Title
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Abstract

Using census block groups data on socio-demographics, land use, and travel behavior, we test the cutoffs suggested in the literature for trustworthy estimates and hypothesis testing statistics, and evaluate the efficacy of deleting observations as an approach to improving multivariate normality, in structural equation modeling. The results show that the measures of univariate and multivariate non-normalities will fall into the acceptable ranges for trustworthy maximum likelihood estimation after a few true outliers are deleted. We argue that pursuing a multivariate normal distribution by deleting observations should be balanced against loss of model power in the interpretation of the results.

Structural equation modeling (SEM) has been theoretically and empirically demonstrated to be powerful in disentangling complex causal linkages among variables in social studies, and has become more and more popular in studying the relationships between travel behavior and the built environment(1,2). As with other statistical methods, assuming conceptual plausibility, the inferences of causality in the SEM are based on hypothesis tests on the model and the parameter estimates. If the data meet all the assumptions required by an estimation method, the results are assumed to be trustworthy.

In SEM, one of the main concerns about the data is whether the sample has a multivariate normal distribution, because that determines what estimation method will be used and to what extent the estimates obtained from the most common methods are trustworthy. Generally speaking, real-world data (including those from travel or activity surveys) do not have even univariate, let alone multivariate, normal distributions (3, 4). In such cases, it is dangerous to apply a normal theory-based estimation method to a sample with a multivariate non-normal distribution. To bring a sample closer to compliance, researchers may transform the raw data (5, 6, 7) or delete the "outliers" (1) which contribute most strongly to the departure from the normal distribution and distort the covariance matrix (8, 9).

Transforming (e.g. square root, logarithm, Box-Cox) raw data reduces the multivariate skewness and kurtosis of all variables collectively by reducing the univariate skewness and kurtosis of each individual variable. According to our experience, when the univariate non-normality is severe, transformation will substantially reduce the univariate skewness and kurtosis. When univariate non-normality is moderate or slight, transformation has only a minor effect. However, slight non-normality of the individual variables may still lead to a large multivariate skewness or kurtosis. Therefore, transformation alone is unlikely to lead to a multivariate normal distribution. In addition, transformation implies that the relationships of one variable with others are assumed to be curvilinear instead of linear. This assumption may not be true. It is not uncommon for a researcher to find that the hypothesized SEM is degraded in terms of model fit indices, or even worse, that the model is empirically unidentifiable after some variables are transformed. In this case, even if transformation improves normality *per se*, it is not helpful for improving the model as a whole. Besides, the curvilinearity makes the interpretation of coefficients more difficult. Therefore, the role of transformation needs to be assessed on a case-by-case basis.

In contrast to transformation, deleting outliers focuses on lowering the multivariate skewness and kurtosis of the original raw data. In AMOS™ (a software package for structural equation modeling), multivariate normality is measured by Mardia's multivariate kurtosis (10). The outliers are indicated by their Mahalanobis distances, which represent the squared distance, in standard units, of the vector of an observation from the vector of sample means for all variables. The larger the distance is, the larger the contribution an observation is making to Mardia's multivariate kurtosis and hence to the departure from multivariate normality. Deleting an outlier will decrease Mardia's multivariate kurtosis. Outliers can be deleted until the multivariate kurtosis index reaches the desired level. One advantage of deleting outliers over transforming the data to achieve normality is that it retains the assumption of linearity. The disadvantage of deleting outliers is obvious: it means loss of observations, and hence information and model power. At the extreme, the results could be a Procrustean model trimmed to fit allegedly "typical" cases, but which ignores the departures from "typical" that characterize real empirical data. Therefore, deleting outliers until multivariate normality is reached may also be undesirable.

Even if choosing transformation to improve multivariate normality, a researcher is unlikely to obtain a sample with a multivariate normal distribution without also deleting observations. He will often estimate the parameters and assess the model using the maximum likelihood (ML) approach under some degree of multivariate non-normality. Therefore, what is most important is not whether a sample has a multivariate normal distribution, but to what extent the hypothesis tests on the parameter estimates and the model are trustworthy when the data are multivariate non-normal. In other words, we want to know what level of multivariate non-normality is acceptable for a reasonably accurate (i.e. less biased) estimation of parameters, standard errors and chi-squared statistic. If this level of multivariate normality is reached by deleting outliers, to what extent is model power lost due to the loss of the information in the deleted observations?

