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Facility-Level and System-Level Stochastic Optimization
of Bridge Maintenance and Replacement Decisions Using History-Dependent Models

by

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of the

University of California, Berkeley

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Charles-Antoine Robelin

Abstract

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of Bridge Maintenance and Replacement Decisions Using History-Dependent Models

by

Charles-Antoine Robelin

Doctor of Philosophy in Engineering-Civil and Environmental Engineering

University of California, Berkeley

Professor Samer M. Madanat, Chair

This dissertation addresses the determination of optimal decisions for bridge maintenance and repair both for one facility and for a system of heterogeneous facilities. Deterioration models are used to predict the future condition of facilities, which is required in the optimization. More specifically, deterioration models decrease or capture the uncertainty regarding future condition. This dissertation concentrates on the use of deterioration models that take into account aspects of the history of deterioration and maintenance.

The first part of the dissertation presents the optimization of bridge inspection, maintenance, and repair decisions for a system of heterogeneous facilities, using a deterioration model from the literature; this model is non-Markovian, and its formulation makes the optimization problem computationally complex. The determination of exact optimal solutions is unlikely to be achieved in polynomial time, and we derive bounds on the optimal cost.

We show in a case study that these bounds are close to the optimal cost, which indicates that the corresponding policies are near optimal.

The second part of the dissertation presents the optimization of maintenance and repair decision for a system of heterogeneous bridge decks, using a Markovian deterioration model. The dependence of this model on history is achieved by including aspects of the history of deterioration and maintenance as part of the state space of the model. We present an approach to estimate the transition probabilities of the model, using Monte Carlo simulation. This model is then used to formulate the problem of optimizing maintenance and repair decisions for one bridge deck as a finite-state, finite-horizon Markov decision process. Numerical simulations show that the benefits provided by this augmented-state model, compared to a simpler Markovian model, are substantial.

Based on the facility-level results, optimal maintenance and repair decisions are determined for a system of heterogeneous facilities. Recommendations are provided for each facility, and we provide formal proofs of optimality in the continuous case. A numerical study shows that the results obtained in the discrete-case implementation seem to be valid approximations of the continuous-case results. The computational efficiency of the system-level solution makes this approach applicable to systems of realistic sizes.

Professor Samer M. Madanat
Dissertation Committee Chair

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Chapter 1

Introduction

This dissertation addresses the problem of determining optimal maintenance and repair decisions in the context of civil infrastructure systems, for a bridge or a system of several bridges. This problem consists of making sequential resource allocation decisions based on the evolution of the system over time, which is stochastic in nature.

Civil infrastructure management is the process by which agencies monitor, maintain, and repair deteriorating systems of facilities, within the constraints of available resources. More specifically, the management process refers to the set of decisions made by an infrastructure agency over time to maximize system performance. The basic maintenance and repair (M&R) decisions an agency has to make are: in every time period, what M&R activity should be performed on each facility in the system?

Based on the recent Status of the Nation's Highways, Bridges, and Transit (FHWA, 2002), the average year of construction of the bridges in the United States was determined to be 1963. In 2002, 50% of the daily traffic in the U.S. utilized bridges that were more than forty years old. Of the 586,000 bridges in the nation, 28% were deficient, where half

of these were structurally deficient. The deteriorating bridge population, as well as the limited amount of funds available for maintenance and inspection, led to the development of bridge management systems to optimize the use of available funds by helping agencies make maintenance and repair decisions.

Experience with infrastructure management systems in the United States shows that the benefits of systematic approaches to facilities management have been substantial in practice. For example, the Arizona Department of Transportation reported that the implementation of their Pavement Management System (PMS) to optimize pavement rehabilitation expenditures has saved over \$200 million in maintenance and rehabilitation costs over a five-year period (OECD, 1987). These savings were achieved because the maintenance and rehabilitation resource allocation decisions were made by the PMS with the objective of minimizing the life-cycle costs of the pavement sections in the network. In the area of bridges, the Intermodal Surface Transportation Efficiency Act (ISTEA) of 1991 required every state Department of Transportation and metropolitan planning organization to implement a bridge management system in order to optimize the allocation of resources for maintenance planning. This federal mandate has recently been waived (Schweppe, 2001). Despite the waiver, government agencies are still faced with aging infrastructure and limited resources. Thus, there is a definite need for the optimization of the allocation of resources for maintenance planning.

1.1 Motivation

This dissertation considers maintenance and repair decisions for a bridge or a system of several bridges. The result of the optimization presented in this dissertation is a set of maintenance and repair policies, i.e., a set of recommendations of maintenance or repair

actions for each facility, depending on the year and the condition of each facility. The cost of a maintenance action of a given type increases as the condition of the facility on which it is performed deteriorates. Optimal decisions are based on this fact, as well as the relationship between the costs and effectiveness of maintenance actions of different types, and on trade-offs between immediate and future costs.

Evaluating the cost effectiveness of maintenance and repair decisions requires the estimation of the future condition of the facilities. This estimation is done through the use of deterioration models. Given the inherent stochasticity associated with the deterioration, the purpose of a deterioration model is to decrease the uncertainty on the prediction of the future condition. There exists a wide range of deterioration models, differing in their required inputs and their prediction capabilities.

The selection of a deterioration model for use in the optimization of maintenance and repair decisions is generally based on the trade-off between the desired level of realism and the computational complexity. The more realistic a model, the lower the uncertainty in the prediction of the future deterioration is but the higher the complexity of the optimization is. This dissertation concentrates on the use of deterioration models that take into account aspects of the history of maintenance and deterioration. We present the use of these history-dependent models in computationally efficient formulations of the problem of optimizing maintenance and decisions for one facility and for a system of several facilities.

Figure 1.1 shows the reduction in the future condition prediction uncertainty if the age of the facility is included as an explanatory variable of the deterioration model. More precisely,

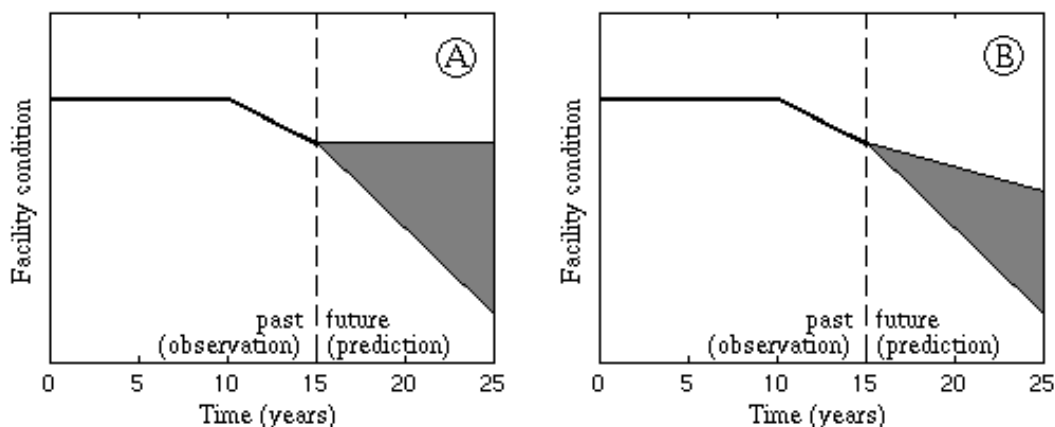


Figure 1.1: Range of condition predictions, starting from year 15, according to two models based on the same physical processes: (A) model including the current condition; (B) model including the current condition and the age of the facility.

the shaded regions indicate the range of condition prediction after year 15. The two graphs compare the ranges of predictions of two models based on the same physical processes, assuming that the information provided to the deterioration models is:

- on the left-hand side, the condition of the facility in year 15;
- on the right-hand side, the condition of the facility in year 15, as well as the age of the facility (i.e., 15 years).

This example shows one way to include information about history in a deterioration model; other possibilities are presented in this dissertation.

1.2 Outline

The remainder of the dissertation is organized as follows:

Chapter 2 reviews existing bridge management systems and optimization models. We present five bridge management systems used or designed for use by agencies in charge of the management of bridges. We also compare models of optimization of bridge maintenance and repair decisions from the literature.

Chapter 3 addresses inspection, maintenance, and repair decision making for a system of heterogeneous facilities, using non-Markovian models of deterioration. Since the determination of the exact solution to the optimization problem rapidly increases with the number of facilities in the system, we derive bounds on the optimal cost and near-optimal policies.

Chapter 4 addresses maintenance and repair decision making for a single facility. We present the derivation of an augmented-state Markovian model of deterioration, taking into account aspects of the history of deterioration and maintenance. We use this model in the optimization of maintenance and repair decisions, formulating the problem as a finite-state, finite-horizon Markov decision process. The benefits of including the additional information (i.e., regarding history) are evaluated through simulation.

Chapter 5 addresses maintenance and repair decision making for a system of heterogeneous facilities, based on the facility-level results from chapter 4. Optimal policies are determined for all facilities in the system. We present formal proofs of the results in the continuous case. The optimization is implemented in the discrete case, and a parametric study shows that the discrete-case results can be considered a valid approximation of the continuous-case results.

Chapter 2

Review of Bridge Management

Systems and Optimization Models

Given available resources and a set of possible maintenance and repair (M&R) actions, the objective of infrastructure management is to determine the optimal M&R decisions in the current year and in future years. The solution of this problem is based on the consequences of possible actions on the future condition of the system. Since information about future conditions is not available at the time of decision making, deterioration models are used to predict future deterioration. This framework is common to all existing bridge management optimization methods, although the actual formulation of the optimization and the deterioration models differ.

In the present section, the differences, characteristics, and limitations of bridge management systems (BMS) implemented by agencies, or designed for implementation, are presented first. Optimization models of bridge maintenance and repair, found in the literature, are reviewed in the second part of this section.

2.1 Characteristics and Limitations of Existing Bridge Management Systems

Pontis (Golabi and Shepard, 1997), Bridgit (Hawk, 1994), the North Carolina Bridge Management System (Al-Subhi et al., 1990) and the Indiana Bridge Management System (Sinha et al., 1988; Saito and Sinha, 1989a,b; Jiang and Sinha, 1989; Gion et al., 1992) are four major BMS that have been developed in the United States. Their purpose is to support M&R decision making for a system of several bridges, considering constraints of available budget and system performance. The types of deterioration models used in these four BMS are similar; however, the optimization approach in Pontis differs from that in Bridgit, the North Carolina Bridge Management System (NCBMS), and the Indiana Bridge Management System (IBMS).

- In Pontis, the optimization approach is top-down. Maintenance and repair optimization is performed at the system level. Actions are recommended for fractions of the bridge population. Actual bridges on which actions are performed are then selected, either manually or by a subroutine. This final selection may differ slightly from the optimization done earlier at the system level. This approach overcomes dimensionality problems in a very effective manner. Pontis considers populations of bridge components rather than individual bridges, which makes it well suited for large systems. However, some of the assumptions required in this approach are unrealistic. Parameters, recommendations, and facility conditions must be aggregated over the bridge population. This may be problematic, as a group of bridges is usually less homogeneous than a system of pavement sections. Moreover, bridge-specific information or environmental factors cannot be taken into account. The version of Pontis described in Golabi and Shepard (1997) divides the

bridges into components, which does not allow the interactions between the components of a bridge to be considered, with the following two consequences: it leads to inaccurate modeling of deterioration, and it prevents the optimization model from favoring practical maintenance strategies. For example, if a bridge deck is to be maintained and its substructure will need maintenance in the near future, it is logical to group the maintenance actions in order to minimize the closure of the bridge. Based on conversations with engineers at the Federal Highway Administration and at the California Department of Transportation, the aforementioned limitations are seen as the main reasons why bridge management systems such as Pontis are not used to their fullest capacity by state agencies in the United States.

- In Bridgit, the NCMBS, and the IBMS, the optimization approach is bottom-up. Maintenance and repair optimization is performed for every facility. Actions are evaluated for individual bridges. These recommendations are then aggregated and certain actions are selected to take into account system level constraints, such as budget or overall performance. The general layout of the optimization in these BMS occurs in two steps. The first step is to define possible sets of actions at the bridge level, called life-cycle activity profiles. The second step selects one life-cycle activity profile for each bridge to maximize the total effectiveness, under constraints of budget and overall performance. This step requires the use of integer programming. The maintenance costs calculated in these BMS are called life-cycle costs. However, the optimization is performed over a limited period, which we refer to as the optimization period (five years in the IBMS, twenty years in Bridgit). Beyond this period, maintenance actions are fixed. It is possible to address this issue by extending the optimization period. However, this would require increasing the number of alternatives for each bridge, which may make the problem intractable.

2.2 Review of Bridge Management Optimization Models

Optimization models present in the literature typically have a higher degree of complexity than bridge management systems. However, they usually cannot be readily implemented and can be viewed as prototypes for future BMS. As was the case for the BMS presented above, the objective of these models is to optimize M&R decisions, based on the knowledge of the current condition of the system through inspections and on the prediction of future condition through the use of deterioration models. While the general framework of the optimization models is similar, these models differ from each other in many aspects: representation of the facility condition, scope of optimization, decision variables, deterioration models, layout of the optimization, and assumptions about the knowledge of the current condition.

Representation of the facility condition. In pavement management, a facility is usually evaluated in terms of its serviceability, i.e., the level of service it provides to its users with respect to the function it is required to perform. For example, highway pavements are evaluated in terms of their ability to provide riders with a safe and comfortable ride, which also limits fuel consumption (Watanatada et al., 1987). Such an indicator is also used in the case of highway bridge decks, for example, in Pontis. Other models, such as in Mori and Ellingwood (1994a) and Kong and Frangopol (2003), consider the probability of failure (or a quantity directly related to the probability of failure) as the measure of the condition of highway bridges. In these models, the condition is represented as a continuous variable, whereas most models considering serviceability use discretized states.

Scope of the optimization. The optimization is performed on a given system for a given planning horizon. The system can be composed of one facility or a system of several

facilities.

The facility-level problem deals with one bridge only and is obviously less complex than the system-level problem. It is also less realistic, especially in the presence of budget constraints. Namely, a budget is usually provided for several facilities and not for a single facility. In practice, a facility-level bridge management system is not very useful. However, most advances in deterioration or optimization models are first explored at the facility level (Madanat and Ben-Akiva, 1994; Mori and Ellingwood, 1994b; Guillaumot et al., 2003; Chung et al., 2003; Kong and Frangopol, 2003).

The system-level problem represents the realistic case where a budget is used for the management of several facilities. Arbitration between actions on different facilities must be performed. Some facility-level models have been extended to the system level. For example, the model developed Smilowitz and Madanat (2000) can be considered a system-level extension of Madanat and Ben-Akiva (1994). However, many facility-level models, in particular the reliability-based models from Mori and Ellingwood (1994b) and Chung et al. (2003), have not been extended to the system level.

Decision variables. The main objective of bridge management optimization models is to optimize the maintenance of a bridge or of a system of bridges, i.e., to help with the decision of which maintenance action to apply on a given facility in a given year.

The decision to inspect a facility in a given year can also be part of the decision variables (Madanat and Ben-Akiva, 1994; Mori and Ellingwood, 1994b; Smilowitz and Madanat, 2000; Chung et al., 2003).

Pontis, Bridgit, and the model developed in Guignier and Madanat (1999) include improvement as a decision variable. Improvements are different from regular maintenance and repair

actions. The goal of maintenance and repair actions is to restore a facility to its original condition, stop its deterioration, and/or decrease its deterioration rate over a limited period of time. Improvements alter the functionality of a facility, e.g. increase the vertical or horizontal clearance, increase the load limit, etc. Improvements and maintenance actions are optimized separately in Pontis and Bridgit, and jointly in Guignier and Madanat (1999). This latter model showed that the joint optimization results in cost savings.

Deterioration model. The optimization of bridge maintenance and repair decisions is based on predictions of future condition. Deterioration models are used for these predictions. Many models present in the literature of optimization of maintenance are time-independent: the models from Madanat and Ben-Akiva (1994), Smilowitz and Madanat (2000), Jiang et al. (2000), and Durango and Madanat (2002). This is also the case for some of the bridge management systems described earlier: Pontis, Bridgit, and the Indiana bridge management system. This means that the predicted future condition of a facility only depends on its current condition and is independent from its past condition. In other words, using states to represent the condition of a facility, the probability for an element to transition from an initial state A to a lower state B does not depend on the time spent in state A. This assumption is linked to the use of Markov chains to represent the deterioration of a facility. A transition probability matrix is defined for each maintenance action. These models recognize that the deterioration of a facility is a stochastic process. However, they consider that this process is known. The formulation developed in Durango and Madanat (2002) explicitly accounts for the uncertainty in the model definition. The definition of the model is updated over time, as information about the actual deterioration becomes available. It should be noted that this deterioration model is also time-independent, in the sense

that the deterioration of the facility from time period n to time period $n+1$ only depends on its condition at n . However, the dependence, expressed by a transition probability matrix, changes over time.

Other models use time-dependent deterioration models (Mori and Ellingwood, 1994a; Kong and Frangopol, 2003). These models are based on physical processes such as corrosion. Moreover, the deterioration is expressed by continuous variables instead of the discretized states described earlier. For example, the deterioration in Mori and Ellingwood (1994b) is composed of two phases, a period before the initiation of damage and a period of linear deterioration. The influence of maintenance is modeled by stopping the deterioration for a period of time and/or decreasing the deterioration rate. The parameters of the models are represented by known, random variables. The uncertainty in the model is not taken into account in the optimization models described in Mori and Ellingwood (1994b) and Kong and Frangopol (2003). A method to update the model parameters using available data is provided in Enright and Frangopol (1999), but it has not been implemented in an optimization model.

