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Rational Bubbles: A Test

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Abstract

This paper presents a no rational bubble restriction for equity markets that is extremely robust with respect to the specification of the stock's expected return process. If the discount factor and the dividend growth rate are stationary stochastic processes, then the fundamental value-dividend ratio is a stationary stochastic process.

Actual aggregate equity price index-dividend ratios have a large autoregressive root. Monte Carlo results presented here make it seem very unlikely that the traditional expected present value model driven by log-normally distributed dividend growth and discount factors can explain the data.
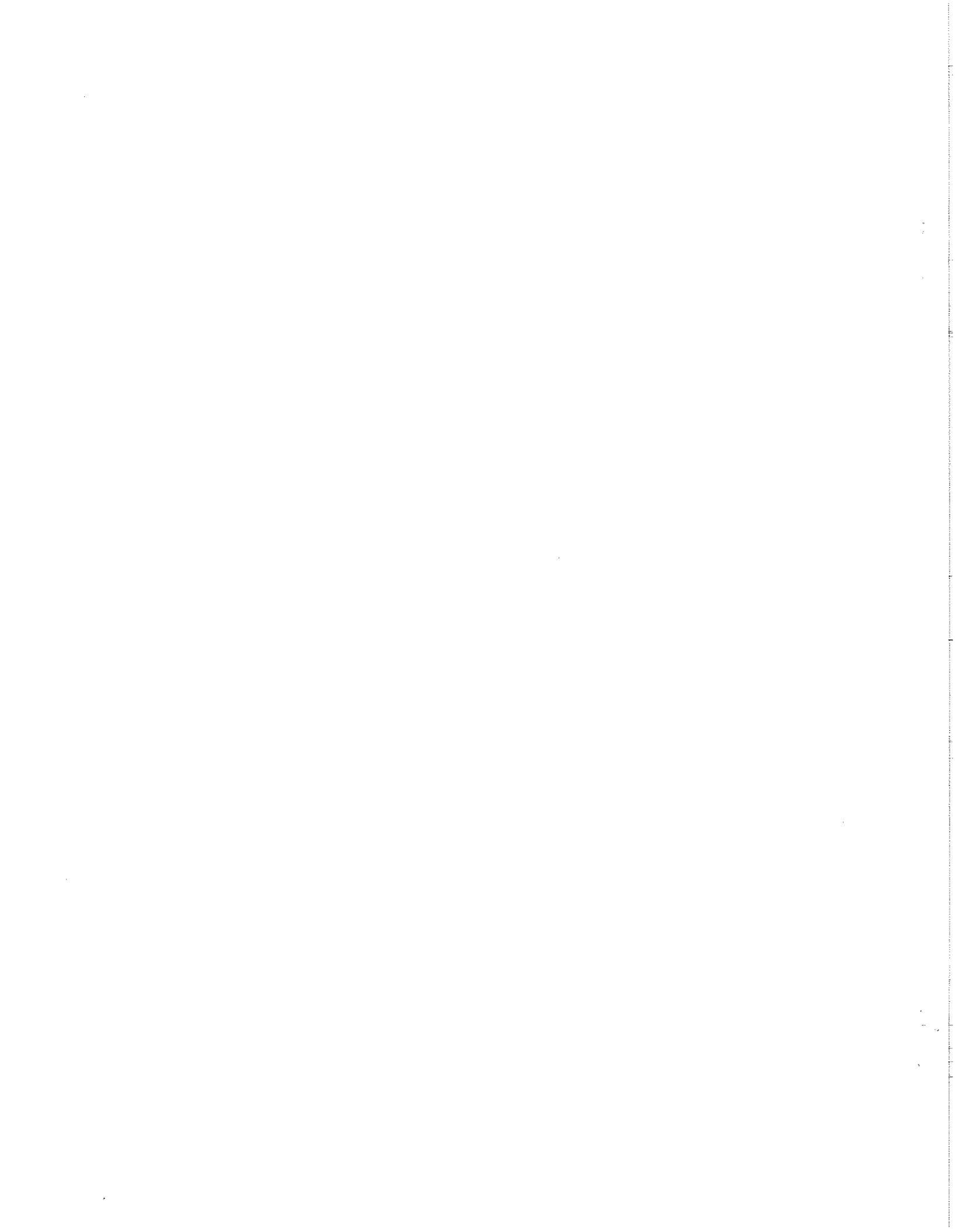
Introduction

The fundamental value of a stock is the sum of the expected discounted dividend sequence. Rational bubbles earn the expected rate of return so they must be expected to grow faster than the stock's fundamental value. A rational bubble causes the stock's price to diverge from its fundamental value. It seems like the divergence should be easy to spot. But as Flood and Hodrick (1990) note pessimistically in a recent assessment of the empirical literature on bubbles,

> Whether the actual volatility of equity returns is due to time variation in the rational equity risk premium or to bubbles, fads and market inefficiencies is an open issue. Bubble tests require a well-specified model of equilibrium expected returns that has yet to be developed, and this makes inference about bubbles quite tenuous.

This paper presents a no rational bubbles restriction for equity markets that is extremely robust with respect to the specification of an equilibrium returns model. It allows for a stochastic discount factor, but it does not require a well-specified model of equilibrium expected returns.

If the discount factor and dividend growth factor are stationary stochastic processes, then the fundamental value-dividend ratio is a stationary stochastic process, eg, see the appendix in Cochrane (1991a), or *theorem 3.34* in White (1984).

Actual dividend growth rates and equity returns look stationary. In fact they have very little serial correlation. Price-dividend ratios, in contrast, contain a very large autoregressive root. Augmented Dickey-Fuller (1979) tests cannot reject the unit root null hypothesis at the 5% level. The upper bounds of local to unity confidence intervals calculated using the tables in Stock (1990) exceed one and the lower bounds are near *0.8*.

