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ASSESSING BENEFITS OF COORDINATION ON SAFETY IN AUTOMATED HIGHWAY SYSTEMS

by

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ABSTRACT

In this report, we present a methodology for assessing the benefits of different vehicle coordination strategies on the safety of a platoon during emergency braking. One can say that a coordinated braking strategy B is more beneficial than a strategy A, if strategy B leads to a larger reduction in the probability of a collision, the expected number of collisions, and the expected relative velocity at impact as compared to strategy A. We consider an emergency braking scenario, in which the lead vehicle brakes at its maximum capability and the following vehicles brake while obeying a vehicle following control law. The sequence of maximum deceleration of vehicles in the platoon is assumed to be a sequence of independent and identically distributed random variables; this distribution is assumed to be discrete and known. Due to coordination, however, the "effective" deceleration of a following vehicle may not necessarily be its maximum value.

The problem of assessing the benefits of coordination can be formulated as three subproblems: the first subproblem deals with determining the probability distribution of the "effective" deceleration of following vehicles during emergency braking. It is intuitive that the smaller the variance of this distribution, the greater the safety benefits are. The second subproblem deals with determining the probability of an intervehicular collision, the expected number of collisions and the expected relative velocity at impact as a function of the difference in the braking capabilities of successive vehicles in a platoon. The probability of an intervehicular collision and the expected number of primary collisions are computed via a Markov chain. Here, the asymptotic behavior (as the size of the string increases indefinitely) of the probability of an intervehicular collision and the expected number of primary collisions is computed. The third subproblem deals with conducting Monte Carlo simulation to

demonstrate the safety benefits of coordination during emergency braking and the viability of our analytical approach to estimate them.

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CHAPTER I

INTRODUCTION

In the design of an Automated Highway System (AHS), the issue of safety is central to preventing the loss of life and damage to the infrastructure. Motivated by this issue, this report deals with development of a methodology for assessing the benefits of coordination on the safety in AHS. In this report, we consider a platoon consisting of "n" vehicles, whose maximum deceleration form a set of independent, identically distributed random variables that follow a known, discrete probability distribution. We obtain this deceleration from Godbole and Lygeros [3], who have studied the uncoordinated braking problem earlier. Much of the basic formulation of this problem comes from their work. We consider a scenario, where the initial following distance and velocity of all vehicles is identical and the lead vehicle brakes at its maximum possible deceleration. We are interested in determining the probability of a collision, the expected number of collisions and the expected relative velocity at impact as a function of the probability distribution of maximum deceleration of vehicles, following distance, and possibly the coefficient of restitution.

In this report, we are interested in assessing the benefits of safety during such a scenario when vehicles coordinate their actions. It is intuitive that if all vehicles brake at the same deceleration, there will not be any collision. This corresponds to a case when all vehicles communicate the value of their maximum braking deceleration (for this case, every vehicle must be cognizant of this value), and the lead vehicle brakes at the least value of the maximum deceleration of all vehicles (say, d_1) in the platoon. In this scenario, the "effective" deceleration of all vehicles is d_1; the corresponding probability distribution has zero variance. The benefits of coordination in this scenario is clear. Qualitatively, it is clear that if the variance of the probability distribution of "effective" braking is small, the corresponding safety quantities such as the probability of a collision, expected number of collisions and the expected relative velocity at impact is small.

While this coordination scheme is "ideal", in that, it offers the best possible safety benefits, it may not be practical for many reasons: firstly, vehicles may not know their maximum braking capability; even in the case, where vehicles know their braking capability, it may be that the lead vehicle is compelled to brake at its maximum deceleration or that a collision can be avoided if the first vehicle braked as hard as it can, while all others follow their preceding vehicles according to a vehicle following law. In some cases, where autonomous vehicle following laws are used, communication of braking capability may not be possible; in this case, the effective deceleration of the ith vehicle in the platoon is the minimum of the effective deceleration of the preceding vehicle and its maximum braking deceleration.

The fact, that coordination renders effective braking of a vehicle different from its maximum braking capability, leads us to a natural way of analyzing the problem: the first step is that of determining how the coordination of vehicles alters the probability distribution of "effective" braking; the second step is that of determining how one can compute the probability of a collision, the expected number of primary collisions and relative velocity at impact in a platoon of "n" vehicles, from the knowledge of the probability distribution of "effective" braking and from the braking differential that guarantees a collision/no-collision between successive vehicles; the third step is that of conducting Monte Carlo simulation to demonstrate the safety benefits of coordination during emergency braking and the viability of our analytical approach to estimate them.

In this report, we consider a specific scheme of coordinated braking, where all vehicles employ the vehicle following control law developed in the paper by Hedrick et. al. [4]. We also consider a variant of this scheme for emergency braking, which is applicable to autonomous vehicle following schemes as well. In this variant, a following vehicle that is saturated in braking, while employing the vehicle following law, will be made the reference vehicle for its following vehicles; in other words, the platoon is naturally broken into subplatoons, where none of the following vehicles in the subplatoon saturate in braking. As a consequence, the effective deceleration of the leaders of the subplatoon form a monotonically decreasing sequence of numbers. Since the distribution is discrete, it is clear that the probability that a vehicle brakes at the smallest of the set of possible decelerations is almost surely unity; in this sense, there is almost surely perfect coordination among vehicles at the tail of the platoon. It is intuitive that the safety benefits of this scheme asymptotically approximate that of the "ideal" coordination scheme.

We have conducted a Monte Carlo simulation study to demonstrate the safety benefits of coordination during emergency braking; we have used the probability distribution of "maximum" braking from Godbole and Lygeros [3] for this purpose.

CHAPTER II

MODELING OF EFFECTIVE BRAKING PROBABILITY DISTRIBUTION

We assume a point mass model for every vehicle in the platoon. By referring to i as the index for a vehicle in the platoon, we mean that there are "$i - 1$" vehicles ahead of it in the platoon; a vehicle with index 1 will be the lead vehicle and the one with index n will be the last vehicle in a platoon of "n" vehicles. Let λ_i be the effective deceleration of the ith vehicle in response to the hard braking by the lead vehicle in the string. Let d_i represent its maximum deceleration. We assume that the sequence, $\{d_i,\ i \geq 1\}$ is a sequence of independent and identically distributed random variables, whose probability distribution is discrete and is known a-priori [3]. Suppose that this variable is random and takes one of "r" possible discrete values, namely, $D_1, D_2, \ldots, D_r$; without any loss of generality, we will assume that $D_1 < D_i < D_{i+1} < D_r,\ i = 2, \ldots r-2$. We will represent the probability that $d_i = D_j$ as p_j for all $j = 1, \ldots, r$, i.e.,

$$p_j := prob\{d_i = D_j\}, \quad j = 1, \ldots, r. \tag{2.1}$$

It is clear that $p_1 + \ldots + p_r = 1$. We will also assume that $D_1, \ldots, D_r$ form an arithmetic progression, with δ being the common difference. A typical probability distribution that we use for estimation of safety parameters is given in the paper by Godbole and Lygeros [3] as shown in figure 1.

A. Assumptions for Analysis

We make a distinction between a primary collision and a secondary collision. A collision between two vehicles in a string is referred to as a primary collision if it

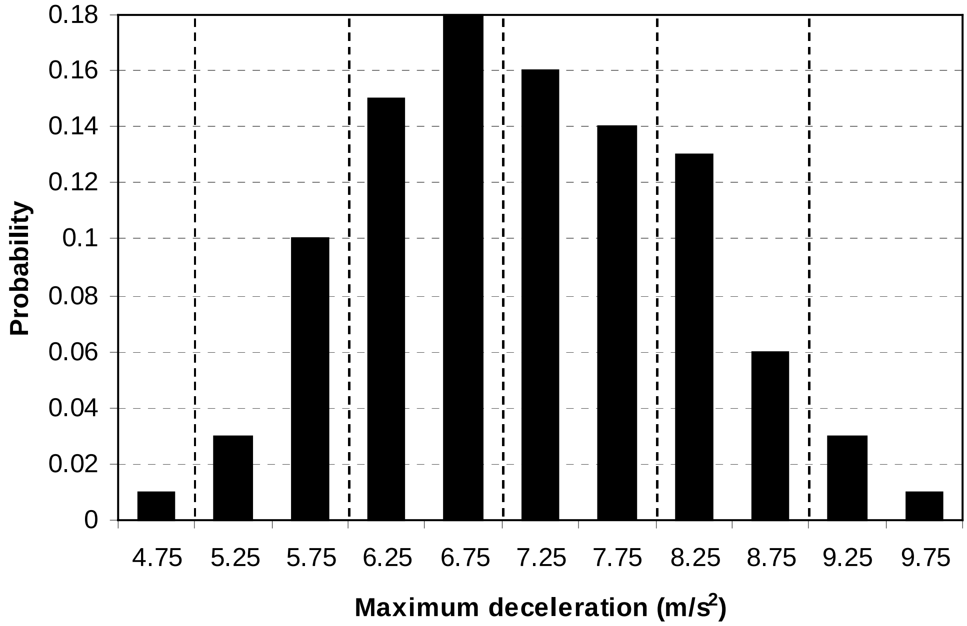


Fig. 1. Probability Distribution for the Random Variable d_i

occurs irrespective of whether preceding vehicles are involved in a collision; otherwise, it is referred to as a secondary collision.