Layout of the optimization. The optimization model can be set up in several ways. The first formulation described here is common to the problems posed as Markov decision processes, the model by Madanat and Ben-Akiva (1994) at the facility level and the model by Smilowitz and Madanat (2000) at the network level. The objective of these models is to minimize the total cost under a budget constraint. The total cost is composed of the agency costs and the user costs. The agency costs represent the actual cost of the maintenance, while the user costs are an expression of the condition of the facilities in

monetary units. In Madanat and Ben-Akiva (1994) and in Smilowitz and Madanat (2000), the cost of maintenance also includes the cost of inspections. These problems are solved using dynamic programming at the facility level and linear programming at the system level. At the system level, the dimensionality is overcome by aggregating homogeneous facilities into groups, and providing recommendations for fractions of the population of these groups.

A different approach is presented in Mori and Ellingwood (1994b). This facility-level model minimizes the sum of the agency costs and the expected failure costs, under the following safety constraint: the probability of failure of the facility within the planning horizon must be kept under a user-defined value. In this model, agency costs also include inspection costs. In Kong and Frangopol (2003), the costs to be minimized are only the maintenance costs, but they are estimated in a more complex manner and depend on the condition of the facility. The constraint serves the same purpose as in Mori and Ellingwood (1994b), but uses a slightly different formulation: the reliability index of the structure must be kept above a predetermined target level.

In the BMS with a bottom-up approach described earlier (Bridgit, the Indiana BMS, and the North Carolina BMS), the objective of the optimization is to maximize the effectiveness, under constraints of budget and overall performance. Effectiveness is generally defined as a monetary measure of the benefits of maintenance policies with respect to their costs, with slight variations depending on the model. Referring to the von Neumann-Morgenstern decision theory, Val and Stewart (2003) justify that minimizing the expected life-cycle cost is equivalent to maximizing the expected utility expressed in monetary values.

Knowledge of current condition. The current condition of the system is estimated through inspections. In the United States, bridges are usually inspected every two years (CFR, 2002). In the model from Kong and Frangopol (2003) (as well as in the bridge management systems described earlier), inspections are assumed to be performed periodically and are not part of the decision variables. Moreover, the current condition of the system is considered known exactly in these models. This is not realistic, as the condition of bridges is assessed predominantly through visual inspections (AASHTO, 2000), which are subject to errors. In the presence of uncertainty with respect to the bridge elements' conditions, a conservative approach is to assume the worst possible condition. Such an approach may lead to unnecessary maintenance actions, thus yielding higher M&R costs. To reduce this uncertainty, it is possible to inspect the bridge components' conditions more frequently. Thus, a trade-off exists between M&R costs and inspection costs.

An improved model is presented in Mori and Ellingwood (1994b) for concrete structures and in Chung et al. (2003) for steel bridges. In these reliability-based models, inspections are part of the decision variables and are assumed to be error-free. While the model in Chung et al. (2003) uses a more complex and more realistic deterioration model, it considers only one maintenance action. The probability of performance of a maintenance action is provided by a model and is not part of the decision variables. The model in Mori and Ellingwood (1994b) introduces a threshold of detectability of damage. If the damage is smaller than the threshold, it is not detected, whereas if it is larger than the threshold, it is detected and repaired with probability 1. This model also considers partial inspections. Each scheduled inspection starts with the inspection of two small portions of a facility. If no damage occurrence greater than a predetermined threshold is detected in one or both portions, no

further inspection or repair is performed until the following scheduled inspection. Otherwise, the entire facility is inspected and all detected damage occurrences are repaired.

The models developed in Madanat and Ben-Akiva (1994) at the facility level and Smilowitz and Madanat (2000) at the system level also jointly optimize inspections and maintenance. However, these models differ from the previous ones, as they explicitly recognize the presence of random errors in the inspection results. The methodology developed in these models is referred to as a *latent Markov decision process*: the Markov decision process is not defined over the space of the condition states of the system, which is unknown, but over the space of the information states. The information state contains all measured condition states and actions performed since the beginning of the planning horizon and the initial condition of the system, which is assumed known. The model developed in Guillaumot et al. (2003) optimizes jointly M&R actions and inspections and does not assume error-free inspections. It also considers uncertainty in the deterioration model definition, as was introduced in Durango and Madanat (2002) and Durango (2004).

2.3 Limitations of Bridge Management Systems and Optimization Models

Optimization models using more realistic, history-dependent deterioration models consider only one facility (Mori and Ellingwood, 1994b) or a system of homogeneous facilities (Kong and Frangopol, 2003). Moreover, the set of decision variables considered in this latter model is limited. This is also the case in existing bridge management systems such as Bridgit, the Indiana BMS and the North Carolina BMS.

On the other hand, existing optimization models considering a large set of decision variables for a system of facilities use Markovian models of deterioration. The limitation of these Markovian models is the memoryless assumption. Although parts of this assumption may be valid for certain bridge states, namely those where the deterioration is primarily governed by mechanical processes, Mishalani and Madanat (2002) have shown empirically that it is unrealistic for bridge states where the deterioration is primarily governed by chemical processes. These optimization models are based on a top-down approach. The limitations identified in the case of Pontis, in relation to the aggregation required for this approach, also apply to these models.

This dissertation presents optimization models that address these limitations. The optimization of maintenance and repair decisions is first formulated using the realistic, reliability-based, history-dependent model presented in Kong and Frangopol (2003). The optimization is done for a system of facilities, considering a realistic set of decision variables.

Markovian models of deterioration allow for the use of standard optimization techniques, such as dynamic programming and linear programming. The Markovian models we present also take into account aspects of the history of deterioration and maintenance, which increases their prediction capability. These models are implemented in facility-level maintenance and repair optimization. The optimization is also extended to a system of heterogeneous facilities, providing action recommendations for each facility.

Chapter 3

Bottom-Up Optimization Using Non-Markovian Deterioration Models

The limitations identified in the literature review point to the need for a method to optimize maintenance and replacement decisions for a system of bridges, with the following objectives:

- Bridge-specific attributes, including environmental factors and probability of failure, must be taken into account in the optimization.
- For each bridge or each component of each bridge, a set of recommended actions must be determined.
- System-level constraints (budget, overall performance) must be taken into account.
- History-dependent deterioration models, whether physical or empirical, should be used.

3.1 Definitions and Assumptions

System. The system considered is a system of highway bridges. The system is managed by a single agency, such as a state Department of Transportation. While the condition of the bridges obviously changes over time, the system remains constant over the planning horizon; no bridges are built or decommissioned.

Budget. For accounting and fiscal reasons, an agency usually has a yearly budget available for the maintenance of the system. In some cases, the available budget has specific funds dedicated to a certain type of activity. For example, a portion of the budget can only be used for replacement of facilities, while another part may be reserved for routine maintenance. In the present model, such refinements are not taken into account and the yearly budget is available for all activities. However, including this refinement does not increase significantly the complexity of the problem. In the present chapter, the budget is not assumed to be cumulative, i.e., the agency cannot save part of its budget in a given year for use in subsequent years.

Costs. The agency incurs costs when maintenance actions or inspections are performed. Moreover, maintenance actions on a bridge usually imply the closure of some or all of its lanes. This leads to delays to the users and/or costs associated with detours. This is particularly important in the case of bridges. In a highway network, bridges are usually capacity constraining, due to their high cost of construction relative to regular highway lanes. Moreover, convenient detours may not be available. In pavement management, the influence of the facility condition on the users is usually modeled by user costs. The user costs represent vehicle wear and tear, fuel consumption due to rough roads, and a translation in monetary values of the riding discomfort. In the case of bridges, users are

assumed indifferent to the facility condition, as long as there is no failure. Bridges are usually short compared to the total distance of a trip, and the roughness of their surface does not influence significantly the total fuel consumption and the overall driving comfort. Thus, user costs consist of delays, closures, and detours associated with the performance of maintenance actions.

Measure of facility condition and modeling of bridges. The only bridge component considered is the deck. The condition of a bridge deck is measured by its reliability index β , which is linked to its probability p of failure by the following relationship:

$$p = \Phi(-\beta)$$

where Φ is the cumulative standard normal distribution. The memoryless assumption made in all current system-level models is relaxed; the deterioration model is continuous, stochastic, and time dependent. The epistemic uncertainty in the deterioration model is not taken into account.

Modeling of maintenance. Although the deterioration of the deck is a continuous process and is modeled as a continuous process, maintenance decisions are made at discrete points in time. Due to the presence of yearly budget constraints, the most logical time unit for maintenance decisions is the year. This assumption is consistent with the typical rate of deterioration of bridges. Within a year, the actual point in time at which the maintenance action is performed is not important, and a maintenance action is said to be performed “in year t .” Hence, the planning horizon is broken down into periods of one year. The planning horizon is assumed to start at the beginning of year 0 and to end at the end of year $T - 1$, where T is the length of the planning horizon.

Knowledge of facility condition. It is assumed that inspections are error free and provide perfect information about the condition of the inspected facility.

3.2 Problem Formulation

A bottom-up approach is used to take into account bridge-specific information and prevent aggregation problems. Given the initial condition of the system, the objective of the agency is to minimize the sum, over the planning horizon, of the expected cost of maintenance, inspection, and failure. When maintenance is performed, user costs are incurred at the same time as agency costs. Thus, the sum of user and agency costs is denoted as maintenance costs.

Two formulations were tried:

- In the deterministic formulation, the set of decision variables is a set of binary variables. Each binary variable represents the decision to perform a given action on a given facility in a given year. The result of the optimization is a set of recommendations such that the reliability and budget constraints are satisfied. From a practical point of view, this means that the optimization is performed at the beginning of the planning horizon, and the recommendations are applied regardless of the actual reliability index of the facilities. This formulation is described in appendix A.
- The second formulation, based on the same assumptions, is motivated by the fact that the information about the past and current condition of the facilities, provided by inspections, is not used in the deterministic formulation. If this information is used, the maintenance can be further optimized. The formulation, referred to as *partially stochastic*, is described below.

The M&R actions considered in the partially stochastic model can be grouped into two gen-

eral categories. The actions of the first category, which will be called *maintenance actions*, are performed according to a schedule, regardless of the condition of the facilities. The frequency of the performance of maintenance actions is a decision variable, for each bridge and each type of maintenance action. The actions of the second category, which will be called *repair actions*, are performed when the condition of a facility reaches a predetermined level. The distinction between time-based and condition-based triggers of M&R actions was first presented in Kong and Frangopol (2003). The level at which a repair action is performed is a decision variable for each bridge and each type of repair action. In that case, the result of the optimization is referred to as a *policy*: the recommendation of action depends on the facility and the time, as well as on the realization of the condition of the facility.

The following notation is used:

- n : number of facilities in the system;
- T : number of years in the planning horizon;
- M_i : number of types of maintenance actions that can be applied on facility i ;
- R_i : number of types of repair actions that can be applied on facility i .

The decision variables are:

- $1_{i,t,m}^{\text{maintenance}}$: binary variable equal to 1 if the maintenance action of type m is scheduled on facility i in year t , 0 otherwise, $i \in \{1, \dots, n\}$, $m \in \{1, \dots, M_i\}$.
- $1_{i,t}^{\text{inspection}}$: binary variable equal to 1 if an inspection is scheduled on facility i in year t , 0 otherwise, $i \in \{1, \dots, n\}$.
- $\beta_{i,t,r}^{\text{repair}}$: target reliability index used for the repair action of type r on facility i in year t , $i \in \{1, \dots, n\}$, $r \in \{1, \dots, R_i\}$. For a given facility i in condition c at the beginning of year t , the repair action to be performed in year t is of type r such that $\beta_{i,t,r}^{\text{repair}} \geq c$ and

for any $r' \neq r$ in $\{1, \dots, R_i\}$, $\left[\beta_{i,t,r'}^{\text{repair}} < c \text{ or } \beta_{i,t,r'}^{\text{repair}} > \beta_{i,t,r}^{\text{repair}} \right]$. In other words, the type of the repair action performed is r , such that $\beta_{i,t,r}^{\text{repair}}$ is the lowest target reliability index greater than the condition c of the facility.

- $I_{i,r,m}$: number of years after repair action r has been performed on facility i during which maintenance action m is not performed, regardless of the value of $1_{i,t,m}^{\text{maintenance}}$. The purpose of this variable is to prevent maintenance actions to be performed in a period of a few years following a repair action. Let us consider some examples of the use of $I_{i,r,m}$, assuming that repair action r has been performed on facility i in year t and that no other repair action has been performed. If $I_{i,r,m} = 1$ and $1_{i,t,m}^{\text{maintenance}} = 1$, then maintenance action m is not to be performed in year t , even though it was scheduled. If $I_{i,r,m} = 1$ and $1_{i,t+1,m}^{\text{maintenance}} = 1$, then maintenance action m is to be performed in year $t + 1$ as scheduled. If $I_{i,r,m} = 0$, then maintenance actions of type m are performed as scheduled, even in year t . In the general case, where several repair actions r_1, r_2, \dots, r_k have been performed on facility i in years t_1, t_2, \dots, t_k , maintenance action m is to be performed on facility i in year t if and only if $\left[1_{i,t,m}^{\text{maintenance}} = 1 \text{ and } t_i + I_{i,r_p,m} \leq t, p \in \{1, \dots, k\} \right]$.

The formulation of the optimization problem is shown below. In the formulation, the time variable is usually denoted by t in the discrete case (e.g., time at which maintenance and replacement decisions are made), and by t_{cont} in the continuous case (e.g., in the deterioration).

$$\min \sum_{i=1}^n \left[F_i(T) C_i^{\text{failure}} + \sum_{t=0}^{T-1} \alpha^t \left[1_{i,t}^{\text{inspection}} C_i^{\text{inspection}} + E \left[C_{i,t}^{\text{maintenance+repair}} \right] \right] \right] \quad (3.1)$$

subject to

$$F_i(t) \leq P_i^{\text{acceptable}}, \quad i \in \{1, \dots, n\}, t \in \{0, \dots, T-1\} \quad (3.2)$$

$$\beta_i(t_{\text{cont}}) = f_i \left(t_{\text{cont}}, t_i^{\text{repl}}(t_{\text{cont}}), \left\{ A_{i,t}, t \in \left\{ t_i^{\text{repl}}(t_{\text{cont}}), \dots, \lfloor t_{\text{cont}} \rfloor \right\} \right\} \right), \quad (3.3)$$

$$i \in \{1, \dots, n\}, t_{\text{cont}} \in [0, T)$$

$$\sum_{i=1}^n \left[1_{i,t}^{\text{inspection}} C_i^{\text{inspection}} + E \left[C_{i,t}^{\text{maintenance+repair}} \right] \right] \leq B_t, \quad (3.4)$$

$$t \in \{0, \dots, T-1\}$$

In addition to the notation defined earlier, the following notation is used:

- $F_i(t)$: probability of failure of facility i in year t
- C_i^{failure} : cost of failure of facility i
- α : discount factor; $\alpha = 1/(1+r)$, where r is the interest rate
- $C_i^{\text{inspection}}$: cost of inspection for facility i
- $E \left[C_{i,t}^{\text{maintenance+repair}} \right]$: expected cost of maintenance and repair of facility i in year t .

The expected value of the cost is used because of the probabilistic nature of the performance of maintenance and repair actions. Namely, repair actions are performed based on the deterioration, which is a probabilistic process; scheduled maintenance actions are performed only after a given period of time following a repair action. As the closed form of the expected cost is not available in the general case, it will be determined through Monte Carlo simulations, which are described later.

- $P_i^{\text{acceptable}}$: acceptable probability of failure for facility i

- $\beta_i(t)$: reliability index of facility i at time t . By definition of the reliability index, the instantaneous probability of failure of facility i at time t (given it has not failed yet) is $\Phi(-\beta_i(t))$.
- $\lfloor t_{\text{cont}} \rfloor$: largest integer less than or equal to t_{cont}
- $t_i^{\text{repl}}(t_{\text{cont}})$: year of last replacement of facility i before time t_{cont} . If the facility i has not yet been replaced by time t_{cont} , $t_i^{\text{repl}}(t_{\text{cont}}) = 0$
- $A_{i,t}$: set of maintenance and repair actions performed on facility i in year t . This set may be empty. Due to the probabilistic nature of deterioration, this set is not deterministic. However, given a set of actual deterioration parameters and a repair policy, this set can be determined. This will be used in Monte Carlo simulations, which are described later.
- $f_i(t_{\text{cont}}, t_i^{\text{repl}}(t_{\text{cont}}), \{A_{i,t}, t \in \{t_i^{\text{repl}}(t_{\text{cont}}), \dots, \lfloor t_{\text{cont}} \rfloor\}\})$: reliability index of facility i at time t_{cont} , depending on actions performed between year $t_i^{\text{repl}}(t_{\text{cont}})$ and year $\lfloor t_{\text{cont}} \rfloor$. This continuous function of time is the basis for the modeling of deterioration and ensures that the deterioration is time-dependent.
- B_t : budget made available to the agency at the beginning for year t , to be used in year t .

Constraint (3.2) is a reliability constraint. Constraint (3.3) determines the deterioration of the facilities. Constraint (3.4) is the budget constraint and is based on expected costs. Due to the large number of bridges in the system, the total cost for the system in a given year has a very low variance. Thus, if the expected system cost satisfies the budget constraint, the probability that the actual cost exceeds the budget constraint is very low.