Since it is well known that unit root tests often have low power the test statistics only provide weak evidence against the important no bubble alternative. I use Monte Carlos to examine the power of the augmented Dickey-Fuller test. I draw the dividend growth and discount factors from a log-normal distribution and calculate the fundamental value-dividend ratios using the expected present value equation. When the driving discounted dividend growth factor has low serial correlation the derived fundamental value-dividend ratio, on average, also has low serial correlation. The Dickey-Fuller test has excellent power against this class of alternatives.

The Monte Carlo results make it seem very unlikely that the popular expected present value model driven by log-normally distributed dividends and discount factors can explain the data. Almost all of the evidence of "excess volatility" comes from simple constant discount factor models, eg, see LeRoy's (1990) survey. The Monte Carlo results show that these models cannot explain the large low frequency component in observed price-dividend ratios since dividend growth rates have very low serial correlation, *0.25* or less. Of course the low frequency component of price-dividend ratios could be due to a low frequency component in the unobservable stochastic discount factor. But if the discount factor is log-normally distributed, then the serial correlation in the discount rate must match the serial correlation in observable returns (see Section 3). And the serial correlation in observable returns is also very small.

The results in this paper do not provide strong evidence in favor of a bubble. They do provide strong evidence against the popular expected present value model driven by log-normally distributed dividends and discount factors.

Section 1: The Restriction

This section extends the no rational bubble restriction developed by Campbell and Shiller (1987) for constant discount rate expected present value models to stochastic discount rate expected present value models.

Definitions

Fundamental Value

A stock[1] produces a random sequence of dividend payoffs, say $<d>$. Define the fundamental value, F_t, of the stock as the expected value of the sum of the discounted dividend sequence,

$$F_t = \lim_{J \to \infty} E \left[\sum_{j=1}^{J} \{ \prod_{i=1}^{j} D_{t+i} \} d_{t+j} | \Omega_t \right] \tag{1}$$

where $<D>$ denotes the stochastic discount factor sequence and Ω the market information set. The transversality condition,

$$\lim_{J \to \infty} E_t \prod_{i=1}^{J} D_{t+i} F_{t+J} = 0, \tag{2}$$

guarantees that only discounted dividends contribute to the fundamental value.

Rational Bubbles

In a rational expectations equilibrium the stock price, P, equals the expected discounted value of the price plus the dividend next period,

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[1] The definitions and the test developed in this paper hold for any asset. I call the asset a stock and the payoff a dividend since I test for bubbles in the stock market.

$$P_t = E_t [D_{t+1} (P_{t+1} + d_{t+1})]$$

$$= E_t \sum_{j=1}^{\infty} \left\{ \prod_{i=1}^{j} D_{t+i} \right\} d_{t+j} + E_t \prod_{i=1}^{J \to \infty} D_{t+i} P_{t+J} \tag{3}$$

$$= F_t + E_t \prod_{i=1}^{J \to \infty} D_{t+i} P_{t+J}$$

The recursive rational expectations equilibrium condition does not rule out rational bubbles in asset prices. The stock price equals the fundamental value plus (possibly) the contribution of the rational bubble.

Rational bubbles earn the expected rate of return. Rational bubbles must be expected to grow fast enough,

$$B_t = E_t D_{t+1} B_{t+1} = E_t \prod_{i+1}^{J \to \infty} D_{t+i} B_{t+J} \tag{4}$$

to keep the transversality condition from converging to zero.[2]

Testable Restrictions

It seems that since rational bubbles must grow faster than the fundamental value, and therefore diverge from the fundamental value, they would produce many robust testable restrictions. In fact, only a few have been discovered. The problem is that if the fundamental value grows over time (is nonstationary), then a robust decomposition of the observed price into the fundamental

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[2] The conditional covariance between a rational bubble and the discount factor is zero, ie, $E_t D_{t+1} B_{t+1} = E_t D_{t+1} E_t B_{t+1}$ since the fundamental value is an optimal forecast of future discounted dividends. Irrational bubbles add an unrestricted error to the pricing equation (3) that does not satisfy the expected return relationship in equation (4).

value and the bubble is very tricky.

Constant Discount Rate Models

If the discount factor is constant, then the expected present value model is a linear function of the random dividend sequence. Only expected dividend payoffs affect the fundamental value. Risk only enters through a constant premium in the discount factor. The covariance between the asset's payoff and other asset payoffs (the capital asset pricing model) or consumption (the consumption CAPM) does not affect the value of the stock.

Diba and Grossman (1984) and Hamilton and Whiteman (1985) showed that in linear expected present value models the driving process and the fundamental value of the stock have the same order of integration. For example, if the dividend sequence is difference stationary, then the fundamental value sequence is difference stationary. But, the first difference of the observed stock price would not be stationary if the sequence contains a rational bubble since the bubble diverges from the fundamental value.[3]

Campbell and Shiller (1987) strengthened the order of integration restriction by showing that the dividend and fundamental value processes are cointegrated.[4] For example, if the driving dividend

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[3] Hamilton and Whiteman (1985) argued that the order of integration restriction was the only robust (at the time) testable restriction. They showed that more powerful cross-equation restrictions implicitly place restrictions on the dynamics of the unobservable error process.

[4] Dividends and stock prices could be independent random walks and satisfy the same order of integration restriction, but stock prices and the fundamental values would wander apart.

process is difference stationary, then the fundamental value minus the perpetuity value of the dividend, $F_t - \{D/(1-D)\}d_t$, (which Campbell and Shiller label "the spread"), is a stationary process. Campbell and Shiller, and Diba and Grossman (1988b) conduct a variety of unit root and cointegration tests to see if the "spread" is stationary. Diba and Grossman find little support for a bubble and Campbell and Shiller get mixed results. The test results are sensitive to the choice of the unobservable discount factor and the series must be deflated since the difference of nominal values grows with inflation. And of course, the restriction is misspecified if the discount factor is stochastic.