In this study, we test the cutoffs suggested in the literature for trustworthy estimates and test statistics, and evaluate the impacts of deleting observations as an approach to improving multivariate normality.

LITERATURE REVIEW

A multivariate normal distribution implies that each variable in a sample has a univariate normal distribution and each pair of variables has a bivariate normal distribution (11). Multivariate normality can be measured in many ways, but Mardia's coefficients of multivariate skewness and kurtosis or an omnibus measure based on both coefficients (for example, Mardia's PK in PRELIS) are most commonly used in SEM software. In AMOS 5, only Mardia's coefficient of multivariate kurtosis and its critical ratio are available. The critical ratio of Mardia's multivariate kurtosis is asymptotically distributed as N (0, 1). Thus, a sample can be considered to be multivariate normally distributed at the 0.05 level of significance when the critical ratio is smaller than 1.96, indicating that the coefficient of multivariate kurtosis is not significantly different from zero (10).

Univariate normality describes the distribution of only one variable in the sample while multivariate normality describes the joint distribution of all variables in the sample. The univariate normal distribution of each variable is a necessary, but not sufficient, condition for having a multivariate normal distribution (12). However, univariate non-normal distributions for each variable will generally result in a multivariate non-normal distribution.

What if a sample with a moderate size does not have a multivariate normal distribution and the ML estimation is used? Theoretically, non-normality leads to an overestimation of the chi-squared statistic (indicating the degree of discrepancy between the model-implied and sample-derived covariance matrices), potentially leading to false rejection of the model as whole, and the underestimation of standard errors of parameter estimates, leading to inflated statistics and hence possibly erroneous attributions of significance of specific relationships in the model (13). These theoretical predictions about the effects of non-normality on parameter estimates, the chi-squared statistic, and the standard errors of the parameter estimates are supported by Monte Carlo simulation studies, in which the values of the skewness and kurtosis can be well controlled at desired levels. Thus, the marginal differences between the samples having normal and non-normal distributions can be attributed to non-normality.

Muthén and Kaplan (13) designed five combinations of skewness and kurtosis (see Table 1) to study the effects of non-normality on chi-squared statistics and parameter estimates. The chi-squared statistic and model rejection frequencies for the model in Case 4 were 84% and 67% higher, respectively, than those in Case 1. The effects of non-normality on the chi-squared statistic and model rejection frequencies in Case 2, Case 3 and Case 5 were negligible. In all cases, the differences between the parameter estimates and the actual parameters were no more than 4.2% and were negligible. They concluded (p. 187) that “if most variables have univariate skewnesses and kurtoses in the range -1.0 to + 1.0, not much distortion is to be expected” and that “this is largely independent of number of variables and number of categories”, and that “when most skewnesses and/or kurtoses are larger in absolute value than 2.0, and correlations are large (say 0.5 and higher), distortions of ML and generalized least squares (GLS) chi-squares and standard errors are very likely”. Hallow (14) tested the impacts of non-normality which was measured by univariate skewness (-1.25 < skewness < 2.0) and kurtosis (-1.0 < kurtosis < 8.0), and Mardia’s kurtosis (-4.9 < Mardia’s kurtosis < 49.1). The results showed that, compared with the parameter estimates of the base condition (multivariate normal distribution), the parameter estimates were still unbiased. The chi-squared statistics were not significantly inflated by non-normality (for all 12 non-normal conditions). However, at least one standard error of the parameter estimate under each non-normal condition was found to be negatively or positively biased.