Due to the probabilistic nature of the application of repair actions, it is not feasible to find a closed-form of the deterioration and of the expected costs of maintenance and repair for a reasonably long planning horizon. Thus, the expected costs, the deterioration profile,

and the probability of failure will be estimated using Monte Carlo simulations. Simplifying assumptions on the decision variables must be made in order to decrease the complexity of the problem. For example, the maintenance actions may be assumed to be performed at a certain frequency. For one facility and one type of maintenance action, the number of decision variables decreases from T to two: one for the frequency and one for the starting year. The number of types of maintenance and repair actions may also be small, as is the case in Frangopol et al. (2001), where one type of maintenance action is considered and replacement is the only type of repair.

3.3 Implementation of the Solution

For each bridge, the decision variables are:

- the threshold in reliability index for replacement. If an inspection shows that the reliability index of the deck is under this threshold, the deck is replaced,
- the frequency of inspections,
- for each type of maintenance action, the frequency with which the action is scheduled,
- for each type of maintenance action, the number of years after a replacement during which no maintenance action of this type is to be performed. For clarity, this period is denoted as a period of blocked maintenance.

A quadruple with values of the decision variables for one bridge will be referred to as a *policy*. Based on the problem formulation, the solution can be determined in two distinct phases.

3.3.1 Facility Level

For each bridge, a list of possible policies is created. Using Monte Carlo simulation, the cost of each possible policy for one bridge is determined, as well as the probability of failure of the bridge under this policy. If the probability of failure under a certain policy is lower than a predetermined threshold, this policy is kept for the second phase and is considered *feasible at the facility level*. Otherwise, it is considered infeasible and is not kept for the second phase.

3.3.2 System Level

At the beginning of this phase, a list of policies that are feasible at the facility level is available for each bridge, along with the associated costs. One policy per bridge must be selected in order to minimize the total cost, while satisfying the budget constraint for every year of the planning horizon. The set containing one policy per bridge for all bridges will be called a *combination of policies*. If the numbers of bridges and feasible policies per bridge were small, the combinations of policies could be enumerated and the combination leading to the minimum cost could be determined. However, 200 is a realistic order of magnitude estimate for the number of policies per bridge, which yields 200^n combinations for n facilities. Even for a small system of $n = 20$ facilities, these combinations cannot be enumerated in a reasonable amount of time.

Fortunately, bounds for the minimum cost can be found. A lower bound of the minimum cost is achieved by the following combination $C_{\text{lower.b}}$ of policies \mathcal{P}_i :

$$C_{\text{lower.b}} = \left(\mathcal{P}_1^{\text{lower.b}}, \mathcal{P}_2^{\text{lower.b}}, \dots, \mathcal{P}_n^{\text{lower.b}} \right) \quad (3.5)$$

where, for all $i \in \{1, \dots, n\}$,

$$\mathcal{P}_i^{\text{lower.b}} = \underset{p \in \{\text{policies of bridge } i\}}{\text{argmin}} \left[\sum_{t=0}^{T-1} \alpha^t \text{cost}_{i,p,t}^{i+m+r} + \text{cost}_{i,p}^f \times F_{i,p} \right] \quad (3.6)$$

In this formula, $\text{cost}_{i,p,t}^{i+m+r}$ is the expected cost of inspection, maintenance and repair of bridge i in year t under policy p , $\text{cost}_{i,p}^f$ is the cost of failure of bridge i under policy p , and $F_{i,p}$ is the probability of failure of bridge i under policy p . These minima can be determined in a reasonable amount of time. Moreover, if this combination of policies satisfies the budget constraint for every year of the planning horizon, it is the optimal solution. If it does not satisfy the budget constraint, one needs to find a combination of policies that satisfies the budget constraint to determine an upper bound of the minimum cost. A good candidate combination to satisfy the budget constraint is

$$C_{\text{upper.b}} = \left(\mathcal{P}_1^{\text{upper.b}}, \mathcal{P}_2^{\text{upper.b}}, \dots, \mathcal{P}_n^{\text{upper.b}} \right) \quad (3.7)$$

where, for all $i \in \{1, \dots, n\}$,

$$\mathcal{P}_i^{\text{upper.b}} = \underset{p \in \{\text{policies of bridge } i\}}{\text{argmin}} \left[\max_{t \in \{0, \dots, T-1\}} \text{cost}_{i,p,t}^{i+m+r} \right] \quad (3.8)$$

Let us explain the choice of this candidate using a simple example with one bridge and two policies over a planning horizon of three years. Let us assume that the first policy costs 400 (monetary units) in the first year and 0 in the following years and that the second policy costs 200 each year. If the budget made available each year is 200, the first policy is not feasible and the second policy is feasible, although the total cost of the first policy is lower than that of the second policy. This is due to the peaked cost of the first policy. Intuitively, policies with lower peaks in their cost per year are more likely to lead to a feasible combination. It will be shown on a realistic numerical example in section 3.4 that the yearly budget needed for this combination of policies is much smaller than that

needed for the combination $C_{\text{lower.b}}$. If the combination $C_{\text{upper.b}}$ does not satisfy the budget constraint, other combinations can be tried until one that satisfies the budget constraint is found.

3.3.3 Computational Complexity

The problem complexity is polynomial in the length of the planning horizon, with an order not exceeding 2. Thus, the length of the planning horizon can be large without unreasonably increasing the computation time needed to solve the problem. The determination of the lower bound and candidate upper bound described above is linear in the number of bridges. However, depending on the budget, $C_{\text{upper.b}}$ may be infeasible, and there is no guarantee that $C_{\text{upper.b}}$ is actually an upper bound. Moreover, there is no guarantee that $C_{\text{lower.b}}$ is feasible, thus optimal. Finding a meaningful upper bound (i.e., an upper bound that is relatively close to the lower bound) in a systematic manner is not likely to be a polynomial-time problem. Section 3.4 presents a realistic numerical example, for which the two bounds are close to each other.

A computer program was created in the C language to solve the optimization problem. With 20 bridges and approximately 200 policies per bridge, the computational time for the facility level portion is approximately 30 minutes on a standard personal computer, and the system level portion requires just a few seconds.

3.4 Case Study

3.4.1 Parameters

Deterioration model and influence of maintenance actions. Frangopol et al. (2001) presents a deterioration model for bridge decks. The time evolution of the reliability index

β is described, as well as the influence of a maintenance action on the reliability index.

Figure 3.1 shows the parameters that characterize the deterioration without maintenance and the influence of a maintenance action.

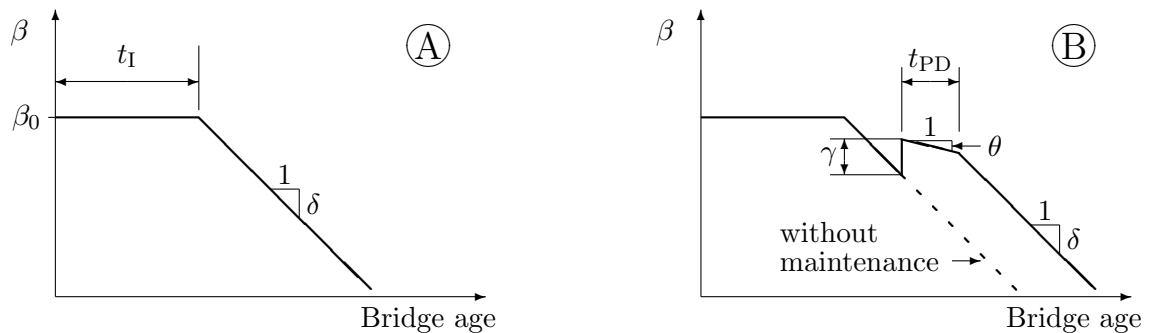


Figure 3.1: Deterioration without maintenance (A); influence of a maintenance action (B).

The reliability index of a new bridge is β_0 . Without maintenance, the reliability index is constant for a time period t_I and then decreases with slope δ . If a maintenance action is performed, the reliability index immediately increases by γ , then decreases with slope θ for a time period t_{PD} . Beyond this period of influence of the maintenance action, the reliability index decreases with slope δ . In Frangopol et al. (2001), the time at which a maintenance action is performed is a random variable, whereas in this study, it is a decision variable. All the other parameters are random variables. The distributions of these random variables are summarized in table 3.1 (Frangopol et al., 2001).

Table 3.1: Distributions of random variables from Frangopol et al. (2001).

Random variable	Distribution type	Characteristics (failure mode: shear)
β_0	Lognormal	mean = 8.5, standard deviation = 1.5
t_I (yr.)	Lognormal	mean = 15, standard deviation = 5
δ (1/yr.)	Uniform	minimum = 0.005, maximum = 0.20
t_{PD} (yr.)	Lognormal	mean = 10, standard deviation = 2
θ (1/yr.)	Uniform	minimum = 0, maximum = 0.05
γ	Lognormal	mean = 0.2, standard deviation = 0.04

In Frangopol et al. (2001), only one bridge and one type of maintenance action are considered, and the only repair action is replacement. However, the purpose of the present study is to demonstrate the capabilities of an optimization model in a more complex situation, both in terms of number of bridges and of types of maintenance actions. The costs for maintenance and replacement, including user costs, are provided in Kong and Frangopol (2003); these values were used as a basis for the present study. Using the same distribution types, the parameters (mean and standard deviation for lognormal; minimum and maximum for uniform) can be changed to represent slightly different bridges. A system of 20 bridges was created in this manner. One additional type of maintenance action was introduced, more costly and with a greater influence than the first one. The objective of this case study is to verify that the developed optimization model provides reasonable results.

Five groups of four bridges were created. Within each group, the four bridge decks have the same deterioration parameters, but their unit costs of maintenance are different. All decks are assumed to have the same area (1,000 m²).

Table 3.2 summarizes the parameters of the deterioration. The random variables β_0 and θ are not included in the table, as the values are the same as those presented in table 3.1.

The parameters for the first group of bridges (1–4) are the same as in Frangopol et al. (2001), except for the maintenance of type 2 that was not included in the paper. The other groups of bridges differ by their deterioration parameters. Bridges 5–8 have faster deterioration than bridges 1–4, and bridges 9–12 have faster deterioration than bridges 5–8. Bridges 13–16 have slower deterioration than bridges 1–4, and bridges 17–20 have slower deterioration than bridges 13–16. The unit costs of maintenance and repair are summarized in table 3.3. The unit costs of maintenance of type 1 and replacement for the first bridge of each group are based on Kong and Frangopol (2003). A higher unit cost was chosen for

Table 3.2: Deterioration parameters for the system of 20 facilities.

Parameter	Bridge				
	1-4	5-8	9-12	13-16	17-20
t_I (yr.)					
mean	10	6	3	12	14
standard deviation	2	1.5	1	2	2
δ (1/yr.)					
minimum	0.01	0.1	0.1	0.01	0.01
maximum	0.3	0.4	0.5	0.2	0.1
<i>Maintenance type 1</i>					
γ					
mean	0.2	0.2	0.2	0.2	0.2
standard deviation	0.04	0.04	0.04	0.04	0.04
t_{PD} (yr.)					
mean	10	8	5	12	14
standard deviation	2	1.5	1	2	2
<i>Maintenance type 2</i>					
γ					
mean	0.6	0.6	0.6	0.6	0.6
standard deviation	0.04	0.04	0.04	0.04	0.04
t_{PD} (yr.)					
mean	10	8	5	12	14
standard deviation	2	1.5	1	2	2

Table 3.3: Unit cost of maintenance and repair for the system of 20 facilities.

Action	Bridge			
	1, 5, 9, 13, 17	2, 6, 10, 14, 18	3, 7, 11, 15, 19	4, 8, 12, 16, 20
Maintenance type 1 (\$/m ²)	216	216	259	259
Maintenance type 2 (\$/m ²)	300	360	360	432
Replacement (\$/m ²)	2,818	2,818	2,818	2,818

the maintenance of type 2, which is more effective than the maintenance of type 1. For the other alternatives, the unit costs of maintenance are different in order to change the ratio between the unit costs of maintenance and replacement and the ratio between the unit costs of the two types of maintenance.

Other parameters. The planning horizon was chosen to be 75 years, and an interest rate of two percent was assumed. The target probability of failure is 10^{-5} over the planning horizon for each bridge. In absence of available data, several values were tried for the cost of failure.

3.4.2 Results

Costs of failure up to five times the unit cost of replacement were tried and did not change the results of the optimization. For each bridge, only one type of maintenance action is allowed, for simplification.

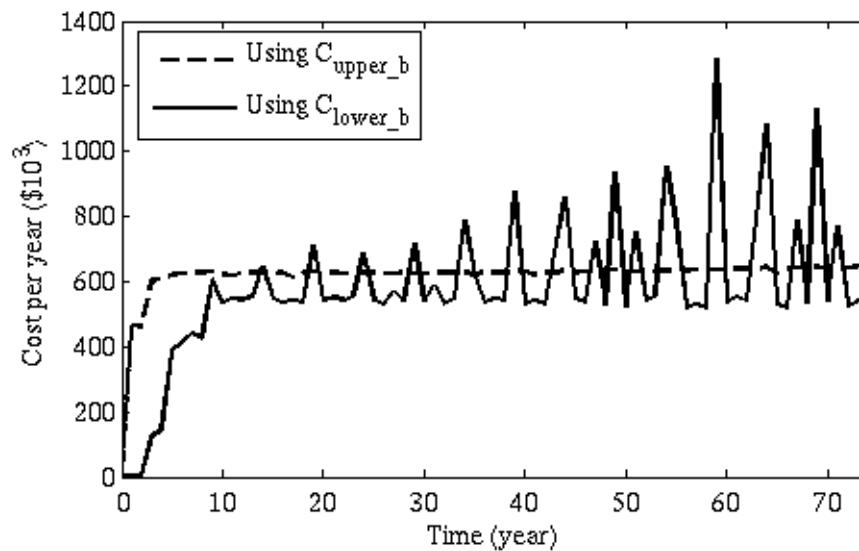
Optimal Policies and Costs

The policies leading to the minimum cost per bridge are described in table 3.4.

The present value of the total cost associated with the combination $C_{\text{lower.b}}$ of policies is 2.0×10^7 dollars over the planning horizon of 75 years. This combination is feasible for any budget above 1,286,000 dollars per year. The present value of the total cost associated with the combination $C_{\text{upper.b}}$ of policies is 2.3×10^7 dollars over the planning horizon of 75 years. The combination $C_{\text{upper.b}}$ is feasible for any budget above 646,000 dollars per year. Figure 3.2 compares the expected yearly costs when applying the combination $C_{\text{lower.b}}$ and when applying the combination $C_{\text{upper.b}}$. In this example, the total cost for the candidate upper bound is only 15 percent higher than the total cost for the lower bound, while the yearly budget required for the lower bound is twice as high as the budget for the candidate upper bound. The candidate upper bound demonstrates that a relatively small premium on the total cost allows for more evenly distributed yearly costs. Further analysis would be required to extend these results to the general case.

Table 3.4: Optimal policies for each facility.

Bridge	Interval between inspections (yr.)	Threshold β for replacement	Interval between maintenance (yr.)		Period of blocked maintenance (yr.)
			Type 1	Type 2	
1	5	4.5	10	—	10
2	5	4.5	10	—	10
3	5	4.5	10	—	10
4	5	4.5	10	—	10
5	3	4.5	—	10	4
6	3	4.5	7	—	6
7	3	4.5	—	10	4
8	3	4.5	7	—	6
9	3	4.5	5	—	2
10	3	4.5	5	—	2
11	3	4.5	5	—	2
12	3	4.5	5	—	2
13	5	4.5	13	—	10
14	5	4.5	13	—	10
15	5	4.5	15	—	10
16	5	4.5	15	—	10
17	1	4.0	20	—	10
18	5	4.5	24	—	10
19	3	4.5	—	—	—
20	3	4.5	—	—	—

Figure 3.2: Comparison of yearly costs resulting from the application of C_{lower_b} and C_{upper_b} .

Discussion

In this example, the optimization model provides logical and intuitive results:

- The faster the deterioration of a bridge, the shorter the recommended interval between maintenance actions is. This can be seen by examining the second bridge in each group. The list of bridges in ascending order of interval between maintenance, and in descending order of deterioration rate, is: 10, 6, 2, 14 and 18.
- The faster the deterioration of a bridge, the shorter the recommended period of blocked maintenance is.
- The smaller the ratio of the unit cost of repair to the unit cost of maintenance, the longer the recommended interval between maintenance is. This can be seen for bridges 13 through 16; maintenance is less costly for bridges 13 and 14 than for bridges 15 and 16 and is thus recommended at a higher frequency. Similarly, the high cost of maintenance for bridges 19 and 20, associated with their slow deterioration, leads to the absence of recommended maintenance, whereas maintenance is recommended for bridges 17 and 18, since it is less costly.
- Maintenance of type 2 is more effective than maintenance of type 1. However, it is also more costly. The smaller the ratio of the unit cost of maintenance of type 2 to the unit cost of maintenance of type 1, the less likely maintenance of type 1 is recommended and the more likely maintenance of type 2 is recommended. This can be seen for bridges 5 through 8. For bridges 5 and 7, the ratio of the cost of maintenance of type 2 to the cost of maintenance of type 1 is 1.4, while for bridges 6 and 8, this ratio is 1.7. Maintenance of type 2 is recommended for bridges 5 and 7, while maintenance of type 1 is recommended for bridges 6 and 8.

These results also contain one peculiarity: bridges 17 and 18 are similar in their deterioration patterns, and their unit costs of maintenance of type 1 are similar. Maintenance of type 1 is recommended for both bridges, so it should be at the same frequency, and the other recommendations (frequency of inspection, threshold for replacement) should be similar; however, this is not the case. These discrepancies occurred because the number of trials of the Monte Carlo simulation was not sufficiently large and the costs to be estimated were very close to each other. Namely, another simulation with 100,000 trials instead of 10,000 trials showed that the total costs were within one percent of each other. With fewer trials, it may happen that the order of the estimates is switched.