Stochastic Discount Rate Expected Present Value Models

If the discount factor is stochastic the expected present value is a nonlinear function of the dividend sequence. The fundamental value is the sum of the expected products of the compound discount factor and the dividend. The covariance between the dividends and the discount factor measures the asset's risk.

Parameterizing the stochastic discount factor expected present value model requires choosing and parameterizing a complete model of equilibrium returns. And as Flood and Hodrick lament there is no agreed upon model of equilibrium returns. But, the basic insight that the growth of the fundamental value must be closely linked to the growth of dividends leads to a cointregation restriction in the nonlinear present value equation that is very robust with respect to the specification of the equilibrium returns process.

Deflating the fundamental value by the current dividend expresses,

$$\frac{F_t}{d_t} \equiv f_t = E_t[D_{t+1}\frac{d_{t+1}}{d_t}(f_{t+1} + 1)] \equiv \lim_{J\to\infty} \sum_{j=1}^{J} E_t \prod_{i=1}^{j} \delta_{t+i} \qquad (5)$$

$$\text{where, } \delta_{t+i} \equiv D_{t+i}\frac{d_{t+i}}{d_{t-1+i}}$$

the fundamental value-dividend ratio as the expected discounted sum of compounded dividend growth. For example, suppose the discounted dividend growth factor is independently and identically log-normally distributed, ie, $\ln\delta \sim N(\mu, \sigma^2)$, then,

$$f_t = \sum_{j=1}^{\infty} E_t[\exp(\sum_{i=1}^{j} \ln\delta_{t+i})] = \sum_{j=1}^{\infty} \exp(j(\mu + \frac{\sigma^2}{2}))$$

$$= \frac{e^{(\mu+\frac{\sigma^2}{2})}}{1 - e^{(\mu+\frac{\sigma^2}{2})}}$$

$$(6)$$

the fundamental value-dividend ratio is stationary.[5] In fact, it is embarrassingly stationary. The fundamental value-dividend ratio is constant. The conditional forecast of the future dividend growth and discount rate is the (constant) unconditional expected rate--current realizations carry no information about future realizations. Generalizing this example to permit a more realistic stationary driving process leads to a more complicated expression that generates a time-varying fundamental value-dividend ratio, but the fundamental value-dividend ratio is still stationary.

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[5] Recall that $Ex = exp(Elnx + \frac{1}{2}var(lnx))$ when x is log-normally distributed. The mean of the log of delta is the difference between continuously compounded expected growth rate of dividends and the continuously compounded expected rate of time preference. A bounded value for the fundamental value-dividend ratio requires a negative exponent. In deterministic models the exponent is negative if the rate of time preference is greater than the dividend growth rate.

A Robust Testable Restriction for Stochastic Discount Rate Expected Present Value Models

If (i) the dividend growth rate is a stationary stochastic process and (ii) the discount factor is a stationary stochastic process, then the fundamental value-dividend ratio is a stationary stochastic process.[6]

This restriction is very robust with respect to the specification of the model of equilibrium expected returns. It does not require a model of the discount factor. It requires a stationary discount rate. And it requires a stationary dividend growth rate. Dividends are observable and can be subjected to the standard time-series tests for unit roots and serial correlation. The discount factor is unobservable, but strong indirect evidence indicates it is stationary. Rearranging the equilibrium pricing condition (3) gives,

$$1 = \frac{E_t[D_{t+1}\{P_{t+1} + d_{t+1}\}]}{P_t} \equiv E_t[D_{t+1}R_{t+1}] \tag{7}$$

an alternative equilibrium condition that the expected discounted return factor, R, (on any asset) equals one. Since observed return sequences look stationary, the alternative equilibrium condition implies that the discount factor sequence is stationary. Furthermore, if the discount factor sequence were nonstationary, then the fundamental value of stocks would be unbounded; but, we observe finite equity prices.

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[6] Cochrane's appendix (1991a) gives the necessary conditions under which the fundamental value-dividend ratio is stationary if the driving processes are stationary. Or, see *Theorem 3.34* in White (1984, p42) which states that a real valued measurable function of stationary variables is stationary.

A Test

Assume the fundamental value-dividend ratio is a covariance stationary process.[7] A rational bubble would make the observed price-dividend sequence, $P_t/d_t \equiv \{F_t + B_t\}/d_t$, nonstationary. I test for a unit root in the price-dividend ratio sequence.

Section 2: Empirical Evidence

This section presents the results of unit root tests on price-dividend ratios for aggregate stock indices.

Figure 1 shows the log of the equity price and the log of dividends for the popular long annual time-series on the *Cowles S&P Composite* index complied by Shiller (1981) and extended by Mankiw, Romer and Shapiro (1991).[8] The vertical distance between graphs is the log of the price/dividend ratio which should be a stationary process if there are no rational bubbles.

INSERT FIGURE 1

Preliminaries

Table 1 shows the sample means, standard deviations, and autocorrelation coefficients for the price-dividend ratio, *P/d*, the change in the log (growth) of dividends, $ln(d/d_{-1})$, and the log of the return factor, *ln(R)*. In addition to the annual *S&P composite* index I use data on the value weighted annual and quarterly *New York Stock Exchange* index from *CRSP*.

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[7] Stationarity does not imply covariance stationarity.

