Title

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Authors

Anderson, Steven T
Friedman, Daniel
Oprea, Ryan

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Steven T. Anderson† Daniel Friedman‡ Ryan Oprea§

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Abstract

Several investors face an irreversible investment opportunity whose value V is governed by Brownian motion with upward drift and random expiration. The first investor i to seize the opportunity before expiration receives the current V less a privately known cost C_i; the other investors receive nothing. We characterize Bayesian Nash Equilibrium (BNE) for this game, extending previously known results.

We also report a laboratory experiment with 72 subjects randomly matched into 600 triopolies. As predicted in BNE, subjects in triopolies invested at lower values than in monopolies, changes in Brownian parameters significantly altered investment values in monopoly but not in triopoly; and the lowest cost investor in a triopoly usually preempted the others. Evidence was mixed on other BNE predictions, e.g., whether higher cost brings smaller markups. Overall, subjects' earnings came rather close to the BNE prediction.

Keywords: Preemption, Incomplete Information, Irreversible Investment, Laboratory Experiment.

JEL codes: C73, C92, D82, G13

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†Minerals Information Team, U.S. Geological Survey, Reston, VA 20192.*sanderson@usgs.gov*

‡Economics Department, University of California, Santa Cruz, CA, 95064. *dan@ucsc.edu*

§Economics Department, University of California, Santa Cruz, CA, 95064. *roprea@ucsc.edu*

1 Introduction

Early humans were doubtless familiar with first mover advantages: the first group to harvest a grove of ripening figs, or the first to hunt a herd of antelope, ate better than those who moved later. They were doubtless also aware of the fact that the timing of the harvest had an effect on the quality of the harvest; figs which are harvested now may grow riper if left awhile longer, or may be eaten by worms. Such tradeoffs persist to the present day, among high tech firms entering a new product niche (for example, internet-savvy cell phones or fuel-cell powered cars), among retailers opening big box stores, and among academic researchers investigating a new hot topic. The intervening years offer innumerable other examples in which (a) the value of some opportunity fluctuates over time, (b) several different individuals (or teams) choose the time at which to seize it, and (c) the first to do so gains a valuable advantage.

In this paper we study such situations both theoretically and empirically. We formalize them as preemption games, using standard simplifications to put the strategic issues into sharp focus. Section 2 lays out the assumptions: the opportunity is available to a known number $n + 1$ of investors, each of whom has a privately known avoidable cost of investing, and the first mover preempts the entire value of the opportunity, which evolves according to Brownian motion with known parameters. These assumptions highlight the tension between waiting for the opportunity to ripen and moving quickly to be first.

The theoretical results in section 2 build on earlier work. Dixit and Pindyck (1994, henceforth denoted DP94) present a duopoly model of irreversible investment with a first mover advantage. They describe their model as “a particularly simple example,” and remark that more general oligopoly models “...present formidable difficulties” (p. 309ff). Lambrecht and Perraudin (2003, henceforth LP03) nevertheless are able to characterize the Bayesian Nash equilibrium (BNE) of the duopoly preemption game, given certain restrictions on the cost distribution.

Our theoretical contribution appears in Section 2. We characterize the Bayesian Nash equilibrium for an arbitrary number of players. The derivation, which builds on Anderson (2003) as well as LP03, shows that investors' BNE strategies take the form of a threshold value: at any higher value, the opportunity is seized immediately. We obtain formulas for BNE threshold functions for arbitrary n and connect them to known results from the theory of first price auctions and the theory of real options. We also characterize simpler constrained equilibria which are relevant to our experimental findings.

Section 3 describes a new experiment informed by the theory, using software created expressly for the purpose. It presents the main treatments—Competition (tripololy) vs Monopoly, and High vs Low Brownian parameters—and obtains five testable hypotheses. Section 4 explains other aspects of the laboratory implementation.

Section 5 presents the results. The first three hypotheses fare quite well: the triopoly market structure leads to much lower investment values than the monopoly structure; the Brownian parameters have a major impact in the predicted direction in Monopoly but (as predicted in BNE) not in Competition; and the lowest cost investor indeed is far more likely to preempt than her rivals. Evidence is mixed on the other two hypotheses. Subjects may use simpler strategies than in BNE—for example, the markup of threshold over cost may be constant rather than decreasing in cost. Subjects appear to pick strategies close to the (constrained) optimal constant markup, and to lose very little payoff relative to the empirical best response.

Following a concluding discussion, Appendix A collects mathematical details, Appendix B discusses the lesser-known econometric techniques, Appendix C reports robustness tests and alternative specifications, and Appendix D contains the instructions to subjects. A companion paper, Oprea, Friedman and Anderson (2007, henceforth OFA07) focuses on the monopoly ($n = 0$) case. It reports a related laboratory experiment and contains more details on the software and econometric techniques common to both experiments.

2 Theoretical Results

This section analyzes two situations. In the first, called monopoly, a single investor i has sole access to an investment opportunity. In the second, called competition, two or more investors with private information concerning their own costs have access to the same opportunity, and the first to seize it renders it unavailable to the others.

2.1 Monopoly

An investor i with discount rate $\rho > 0$ can launch a project whenever she chooses by sinking a given cost $C_i > 0$. The present value V of the project follows a geometric Brownian motion with drift parameter $\alpha < \rho$ and volatility parameter $\sigma > 0$:

$$dV = \alpha V dt + \sigma V dz, \tag{1}$$

where z is the standard Wiener process. That is, the value follows a continuous time random walk in which the appreciation rate has mean α and standard deviation σ per unit time. At times $t \geq 0$ prior to launching the project, the investor observes $V(t)$ (and previous values $V(s)$ for $0 \leq s \leq t$). If she invests at time t, then she obtains payoff $[V(t) - C_i]\, e^{-\rho t}$. The project is irreversible and generates no other payoffs. Thus, the task is to choose the investment time so as to maximize the expected payoff.

The solution goes back to Henry (1974) and has been widely known since McDonald and Siegel (1986); see Chapter 5 of DP94 or OFA07 for more recent expositions. It turns out that the optimal policy takes the form: wait until $V(t)$ hits the threshold

$$V_M^*(C_i) = (1+w)C_i, \tag{2}$$

then launch immediately. Note that the threshold is proportional to cost, and that the wait option premium $w \geq 0$ is an algebraic function of the volatility, drift and discount parameters σ, α and ρ. Specifically,

$$w = \frac{1}{\beta - 1}, \;\; \texttt{where} \;\; \beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{3}$$

2.2 Competition

Now consider the case that each investor has $n \geq 1$ rivals. All investors $i = 1, 2, ..., n+1$ have access to the same investment opportunity, whose value V again evolves according to geometric Brownian motion (1). Each investor i again knows her own cost C_i, but doesn't know the other investors' costs C_j, $j \neq i$. She regards them as drawn independently from a cumulative distribution function, $H(C)$, with a positive continuous density function $h(C)$ on support $[C_L, C_U]$. The first investor to launch obtains payoff $[V(t) - C_i]e^{-\rho t}$ and the other investors obtain zero payoff. All this is common knowledge.

As shown in Appendix A.2, there is a unique symmetric Bayesian Nash Equilibrium (BNE) for the game. It is characterized by a monotone increasing function $V^*(C_i)$ that maps the investor's cost into a threshold value, above which she immediately invests. Here we sketch the derivation and offer some intuition.

The objective function takes the form:

$$F(m|C_i, n) = [V^*(m) - C_i] \left[\frac{V}{V^*(m)}\right]^\beta \left[\frac{1 - H(m)}{1 - H(\hat{C})}\right]^n, \tag{4}$$

where m, V and $\hat{C}$ are defined as follows. The natural choice variable is the threshold value, but since $V^*(C_i)$ is increasing, we can more conveniently write the choice variable as $m \in [C_L, C_U]$, interpreted as the cost-type that the investor chooses as her potential "masquerade".

The first factor in the objective function (4) is simply the profit $[V^*(m) - C_i]$ obtained at the time of successful investment. The second factor, $\left[\frac{V}{V^*(m)}\right]^\beta$, accounts for the time cost of delaying investment and the expiration hazard, given that V is the current value of the investment project. Appendix A.1 shows that the monopolist's value function consists of only these first two factors.

The third and final factor, $\left[\frac{1-H(m)}{1-H(\hat{C})}\right]^n$, is the probability that i has the lowest investment cost, conditioned on the fact that none of the n rivals has already invested. That conditioning is reflected in the denominator, where $\hat{C}$ is the cost corresponding to $\hat{V} \geq V$, the "highest peak" so far achieved by the random walk.

The key step in obtaining the BNE comes from the best response property that investor i maximizes (4) at $m = C_i$. It is straightforward to show that the associated first-order condition can be expressed as the following ordinary differential equation (ODE):

$$V^{*\prime}(C_i) = \frac{[V^*(C_i) - C_i]\, V^*(C_i)}{V^*(C_i) - \beta\, [V^*(C_i) - C_i]} \times \frac{nh(C_i)}{[1 - H(C_i)]} \texttt{ subject to } V^*(C_U) = C_U. \tag{5}$$

This boundary-value problem has a unique solution that characterizes the symmetric BNE investment timing rule for our preemption game. Apparently that solution, like those of most non-linear ODE's, is not expressible in terms of standard functions. However, given a specific cost distribution H and specific values of n and β, it can be computed numerically by direct integration from the boundary value.

Theorem 1. *Let the cumulative distribution function H have a positive, continuous density h on its support $[C_L, C_U]$. Let it be common knowledge among all investors $i = 1, ..., n+1$ that i's cost C_i is drawn independently from H and that the realization is observed only by investor i. Then (a) boundary value problem (5) has a unique solution V^* on $[C_L, C_U]$, and (b) the premption game has a symmetric Bayesian-Nash equilibrium in which each investor i's investment threshold value is V^* evaluated at the realized cost C_i.*

All proofs appear in the appendix.

Lambrecht and Perraudin (2003) use slightly different methods to derive an ODE that is consistent with Equation (5) for the special case that (a) there are only two investors (duopoly), and (b) H satisfies a restriction on the modified hazard rate $\frac{C_i[h(C_i)]}{[1-H(C_i)]}$. Their methods rule out asymmetric

BNE. Our method, adapted from Anderson (2003), considers only symmetric BNE, but it does have several compensating advantages. It covers $n+1 > 2$ investors, allows arbitrary hazard rates, and leads to a more streamlined and unified analysis of useful special cases.

2.3 Special Cases, Intuition, and Bounds

Consider again the special case $n = 0$, monopoly. The last factor in objective function (4) then disappears. At first it seems that the RHS of the ODE (5) is identically zero, but Appendix A.2 shows that the denominator $V^*(C_i) - \beta[V^*(C_i) - C_i] = 0$, and that this last equation characterizes the monopoly solution.

The Appendix also shows that, at any realized cost C_i, the monopoly solution $V_M^*(C_i)$ is an upper bound on the competitive solution $V^*(C_i)$. The intuition is simple but revealing. In competition, an investor increases her threshold up to the point that the greater profit when not preempted just balances the greater threat of preemption by other investors or by "Nature". The threat of premption by other investors disappears when $n = 0$, and so the monopolist finds the balance at a higher threshold.

The special case $\beta = 0$ is particularly instructive. In this case, the "net discount rate" is zero, i.e., there is no preemption threat from "Nature." (By Equation (3), this special case arises when $\rho = 0$, thus eliminating the expiration hazard and the time cost of delay.) Then the middle factor of the objective function (4) disappears, so the expression simplifies to

$$F\left(m|C_i, n\right) = \left[V^*(m) - C_i\right]\left[\frac{1 - H(m)}{1 - H\left(\hat{C}\right)}\right]^n. \tag{6}$$

The ODE (5) then collapses to the well-known ODE for bid functions in first price auctions:

$$V^{*\prime}\left(C_i\right) = \left[V^*\left(C_i\right) - C_i\right] \times \frac{nh\left(C_i\right)}{\left[1 - H\left(C_i\right)\right]} \quad \texttt{subject to} \quad V^*\left(C_U\right) = C_U. \tag{7}$$

Thus, with $\beta = 0$, we obtain the isomorphism noted by Milgrom and Weber (1985), among others, between threshold values in preemption games and bids in auctions.

The solution $\bar{V}(C_i)$ to (7) also is an upper bound for the solution to the more general ODE (5). The intuition is again quite simple. When $\beta = 0$, there is no threat of preemption by "Nature", so the investor finds the balance between greed (a larger profit margin) and fear (of preemption) at a higher threshold than when $\beta > 0$.

The BNE threshold $V^*(C_i)$ also has a natural lower bound, given by the classic Marshallian ($NPV = 0$) investment rule $V^0(C_i) = C_i$. Note that (5) and (7) impose the condition that this lower bound is binding at the highest possible cost C_U. The intuition is compelling. An investor with the highest possible cost faces Bertrand competition: even a single rival will find it profitable to undercut any positive markup she might seek.