Evaluation of the number of trials. A qualitative method to evaluate whether the number of trials is sufficient to provide reliable results is as follows: for one bridge and one policy, a simulation consists of n_{trials} trials and provides an average total cost \bar{X} . If processes were deterministic and the simulation for the same bridge and the same policy were run twice, the two simulations would provide the same values for the total cost. However, in presence of probabilistic processes, the results of the two simulations are different. A larger number of trials produces a greater likelihood of similarity between the results. For one bridge and one policy, the variance and the mean of the results of the simulation can be estimated, which provides the coefficient of variation of the results. If this positive value is much smaller than 1, it indicates qualitatively that the number of trials is sufficient to provide reliable results.

Expected value as indicator of the cost. In this study, the Monte Carlo simulations were performed using 10,000 trials, which is quite a large number. Nonetheless, in an earlier paragraph, it was mentioned that this large number might not be sufficient to provide very

accurate estimates of expected costs and probabilities of failure. This is likely due to the large variance of the results between trials. This large variance is introduced by the presence or absence of replacement over the planning horizon, because replacements are very costly compared to maintenance. This also highlights a limitation in the approach that was used: the expected value may not be the most relevant indicator of cost.

Let us consider a simple and extreme example, with one bridge over a planning horizon of 50 years without discount rate. The deterioration of the deck is as follows:

- its initial reliability index is 8,
- it starts deteriorating immediately,
- its deterioration rate is uniformly distributed between 0.1 and 0.2 units of reliability index per year.

The policy is as follows:

- no maintenance is performed. In that case, the cost is zero;
- the deck is replaced when the reliability index reaches 4. In that case, the cost of replacement is 1,000 (monetary units).

If the actual deterioration rate is between 0.1 and 0.125 (which happens with probability 0.25), the bridge will not need to be replaced, whereas if the actual deterioration rate is between 0.125 and 0.2 (which happens with probability 0.75), the bridge will be replaced once in the planning horizon. This means that the cost over the planning horizon is 0 with probability 0.25 and 1,000 with probability 0.75. Thus, the estimated cost varies considerably between individual trials. Moreover, the average cost over several trials does not describe completely the actual costs. Thus, considering only the expected value as an indicator of costs leads to a significant loss of information.

The application of these remarks is not limited to the present example, or to the model

presented in this chapter. They also apply to the models developed in the literature, in which expected costs are considered in the optimization.

3.5 Summary

In the present chapter, the problem of optimizing inspection, maintenance, and repair decisions for a system of bridge decks was formulated, using a history-dependent, mechanistic model of deterioration. A solution was implemented, leading to bounds on the optimal cost and providing recommendations for actions that were shown to be close to optimal on a numerical example.

The complexity of the optimization method, a direct consequence of the complexity of the deterioration models used, is such that determining the optimal solution or even provably close bounds on the optimal cost is unlikely to be a polynomial-time problem. This limitation is a motivation for the Markovian approach presented in subsequent chapters.

Chapter 4

Facility-Level Optimization: Augmented-State Markov Decision Process

The objective of this chapter is to develop a bridge-component maintenance and repair (M&R) optimization approach that uses a Markovian deterioration model, while accounting for aspects of the history of deterioration and maintenance. These aspects are: the initial condition of the facility and the time since the performance of the latest maintenance action. Such a model represents a compromise between simple deterioration models allowing the use of standard optimization techniques, and realistic deterioration models whose complexity prevents efficient optimization of maintenance and replacement decisions, as was shown in the previous chapter.

Compared to Markovian models whose state is the current condition of the facility, the model developed in the present chapter includes additional information. Numerical simulations show that the benefits of including this additional information are significant.

4.1 Formulation of a History-Dependent Markovian Deterioration Model

The objective of the present section is to develop a model of the deterioration of a bridge deck with the two following characteristics: the model is Markovian, and it takes into account aspects of the history of deterioration and maintenance.

4.1.1 Definitions and Assumptions

The system considered is a single bridge deck managed by an agency such as a state Department of Transportation. Maintenance and replacement decisions are made by the agency at discrete points in time. Due to the presence of yearly budget constraints, the most logical time unit for M&R decisions is the year. As a consequence, the model of deterioration is also discretized in time intervals of one year. This assumption is consistent with the typical rate of deterioration of bridges. Within a year, the actual point in time at which the action (maintenance or replacement) is performed is not important, and an action is said to be performed “in year n .”

The condition of the deck is represented by its reliability index β . By definition of the reliability index, the instantaneous probability of failure of the deck (given it has not failed yet) is $\Phi(-\beta)$, where $\Phi(\cdot)$ is the standard normal cumulative distribution.

Several types of maintenance actions can be performed on the bridge deck. Maintenance actions of different types have different influences on the condition of the deck, as well as different costs. The deck can also be replaced, in which case it is then considered new.

4.1.2 Definition of the State of the Markov Chain

In earlier Markovian deterioration models, the state is an integer representing the condition of the deck. In the present model, the reliability index β of the deck is part of the state space of the Markov chain; additional variables that are known to the decision maker, and can provide an advantage in selecting the optimal actions, are also included as part of the state space. This process is known as state augmentation (Bertsekas, 2001). The variables added to the state space are:

- m : an integer indicating the type of the latest action (maintenance or replacement) performed on the deck (or 0 if no action has been performed since the deck was new);
- τ : the time since the latest action (or the time since the deck was new, if no action has been performed yet);
- β^0 : the reliability index of the deck when the latest action was performed, or when the deck was new, if no action has been performed yet. This variable, causing numerical issues in the implementation, is not included in the case study (sec. 4.3). The numerical issues are discussed in section 4.3.5.

Note that, when the deck is replaced, the value of β^0 before replacement and after replacement may be different. The choice of these variables is based on the precision and ease of their measurement, as well as on their contribution to the reduction of the uncertainty in modeling the deterioration. The number of additional variables must also remain reasonably low, so that the model can be implemented.

The state $x = (\beta, \beta^0, m, \tau)$ consists of two real numbers in the general case (β and β^0) and two integers (m and τ), since there is a finite number of different types of maintenance actions and the unit of time is the year. In practice, the set of possible values for each variable can actually be restricted to small intervals while maintaining the full functionality

of the model. For example, typical values of β are integers between 1 and 15.

4.1.3 Estimation of the Transition Probabilities

Transition probabilities represent the probability for a facility that is in state x_t at time period t to be in state x_{t+1} at the following time period, where

$$x_t \triangleq (\beta_t, m_t, \tau_t, \beta_t^0), \quad x_{t+1} \triangleq (\beta_{t+1}, m_{t+1}, \tau_{t+1}, \beta_{t+1}^0) \quad (4.1)$$

This transition probability is denoted as

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1}, \beta_{t+1}^0 \mid \beta_t, m_t, \tau_t, \beta_t^0) \quad (4.2)$$

Note that x_t and x_{t+1} can be any elements of the state space and may or may not be equal. The original deterioration model of the facility, which is stochastic, is used to estimate the transition probabilities for the resulting Markovian model. In order to accommodate any original deterioration model, Monte Carlo simulation is used to estimate the transition probabilities. Namely, a large number of deterioration profiles are generated using the original deterioration model, and the counts shown on the right-hand side of equation (4.3) are determined.

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1}, \beta_{t+1}^0 \mid \beta_t, m_t, \tau_t, \beta_t^0) = \frac{N_{(\beta_t, m_t, \tau_t, \beta_t^0), (\beta_{t+1}, m_{t+1}, \tau_{t+1}, \beta_{t+1}^0)}^{\text{transitions}}}{N_{(\beta_t, m_t, \tau_t, \beta_t^0)}^{\text{total}}} \quad (4.3)$$

where

- $N_{(\beta_t, m_t, \tau_t, \beta_t^0)}^{\text{total}}$ is the number of occurrences of the following situation, among all time steps of all Monte Carlo trials: “the deck is in state $(\beta_t, m_t, \tau_t, \beta_t^0)$;
- $N_{(\beta_t, m_t, \tau_t, \beta_t^0), (\beta_{t+1}, m_{t+1}, \tau_{t+1}, \beta_{t+1}^0)}^{\text{transitions}}$ is the number of occurrences of the following situation, among all time steps of all Monte Carlo trials: “the deck is in state $(\beta_t, m_t, \tau_t, \beta_t^0)$ at a time step and in state $(\beta_{t+1}, m_{t+1}, \tau_{t+1}, \beta_{t+1}^0)$ at the following time step.”

The pseudocode for the estimation of the transition probabilities using Monte Carlo simulation is presented below.

Estimation of transition probabilities using Monte Carlo simulation

- 1 Define parameters: BETA (set of possible values for β and β^0), M (set of possible values for m), TAU (maximum value for τ), N (number of Monte Carlo trials)
- 2 Initialize Ntotal[beta,m,tau,betaIni] at 0 for all values of beta in BETA, m in M, and tau in TAU, betaIni in BETA
- 3 Initialize Ntr[beta1,m1,tau1,betaIni1,beta2,m2,tau2,betaIni2] at 0 for all values of beta1 and beta2 in BETA, m1 and m2 in M, and tau1 and tau2 in TAU, betaIni1 and betaIni2 in BETA
- 4 Repeat N times:
 - 5 Repeat for all values of m in M:
 - 6 Repeat for all values of betaIni in BETA:
 - 7 Draw an instance of a set of deterioration parameters
from their known distributions
 - 8 Repeat for tau from 0 to TAU:
 - 9 Determine the (continuous) condition betaCont[tau]
 - 10 Determine the corresponding discretized condition beta[tau],
where beta[tau] is in the set BETA
 - 11 End repeat
 - 12 Repeat for tau from 1 to TAU-1:
 - 13 Increment Ntotal[beta[tau],m,tau,betaIni] by 1
 - 14 Increment Ntr[beta[tau],m,tau,betaIni,...
beta[tau+1],m,tau+1,betaIni] by 1
 - 15 End repeat
 - 16 End repeat
 - 18 End repeat
- 19 End repeat
- 20 Ntr[beta1,m1,tau1,betaIni1,beta2,m2,tau2,betaIni2] divided by Ntotal[beta1,m1,tau1,betaIni1] is an estimate of the probability of transition from state (beta1,m1,tau1,betaIni1) to state (beta2,m2,tau2,betaIni2)

Several simplifications are possible in order to decrease the computation time necessary to estimate all transition probabilities. For example, $P(\beta_{t+1}, \beta_{t+1}^0, m_{t+1}, \tau_{t+1} | \beta_t, \beta_t^0, m_t, \tau_t) = 0$ if $m_{t+1} = m_t$ and $\tau_{t+1} \neq \tau_t + 1$, or if $m_{t+1} \neq m_t$ and $\tau_{t+1} \neq 0$, for any $\beta_t, \beta_{t+1}, \beta_t^0$ and β_{t+1}^0 .

4.2 Augmented State Markov Decision Process

The model of deterioration of a bridge deck developed in the previous section can be used in the optimization of maintenance and replacement decisions for that deck.

4.2.1 Definitions and Assumptions

System. As described earlier, the system considered is a bridge deck managed by an agency. The agency incurs costs when maintenance actions are performed or when the deck is replaced. User costs are also taken into account, in the same manner as described in the previous chapter (sec. 3.1). User costs consist of delays, closures, and detours associated with the performance of maintenance actions and deck replacement.

Planning horizon. The agency is responsible for the maintenance and replacement of the bridge for the duration of the planning horizon (T years). The planning horizon is assumed to start at year 0 and is broken down into periods of one year. The agency makes maintenance and replacement decisions every year.

Knowledge of current condition. It is assumed that there is perfect information regarding the past and present reliability index of the bridge deck. From a practical point of view, this means that inspections are carried out frequently, such as on a yearly basis, and that the inspections are error free.

4.2.2 Problem Formulation

Given the deterioration model described earlier, the optimization problem can be formulated as a finite-state discrete-time Markov decision process, with a finite horizon (Bertsekas, 2001). This formulation is also referred to as *dynamic programming*. The optimization

problem consists of minimizing the expected cost of maintenance and replacement, subject to the following reliability constraint: the reliability index of the facility must remain above a user-defined threshold.

The following notation is used:

- X : state space of the Markov chain representing the deterioration of the deck. X is the set of all possible values for $(\beta, \beta^0, m, \tau)$, as defined in the previous section.
- U : set of all possible M&R actions, i.e., all types of maintenance actions, replacement, or do-nothing.
- $c_u(x)$: cost of action u performed on bridge deck in condition β .
- α : discount factor, $\alpha = 1/(1 + r)$ where r is the discount rate.
- $V_t(x)$: minimum cost-to-go for the agency to manage the bridge deck from year t to the end of the planning horizon, starting from state x in year t .
- μ : set of optimal decisions. $\mu_t(x)$ is the optimal decision when the bridge deck is in state x in year t .

The problem formulation is as follows:

$$\begin{aligned} \forall x \in X, V_t(x) &= \min_{u \in U} \left\{ c_u(x) + \alpha \sum_{y \in X} P(y|x) V_{t+1}(y) \right\} && \text{if } t \in \{0, \dots, T-2\} \\ &= V^s && \text{if } t = T-1 \end{aligned} \quad (4.4)$$

where V^s captures the usefulness of the bridge deck past the planning horizon. For practical purposes, the influence of the actual value of V^s is limited if the discount rate r is strictly positive.

Reliability constraint. The cost $c_u(x)$ is used to implement the reliability constraint: for each state x corresponding to an unacceptable reliability index, the cost of action u is chosen to be infinite for each action $u \in U$.

4.2.3 Solution

The solution to the problem formulated above is composed of the set of $V_t(x)$ and the set of $\mu_t(x)$, for all time steps t and all states x , where $\mu_t(x)$ is the optimal action to be taken at time step t if the facility is in state x . The function μ is referred to as a *policy*, and is defined mathematically as a mapping from the set composed of the state space and the time space ($X \times \{0, \dots, T\}$) to the set U of actions.

The solution is obtained by recursion, starting from the final year of the horizon. This algorithm, referred to as the *backward recursion algorithm* (Bertsekas, 2001), is described by the pseudocode below.

Backward recursion algorithm

```

1  Define parameters (similar notations as defined in the problem formulation and
   in the pseudocode for the Monte Carlo simulation used to estimate the transition
   probabilities). In particular, define the threshold of reliability index, and the cost
   of actions  $c[u, \text{beta}]$  accordingly.
2  Initialize  $V[T-1, x]$  to  $VS$  for all possible values of  $x=(\text{beta}, m, \text{tau}, \text{betaIni})$ 
3  Repeat for  $t$  from  $T-2$  to  $0$  (step  $-1$ ):
4      Repeat for all possible values of  $x=(\text{beta}, m, \text{tau}, \text{betaIni})$ :
5           $V_{tmp} \leftarrow \infty$ 
6          Repeat for all possible actions  $u$ :
7              If  $c[u, \text{beta}] + \alpha * \sum_{y \in X} P(y|x, u) * V[t+1, y] < V_{tmp}$ 
8                   $V_{tmp} \leftarrow c[u, \text{beta}] + \alpha * \sum_{y \in X} P(y|x, u) * V[t+1, y]$ 
9                   $\mu_{tmp} \leftarrow u$ 
10             End if
11         End repeat
12          $V[t, x] \leftarrow V_{tmp}$ 
13          $\mu[t, x] \leftarrow \mu_{tmp}$ 
14     End repeat
15 End repeat

```

The minimum discounted cost to manage the bridge deck over the whole planning horizon is

$V_0(x_0)$, where x_0 is the initial state of the bridge deck. Using the notation of the pseudocode, the minimum discounted cost is $V[0, x_0]$.

4.3 Case Study

The objective of this section is to compare the policies derived using the augmented state Markovian model proposed in the present chapter and the policies derived using a simpler Markovian model. The state of the simpler Markov chain is the condition of the facility in the current year. This section also provides some insight regarding a numerical issue arising in the implementation of the optimization method described in the previous section.

4.3.1 Definition of Alternative Optimization Methods

Terminology

Several Markovian optimization methods are considered in the present section. For simplicity, the number of variables (or dimensions) of their state space is used to refer to each method.

In the optimization method described in section 4.2, the state of the Markov chain is composed of four variables:

- β : an integer representing the current discretized reliability index of the deck, and
- m : an integer indicating the type of the latest action (maintenance or replacement) performed on the deck (or 0 if no action has been performed since the deck was new), and
- τ : the time since the latest action (or the time since the deck was new, if no action has been performed yet), and

- β^0 : the reliability index of the deck when the latest action was performed, or when the deck was new if no action has been performed yet.

This method is referred to as **4d** model.

Given the numerical issues discussed in section 4.3.5, the augmented-state model implemented in the present section is a slightly modified version of the **4d** model. Namely, β^0 is not considered, and the state consists of three variables instead of four. This model is referred to as the **3d** model.

The state of the simple Markovian model used as a comparison is composed of one variable only, the current condition of the deck. This model is referred to as the **1d** model.

Definition of the 3d model

In the same manner as for the **4d** model, transition probabilities are estimated in the **3d** model, and optimal policies are determined using dynamic programming.

