)}{P_D(\omega)}\right) d\omega \;=\; L_D \int_{\omega=0}^{1} \min\left\{\frac{w_U\, l_U(\omega)}{P_D(\omega)}\,,\, 1\right\} d\omega \tag{21}$$

We prove here that τw_U decreases as trade costs τ decrease, which also implies that foreign labor content $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$ increases when τ decreases. To prove this result, we show that we arrive at a contradiction if we assume that τw_U increases when τ decreases. The right-hand-side term of expression (21) would decrease since it is a strictly decreasing function of τw_U: country D sources less from U if trade-cost-adjusted wages increase in U. On the other hand the term on the left would increase because of higher income $L_U w_U$, a lower import threshold ω_U^* (since goods from D would become relatively cheaper) and higher foreign value-added content $1 - \frac{w_U\, l_U(\omega)}{P_D(\omega)}$. Hence it must be that τw_U decreases when τ decreases.

Trade elasticity: For country D, it is easy to check that the elasticity is the same as in Eaton and Kortum (2002):

$$\frac{\omega_D^*}{1-\omega_D^*} = \frac{\tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D+1)^{-(\theta_U-\theta_D)\xi}}{\tilde{T}_D}$$

Hence:

$$\varepsilon_D^F \equiv \frac{d\log\left(\frac{\omega_D^*}{1-\omega_D^*}\right)}{d\log\tau} = -\xi$$

For country U, we take the derivative of $\tau P_D(\omega_U^*) = P_U(\omega_U^*)$ with respect to $\log\tau$, which gives:

$$1 + \frac{\partial \log P_D}{\partial \log \tau}\cdot\left[1 + \frac{d\log A(\omega_U^*)}{d\log\tau}\right] = \frac{\partial \log P_U}{\partial \log\tau}\ \frac{d\log A(\omega_U^*)}{d\log\tau}$$

In the expression above, the partial derivative $\frac{\partial \log P_U}{\partial \log\tau}$ is equal to unity. The partial derivative $\frac{\partial \log P_D}{\partial \log\tau}$ is lower than one and equals the share of value coming from U for the threshold variety ω_U^*:

$$\frac{d\log P_D}{d\log\tau} = \frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}$$

After solving for $\frac{d\log A(\omega_U^*)}{d\log\tau}$, we find:

$$\frac{d\log A(\omega_U^*)}{d\log\tau} = \frac{1+\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}{1-\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}$$

The trade elasticity in final goods for country U is then:

$$\varepsilon_U^F = \frac{d\log\left(\frac{1-\omega_U^*}{\omega_U^*}\right)}{d\log\tau} = \frac{1}{\xi}\ \frac{d\log A(\omega_U^*)}{d\log\tau} = \frac{1}{\xi}\ \frac{1+\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}{1-\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}$$

The lower the trade costs, the higher the trade elasticity. Because lower trade costs leads to more fragmentation, the foreign labor content for the marginal variety increases. Note that, when trade becomes frictionless, the foreign labor content for this marginal variety converges to unity and the trade elasticity ε_U^F goes to infinity.

Vertical specialization and the value-added content of trade: We focus here on Johnson and Nogura (2012a)'s "VAX ratio", the ratio of the value-added content of exports and gross exports. For country D, this corresponds to:

$$\begin{aligned} VAX_D &= \frac{1}{1-\omega_U^*}\int_{\omega_U^*}^1 \left(1-\frac{w_U\, l_U(\omega)}{P_D(\omega)}\right) d\omega \\ &= \frac{1}{1-\omega_U^*}\int_{\omega_U^*}^1 \left(1-\frac{(\theta_D+1)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}{1-(\theta_U-\theta_D)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}\right) d\omega \end{aligned}$$

where $1-\omega_U^*$ is the share of imported goods by consumers in U and where $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$ is the share of foreign labor in the production of variety ω in country D.