[8] I thank David Romer for the data. I added 1988 dividends.
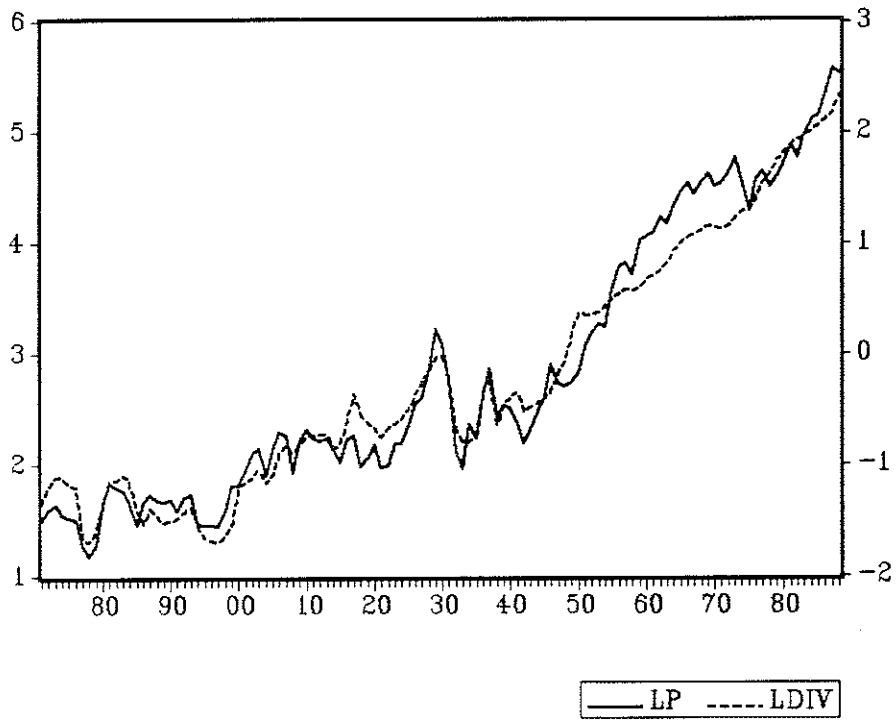
FIGURE 1

Log Price and Log Dividend

TABLE 1

SUMMARY STATISTICS

Sample	variable	mean	sd	$\rho 1$	$1/\sqrt{n}$
1872-1988	Cowles S&P data				
	P/D	21.28	5.15	0.83	0.09
	$\ln(D/D_{-1})$	0.03	0.13	0.25	
	$\ln(R)$	0.08	0.18	0.07	
1927-1989	CRSP VWNYSE data				
	P/D	23.14	5.79	0.81	0.13
	$\ln(D/D_{-1})$	0.04	0.14	-0.01	
	$\ln(R)$	0.10	0.20	0.10	
1926.2-1989.4	CRSP VWNYSE quarter data				
	P/D	95.52	25.27	0.75	0.06
	$\ln(D/D_{-1})$	0.01	0.21	-0.58	
	$\ln(R)$	0.02	0.11	-0.05	

INSERT TABLE 1

The price-dividend ratios for all of the series display substantial sample serial correlation (*0.7* to *0.8*). The dividend growth rate and the rate of return, in contrast, show very little serial correlation.[9]

Unit Root Tests for the Price-Dividend Ratio

Table 2 presents the results of the unit root tests. I ran standard augmented Dickey-Fuller (ADF) regressions on the change in the logarithm of the price-dividend ratio. The first column in Table 2 identifies the series. The second column contains the results of the unit root tests. First comes the estimated coefficient, b, on the lagged level of the log of the series. Under the null hypothesis that the sequence has a unit root, the coefficient equals zero, ie, H_φ: $1+\beta = 1$, so $\beta = 0$. Directly under the estimated coefficient is the augmented Dickey-Fuller "t" statistic, $\tau(ADF)$. Beneath the Dickey-Fuller test statistic is the ninety-five percent local-to-unity confidence interval, $\alpha(95\%)$, for one plus the estimated coefficient computed from Table A1 in Stock (1990).

INSERT TABLE 2

The data do not reject the unit root null hypothesis for any of the series at the *5%* level.[10] Only

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[9] Quarterly dividend growth does show substantial serial correlation. Since it doesn't show up in the annual data I presume this is high frequency dynamics related to the dividend seasonal.

[10] The critical values from Table 8.5.2 in Fuller (1976) are: 100 observations, $\tau(5\%) = -2.89$, $\tau(10\%) = -2.58$, 50 observations, $\tau(5\%) = -2.93$, $\tau(10\%) = -2.60$.

TABLE 2

DICKEY-FULLER REGRESSIONS

Δy_t	by_{t-1} $\tau(ADF)$ $((\alpha)(.95)_L, \alpha(.95)_U)$	$a_1\Delta y_{t-1}$ t	$a_2\Delta y_{t-2}$ t	$a_3\Delta y_{t-3}$ t	$a_4\Delta y_{t-4}$ t	c t	R^2	n	smpl
$\Delta\ln\left(\dfrac{SP}{SPD}\right)$	-0.171 -2.794 (0.78, 1.012)	0.031 0.303	-0.158 -1.568	0.082 0.835	0.044 0.453	0.522 2.819	0.13	113	1876-1988
$\Delta\ln\left(\dfrac{VWP}{VWD}\right)$	-0.160 -1.698 (0.807, 1.064)	0.056 0.378	-0.106 -0.733	-0.143 -1.039	0.008 0.054	0.500 1.710	0.13	59	1931-1989
$\Delta\ln\left(\dfrac{VWPQ}{VWDQ}\right)^{F1}$	-0.097 -2.467 (0.916, 1.009)	-0.272 3.865	0.121 -1.748	-0.111 -1.633	0.312 4.585	0.439 2.470	0.63	247	1928.2-1989.4

F1 I included eight lags in this regression. Table 2 only reports sample statistics on the first four lags.

the long annual time-series for the *S&P* rejects the unit root null at the *10%* level.[11]

The confidence intervals reveal substantial sampling variability, but confirm that the price-dividend series contains a large, if not unit, autoregressive root. The least lower bound of the confidence intervals is *0.78*. And while two of the three confidence intervals don't allow much room for explosive bubbles the greatest upper bound is *1.06*.