The estimation of the transition probabilities cannot be conducted as described in section 4.1.3 for the **4d** model, because the value of the reliability index at the time of the previous action is not part of the state in the **3d** model. The probabilities to be estimated are of the following form:

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t) \quad (4.5)$$

where β_t and β_{t+1} represent the reliability index of the facility at time t and $t + 1$, respectively; m_t and m_{t+1} represent the mode of deterioration at time t and $t + 1$, respectively; and τ_t and τ_{t+1} represent the time since the previous action (or the time since the facility was new if no action has been performed yet) at time t and $t + 1$, respectively.

If an action is performed in year t (maintenance or replacement), then the probability (4.5) can be estimated in the same manner as in the **4d** model. However, if no action is performed

in year t , i.e., if $\tau_{t+1} > 0$, then we have the following conditional probability:

$$\underbrace{P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t)}_{P_3} = \sum_{b^0 \in B^0} \underbrace{P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t, b^0)}_{P_4} P(\beta^0 = b^0) \quad (4.6)$$

where B^0 is the set of all possible values for the condition β^0 of the facility at the time of the previous action. In this expression, P_3 is a transition probability to be estimated in the 3d model. We will show that P_4 actually corresponds to a transition probability in the 4d model, i.e., we will show that, when τ_{t+1} ,

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t, b^0) = P(\beta_{t+1}, m_{t+1}, \tau_{t+1}, b^0 | \beta_t, m_t, \tau_t, b^0) \quad (4.7)$$

In general, we can write the following relation, for a given value b^0 :

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t, b^0) = \sum_{b' \in B^0} P(\beta_{t+1}, m_{t+1}, \tau_{t+1}, b' | \beta_t, m_t, \tau_t, b^0) \quad (4.8)$$

We have seen earlier that, when $\tau_{t+1} > 0$ (i.e., when no action is performed in the current year), the condition β^0 of the facility at the time of the previous action cannot change from year t to year $t + 1$. This means that, when $\tau_{t+1} > 0$, we have, for $b' \neq b^0$,

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1}, b' | \beta_t, m_t, \tau_t, b^0) = 0 \quad (4.9)$$

Equation (4.7) is justified by equations (4.8) and (4.9). Therefore, we have

$$\underbrace{P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t)}_{P_3} = \sum_{b^0 \in B^0} \underbrace{P(\beta_{t+1}, m_{t+1}, \tau_{t+1}, b^0 | \beta_t, m_t, \tau_t, b^0)}_{P_4} P(\beta^0 = b^0) \quad (4.10)$$

where P_3 is the probability to be estimated in the 3d model, and P_4 corresponds to a transition probability in the 4d model, which can be estimated. However, the probability $P(\beta^0 = b^0)$ cannot be estimated accurately, since it actually depends on the maintenance and replacement policies.

In the case of the present study, this issue is not very important, since the values of $P(\beta_{t+1}, m_{t+1}, \tau_{t+1}, b^0 | \beta_t, m_t, \tau_t, b^0)$ are found not to vary by a large amount for different values of b^0 , all other parameters being equal. Therefore, for the purpose of the estimation, the condition β^0 of the facility at the time of the previous action was assumed to be uniformly distributed over its set of possible values. The pseudocode for the estimation of the transition probabilities in the 3d model is as follows:

Estimation of transition probabilities in the 3d model

- 1 Define parameters: BETA (set of possible values for β and β^0), M (set of possible values for m), TAU (maximum value of τ), N (number of Monte Carlo trials)
- 2 Initialize Ntotal[beta,m,tau] at 0 for all values of beta in BETA, m in M, and tau in TAU
- 3 Initialize Ntr[beta1,m1,tau1,beta2,m2,tau2] at 0 for all values of beta1 and beta2 in BETA, m1 and m2 in M, and tau1 and tau2 in TAU
- 4 Repeat N times:
 - 5 Repeat for all values of m in M:
 - 6 Repeat for all values of betaIni in BETA:
 - 7 Draw an instance of a set of deterioration parameters
from their known distributions
 - 8 Repeat for tau from 0 to TAU:
 - 9 Determine the (continuous) condition betaCont[tau]
 - 10 Determine the corresponding discretized condition beta[tau],
where beta[tau] is in the set BETA
 - 11 End repeat
 - 12 Repeat for tau from 0 to TAU-1:
 - 13 Increment Ntotal[beta[tau],m,tau] by 1
 - 14 Increment Ntr[beta[tau],m,tau,beta[tau+1],m,tau+1] by 1
 - 15 End repeat
 - 16 End repeat
 - 17 End repeat
- 18 End repeat
- 19 Ntr[beta1,m1,tau1,beta2,m2,tau2] divided by Ntotal[beta1,m1,tau1] is an estimate of the probability of transition from state (beta1,m1,tau1) to state (beta2,m2,tau2)

Unlike for the transition probability estimation, the general structure of the backward recursion algorithm determining optimal maintenance and replacement policies does not change between the 4d and the 3d models. The algorithm described in section 4.2.3 can be applied,

provided the instances of `betaIni` are removed.

Definition of the 1d model

In the same manner as for the 4d model, transition probabilities are estimated in the 3d model, and optimal policies are determined, using dynamic programming.

The issue described earlier for the 3d model arises in the estimation of the transition probabilities. Namely, it can be shown that, for a year t in which no action is performed,

$$\underbrace{P(\beta_{t+1}|\beta_t)}_{P_1} = \sum_{b^0} \sum_{\tau} \sum_m \underbrace{P(\beta_{t+1}, m, \tau + 1, b^0|\beta_t, m_t, \tau, b^0)}_{P_4} P(m, \tau, b^0) \quad (4.11)$$

where the notation is the same as in the previous section, P_1 is the transition probability to be estimated in the 1d model, and P_4 corresponds to a transition probability in the 4d model. The probability $P(m, \tau, b^0)$ cannot be estimated accurately and actually depends on the maintenance and replacement policies. The approach followed to estimate $P(m, \tau, b^0)$ consisted of simulating different realistic maintenance policies, such as performing maintenance every k years, where k is taken in a realistic set of values, and aggregating the results, assuming all the considered maintenance policies were equally likely.

In the estimation for the 3d and the 4d models, one trial of the Monte Carlo simulation consisted in drawing a set of deterioration parameters, and simulating the deterioration over time of that particular facility, until the condition reached the lowest admissible condition. If the estimation of the transition probabilities in the 1d model were a repetition of N such trials, the resulting estimates would be biased, since the age (denoted by τ in the other models) is not a parameter in the 1d model. Namely, the facilities with a higher rate of deterioration reach the lower admissible condition faster and would be underrepresented in the estimation. This issue is referred to as *selection bias* (Greene, 1993). One solution

consists in dividing the estimates from each trial by the sampling rate, which is the number of years of a trial in the present estimation problem. The pseudocode for the estimation of the transition probabilities in the 1d model is as follows:

Estimation of transition probabilities in the 1d model	
1	Define parameters: BETA (set of possible values for β and β^0), N (number of Monte Carlo trials), b_lowest (lowest admissible reliability index)
2	Initialize Ntotal[beta] at 0 for all values of beta in BETA
3	Initialize Ntr[beta_1,beta_2] at 0 for all values of beta_1 and beta_2 in BETA
4	Repeat N times:
6	Draw an instance of a set of deterioration parameters from their known distributions
7	t ← 0
8	b_cont[t] ← beta0
9	While { b_cont[t] > b_lowest } Do:
10	t ← t + 1
11	Determine the (continuous) condition b_cont[t]
12	Determine the corresponding discretized condition beta[t] , where beta[tau] is in the set BETA
13	End while
14	T ← t
15	Repeat for t from 0 to T :
16	Ntotal[beta[t]] ← Ntotal[beta[t]] + 1/ T
17	Ntr[beta[t],beta[t+1]] ← Ntr[beta[t],beta[t+1]] + 1/ T
18	End repeat
19	End repeat
20	Ntr[beta1,beta2] divided by Ntotal[beta1] is an estimate of the probability of transition from state (beta1,m1,tau1) to state (beta2,m2,tau2)

The general structure of the backward recursion algorithm determining optimal maintenance and replacement policies does not change between the 4d and the 1d models. The algorithm described in section 4.2.3 can be applied, provided the instances of **betaIni**, **m** and **tau** are removed.

4.3.2 Data

The implementation of the method described in the present chapter consists in determining transition probabilities, and then using these transition probabilities to determine optimal

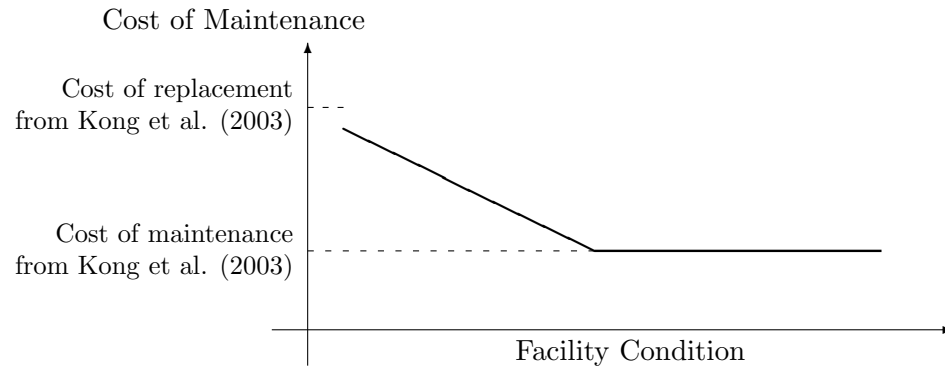


Figure 4.1: Cost of a maintenance action as a function of the condition of the facility on which it is performed.

maintenance and replacement policies. A deterioration model is required as input for the determination of the transition probabilities, and cost data is needed for the derivation of optimal policies. The deterioration parameters and replacement costs used for this case study are the same as those presented in section 3.4, which are taken from Frangopol et al. (2001) and Kong and Frangopol (2003). The values of the cost for maintenance are taken from Kong and Frangopol (2003) but are modified to account for the increase in the cost of a maintenance action of a given type as the condition of the facility on which it is performed deteriorates. The cost profile, shown in figure 4.1, follows the framework presented in Madanat (1988). The same parameters are used for all models (1d, 3d, and 4d).

4.3.3 Comparison of the 3d and 1d Models

The comparison of two optimization methods consists in applying policies determined using each model to two bridge decks having the same deterioration parameters (fig. 4.2). In the present section, the set of random variables mentioned in the box in the upper part of figure 4.2 (box A) is the one presented in section 4.3.2. It is important to note the difference between this set of random variables and the set of deterioration parameters in box B. The

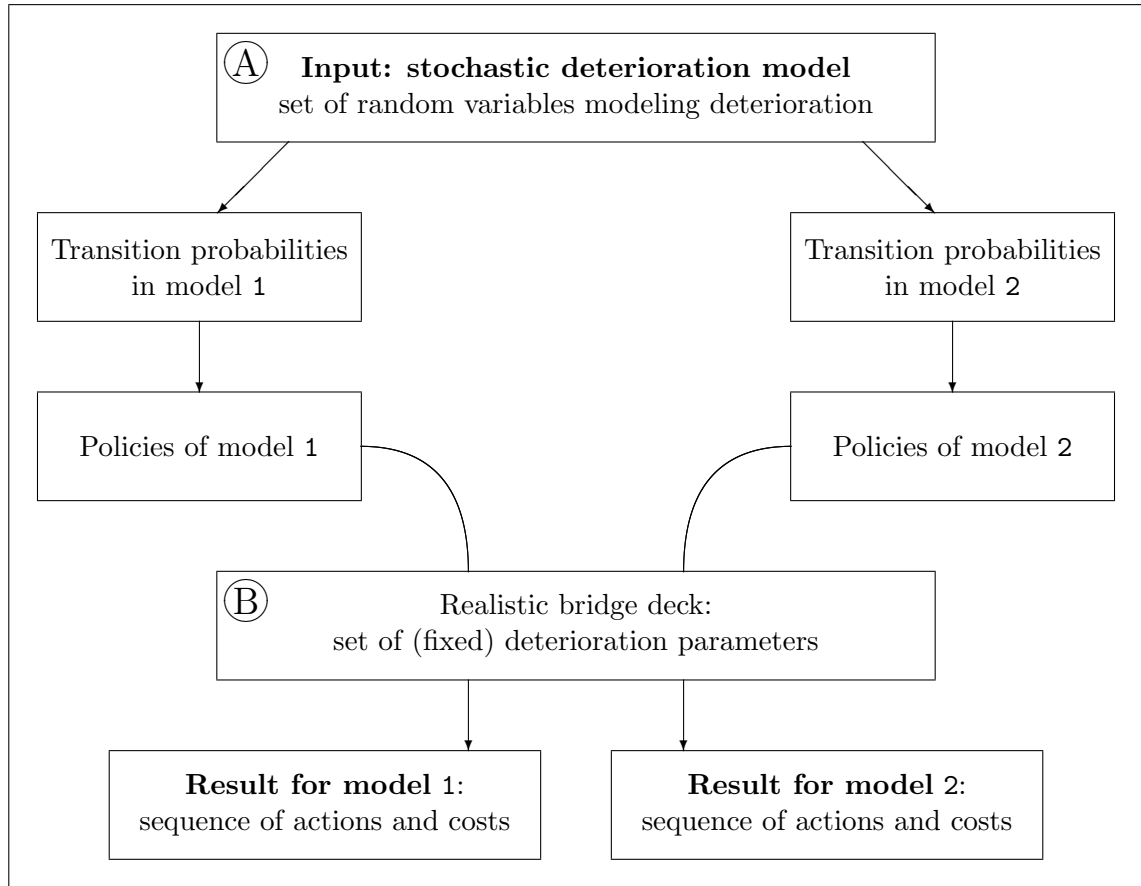


Figure 4.2: Comparison of two optimization methods (called model 1 and model 2)

set of random variables in box A remains the same throughout the present case study (for a given value of the user-defined threshold of reliability), whereas different possibilities are tried for the set of deterioration parameters in box B.

Moreover, in the simulation, the deterioration of the bridge deck is determined by a model such as the one presented in Frangopol et al. (2001), and not using the transition probabilities derived in section 4.1.3. Therefore, the comparison is made on the basis of a model assumed to be an accurate representation of reality, and more importantly, independent from the method used to determine the policies. Specifically, the comparison would be much weaker if it were based on the values of the objective functions (predicted optimal costs and not simulated costs).

Comparison of the 3d and 1d Models for One Facility

The applications of the policies determined using the 3d and 1d models are first compared for one facility. Figure 4.3 shows the evolution of the condition of a bridge deck if the policies of the 1d model are applied (top graph), and if the policies of the 3d model are applied (bottom graph). The figure also shows the sequence of actions performed in each case. The

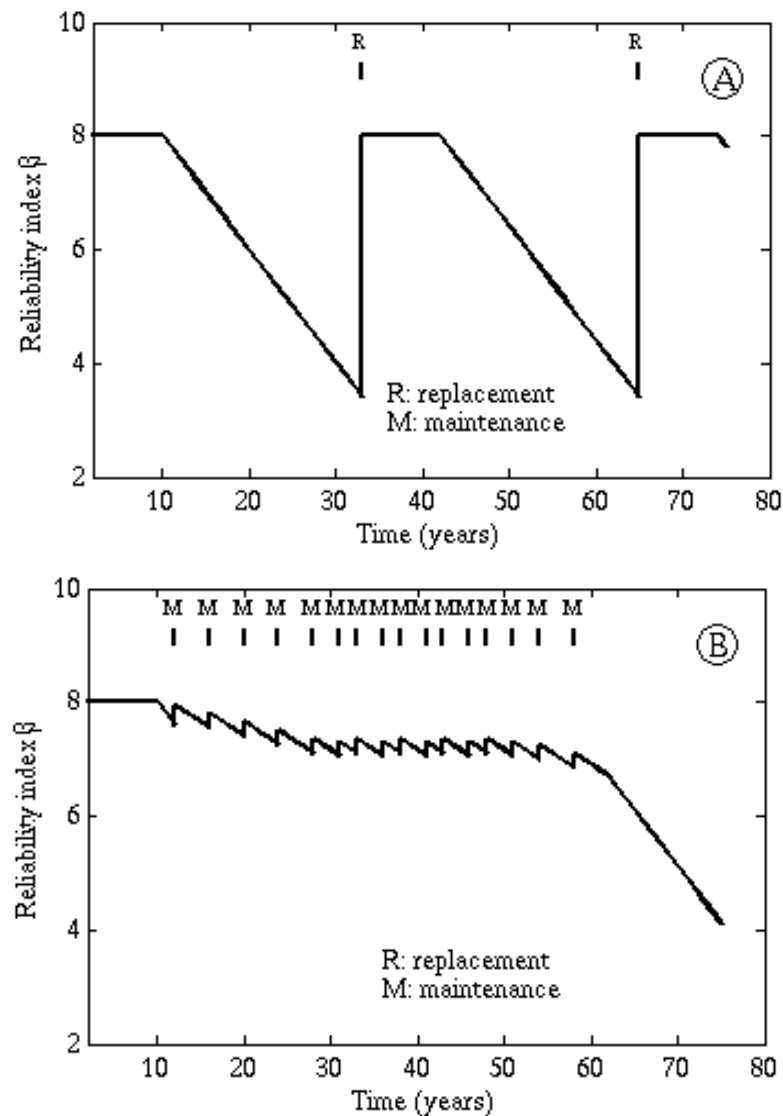


Figure 4.3: Comparison of the application of the policies of the 1d model (A), and of the 3d model (B).

main objective of this comparison is to show the difference in the sequence of actions when

applying the two sets of policies. Cost comparisons are presented in the probabilistic sense in the next section.

