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ABSTRACT

It is widely believed that it is optimal to call a bond as soon as its market price equals its call price. We show that this policy is generally not optimal when there is more than one bond issue outstanding because minimizing the value of a particular bond issue is not the same as maximizing the value of equity. Furthermore, the value of a callable bond can rationally exceed the call price when a firm follows the optimal call policy. These results have important implications for valuing and hedging callable bonds.

1. INTRODUCTION

A well-known and often-cited result in corporate finance is that a firm should call a bond as soon as its market price reaches its call price. Underlying this rule is the notion that a firm should choose its call policy for a particular bond so as to minimize the value of that bond. This implies that a firm is not optimizing if the market price of its bond ever exceeds its call price. Because this result is so widely accepted, we designate it the “textbook” policy.

The textbook policy is correct when a firm has only one outstanding issue of debt.¹ In this case, minimizing the value of the debt is equivalent to maximizing the value of equity. In the much more common case of multiple debt issues, however, the textbook rule is generally not optimal; maximizing the value of equity is not equivalent to minimizing the value of a particular debt issue.

To see this, consider the case of a firm that has two issues outstanding, a callable senior bond and a noncallable junior bond. If the firm calls the senior issue with cash, the junior bond may increase in value due to the promotion of its claim. Thus, minimizing the value of the senior bond is not equivalent to minimizing the value of both bond issues, which means, therefore, that it is not equivalent to maximizing the value of equity. An important implication of this is that the price of a callable bond can exceed its call price when the firm follows the optimal call policy. This provides an economic rationale for why market prices of bonds often exceed their call prices.

Section 2 presents a number of simple examples that demonstrate that the textbook policy need not be optimal when the firm has more than one issue of bonds outstanding. Depending on seniority, the optimal policy may require the firm to call an issue earlier or later than implied by the textbook policy. We show that the optimal call policy results in bond prices and hedging behavior that can be very different from those implied by the textbook policy. We also illustrate that the market value of a callable bond can be significantly higher than the call price when the firm follows the optimal call policy.

¹See, for example, Brennan and Schwartz (1977).

Section 3 discusses refunding calls, or calls that are financed by issuing new debt. The purpose of this section is to show that the textbook policy is not optimal for almost all refunding operations. In other words, the fact that an issuer can finance a call with new debt does not eliminate the effects described in section 2.

Section 4 presents the results of an exploratory empirical analysis of callable bonds. We find that currently-callable bonds frequently sell for more than their call prices. Also, the distribution of credit ratings across bonds that sell for a premium above their call prices is consistent with our analysis. Section 5 summarizes the results and concludes the paper.

2. OPTIMAL CALL POLICY

In this section, we demonstrate that the textbook policy is generally not optimal when the firm has more than one issue of debt outstanding. To present the intuition behind our results more clearly, we focus on the simplest possible examples. The analysis, however, could be extended to more complex and realistic types of capital structures.

We develop our analysis in the standard continuous-time framework of Merton (1974). Let V denote the value of the assets of a firm. The value of the assets is random and has risk-neutral dynamics given by

$$dV = rVdt + \sigma VdZ. \tag{1}$$

Assume that the firm has two zero-coupon bonds outstanding. The maturity date of both bonds is T and the face amounts of the bonds are F_1 and F_2 . The remainder of the capital structure consists of stock. Let K_1 and K_2 be the call prices for the two bonds. For simplicity, we assume that the bonds can only be called at time zero. Note that the call prices K_1 and K_2 must be less than F_1e^{-rT} and F_2e^{-rT} respectively in order for the bonds to ever be called. Finally, for the moment, we assume that bonds are not refunded if called.

2.1 Calling Senior Debt

In this example, we assume that the first bond is senior to the second bond and

that only the senior bond is callable. At time zero, the stockholder must decide whether to call the senior bond at a call price of K_1 . Consequently, the values of the bonds and the equity in the firm at time zero depend on whether the senior debt is called. These values are tabulated below,

	<u>Not Called</u>	<u>Called</u>
Senior Debt	$V - C(V, F_1)$	K_1
Junior Debt	$C(V, F_1) - C(V, F_1 + F_2)$	$V - K_1 - C(V - K_1, F_2)$
Equity	$C(V, F_1 + F_2)$	$C(V - K_1, F_2)$

where $C(V, F)$ denotes the value of a call option on V with strike price F . The value of the senior debt is K_1 in the second column because the bondholders receive a payment of K_1 when the bond is called.

