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# Commodity Money Equilibrium in a Walrasian Trading Post Model: An Elementary Example

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## **Abstract**

Walrasian general competitive equilibrium is considered in a simple example of an exchange economy with commodity-pairwise trading posts and transaction costs. Budget balance is enforced at each trading post separately. Commodity-denominated bid and ask prices at each post allow the post to cover transaction costs through the bid/ask spread. In the absence of double coincidence of wants, the lowest transaction-cost commodity (with the narrowest bid/ask spread) becomes the common medium of exchange, commodity money. Selection of the monetary commodity and adoption of a monetary pattern of trade results from price-guided equilibrium without central direction, fiat, or government.

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# 1 The Theory of Money and the Theory of Value

There is a long-standing problem in monetary theory, Hicks (1935): bringing price theory and monetary theory together — ideally, so that they are mutually reinforcing — at least so that they are consistent with one another. Price theory is the most elementary part of economic theory. It should be possible to derive the foundations of monetary theory from principles of price theory. This several-hundred year old topic remains a recurring challenge to economic theory. <sup>1</sup>

Hence, Nobel Laureate James Tobin (1961) commented: "The intellectual gulf between economists' theory of the values of goods and services and their theories of the value of money is well known and periodically deplored ... our students[']...mastery of the presumed fundamental, theoretical apparatus of economics is put to very little test in their studies of monetary economics..."

Prof. Frank Hahn (1982) described the impasse: "The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it. The best developed model is, of course, the Arrow-Debreu version of a Walrasian general equilibrium. A first, and...difficult...task is to find an alternative construction without...sacrificing the clarity and logical coherence ... of Arrow-Debreu."

Eventually, Tobin (1980) decided that the research program Hahn implicitly recommended would necessarily be unsuccessful: "Social institutions like money are public goods. Models of general equilibrium — competitive markets and individually optimizing agents — are not well adapted to explaining the existence and quantity of public goods... General equilibrium theory is not going to explain the institution of a monetary ... common means of payment."

This paper presents a successful simple example of the project that Hahn sets out (and Tobin says is impossible): To augment the Arrow-Debreu general equilibrium model sufficiently to allow monetary structure to appear as a result, not an assumption, of the model. The essential point of money — the medium of exchange — is that it is a carrier of value held between successive

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<sup>1</sup> Recent contributions to this topic include the overlapping generations model, Wallace (1980) and a vast literature following, the search and matching model, Jones (1976) and Kiyotaki and Wright (1989) with a similarly vast literature, Walrasian general equilibrium with transaction cost models with a smaller literature including Foley (1970), Hahn (1971), Heller and Starr (1976), Howitt (2000), Kurz (1974), Starrett (1973), and Starr (2003a, 2003b).

transactions. A model of the transactions foundations of money then needs to portray a succession of transactions. The Arrow-Debreu model uses a single budget constraint summarizing all buying and selling transactions in a single equation. In order to model the function of a carrier of value between transactions the model will need to distinguish transactions individually. That notion is formalized below, in an oversimplified way, as a trading post model. This paper will demonstrate that the trading post model can generate endogenously a flow of a common medium of exchange.

The history of this notion goes back almost a century before Hahn's remarks, to German-Austrian 19th century monetary theory. Carl Menger (1892) wrote: "[Call] goods ... more or less saleable, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution... Men ... exchange goods ... for other goods ... more saleable....[which] become generally acceptable media of exchange."

What Menger says here is that every traded good is characterized by a bid (wholesale) price and an ask (retail) price. A commodity that acts as a medium of exchange is necessarily repeatedly bought (accepted in trade) and sold (delivered in trade). However, it is costly repeatedly to buy and sell the same good since such transactions repeatedly incur transaction costs; they buy high (at the ask price) and sell low (at the bid price). Therefore a good with a narrow spread between bid and ask (a narrow bid/ask spread, a narrow wholesale/retail margin) is priced to act as a medium of exchange with relatively low cost. It is a natural medium of exchange, a natural commodity money. Formalizing Menger's remark in a simple example is the task of the remainder of this paper.