As a first approximation, we assume that the expected relative velocity at impact is the same as the expected relative velocity during primary collisions; similarly, the expected number of total collisions in string is assumed to be proportional to the expected number of primary collisions. These two assumptions allow us to circumvent the mechanics of collision during analysis; however, we do consider a simple model of collision, one that employs coefficient of restitution, in our numerical simulations. Our numerical results on the metrics of safety include secondary collisions also.

B. The Uncoordinated Braking

In this case, during an emergency maneuver, every vehicle brakes at its maximum possible value, in other words,

$$\ddot{x}_i = -d_i, \quad i = 1, \ldots, n. \tag{2.2}$$

It is clear that the "effective" braking deceleration of the ith vehicle in the platoon is its maximum braking deceleration, i.e., $\lambda_i = d_i, \ i = 1, \ldots, n.$ If a coordinated control law is improperly designed, string instabilities can occur and the braking control effort can saturate for every vehicle at the tail of the string. In such a scenario, $\lambda_i = d_i$ for i sufficiently large and the benefits of coordinated control law are not realizable.

C. The Coordinated Braking

With the coordination scheme given in the paper by Hedrick et. al. [4], if the first vehicle in the platoon brakes at its maximum possible deceleration, i.e.,

$$\ddot{x}_1 = -d_1, \tag{2.3}$$

the following vehicles brake according to the vehicle following law given below:

$$\ddot{x}_i(t) = u_{c,i}(t), \tag{2.4}$$

$$\begin{aligned} u_{c,i}(t) = \alpha \ddot{x}_{i-1} + (1-\alpha)\ddot{x}_1 - k_v(\dot{x}_i - \dot{x}_{i-1}) \\ -k_p(x_i - x_{i-1} + L) - c_v(\dot{x}_i - \dot{x}_1) \\ -c_p(x_i - x_1 + (i-1)L), \end{aligned} \tag{2.5}$$

where α is a non-negative number that is upper bounded by unity and L is the desired spacing distance between two vehicles. We incorporate saturation of control action, i.e., if $u_{c,i}(t) \leq -d_i$, then

$$u_{c,i}(t) = -d_i. \tag{2.6}$$

With an appropriate choice of the gains, $\alpha, k_v, k_p, c_v, c_p$, one can guarantee string stability [8]; this enables us to neglect spacing and velocity errors and model the effective deceleration in the following way:

$$\begin{aligned} \lambda_1 &= d_1, && (2.7) \\ \lambda_i &= min\{\alpha\lambda_{i-1} + (1-\alpha)\lambda_1, d_i\}, \;\; i = 2, \ldots, n. && (2.8) \end{aligned}$$

The feedback terms are neglected to arrive at this simplification; while this may not accurately model the problem, it captures the essence. The problem of understanding how the distribution for λ_i evolves as a function of i is considered in [6]; the probability distribution of each λ_i can be determined from the original probability distribution of d_i. In this report, we consider three variants of coordination, i.e., $\alpha = 0$, $\alpha = 1$, and $0 < \alpha < 1$. The following analysis points out that the maximum safety benefit, due to coordination, corresponds to the case $\alpha = 1$. Let $q_{i,j}$ denote the probability that λ_i takes a value, D_j.

Case 1 ($\alpha = 0$): The effective braking deceleration, $\{\lambda_i\}$, of vehicles in the platoon form a sequence of dependent random variables, with a known probability distribution for the first random variable. In this case, we have

$$\begin{aligned} \lambda_1 &= d_1; && (2.9) \\ \lambda_i &= min\{\lambda_1, d_i\}, \;\; i \geq 2. && (2.10) \end{aligned}$$

Therefore,

$$prob\{\lambda_i \geq D_j\} = prob\{d_1 \geq D_j\} \text{ and } prob\{d_i \geq D_j\} \tag{2.11}$$

$$= (p_j + p_{j+1} + \ldots + p_r)^2 \tag{2.12}$$

where "*prob*" is meant by probability. From the above equation, it follows that

$$q_{i,j} := prob\{\lambda_i = D_j\} = prob\{\lambda_i \geq D_j\} - prob\{\lambda_i \geq D_{j+1}\} \tag{2.13}$$

$$= (p_j + p_{j+1} + \ldots + p_r)^2 - (p_{j+1} + \ldots + p_r)^2 \tag{2.14}$$

$$= p_j^2 + 2p_j(p_{j+1} + \ldots + p_r). \tag{2.15}$$

In this case, the "effective" braking deceleration of all the following vehicles follow the same probability distribution. Shown in figure 2 is a plot of the distribution of the "effective" braking of following vehicles. The expected braking deceleration for

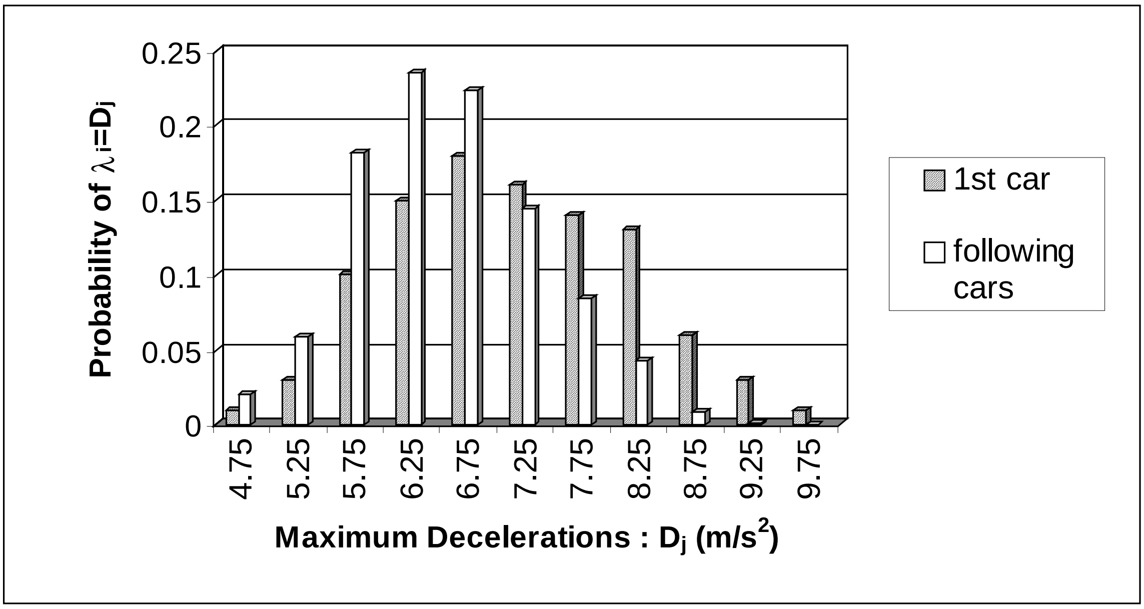


Fig. 2. Probability Distribution for the Random Variable λ_i for the Braking Case 1.

this distribution is 6.56 m/s^2 and the variance is 0.6779 m^2/s^4; the corresponding quantities for the uncoordinated case are 7.15 m/s^2 and 1.0750 m^2/s^4 respectively. Clearly, these two quantities are smaller as compared to the uncoordinated case.