Let V^* denote the time-zero value of V at which it is optimal for the stockholder to call the senior debt. Since the stockholder maximizes his wealth, the value of the equity at time zero is

$$\max(C(V, F_1 + F_2), C(V - K_1, F_2)). \quad (2)$$

The optimal call policy can be determined by finding V^* such that

$$C(V^*, F_1 + F_2) = C(V^* - K_1, F_2). \quad (3)$$

In contrast, the textbook call policy is determined by finding \hat{V} such that

$$\hat{V} - C(\hat{V}, F_1) = K_1. \quad (4)$$

Numerical Example. Let $F_1 = 100$, $F_2 = 100$, $r = .05$, $\sigma^2 = .04$, $T = 1$, and

$K_1 = 94$. The optimal policy is to call the senior debt for values of $V \geq V^*$, where $V^* = 257.1$. The textbook policy is to call the senior debt for values of $V \geq \hat{V}$, where $\hat{V} = 120.7$.

This simple analysis has a number of important implications. First, the optimal value V^* is significantly higher than the value implied by the textbook policy. Thus, the optimal policy is to delay calling the bond far beyond the value of \hat{V} implied by the textbook policy. The intuition for this result is that when the senior debt is called, much of the wealth transferred from senior bondholders accrues to junior debtholders rather than to stockholders, since the junior debtholders have the prior claim. Thus, stockholders have no incentive to call senior debt until the value of V is sufficiently high for the junior debt to be nearly riskless.

A second major implication is that the value of the senior debt can exceed the call price of the bond. This is shown in Fig. 1 which graphs the time-zero value of the senior debt as a function of V . As illustrated, the value of the senior debt reaches $K_1 = 94$ at a value of $V = 120.7$, and is greater than K_1 for the range $120.7 < V < 257.1$. The value of the senior debt can be as high as 95.12, which represents a 1.19 percent premium over the call price. Vu (1986) reports that out of a sample of 41 callable bonds with values in excess of their call price, 22 had a premium in excess of one percent while only one had a premium in excess of two percent.

A third implication of these results is that the market value of the senior debt is higher than implied by the textbook policy. This is intuitive since the optimal policy maximizes the value of the equity instead of minimizing the value of the debt. Fig. 1 shows that the value of the senior debt at time zero is higher when stockholders follow the optimal call policy. Thus, the value of the debt at any time prior to the call date must be higher than implied by the textbook call policy. This feature has important implications for pricing callable debt and determining the actual cost of capital.

Finally, Fig. 1 shows that the value of the senior debt at time zero is discontinuous at $V = 257.1$, jumping from 95.12 when the bond is not called to 94 when the bond is called. This discontinuity arises because some of the wealth of the senior debtholder is transferred to the junior bondholder when the debt is called.

Note, however, that there is no discontinuity in the value of the equity at V^* . This is because the maximizing behavior of the stockholder guarantees that the value of the equity satisfies the continuity condition at V^* .

Because of the discontinuity in the value of the senior debt at time zero, the value of the senior debt prior to the call date, although continuous, can be very sensitive to small changes in the value of V . In fact, the price risk of the senior debt, in both absolute and percentage terms, can be larger than that of the equity in the firm. Furthermore, over some range of V , the value of the senior debt can be shown to be a decreasing function of V . These results illustrate that the behavior of callable bond prices implied by the optimal policy can be quite different from those implied by the textbook policy.