The starting point for a model of money as a medium of exchange is to set up a trading system with many separate transactions, so that there is a role for a carrier of value between them. The model presented here does that in commodity pairwise trading posts. Walras (1874) forms the picture this way (assuming  $m$  distinct commodities): "we shall imagine that the place which serves as a market for the exchange of all the commodities (A), (B), (C), (D) ... for one another is divided into as many sectors as there are pairs of commodities exchanged. We should then have  $\frac{m(m-1)}{2}$  special markets each identified by a signboard indicating the names of the two commodities exchanged there as well as their ... rates of exchange..."<sup>2</sup>

In the example of commodity-pairwise trade below, barter is possible, but monetary trade is the competitive general equilibrium outcome. As Tobin

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<sup>2</sup>Cournot (1838) and Shapley and Shubik (1977) also treat the trading post model.

(1980) notes, "Money is not the only way of avoiding the restrictions of 'double coincidences.'<sup>3</sup> Individuals can exchange their endowments for commodities they do not wish to consume ... and hold those for later ... exchange." Thus any good may be used as a medium of exchange. The puzzle is to find out why that function tends to be specialized in a single instrument. The present example gives one very elementary answer.

## 2 Households, Trading Posts, Transaction Costs

This paper will focus on the following simple example. Consider a pure exchange trading post economy with ten commodities denoted 1, 2, 3, ..., and 10.

### 2.1 Households

Let  $[i,j]$  denote a household endowed with good  $i$  who prefers good  $j$ ;  $i \neq j$ ,  $i,j = 1, 2, \dots, 10$ . Household  $[i,j]$ 's endowment is 1 unit of commodity  $i$ .  $[i,j]$ 's utility function is  $u^{[i,j]}(x_1, x_2, x_3, \dots, x_{10}) = \sum_{k \neq j} x_k + Ax_j$ ,  $A \gg 1$ . That is, household  $[i,j]$  values goods 1, 2, 3, ..., 10 as linear substitutes, with good  $j$  being many times more desirable to  $[i,j]$  than any other.

Consider a population denoted  $\Lambda$  of households including several households endowed with each good and each household desiring a good different from its endowment. Thus, there are four households endowed with good 1, preferring respectively, goods 2, 3, 4, and 5:  $[1,2]$ ,  $[1,3]$ ,  $[1,4]$ ,  $[1,5]$ . There are four households endowed with good 2, preferring respectively goods 3, 4, 5, 6:  $[2,3]$ ,  $[2,4]$ ,  $[2,5]$ ,  $[2,6]$ . The roll call of households proceeds so forth, through  $[9, 10]$ ,  $[9,1]$ ,  $[9, 2]$ ,  $[9, 3]$  and finally  $[10, 1]$ ,  $[10, 2]$ ,  $[10, 3]$ , and  $[10,4]$ .

Population  $\Lambda$  displays absence of double coincidence of wants. For each household endowed with good  $i$  and desiring good  $j$ ,  $[i,j]$ , there is no precise mirror image,  $[j,i]$ . Nevertheless, there are four households endowed with one unit of commodity 1, and four households strongly preferring commodity 1 to all others. That is true for each good. Thus gross supplies equal gross

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<sup>3</sup> The notion of "double coincidence of wants", Jevons (1875), posits that barter requires pairwise mutually improving trades. In barter, an exchange of good  $i$  for good  $j$  is supposed to include one trader with an excess supply of  $i$  and an excess demand for  $j$ , and a second trader with the opposite unsatisfied supply and demand.

demands, though there is no immediate opportunity for any two households to make a mutually advantageous trade. This is precisely the setting where money is suitable to facilitate trade, Jevons (1875).