These results are summarized in the following

Theorem 2. *Under the hypotheses of Theorem 1, the BNE threshold $V^*(C_i)$ is bounded below by the Marshallian threshold function $V^0(C_i) = C_i$, and is bounded above by the monopoly threshold $V_M^*(C_i)$. It is also bounded above by the solution $\bar{V}(C_i)$ to (7) and is tangent to $\bar{V}(C_i)$ at the upper endpoint C_U.*

Our experiment employs the uniform distribution H on $[C_L, C_U]$. In this case, (5) reduces to

$$V^{*\prime}(C_i) = \frac{[V^*(C_i) - C_i]\,V^*(C_i)}{V^*(C_i) - \beta\,[V^*(C_i) - C_i]} \times \frac{n}{C_U - C_i} \quad \texttt{subject to } V^*(C_U) = C_U, \tag{8}$$

and if we again take $\beta = 0$ then we get the well-known ODE from Vickrey (1961)

$$V^{*\prime}(C_i) = [V^*(C_i) - C_i] \times \frac{n}{C_U - C_i} \quad \texttt{subject to } V^*(C_U) = C_U, \tag{9}$$

with analytic solution

$$V^{**}(C_i) = \frac{nC_i + C_U}{n+1}. \tag{10}$$

Corollary 1. *Let H be the uniform distribution on $[C_L, C_U]$, let n be an integer ≥ 1, and let $V^{**}(C_i)$ be given by equation (10). For $\beta > 0$, the Bayesian-Nash equilibrium threshold $V^*(C_i)$, the solution to (8) on $[C_L, C_U]$, is bounded above by V^{**}, tangent to V^{**} at C_U, and bounded below by the diagonal function $V^0(C_i) = C_i$.*

Thus when the cost distribution is uniform, an upper bound (and a good approximation) of the BNE threshold V^* is the function

$$V^U(C_i) = \min\left\{\left[\frac{nC_i + C_U}{(n+1)}\right]; V_M^*\right\}, \tag{11}$$

where the monopolist threshold V_M^* is given by equations (2) and (3). The monopolist threshold binds in (11) at lower cost realizations for high values of β.

2.4 Constant Markup Equilibrium

The BNE strategy is not as complex as some sorts of feasible strategies. For example, it is not contingent on time elapsed, nor on the current value of the Brownian motion (as long as it is below the threshold!), nor on the history observed so far. However, the BNE strategy relies on a non-linear function of realized cost. It may be too complex for human investors to discover.

Therefore it may be worth analyzing a restriction of the preemption game to simple 1-dimensional strategy spaces. Here each investor chooses a constant additive markup, i.e., a profit aspiriation $k \geq 0$, and sets the threshold $V^R(C_i, k) = C_i + k$. In all other respects, the game is the same as before.

A symmetric Nash equilibrium (NE)[1] of the restricted game is a markup k^* that is a best response to itself. To characterize it, suppose that other investors choose markup $k > 0$ and let investor i consider possible deviations $k_i = k + x$ for any real number x. Her objective function is the expected payoff

$$E_H F^R(x|k,n) = (k+x)\int_{C_L}^{C_U}\left[\frac{C_L}{C+k+x}\right]^{\beta}[1-H(C+x)]^n\,h(C)dC. \tag{12}$$

It is logically straightforward but a bit messy to obtain the NE. Take the derivative of Equation (12) with respect to x and evaluate it at $x = 0$. As explained in the Appendix, any symmetric NE k^* must be a root of the resulting expression, and can be found by Newton's method.

Again, we are especially interested in the case of uniform distributions H and triopoly $(n = 2)$. The results are summarized in the following

Theorem 3. *Let H be the uniform distribution on $[C_L, C_U]$. Then for $n + 1 = 3$ total investors and every $\beta \in [0, \infty)$, there is a unique symmetric Nash equilibrium $k^*(\beta) > 0$ of the restricted preemption game.*

2.5 Numerical Example

Figure 1 presents numerical solutions to shows the numerical solution V^* to the boundary value problem equation (8) for triopoly $(n = 2)$, given the uniform cost distribution on $[C_L, C_U] = [50, 80]$, for $\beta = 2.25$ and $\beta = 3.00$, corresponding respectively to monopoly premiums $w = 0.8$ and $w = 0.5$. Because of the monopoly premiums induced in each of these cases, we refer in the figures to $\beta = 2.25$ as High and $\beta = 3.00$ as Low parameters. Figure 1 (a) shows these numerical solutions with the corresponding monopoly thresholds V_M^*.

Figure 1 (b) shows more detail on the BNE threshold functions and presents the Marshallian (NPV=0) and Vickrey rule for comparison. One can see that the solutions for $\beta = 2.25, 3.00$ lie

[1] We drop "Bayesian" since the restricted strategies are not contingent on cost type.

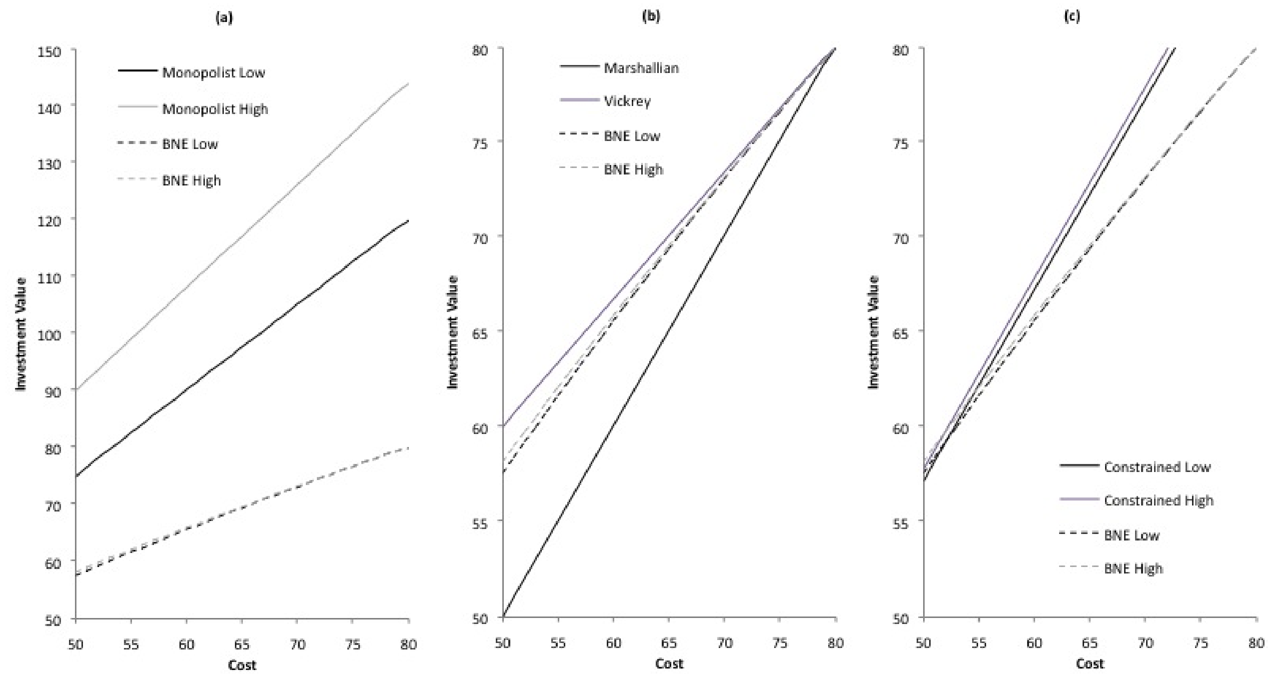

Figure 1: Numerical values for a parameter set with $\beta = 2.25$ (called High because it induces a relatively high option premium) and a parameter set with $\beta = 3.00$ (called Low).

close together and that, as shown in Theorem 2, both are tangent to the Vickrey bound at the upper endpoint. The Marshallian rule serves as a lower bound on the BNE. Because the Vickrey rule is everywhere smaller than V_M^* under both parameter sets, it serves as an upper bound to the BNE.

The mappings of the numerical solutions to Equation (42) for $\beta = 2.25$ and $\beta = 3.00$ are added in Figure 1 (c). The Nash equilibrium markups for the restricted (1-dimensional strategy set) game are $k^* = 7.81$ and $k^* = 7.15$ respectively for $\beta = 2.25$ and $\beta = 3.00$. Note that these are constant markup rules in that they are parallel to the zero profit Marshallian threshold. Figure 1 (c) shows that the (restricted) equilibrium constant markups are fairly close to the (unrestricted) BNE markups at the low end of the cost range. However, the constant markups are higher than the BNE markups in the upper three quarters of the cost range, and the divergence increases in cost.

2.6 Discrete Approximations

Brownian motion is an idealization. As in OFA07, our experiment uses a close binomial approximation of the continuous time process. Specifically, it has a fixed interval $\Delta t = 0.00\bar{3}$ minutes (i.e., 200 milliseconds) for each discrete step of the value path, and three binomial parameters:

- the step size $h > 0$ of the proportional change in value, i.e., the current value V becomes either $(1+h)V$ or $(1-h)V$ at the next step;
- the uptick probability $p \in (0,1)$, i.e., the probability that the next step is to $(1+h)V$ rather than to $(1-h)V$; and
- the expiration probability $q \in (0,1)$, i.e., the probability that the current step is the last, and the opportunity disappears.

The deviation of the uptick probability p from 0.5, times the distance $2h$ between an uptick and downtick, corresponds to the Brownian drift rate α:

$$\alpha = \lim_{\Delta t \to 0} \frac{(2p-1)\,h}{\Delta t}. \tag{13}$$

The Brownian volatility σ comes mainly from the stepsize h but when p differs from 0.5 we must also account for binomial variance $p\,(1-p)$. The exact expression is

$$\sigma^2 = \lim_{\Delta t \to 0} \frac{4p\,(1-p)\,h^2}{\Delta t}. \tag{14}$$

OFA07 explains in some detail why the Brownian discount is given by

$$\rho = \frac{-\ln(1-q)}{\Delta t}. \tag{15}$$

3 Treatments and Hypotheses

Two treatment variables allow us to test the major predictions of the model. The first treatment involves the binomial parameters that govern the value process. We fix the time step at $\Delta t = 0.00\bar{3}$ (in minutes) and the step size at $h = 0.03$. The Low parameter vector is $p = 0.524$ and $q = 0.007$, corresponding to option premium $w \approx 0.5$ and $\beta \approx 3.0$. The High parameter vector is $p = 0.513$ and $q = 0.003$, corresponding to $w \approx 0.8$ and $\beta \approx 2.25$. In OFA07, the same two parameter configurations were labeled Medium A and High respectively. These configurations differ considerably from each other, yet both yield value paths "in the money" often enough, and jagged enough, to maintain subjects' interest. Note that these two parameter sets generate predictions which correspond with the "Low" and "High" cases examined in the numerical examples above.

The second treatment variable is market structure. In the Monopoly treatment, subjects made investment decisions with no rivals ($n = 0$) and therefore no risk of preemption. In the Competition treatment, subjects competed in triopolies ($n = 2$). Each period the subjects were randomly reassigned to one of three or four separate markets, each with three investors.

Each period each subject's cost was drawn independently from $U[50, 80]$, the uniform distribution with support [50, 80]. Each session began with a 10 period Monopoly block, continued with 25 periods of Competition, and ended with MonopolyII, another 10 period Monopoly block. The data analysis will focus on the Monopoly and Competition blocks; see Appendix A3 for an analysis of MonopolyII data, which generally leads to parallel (but somewhat more diffuse) results.

Figure 2 plots the theoretical benchmarks as markups, i.e., as optimal threshold value less cost. Dotted lines represent Monopoly markup, $V_M^*(C_i) - C_i$, and solid lines are BNE markups in Competition, $V^*(C_i) - C_i$ from the numerical solution to equation (8) with $n = 2$ (triopoly) and H from $U[50, 80]$, for $\beta = 3.0$ and 2.25 (Low and High parameters).

Figure 2 shows that, for either parameter vector, the Competitive markups are everywhere much lower than the Monopoly markups. Our first hypothesis is that the markups observed in the experiment will have the same ordering. We express the hypothesis directly in terms of investment value, i.e., the observed value at which an investor chooses to launch a project.