2.2 Calling Junior Debt

Now assume that only the junior debt is callable. The stockholder again chooses V^* to maximize the value of equity. The values of the bonds and the equity are given by

	<u>Not Called</u>	<u>Called</u>
Senior Debt	$V - C(V, F_1)$	$V - K_2 - C(V - K_2, F_1)$
Junior Debt	$C(V, F_1) - C(V, F_1 + F_2)$	K_2
Equity	$C(V, F_1 + F_2)$	$C(V - K_2, F_1)$

The value of the equity at time zero is maximized by choosing V^* such that

$$C(V^*, F_1 + F_2) = C(V^* - K_2, F_1). \quad (5)$$

In contrast, the textbook policy is determined by solving for \hat{V} such that

$$C(\hat{V}, F_1) - C(\hat{V}, F_1 + F_2) = K_2. \quad (6)$$

Numerical Example. Let $K_2 = 94$. Using the same parameter values as before, the optimal strategy is to call the junior debt for values of $V \geq V^*$, where $V^* = 257.1$. The textbook policy is to call the junior debt for values of $V \geq \hat{V}$, where $\hat{V} = 257.4$.

The optimal policy again differs from the textbook policy. In particular, the textbook policy is to call the junior debt when V reaches the value 257.4, which is slightly higher than for the optimal policy. Thus, it is optimal to call the junior debt earlier than is implied by the textbook policy. This is again consistent with the results of Vu (1986) who shows that firms frequently call bonds earlier than implied by the textbook policy. These results, in conjunction with those above, demonstrate that the optimal value of V^* may be greater than, or less than, the value implied by the textbook policy.

The optimal policy again has a number of implications for pricing and hedging junior debt. For example, it is easily shown that the value of the junior debt is greater when stockholders follow the optimal policy than when they follow the textbook policy. This is because the optimal policy maximizes the value of the equity rather than minimizing the value of the junior debt. One difference between the senior and junior debt examples is that the value of the junior debt cannot be greater than its call price. In fact, it can be shown that the left limit of the value of the junior debt at time zero as V approaches V^* is strictly less than the call price K_2 . Of course, the value of the junior debt at $V = V^*$ is K_2 . Since the value of the junior debt at time zero is discontinuous at V^* , the value of the junior debt prior to the call date may again be more sensitive to the value of V than the value of the equity.

2.3 Calling Debt of Equal Seniority

In this example, we demonstrate that the optimal policy can differ from the textbook policy even when the bonds have the same seniority. Assume that only one of the bonds is callable, say the first. The values of the bonds and the equity are given by

	<u>Not Called</u>	<u>Called</u>
Callable Bond	$(F_1/(F_1 + F_2)) \times (V - C(V, F_1 + F_2))$	K_1
Noncallable Bond	$(F_2/(F_1 + F_2)) \times (V - C(V, F_1 + F_2))$	$V - K_1 - C(V - K_1, F_2)$
Equity	$C(V, F_1 + F_2)$	$C(V - K_1, F_2)$

The optimal call policy V^* is determined by solving the equation

$$C(V^*, F_1 + F_2) = C(V^* - K_1, F_2), \quad (7)$$

which is identical to (3). Thus, using the same numerical values as before, the optimal call policy is to call when $V \geq V^*$, where $V^* = 251.7$. The textbook policy is again to call when the uncalled value of the bond just equals its call price K_1 . In this example, this occurs when V reaches 241.4. Thus, the optimal policy delays the call past the point implied by the textbook policy.

The value of the callable bond can again be greater than the call price. For example, the value of the callable bond at time zero can be as high as 94.55, or .59 percent higher than the call price. This result also holds for the value of the callable bond prior to the call date. As in the senior debt example, the value of the callable debt at time zero is a discontinuous function of V and the properties of its price are similar to those described earlier.

3. OPTIMAL CALL POLICY WITH REFUNDING

In this section, we demonstrate that the textbook policy is generally not optimal even when the called bonds are refunded with bonds of the same seniority. Similar results are obtained when called debt is assumed to be refunded by debt of lower

seniority or by new equity.