## 2.2 Trading posts

For each pair of distinct commodities, there is a trading post where those two commodities are traded. The notation  $\{1,2\}$  represents the trading post where good 1 is traded for good 2 and (vice versa) good 2 is traded for 1. Operating the trading post is a resource-using activity. The proprietor of the trading post provides his labor to operate the post. He receives in compensation a portion of the goods traded at the post. For ease of notation, his labor will be compensated unit-for-unit by goods traded at the post. Denote this cost of operating trading post  $\{i,j\}$  as  $C^{\{i,j\}}$ .

## 2.3 Trading posts and transaction costs

The notion of a trading post here is a complication of the Arrow-Debreu model, and a simplification of the trading possibilities — most of them inactive — in actual economies. The Arrow-Debreu model includes delivering goods and services to a single (centralized) market, receiving an accounting credit for the delivery, and withdrawing goods and services of equal value. The trading post model further decentralizes this process. In the trading post model, there is a choice of delivering a good (or service) in exchange separately and distinctly for any of the other goods or services.

Accounts must balance at each trading post — that is, you pay for what you get not only over the course of all trade (as in the Arrow-Debreu model) but at each trading post separately. This is a note of realism; that is how budget constraints apply in actual transactions. A household delivers supplies to the trading post, and they are evaluated at the post's bid price. The household takes its demands from the post, and they are evaluated at the post's ask price. Budget balance requires that the values be equal. You pay for what you get at each post separately with deliveries to the post valued at the post's bid price and withdrawals from the post at the post's ask price.

The notion of a trading post for each good in exchange for each alternative sets up many more specialized trading institutions than we expect actually to

see in any economy, but it is a convenient formalization. With  $N$  commodities, using the  $N(N - 1)/2$  trading post model (45 trading posts when  $N = 10$ ) as a basis for deriving the use of a common medium of exchange represents two notions: (i) that a meaningful discussion of means of payment, money, depends on the notion that goods do not trade for all other goods simultaneously; segmentation of the market is part of monetization, Alchian(1977); (ii) monetary trade is an equilibrium outcome based on individual optimization and market clearing, where barter (without a double coincidence of wants restriction) could be chosen as an alternative.

The resources that go into the operating cost of a trading post are presented here only as the labor of the post proprietor. This is intended as a convenient (overly simple) representation of transaction costs. In actual economies these costs include the inputs of trading firms such as brokers, retailers, shippers, etc. and the non marketed resources of households and firms used in the transactions process. The latter are typically not priced explicitly, but they enter into household and firm trading decisions. They are summarized here as part of the bid/ask spread. This representation is unrealistic but convenient and effective.

## 3 Transaction Costs and Prices at a Trading Post

### 3.1 Transaction Costs

Consider trading posts with a linear transaction cost structure. Thus, let the cost structure of trading post  $\{1,2\}$  be:

$$C^{\{1,2\}} = .1 \times (\text{volume of goods 1 and 2 purchased})$$

Marginal cost of trading 1 for 2 (in equal quantities) is 0.1 times the gross quantity traded. The trading post expects to cover its transaction costs through the bid/ask spread. Assume similar cost functions at almost all of the trading posts. There is one group of exceptions; trading good 10 is assumed to be costless. Thus,

$$C^{\{10,j\}} = .1 \times (\text{volume of good } j \text{ purchased}), \text{ for } j = 1, 2, \dots, 9.$$

### 3.2 Bid and ask prices

Trading post  $\{1,2\}$  accepts good 1 in exchange for good 2 and accepts good 2 in exchange for good 1. Prices are expressed as a rate of exchange between goods 1 and 2. That is, good 1 is priced in units of good 2 and good 2 is priced in units of good 1. In order to cover the post's operating costs, the prices at which the public buys (ask or retail prices) are higher than those at which the public sells (bid or wholesale prices). The difference between buying and selling prices covers operating costs.