Hypothesis 1. *(Structure) As compared to Monopoly, the Competition treatment significantly reduces investment values.*

Another striking aspect of Figure 2 is that Monopoly line for the High parameter vector is far above the corresponding Low line, while under Competition the two lines are virtually identical. The second hypothesis is that these theoretical orderings will be seen in the experimental data:

Hypothesis 2. *(Parameters) Investment values in the High/Monopoly data significantly exceed those in the Low/Monopoly data, but the data have the same distribution in High/Competition as in Low/Competition.*

Recall that the symmetric BNE strategy $V^*(C_i)$ is increasing in cost C_i. A direct implication is that the investment opportunity is always seized by the lowest cost investor. Allowing for some behavioral noise, we obtain the following cost sorting hypothesis:

Hypothesis 3. *(Efficiency) Under Competition, the most efficient (lowest cost) firm is the one most likely to preempt the others.*

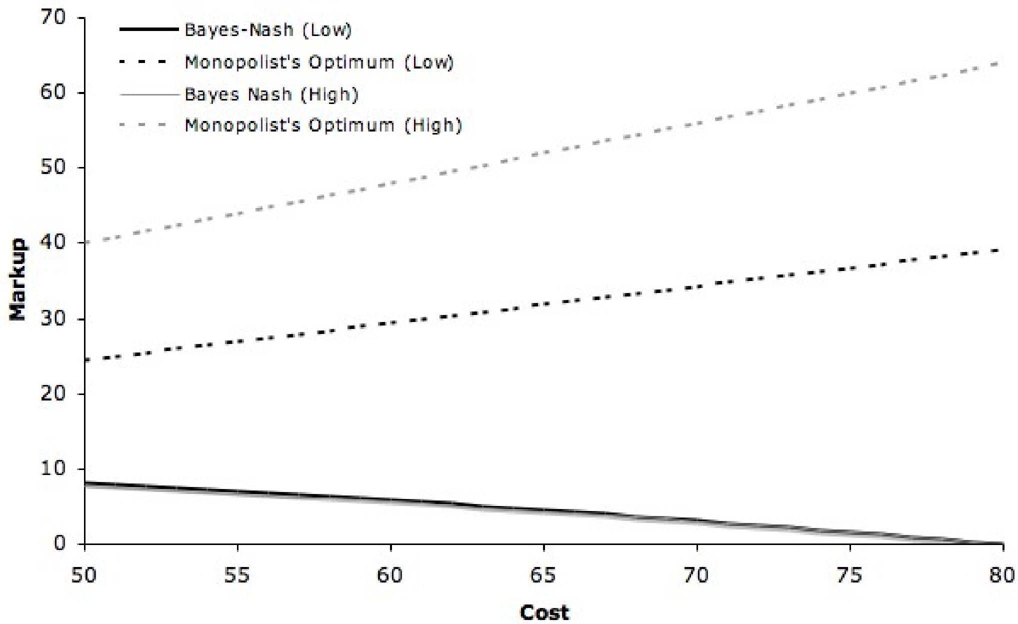

Figure 2: Optimal investment values and numerically estimated Bayesian-Nash strategy functions for each parameter set shown in Cost-Markup space.

A final observation from Figure 2 is that the markup under Competition is decreasing in costs. The reason is that the slope $V^{*\prime}(C_i) < 1$; indeed, as can be seen from equation (11) and the surrounding discussion, that slope is about 2/3, so the optimal markup slope in Competition is about -1/3.

Hypothesis 4. *(Monotonicity) Under Competition, observed markups are decreasing in cost.*

These hypotheses assume rapid convergence of behavior towards optimum or Bayesian Nash Equilibrium. That assumption implies something that we can easily test: that the observed relationship between cost and investment value is time-invariant.

Hypothesis 5. *(Equilibrium) In each treatment, the observed relationship between cost and investment is stable over time.*

A possible alternative hypothesis is that observed behavior gradually adjusts so that the last few observations are significantly closer to prediction than are earlier observations.

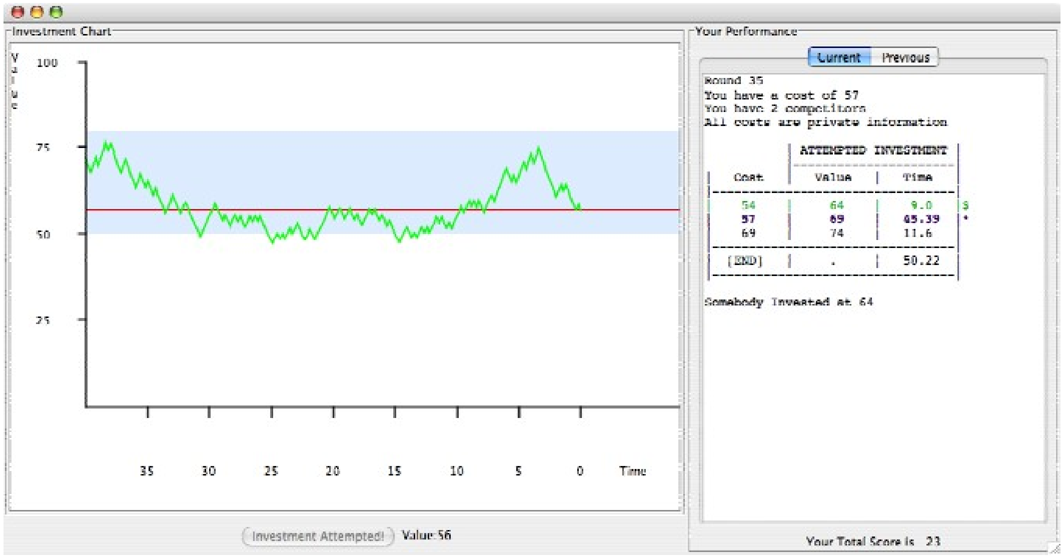

Figure 3: Subject screen under Competition at the end of a round.

4 Implementation

Experiments were conducted using Investment Timing, the same customized software used in OFA07. Figure 3 shows the user interface. The lighly shaded blue band indicates the cost range, $[50, 80]$, which was held constant throughout the session and announced publicly. The horizontal red line represents the subject's own cost that period; its status as private information was also announced publicly.

The current value of investment, $V(t)$, was represented by a jagged green line that evolved from the right, as on a seismograph. During each period the green value line was initialized at 50 (the lower bound of the cost distribution)[2] and evolved from there according to the binomial parameter vector, High or Low, chosen for that session. The screen rescaled if the value line ever rose out of the given bounds.

Subjects were not allowed to invest when the value line was below their own cost, to prevent negative earnings, nor could they invest after the random ending time. At all other times, subjects could attempt to invest by tapping the space bar at their computer terminal.

In the Monopoly treatment, an investment attempt prior to period end was always successful, immediately netting a subject $V(t) - C_i$ points. Subjects in the Competition treatment were not told whether or when their competitors invested until after the period was over. This semi-strategy

[2]Unlike the experiment reported in OFA07, where the initialization was at realized cost, typically higher than 50.

		Parameters		
Treatment	**h**	**p**	**q**	**Replications**
Low	0.03	0.524	0.007	36 subjects
High	0.03	0.513	0.003	36 subjects
		Blocks/Periods		
Treatment	**1-10**	**11-35**	**36-45**	
Low	Monopoly	Competition	Monopoly	
High	Monopoly	Competition	Monopoly	

Table 1: Experimental Design.

method gives us access to more data, while still giving subjects the real-time choice experience that we feel helps them adapt to the stochastic environment.[3]

After the period ended, subjects were told the time each subject in their group attempted investment, the value at which they attempted investment, the costs of each competitor, and the resulting profits: $V(t) - C_j$ to the subject who invested first, and zero to the others. In the Monopoly treatment, of course, there were no other subjects in the group.

All cost draws, value sequences and period endings were made only once for each parameter set, and repeated in all sessions for that treatment. This procedure permits sharper tests of the hypotheses. For example, a different sequence of cost draws in Competition than in Monopoly might itself have a significant impact that would confound inferences regarding the structure treatment. The realized cost distributions are nearly identical across the two parameter sets. In one session under High parameters, a software malfunction during period 30 (towards the end of the Competition block) lead to 3 missing periods which have been dropped from the dataset.

Experiments were conducted at the University of California, Santa Cruz using inexperienced undergraduate subjects recruited from an online database. Subjects were paid a $5 showup fee. Subjects were paid 5 cents per point earned in Monopoly periods and (to maintain the same expected payout rate) 15 cents per point in Competition periods. Subjects earned an average of $19.56. Table 1 summarizes the design.

[3]Implementing the full strategy method would require us to constrain the strategy space, e.g., to a choice of threshold $V(C)$, excluding a priori non-stationary and other sorts of strategies that subjects might use. Unfortunately our semi-strategy method censors choices in periods that end before a subject attempts to invest.

5 Results

Data analysis must account for two complications. First, the method of inducing impatience produces random ending times, so observed investment values have random right hand censoring. Therefore any estimate using observed investment values alone would suffer downward bias. Second, because subjects cannot invest at values below their costs, the data are left truncated at cost.

To cope with these complications, we generate product limit (PL) estimates of the empirical distribution function of investment values from each treatment. As explained in the Appendix B, the PL procedure provides maximum likelihood non-parametric estimates of the cumulative distribution function (CDF). The estimates are graphed for each treatment in Figure 4.

The top panel of Figure 4 suggests that investment behavior is considerably different under Competition than under Monopoly. For example, the median investment values (where the graph crosses the horizontal line at 0.5) are about 66 or 67 for both Competition treatments versus about 80 for Low/Monopoly and about 87 for High/Monopoly. The observed ordering seems consistent with the first two hypotheses.

The bottom panels of Figure 3 compare observed distributions (solid lines) with the predicted distributions. The dashed lines indicate the PL estimates for optimal Monopoly behavior according to equations (2, 3) and BNE Competition behavior according to the numerical solutions to equation (8), given the realized cost draws. In the Competition panel, the predictions for High and Low parameters are very close together. The CDFs for observed investment values for High and Low parameters are also close to each other, and only slightly more diffuse than predictions. In the Monopoly panel, the predictions for High parameters are about 15-20 points higher (i.e., to the right of) those for Low parameters at each percentile. Above the 20th percentile, the CDFs for observed investment values have the same ordering and about the same spacing above the 80th percentile, but for the most part they fall well below (i.e., to the left of) the theoretical predictions.

5.1 Tests of Treatments

The PL estimates can be compared statistically using a variant of the Mann-Whitney test called the log-rank test; again see Appendix B. That test confirms that the differences between Monopoly and Competition are significant at the one percent level for both High parameters and for Low parameters. As a supplementary test to control for within-subject variance, we also estimate the

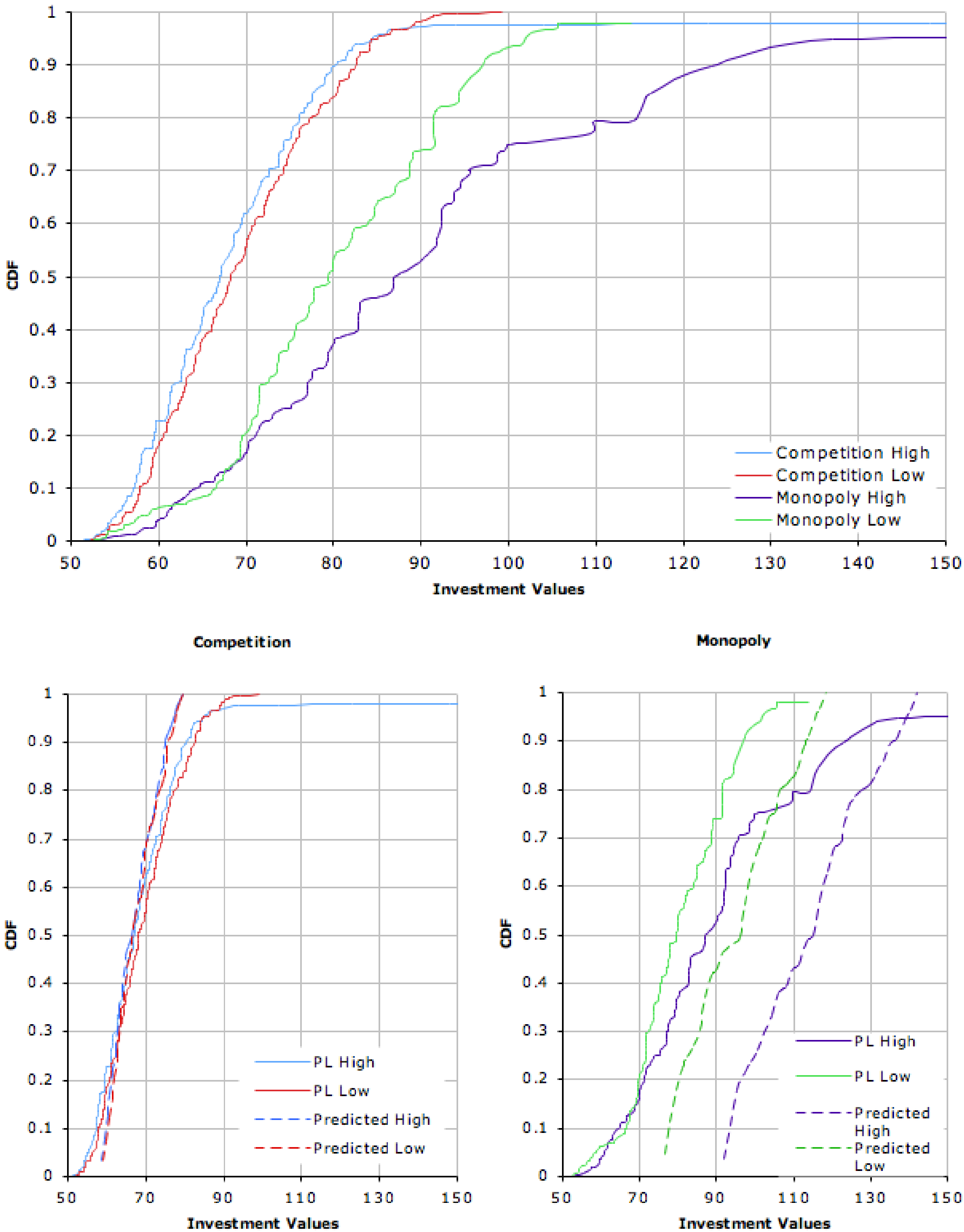

Figure 4: Product limit CDFs.

product-limit mean investment value for each subject under each treatment and market structure. Comparing the populations of estimates using Mann-Whitney tests confirms that investment values are higher under Monopoly than under Monopoly for both Low parameters and High parameters (both with $p = 0.000$). We report these observations as a first finding.