The sequence of M&R actions obtained by application of the policies of the 1d model is very different from the sequence obtained by application of the policies of the 3d model. Namely, using policies of the 1d model, the deck is replaced twice, at years 32 and 64, and maintenance is never performed. Using the policies of the 3d model, maintenance is performed regularly, every three to four years. Maintenance is not performed at the end of the planning horizon, since the final condition of the deck is indifferent, provided it is above the user-defined threshold of reliability index. The deck is not replaced over the planning horizon.

Using the policies of the 3d model, the performance of maintenance actions at almost regular intervals is a result of the optimization and was not provided as an input to the model. A possible intuitive explanation for this fact is as follows. By construction, the state space of the 3d model captures more detail than the state space of the 1d model. In particular, the combination of values for the condition of the facility and for the time since the previous maintenance action is possible in the augmented-state model and not in the 1d model. This combination allows for more selective recommendations using the 3d model. For example, if the current condition is 5, the recommendation using the 3d model may be to perform maintenance if the previous maintenance action was performed five years before or earlier, and to do nothing if the previous maintenance action was performed less than five years before. In the same situation, if the current condition is 5, the simple model provides only one recommendation, regardless of the time since the previous action. Thus, the performance of maintenance at regular intervals cannot be recommended by the 1d model. This behavior is not limited to the example presented in this section. The application of the

policies with many other deterioration parameters produced the same pattern of sequences of actions. More precisely, a large number of deterioration parameters were drawn from the distributions presented in Frangopol et al. (2001), thus creating a large number of *test* bridge decks, and the pattern of the sequences of actions described in this section occurred in the majority of the cases.

Comparison of the 3d and 1d Models in the Probabilistic Sense

In the previous paragraph, the comparison of the two models was mostly qualitative, in terms of the general patterns of the sequence of actions. The purpose of the present comparison is to analyze the difference in costs resulting from the application of the policies of the 3d and 1d models, based on trials on a large number of test bridge decks representing a realistic range of deterioration parameters.

The input deterioration model, as described in figure 4.2, remains the same, which implies that the policies remain the same as well, for each model (and for a value of the user-defined threshold of reliability index). These two sets of policies are applied to a variety of different bridge decks (box B). To create a realistic set of different facilities, the deterioration parameters for each bridge are drawn from the distributions provided in Frangopol et al. (2001). The distributions used as input to determine the optimal policies and the distributions used to draw the population of test facilities are the same. This is not a coincidence, but rather, a consequence of the following assumption: the stochastic deterioration model is assumed to be known and is assumed to be the same for each test bridge deck.

Comparison of average costs. The average total cost over the planning horizon when applying the policies of the 3d model is determined for a large number of different test bridge decks. This average is also determined in the case of the policies of the 1d model.

Moreover, these simulations are done for several different values of the user-defined threshold of reliability index. The pseudocode for the determination of the policies and the simulations is presented below.

Determination of optimal policies and simulation of the application of the policies to a population of bridges, for one model

- 1 Define distributions \mathcal{D} of random variables modeling the deterioration (box A in fig. 4.2)
- 2 Determine the transition probabilities (pseudocode provided earlier)
- 3 Define parameters: N (number of trials), B (set of user-defined thresholds of reliability index)
- 4 Repeat for all values of \mathbf{b} in B :
- 5 Determine the optimal policies $\mathcal{P}(\mathbf{b})$, with \mathbf{b} as the user-defined threshold of reliability index (pseudocode provided earlier)
- 6 End Repeat
- 7 Initialize `totalcost[b,i]` at 0 for all values of \mathbf{b} in B and for i from 1 to N
- 8 Repeat for all values of \mathbf{b} in B :
- 9 Repeat for i from 1 to N :
- 10 Draw an instance of a set of deterioration parameters (box B in fig. 4.2) from the distributions \mathcal{D}
- 11 Simulate the evolution of the condition of the facility, applying actions according to policies $\mathcal{P}(\mathbf{b})$ (recording yearly costs)
- 12 Record simulated total cost in `totalcost[b,i]`
- 13 End repeat
- 14 End repeat

This simulation is done for the 3d and 1d models. As shown by the results in figure 4.4, the average cost when using the policies of the 3d model is approximately 30 percent lower than the average cost when the policies of the 1d model, based on 100,000 trials.

Probability of lower cost. Using the simulation described earlier, it is possible to obtain the empirical distribution of the simulated costs. This, in turn, allows for the determination of the probability of a lower cost when applying the policies of the 3d model than when applying the policies of the 1d model. The empirical distribution of the simulated costs is shown in figure 4.5. It can be noted that, for a given model, the mean simulated cost increases as the threshold of reliability index increases (i.e., as the facility becomes more

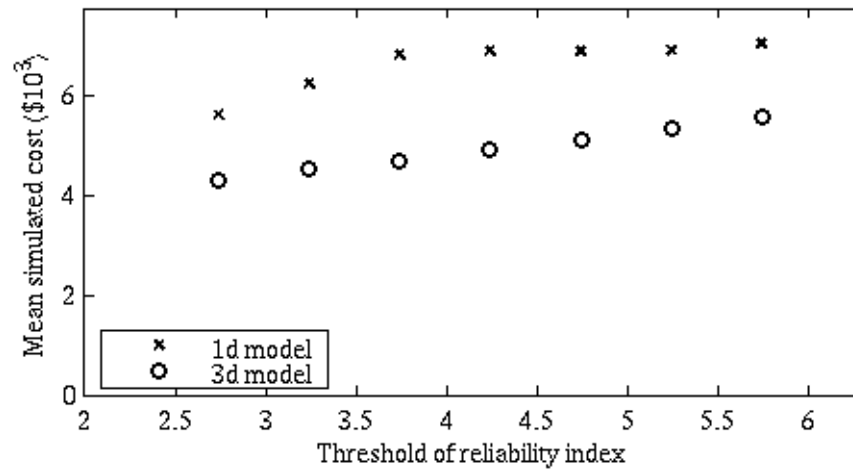


Figure 4.4: Comparison of the mean simulated total costs, when applying the policies of 3d and 1d models, for different values of threshold of reliability index.

reliable), which is intuitive. Using these joint distributions, it can be determined that the

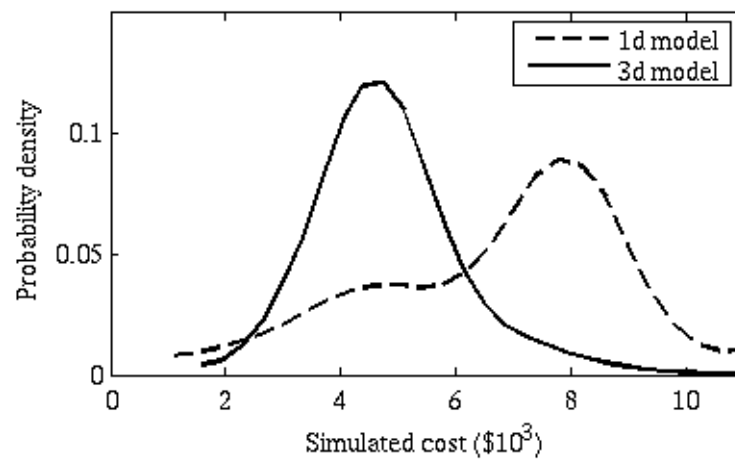


Figure 4.5: Comparison of the distribution of simulated costs, when applying the policies of the 3d model and when applying the policies of the 1d model.

probability of a lower cost when applying the policies of the 3d model than when applying the policies of the 1d model is 0.75. Moreover, a Kolmogorov-Smirnov test run on the sample costs allows to reject the following hypothesis: “the sample costs when applying the policies of the 3d model and the sample costs when applying the policies of the 1d model are drawn from the same distribution.” This formally proves that the difference in the

distribution of the costs intuitively apparent in figure 4.5 is indeed statistically significant.

4.3.4 Computational Complexity

The computation times needed for the implementation of the 4d model are very short on a computer with a 1.7GHz processor, 1GB RAM, running Unix. The computation time for the estimation of the transition probabilities is approximately one minute, with 100,000 Monte Carlo trials. The computation time for the determination of optimal policies is of the order of a few seconds, with a state space of size 11,760 and a time horizon of 75 years. The computation times needed for the implementation of the 3d and 1d models are even shorter. In the 4d model, the state space consists of 14 different values for the current condition, two types of actions (maintenance and replacement), 30 different values for the time since the previous action, and 14 different values for the condition after the previous action. The limitation for the state space is actually not the computation time, but the memory needed to implement the backward recursion algorithm.

4.3.5 Numerical Issues

The implementation of the 3d and 4d models exhibits a counterintuitive trend: in most cases, the mean simulated cost is lower when applying the policies of the 3d model than when applying the policies of the 4d model, as shown in figure 4.6. This is not intuitive, since the 4d model is able to capture more detail than the 3d model, and therefore should provide lower or similar costs. After a thorough investigation, it can be concluded that this issue does not stem from a programming mistake, but from policies “correctly” recommended by the 4d model, which actually lead to higher costs when applied to test facilities.

Another cause could be the size of the state space in the 4d model, which could prevent

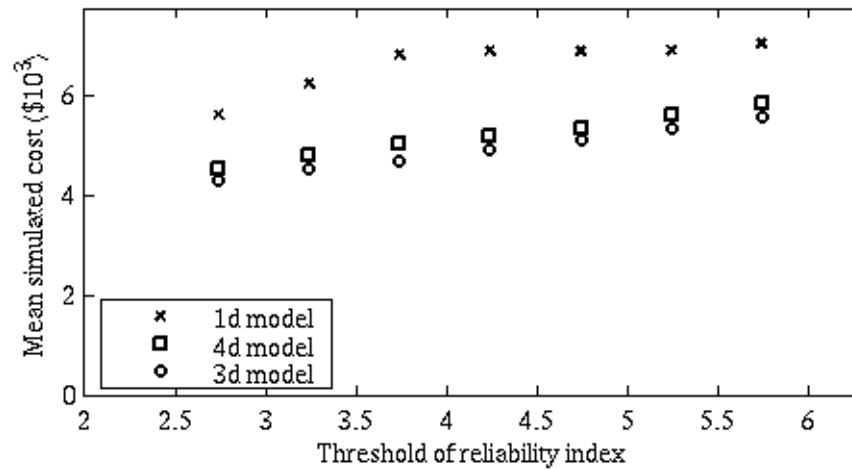


Figure 4.6: Comparison of the mean simulated total costs, when applying the policies of the 4d, 3d, and 1d models.

correct estimation of the transition probabilities if the number of trials of the Monte Carlo is not sufficiently large. However, the estimation was conducted several times with 1,000,000 Monte Carlo trials, and the estimates for the transition probabilities were found to be very close to each other. The resulting policies determined by the 4d model were always identical. Finally, examples with a smaller state space were created, in order to be able to explore the mechanism leading to this issue by hand. However, the issue did not arise with small state spaces, which seems to indicate that this is a numerical problem.

For these reasons, the augmented-state model used in the present case study is the 3d model instead of the 4d model. The 3d model also takes into account aspects of the history of deterioration and maintenance, which is the main feature studied in this chapter.

4.4 Summary

In this chapter, the derivation of a Markovian model of bridge deck deterioration is presented. The model takes into account aspects of the history of deterioration and mainte-

nance, through the use of state augmentation. This model is used in the formulation of the optimization of maintenance and replacement decisions for one facility as a finite-state, discrete-time Markov decision process, with a finite horizon.

The optimal policies determined with the augmented-state model are compared to the policies determined with a simpler Markovian model, and numerical results indicate that the benefits of including additional information in the model are significant. Moreover, this comparison is valid, since it is made on the basis of a model assumed to represent reality, independently from the method used to determine the optimal policies.

An agency, such as a state Department of Transportation, is responsible for the management of a system of a large number of bridges. The objective of the next chapter is to extend the facility-level optimization method described in the present chapter to the system level. This extension is likely to benefit from the short computation times needed to solve the facility-level problem.

Chapter 5

System-Level Optimization

This chapter uses the results of the optimization at the facility level, described in the previous chapter, to provide optimal maintenance and replacement decisions for a system of heterogeneous facilities. The computational results show that this method is applicable to systems of realistic sizes.

5.1 Problem Definition

The objective of the previous chapter was to derive optimal maintenance and replacement policies for one facility. The input to the optimization method was a threshold for the reliability index (or equivalently, for the probability of failure). The results obtained were the minimum expected cost for maintaining the facility over the planning horizon, as well as a set of optimal policies, i.e., maintenance and replacement recommendations for each year, depending on the realization of the state of the facility in the considered year. At the system level, the result of the optimization is a set of optimal policies for each facility, i.e., maintenance and replacement recommendations for each facility and each year, depending on the realization of the state of each facility in the considered year.

5.1.1 Definitions and Assumptions

The system considered in the present chapter is a system of facilities managed by a single agency, such as a state Department of Transportation. Although the condition of each bridge in the system changes over time, the system remains constant over the planning horizon: bridges are neither added to the system nor decommissioned. The system can be composed of heterogeneous bridges.

The only bridge component considered in this study is the bridge deck. The assumptions about the deterioration of each bridge deck are the same as those made in the facility-level optimization: the deterioration is modeled using a Markov chain whose state includes aspects of the history of deterioration and maintenance, and whose transition probabilities are estimated using a mechanistic model of deterioration.

As in previous chapters, the planning horizon is broken down into periods of one year. Maintenance and replacement decisions are made every year for every facility.

The condition of each facility is assumed to be known perfectly, which in practice means that inspections are carried out frequently and are error free.

5.1.2 Formulation

In previous chapters, the reliability threshold was expressed in terms of reliability index β . In the present chapter, it is expressed in terms of probability of failure and is denoted by p . These two expressions are equivalent, since there is a one-to-one relationship between the probability of failure p and the reliability index β :

$$p = \Phi(-\beta)$$

where Φ is the cumulative standard normal distribution.

At the facility level, the optimization consists of minimizing the cost of maintenance and replacement, subject to a reliability constraint. A direct extension of this formulation at the system level would consist of minimizing the cost of maintenance and replacement for all facilities, subject to a reliability constraint for each facility. This would require the user of the optimization method to provide a value for the threshold of the probability of failure for each facility. An intuitive choice could be to take the same value of the threshold for all facilities. With a slightly different formulation, it can be proven that this choice is actually optimal in the context of reliability-based optimization.

Following an idea similar to that of duality, let us define the set of decision variables as the set (p_1, \dots, p_n) of thresholds of probability of failure for each facility, where n is the number of facilities. Let us formulate the optimization as a minimization of a quantity $J_{(p_1, \dots, p_n)}$, expressing the performance in terms of probability of failure or risk, subject to a cost constraint. In the context of reliability, it is relevant to choose $J_{(p_1, \dots, p_n)}$ to be the highest probability of failure threshold among all facilities, denoted by

$$J_{(p_1, \dots, p_n)} = \max_{i \in \{1, \dots, n\}} p_i \quad (5.1)$$

Another expression of the performance in terms of risk could be the average probability of failure over all facilities. However, this expression is not suitable in the context of reliability-based optimization, because the following unsafe situation would be possible: the majority of the facilities with low probabilities of failure and a few facilities with probabilities of failure close to 1.

The formulation of the system-level optimization is as follows:

$$\begin{aligned} & \min_{(p_1, \dots, p_n) \in [0, 1]^n} \max \{p_1, \dots, p_n\} \\ & \text{subject to} \\ & \sum_{i=1}^n f_i(p_i) \leq B \end{aligned} \tag{5.2}$$

where B is the maximum total cost (budget), and $f_i(p_i)$ is the optimal cost of maintenance and replacement for facility $i \in \{1, \dots, n\}$, determined using the facility-level optimization method described in the previous chapter.

It is assumed that $f_i(p_i)$ is defined for a continuous interval of p_i 's, i.e., $[0, 1]$. This assumption cannot be achieved in practice, since it would require solving an infinite number of facility-level problems; however, section 5.3 shows that the discrete case implementation of this optimization problem can confidently be considered a good approximation of the idealized continuous case. Moreover, the functions f_i are assumed to be continuous. Even if these functions are likely to be only piecewise continuous in practice, section 5.3 shows that the sum of these functions is smooth enough so that approximating it by a continuous function is unlikely to lead to large errors.

For each facility, the function f_i is nonincreasing. Intuitively, if larger probabilities of failure are allowed for a facility (i.e., larger values of p_i), less actions are required on the facility, thus leading to a lower cost $f_i(p_i)$. More formally, if $p_i < p'_i$, then the set of feasible actions with the threshold of probability of failure p_i is included in the set of feasible actions with the threshold of probability of failure p'_i . Therefore, the formulation indicates that decreasing the threshold p_i of the probability of failure for each facility increases the total cost, and that the objective of decreasing the p_i 's is constrained by the budget.

5.2 Continuous-Case Solution

The solution is first determined in the case where the facility-level optimal cost, $f_i(p_i)$, is defined for a continuous interval of p_i 's, for each facility $i \in \{1, \dots, n\}$.

5.2.1 Optimal Solution

If the following two conditions are met:

$$\begin{aligned} (i) \quad & \text{for all } (i, j) \in \{1, \dots, n\}^2, \quad p_i = p_j \\ (ii) \quad & \sum_{i=1}^n f_i(p_i) = B \end{aligned} \tag{5.3}$$

then the set (p_1, \dots, p_n) of thresholds of probabilities of failure of n heterogeneous facilities is an optimal solution of (5.2).

The optimal solution p^* can be expressed as

$$p^* = \min_{p \in [0,1]} \left\{ p : \sum_{i=1}^n f_i(p) = B \right\} \tag{5.4}$$

If the functions f_i are strictly decreasing, the conditions (5.3) are also necessary. In that case, the function $F : p \mapsto F(p) = \sum_{i=1}^n f_i(p)$ can be inverted, and the optimal solution p^* has a simple expression:

$$p^* = F^{-1}(B) \tag{5.5}$$

where F^{-1} is the inverse function of F .