Trading post  $\{1,2\}$  may post a bid price for good 1 of 0.9 units of good 2 and a bid price for good 2 of 0.9 units of good 1 (the symmetry is convenient but inessential). Then on an exchange of one unit of 1 for 2 (or one unit of 2 for 1), the trading post keeps 0.1 unit of 1 (or 0.1 unit of 2). Each side of the trade is priced separately. When the trading post engages (as it must in equilibrium) in both sides of this trade it retains 0.1 unit of each good to cover costs. Marginal cost pricing leads to bid prices depicted in Table 1, below.

Each entry in the table represents the bid price (denominated in units of the row good) for delivery of a unit of the column good. Thus, the diagonal is blank — no good is bought or sold for itself. The prices in this example show that selling one unit of good 1 for good 2 pays 0.9 units of 2. Conversely, selling one unit of good 2 for good 1 pays 0.9 units of 1. Reflecting marginal costs, the bid price of good 10 is unity. Consider trade at the  $\{1,2\}$  trading post. Suppose one trader delivers one unit of 1 and a second trader delivers one unit of 2. The post pays out .9 good 2 to the first and .9 good 1 to the second. Trade at the post clears. The remainder, .1 good 1 and .1 good 2, stays with the trading post covering its operating costs.

At trading post  $\{i,j\}$ , the ask price of  $j$  (denominated in  $i$  per unit  $j$ ) is the inverse of the bid price of  $i$  (denominated in  $j$  per unit  $i$ ). The bid price of goods 1, ..., 9 in Table 1 is 0.9, implying that the ask price is 1.11. Denote the bid price of good  $i$  at  $\{i,j\}$  as  $q_i^{\{i,j\}}$ . Then the ask price of  $j$  is  $[q_i^{\{i,j\}}]^{-1}$ . Denote the purchase of  $i$  by a typical household as  $b_i$ , sale of  $j$  as  $s_j$ . Then the budget constraint facing a typical household at  $\{i,j\}$  is  $s_j q_j^{\{i,j\}} \geq b_i [q_i^{\{i,j\}}]^{-1}$ .

In an economy of  $N$  commodities there are  $N(N-1)/2$  trading posts each with two posted prices (bid for one good in terms of a second, and bid price of the second in units of the first) totaling  $N(N-1)$  pairwise price ratios. In this paper's example, with 10 commodities, there are 90 posted bid prices in Table 1.



Table 1: Example: Marginal Cost Pricing Starting Bid prices at trading posts

selling:	1	2	3	4	5	6	7	8	9	10
buying:	1	X	.9	.9	.9	.9	.9	.9	.9	1
	2	.9	X	.9	.9	.9	.9	.9	.9	1
	3	.9	.9	X	.9	.9	.9	.9	.9	1
	4	.9	.9	.9	X	.9	.9	.9	.9	1
	5	.9	.9	.9	.9	X	.9	.9	.9	1
	6	.9	.9	.9	.9	.9	X	.9	.9	1
	7	.9	.9	.9	.9	.9	.9	X	.9	1
	8	.9	.9	.9	.9	.9	.9	.9	X	1
	9	.9	.9	.9	.9	.9	.9	.9	.9	X
	10	.9	.9	.9	.9	.9	.9	.9	.9	X

### 3.3 Pricing at inactive trading posts

The principal function of the price system is to provide incentives that communicate what goods should be produced. Less conspicuous, but obvious and equally important, prices provide the incentives that determine which goods should not be produced. Goods provided at zero quantity are not produced because the (implicit) prices at which they would trade make the equilibrium quantity zero.

In a model with transaction costs, the transaction process is a production activity. The price system here must answer the question: which trading posts operate at positive trading volume? In actual economies, most conceivable pairwise commodity trades do not occur. Professors would like to trade lectures for food, but find that the implicit market prices for this exchange are unattractive. Better to trade lectures for money and money for food.