Consider the case where the senior debt is callable at a price of K_1 and a proportion α of the funds used to make the call are obtained by issuing new zero-coupon senior debt. The proportion α can be greater than, equal to, or less than one. The value of the bonds and the equity at time zero are given by

	<u>Not Called</u>	<u>Called</u>
Senior Debt	$V - C(V, F_1)$	K_1
New Senior Debt	0	$V - (1 - \alpha)K_1 - C(V - (1 - \alpha)K_1, F^*)$
Junior Debt	$C(V, F_1) - C(V, F_1 + F_2)$	$C(V - (1 - \alpha)K_1, F^*) - C(V - (1 - \alpha)K_1, F^* + F_2)$
Equity	$C(V, F_1 + F_2)$	$C(V - (1 - \alpha)K_1, F^* + F_2)$

where F^* is the face value of the new senior debt such that it sells for a price of αK_1 at time zero when $V = V^*$. The optimal call policy is found by solving

$$C(V^*, F_1 + F_2) = C(V^* - (1 - \alpha)K_1, F^* + F_2), \quad (8)$$

for the value of V^* . This is easily done numerically by first picking a trial value of V^* , solving for the corresponding value of F^* , and then checking whether (8) is satisfied. As before, the optimal policy is to call and refund the senior debt when $V \geq V^*$. The textbook policy is again given by solving (4) for \hat{V} .

Numerical Example. The senior debt example given in the previous section corresponds to the case where $\alpha = 0$. Using the same parameter values, V^* and the corresponding maximum premium of the value of the senior debt over the call price are shown in Table 1 for various values of α .

Table 1 illustrates that the textbook policy is optimal only in the special case where $\alpha = 1$. For any value of α less than one, the optimal value of V^* is greater than that implied by the textbook policy. For any value of α greater than one, the optimal value of V^* is less than implied by the textbook policy. This example also shows that the difference between the optimal and textbook policies can be very large even when nearly 100 percent of the called debt is refunded. For example, when 99 percent of the debt is refunded, the difference between the optimal value of V^* and the value implied by the textbook policy is 7.0. In addition, the value of the maximum premium .51 is almost half as large as when none of the called debt is refunded.

These results demonstrate that the implications of our analysis are robust to refunding considerations. Although the textbook policy can be optimal, its applicability is the exception rather than the rule. In actual debt markets, the condition $\alpha = 1$, which may be called the refunding-neutrality condition, can be satisfied by a set of refunding issues that match all remaining payments and option features of the retired issue. However, given the convention of refunding with one coupon bond issue that sells for par, it is most unlikely that the refunding issue will match the retired bond in all particulars. Furthermore, because of asset-liability management considerations, issuers may not even want to match the refunding bond's terms with those of the retired issue.

4. AN EXPLORATORY EMPIRICAL ANALYSIS

One clear implication of the examples presented earlier is that the market price of a callable corporate bond can exceed its call price. To examine this implication, a sample was collected of all currently-callable industrial bonds listed in the August 1992 edition of *Moody's Bond Record*.²

Restricting the sample to industrial firms avoids the complicating effects that government regulation may have on the call policies of public utilities, municipalities, or banks.³ At the time of writing this paper, the most recent available edition

²Bonds in default are excluded from the sample.

³Equity holders of public utilities, for example, may not find it worthwhile to call a

of *Moody's Industrial Manual*, from which this supplemental information was required, covered the period from 8/1/91 through 7/31/92. Therefore, the August 1992 edition of *Moody's Bond Record*, providing prices as of 7/31/92, was the most recent edition consistent with the information from the industrial manual.

The sample of 727 issues reveals that the central implication of the textbook policy is frequently violated in practice; market prices exceed call prices in 258 cases, or in 35.5 percent of the sample. For these 258 bonds, Table 2 shows that the difference between the market price and the call price, henceforth called the premium over the call price, averages \$1.83 per \$100 face value and has a median value of \$.90 per \$100 face value.

Moody's Bond Record contains two types of price quotations, bid prices and sale prices. As the labels imply, the former are price quotations while the latter are transaction prices. To be certain that the existence of these premiums over the call price is not due to stale price quotations, Table 2 also reports separate summary statistics for bid prices and sale prices.

Neither the means nor the medians of the two samples are significantly different from each other at the 5 percent level. Nevertheless, the averages and maximums of the two samples suggest that the sample of bid prices contains more particularly large premiums over the call price than does the sample of sale prices. However, it is clear that premiums exist and cannot be explained away as artifacts of stale prices.