A corner solution also exists when the budget is very low, but it is very unlikely to happen in practice. If the budget B is so low that it does not allow the threshold of probability of failure of all n facilities to be strictly less than 1, then the optimal solution is to use the entire budget. In that case, some facilities will have a threshold of probability of failure strictly less than 1, while at least one facility will have this threshold at 1.

Given the nature of the problem, it is impossible to maintain the threshold of probabilities of failure of all facilities at 0 with a finite budget B . Therefore, there is no corner solution corresponding to large values of the budget B .

5.2.2 Optimal Policies at the Facility Level

Once the optimal value p^* for the threshold of probability of failure has been determined at the system level, optimal maintenance and replacement policies can be devised for each facility by solving the optimization problem described in the previous chapter for each facility. The threshold of probability of failure, which is an input to that optimization method, is chosen to be p^* (or equivalently, the threshold of reliability index is chosen to be $-\Phi^{-1}(p^*)$).

5.2.3 Intuition for the Proof of System-Level Results

The second condition in (5.3), which indicates that the entire budget is used at optimality, is intuitive. In order to obtain an intuitive explanation for the first condition, which indicates that all p_i 's are equal at optimality, let us consider a system of two facilities, and the following situation: the threshold of probability of failure of facility 1, p_1 , is strictly lower than that of facility 2, p_2 , as shown in figure 5.1, and the combination of decision variables (p_1, p_2) is feasible (i.e., $f_1(p_1) + f_2(p_2) \leq B$).

It is easy to show that this combination cannot be optimal. Since $f_1(p_1)$ decreases as p_1 is increased, and $f_2(p_2)$ increases as p_2 is decreased, it is possible to decrease p_2 by a small amount δ_2 and to increase p_1 by δ_1 so as to keep $f_1(p_1) + f_2(p_2) = f_1(p_1 + \delta_1) + f_2(p_2 - \delta_2)$. Thus, the new combination $(p_1 + \delta_1, p_2 - \delta_2)$ is feasible, and the objective function is improved. This gives an intuitive explanation as to why p_1 and p_2 must be equal at

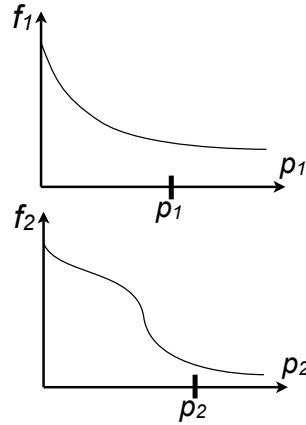


Figure 5.1: System of two facilities with thresholds of probability of failure such that $p_1 < p_2$.

optimality.

5.2.4 Proof of the System-Level Results

This section provides the proof that conditions (5.3) are sufficient conditions of optimality for problem (5.2), as well as the proof of the existence of the solution to that problem. The expression of the solution is given by Equation (5.4) in the general case, and by Equation (5.5) in the case where the function f_i is strictly decreasing for each facility. Clarifications about the notation used in these proofs can be found in appendix B. That appendix also includes the proof that conditions (5.3) are necessary if the function f_i is strictly decreasing for each facility.

Proof of Sufficiency Conditions

We will prove that, if $(p_1, \dots, p_n) \in (0, 1]^n$ verifies the following conditions:

- (i) for all $(i, j) \in \{1, \dots, n\}^2$, $p_i = p_j$, and
- (ii) $\sum_{i=1}^n f_i(p_i) = B$,

then (p_1, \dots, p_n) is an optimal solution of (5.2).

We will consider $(p_1, \dots, p_n) \in (0, 1]^n$ such that

$$\forall (i, j) \in \{1, \dots, n\}^2, p_i = p_j, \text{ and} \quad (5.6)$$

$$\sum_{i=1}^n f_i(p_i) = B \quad (5.7)$$

From assumption (5.7), (p_1, \dots, p_n) is a feasible solution of optimization problem (5.2).

Let us show that, if $(q_1, \dots, q_n) \in (0, 1]^n$ is a feasible solution of (5.2), then

$$\max\{q_1, \dots, q_n\} \geq \max\{p_1, \dots, p_n\} \quad (5.8)$$

Consider a feasible solution $(q_1, \dots, q_n) \in (0, 1]^n$ of optimization problem (5.2). Two cases are possible:

- (i) For all $(i, j) \in \{1, \dots, n\}^2$, $q_i = q_j$. We define c such that, for all $i \in \{1, \dots, n\}$, $q_i = c$.
- (ii) There exists $(i, j) \in \{1, \dots, n\}^2$ such that $q_i \neq q_j$. In this case, $\min\{q_1, \dots, q_n\} < \max\{q_1, \dots, q_n\}$. We define $(i_{\min}, i_{\max}) \in \{1, \dots, n\}^2$ such that $q_{i_{\min}} = \min\{q_1, \dots, q_n\}$ and $q_{i_{\max}} = \max\{q_1, \dots, q_n\}$. Note that i_{\min} and i_{\max} exist, but are not necessarily unique. If they are not unique, the choice of a particular i_{\min} or i_{\max} does not matter.

Let g be the following function:

$$g : (0, 1] \longrightarrow \mathbb{R}$$

$$x \longmapsto \sum_{i=1}^n f_i(q_i) - \sum_{i=1}^n f_i(x)$$

For all $i \in \{1, \dots, n\}$, $q_i \leq q_{i_{\max}}$; thus, $f_i(q_i) \geq f_i(q_{i_{\max}})$, because f_i is a nonincreasing function on $(0, 1]$. Therefore, $g(q_{i_{\max}}) = \sum_{i=1}^n (f_i(q_i) - f_i(q_{i_{\max}})) \geq 0$. Similarly, $g(q_{i_{\min}}) \leq 0$. We have:

- (a) g is continuous on $(0, 1]$,
- (b) $g(q_{i_{\min}}) \leq 0$, and
- (c) $g(q_{i_{\max}}) \geq 0$.

By the intermediate value theorem, there exists $c \in [q_{i_{\min}}, q_{i_{\max}}]$ such that $g(c) = 0$.

In both cases, we have the following

$$c \leq \max\{q_1, \dots, q_n\}, \text{ and} \quad (5.9)$$

$$\sum_{i=1}^n f_i(c) = \sum_{i=1}^n f_i(q_i) \quad (5.10)$$

Moreover, (q_1, \dots, q_n) is a feasible solution of optimization problem (5.2). Therefore,

$\sum_{i=1}^n f_i(q_i) \leq B$, and

$$\sum_{i=1}^n f_i(c) \leq B \quad (5.11)$$

We define $p \in (0, 1]$ such that, for all $i \in \{1, \dots, n\}$, $p_i = p$. From assumption (5.7),

$$\sum_{i=1}^n f_i(p) = B \quad (5.12)$$

Let h be the following function:

$$\begin{aligned} h : (0, 1] &\longrightarrow \mathbb{R} \\ x &\longmapsto \sum_{i=1}^n f_i(x) \end{aligned}$$

Since f_i is nonincreasing on $(0, 1]$ for all $i \in \{1, \dots, n\}$, h is a nonincreasing function on $(0, 1]$.

From (5.11) and (5.12), we have $B = h(p) \geq h(c)$, which means that $p \leq c$. Using (5.9), we

have

$$\max\{p_1, \dots, p_n\} = p \leq c \leq \max\{q_1, \dots, q_n\} \quad (5.13)$$

which proves the sufficiency conditions. \square

Proof of Existence of an Optimal Solution to Problem (5.2)

Let F be the following function:

$$\begin{aligned} F : (0, 1] &\longrightarrow \mathbb{R} \\ p &\longmapsto \sum_{i=1}^n f_i(p) \end{aligned}$$

Let G be the following function:

$$G : \mathbb{R}^+ \longrightarrow (0, 1]$$

$$b \longmapsto \min_{p \in (0, 1]} \{p : F(p) = b\}$$

We will show that $G(B)$ is an optimal solution of (5.2).

Let us show first that $G(b)$ is defined for all $b \in \mathbb{R}^+$. This is done in two steps:

- we first show that $I(b) = \{p \text{ s.t. } F(p) = b\}$ is not an empty set, which proves that $\inf(I(b))$ exists.
- we then show that $\inf(I(b)) \in I(b)$, i.e., that $\min(I(b))$ exists.

Existence of an infimum of $I(b)$. If a probability of failure of 1 is allowed for each facility, then the cost of maintenance and repair is 0. Therefore, $F(1) = 0$. Since F is continuous, and $F(1) = 0$ and $F(p) \rightarrow +\infty$ as $p \rightarrow 0$, then for any $b \in \mathbb{R}^+$, there exists $p \in (0, 1]$ such that $F(p) = b$. Therefore, $I(b) = \{p \text{ s.t. } F(p) = b\}$ is not empty, and has an infimum.

Existence of a minimum of $I(b)$. For a given $b \in \mathbb{R}^+$, the set $I(b)$ is bounded below by $m(b) = \inf(I(b)) > 0$ because $F(p) \rightarrow +\infty$ as $p \rightarrow 0$. The set $I(b)$ is bounded above by $M(b) = \sup(I(b))$. We will show that $\inf(I(b)) \in I(b)$, which means that the infimum of the set $I(b)$ is actually a minimum. There are two cases:

- (i) Case $M(b) = m(b)$. Since $I(b)$ is not empty, the condition $\inf(I(b)) = \sup(I(b))$ indicates that the set $I(b)$ is reduced to a single point, $\inf(I(b)) = m(b) = M(b) = \sup(I(b))$.
- (ii) Case $M(b) > m(b)$. Consider the sequence $(u_k)_{k \in \mathbb{N}}$ defined as follows. For a given $b \in \mathbb{R}^+$, and for all $k \in \mathbb{N}$,

$$u_k = m(b) + \frac{M(b) - m(b)}{k + 1}$$

This sequence converges to $m(b)$. Moreover, $M(b) - m(b) > 0$, which means that for all

k , $u_k \leq m(b)$. For all k , we also have

$$\begin{aligned} M(b) - u_k &= M(b) - m(b) - \frac{M(b) - m(b)}{k+1} \\ &= (M(b) - m(b)) \frac{k}{k+1} > 0 \end{aligned}$$

Therefore, $m(b) < u_k < M(b)$. This means that $u_k \in I(b)$, and $F(u_k) = b$. Since F is continuous, we have

$$F(m(b)) = F\left(\lim_{k \rightarrow \infty} u_k\right) = \lim_{k \rightarrow \infty} b = b \quad (5.14)$$

By definition of $I(b) = \{p \text{ s.t. } F(p) = b\}$, $m(b)$ belongs to the set $I(b)$.

In both cases, $m(b) \in I(b)$, and $m(b) = \min I(b) = G(b)$, which means that $G(b)$ is defined. $G(B)$ is an optimal solution. Since $F(G(B)) = B$, then $p^* = G(B)$ is an optimal solution to problem (5.2), according to the sufficiency conditions (5.3).

If the functions f_i are strictly decreasing, then the function F is strictly decreasing and continuous, therefore invertible. In that case, $I(b)$ consists of a single point, and we have $G(b) = F^{-1}(b)$. □

5.3 Discrete-Case Solution

5.3.1 Empirical Evidence of the Validity of the Discrete-Case Results

In practice, the values $f_i(p_i)$ cannot be determined for a continuous interval of p_i , because this would require solving the facility-level optimization problem described in the previous chapter an infinite number of times. The values $f_i(p_i)$ can therefore be determined only for a finite set of values of p_i , for each facility. The present section aims to show empirically that the results of the discrete case implementation represent a good approximation of the continuous-case solution.

The numerical application considers a system of 742 heterogeneous bridges. The deterioration of each bridge deck is modeled according to Frangopol et al. (2001), and maintenance and replacement cost information is adapted from Kong and Frangopol (2003). A system of heterogeneous bridges is created by changing the parameters provided in these articles within reasonable ranges. The facility-level optimization problem is solved for each facility, for various values of the threshold of probability of failure. The condition of each facility, measured by its reliability index β , is discretized, and various discretization step sizes are tried.

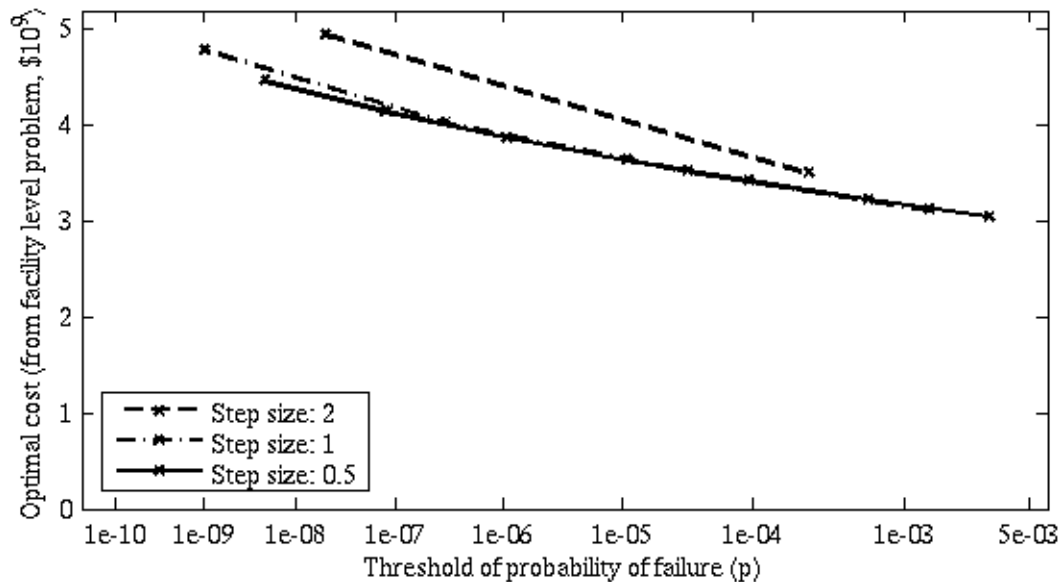


Figure 5.2: Aggregation of results of facility-level optimization for 742 bridges.

The results for the system of 742 bridges is shown in figure 5.2. For each of the three discretization step sizes, the optimization problem is solved for different values of the threshold p of probability of failure (two different values with step size 2, four different values with step size 1, and seven different values with step size 0.5). For a given discretization step size, it can be noted that the sum of the facility-level optimal costs decreases as the threshold p

of probability of failure increases, which is intuitive, and was one of the assumptions made in the continuous case. Moreover, the variation of the optimal cost with respect to p is relatively smooth; therefore, it is relevant to interpolate between the results for different values of p , and the continuity assumption made in the continuous case is likely to be verified.

The graph also shows that, for a given value of p , the sum of the facility-level optimal costs decreases as the discretization step size decreases. This is intuitive, because the model becomes finer, as the step size decreases, thus allowing for improvements in the optimization. It can also be noted that, for any given value of p , the difference in cost between step size 1 and step size 0.5 is much smaller than the difference between step size 2 and step size 1, which suggests that the results “converge” as the step size decreases to 0.

These arguments, derived on a system of significant size, provide empirical evidence that the results found in the discrete-case implementation can confidently be considered valid approximations of the results in the continuous case.

5.3.2 Implementation and Derivation of Facility-Level Policies

The facility-level optimization problem is first solved for each facility, for different values of the threshold p of probability of failure (or equivalently, for different values of the reliability index). This provides the optimal cost $f_i(p)$ for a discrete set of values of p , for each facility. After summing these values over all facilities and interpolating, the function F , defined as follows, is known.

$$F : (0, 1] \longrightarrow \mathbb{R}$$

$$p \longmapsto \sum_{i=1}^n f_i(p)$$

Given a user-defined value B of the budget, the optimal value p^* of the threshold of probability of failure can be expressed as:

$$p^* = \min \{p : F(p) = B\}$$

For each facility, solving the facility-level optimization with the threshold of probability of failure taken as p^* provides a set of maintenance and replacement policies. This set of policies is optimal at the system level. Note that the optimization described in the previous chapter utilizes a threshold of reliability index instead of a threshold of probability of failure. The threshold of reliability index corresponding to the threshold p^* of probability of failure is as follows:

$$\beta^* = -\Phi^{-1}(p^*)$$

where Φ is the cumulative standard normal distribution.

5.3.3 Computational Complexity

The solution is such that the computational complexity of the system-level problem is low. Namely, the fact that the threshold of probability of failure is the same for all facilities at system-level optimum reduces the optimization problem for n facilities to n independent facility-level problems. Therefore, the complexity is proportional to the number of facilities in the system, i.e., $O(n)$. For each facility, the facility-level optimization problem is solved a small number of times, as seen earlier. Since the time to solve one facility-level problem is of the order of a few seconds on a personal computer, typical values of computation times for the system-level problem are five hours per thousand facilities. These computation times indicate that the present system-level optimization method can be applied to systems of the size of that managed by a state Department of Transportation. For example, if applied to the system managed by Caltrans, composed of 24,000 bridges (California Department of

Transportation, 2006), the optimization would require a computation time of approximately five days, which is very short compared with the time scale of maintenance and replacement decisions.

5.4 Summary

In this chapter, we address the problem of optimization of maintenance and repair decisions for a system of heterogeneous facilities. The facility-level results from chapter 4 are used in this reliability-based formulation.