Case 2 ($\alpha = 1$): While setting $\alpha = 1$ in the constant spacing vehicle following law may not render the string stable, this case can be implemented in a string stable manner and corresponds exactly to the variant of the control law by Hedrick et.al. [4] discussed earlier for emergency braking. Mathematically, we can describe the situation here as follows:

$$\lambda_1 = d_1; \tag{2.16}$$

$$\lambda_i = min\{\lambda_{i-1}, d_i\}, \quad i \geq 2. \tag{2.17}$$

In this case, a following vehicle saturated in braking while employing the vehicle following law, will be the reference vehicle for its following vehicles; this will result in subplatoons, where none of the following vehicles in the subplatoon saturate in braking, while the effective deceleration of the leaders of the subplatoon form a decreasing sequence. Therefore, the probability distribution can be computed as follows:

$$\begin{aligned}
prob\{\lambda_i \geq D_j\} &= prob\{d_k \geq D_j, \quad k = 1, \ldots, i\} && (2.18)\\
&= (p_j + p_{j+1} + \ldots + p_r)^i && (2.19)\\
q_{i,j} := prob\{\lambda_i = D_j\} &= prob\{\lambda_i \geq D_j\} - prob\{\lambda_i \geq D_{j+1}\} && (2.20)\\
&= (p_j + \ldots + p_r)^i - (p_{j+1} + \ldots + p_r)^i. && (2.21)
\end{aligned}$$

In this case, as shown in figure 3, the effective braking distribution for ith vehicle will be different from that of kth vehicle, if $i \neq k$. As $i \to \infty$, the distribution for λ_i almost surely converges to the unit distribution, i.e., $prob\{\lambda_i = D_1\} = 1$, and

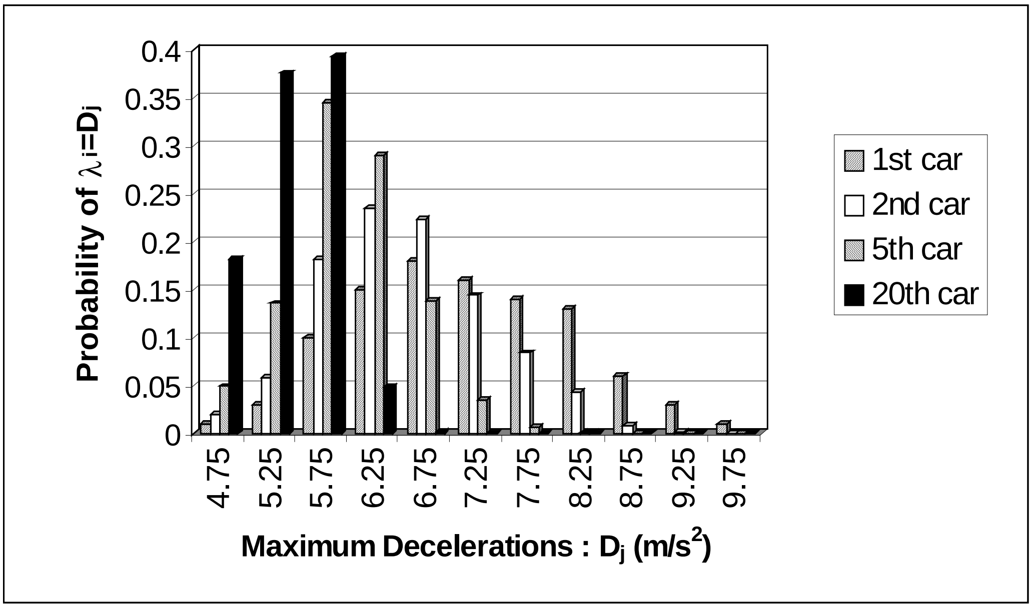


Fig. 3. Probability Distribution for the Random Variable λ_i for the Braking Case 2.

$prob\{\lambda_i > D_1\} = 0$.

Case 3 ($0 < \alpha < 1$)**:** Let $z_i := \lambda_i - d_1,\ \ \bar{d}_i := d_i - d_1,\ \ i \geq 1$. Then,

$$z_{i+1} = \min\{\alpha z_i, \bar{d}_{i+1}\}. \tag{2.22}$$

Clearly, $\{\bar{d}_i, i \geq 2\}$ will form a sequence of independent and identically distributed random variables; the probability distribution will be discrete and symmetric. Let $[-D, D]$ represent the range of values $\bar{d}_i$ will assume; it is clear that $-D \leq z_i \leq 0$ for all i. Let $C_d(x)$ represent the probability that $\bar{d} \geq x$ where $\bar{d}$ represents the difference in the maximum braking capability of any two vehicles picked randomly. Let $C_n(x)$ represent the probability that $z_n \geq x$, then,

$$C_{n+1}(x) \quad = \quad C_n(\frac{x}{\alpha})C_d(x). \tag{2.23}$$

From the observation that $-D \leq z_i \leq 0$ for all i, because the effective deceleration is smaller than equal to its maximum deceleration it follows that $C_n(x) \equiv 0$ for all $x > 0$ and $C_n(x) \equiv 1$ for all $x \geq -D$. Since $z_2 = \bar{d}_2$, it follows that $C_2(x) \equiv C_d(x)$. Therefore,

$$C_n(x) \quad = \quad \prod_{k=1}^{n-1} C_d(\frac{x}{\alpha^{k-1}}). \tag{2.24}$$

For $x = 0$, $C_d(x) = \frac{1}{2} < 1$, therefore, $C_n(x) \rightarrow 0$ as $n \rightarrow \infty$. For any $x < 0$, there exists a $N(x) > 0$, such that $C_m(x) = C_n(x)$ for all $m, n > N(x)$. Therefore, the probability distribution of random variables, z_i, converges pointwise (in fact, uniformly) to a distribution, $C^*(x)$. In fact, one can express $C^*(x)$ as: $C^*(x) := \Pi_{k=0}^{N-1} C_d(\frac{x}{\alpha^k})$ for all x lying in the interval, $D_N := [-\alpha^{N-1}D,\ -\alpha^N D]$, and since the interval $[-D, 0]$ is the union of intervals $D_N,\quad N = 1, 2, \ldots$, the function $C^*(x)$ is well defined. Moreover, the function is continuous, since continuity is preserved at the ends of each interval, and at 0, the limit is zero from the left; earlier, it has been established that the function $C^*(x)$ tends to zero as x approaches zero from the right. As a consequence, λ_i, as $i \rightarrow \infty$, converges distributionally and does not converge to a value as in the case, $\alpha = 1$. Asymptotically, the maximum safety benefit corresponds to the case $\alpha = 1$.

CHAPTER III

DETERMINATION OF SAFETY PARAMETERS FROM THE EFFECTIVE BRAKING PROBABILITY DISTRIBUTION OF FOLLOWING VEHICLES

Consider a platoon consisting of "n" vehicles as shown in figure 4:

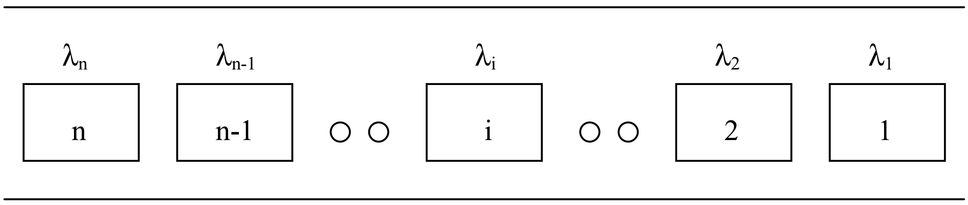


Fig. 4. A Platoon of "n" Vehicles

Consider a subplatoon or a substring, S_k, consisting of the last $(n-k+1)$ vehicles in the subplatoon. Let all vehicles take a value for their maximum deceleration capability from a given set of "r" values, $D_1, D_2, \ldots, D_r$. Recall that the probability that the effective deceleration, λ_i, of the ith vehicle takes the value D_j is denoted by $q_{i,j}$. Before we proceed further, we make a definition of a violation in a string S_k as follows: we define that S_k has a violation if there is some i lying between k and $n-1$, both inclusive, such that $\lambda_{i+1} < \lambda_i$. Furthermore, we say that S_k has l violations if there exist $i_1, i_2, \ldots, i_l$ such that $k \leq i_1 < i_2 < \ldots < i_l < n$ and $\lambda_{i_j+1} < \lambda_{i_j}$, $j = 1, 2, \ldots, l$. *We make the assumption that the fewer the number of violations, the fewer are the total number of collisions in a platoon.* This assumption enables us to circumvent the mechanics of collision; as such, the subsequent analysis must be considered as a first approximation to the determination of the expected total number of collisions and the expected relative velocity at impact. This approximation is better if vehicles are so closely packed that a collision will definitely occur if the effective deceleration of a following vehicle is less than that of its predecessor.