How is this choice formalized with the help of the price system? A trading post becomes unattractive in equilibrium, and will have zero trading volume, when its bid/ask spread is wide enough to discourage trade. When a trading post is inactive, how can we know its bid and ask prices? This is essentially the issue of pricing corner solutions in output in general equilibrium. Any good produced and consumed at zero volume in equilibrium has the same problem as the inactive trading posts in this example. If there's no trade, no production, and no consumption, how can we know the price? Conversely, it's the price that determines zero volume. Zero volume goods in actual economies include pure platinum-body automobiles and combination Swiss-Army-Knife-cell phones. Most zero-volume goods are not priced explicitly, but the prices implicit in their costs of production and demand conditions result in zero volume. Thus, if a professor of economics goes to the supermarket and asks to pay for his shopping cart of groceries with an economics lecture (spot or future), the manager can be expected to say, "With respect, sir, I don't see how we can arrange that." The translation of this reply to the language of prices is: the bid price of an economics lecture at the trading post {economics lecture, grocery} is very near zero (though the money price is in the hundreds of dollars at the nearby university). The reason for this anomaly is the transaction costs implicit in retrading an economics lecture. The supermarket should be delighted to accept an economics lecture (at wholesale) and retrade it at retail for additional groceries (wholesale), but the transaction costs incurred in arranging this trade are prohibitive. That message is conveyed in the price system by a wide bid/ask spread and by the manager who says "I don't see how we can arrange that."

A barter equilibrium would be an outcome where most pairwise trading posts operate at a positive trading volume. Conversely, most of the 45 trading posts posited here may be inactive (have zero trade) in equilibrium, but it is the price system in equilibrium that determines their inactivity. In a monetary equilibrium, trading activity concentrates on the 9 trading posts that deal in a single one of the 10 goods, the commodity 'money' traded for the other 9 goods. The money prices of a good then are the bid and ask prices at the trading post where it is traded for the commodity 'money.'

## 4 Marginal cost pricing equilibrium

An array of prices  $q_i^{o\{i,j\}}$  and trades  $b_i^{oh}, s_j^{oh}$  for  $h \in \Lambda$  is said to be a *marginal cost pricing equilibrium* if each household  $h \in \Lambda$  optimizes utility subject to budget at prevailing prices, each trading post clears, and trading posts cover marginal costs through bid/ask spreads at prevailing trading volume. This description purposely leaves unspecified whether marginal costs are recouped through the bid/ask spread on  $i, j$ , or both. More formally a marginal cost pricing equilibrium under the transaction cost function above consists of  $q_i^{o\{i,j\}}, q_j^{o\{i,j\}}, 1 \leq i, j \leq 10, i \neq j$ , so that:

For each household  $h \in \Lambda$ , there is a utility optimizing plan  $b_n^{oh\{i,j\}}, s_n^{oh\{i,j\}}$  so that

$$b_i^{oh\{i,j\}} [q_j^{o\{i,j\}}]^{-1} \leq s_j^{oh\{i,j\}} q_j^{o\{i,j\}} \quad (\text{budget balance}),$$

$$\sum_h b_n^{oh\{i,j\}} \leq \sum_h s_n^{oh\{i,j\}}, \quad n = i, j \quad (\text{market clearing}),$$

$$\begin{aligned} & \text{For } i, j \neq 10, \quad 0.1 + 0.1 [q_j^{o\{i,j\}}]^{-1} \\ & = [q_j^{o\{i,j\}}]^{-1} \{ [q_i^{o\{i,j\}}]^{-1} - q_j^{o\{i,j\}} \} + \{ [q_j^{o\{i,j\}}]^{-1} - q_i^{o\{i,j\}} \} \quad (\text{transaction} \\ & \text{cost coverage}). \end{aligned}$$

$$\begin{aligned} & \text{For } i \neq 10, j = 10, \quad 0.1 [q_i^{o\{i,10\}}]^{-1} \\ & = \{ [q_i^{o\{i,10\}}]^{-1} - q_{10}^{o\{i,10\}} \} + [q_i^{o\{i,10\}}]^{-1} \{ [q_{10}^{o\{i,10\}}]^{-1} - q_i^{o\{i,10\}} \} \quad (\text{trans-} \\ & \text{action cost coverage}). \end{aligned}$$

The concluding expressions are a marginal cost pricing condition; the bid/ask spreads times trading volumes at prevailing prices summed over  $i$  and  $j$  at trading post  $\{i,j\}$  should equal the sum of the marginal costs multiplied by implied trading volumes.