Exact solutions are determined in the continuous case, and we provide formal proofs of optimality. A numerical study also shows that the results obtained in the discrete-case implementation of the solution seem to be valid approximations of the continuous-case results.

The computational complexity of the determination of the optimal solution is low, which makes this approach suitable for systems of realistic sizes. Moreover, we designed an approach that provides recommendations of optimal maintenance and repair decisions for each facility in the system. This characteristic was considered by maintenance engineers to be very important for implementation purposes, rather than providing recommendations for fractions of the bridge population, as is the case in Pontis for example.

Chapter 6

Conclusion

6.1 Contributions

This dissertation addresses the determination of optimal decisions for bridge maintenance and repair both for one facility and for a system of heterogeneous facilities. Deterioration models are used to predict the future condition of facilities, which is required in the optimization to evaluate maintenance and repair policies. Given the inherent stochasticity associated with bridge deterioration, the purpose of deterioration models is to decrease or capture the uncertainty regarding future condition. More precise deterioration models allow for more discriminating M&R decisions, yielding greater benefits (lower costs or higher performance).

This dissertation concentrates on the use of deterioration models that take into account aspects of the history of deterioration and maintenance. Such deterioration models have not been widely used in optimization methods present in the literature, although their benefits in reducing the uncertainty in the prediction of facility condition are substantial.

In the first part of the dissertation, we formulate the problem of optimizing inspection, maintenance, and repair decisions for a system of heterogeneous facilities, using a non-Markovian model of deterioration. A bottom-up approach is followed, which provides individual recommendations for each bridge. The deterioration model we consider has been used in an optimization method present in the literature; however, the optimization in that article had a very small set of decision variables and was limited to one facility or to a system of homogeneous facilities. The computational complexity of the optimization problem we propose is such that the determination of the exact solution is not likely to be achieved in polynomial time. Therefore, we determine bounds on the optimal cost, which are shown to be close to each other in a numerical example.

In the second part of the dissertation, we provide optimal maintenance and repair decisions for a system of heterogeneous facilities. For each facility, the formulation of the optimization is reliability based. We define a deterioration model that takes into account aspects of the history of deterioration and maintenance, while allowing for the use of standard optimization techniques. The policies determined by the proposed approach are compared to the policies determined by a model comparable to those present in the literature. The application of the policies of the model proposed in this dissertation provides significant benefits.

The results of the facility-level optimization are incorporated in a new reliability-based formulation of the problem of optimizing M&R decisions for a system of heterogeneous facilities, providing optimal decisions for each facility in the system. The computational efficiency of the system-level solution makes the formulation suitable for systems of realistic sizes.

6.2 Future Work

The directions we identified for future research include the following:

1. In the system-level formulation presented in chapter 5, the budget is expressed as a constraint over the entire planning horizon. This may be seen as a limitation if the budget available to the agency is yearly and not cumulative, i.e., if the agency must use the budget for a given year in that year only and cannot save part of it for future use. However, the policies described in chapter 4 seem to lead to the performance of maintenance actions at regular intervals. This is likely to lead to stable yearly costs, especially if the number of facilities in the system is large. Moreover, maintenance actions were shown to be performed on facilities in good condition, which means that they could be delayed by a few years without increasing the risk of failure of the facilities to unacceptable levels. These results could be investigated, to determine if the policies found are robust to small changes in the timings of the actions. If it were the case, policies could be defined in the form “ n maintenance actions must be performed over the next 75 years,” and the actual timing of the actions could be determined to meet yearly budget constraints. Such results could be very beneficial to agencies in charge of the maintenance of infrastructure systems.
2. The formulations presented in chapters 4 and 5 assume perfect information regarding facility condition. In practice, this implies that inspections are carried out every year and are error free. Inspection decisions have been included in the optimization in earlier studies considering simpler Markovian models. Measurement uncertainties have also been taken into account. Provided accurate data is available, these refinements would be beneficial to the formulation presented in this dissertation, since it seems promising to link the notions of inspection decisions and of history of deterioration and maintenance.

3. The reliability-based formulation for a system of facilities presented in chapter 5 is likely to be applicable to other systems: fleets of vehicles, other civil infrastructure systems, supply chain of time-critical goods, assembly lines, etc.
4. Although the reliability-based approach presented in this dissertation seems to be preferred by maintenance engineers, rather than the serviceability-based approach followed by existing bridge management systems such as Pontis, models considering the probability of failure of bridge components are not widely available in the literature. Deterioration models play a critical role in the optimization of maintenance and repair decisions. The increasing interest in small and inexpensive measuring devices, such as Micro-Electro-Mechanical Systems (MEMS), offers great potential to the development of improved deterioration models. These sensors can provide precise and frequent measurements of the moisture content, the temperature, as well as the concentration of chloride, sodium, and potassium ions in concrete. This data can be used to estimate the structural health of a bridge deck. Accelerometers can also provide information about modes of vibration of structures, which are relevant in the analysis of structural health (Pakzad et al., 2005). This data on probability of failure of bridge decks, associated with information on traffic, environmental conditions, and physical characteristics of the bridge, can be used to improve deterioration models.

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Appendix A

Bottom-Up Optimization: Deterministic Formulation

In the deterministic formulation, the set of decision variables is a set of binary variables. Each binary variable represents the decision to perform a given action on a given facility in a given year. The mathematical formulation is as follows.

$$\min_{\{1_{i,t,a}, i \in \{1, \dots, n\}, t \in \{0, \dots, T-1\}, a \in \{1, \dots, A\}\}} \sum_{i=1}^n \left[F_i(T) C_i^{\text{failure}} + \sum_{t=0}^{T-1} \sum_{a=1}^A \alpha^t E [1_{i,t,a} C^a(i, \beta_i(t))] \right] \quad (\text{A.1})$$

subject to

$$F_i(t) \leq P_i^{\text{acceptable}} \quad i \in \{1, \dots, n\}, t \in \{0, \dots, T-1\} \quad (\text{A.2})$$

$$\sum_{i=1}^n \sum_{a=1}^A \alpha^t E [1_{i,t,a} C^a(i, \beta_i(t))] \leq B_t, \quad t \in \{0, \dots, T-1\} \quad (\text{A.3})$$

$$\beta_i(t_{\text{cont}}) = f_i \left(t_{\text{cont}}, t_i^{\text{repl}}(t_{\text{cont}}), \{1_{i,t,a}, t \in \{t_i^{\text{repl}}(t_{\text{cont}}), \dots, [t_{\text{cont}}]\}, a \in \{1, \dots, A\}\} \right) \quad (\text{A.4})$$

$$i \in \{1, \dots, n\}, t_{\text{cont}} \in [0, T)$$

$$1_{i,t,a} \in \{0, 1\}, \quad i \in \{1, \dots, n\}, t \in \{0, \dots, T-1\}, a \in \{1, \dots, A\} \quad (\text{A.5})$$

The following notation is used:

- n : number of bridges in the system
- T : number of years in planning horizon
- A : number of possible maintenance actions
- $1_{i,t,a}$: binary variable taking value 1 if action a is performed on facility i in year t , 0 otherwise
- C_i^{failure} : cost of failure of facility i
- $F_i(t)$: probability of failure of facility i over t years
- α : discount factor, $\alpha = 1/(1+r)$ where r is the interest rate
- $\beta_i(t)$: reliability index of facility i at time t . By definition of the reliability index, the instantaneous probability of failure of facility i at time t (given it has not failed yet) is $\Phi(-\beta_i(t))$.
- $C^a(i, \beta_i(t))$: cost of action a performed on facility i when its reliability index is $\beta_i(t)$
- $\lfloor t_{\text{cont}} \rfloor$: largest integer less than or equal to t_{cont}
- $t_i^{\text{repl}}(t_{\text{cont}})$: year of last replacement of facility i before time t_{cont} . If the facility i has not yet been replaced by time t_{cont} , $t_i^{\text{repl}}(t_{\text{cont}}) = 0$
- $f_i(t_{\text{cont}}, t_i^{\text{repl}}(t_{\text{cont}}), \{1_{i,t,a}, t \in \{t_i^{\text{repl}}(t_{\text{cont}}), \dots, \lfloor t_{\text{cont}} \rfloor\}, a \in \{1, \dots, A\}\})$: reliability index of facility i at time t_{cont} , depending on maintenance performed between year $t_i^{\text{repl}}(t_{\text{cont}})$ and year $\lfloor t_{\text{cont}} \rfloor$. This continuous function of time is the basis for the modeling of deterioration and ensures that the deterioration is time-dependent.
- $P_i^{\text{acceptable}}$: acceptable probability of failure for facility i
- B_t : budget made available to the agency at the beginning for year t , to be used in year t .

Constraint (A.2) sets an upper bound to the probability of failure of each facility over the planning horizon. Constraint (A.3) is the budget constraint. Constraint (A.4) represents

the component deterioration and constraint (A.5) defines the decision variables as binary. No discount rate is applied to the cost of failure, as failure is a very undesirable event.

The result of the optimization is a set of recommendations such that the reliability and budget constraints are satisfied. From a practical point of view, this means that the optimization is performed at the beginning of the planning horizon, and the recommendations are applied regardless of the actual reliability index of the facilities.

The ideal case would be to consider an infinite planning horizon, in order to optimize the life-cycle cost of the system. However, this would require an infinite computation time. Let us justify in an informal manner the choice of the length of the planning horizon. Let us compare two instances of the problem: one with an infinite planning horizon and one with a finite planning horizon T . The difference between the two cases is the costs in years $t > T$, which are undefined in the finite case. If a positive interest rate is applied, the further in the future a cost is incurred, the smaller its present value. In the cost minimization we are considering, this can be expressed as: if a cost is incurred at year $t + \tau$, the larger τ is, the less this cost influences decisions made at year t . This can be extended as: if τ is sufficiently large, the decision in year t is the same whether a (bounded) cost is to be incurred in year $t + \tau$ or not. Intuitively, for any α strictly positive and any positive integer T_{agency} , there exists a positive integer T_0 such that the optimal maintenance decisions between year 0 and year $T_{\text{agency}} - 1$ are the same whether the problem has an infinite planning horizon or a finite planning horizon of length greater than T_0 . This statement, which can be seen as a property of convergence of the optimal values as the planning horizon tends to infinity, needs to be proved formally, at least for simple cases. If it is true, it means that, after having chosen the interest rate α and the length T_{agency} of the period for which the agency wants to determine the recommended actions, the problem can be solved using a planning horizon of length T_0 .

This also means that a constraint on the final condition of the system is not required, as the final condition would not influence decisions at the beginning of the planning horizon. Due to the inherently stochastic deterioration of bridges and the absence of reliable long-term forecasts on budgets, agencies are not likely to be interested in decisions more than 10 years in advance. Thus, a reasonable value of T_{agency} can be assumed to be 10 years.

Appendix B

Notation and Additional Proofs for System-Level Result

B.1 Notation

For any integer n , the notation $\{1, \dots, n\}$ represents the set of integers between 1 and n , 1 and n being included. The set \mathbb{R}^+ is the set of nonnegative real numbers. For any set $E \subset \mathbb{R}$, E^n indicates the set of n -tuples of elements of E . For any $(p_1, \dots, p_n) \in \mathbb{R}^n$, the notation $\max\{p_1, \dots, p_n\}$ represents the largest element of the n -tuple (p_1, \dots, p_n) . For any real numbers a and b such that $a \leq b$, the set of real numbers x such that $a \leq x \leq b$ is denoted by $[a, b]$; the set of real numbers x such that $a < x < b$ is denoted by (a, b) ; the set of real numbers x such that $a \leq x < b$ is denoted by $[a, b)$.

B.2 Proof of Necessary Conditions for Strictly Decreasing

f_i 's

In this section, the functions f_i are assumed to be strictly decreasing. We will prove that, if $(p_1, \dots, p_n) \in (0, 1]^n$ is an optimal solution of (5.2), then

(i) for all $(i, j) \in \{1, \dots, n\}^2$, $p_i = p_j$, and

(ii) $\sum_{i=1}^n f_i(p_i) = B$.

Let $(p_1, \dots, p_n) \in (0, 1]^n$ be an optimal solution of (5.2).

Let us prove that for all $(i, j) \in \{1, \dots, n\}^2$, $p_i = p_j$. Suppose that

$$\min \{p_1, \dots, p_n\} < \max \{p_1, \dots, p_n\} \quad (\text{B.1})$$

We will show that assumption (B.1) leads to a contradiction.

The minimum of $\{p_1, \dots, p_n\}$ is reached by at least one element. Let us denote by i_{\min} the index of such an element, i.e., $\min \{p_1, \dots, p_n\} = p_{i_{\min}}$. Similarly, let us define i_{\max} such that $\max \{p_1, \dots, p_n\} = p_{i_{\max}}$. Let g be the following function:

$$\begin{aligned} g : (0, 1] &\longrightarrow \mathbb{R} \\ x &\longmapsto \sum_{i=1}^n f_i(p_i) - \sum_{i=1}^n f_i(x) \end{aligned}$$

For all $i \in \{1, \dots, n\}$, $p_i \leq p_{i_{\max}}$; thus, $f_i(p_i) \geq f_i(p_{i_{\max}})$, because f_i is a decreasing function on $(0, 1]$. Therefore, $g(p_{i_{\max}}) = \sum_{i=1}^n (f_i(p_i) - f_i(p_{i_{\max}})) \geq 0$. Similarly, $g(p_{i_{\min}}) \leq 0$.

We have:

(i) g is continuous on $[p_{i_{\min}}, p_{i_{\max}}]$,

(ii) $g(p_{i_{\min}}) \leq 0$, and

(iii) $g(p_{i_{\max}}) \geq 0$.

By the intermediate value theorem, there exists $c \in [p_{i_{\min}}, p_{i_{\max}}]$ such that $g(c) = 0$.

Moreover, since f_i is strictly decreasing on $(0, 1]$ for all $i \in \{1, \dots, n\}$,

- (i) $f_{i_{\min}}(p_{i_{\min}}) > f_{i_{\min}}(p_{i_{\max}})$, and
(ii) for all $i \in \{1, \dots, n\}$ such that $i \neq i_{\min}$, $f_i(p_i) \geq f_i(p_{i_{\max}})$.

Therefore,

$$g(p_{i_{\max}}) = \sum_{\substack{i=1 \\ i \neq i_{\min}}}^n \underbrace{(f_i(p_i) - f_i(p_{i_{\max}}))}_{\geq 0} + \underbrace{f_{i_{\min}}(p_{i_{\min}}) - f_{i_{\min}}(p_{i_{\max}})}_{> 0} > 0 \quad (\text{B.2})$$

Thus, c is necessarily in $[p_{i_{\min}}, p_{i_{\max}})$. Note that this interval is not empty, since $p_{i_{\min}} < p_{i_{\max}}$.

Let us define $(q_1, \dots, q_n) \in (0, 1]^n$ such that, for all $i \in \{1, \dots, n\}$, $q_i = c$.

Since $g(c) = 0$, we have $\sum_{i=1}^n f_i(q_i) = \sum_{i=1}^n f_i(p_i) \leq B$, so (q_1, \dots, q_n) is a feasible solution of optimization problem (5.2). Moreover,

$$\max \{q_1, \dots, q_n\} = c < p_{i_{\max}} = \max \{p_1, \dots, p_n\} \quad (\text{B.3})$$

which contradicts that (p_1, \dots, p_n) is an optimal solution of (5.2).

Therefore, assumption (B.1) leads to a contradiction, and

$$\min \{p_1, \dots, p_n\} \geq \max \{p_1, \dots, p_n\} \quad (\text{B.4})$$

which means that $p_1 = p_2 = \dots = p_n$.

We now prove that $\sum_{i=1}^n f_i(p_i) = B$. Suppose that

$$\sum_{i=1}^n f_i(p_i) < B \quad (\text{B.5})$$

We will show that assumption (B.5) leads to a contradiction.

We have shown that $p_1 = \dots = p_n$. Let us define $p \in (0, 1]$ such that, for all $i \in \{1, \dots, n\}$,

$p_i = p$, and $\epsilon \in \mathbb{R}$ such that

$$\epsilon = B - \sum_{i=1}^n f_i(p_i) > 0 \quad (\text{B.6})$$

Let h be the following function:

$$\begin{aligned} h : (0, 1] &\longrightarrow \mathbb{R} \\ x &\longmapsto \sum_{i=1}^n f_i(x) \end{aligned}$$

The function h is continuous on $(0, 1]$. By continuity of h at point $p > 0$, there exists $\delta > 0$ such that $p - \delta > 0$ and, for all $x \in (p - \delta, p + \delta)$, $|h(x) - h(p)| < \epsilon$. Therefore,

$$h(p - \delta/2) < \epsilon + h(p) = B \tag{B.7}$$

Let us define $(r_1, \dots, r_n) \in (0, 1]^n$ such that, for all $i \in \{1, \dots, n\}$, $r_i = p - \delta/2$.

$$\sum_{i=1}^n f_i(r_i) = h(p - \delta/2) < B \tag{B.8}$$

Therefore, (r_1, \dots, r_n) is a feasible solution of optimization problem (5.2). Moreover,

$$\max\{r_1, \dots, r_n\} = p - \delta/2 < p = \max\{p_1, \dots, p_n\} \tag{B.9}$$

which contradicts that (p_1, \dots, p_n) is an optimal solution of (5.2).

Therefore, assumption (B.5) leads to a contradiction, and we have $\sum_{i=1}^n f_i(p_i) \geq B$. Since (p_1, \dots, p_n) is a feasible solution of (5.2), $\sum_{i=1}^n f_i(p_i) \leq B$. Therefore, $\sum_{i=1}^n f_i(p_i) = B$. \square