Assuming that a collision occurs whenever $\lambda_{i+1} < \lambda_i$, we define the probability of "l" violations in the given string as

$$P_l(k,j) := prob\{\lambda_k = D_j,\ S_k\ has\ l\ violations\}. \tag{3.1}$$

Consequently, we can write $P_l(k-1,j)$ as

$$\begin{aligned} P_l(k-1,j) &= prob\{\lambda_{k-1} = D_j, \lambda_k \geq D_j,\ S_k\ has\ l\ violations\} \\ &+prob\{\lambda_{k-1} = D_j, \lambda_k < D_j,\ S_k\ has\ l-1\ violations\} \\ &= q_{k-1,j}(P_l(k,j) + P_l(k,j+1) + \cdots + P_l(k,r)) \\ &+q_{k-1,j}(P_{l-1}(k,1) + \cdots + P_{l-1}(k,j-1)). \end{aligned} \tag{3.2}$$

Now, we can express the recursive relation in the following Markov chain form.

$$\begin{pmatrix} P_l(k-1,1) \\ P_l(k-1,2) \\ \vdots \\ P_l(k-1,j) \\ \vdots \\ P_l(k-1,r) \end{pmatrix} = A \cdot \begin{pmatrix} P_l(k,1) \\ P_l(k,2) \\ \vdots \\ P_l(k,j) \\ \vdots \\ P_l(k,r) \end{pmatrix} + B \cdot \begin{pmatrix} P_{l-1}(k,1) \\ P_{l-1}(k,2) \\ \vdots \\ P_{l-1}(k,j) \\ \vdots \\ P_{l-1}(k,r) \end{pmatrix},$$

where

$$A = \begin{pmatrix} q_{k-1,1} & q_{k-1,1} & \cdots & q_{k-1,1} & \cdots & q_{k-1,1} \\ 0 & q_{k-1,2} & \cdots & q_{k-1,2} & \cdots & q_{k-1,2} \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & q_{k-1,j} & \cdots & q_{k-1,j} \\ \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & q_{k-1,r} \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 \\ q_{k-1,2} & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ q_{k-1,j} & \cdots & q_{k-1,j} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ q_{k-1,r} & \cdots & q_{k-1,r} & \cdots & q_{k-1,r} & 0 \end{pmatrix}.$$

It follows that the probability of "l" violations in the string, denoted by $\overline{P_l}$, can be obtained as

$$\overline{P_l} = \sum_{j=1}^{r} P_l(1, j). \tag{3.3}$$

We associate, with each subplatoon, S_k, an "r" dimensional state, $P_l(k, j)$, $j = 1, \ldots, r$. The state $P_0(k, j)$ is the probability that the deceleration, λ_k, is D_j and the deceleration of following vehicles in the subplatoon, S_k, form a monotonically increasing sequence; in other words, it is the probability of no collision in the subplatoon. Therefore, probability of no collision (zero violation) for the platoon, $\overline{P_0}$, can be obtained as

$$\overline{P_0} = \prod_{j=1}^{r} P_0(1, j). \tag{3.4}$$

It is clear that $P_0(N-1,j) = q_{n-1,j}[q_{n,j} + q_{n,j+1} + \cdots + q_{n,r}], \quad j = 1, \ldots, r$. This end point condition can be used to calculate the state for the platoon, S_1. The probability of a collision is, therefore, given by

$$P_{collision} = 1 - P_0(1,1)P_0(1,2)\ldots P_0(1,r). \tag{3.5}$$

The expected number of violations (primary collisions) is given by

$$\sum_{l=1}^{n-1} l \cdot \overline{P_l}. \tag{3.6}$$

Since subsequent collisions, though possible, are not accounted for, in this calculation, the above equation provides a lower bound on the expected number of collisions. In other words, the expected number of total collisions in a platoon will be greater than or equal to the expected number of primary collisions. We will now focus on the problem of determining the expected relative velocity at impact, making the same strong assumption for guaranteeing a collision. To compute this quantity, we define a violation of order "m" as follows:

A substring, S_k, has a violation of order m, if there exists an index, i, lying between k and $n-1$, both inclusive, such that $\lambda_i = D_j$ for some $j > m$ and $\lambda_{i+1} = D_{j-m}$. In other words, the effective deceleration of the following vehicle is m notches smaller than that of its predecessor. The method of computing expected relative velocity is clear: we will first find the probability of the occurrence of a violation of order m; we will compute the relative velocity at impact for this scenario and from these two quantities, we will compute the expected relative velocity at impact.

As we have stated earlier, initially, all vehicles start with the same following distance and velocity. Assuming that the following distance is reasonably small, the expected relative velocity at impact, given that there is a violation of order m, is proportional to $\sqrt{m\delta}$, where the constant of proportionality involves the initial intervehicular separation; we will represent the constant of proportionality to be β.

If we define $P_{l,m}(k,j)$ as the probability that there are "l" violations of order m in the subplatoon, S_k, giving that $\lambda_k = D_j$, i.e.,

$$P_{l,m}(k,j) := prob\{\lambda_k = D_j,\ S_k\ has\ l\ violations\ of\ order\ m\}, \tag{3.7}$$

$P_{l,m}(k-1,j)$ can be written as

$$\begin{aligned} P_{l,m}(k-1,j) = prob\{&\lambda_{k-1} = D_j, \lambda_k = D_{j-m}, S_k\ has\ l-1\ violations\ of \\ &order\ m\} \\ &+prob\{\lambda_{k-1} = D_j, \lambda_k \neq D_{j-m}, S_k\ has\ l\ violations\ of\ order\ m\} \\ &= q_{k-1,j}[P_{l-1,m}(k,j-m) + \sum_{i=1,i\neq j-m}^{r} P_{l,m}(k,i)]. \end{aligned} \tag{3.8}$$

Let $\mu(n,m)$ be the expected number of violations of order m for the given entire string with the size of platoon n. Then

$$\mu(n,m) = \sum_{l=1}^{n-1}\sum_{j=1}^{r} l \cdot P_{l,m}(1,j). \tag{3.9}$$

Therefore, the expected relative velocity, Δv, at impact is given by:

$$\Delta v = \frac{\sum_{m=1}^{r-1} \mu(n,m) \cdot \beta\sqrt{m\delta}}{\sum_{m=1}^{r-1} \mu(n,m)}. \tag{3.10}$$

In the above equation, β is the constant of proportionality discussed in the earlier paragraph.

A. The Uncoordinated Case

In this case, since $\lambda_i = d_i$, $q_{i,j} = p_j$ the Markov chain form becomes

$$\begin{pmatrix} P_l(k,1) \\ P_l(k,2) \\ \vdots \\ P_l(k,j) \\ \vdots \\ P_l(k,r) \end{pmatrix} = \begin{pmatrix} p_1 & p_1 & \cdots & p_1 & \cdots & p_1 \\ 0 & p_2 & \cdots & p_2 & \cdots & p_2 \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & p_j & \cdots & p_j \\ \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & p_r \end{pmatrix} \cdot \begin{pmatrix} P_l(k+1,1) \\ P_l(k+1,2) \\ \vdots \\ P_l(k+1,j) \\ \vdots \\ P_l(k+1,r) \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 \\ p_2 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ p_j & \cdots & p_j & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ p_r & \cdots & p_r & \cdots & p_r & 0 \end{pmatrix} \cdot \begin{pmatrix} P_{l-1}(k+1,1) \\ P_{l-1}(k+1,2) \\ \vdots \\ P_{l-1}(k+1,j) \\ \vdots \\ P_{l-1}(k+1,r) \end{pmatrix}.$$