For a given ω, the term in the integral sum increases with trade-cost-adjusted wages τw_U, which itself increases with τ (larger domestic value added share as trade costs increase). This is a direct effect. There is also a composition effect: ω_U^* increases with τ, so that country D only exports high-value-added goods (varieties ω closer to one) when trade costs are higher. This second effect also leads to an increase in the VAX ratio when trade costs increase.

A similar intuition holds for the VAX ratio for country U, as described in the text. The VAX ratio for country U equal the one for country D in this two-country example because we have a trade balance in gross flows as well as in the value-added content of trade.

Gains from trade for country D: For country D, the wage and the price index under autarky are normalized to zero (in log). Hence the log of the price index with trade also reflects the gains from trade:

$$\Delta \log\left(\frac{1}{P_D}\right) = \log P_D = \left[\int_0^1 \log P_D(\omega) d\omega\right]$$

Using expressions above for prices, we obtain:

$$\Delta \log\left(\frac{1}{P_D}\right) = \int_0^{\omega_D^*} \log\left(\tau w_U A(\omega)\right) d\omega + \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega$$

with:

$$\Theta(\tau w_U A(\omega)) = \left[1 - (\theta_U - \theta_D)\left(\tau w_U A(\omega)\right)^{-\frac{1}{\theta_U - \theta_D}}\right]^{\theta_D + 1}$$

and:

$$\Theta(\tau w_U A(\omega_D^*)) = \tau w_U A(\omega_D^*)$$

at the threshold ω_D^*.

The expression for the gains from trade can be rewritten:

$$\begin{aligned}
\Delta \log\left(\frac{1}{P_D}\right) &= -\int_0^{\omega_D^*} \log\left(\tau w_U A(\omega)\right) d\omega - \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \\
&= -\int_0^{\omega_D^*} \log\left(\frac{A(\omega)}{A(\omega_D^*)}\right) d\omega - \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \\
&\quad -\omega_D^* \log\Theta\left(\tau w_U A(\omega_D^*)\right)
\end{aligned}$$

There are three terms in the above formula. The first term $\frac{1}{\xi}\log\left(1-\omega_D^*\right)$ corresponds to the Arkolakis et al (2012) formula based on final demand trade: the log of the gains from trade are proportional to the log of the domestic content of consumption, where the proportionality coefficient is the inverse of the trade elasticity ξ.

After integrating by parts, we can see that it equals the ratio of the change in domestic consumption share and the trade elasticity ξ for country D:

$$\begin{aligned}
-\int_0^{\omega_D^*} \log\left(\frac{A(\omega)}{A(\omega_D^*)}\right) d\omega &= \int_0^{\omega_D^*} \frac{\partial \log A(\omega)}{\partial \log \omega} d\omega \\
&= \frac{1}{\xi}\int_0^{\omega_D^*} \frac{d\omega}{1-\omega} \\
&= -\frac{1}{\xi}\log\left(1-\omega_D^*\right)
\end{aligned}$$

The second and third terms reflect additional gain from fragmentation:

$$-\int_{\omega_D^*}^1 \log \Theta\left(\tau w_U A(\omega)\right) d\omega \;-\; \omega_D^* \log \Theta\left(\tau w_U A(\omega_D^*)\right) \;=\; -\int_{\omega_D^*}^1 \frac{\partial \log \Theta}{\partial \log A} \frac{\partial \log A}{\partial \log \omega}\, d\omega$$

$$= \int_{\omega_D^*}^1 \frac{\partial \log \Theta}{\partial \log A} \frac{1}{\xi} \frac{1}{1-\omega}\, d\omega \;=\; \frac{1}{\xi} \int_{\omega_D^*}^1 \frac{w_U\, l_U(\omega)}{P_D(\omega)} \frac{d\omega}{1-\omega} \;>\; 0$$

using the equality between $\frac{\partial \log \Theta}{\partial \log A}$ and the foreign labor content $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$. Hence, for country D, Arkolakis et al (2012) formula underestimates the gains from trade.