### 4.1 Starting trades

Start with the population  $\Lambda$  described in Section 2.1, and the transaction costs described in Section 3.1 summarized as the price array in Table 1. Each household decides what trade it wants to make. Household  $[1,2]$  goes to trading

post  $\{1,2\}$  and sells its good 1 for good 2. All the other households behave similarly.

This pattern of trade poses a problem. It is not an equilibrium. At each of the active trading posts, there is an excess demand. Trading post  $\{1,2\}$  has a supply of good 1 and a demand for good 2. It needs some trader to come with a complementary supply of good 2 and a demand for good 1. All of the active trading posts have a similar problem.

## 4.2 Price adjustment

Seeing an excess demand for one of their goods and an excess supply of the other at each post, the price mechanism (aided by trading post managers or the Walrasian auctioneer) adjusts prices as in Table 2.

Goods in excess demand have their bid prices increased to unity. Goods in excess supply are discounted to bear the full transaction cost of both sides of their trade.

## 5 Monetary Equilibrium

But the adjusted prices in Table 2 do not lead to a barter equilibrium. Household  $[1,2]$  considers trading 1 for 2 at trading post  $\{1,2\}$ . But  $\{1,2\}$  has  $q_1^{\{1,2\}} = 0.8$ , representing a discount of 20% on  $[1,2]$ 's endowment of good 1. There appears to be a more attractive alternative. At the posted prices  $[1,2]$  can exchange 1 for 10 without discount at  $\{1, 10\}$  and then 10 for 2 with a 10% discount at  $\{2, 10\}$ , a far better deal. At Table 2's prices, households want to treat good 10 as commodity money.

Completing the symmetry of this trading array, the Walrasian auctioneer can adjust the price matrix to Table 3.

Table 3 represents a symmetric price system that guides the economy into using good 10 as the common medium of exchange. Goods 1, 2, ..., 9 trade for good 10 at a price of .9. The discount of .1 unit of n on each of four trades covers the post's operating costs. Typical trading behavior for household  $[i, j]$  (for  $i, j \neq 10$ ) is  $s_i^{[i,j]\{i,10\}} = 1$ ,  $b_1^{[i,j]\{i,10\}} = 0.9$ ,  $s_1^{[i,j]\{j,10\}} = 0.9$ ,  $b_j^{[i,j]\{j,10\}} = 0.9$ . All other trades for  $[i,j]$  are nil. For households desiring to consume or to supply good 10 (not merely trade it), the equilibrium trades are

Table 2: Example: Marginal Cost Pricing - Adjusted bid prices at trading posts

selling:	1	2	3	4	5	6	7	8	9	10	
buying:	1	X	1	1	1	1	.9	.8	.8	.8	.9
	2	.8	X	1	1	1	1	.9	.8	.8	.9
	3	.8	.8	X	1	1	1	1	.9	.8	.9
	4	.8	.8	.8	X	1	1	1	1	.9	.9
	5	.8	.8	.8	.8	X	1	1	1	1	1
	6	.9	.8	.8	.8	.8	X	1	1	1	1
	7	1	.9	.8	.8	.8	.8	X	1	1	1
	8	1	1	.9	.8	.8	.8	.8	X	1	1
	9	1	1	1	.9	.8	.8	.8	.8	X	1
	10	1	1	1	1	.9	.9	.9	.9	.9	X

$s_i^{[i,10]\{i,10\}} = 1$ ,  $b_{10}^{[i,10]\{i,10\}} = .9$ ,  $s_{10}^{[10,j]\{j,10\}} = 1$ ,  $b_{10}^{[10,j]\{j,10\}} = 1$ . The arrangement is a market clearing equilibrium with all trade going through good 10.