Finding 1. *Consistent with Hypothesis 1, aggregate investment values controlling for binomial parameters are significantly lower under Competition than under Monopoly. The same is true for individual subjects' investment values.*

Our second hypothesis predicts that investment values will be sensitive to binomial parameters under Monopoly but not under Competition. Comparing CDF estimates across parameter sets seems to support this conjecture. Recall from Figure 4 that investment values are typically larger under High parameters than low parameters in the Monopoly block. This difference is significant at the one percent level according to the relevant log-rank test. Visually one can see that in Competition, the Low investment values exceed High by about 1-3 points at most percentiles (the opposite direction from that one might expect) but the log-rank test indicates that the difference is insignificant ($p = 0.1090$). The same story holds if we compare by-subject PL mean values. Investment values are higher under High parameters under Monopoly (Mann-Whitney $p = 0.0042$) though not under Competition ($p = 0.2460$). Together these test results provide us with a second finding.

Finding 2. *Consistent with Hypothesis 2, investment values in Monopoly periods are significantly larger under High parameters than Low parameters, and there is no significant difference in Competition periods.*

5.2 Cost and Investment in Competition

Our third hypothesis is that the lowest cost investor (the efficient one) will usually preempt her rivals. Figure 5 shows the fraction of times an investment is made by the lowest, middle and highest cost investor for each parameter set. In both treatments the lowest cost investor wins roughly 80 percent of the time while the highest cost investor wins less than 5 percent of the time. Note that these tendencies don't much vary across parameter sets. Equally important, higher cost subjects also tend to preempt lower cost subjects only when costs are very close. On average, the difference between the middle and low cost draws is $(C_U - C_L)/(n+1) = (80-50)/(3+1) = 7.5$ points but in periods when a second lowest cost subject preempts the lowest cost subject, the median difference

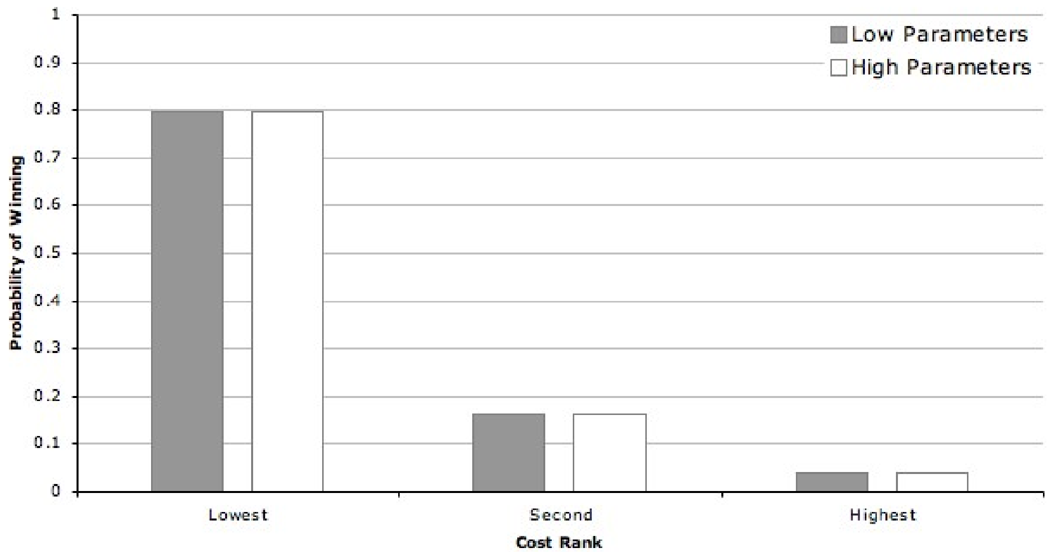

Figure 5: Probability of winning investment as a function of cost rank.

is only 2.5. Likewise, when a highest cost subject preempts the lowest cost subject, the median difference is only 4, as compared to an a priori average of $2 \times 7.5 = 15$ points. To summarize,

Finding 3. *Consistent with Hypothesis 3, the lowest cost investors usually preempt the other investors and the highest cost investor rarely preempts the other investors.*

Hypothesis 4 predicts that under Competition, the target markup $M = V(C) - C$ is decreasing in the cost of investment C. To test the hypothesis, we use the Cox proportional hazard model, a semi-parametric estimate of marginal effects on the instantaneous probability (hazard rate) of an event. Although proportional hazard models are typically used in time-to-event studies, they are easily adapted to our value-at-event data. The Cox model is especially useful for our data because it makes no assumptions violated by the left hand truncation and right hand censoring. We estimate:

$$h_{it}(M) = h_0(M)e^{a_i + bC_{it}} \tag{16}$$

where t indexes periods and a_i is allowed to vary with subject i.[4]

Table 2 collects the estimation results in columns labelled (1). Most importantly, and a bit

[4]Because we allow a to vary across subjects, this model can be thought of as a random effects estimator for the Cox model. Models in this family are often called shared frailty models. We use the standard assumption that a_i has the gamma distribution with mean 1.

	Low Parameters		High Parameters	
Parameter	(1)	(2)	(1)	(2)
C_{it}	0.0053 ± 0.0057	0.0054 ± 0.0057	0.0053 ± 0.0067	-0.0012 ± 0.0057
$Elapsed_t$		0.0314*** ± 0.0060		0.0432*** ± 0.0073
N obs	567	567	472	472

Table 2: Cox model regressions for markups. Elapsed indicates elapsed periods under Competition. Fitted values ($\pm$ standard errrors) for equations 16 and 17 are shown respectively in columns labelled (1) and (2). One, two and three stars represent significance at the ten percent, five percent and one percent levels.

	Low Parameters		High Parameters	
Parameter	(1)	(2)	(1)	(2)
C_{it}	0.021 ± 0.0230	-0.136** ± 0.0525	-0.050 ± 0.0474	-0.245* ± 0.1329
Intercept	5.247*** ± 0.5103	6.570*** ± 0.6227	5.534*** ± 0.8379	7.131*** ± 1.3088
Weighted?	No	Yes	No	Yes
N obs	286	286	163	163

Table 3: Regressions for markups. Fitted coefficients (and standard errors) are shown for the upper quartile data of $\hat{v}$. The least squares weights used in columns labelled (2) are the expected BNE earnings for the realized cost. One, two and three stars represent significance at the ten percent, five percent and one percent levels.

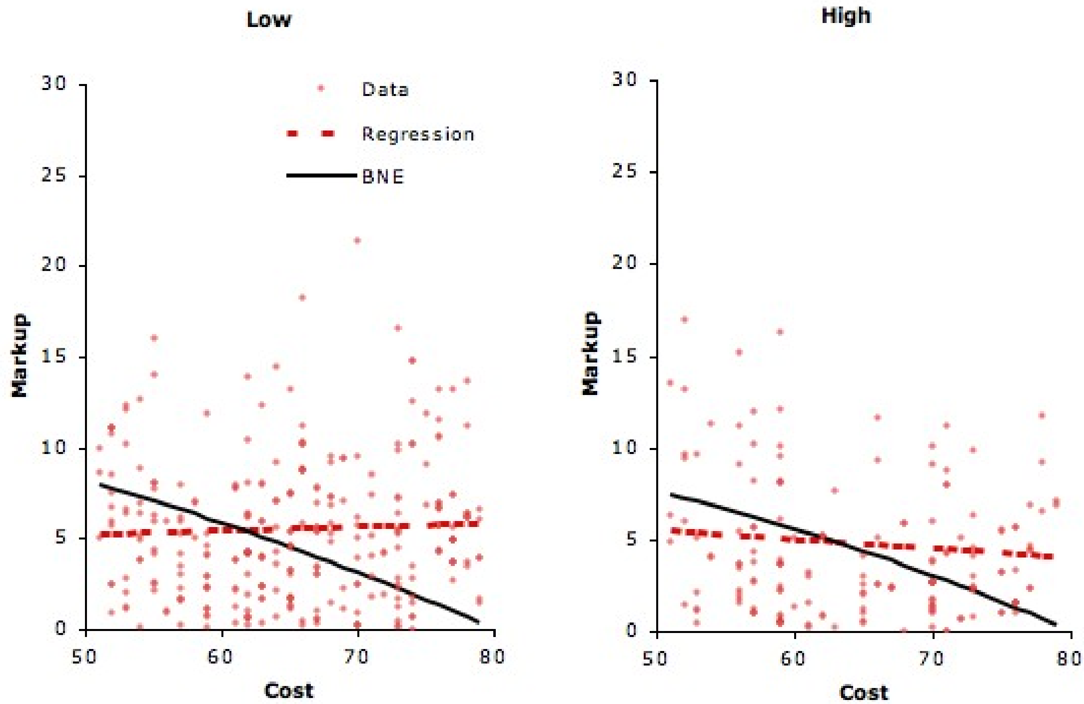

Figure 6: Scatterplots of observed markups and cost by treatment. Data are from Competition periods with $\hat{v}$ in the upper quartile. Red lines are fits from specification (1) in Table (3) . Black lines are fits from specification (2) in Table (3).

surprisingly, the cost coefficent b is insignificantly different from zero, suggesting that markups are independent of cost.

To examine the question more directly, we take a subsample of the data where censoring is not a problem. Note that $\hat{v}$, the maximum value achieved in a given period, is a random variable which is uncorrelated with cost and, because it is unknown to subjects ex ante, is also uncorrelated with the chosen investment threshold. It is however correlated with our ability to observe investment choices: we observe a larger proportion of choices when $\hat{v}$ is higher. Censoring therefore is rare in the set of periods with $\hat{v} > 87$, the upper quartile: of the 456 observations, only 7 are censored. Thus the subsample of the other 449 observations is almost unbiased, and we use it to estimate the slope of the markup function.