Gains from trade for country U: For country U, the price index under autarky is:

$$\log P_U^{aut} = \int_0^1 \log\left(w_U^{aut} A(\omega)\right) d\omega$$

where w_U^{aut} denotes the wage in autarky. With trade, the price index is:

$$\log P_U = \int_0^{\omega_U^*} \log\left(w_U A(\omega)\right) d\omega \;+\; \int_{\omega_U^*}^1 \log \tau \Theta\left(\tau w_U A(\omega)\right) d\omega$$

where $\Theta\left(\tau w_U A(\omega)\right)$ is defined like above:

$$\Theta(\tau w_U A(\omega)) = \left[1 - (\theta_U - \theta_D)\left(\tau w_U A(\omega)\right)^{-\frac{1}{\theta_U - \theta_D}}\right]^{\theta_D + 1}$$

with the following equality at the threshold ω_U^*:

$$\tau \Theta\left(\tau w_U A(\omega_U^*)\right) = w_U A(\omega_U^*)$$

Gains from trade can then be expressed as:

$$\begin{aligned}
\Delta \log\left(\frac{w_U}{P_U}\right) &= \int_{\omega_U^*}^1 \log\left(w_U A(\omega)\right) d\omega - \int_{\omega_U^*}^1 \log \tau \Theta\left(\tau w_U A(\omega)\right) d\omega \\
&= \int_{\omega_U^*}^1 \log\left(\frac{A(\omega)}{A(\omega_U^*)}\right) d\omega - \int_{\omega_U^*}^1 \log\left(\frac{\Theta\left(\tau w_U A(\omega)\right)}{\Theta\left(\tau w_U A(\omega_U^*)\right)}\right) d\omega
\end{aligned}$$

Like above, the first term corresponds to Arkolakis et al (2012) formula:

$$\int_{\omega_U^*}^1 \log\left(\frac{A(\omega)}{A(\omega_U^*)}\right) d\omega = -\frac{1}{\xi} \log \omega_U^*$$

The second term yields:

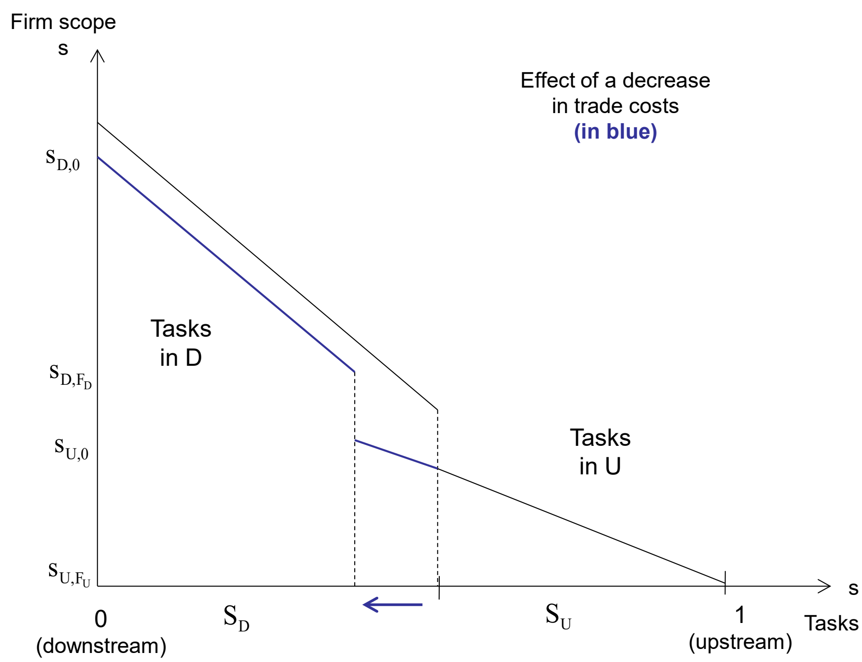
$$\begin{aligned}
-\int_{\omega_U^*}^1 \log\left(\frac{\Theta\left(\tau w_U A(\omega)\right)}{\Theta\left(\tau w_U A(\omega_U^*)\right)}\right) d\omega &= -\int_{\omega_U^*}^1 \frac{\partial \log \Theta}{\partial \log A} \frac{\partial \log A}{\partial \omega} (1-\omega)\, d\omega \\
&= -\int_{\omega_U^*}^1 \frac{\partial \log \Theta}{\partial \log A} \frac{1}{\xi \omega}\, d\omega \\
&= -\frac{1}{\xi} \int_{\omega_U^*}^1 \frac{w_U\, l_U(\omega)}{P_D(\omega)} \frac{d\omega}{\omega} \;<\; 0
\end{aligned}$$