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[11] This is an extremely robust result. Using the level of the series instead of the log gives a rejection at exactly the *10%* level for the *S&P* and no others, and allowing for a time trend leads to no rejections at the *10%* level. One gets similar results with more lags or other test procedures, eg, see Froot and Obstfeld (1991) Table 2.

Section 3: Power

It is well known that unit root tests sometimes have low power against interesting stationary alternatives. If the augmented Dickey-Fuller test cannot reject the unit root null for a stationary sequence generated by a reasonable model, then the empirical results in Section 2 only reflect low power against the alternative of interest. This section examines the power of the test against specific alternatives with Monte Carlos.

The Monte Carlo Design

The Monte Carlo is designed to generate a power function for the ADF test against a specific alternative. I create 1000 samples of 100 and 60 observations. Then I calculate 1000 ADF τ statistics. The cumulative distribution of the τ sample statistics is the power function against that alternative.

The Stationary Alternatives

The critical assumption underlying the data generation process is that the discounted dividend growth factor,

$$lnD + ln(\frac{d}{d_{-1}}) \equiv ln\delta \sim N(\mu, \sigma(s)),$$

is log-normally distributed. This is a common specification in empirical and theoretical work. Most empirical work on bubbles and excess volatility assumes a constant discount factor and lognormally distributed dividends, eg see DeJong and Whiteman (1991), LeRoy and Parke (1990), Mankiw, Romer, and Shapiro (1991), and Froot and Obstfeld (1991). I assume the

discount factor also is log-normally distributed.

I consider two stochastic environments. The first is a "reduced form" in which I simply draw realizations of the log of δ from a normal distribution. The second is a more structural approach. I draw realizations of the log of the discount factor and the dividend growth factor from a joint normal distribution and combine them to form the log of delta.

Given a realization of δ I can calculate the fundamental value-dividend ratio using equation 5.[1]

Reduced Form Specification

This specification explores the relationship between the (first-order) serial correlation in the driving process and the serial correlation in the fundamental value-dividend ratio. Recall that the first-order sample serial correlation in the price-dividend ratio (Table 1) is approximately *0.8*. But the serial correlation in dividend growth or returns is quite low, *0.25* or less.

The specification is:[2]

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[1] The appendix gives the details.

[2] I "calibrated" the distribution to make it comparable to constant discount factor models. I adjusted the variance of u so that the variance of $ln\delta$ was roughly the variance of dividend growth and I adjusted α so that the average fundamental value-dividend ratio was about the sample average price-dividend ratio. Increasing the variance of u does not alter the results.

$$\ln\delta_t = \alpha + \rho\ln\delta_{t-1} + u_t \; ; \quad \rho \in \{0.2,\ 0.5,\ 0.8\}. \tag{8}$$

It turns out that the serial correlation in the fundamental value-dividend ratio is tightly linked to the serial correlation in the driving process. First-order serial correlation of *0.2* in the log of the discounted dividend growth process generates first-order serial correlation of *0.18* in the fundamental value-dividend ratio. A ρ of *0.5* in the input series generates serial correlation of *0.47* in the output series. And ρ of *0.8* generates serial correlation of *0.74* in the fundamental value-dividend ratio.[3]

Figures 2 and 3 show the power functions for 100 observations and 60 observations. Table 3 shows the Dickey-Fuller critical values under the null and the power of the test against the alternatives at the critical values.

Insert Figures 2&3

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[3] It makes almost no difference whether one calculates the first-order serial correlation coefficient on the level of the fundamental value-dividend ratio or the log of the ratio (the largest difference is *0.74* for the level versus *0.76* for the log.) The same is true for the actual data.