TABLE 1 Five Combinations of Skewness and Kurtosis (N = 1000)*

	Skewness	Kurtosis
Case 1	0.08	0.00
Case 2	1.80	0.62
Case 3	5.63	6.65
Case 4	15.42	21.41
Case 5	0.13	13.92

* Source: Muthén and Kaplan (13). The original paper actually used multivariate relative kurtosis, defined as the sample multivariate kurtosis ($b_{2,p}$) divided by the expected value of the sample multivariate kurtosis ($\beta_{2,p}$), which was 0.989, 1.026, 1.277, 1.892 and 1.580 in Cases 1, 2, 3, 4 and 5, respectively. The number of variables (p) in the model was 4. We recalculated the relative kurtosis as Mardia’s multivariate kurtosis based on p = 4 and N = 1000 to be consistent with the multivariate kurtosis used in the present paper. The univariate skewness and kurtosis were the same for each variable in a case. The univariate skewness/ kurtosis measures for the five cases were 0.000/0.000, -0.742/-0.334, -1.217/1.615, -2.028/2.898 and 0.000/2.785, respectively.

Using samples drawn from a non-normal population with $\beta_{1,6}$ (multivariate skewness) = 0 and $\beta_{2,6}$ (multivariate kurtosis) = 63.9, and a normal population with $\beta_{1,6}$ = 0 and $\beta_{2,6}$ = 48, Henly (15) studied the effects of sample size, distribution, and non-normality on the chi-squared statistic. She found that 1) for the cases whose sample size was smaller than 300, even if the sample was multivariate normally distributed, the parameter estimates and standard errors were subject to biases due to the small sample size; 2) when a sample was multivariate normally distributed and its size was bigger than 300, the parameter estimates and standard errors were unbiased; 3) the sample sizes should be at least 600 to obtain unbiased parameter estimates for samples with a multivariate non-normal distribution; and 4) for samples with a multivariate non-normal distribution, regardless of the sample size, the model rejection frequencies were substantially higher than for the corresponding samples with a normal distribution, and the standard error estimates obtained from maximum likelihood estimation appeared not to be useful.

Curran et al. (16) compared the percentage bias of chi-squared statistics and percentages of rejecting models under moderate non-normality (skewness = 2, kurtosis = 7) and extreme non-normality (skewness = 3, kurtosis = 21), with those under a normal distribution (skewness = 0, kurtosis = 0), and found that the chi-squared statistics were positively biased by non-normality. Lei and Lomax (17) tested the impacts of non-normality with six combinations of univariate skewness (0 < skewness < 1.74) and kurtosis (0 < kurtosis < 3.8). They found that, compared with a normal distribution, non-normality did not have significant impacts on the means of standard errors for the parameter estimates and on the parameter estimates, but had a significant impact on the chi-squared statistics. They also found that either skewness or kurtosis led to a significant change in the chi-squared statistic.

In a broad sense, the findings on the consequences of violating the normality assumption in these studies are consistent. In terms of Hallow's and Curran et al.'s findings, the cutoffs for severe non-normality used by Muthén and Kaplan, and Lei and Lomax seem to be too low. The main shortcoming in these studies is that none of the studies gave out the univariate skewness and kurtosis, and multivariate skewness and/or kurtosis simultaneously. It makes it difficult for a researcher to choose a cutoff of non-normality if it exists in the data. We think more evidence is needed to exclude the possible effects of the number of variables on the assessment of multivariate normality of a sample. This can be done only if the normality is measured by multivariate skewness and/or kurtosis.

CONCEPTUAL MODEL AND DATA

For the purpose of testing the impacts of sample size and non-normality on model fit indices, parameter estimates, and standard errors of the parameter estimates, we use year 2000 census block group (BG) data. The study area, Sacramento County, California, has 787 census blocks after deleting five census block groups, which consist of prisons and a military base, from the BG sample.