This term is negative, which means that country U gains less than predicted by the Arkolakis et al (2012) benchmark.

Firm scope in the two-country case Combining equations (19) and (39), we obtain firm scope for each country depending on its position on the chain. Combining with equation (34), we can express firm scope as a function of the position of each tasks along the range of tasks. We obtain a linear relationship between firm scope and the position of a task within each country, with a discontinuous jump between countries.

In Figure 3 we show the effects of trade cost change on firm scope, using a 2-country case as an illustration, as illustrated in Proposition 2. The scope of the most downstream firm in the downstream country is denoted $s_{D,0}$, while the most upstream firm in the downstream country is s_{D,F_D}. The most up- and down-stream firms in the upstream countries are denoted $s_{U,0}$ and s_{U,F_U} respectively. The reduction in trade costs reduces firm scope in the downstream country but increases average firm scope in the upstream country through an extensive margin of additional (relatively large) firms in the upstream country. In principle firm scope in the upstream country can also increase through an intensive margin, governed by equation (32) in the text.

Figure 3: Changing trade costs and the effects on firm scope



B) Indexes U and N

Model-based interpretation

Index U. How to interpret 'U' in the model? First, suppose that products k correspond to stages f. When f is strictly positive, i.e. when it does not refer to the most downstream stage in the n^{th} country $i(n)$, then all sales are made to the next firm $f - df$ in the chain:

$$U_{i(n),f+df} = df + U_{i(n),f}$$

If $f = 0$ and $i(n)$ is not the most downstream country $i(1)$, then all sales go towards the most upstream firm in the next country in the chain. After integrating, we obtain that the model counterpart of U_{if}

corresponds to the total range of firms located downstream:

$$U_{i(n),f} = f + \sum_{n'<n} F_{i(n')}$$

summing across all downstream countries $i(n')$ with $n' < n$.

In terms of the model, we can also interpret $U_{i,k}$, for any country i, as a semi-elasticity of required quantities w.r.t. to transaction costs. Formally, U_{if} corresponds to:

$$U_{i,f} = \sum_j \frac{\partial \log q_i(f)}{\partial \gamma_j}, \tag{22}$$

which follows from (7). Because γ_j governs proportional iceberg "melt" that arises in firm-to-firm transactions, it summarizes the degree to which the additional quantities required of upstream firms rise with the upstreamness of their position.

Index N. We can also interpret N in light of our theoretical framework. In the model, the amount of input purchased by other firms corresponds to the price of the good minus the labor cost incurred at each stage, i.e.: $\frac{w_i c_i(s_i(f))df}{p_i(f)}$, where $p_i(f)$ denotes the price of the good in country i at stage f. The model counterpart of index N would thus correspond to the following recursive definition:

$$N_{i,f} = df + \left(1 - \frac{c_i(s_i(f))df}{p_i(f)}\right) N_{i,f+df}$$

with a similar equation when the chain crosses a border. The solution to this differential equation equals the average of the number of production stages required to produce a good at stage f in country i.