Figure 6 displays scatterplots of the observed markups in that subsample. Neither panel suggests a negative relationship between cost and markups. The dashed red lines represent linear fits to the data from the regressions of markup on cost, reported in columns labelled (1) in Table (3), and they also are very nearly flat, not downward sloping. So far, the evidence all seems contrary to

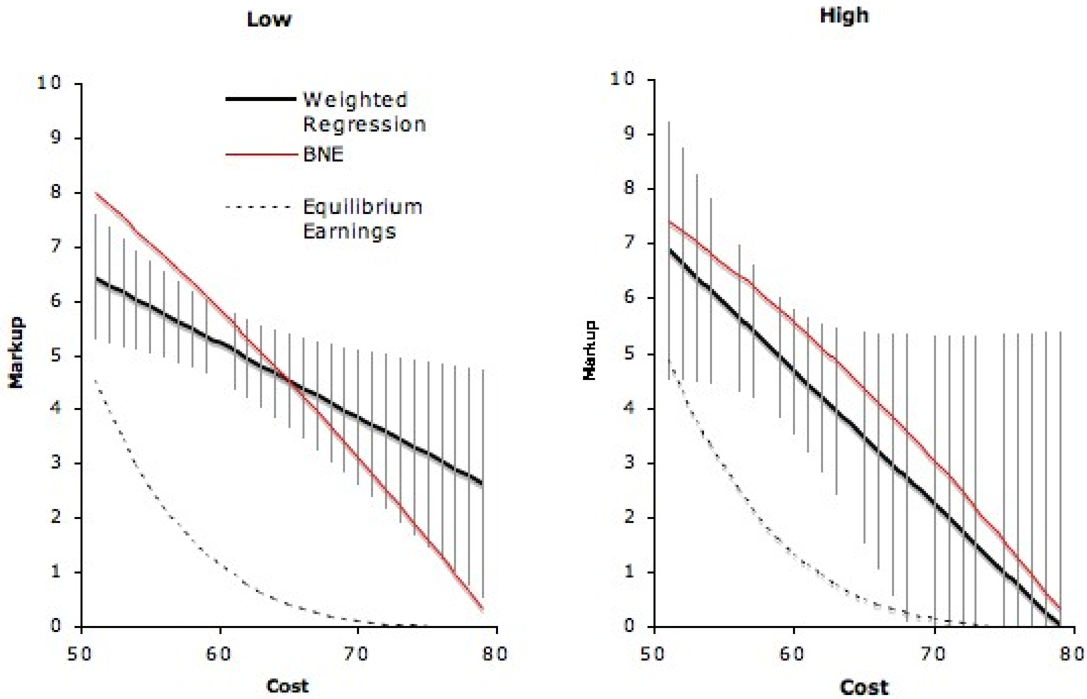

Figure 7: Weighted regressions. Black lines are fits from specification (2) in Table (3), and the thin vertical lines show 95 % confidence intervals. The dotted line shows the weights, the expected earnings in BNE. The red line shows the BNE threshold function.

Hypothesis (4).

But different costs create different incentives to optimize. The dashed line in Figure 7 shows that maximum profit falls sharply at higher cost. With costs greater than 65, expected earnings are lower than a half of a point (7.5 cents) and with costs above 70 expected earnings are virtually zero. Could the observed behavior be consistent with noisy BNE strategies, with noisier behavior from investors with less at stake?

In order to explore this possibility we run a weighted regression for each treatment, with weights proportional to the expected earnings at BNE. The cofficient estimates are shown in columns labelled (2) in Table 3 and the fitted threshold functions are shown in Figure 7. The Figure also includes vertical lines for 95% confidence intervals, and a red line showing the BNE threshold function.

The results reveal that, from the "payoff space" perspective, there might well be a negative relationship between markup and cost. For the Low treatment, the confidence intervals just barely exclude the BNE prediction (and just barely exclude a constant markup of about 6 points). The

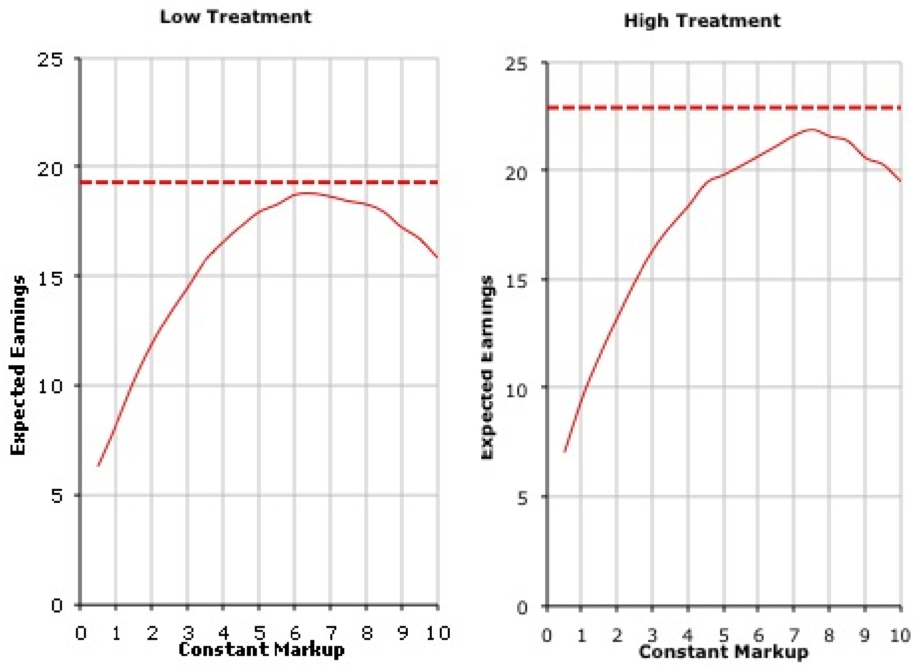

Figure 8: Expected earnings for constant markup strategies. The graph shows average earnings (in cents per period) over 250 Monte Carlo trials for each constant markup played against two rivals using BNE threshold strategies. The dashed horizontal line shows earnings in BNE.

sample includes just 163 High treatment observations, and here the fitted line is quite close to the BNE prediction (but now the confidence intervals include constant markups of around 5 points). In the High treatment we cannot reject the joint hypothesis that the intercept and slope are equal to the BNE values ($p = 0.3906$). In the Low treatment we have only marginal evidence against the same joint hypothesis ($p = 0.076$).

Finding 4. *Evidence on Hypothesis 4 is mixed. Unweighted data indicate that subjects use constant markup strategies. Payoff-weighted data, however, are consistent with noisy versions of the BNE strategies, in which markups decrease in realized cost.*

5.3 Constant Markups and Dynamics

There is enough evidence favoring constant markup strategies to warrant further analysis. The first question is the opportunity cost. How much money do subjects leave on the table when they use a simple constant markup rule in Competition instead of the more complex equilibrium strategy? To investigate, we simulated subjects playing a range of constant markup strategies against two competitors playing BNE strategies over 25 periods. In each of 250 Monte Carlo simulations for

	Low Parameters		High Parameters	
Parameter	(1)	(2)	(1)	(2)
Intercept	7.014*** ± 0.8748	7.091*** ± 0.6141	8.171*** ± 1.0280	7.839*** ± 1.0555
$Elapsed_t$	-0.110*** ± 0.0339	-0.111*** ± 0.0321	-0.211*** ± 0.0578	-0.215*** ± 0.0554
C_{it}	0.004 ± 0.0255		-0.026 ± 0.0489	

Table 4: Regressions and weighted regressions on data with upper quartile of $\bar{v}$. Standard errors are shown in parentheses under estimates. One, two and three stars represent significance at the ten percent, five percent and one percent levels.

each parameter set, we calculated earnings for a subject using 20 different constant markups: 0.5, 1, 1.5, ..., 9.5, 10.0.

The results are collected in in Figure 8. Expected payoffs are unimodal and suggest unique best response markups of about 6.5 for Low parameters and 7.5 for High. Perhaps the most striking result is the optimal constant markup choice yields payoffs that are quite close to the BNE payoffs. For both High and Low parameters, the difference is less than one expected cent per period or \$0.25 over an entire session. Thus, although some money is left on the table by choosing a simple constant markup strategy, that amount can be quite small.

The results are similar when the two competitors also use constant markups; simulations suggest that with Low (respectively High) parameters, a constant markup of 6.5 (respectively 7.5) is a best response to two rivals using the same strategy. These are fairly close to the NE value 7.1 (respectively 7.8) computed numerically using derivations from Theorem 3. Also, against the empirical distribution of choices, the best constant markup is 7.5 for both High and Low parameters. It might also be worth mentioning that the (unconstrained) BNE strategy earns very slightly *less* against the empirical distribution than does the average subject.

We have already seen some evidence on a second question. How do subjects' chosen markups (if assumed constant but noisy) compare to the constrained optimum? The intercept estimates in columns (1) of Table (3) indicate that chosen markups are statistically significantly lower than respective constrained optima.

More evidence can be obtained from the Cox proportional hazard model applied to the entire data set. At the same stroke we can address a final question: are the markups stable over time as in Hypothesis 5, or do they trend towards (or away from) the constrained best response?

Table (2) includes columns labelled (2) that include as an explanatory variable *Elapsed*, the number of periods since the first period of Competition. Thus the equation fit to the data is

$$h_{it}(M) = h_0(m)e^{a_i + bC_{it} + \gamma Elapsed_t}. \tag{17}$$

We see strong evidence that markups decrease over time, especially with High parameters. It is worthwhile to note however that diagnostic tests suggest that the proportional hazard assumption which Cox models rely on may not hold under for this dynamic specification under High parameters. To check robustness and gain further insight, we return again to the almost uncensored subsample (with $\hat{v} > 87$), and run the regression:

$$M_{it} = a + b \times Elapsed_t + \kappa \times C_{it} + \epsilon_{it} \tag{18}$$

where ϵ_{it} is a disturbance term and standard errors are clustered at the subject level.

Results are displayed in Table 4 in columns labelled (1). The significant and negative coefficient on *Elapsed* provides further evidence that markups are decreasing in time. It is notable that the magnitude of the coefficient on *Elapsed* is somewhat larger under High than Low parameters again suggesting a somewhat larger drop in markups over time. As before, this unweighted specification suggests that markups are more or less independent of cost. Dropping the insignificant cost variable in columns (2) gives us similar results.

Finding 5. *Contrary to Hypothesis 5, subjects in Competition reduce their markups over time. There is weak evidence that this effect is stronger in the High treatment than the Low treatment.*

Table 4 also provides a surprising answer to the question regarding constrained equilibrium. In the Low treatment subjects begin with a markup of $a = 7.09$ which is insignificantly different from the constrained equilbrium value of 7.15. In the High treatment subjects begin with a markup of 7.83 which again is remarkably close to the constrained equilibrium value of 7.81.

Finding 6. *Subjects' average initial markups are very close to the (constrained) equilibrium, and their average earnings are quite close to expected earnings in unconstrained BNE.*

6 Discussion

Our exploration of preemption under uncertainty uncovered several new regularities. On the theoretical side, we were able to extend previous work to obtain precise predictions of behavior in competition. In Bayesian Nash equilibrium (BNE), each investor waits until the value of the investment opportunity hits a specific threshold that depends on that investor's private cost. We characterized the BNE threshold function for an arbitrary number of competitors and over relevant parameter ranges, and obtained useful approximations. For example, in triopoly with uniformly distributed costs, the BNE markup of threshold over cost decreases by about \$1 for each \$3 increase in cost.

The laboratory experiment confirmed several of the theoretical predictions. Observed investment thresholds indeed were much lower in triopoly competition than in monopoly, and changes in the parameters driving the stochastic value process had a strong effect (in the predicted direction) in monopoly but no detectable effect in competition. We also confirmed the sort of efficiency predicted in BNE: lower cost investors are far more likely to preempt than their higher cost rivals.

Other laboratory findings offered less support for the BNE theory. Although there is some weak evidence that triopoly competitors with low cost tend to reduce their markups as cost increases, the overall pattern arguably is more consistent with very simple constant markup strategies.

Equilibrium theory is silent on the process that might lead subjects towards BNE strategies. Our subjects showed no clear tendency to move closer to the BNE strategies over time, and the data suggest that they would gain little by doing so. Despite using apparently simple (and rather noisy) strategies, their average earnings were not far from those associated with BNE.

Future work can examine many interesting areas outside the scope of the current paper. On the theoretical side, we conjecture that the (rather mild) technical assumptions we used to obtain BNE are unnecessary. Future laboratory work could investigate behavior for different market structures. We focused on triopoly ($n = 2$) and suspect that higher values of n will push behavior to noisy approximations of the Marshallian threshold. We skipped the duopoly case ($n = 1$) to gain more separation, but future work could investigate it. Perhaps, as in many other laboratory duopolies, one will find attempts to collude (for recent examples see Dufwenberg and Gneezy (2000) and Huck, Normann and Oechssler (2004)).

Two other twists on the laboratory procedures could prove interesting. Our game can be thought of as a Dutch auction with a Brownian clock. One could give the the first mover $\gamma(V - C_i)$, and soften the first mover advantage by varying the parameter γ between $1/(n + 1)$ (equivalent to monopoly at lower stakes) and 1 (full preemption, as in the current experiment). See Levin and Peck (2003) for a related theoretical analysis, and see Brunnermeier and Morgan (2004) for analysis of games with an ordinary clock but more complex payoffs.

In all such laboratory games, an expanded strategy method might offer new insights into players' reasoning. After receiving her cost draw, each player could be offered a menu including the option to seize the opportunity manually (as now) or to program a threshold agent (by filling in the threshold value) or to program any other sort of parametric agent thought to represent an attractive strategy.