Figure 1 illustrates the conceptual model (18, 19). Table 2 shows the descriptive statistics of the variables (to place the magnitudes of *job accessibility*, *income per capita* and *median rent* roughly in the same range, the raw values of these three variables are divided by 10000, 10000 and 1000, respectively) of the initial sample. *Median rent* (median asking rent per month) and *income per capita* (per year) are imported directly from the data CD we purchased from Geolytics, Inc. *Percentage of rental housing units* is defined as the percentage of renter-occupied housing units out of the total occupied housing units. *Education attainment* is calculated by dividing total persons who have a bachelor's or higher degree by the total population in a census tract. *Workers per capita* is computed by dividing the total number of employed persons by total population. *Household size* is calculated by dividing the total population by the total number of households. *Autos per capita* is computed by dividing the total number of vehicles by the

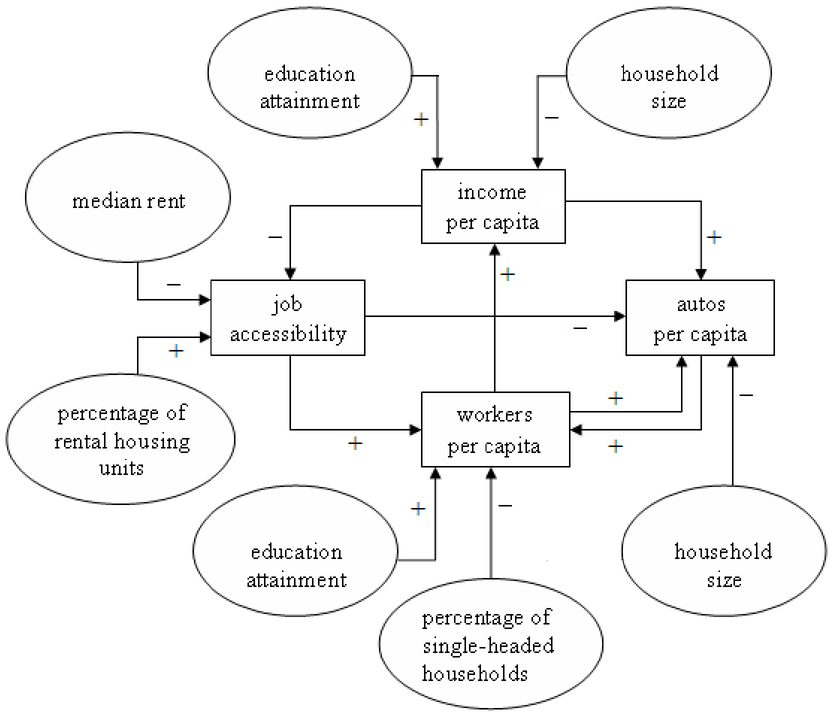

Figure 1 Conceptual causal relationships among job accessibility, employment, income and auto ownership

TABLE 2 Descriptive Statistics of the Variables (N = 787)

Variables	Minimum	Maximum	Mean	Standard Deviation
Percentage of rental housing units	0.00	100.00	40.58	25.50
Median rent ($1000/month)	0.00	1.88	0.75	0.23
Percentage of single-headed households with children	0.00	52.77	16.62	9.18
Education attainment (proportion with 4-year college degree or better)	0.00	0.65	0.17	0.13
Household size	1.18	9.40	2.71	0.70
Job accessibility (original index divided by 10,000)	1.47	7.28	4.19	0.88
Workers per capita	0.00	0.82	0.45	0.11
Income per capita ($10,000/year)	0.45	7.20	2.19	1.01
Autos per capita	0.02	1.13	0.64	0.15

total population. *Percentage of single-headed households with children* (*Percentage of single-headed households* in Figure 1) is defined as the percentage of single-headed households with children out of the total number of households. Job accessibility is calculated as:

$$CBGJobAccessibility_m = \sum_{i=1}^{n_m} Job_i + \Big(\sum_{\substack{i=1 \\ TAZi \subset CBGm}}^{n_m} \sum_{\substack{j=1 \\ TAZj \not\subset CBGm}}^{N-n_m} Job_j * (1/TravelTime_{ij}) \Big) / n_m \qquad (1)$$

where $CBGJobAccessibility_m$ is the job accessibility in census block groups m; n_m is the number of traffic analysis zones (TAZs) contained in census block groups (CBG) m ($TAZ_i \subset CBG_m$); Job_i is the number of jobs in TAZ i, and $TravelTime_{ij}$ is the travel time from TAZ i to j in the three-hour AM peak period on the highway network (17).