There is a strong connection between the N and U index. Since the number of stages between firm f' in the m^{th} country $i(m)$ and firm f in the n^{th} country $i(n)$ corresponds to $U_{i(m),f'} - U_{i(n),f}$, we obtain formally:

$$N_{i(n),f} = \frac{1}{q_{i(n)}(f)p_{i(n)}(f)} \left[\int_{(m,f')>(n,f)} (U_{i(m),f'} - U_{i(n),f})\, q_{i(m),f'} c_{i(m)}(s_{i(m)}(f')) \right]$$

where the integral is taken across all upstream firms either in $i(n)$ at a more upstream stage $f' > f$ or in more upstream countries $i(m)$ with $m > n$, and where the price $p_i(f)$ can be itself re-expressed as the sum of all costs incurred in upstream stages, adjusting for quantities: $q_{i(n)}(f)p_{i(n)}(f) = \int_{(m,f')>(n,f)} q_{i(m)}(f')c_{i(m)}(s_{i(m)}(f'))$.

The connection between the two indexes N and U is clearest if we look at the most downstream stage. For the most downstream country $i = 1$ and the most downstream firm $f = 0$ in the country, index N corresponds to a weighted average of U:

$$N_{i(1),f=0} = \frac{1}{q_1(0)p_1(0)} \left[\sum_j \int_{f'=0}^{F_j} U_{j,f'}\; q_i(f')c_j(s_j(f'))\, df' \right]$$

with the price $p_{i=1}(0) = \sum_j \int_{f'=0}^{F_j} q_i(f')c_j(s_j(f'))df'$ being the sum of all upstream costs.

As for U, we can also use the model to interpret N_{ik} as a semi-elasticity of w.r.t. to transaction costs, looking at prices instead of quantities. Formally, N_{if} corresponds to:

$$N_{if} = \sum_j \frac{\partial \log p_i(f)}{\partial \gamma_j}$$

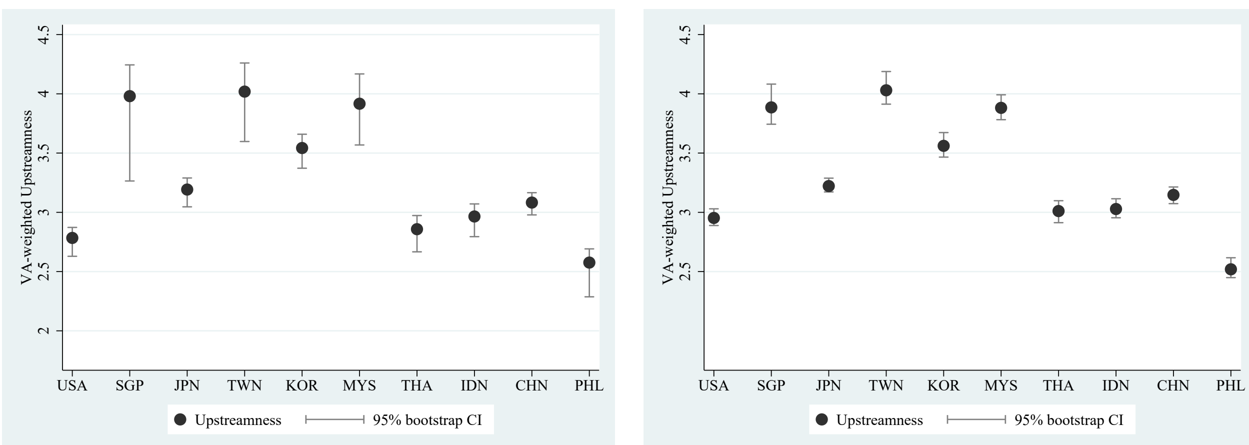
Sensitivity to Rest-of-the-World

The IDE-JETRO tables report transactions between IDE-JETRO countries and the rest of the world, but not input-output links within the rest of the world. However, the definitions of upstreamness U and stages N for Asian countries also depend on values for the rest of the world. In our baseline computations, we assign average industry values for U and N to the rest-of the world.