This paper claims that callable bond prices can exceed their call prices whenever the bonds are equal or senior to the issuer's other outstanding bonds. Of the 258 issues selling at a premium to their call prices, 82 percent are equal or senior to other public debt of the same issuer. Furthermore, this percentage underestimates the presence of other issues because private debt is not included. In any case, callable bonds selling at a premium to their call prices cannot be characterized as solitary issues for which the analysis of this paper does not apply.

Our results have another important empirical implication. The extent to which

bond if that action would result in a lower computed cost of capital for rate setting purposes. See Kalotay (1979).

a call of a particular issue transfers value to equal or junior debt depends on the bonds' credit quality. The effect will not be large for nearly riskless debt but can be expected to increase as credit quality deteriorates. This argues that bonds selling at a premium to their call prices are likely to have relatively low credit ratings. On the other hand, since bonds with particularly low credit ratings will not have values anywhere near their call prices, these bonds will not appear in the sample of bonds selling at a premium to their call prices.⁴

Table 3 shows the rating frequency distributions for the sample of bonds with market prices above their call price and for the sample of bonds with market prices less than or equal to their call price. The two distributions are quite different and a χ^2 test easily rejects the independence of rating and the relation between market and call prices. Furthermore, the differences between the distributions are consistent with the theoretical insights of this paper. Particularly high credits, namely Aaa and Aa, for which the transfer of value associated with calling the bond is small, appear less frequently in the sample of bonds selling above their call price. Bonds of lower quality, namely A and Baa, appear more frequently in that sample because the transfer of value is relatively large. Finally, bonds of the lowest quality, namely Ba and below, appear less frequently simply because their prices are likely to be well below par, call option or not.

5. CONCLUSION

This paper demonstrates that the well-known policy of calling a bond when its value equals its call price is generally not an optimal policy. The reason for this is that minimizing the value of the bond is not the same as maximizing the value of the equity when the firm has more than one issue of debt outstanding. We provide a number of simple examples illustrating that the optimal policy is to call bonds at values of the firm greater than or less than that implied by the textbook policy. In addition, we show that the price of a callable bond can be significantly greater than the call price when the firm follows the optimal strategy.

These results have a number of important implications. First, they provide an

⁴Call prices are almost always greater than or equal to the face value of the bond.

explanation for the long-standing puzzle as to why callable bonds trade at prices above their call prices. Second, they suggest that the price behavior of corporate bonds may be far different from that implied by standard valuation models. In particular, callable bonds may be more sensitive to changes in the firm's asset value than previously recognized. Future research should be directed towards determining optimal call policies for firms with more realistic capital structures than those considered here.

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Table 1

The textbook vs. the optimal call policy for the senior debt issue across different levels of refunding. The call policies are stated in terms of the level of the firm's assets at which the senior debt issue is called at 94. The premium over call represents the maximum difference between the value of the senior debt and the call price on the date at which the bond can be called.

Percent Refunded	Textbook Policy	Optimal Policy	Premium Over Call
0	120.7	257.1	1.12
25	120.7	255.7	1.12
50	120.7	250.9	1.12
75	120.7	235.8	1.12
90	120.7	206.4	1.12
99	120.7	127.7	.51
100	120.7	120.7	.00
101	120.7	116.2	.00
110	120.7	101.4	.00
125	120.7	96.5	.00
150	120.7	94.8	.00

Table 2

Summary statistics for the premium of market prices over call prices. These summary statistics are based on a sample of currently-callable bonds with market prices exceeding their respective call prices. All dollar entries are per \$ 100 face value.

	Mean	Standard Deviation	Minimum	Median	Maximum	Number
Bid Prices	\$ 2.00	\$ 2.84	\$.03	\$.95	\$ 14.13	185
Sale Prices	\$ 1.39	\$ 1.86	\$.01	\$.89	\$ 6.13	73
All Prices	\$ 1.83	\$ 2.61	\$.01	\$.90	\$ 14.13	258

Table 3

Rating frequency distributions for the sample of currently-callable bonds with market prices in excess of the call price, and the sample of currently-callable bonds with market prices less than or equal to the call price.