ASSESSMENT OF MULTIVARIATE NORMALITY

In AMOS 5, univariate and multivariate normalities are evaluated in one step. Table 3 shows the results of the assessment of the univariate normality for each variable and the multivariate normality of the initial sample (N = 787). According to Muthén and Kaplan's cutoffs, *Median rent* and *income per capita* are slightly non-normal while *household size* is severely non-normal in terms of skewness, and the kurtosis of *household size* indicates a severe non-normal distribution of this variable. By Curran et al.'s standards, the univariate variate skewness and kurtosis are moderate. However, the large multivariate kurtosis implies that the sample has a severely multivariate non-normal distribution.

To reach a multivariate normal distribution, we deleted five observations at a time, based on the Mahalanobis distance. After the first five observations were deleted, the multivariate kurtosis dropped sharply from 101.61 to 31.99. However, the critical ratio falls at 1.96 only after 137 observations are deleted. Thus, 17% of the observations had to be deleted in order to achieve the desired critical ratio. From the table, it can be seen that, when the critical ratio of multivariate kurtosis is smaller than 1.96, the absolute values of univariate skewness and kurtosis for all variables are not necessarily close to 0, but are equal to or smaller than 1. In this respect, it appears that measuring only multivariate kurtosis may be good enough for the purpose of assessing multivariate normality.

It is noted that, among the deleted observations, only six can be considered to be outliers in terms of their Mahalanobis distances being much larger than those of the other observations. Deleting these outliers lowers not only univariate non-normality but also multivariate non-normality (by 86.4%). Therefore, it is these six true outliers that lead to severe univariate and multivariate non-normality. Aside from these, the marginal and cumulative contributions of the other deleted observations to the departure of the sample from normality are relatively small.

TABLE 3 Assessment of Normality of Initial Sample

Variables	Skewness (N = 787)	Critical Ratio of kewness (N = 787)	Kurtosis (N = 787)	Critical Ratio of Kurtosis (N = 787)	Skewness (N = 650)	Critical Ratio of Skewness (N = 650)	Kurtosis (N = 650)	Critical Ratio of Kurtosis (N = 650)
Percentage of rental housing units	0.54	6.13	-0.62	-3.57	0.47	4.89	-0.62	-3.24
Median rent	1.10	12.55	3.47	19.87	0.84	8.70	1.00	5.21
Percentage of single-headed households with children	0.82	9.44	1.49	8.51	0.50	5.18	-0.07	-0.38
Education attainment	0.93	10.68	0.31	1.77	0.83	8.64	-0.05	-0.25
Household size	2.23	25.56	15.93	91.23	0.20	2.04	-0.42	-2.16
Job accessibility	-0.11	-1.21	0.44	2.54	-0.26	-2.74	0.16	0.81
Workers per capita	-0.24	-2.79	0.57	3.28	-0.22	-2.25	-0.14	-0.74
Income per capita	1.25	14.29	2.56	14.65	0.47	4.92	-0.35	-1.83
Autos per capita	-0.62	-7.08	0.69	3.92	-0.45	-4.68	-0.21	-1.07
Multivariate			101.61	101.29			2.06	1.97