As mentioned earlier, the probability of a collision is given by equation (29). To get the expression of $P_0(1,j)$, we first consider the relationship between $P_0(n-2,j)$ and $P_0(n-1,j)$. Since we know that from the definition of $P_0(k,j)$, $P_0(n-1,j)$ can be written as:

$$P_0(n-1,j) = p_j(p_j + p_{j+1} + \cdots + p_r). \tag{3.11}$$

Therefore, $P_0(n-2,j)$ is given by

$$\begin{pmatrix} P_0(n-2,1) \\ P_0(n-2,2) \\ \vdots \\ P_0(n-2,j) \\ \vdots \\ P_0(n-2,r) \end{pmatrix} = \begin{pmatrix} p_1 & p_1 & \cdots & p_1 & \cdots & p_1 \\ 0 & p_2 & \cdots & p_2 & \cdots & p_2 \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & p_j & \cdots & p_j \\ \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & p_r \end{pmatrix} \cdot \begin{pmatrix} P_0(n-1,1) \\ P_0(n-1,2) \\ \vdots \\ P_0(n-1,j) \\ \vdots \\ P_0(n-1,r) \end{pmatrix},$$

or equivalently,

$$\begin{pmatrix} P_0(n-2,1) \\ P_0(n-2,2) \\ \vdots \\ P_0(n-2,j) \\ \vdots \\ P_0(n-2,r) \end{pmatrix} = \begin{pmatrix} p_1 & p_1 & \cdots & p_1 & \cdots & p_1 \\ 0 & p_2 & \cdots & p_2 & \cdots & p_2 \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & p_j & \cdots & p_j \\ \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & p_r \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2(1-p_1) \\ \vdots \\ p_j(1-p_1-\cdots p_{j-1}) \\ \vdots \\ p_r^2 \end{pmatrix}.$$

Thus, we can obtain

$$\begin{pmatrix} P_0(1,1) \\ P_0(1,2) \\ \vdots \\ P_0(1,j) \\ \vdots \\ P_0(1,r) \end{pmatrix} = A^{n-2} \cdot \begin{pmatrix} p_1 \\ p_2(1-p_1) \\ \vdots \\ p_j(1-p_1-\cdots p_{j-1}) \\ \vdots \\ p_r^2 \end{pmatrix},$$

where

$$A = \begin{pmatrix} p_1 & p_1 & \cdots & p_1 & \cdots & p_1 \\ 0 & p_2 & \cdots & p_2 & \cdots & p_2 \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & p_j & \cdots & p_j \\ \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & p_r \end{pmatrix}.$$

Since all the eigenvalues of A above lie between 0 and 1 it is clear that the probability of zero violation in the string goes to zero as $n \to \infty$. Therefore, it follows that the probability of a collision becomes 1 as $n \to \infty$. The expected number of primary collisions and the expected relative velocity at impact are given by equation (30) and (34).

B. The Coordinated Cases I and II : $\alpha = 0$ and $\alpha = 1$

Using the probability distributions shown in equations (15) and (21), respectively, we compute quantitatively the expected number of primary collisions and expected relative velocity at impact with $\beta = 2$ and $\delta = 0.5$ corresponding to these two cases in the same manners as the uncoordinated case. We plot these two quantities as a function of size of platoon.

As one expects, the expected number of primary collisions and the expected relative velocity at impact in coordinated braking cases are smaller than the ones in uncoordinated case. In figure 6, the graph corresponding to a platoon of size 2 may seem anomalous because the expected relative velocity at impact for the uncoordinated case is smaller than its coordinated counterparts. Referring to figure 2, it is clear that if $d_1 = 9.75\ m/s^2$, the probability that $\lambda_2 < 9.75\ m/s^2$ is greater with a coordinated braking as opposed to an uncoordinated braking scenario. This is because the probability distribution for effective braking of the second vehicle, i.e., for λ_2, skews to the left. Although the probability of a collision decreases for a coordinated case, the probability of a high relative velocity at impact is much higher with a coordinated control law for platoons of size 2.

This apparent anomaly, however, does not continue with platoons of larger sizes, because the distribution of effective braking of following vehicles is better. For example, the probability that the effective braking takes a value of $8.75, 9.25$ or $9.75\ m/s^2$ in coordinated case 1 is much smaller than the corresponding uncoordinated case, i.e. the probability of a collision of high relative velocity at impact between two following vehicles is significantly smaller.

The apparent anomaly for a platoon of size 2 may have a significant bearing on mixed traffic safety where ACC vehicles mingle with manually controlled traffic.

It is natural to expect the relative velocity at impact to saturate as the size of platoon increases and this is what we observe in figure 6. Similarly, one would expect the expected number of collisions to increase monotonically for the uncoordinated as well as coordinated case 1 and this is the case as well. However, one would expect the total number of collisions with the coordinated case 2 to saturate as the size of the platoon increases. The convergence in probability of the effective braking distribution is very slow, as can be seen in figure 3. Hence, it looks in figure 5, as if the expected number of collisions is also increasing linearly with the size. If one were to continue the calculations, as we have done, the expected number of collisions saturates (for platoons of size 500 approximately).

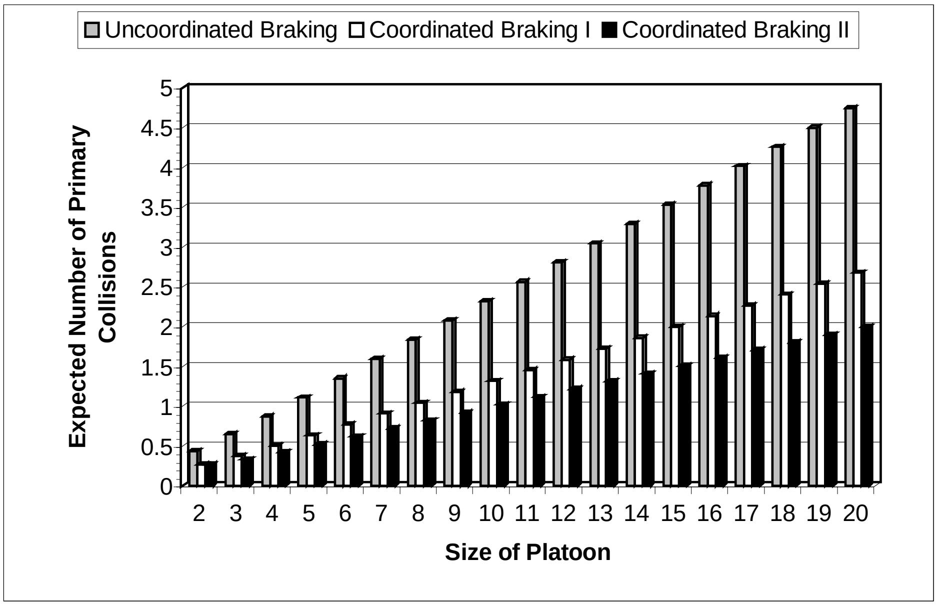


Fig. 5. Analytical Result of the Expected Number of Primary Collisions

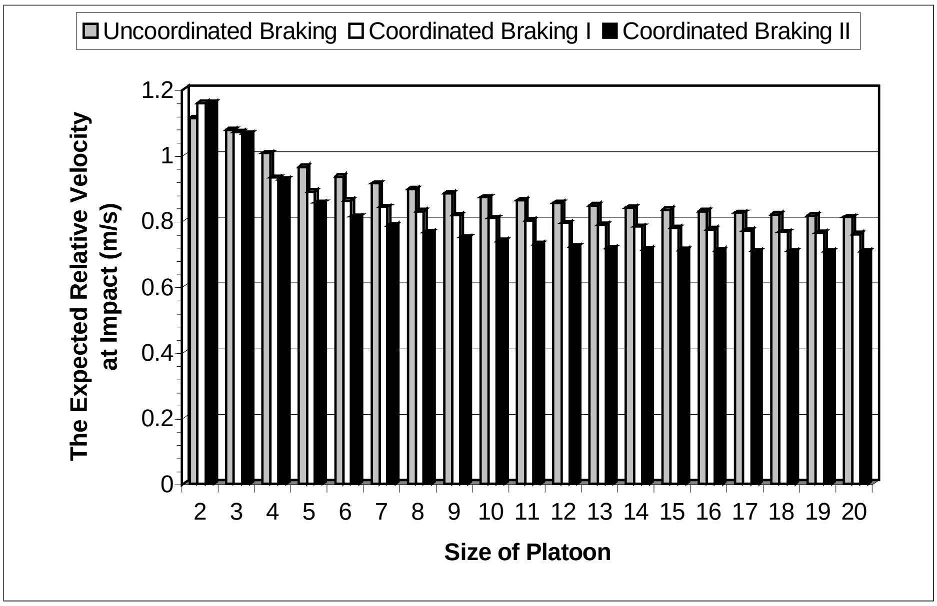


Fig. 6. Analytical Result of the Expected Relative Velocity at Impact

CHAPTER IV

MONTE CARLO SIMULATIONS AND RESULTS

The Monte Carlo simulations enable us to compute the three variables of interest, namely, the probability of a collision, the expected number of collisions and expected relative velocity at impact.