FIGURE 2
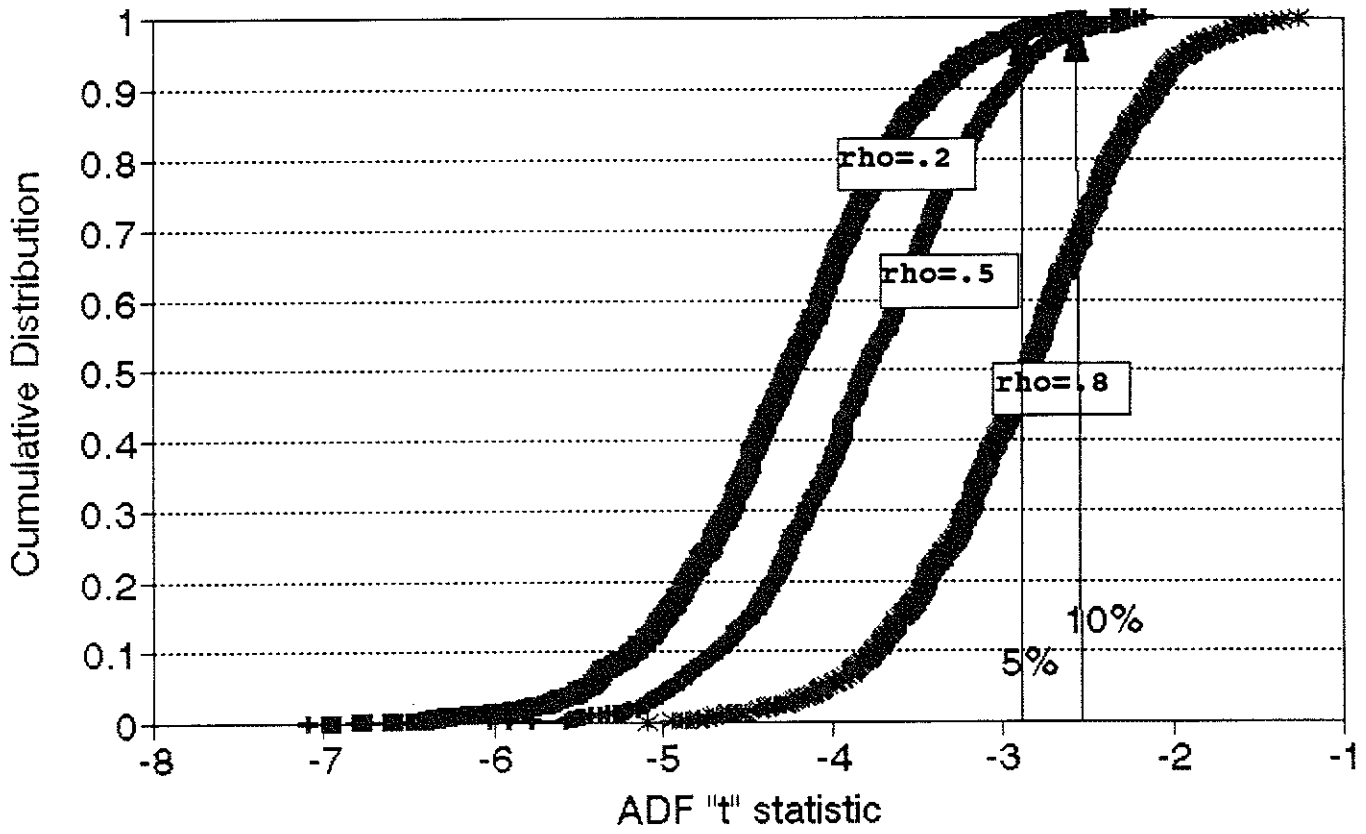
Power of ADF Test
100 Observations
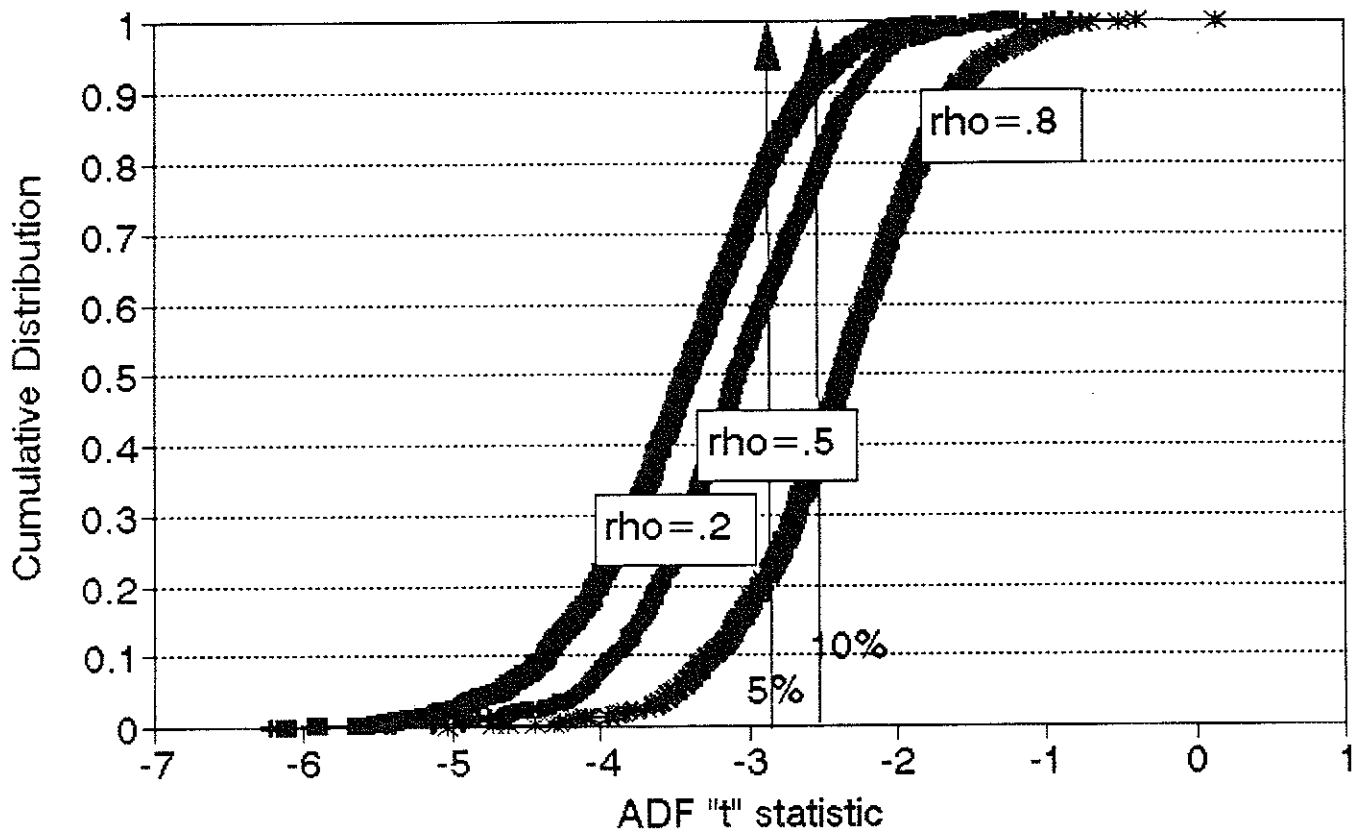
FIGURE 3

Power of ADF Test
60 Observations

Table 3

		100	Obvs	60	Obvs
Critical		5%	10%	5%	10%
Values	τ	-2.89	-2.58	-2.93	-2.60
Power	$\rho(0.2)$	0.98	0.995	0.77	0.90
	$\rho(0.5)$	0.93	0.98	0.58	0.75
	$\rho(0.8)$	0.48	0.67	0.19	0.34

The ADF test has excellent power until the serial correlation in the log of δ exceeds 0.5.