TABLE 4 Unstandardized Parameter Estimates and Their Standard Errors

Dependent Variable	Explanatory Variable	Estimate (N = 787)	Standard Error (N = 787)	*P*-value (N = 787)	Estimate (N = 781)	Standard Error (N = 781)	*P*-value (N = 781)	Estimate (N = 650)	Standard Error (N = 650)	*P*-value (N = 650)
Job accessibility	*Percentage of rental housing units*	0.225	0.011	***	0.221	0.011	***	0.205	0.011	***
Job accessibility	*Median rent*	-0.140	0.087	0.107	-0.139	0.079	0.079	-0.198	0.088	0.025
Job accessibility	*Income per capita*	0.279	0.030	***				0.288	0.039	***
Workers per capita	*Percentage of single-headed households with children*	-0.001	0.000	***	-0.001	0.000	***	-0.001	0.000	***
Workers per capita	*Education attainment*	0.216	0.035	***	0.253	0.033	***	0.273	0.033	***
Workers per capita	*Job accessibility*	0.013	0.004	0.003	0.009	0.004	0.012	0.002	0.004	0.604
Workers per capita	*Autos per capita*	0.333	0.036	***	0.292	0.033	***	0.334	0.031	***
Income per capita	*Education attainment*	3.094	0.478	***	2.732	0.532	***	1.691	0.451	***
Income per capita	*Workers per capita*	7.598	0.855	***	8.279	0.968	***	7.873	0.720	***
Autos per capita	*Household size*	-0.074	0.006	***	-0.106	0.007	***	-0.120	0.008	***
Autos per capita	*Job accessibility*	-0.117	0.007	***	-0.132	0.008	***	-0.133	0.010	***
Autos per capita	*Workers per capita*	0.454	0.035	***	0.380	0.033	***	0.395	0.035	***
Autos per capita	*Income per capita*	0.056	0.005	***	0.053	0.005	***	0.052	0.006	***