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A Mathematical Details

A.1 Optimal Monopoly

Let the gross value of investment V be governed by the stochastic differential equation

$$dV = \alpha V dt + \sigma V dz, \tag{19}$$

where z is the standard Wiener process. Following Chapter 5 of Dixit and Pindyck (1994) and Appendix A of OFA07, we will show that expected discounted profit $E\left[(V-C)\,e^{-\rho t}\right]$ is maximized when the given cost C is sunk as soon as V exceeds $V_M^* = (1+w^*)\,C$, for

$$w^* = \frac{1}{\beta - 1}, \;\; \texttt{where} \;\; \beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{20}$$

Let $F(V)$ be the value of the wait option, i.e., the maximized value of $E\left[(V-C)\,e^{-\rho t}\right]$, given initial project value $V \geq 0$. Prior to investment, the Bellman equation equates the expected return on the opportunity to the expected rate of appreciation,

$$\rho F(V)\,dt = E_t\,[dF]\,. \tag{21}$$

The second order Taylor expansion of dF yields

$$dF = F'(V)\,dV + \frac{1}{2}F''(V)\,(dV)^2\,. \tag{22}$$

Expand dV using Ito's Lemma and equation (19), recalling that $E_t dz = 0$ and $E_t[dz]^2 = dt$, to obtain

$$E_t\,[dF] = \alpha V F'(V)\,dt + \frac{1}{2}\sigma^2 V^2 F''(V)\,dt. \tag{23}$$

Insert equation (23) into (21) and divide by dt to obtain

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \alpha V F'(V) - \rho F(V) = 0. \tag{24}$$

We now derive the value function $F(V)$ and the optimal threshold value V^* by solving the second order ordinary differential equation (24) subject to the boundary conditions

1. $F(0) = 0$,
2. $F(V^*) = V^* - C$, and
3. $F'(V^*) = 1$.

The first boundary condition is implied by geometric Brownian motion; by (19), if $V(0) = 0$ then $V(t) = 0$ for all $t > 0$. The second condition is called value matching: if the initial value of the project makes it worthwhile to launch immediately, then the realized value is simply that value less the cost.[5] The third condition is called smooth pasting. It rules out a kink in the Bellman

[5] Actually, the condition says more. When she invests, the investor gains V but loses the wait option $F(V)$. Thus she should set the threshold V^* so that the gain net of opportunity cost, $V - F(V)$, is just equal to the out-of-pocket cost C.

value function F at the threshold, which (it can be shown) would permit profitable arbitrage at points arbitrarily close to V^*.

To solve the problem, suppose it has a solution of the general form $F(V) = AV^\beta$. Insert this into (24) and cancel the common factor AV^β to obtain the quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho = 0. \tag{25}$$

A straightforward calculation shows that the larger root of (25) is

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{26}$$

To complete the derivation, substitute the general solution $F(V) = AV^\beta$ into boundary condition 2 and re-arrange slightly to obtain

$$C = \left(1 - AV^{\beta-1}\right)V. \tag{27}$$

Inserting the general solution in boundary condition 3 and rearranging yields $AV^{\beta-1} = 1/\beta$. Substituting this last expression into (27) yields $C = (1-1/\beta)\,V$ which is easily rearranged to obtain the desired expression for the monopolist's optimal threshold:

$$V_M^* = \left(1 + \frac{1}{\beta-1}\right)C = \left(\frac{\beta}{\beta-1}\right)C. \tag{28}$$

To obtain the value $F(V) = AV^\beta$ in useful form, rewrite the second boundary condition as $A = (V^* - C)\,/\,(V^*)^\beta$ and evaluate F at V, the current value of the Brownian process. The result is

$$F_M(C|V) = [V^*(C) - C]\left[\frac{V}{V^*(C)}\right]^\beta. \tag{29}$$

To clean up loose ends, note that boundary condition 1 is used to show that the smaller (negative) root of the quadratic equation (25) is irrelevant and that $F(V) = AV^\beta$ is indeed the general solution of (24). That second order differential equation would have a unique solution given only one other fixed boundary condition, but since V^* is endogenous (i.e., is a free boundary) we need a third condition to determine a specific solution.

A.2 Bayesian Nash Equilibrium in the Preemption Game

Following the approach of A03, start with monopolist value function (29) and adjust it for the probability that one of the other n investors $j \neq i$ will preempt investor i. Under the assumption (to be verified later) that all investors use the same increasing threshold function, that probability is precisely the probability that i's cost is lowest, $\left[\frac{1-H(C_i)}{1-H(\hat{C})}\right]^n$. As explained in the text, the denominator reflects conditioning on the fact that no investor has yet preempted; $\hat{V}$ denotes the highest value of the process so far observed, from which one infers (via the increasing threshold function) a lower bound $\hat{C}$ on rivals' costs.

With this adjustment, one obtains Equation (4) as the the competitor's objective function. For convenience, we reproduce it here:

$$F\left(m|C_i, n\right) = \left[V^*\left(m\right) - C_i\right]\left[\frac{V}{V^*\left(m\right)}\right]^\beta \left[\frac{1-H\left(m\right)}{1-H\left(\hat{C}\right)}\right]^n. \tag{30}$$

Use the product rule to take the derivative of Equation (30) with respect to m and cancel like terms (or, alternatively, take the derivative of $\ln F$) and evaluate at the "truth-telling" point $m = C_i$, to obtain the following FOC:

$$\frac{V^{*\prime}\left(C_i\right)}{\left[V^*\left(C_i\right) - C_i\right]} - \frac{\beta V^{*\prime}\left(C_i\right)}{V^*\left(C_i\right)} - \frac{nh\left(C_i\right)}{\left[1-H\left(C_i\right)\right]} = 0. \tag{31}$$

Solve (31) for $V^{*\prime}$ to obtain Equation (5), reproduced here for convenience:

$$V^{*\prime}(C_i) = \frac{\left[V^*\left(C_i\right) - C_i\right]V^*\left(C_i\right)}{V^*\left(C_i\right) - \beta\left[V^*\left(C_i\right) - C_i\right]} \times \frac{nh(C_i)}{\left[1-H\left(C_i\right)\right]} \ \texttt{subject to}\ V^*\left(C_U\right) = C_U. \tag{32}$$

As noted in the text, the boundary value $V^*(C_U) = C_U$ comes from the economics of the situation. At the highest possible cost realization, the existence of rivals known to have equal (or lower) cost induces Bertrand competition and drives the markup to zero.

Proof of Theorem 1. The first task is to show that that there is a unique differentiable function $V^* : [C_L, C_U] \to \mathbf{R}$ satisfying the ODE and boundary value (32). The second task is to verify that V^* is an increasing function. For both tasks, the key is to show that the RHS of (32) is positive and finite, in particular that the denominator $V^*(C_i) - \beta[V^*(C_i) - C_i]$ is positive and bounded away from zero on $[C_L, C_U]$. We do so using the following

Lemma 1. *Assume that rivals use a threshold function with inverse g such that $g' > 0$ and that the hypotheses of Theorem 1 hold. Let the threshold value y maximize the competitor's payoff given cost realization $C \in [C_L, C_U]$. Then $y < V_M^*(C)$.*

Proof. The derivation of Equation (28), together with equation ((29), shows that the expression $[x - C]\left[\frac{V}{x}\right]^\beta$ is maximized at $x = V_M > C$. Therefore, it must satisfy the FOC

$$0 = \frac{1}{x - C} - \frac{\beta}{x}. \tag{33}$$

But by (30) or (31), the threshold value $x = y$ must satisfy the FOC

$$0 = \frac{1}{x - C} - \frac{\beta}{x} - \frac{nh[g(x)]g'(x)}{1 - H[g(x)]}. \tag{34}$$

By (33), at $x = V_M^*$ the RHS of (34) reduces to $-\frac{nh[g(x)]g'(x)}{1-H[g(x)]} < 0$. Since the RHS of (33) is negative for $x > V_M$, no value of $x \geq V_M$ can satisfy the FOC (34). On the other hand, since the first RHS term in (34) goes to $+\infty$ as $x \searrow C$ while the other terms remain bounded, the continuity of the RHS in x guarantees a solution to (34) at some value $x = y \in (C, V_M)$. □

Remark 1. Without additional assumptions (such as an increasing modified hazard rate, as in LP03) there can be multiple solutions y to (34). In this case it is important for later purposes to select the solution y that maximizes (30).

To continue the proof of Theorem 1, apply Lemma 1 to an arbitrary cost $C_i \in [C_L, C_U]$, and conclude that $y = V^*(C_i) < V_M^*(C_i)$. Divide both sides of this inequality by $C_i > 0$ and use Equation (28) to conclude that

$$\frac{V^*(C_i)}{C_i} < \frac{V_M^*(C_i)}{C_i} = \frac{\beta}{(\beta - 1)}. \tag{35}$$

Cross-multiply and solve for β to obtain $\beta < \frac{V^*(C_i)}{V^*(C_i) - C_i}$. Cross-multiply once more to conclude that $0 < V^*(C_i) - \beta\left[V^*(C_i) - C_i\right]$. Thus the denominator in (32) is indeed positive at an arbitrary cost realization.

Since the denominator is continuous on the closed interval $[C_L, C_U]$, it achieves a positive minimum value and thus is bounded away from zero. It now is clear that the RHS of the ODE (32) is Lipshitz continuous. Standard theorems (see for instance Chapter 8 of Hirsch and Smale, 1974) then guarantee that a solution to the boundary problem (32 or 5) exists and is unique. Thus the first task is accomplished. The second task is now trivial: the RHS of (32) has been shown to be positive, so $V^{*\prime} > 0$ and V^* is increasing.

The only remaining task is to verify that it is a BNE for each investor to use the threshold function V^*, but this follows by construction. Writing out V^* by Euler's method (integrating (32) backward from the boundary value $V^*(C_U) = C_U$) and taking the inverse g, one verifies the hypothesis of Lemma 1 that g' (the reciprocal of the RHS of (32)) is positive. The argument of Lemma 1 (in light of Remark 1) then shows that that strategy maximizes expected payoff. That is, assuming that other investors $j \neq i$ play the threshold strategy given by V^*, it is a best response for investor i to do so as well. □

Remark 2. Theorem 1 imposes the hypothesis that H have a continuous positive density on the entire support interval, and that the upper endpoint is finite. Such distribution functions are a dense subset of all distributions, so this restriction is not especially onerous. It may also be unnecessary. For an arbitrary distribution H, one could take a sequence of distributions with positive densities converging to H. At points C where H has density zero, both the numerator factor $h(C)$ and the denominator factor $V^*(C) - \beta\,[V^*(C) - C]$ in (32) go to zero in the limiting distribution. An analysis of this situation, perhaps using L'Hospital's rule, would be of interest.

Proof of Theorem 2. The Marshallian lower bound is obvious: a threshold below cost implies negative profit, but zero profit can be assured in this game by never investing. Hence the BNE threshold must be at least the realized cost. Lemma 1 demonstrates that V_M^* is an upper bound for the BNE threshold.

Let $\bar{V}$ be the auction bid function, the BNE threshold for $\beta = 0$ as given by Equation (10). Fix $C \in [C_L, C_U)$. From Equation (6), $z = \bar{V}(C)$ is the unique interior maximizer of $[z - C]\left[\frac{1-H(g(z))}{1-H(\hat{C})}\right]^n$. Therefore, z must satisfy the FOC

$$0 = \frac{1}{z-C} - \frac{nh[g(z)]g'(z)}{1-H[g(z)]}, \tag{36}$$

where $g^{-1} = \bar{V}$. However, by Equation (31) the threshold value $z = y$ for $\beta > 0$ must satisfy the FOC

$$0 = \frac{1}{z-C} - \frac{\beta}{z} - \frac{nh[g(z)]g'(z)}{1-H[g(z)]} \tag{37}$$

By Equation (36), at $z = \bar{V}(C)$ the RHS of Equation (37) reduces to $-\frac{\beta}{z} \leq 0$. Reasoning parallel to that in Lemma 1 guarantees a solution to (37) at some value $z = y \in (C, \bar{V}]$. Hence we have established that $\bar{V}(C)$ is an upper bound for $V^*(C)$.

There remains only to show that $\bar{V}(C)$ and $V^*(C)$ are tangent at the upper endpoint C_U. Of course, the boundary conditions defining these functions ensure that $\bar{V}(C_U) = C_U = V^*(C_U)$. Since the slope of $\bar{V}$ is given by the RHS of (32) when $\beta = 0$, it suffices to verify that the RHS of equation (32) evaluated at C_U is independent of β. The expression in question is

$$\begin{aligned}
V^{*\prime}(C_U) &= \lim_{C \nearrow C_U} \frac{[V^*(C) - C]V^*(C)}{V^*(C) - \beta[V^*(C) - C]} \times \frac{nh(C)}{1-H(C)} \\
&= \frac{nh(C_U)V^*(C_U)}{V^*(C_U) - \beta[V^*(C_U) - C_U]} \lim_{C \nearrow C_U} \frac{V^*(C) - C}{1 - H(C)} \\
&= \frac{nh(C_U)}{1 - \beta[1 - \frac{C_U}{C_U}]} \frac{V^{*\prime}(C_U) - 1}{[-h(C_U)]} = -n[V^{*\prime}(C_U) - 1],
\end{aligned} \tag{38}$$

using L'Hospital's rule. Hence $V^{*\prime}(C_U) = \frac{n}{n+1}$, independent of β (and H!). $\square$

A.3 Nash Equilibrium for Constant Markup Strategies

We begin the analysis again with monopoly ($n = 0$). With the restricted strategy set, the monopolist's value function simplifies to $F_M^R(k, C_i, V) = k[\frac{V}{C_i+k}]^\beta$, with expected value

$$E_H F_M^R = kV^\beta \int_{C_L}^{C_U} (C + k)^{-\beta} h(C) dC. \tag{39}$$

Standard arguments confirm that the value function (39) has a unique maximum $k_M^* > 0$ as long as $\beta > 1$ (a necessary condition in monopoly, as noted in section A.1 above).