To explore the sensitivity of aggregate upstreamness to alternative treatments, we construct a hundred bootstrap samples where we assign a random country's upstreamness index to the rest of the world, randomly drawing among the 10 IDE-JETRO countries. Results are reported in Figure 4(a), which provides 95% confidence intervals across these bootstrap samples. One can see that the levels and rankings in upstreamness across countries are well preserved, with the widest confidence intervals for the smallest countries in terms of GDP: Singapore, Taiwan and Malaysia.

In Figure 4(b), we instead draw countries randomly but with a sampling probability that is proportional to GDP, since larger countries are more representative of transactions with the Rest-of-the-World (transactions are larger with larger countries). Confidence intervals are greatly tightened, and averages are very close to our baseline specification.

Figure 4: Aggregate upstreamness index – bootstrap replacements for rest-of-world



(a) Equal weights (b) Weighted by GDP

Indexes U and N across industries

Figure 5 plots cross-country averages of index U_{ik} and N_{ik} in the IDE-JETRO data. As expected, primary commodities such as ores and metals tend to be upstream while finished goods tend to be downstream. Moreover, one can also see that indexes U and N are not strongly correlated and capture different moments and industry characteristics. More details on these indexes can be found in Fally (2012), who documents in particular the robustness of these indexes to aggregation biases. In Figure 6, we plot the upstreamness index U_{ik} evaluated for our five largest countries. Index U_{ik} is highly correlated across industries but there are still large differences across countries. Overall, we find that industry fixed effects can explain about 70% of the variance in U_{ik} across countries and industries.

Figure 5: Average of indexes U_{ik} and N_{ik} by industry, year 2000

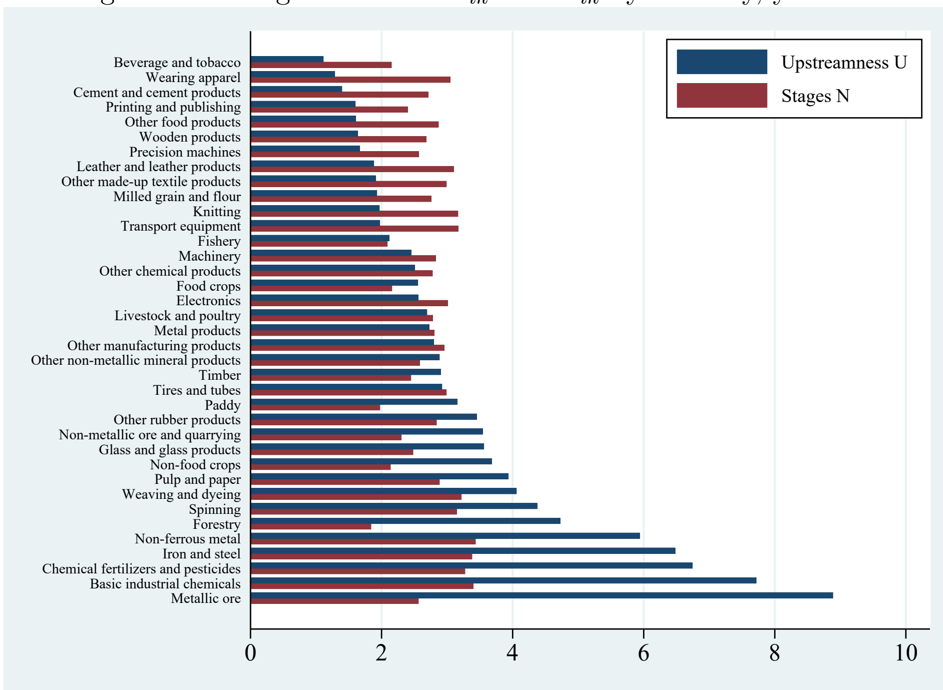


Figure 6: Index U_{ik} across industries for five largest countries, year 2000