A. The Necessity for Monte Carlo Simulations

For the purpose of illustration, we set the size of a platoon to be 10 vehicles following one lead vehicle. Each vehicle has random maximum braking capability distribution with the maximum deceleration of every vehicle capable of taking any of the 11 different values. In reality, it takes an enormous amount of time to simulate all the possible combinations, 11^{11} cases. Moreover, since we are seeking the variables of interest as a function of 5 desired intervehicular distances and 7 coefficients of restitution, we need 35 more combinations, which results in $35 \cdot 11^{11}$ cases. Therefore, as an alternative, we use Monte Carlo simulation to illustrate the benefits of coordination.

B. Interpretation of Monte Carlo Simulations

Monte Carlo simulations aim to compute the expected values of a random variable, say X, empirically. Let X_t be the true expected value of X, and let X_m be the mean of "m" randomly drawn samples (obtained via Monte Carlo simulation) of X. Then, an inequality due to Hoeffding [5], for random variables with bounded mean and variance, indicates that

$$prob\{|X_m - X_t| > \varepsilon\} \leq 2e^{-2m\varepsilon^2}. \tag{4.1}$$

This inequality indicates that the probability that the empirical mean of X obtained after m iterations deviates from its true mean by ε or more is smaller than $2e^{-2m\varepsilon^2}$. It is in the sense that the empirical quantities of interest are taken in the subsequent sections.

C. Empirical Evaluation of Quantities of Interest

The Monte Carlo simulation scenario is as follows: The initial velocity for all vehicles is $30m/s$ and the initial acceleration is zero. After a second, the lead vehicle starts to brake at its randomly assigned maximum braking capability. Then, the following vehicles obey coordinated or uncoordinated braking laws depending upon the case of study. We have conducted simulations with a combination of 5 different desired intervehicular distances between $1m$ and $10m$ and with 7 different coefficients of restitution ranging between 0 and 1 for 11 random maximum possible decelerations assigned to each vehicle. Each iteration is done for 11 seconds, which ensures that all the vehicles come to rest during that time. At the end of each simulation, we gather the following data; (1) if there was any collision during simulation (2) if there were any collisions in the platoon during simulation, we record the number of collisions with relative velocity at impact in each of the specified ranges, 0 to 1, 1 to 2, 2 to 3, 3 to 4, 4 to 5, and over $5m/s$.

To compute empirically, the quantities of interest, we gather the number of data points with the relative velocity in each of the six intervals over all simulation runs. We determine the average number of data points in each of the intervals over the total number of simulations performed. We performed a minimum of 100 simulations; after every following simulation, we check if the empirical average deviates from the previously calculated value by less than 0.001 in each of the six ranges. If the empirical averages from successive simulation runs deviate by less than 0.001 in each of the six ranges, we stop performing additional simulations; otherwise, we continue.

The algorithm for this simulation program is shown in figure 7. Let $N_{i,j}$ be the data points of the relative velocity at impact corresponding to within jth interval at the ith iteration and $\sigma_{i,j}$ be the average of $N_{i,j}$ during i iterations where $j = 1 \cdots 6$. Then,

$$\sigma_{i,j} = [\sum_{k=1}^{i} N_{k,j}]/i. \tag{4.2}$$

and the convergence criteria is given by

$$i \geq 100 \ \ and \ \ |\sigma_{i,j} - \sigma_{i-1,j}| < 0.001, \ \ j = 1, \ldots, 6. \tag{4.3}$$

After we stop the simulations, we calculated the expected average relative velocity at impact, V_{avg}, given by

$$V_{avg} = (\sum_{j=1}^{6} v_j \cdot \sigma_{i,j}) / \sum_{j=1}^{6} \sigma_{i,j}, \tag{4.4}$$

where $v_j = j - 0.5$.

The probability of a collision, $P_{collision}$, of a platoon is given by assigning a binary value 0 or 1 corresponding to no collisions or a collision respectively in each iteration. Assuming that the simulation converges at the ith iteration and the total number of the binary numbers is N_{total} during i iterations, then the expected probability of a collision in platoon is given by

$$P_{collision} = N_{total} \cdot 100/i. \tag{4.5}$$

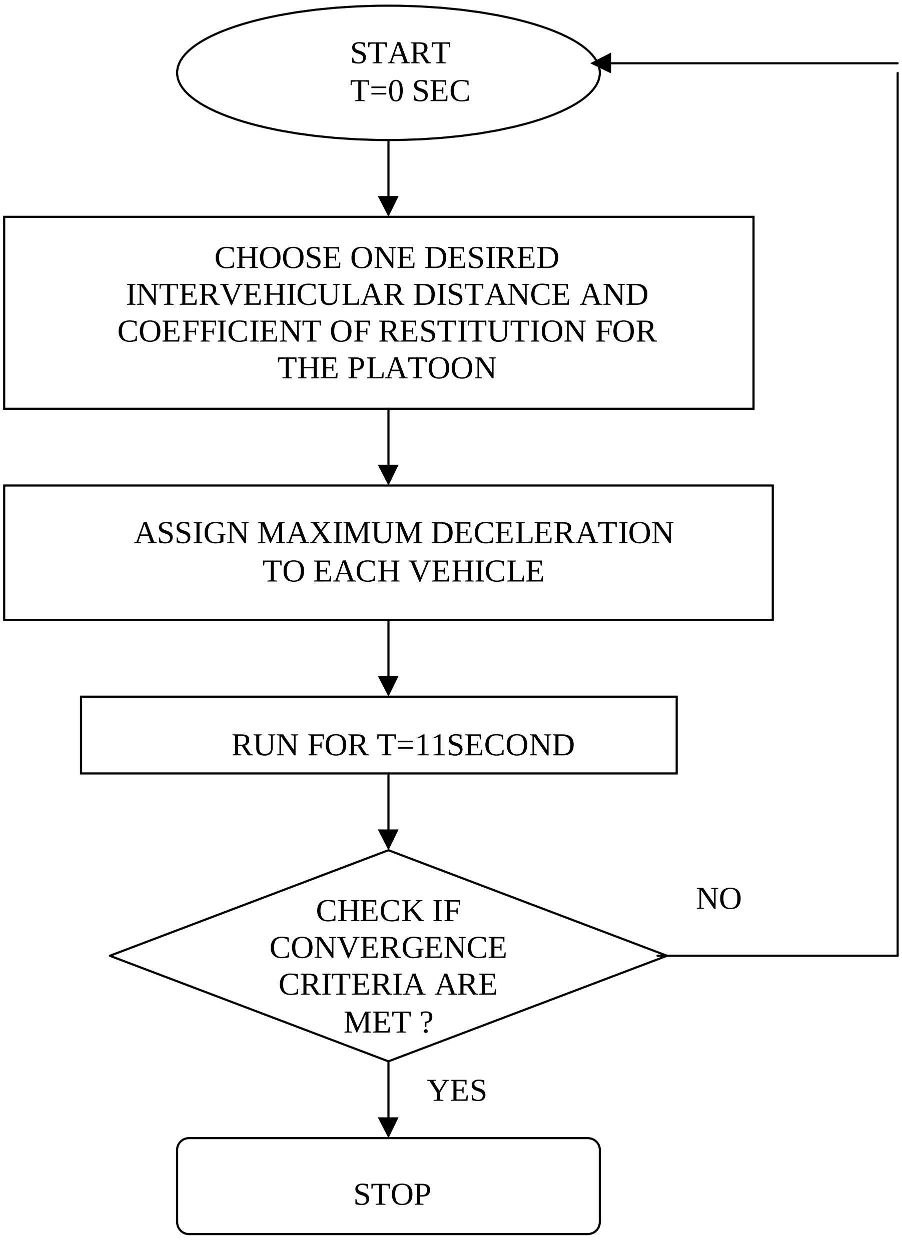


Fig. 7. Algorithm Diagram for the Monte Carlo Simulation

D. Results and Discussion

From the Monte Carlo simulation, we get the trend of our quantities of interest as a function of the desired intervehicular distance, L, and the coefficient of restitution, e, for both uncoordinated and coordinated braking cases.