Proof of Theorem 3. With $n \geq 1$ other investors, we seek a symmetric Nash equilibrium markup k^*. As noted in the text, investor i's objective function is (12), reproduced here for convenience:

$$E_H F^R(x|k, n) = (k + x) \int_{C_L}^{C_U} \left[\frac{C_L}{C + k + x}\right]^\beta [1 - H(C + x)]^n h(C) dC. \tag{40}$$

To explain, note that the initial value of the Brownian value is C_L by convention. Thus the integrand is simply the prior probability of preemption by "Nature" or other investors given realized cost C. Of course, the factor outside the integral is simply the profit earned when not preempted. Hence (12) or (40) is the expected profit for constant markup $(k + x)$.

Taking the derivative of Equation (40) with respect to x, and imposing the incentive condition that (40) is maximized at $x = 0$, yields the following first order condition:

$$\begin{aligned}[E_H F]_x = &\int_{C_L}^{C_U} (C_i + k)^{-\beta} [1 - H(C_i)]^n h(C_i) dC_i \\ &- k\beta \int_{C_L}^{C_U} (C_i + k)^{-1-\beta} [1 - H(C_i)]^n h(C_i) dC_i \\ &- kn \int_{C_L}^{C_U} (C_i + k)^{-\beta} [1 - H(C_i)]^{n-1} [h(C_i)]^2 dC_i = 0.\end{aligned} \tag{41}$$

A solution of Equation (41) characterizes the NE k^*, and makes it possible to obtain comparative statics in n, β and H. Again, we are especially interested in the case of uniform distributions H and triopoly ($n = 2$). In this case the equation simplifies to:

$$\begin{aligned}&2(C_U + k)^{3-\beta} - (3 - \beta)(2 - \beta)(C_U - C_L)^2 (C_L + k)^{1-\beta} - 2(3 - \beta)(C_U - C_L)(C_L + k)^{2-\beta} \\ &- 2(C_L + k)^{3-\beta} - k(3 - \beta)(2 - \beta)(1 - \beta)(C_U - C_L)^2 (C_L + k)^{-\beta} = 0,\end{aligned} \tag{42}$$

The solution k^* can be found numerically using Newton's method. □

B Econometric Details

Our description of the product limit estimator and log rank test closely mirror the descriptions provided in an appendix of OFA07. We include it here for completeness.

B.1 Product Limit Estimator

The product-limit estimator produces an estimate of the distribution function, $F(w_i) = prob(x \leq w_i)$ while taking account of random right censoring. Consider a sample consisting of n observations. In uncensored observations denote v as the observed investment value and in censored cases denote v as the highest value available before censoring, $\bar{v}$. We can then construct an n-vector of these values $v = (v_0, v_1, ...v_n)$ ordered so that $i < j$ if and only if $v_i < v_j$. Include in this vector $v_0 = 50$, the lowest value possible at investment.

The product-limit estimate of $F(v_i)$ exploits the fact that the complement of the distribution function can be written as a product of conditional probabilities. Note that $1 - prob(x \geq v_i) = 1 - prob(x \geq v_i | x \geq v_{i-1}) \times prob(x \geq v_{i-1})$. Recursively then,

$$F(v_i) = 1 - \prod_{j=1}^{i} prob(x \geq v_j | x \geq v_{j-1}) \tag{43}$$

Let c_i denote the number of censored observations smaller than v_i and let u_i denote the number of uncensored observations smaller than v_i. Finally, define $n_i = n - c_i - u_i$, the number of observations equal to or greater than v_i. The product-limit estimate of $p(x \geq v_i | x \geq v_{i-1})$ is the proportion of investments greater than v_{i-1} which are also greater than v_i:

$$\hat{p}(x \geq v_i | x \geq v_{i-1}) = \frac{n_i - u_i}{n_i} \tag{44}$$

The product-limit estimator is then, following (43), the cumulative product of these individual conditional probabilities at each v_i

$$\hat{F}(x) = 1 - \prod_{v_i < x} \frac{n_i - u_i}{n_i} \tag{45}$$

Without censoring (that is when all v_i denote option premia at investment) it can be shown that $\hat{F}(v_i)$ is simply the empirical distribution function – the proportion of investments which are lower than v_i for each v_i. Kaplan and Meier (1958) show that the product-limit estimator is the maximum likelihood *non-parametric* estimator of the distribution function in environments with censoring problems analogous to ours.

B.2 Log Rank Tests

Typically, hypothesis tests comparing product-limit distribution functions are conducted using a log-rank test. Consider two samples, labeled $j = 1, 2$. In what follows we will sometimes pool the two samples and construct the statistics described above, in which case we omit the j subscript. In other cases we'll use statistics described above computed only for pool j in which cases we include the subscript. That implies, for instance, that $n_i = n_{i1} + n_{i2}$. We'll also introduce d_i, defined as the total number of investments made at premium v_i where $d_i = d_{i1} + d_{i2}$.

Under the hypothesis that the two samples are the same at v_i, expected investments in group 1 are $n_{i1}d_i/n_i$ while the actual observed investment is simply d_{i1}. As with many non-parametric techniques, the log-rank test relies on a test statistic based on the difference between these observed and expected statistics. To construct the test statistic, the log-rank test computes the hypergeometric variance for the number of investments at premium v_i as

$$s_i^2 = \frac{n_{i1}n_{i2}(n_i - d_i)d_i}{n_i^2(n_i - 1)} \tag{46}$$

The test statistic for the log rank test is then

$$z = \frac{\sum_i^n (d_{i1} - \frac{n_{i1}d_i}{n_i})}{\sqrt{\sum_i^n s_i^2}} \tag{47}$$

which is approximately distributed standard normal under the hypothesis that the hazard rates for the two samples are equal .

C Supplementary Data Analysis

C.1 Monopoly Data

The design in our monopoly blocks closely mirrors the design used in OFA in several important respects. Subject made decisions on a similar interface and parameters in the Low and High treatments here are precisely the ones used in the Medium B and High treatments in OFA. There are, however, several notable differences.

First, in OFA the value line initiated at the subject's cost draw ($V = C_i$) whereas in our experiment the line initiates at the lower bound of the cost range ($V = 50$). The design choice in OFA was made to optimize the number of periods in which subject investments were observed. The deviation here was made to create comparability with the preemption periods in which the value line had to begin at a common point for three subjects with three different and private cost draws. This design decision has the cost of increasing the rate at which subject decisions are censored. It also means that a number of subjects in any given round do not face a feasible investment choice since V never reaches their cost.

Second, the cost range used in OFA was somewhat larger than that used here. Costs are drawn from $U[50, 110]$ in OFA but are drawn from $U[50, 80]$ in our experiment.

Finally, in OFA subjects experienced 80 contiguous periods of investment. In our experiment subjects experience 20 investment periods broken up by a 25 period block of preemption periods after period 10. In the Low treatment, there is little difference between the early block and the later block. The PL mean is 80.87 and 80.47 for the early and late blocks respectively, indicating no ordering effect. In the High treatment, however, there is a significant difference. In the early block values the PL mean value is 95.85, dropping to 78.95 following the competition periods. This drop is significant at the one percent level by a log-rank test.

It is important to note that these trends run in the opposite direction of those observed in OFA where investment values tend to increase over time which we attribute to contamination from the intervening competition periods. It is for this reason we drop the final block from the analysis in the body of the paper. However, as we demonstrate below, this decision does not alter the comparative statics reported in the text.

Under the complete data set, PL estimated mean investment values are 69.41 under competition and 80.69 under monopoly in the Low treatment. Similarly, mean investment values are 69.58 under competition and 87.4 under monopoly in the High treatment. Using log-rank tests these differences are significant at the one percent level in each treatment. Thus our first finding – that competition lowers investment values – continues to hold with the final block included in the data. Our other major finding – that binomial parameters exert an influence over monopoly behavior – is likewise confirmed at the one percent level by a log rank test.

C.2 A Directional Learning Model

What causes lowering of markups over time? To investigate, we estimate a directional learning model in the tradition of Selten and Buchta (1998) and Cason and Friedman (1999). We classify all the different kinds of ex post losses that an investor could suffer, hypothesize that adjustment of markup is proportional to the magnitude of each kind of loss, and estimate the sensitivities to each kind.

One way to lose earnings is to be preempted by a competitor. Denoting the value chosen by the winner as v^w, the loss in this case is $Lose(v^w - c)$ where $Lose$ is a dummy taking a value of 1 when the subject attempts but fails to invest. We hypothesize that this source of loss causes subjects to decrease their markup in future periods.

Another way to lose potential earnings is to successfully invest but to realize that one might have invested successfully at a higher value. Denoting v^s as the value of the second investor to invest, this loss can be measured by $Win(v^w - v^s)$ where Win is a dummy taking a value when the subject successfully invest. We hypothesize that this would cause subjects to increase markups instead.

A third type of loss is to fail to attempt investment at all because the round ends before the subject chooses to attempt. In such case a subject loses $Random(\bar{v} - c)$ where $Random$ is a dummy taking a value of 1 if the subject has the opportunity to invest but all subjects fail to attempt investment.

A final way to lose earnings, though not due to strategic failure, is to miss the opportunity to invest at all. This occurs if the value line never reaches the subject's cost ($\bar{v} < c$). We will represent this with a dummy $NoChance$.

We also include the term $c_t - c_{t-1}$ as a robustness test of our constant markup findings. Under the hypothesis that markups are decreasing in costs, a coefficient on this term should be significantly negative.

Estimating the model is complicated by the fact that we do not observe a subject's investment attempt (or his markup M) in periods where $Random$ or $NoChance$ are equal to one. If we had access to target markups in all periods, we could include all of the covariates described in section (5.3) in the following linear model

$$M_t - M_{t-1} = \alpha Lose_{t-1}(v^w_{t-1} - c_{t-1}) + \beta Win_{t-1}(v^w_{t-1} - v^s_{t-1}) + \kappa NoChance_{t-1}$$
$$+\delta Random_{t-1}(v^-_{t-1} - c_{t-1}) + \phi(c_t - c_{t-1}) + \epsilon_{t-1}$$

where ϵ is distributed $N(0, \sigma^2)$. An obvious problem with estimating such a model is that in many periods, we do not observe a subject's investment attempt and therefore fail to observe M. In particular this happens in cases where *Random* or $NoChance$ are equal to one. We deal with this by instead estimating

$$\hat{M}_t - \hat{M}_{t-1} = \alpha Lose_{t-1}(v^w_{t-1} - c_{t-1}) + \beta Win_{t-1}(v^w_{t-1} - v^s_{t-1}) + \kappa NoChance_{t-1}$$
$$+\delta Random_{t-1}(v^-_{t-1} - c_{t-1}) + \phi(c_t - c_{t-1}) + \epsilon_{t-1}$$

where $\hat{M}$ is the observed M in periods in which it is observed and is a latent variable in those periods in which it is not observed.

Consider a period t in which M is observed and suppose that period N_t period precede it in which M is not observed. Then we can rewrite our latent model entirely in terms of observables:

$$M_t - M_{t-N_t-1} = \alpha Lose_{t-N_t-1}(v^w_{t-N_t-1} - c_{t-N_t-1}) + \beta Win_{t-N_t-1}(v^w_{t-N_t-1} - v^s_{t-N_t-1})$$
$$+ \sum_{i=t-N_t}^{t-1} [\kappa NoChance_i + \delta Random_i(\bar{v}_i - c_i)] + \phi(c_t - c_{t-N_t-1}) + \sum_{k=t-N_t-1}^{t-1} \epsilon_k$$

As long as we limit attention to this linear versions of the model, we can estimate such a function via standard weighted least squares. First, for each subject, we set M_0 equal to the markup used in the subject's first observed investment attempt, dropping previous data. Second, we form the variable $M_t - M_{t-N_t-1}$ and estimate

$$M_t - M_{t-N_t-1} = \alpha Lose_{t-N_t-1}(v^w_{t-N_t-1} - c_{t-N_t-1}) + \beta Win_{t-N_t-1}(v^w_{t-N_t-1} - v^s_{t-N_t-1})$$
$$+ \sum_{i=t-N_t}^{t-1} [\kappa NoChance_i + \delta Random_i(\bar{v}_i - c_i)] + \phi(c_t - c_{t-N_t-1}) + \psi_t$$

where ψ_t is distributed $N(0, \sigma^2 N_t)$. In order to control for within subject correlation, we estimate this model with clustered standard errors at the subject level.