***: *p*-value < .001

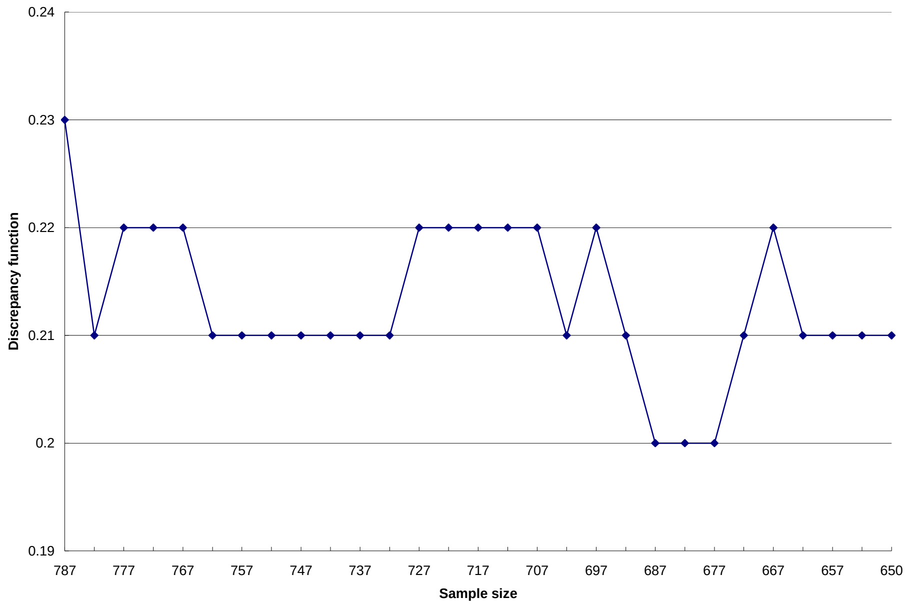

Figure 2 Discrepancy function F_{ml} under different multivariate kurtoses

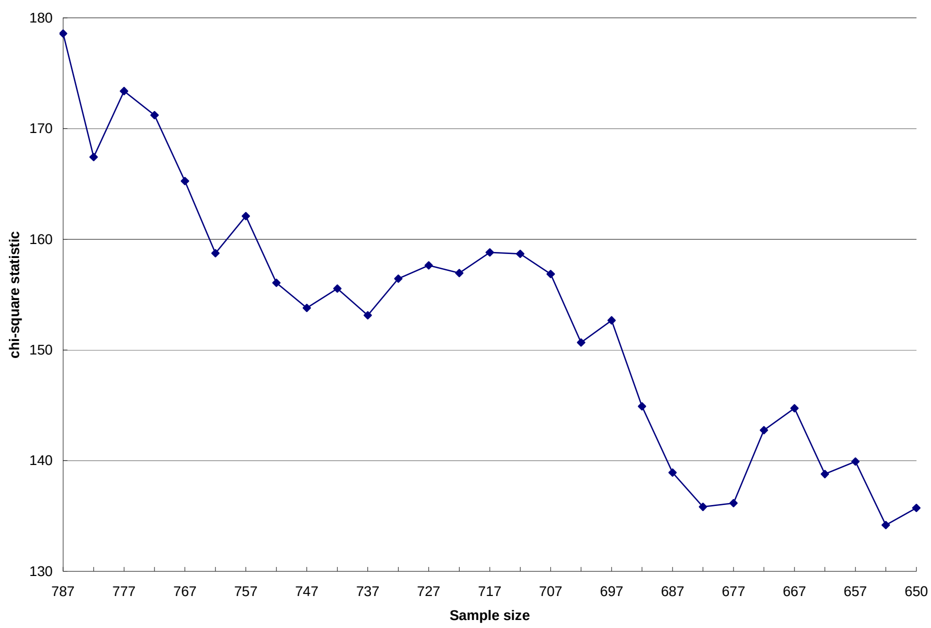

Figure 3 Chi-squared statistic (N-1)F_{ml} under different multivariate kurtoses

TABLE 5 Assessment of Model Fit

Model Fit Index	Independence Model (N = 787)	Hypothesized Model (N = 787)	Saturated Model (N = 787)	Independence Model (N = 650)	Hypothesized Model (N = 650)	Saturated Model (N = 650)
χ^2	4103.26	178.59	0.00	4188.82	135.73	0.00
Degrees of freedom (d.f.)	36.00	9.00	0.00	36.00	9.00	0.00
Goodness-of-fit index (GFI)	0.42	0.96	1.00	0.37	0.96	1.00
Normed fit index (NFI)	0.00	0.96	1.00	0.00	0.97	1.00
Incremental fit index (IFI)	0.00	0.96	1.00	0.00	0.97	1.00
Comparative fit index (CFI)	0.00	0.96	1.00	0.00	0.97	1.00
Akaike information criterion (AIC)	4121.26	250.59	90.00	4206.82	207.73	90.00
Expected cross-validation index (ECVI)	5.24	0.32	0.16	6.48	0.32	0.14

THE PARAMETER ESTIMATES AND STANDARD ERRORS

Using ML estimation, the hypothesized model is estimated. Tracking the changes of sample size and multivariate normality, we find that the *p*-value of the coefficient estimate of *median rent* on *job accessibility* becomes smaller and the *p*-value of *job accessibility* on *workers per capita* becomes larger with the decrease of the critical ratio of Mardia's kurtosis. From Table 4, we can see that the direct effect of *median rent* on *job accessibility* is not significant at the 0.1 level in the sample of 787 observations while it is significant at the 0.05 level when the sample size is reduced to 650. The direct effects of *job accessibility* on *workers per capita* become insignificant ($p = 0.604$) when a multivariate normal distribution is reached (N = 650). Five of the parameter estimates have a variation in magnitude of more than 20%. After the sample size is reduced, four of 13 parameter estimates have larger standard errors, and two of 13 have smaller standard errors.

THE MODEL FIT INDICES

Following the principles suggested by Bollen and Long (20), Hoyle and Panter (21), and Shah and Goldstein (22), we report the model fit indices from several different index families. From Table 5, we can see that the changes in the sample size and multivariate normality do not have effects on the goodness–of-fit (GFI), normed fit (NFI), incremental fit (IFI), and comparative fit indices (CFI) and the expected cross-validation index (ECVI), while they lead to a decrease of 24% in the chi-squared statistic and a decrease of 17% in the Akaike information criterion (AIC). In terms of these model fit indices, the achievement of multivariate normality improves the model fit.