1. The Uncoordinated Case

As shown in figure 8, one can see clearly that the expected number of collisions is inversely proportional to intervehicular distances for each value of the coefficient of restitution. When the desired intervehicular distance ranges from $1m$ to $4m$, the expected number of collisions increases as the coefficient of restitution increases from 0 to 0.4 and decreases with the coefficient of restitution thereafter. As the intervehicular distance increases, the expected number of collisions decreases significantly and is not significantly affected by the coefficient of restitution. The minimum expected number of collisions in all cases corresponds to the case when coefficient of restitution is zero.

The expected relative velocity at impact, increases with desired intervehicular distance as shown in figure 11. Since there is a loss of kinetic energy for all cases except for perfectly elastic collisions, the linear momentum is conserved during impact in a platoon, and it follows that the expected average relative velocity at impact is most significant in case of $e = 1$ for all intervehicular distances.

For most cases with 2 variables, intervehicular distance and coefficient of restitution, the expected probability of a collision is over 99 percentage as shown in figure 14. In case of $L = 10$, probability drops slightly, since there is some space between vehicles for vehicles to stop completely before colliding with their predecessors.

2. The Coordinated Cases

As shown in figures 9, 12, and 15 for the first control scheme and in figures 10, 13, 16 for the second control scheme, we can see that all the quantities of interest, the expected number of collisions, the expected relative velocity at impact, and the expected probability of a collision have basically same trend with respect to uncoordinated braking law case; however, they are significantly smaller for the coordinated braking strategies.

3. Comparison on Monte Carlo Simulation Results with Analytical Approach Results

As mentioned earlier, Monte Carlo simulations are conducted to demonstrate the viability of our analytical approach to approximate the three metrics of safety considered here. To do so, we consider the proportion of metrics for the three braking strategies corresponding to a platoon of size 10. From figures 5 and 6, the proportion is 2.317 : 1.315 : 1.02 for expected number of collisions and 0.872 : 0.808 : 0.738 for the expected relative velocity at impact. The corresponding results obtained via Monte Carlo simulations for the case $e = 0.4$ is 187.33 : 98.10 : 29.43 for expected number of collisions, which corresponds to $L = 1$, and 5.006 : 4.489 : 3.773 for the expected relative velocity at impact, which corresponds to $L = 10$. These results agree reasonably well.

Fig. 8. Expected Number of Collisions with the Uncoordinated Braking Control Scheme