Covariate	Estimates
$Lose_{t-1}[v^w_{t-1} - c_{t-1}]$	-0.403^{***} ± 0.082
$Win_{t-1}[v^w_{t-1} - v^s_{t-1}]$	0.058 ± 0.041
$Random_{t-1}[\bar{v}_{t-1} - c_{t-1}]$	0.330 ± 0.205
$NoChance_{t-1}$	-0.394^{**} ± 0.157
$c_t - c_{t-1}$	0.019 ± 0.026

Table 5: Estimated adjustment parameters.

Estimation results with clustering included at the subject level are presented in Table 5. The coefficient on $Lose \times [v^w - c]$ is significant and negative as expected as is the coefficient on *NoChance*. All other loss variables are insignificant. Also insignificant is the coefficient on $c_t - c_{t-1}$, giving us further evidence against Hypothesis 4. Thus, the drop in markups over time is driven largely by adjustment in response to preemption. Moreover, subjects adjust their markups downwards after experiencing periods in which the investment opportunity never becomes profitable.

Simulations using the significant parameters predicts substantial decreases in markups over time. Indeed the decrease is somewhat stronger than that observed in our data. Note however that the R^2 on this model is quite low (0.0484) indicating that other factors including subject heterogeneity likely exert a great deal of influence over the pattern adjustment. Moreover, because of our censoring problem our empirical strategy limits us to linear specifications. However our model does indicate that the experience of being preempted and of losing the opportunity to attempt investment significantly influence subjects' markups over time.

C.3 Tests of Proportional Hazards Assumption in Cox Models

We report estimation results from four Cox hazard models in Table 2. A central assumption in Cox models (in our context) is that the effects of covariates on hazard rates are unchanging over values of V. This is often called the proportional hazards assumption. Grambsch and Therneau (1994) show that an adjustment of the model's residuals (calculated ala Schoenfeld (1982)) can be interpreted as the log hazard-ratio function which permits global tests of the proportional hazards assumption. Adjustments suggested in Therneau and Grambsch (2000) are applied in our case because our Cox models are in fact shared frailty models. The null hypothesis that the log hazard ratio is constant is tested with a likelihood ratio test. We can reject the proportional hazards assumption at the 5 percent level in only one specification. Under High parameters, specification (2) marginally fails this test ($p = 0.037$).

D Instructions to Subjects

Instructions were presented to subjects in two parts. Part 1 pertained to the Monopoly treatment and Part 2 to the Competition treatment. Subjects were given part 1 at the beginning of the session and part 2 after the completion of period 10, just before the Competition block began. Part 1 of the instructions were complemented with a projected display of the computer interface on a screen.

D.1 Part 1

You are about to participate in an experiment in the economics of decision-making. The National Science Foundation and other agencies have provided the funding for this project. If you follow these instructions carefully and make good decisions, you can earn a CONSIDERABLE AMOUNT OF MONEY, which will be PAID TO YOU IN CASH at the end of the experiment.

Your computer screen will display useful information. Remember that the information on your computer screen is PRIVATE. To insure best results for yourself and accurate data for the experimenters, please DO NOT COMMUNICATE with the other participants at any point during the experiment. If you have any questions, or need assistance of any kind, raise your hand and one of the experimenters will come.

In the experiment you will make investment decisions over several rounds. At the end of the last period, you will be paid $5.00, plus the sum of your investment earnings over all rounds.

The Basic Idea. Each round you will decide when (if ever) to seize an investment opportunity. At the beginning of the round you will be assigned a cost, C of investing. The value V of investing will change randomly over time. You earn V-C points if you seize the opportunity before it disappears. If you wait longer, V might go higher, earning you more points. Or V might go lower. The opportunity to invest might evaporate before you seize it, in which case you earn 0 points that round.

Investor Screen Information. Your cost C is shown on your screen as a horizontal red line, as in Figure 1. The value V of the investment is shown as a jagged green line that scrolls from left to right, with the rightmost tip (the leading edge) representing the current value V. Previous values move left, as on a ticker tape. At the start of each round the value line V starts at 50 and randomly evolves from there.

When you want to invest, press the SPACE BAR.

Other useful messages appear in the window to the right labeled Your Performance. For example, in Figure 1, that window tells you the round number, your cost, and that you have no competitors for the investment opportunity. You can see messages and results from previous rounds by clicking the Previous button at the top of the window.

The round will continue until the investment opportunity disappears even if you have already invested.

Feedback

After the period is over, you will be shown a chart, reproduced in Figure 1, repeating your cost, the value you invested at and the number of seconds that elapsed before you invested. The final line in the chart, marked [End] shows you how many seconds the round lasted.

Payment. Points translate into dollars according to a formula written on the board. You will be paid in cash at the end of the experiment for the points earned in all rounds plus the $5 show-up fee. For example, if the formula is $0.02 per points in excess of 1000, and if you earn 1682 points, then your cash payment is $5.00 + $(1682 1000)*0.02 = $5.00 + $13.64 = $18.64.

Details. In case you want to know, here are a few details of how V unfolds. You can skip these if you prefer to learn just from experience.

- The round is a series of many ticks (e.g., 5 ticks per second).
- Each tick the value V moves randomly up or down by a fixed percentage, e.g., 3%.
- Upticks are slightly more likely than downticks, e.g., each tick is up with probability 51% or down with probability 49%.
- The round ends (the investment evaporates) with a small probability each tick, e.g., ? of 1%.
- The actual values (for ticks per second, tick size, uptick probability, and evaporation probability) will be written on the board before the experiment begins.
- The value always starts at 50.
- The computer will not allow you to seize the investment opportunity when V is less than C, because that would give you a negative number of points.

Frequently Asked Questions

Q1. Is this some kind of psychology experiment with an agenda you haven't told us?

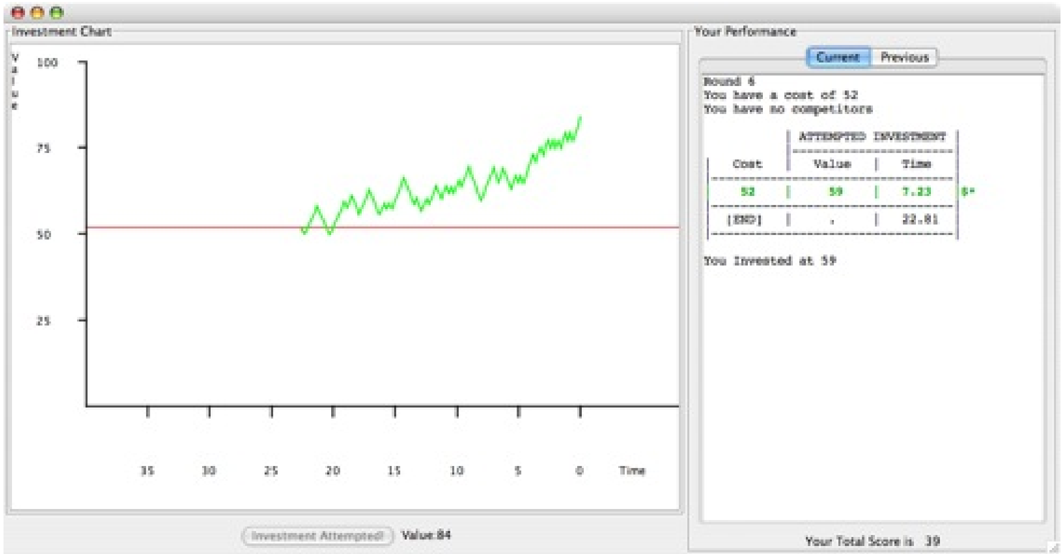

Figure 9: Graphic included in first part of instructions.

Answer: No. It is an economics experiment. If we do anything deceptive, or don't pay you cash as described, then you can complain to the campus Human Subjects Committee and we will be in serious trouble. These instructions are meant to clarify how you earn money, and our interest is in seeing how people make investment timing decisions.

Q2. How long does a round last? Is there a minimum or maximum?

Answer: The length of time is random. In the example, the probability is 0.005 that any tick is the last, and there are 5 ticks per second. In this case, the average length of a round is 200 ticks or 40 seconds. Many rounds will last less than the average, and a few will last much longer. Rounds longer than 7 minutes are so unlikely that you probably will never see one. The minimum length is one tick, but it is unlikely you will ever see a round quite that short!

Q3. How many rounds will there be?

Answer: Lots. We arent supposed to say the exact number, but there will be a number of rounds.

Q4. Are there patterns in upticks and downticks?

Answer: No. Weve tried very hard to make it random. No matter what the recent history of upticks and downticks, the probability that the next tick is up is always the same (and is written on the board).

(Figure 9 here)

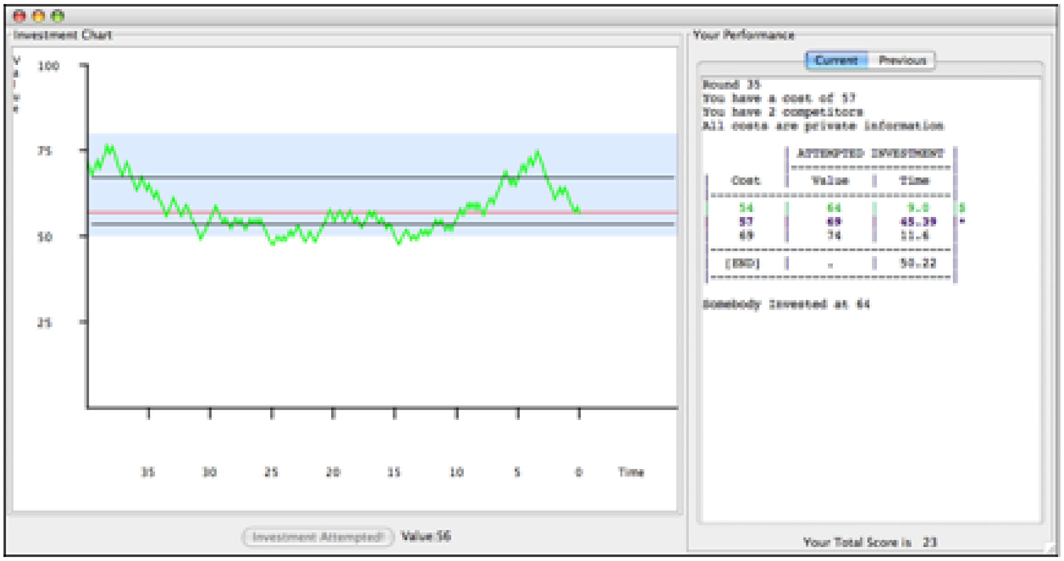

Figure 10: Graphic included in second part of instructions.

D.2 Part 2

In this part of the experiment everything will be as before except you will now be grouped each period with 2 other investors. Each investor in your group will secretly decide when to attempt to invest by pressing the space bar. You will not find out who invested when until after the period is over.

Each participant will be assigned a cost each period randomly chosen between 50 and 80 with equal likelihood. This range is shaded in blue on your display. During the period you will see your cost as a red line and each of the other participants costs (in your group) as black lines.

After the period is over, we will reveal who attempted to invest first, second and third in your group. Whoever attempted to invest first will have successfully invested and will get V-C points. Whoever attempted to invest second and third will fail to invest and will get zero points. This information will be revealed in a table on the right side of the screen when the period is over. The table will have a line for each participant and will be ordered by cost. The participant who successfully invested will be listed in green and will have a dollar sign beside it ($). Your information will be in bold type and will have an asterisk (*) beside it. If you are the successful investor, your information will have both a dollar sign and asterisk beside it and will be in both bold and green.

(Figure 10 here)

In the example above, you had a cost of 57 and attempted to invest at a value of 69 after 45.39

seconds. Another participant had a cost of 54 and attempted to invest at a value of 64 after 9 seconds. A third participant had a cost of 69 and attempted to invest at a value of 74 after 11.6 seconds. Because the first participant attempted to invest at the earliest time, he or she successfully invested and will earn 64-54=10 points. Note that this participants information is in green and has a dollar sign by it. You and the other participant earn nothing this period.

The fourth line marked with [End] gives you the number of seconds the period lasted. In the example above, the period lasted a total of 50.22 seconds.

After each period, you will be randomly matched into a new group. You will never be matched in the same group twice.