Rating	Market > Call		Market ≤ Call	
	Number	Percent	Number	Percent
Aaa	19	7.7	62	13.8
Aa	28	11.3	78	17.3
A	105	42.3	111	24.7
Baa	40	16.1	58	12.9
Ba	21	8.5	42	9.3
B	31	12.5	61	13.6
Caa	4	1.6	20	4.4
Ca	0	.0	18	4.0
Total Rated	248	100.0	450	100.0
Not Rated	10		19	
Total	258		469	

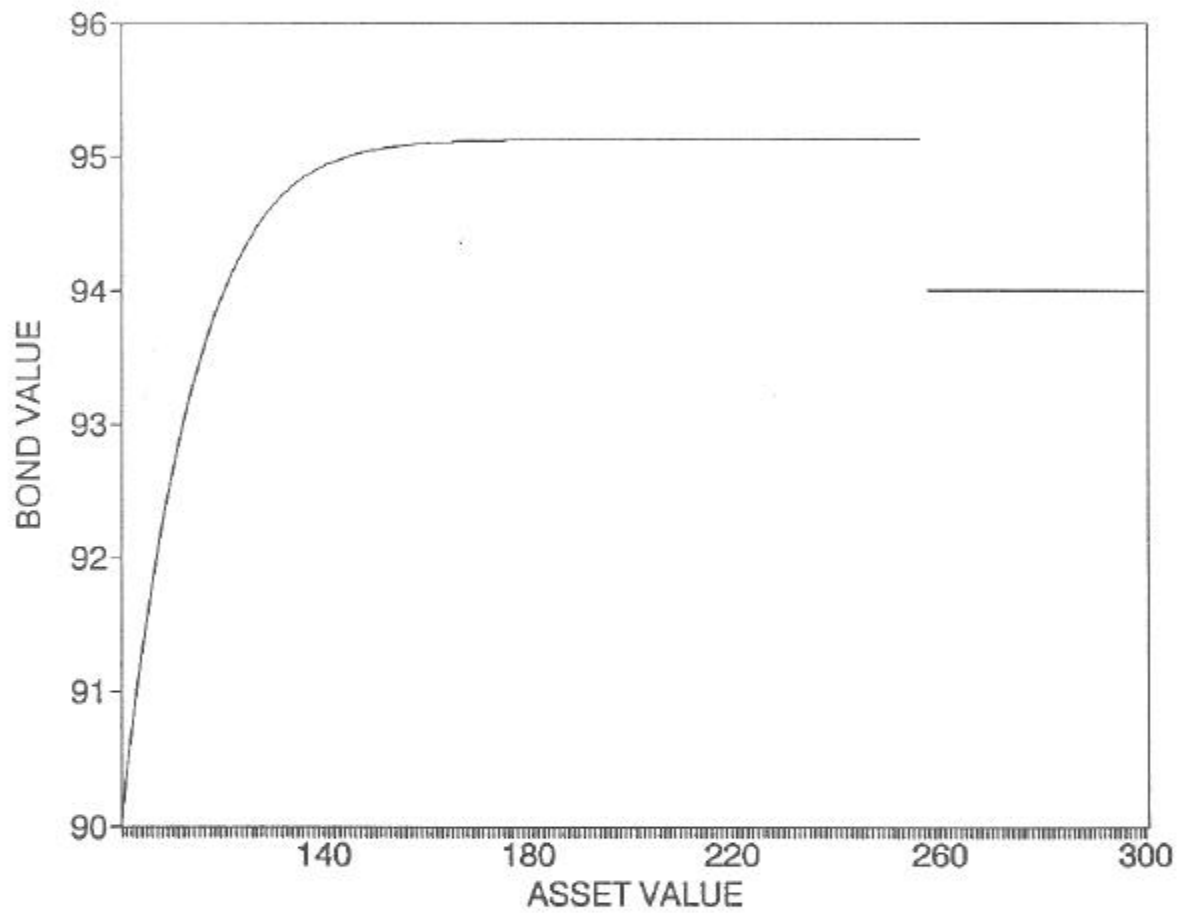


Fig. 1. Graph of the value of the senior debt as a function of the value of the underlying assets of the firm on the date that the bond is callable. The call price for the senior debt is 94.