The Monte Carlo results from this specification essentially rule out constant discount factor expected present value models with log-normally distributed dividend growth factors as an explanation of the large low frequency component in the price-dividend ratio. The sample serial correlation in dividend growth rates is simply too small to generate enough serial correlation in the fundamental value-dividend ratio for the ADF test to have low power. If the discounted dividend growth rate is normally distributed, then high serial correlation in the fundamental value-dividend ratio must come from serial correlation in the discount factor.

Bivariate Process

To explicitly analyze the stochastic discount factor case I consider an observable bivariate

representation for the components of δ. I substituted realized returns for the unobservable

discount rate and parameterized the data generating process using the estimates of the bivariate

system:

Table 4

Dep Var	lnR_{-1}	lnR_{-2}	lnd_{-1}/d_{-2}	Const	SERegr	R^2
lnR_{-1}	-0.017 (0.121)	-0.271 (0.094)	0.217 (0.168)	0.099 (0.019)	0.171	0.076
lnd/d_{-1}	0.085 (0.090)	-0.071 (0.071)	0.198 (0.125)	0.023 (0.014)	0.127	0.080

from the *Cowles S&P* data. I chose the variance-covariance matrix of the errors to match the

residual variance-covariance matrix (the correlation between the residuals is *0.65*).[4]

Figure 4 shows the power function for this specification.

Insert Figure 4

The ADF test rejects the null at the *5%* level *98%* of the time with *100* observations and *78%*

of the time with only *60* observations.

The specific Monte Carlo results depend on the parameterization I chose using returns as a proxy

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[4] I experimented with higher-order vector autoregressions. Adding more lags does not produce any more significant coefficients. And the impulse-response functions all look alike so I chose this fairly parsimonious representation.

FIGURE 4

Power of ADF Test
ln(D) = -ln(R)
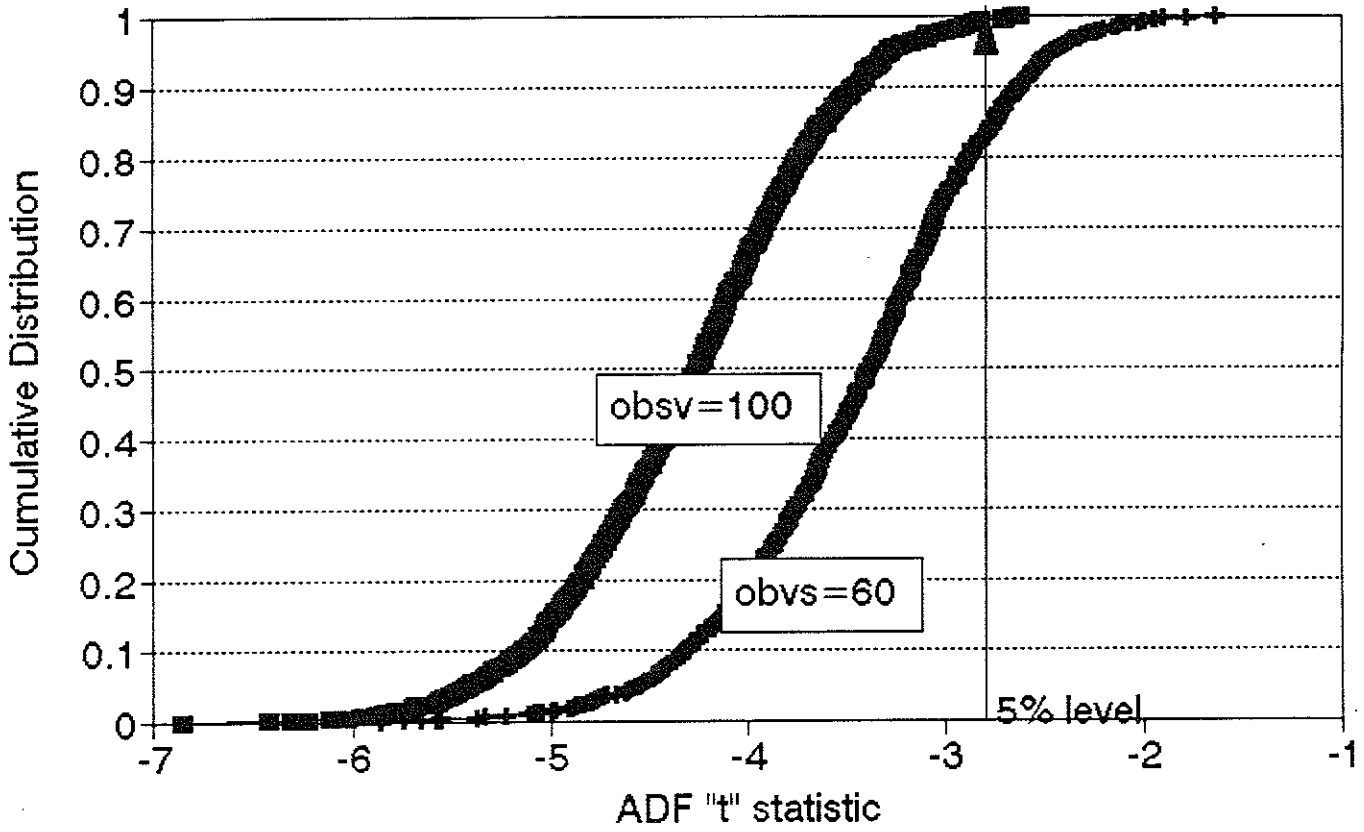
for the discount rate. However, the augmented Dickey-Fuller test will have good power against any alternative generated with a log-normally distributed discount factor that satisfies the rational expectations pricing restriction. The reason is that the serial correlation of the driving input series basically determines the power. And the serial correlation in the unobservable discount rate must match the serial correlation in observable returns.