In ML estimation, the chi-squared statistic is the product of (N-1) and the discrepancy function (F_{ml}). Thus, it is very sensitive to sample size (N) and cannot well reflect the effects of non-normality on the discrepancy function when the change in non-normality is accompanied by a change in sample size, as in this study. Monte Carlo simulation studies have demonstrated that, when sample size is controlled, multivariate non-normality will lead to an inflation of the chi-squared statistic (12, 15, 16). Figure 2 shows the change in the discrepancy function when outliers are deleted. The F_{ml} is largest, at 0.23, for N = 787 and fluctuates slightly around 0.21 when the sample size becomes smaller. This suggests that deleting outliers improves multivariate normality but may or may not lead to a smaller discrepancy function and chi-squared statistic (see Figure 3).

DISCUSSION AND CONCLUSIONS

This study, as well as our previous study (19), show that even when the cutoff for univariate skewness and/or kurtosis recommended by Muthén and Kaplan is achieved for most (but not all) of the variables in the data, the multivariate kurtosis can be large and multivariate non-normality can be extreme (critical ratio >> 1.96). Some outliers may be included in the sample. It is possible that the estimates of the standard errors and chi-squared statistic are severely biased (15). When a sample with a multivariate normal distribution (critical ratio < 1.96) is obtained by deleting some observations, the univariate skewnesses and kurtoses are within the range between -1 and +1. The estimates based on the reduced sample are unbiased. However, the reduced sample having a multivariate normal distribution does not include some observations containing important information about the covariance among the variables. Based on the SEM parameter estimates and their *p*-values, the covariance structure of the reduced sample seems rather different from that of the original sample, although the sample means are similar. Particularly, the parameter estimate representing the impacts of *job accessibility* on the *workers per capita* (see Table 4) is insignificant. From the perspective of statistics, it implies that the contribution of improving job accessibility to increase employment ratio is very limited. This may lead to underestimation of the role of job accessibility in enhancing employment ratio. Therefore, we suggest that 1.96 should not be used as a steadfast threshold of the critical ratio for practical conformance to multivariate normality. In the sample of this study, after six true outliers are deleted, the multivariate kurtosis drops to 28.78 and the critical ratio is 28.56. All the univerate skewnesses and kurtoses are smaller than the thresholds of the moderate non-normality (skewness = 2, kurtosis = 8) in Curren et al.'s (16) and Hallow's (14) studies. In Muthén and Kaplan's (13) study, even when the multivariate kurtosis was as high as 21.408, the biases for the estimates of parameters and standard errors of the parameter estimates were no more than 5%. This suggests that the biases for the parameter estimates and the standard errors of the parameter estimates in our results may be controlled at an acceptable level when the six true outliers are deleted. At this point, the loss of information due to deleting observations seems to be minimized while the results of the significance tests on the parameter estimates appear to be robust enough to support policy analysis.

It should be emphasized that deletion of the observations in this study is determined only by Mahalanobis distance and is stopped when the critical ratio of the Mardia's coefficient of multivariate kurtosis is smaller than 1.96. The deleted observations can be classified into two groups. The mean of the Mahalanobis distances of the observations in the first group is much larger than the mean of the distances of the observations in the reduced sample. The difference between each distance in the first group is substantial. Therefore, these observations may be considered as true outliers. In this study, there are only six observations in that group. These six outliers greatly increase the multivariate kurtosis and some univariate skewnesses and/or kurtoses, and distort the estimates for parameters, standard errors, and the chi-squared statistic (9). It is these outliers that make the parameter estimate of the direct effect of *job accessibility* on *workers per capita* significant at the 0.01 level in the original sample. From the perspective of lowering the distortion of the estimates, these outliers should be either transformed or deleted.

The mean of the Mahalanobis distances of the observations in the second group is substantially smaller than that in the first group, while still larger than that in the reduced sample. The differences among the distances in the second group are small. The larger distances simply mean that the observations are a little farther from the sample mean than those in the reduced sample. It appears to be improper to consider these observations as outliers. From Table 4, we can see that keeping these observations makes the parameter estimate of the direct effect of *job accessibility* on *workers per capita* is significant at the 0.05 level. Therefore, our suggestion is to keep these observations in the sample even if the critical ratio of the multivariate kurtosis is somewhat larger than desired as a result.

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