Good 10 acts as medium of exchange, commodity money. The trading posts dealing in good 10,  $\{10,1\}$ ,  $\{10,2\}$ ,  $\{10,3\}$ , ...,  $\{10,n\}$ , ...,  $\{10,9\}$ , cover their operating costs. For each good  $n = 1,2,3,\dots,9$ , they find four sellers coming to the post delivering one unit of  $n$  in exchange for 10, and four buyers coming to the post, exchanging good 10 for good  $n$ . The trading post clears.

Household  $[3,4]$ , for example, wants to trade good 3 for good 4. He considers the pricing array above. He considers trading the goods directly at  $\{3,4\}$ . Pricing at  $\{3,4\}$  means that household  $[3,4]$  could deliver good 3 to  $\{3, 4\}$  and

Table 3: Example: A monetary equilibrium, good 10 is money, bid prices at trading posts

selling:	1	2	3	4	5	6	7	8	9	10
buying:	1	X	.8	.8	.8	.8	.8	.8	.8	1
	2	.8	X	.8	.8	.8	.8	.8	.8	1
	3	.8	.8	X	.8	.8	.8	.8	.8	1
	4	.8	.8	.8	X	.8	.8	.8	.8	1
	5	.8	.8	.8	.8	X	.8	.8	.8	1
	6	.8	.8	.8	.8	.8	X	.8	.8	1
	7	.8	.8	.8	.8	.8	.8	X	.8	1
	8	.8	.8	.8	.8	.8	.8	.8	X	1
	9	.8	.8	.8	.8	.8	.8	.8	.8	X
	10	.9	.9	.9	.9	.9	.9	.9	.9	X

receive good 4 after incurring a 20% discount covering the bid/ask spread, using direct trade. Alternatively, [3,4] can trade at {3,10} and at {4,10}. He sells 3 at {3,10} in exchange for 10 and sells 10 at {4,10} in exchange for the 4 he really wants. In this indirect trade, he incurs a 10% discount, saving 10% compared to direct trade, by using monetary trade with good 10 as 'money.' Indirect monetary trade is more attractive because it is less expensive. The lower expense reflects lower resource costs due to the low transaction cost of good 10 and the matching of suppliers and demanders of each good  $n = 1, 2, \dots, 9$ , at the trading posts {10, n} where good 10 is traded. As Jevons (1875) reminds us, the common medium of exchange overcomes the absence of a double coincidence of wants. Thus each household needs to incur the transaction cost on only one side of the monetary trade he enters.

The model here is a single period flow equilibrium model. There are no stocks to account for. Pure flows clear the markets. At trading post  $\{i,j\}$  it is sufficient that outflow of each good be at least equal to inflow. It is sufficient that each seller provide his supplies from endowment or simultaneous purchases elsewhere. There is no time structure and no cash-in-advance (or inventory-in-advance) restriction on trade. Those restrictions would create a demand for inventories — including money stock — but require a time structure and an equilibrium notion that includes both stocks and flows (e.g. Kurz (1974), Heller and Starr (1976)). Hence, in this paper's model, there is no demand for a stock of money (or of anything else) and no money-holding as a transactions demand.

All trading posts except the those dealing in good 10 become inactive. All trade is transacted at the nine posts dealing in 10. Trading posts clear. Good 10 has become the common medium of exchange, commodity money.

## 6 Conclusion

There is a surprise here. Tobin (1961, 1980) and Hahn (1982) despaired of getting a general equilibrium model based on elementary price theory to result in a common medium of exchange. The surprise in the example is that the pricing array in Table 3 leads directly to a monetary equilibrium. Monetary trade is the result of decentralized optimizing decisions of households guided by prices. The price system provides all of the co-ordination required to maintain a common medium of exchange. That's the successful co-ordination by prices we expect in an Arrow-Debreu Walrasian general equilibrium model, Debreu (1959). But the logic of that model is framed for a non-monetary economy. The example here demonstrates — as Menger (1892) said — that the same price system logic can be used to generate a monetary equilibrium with a single common medium of exchange.

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