Let,

$$\xi_{t+1} = D_{t+1}R_{t+1},$$

(9)

represent the realization of the product of the random discount factor times the return. The equilibrium pricing restriction, equation 7, implies that the conditional expectation of random variable ξ is one and that it is serially independent and independent of any element in the information set. Furthermore, the log of ξ also is serially independent and independent of any element of the information set. Therefore, the log of the observable return factor sequence reveals,

$$lnD_{t+1} - ln\xi_{t+1} = lnR_{t+1},$$

the serial correlation in the log of the unobservable discount factor sequence.

These results essentially rule out expected present value models with log-normally distributed discount and dividend growth factors as an explanation of the large low frequency component in the price-dividend ratio.

Conclusion

This paper presented a robust no rational bubble restriction and tests of the restriction. But,

..whether the actual volatility of equity returns is due to time variation in the rational equity risk premium or to bubbles, fads and market inefficiencies is (*still*) an open issue.[1]

The paper narrows the search for an efficient markets model of the data by presenting strong evidence that the popular specification of the fundamental value driven by log-normally distributed dividends and discount factors will not explain the data. Observed dividend growth rates and returns have low serial correlation which will not generate a large autoregressive root in the fundamental value-dividend ratio if the discounted dividend growth factor is log-normally distributed.

The log-normal distribution assumption is a simple tractable specification. Non-normal specifications for the driving dividend and discount factor process can introduce temporal dependence that is not detected with linear methods. These processes could also generate more persistence in fundamental value-dividend ratios.

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[1] Flood and Hodrick (1990) p98, I added "still" to their sentence.

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Appendix: Monte Carlos

No closed-form solution exists to the general expected present value equation,

$$\frac{F_t}{d_t} \equiv f_t = E_t[D_{t+1}\frac{d_{t+1}}{d_t}(f_{t+1} + 1)] \equiv \lim_{J \to \infty} \sum_{j=1}^{J} E_t \prod_{i=1}^{j} \delta_{t+i} \quad \text{(A.1)}$$

$$\text{where, } \delta_{t+i} \equiv D_{t+i}\frac{d_{t+i}}{d_{t-1+i}}$$

Restricting the driving variable, δ, to a log-normal distribution does not give a closed-form solution; but, it gives an expression that is easily evaluated on the computer.

Define,

$$w_{t+i} \equiv \ln\delta_{t+i}$$

$$w(k) \equiv \sum_{i=1}^{k} w_{t+i} \ .$$

Now since δ is log-normally distributed A.1 can be rewritten as:

$$f_t = \sum_{j=1}^{\infty} e^{(E_t w(j) + \frac{1}{2}var_t w(j))} \quad \text{(A.2)}$$

Evaluating A.2 on the computer requires calculating the conditional expectations and variances of $w(j)$ and a rule to truncate the infinite sum.

Consider the bivariate model where the log of the discount and dividend growth factors are

jointly normally distributed. I used the autoregressive representation,[1]

$$z_{t+1} = Az_t + b + e_{t+1},$$ (A.3)

$$where \quad z_{t+1} = \begin{bmatrix} lnD_{t+1} \\ lnD_t \\ ln(\frac{d_{t+1}}{d_t}) \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$e_{t+1} = \begin{bmatrix} e(D) \\ 0 \\ e(d/d) \end{bmatrix} \sim N \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \psi \end{pmatrix}$$ (A.4)

Evaluating the first two moments of z gives,

$$E_z(k) \equiv \sum_{j=1}^{k} E_t z_{t+j} = \sum_{j=1}^{k} A^j z_t + \sum_{j=1}^{k} A^{j-1} b$$ (A.5)

and,

$$var_z(k) \equiv var_t \sum_{j=1}^{k} z_{t+j}$$
$$= \psi + (I+A)\psi(I+A)' + \cdots (I+A+\cdots A^k)\psi(I+A+\cdots A^k)'$$

Now the selector vector,

---

[1] The reduced-form specification is a special case of this representation which can be generalized to represent any finite-order linear ARMA model.

$$\iota = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} ,$$

combines the elements of $z(k)$ to form, $E_\iota w(k) = \iota' E_\iota z(k)$ and $var_\iota w(k) = \iota' var_\iota z(k) \iota$.

Finally substituting the conditional means and variances in the present value equation A.2 gives the fundamental value-dividend ratio. I truncate the sum when $exp(E_\iota w(k) + \frac{1}{2} var_\iota w(k)) < .001$. On average the truncation occurs when the final term amounts to less than .005% of the fundamental value-dividend ratio.

These calculations produce a single observation. The sequence is to make a draw from the distribution in A.4. Then calculate conditional means and variances of the vector process as in A.5 and create the conditional means and variances of the driving variable $w(k)$ using the selector vector. Continue creating means and variances until the truncation limit is reached. Then the sum in A.2 gives the fundamental value-dividend ratio for this realization. One hundred repetitions of this sequence create a sample. From a sample I calculate a set of sample statistics. One thousand sample augmented Dickey-Fuller statistics give the power functions in Section 3.

The program is written in GAUSS386. Generating one thousand sets of sample statistics takes about ten hours on a 486/25 machine.

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