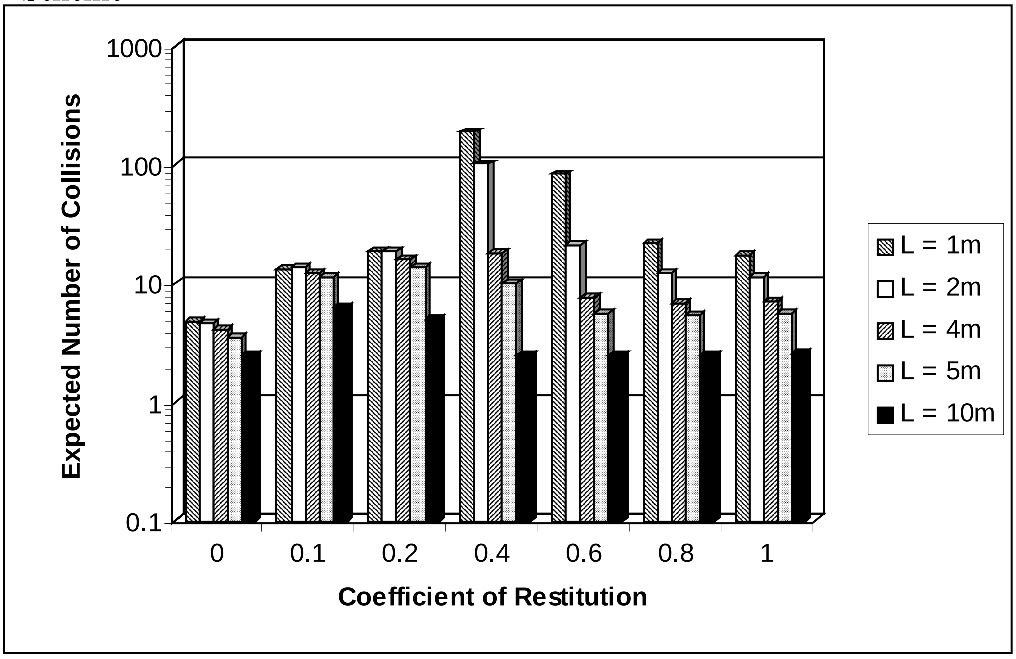


Fig. 9. Expected Number of Collisions with the Coordinated Braking Scheme I

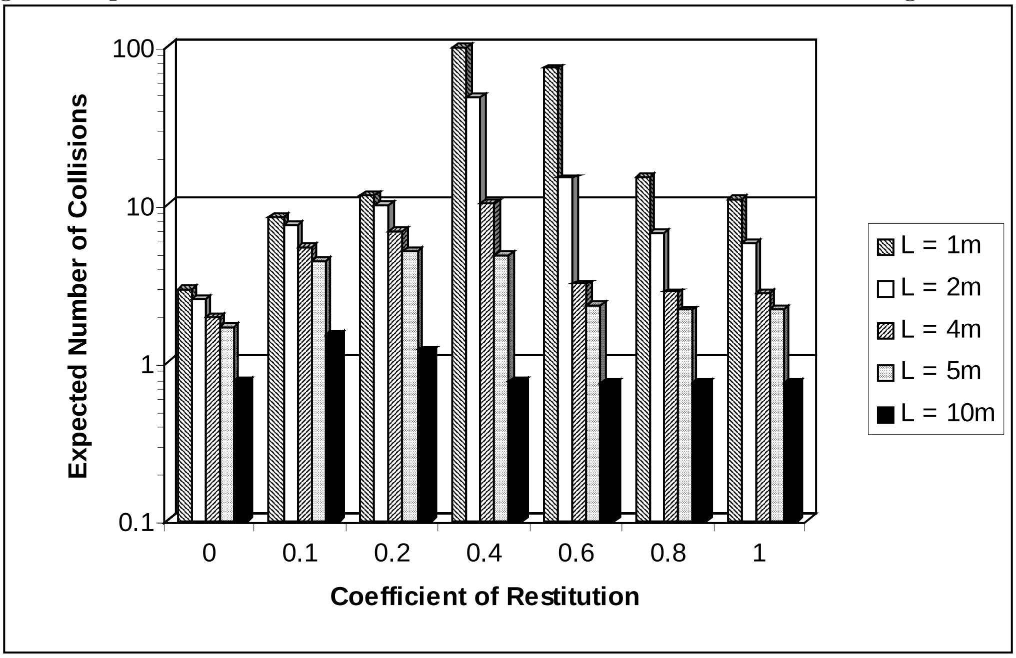


Fig. 10. Expected Number of Collisions with the Coordinated Braking Scheme II

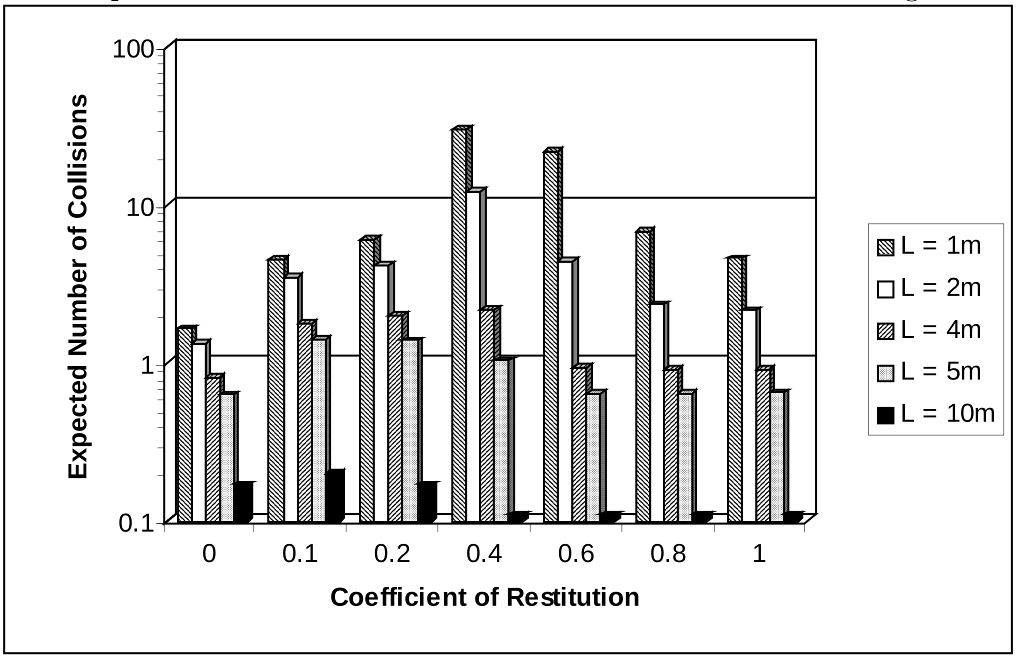


Fig. 11. Expected Relative Velocity at Impact with the Uncoordinated Braking Control Scheme

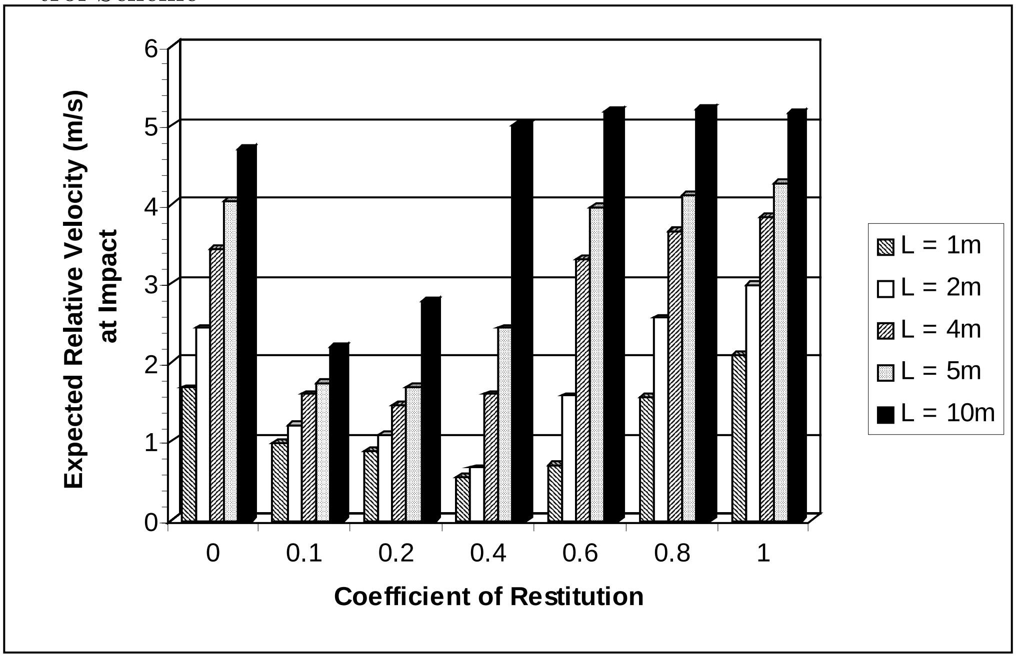


Fig. 12. Expected Relative Velocity at Impact with the Coordinated Braking Scheme I

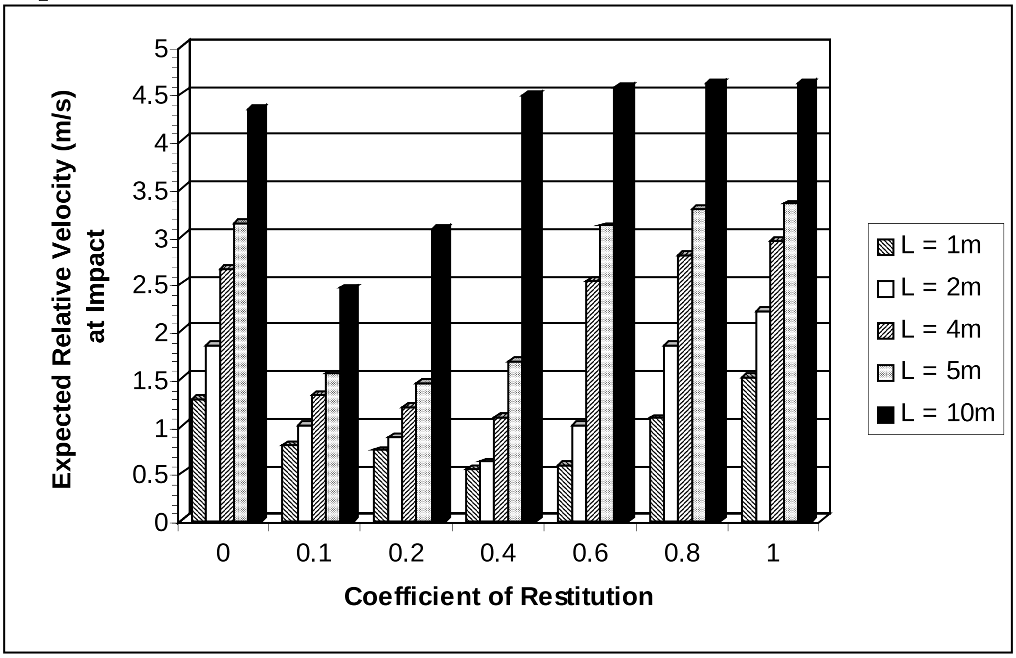


Fig. 13. Expected Relative Velocity at Impact with the Coordinated Braking Scheme II

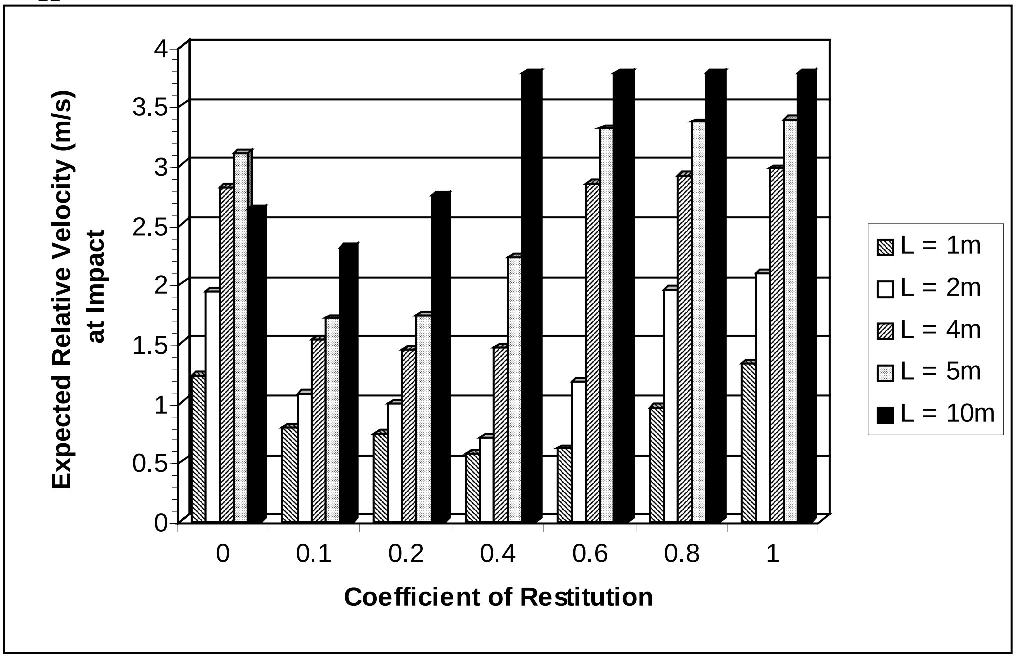


Fig. 14. Expected Probability of a Collision with the Uncoordinated Braking Control Scheme

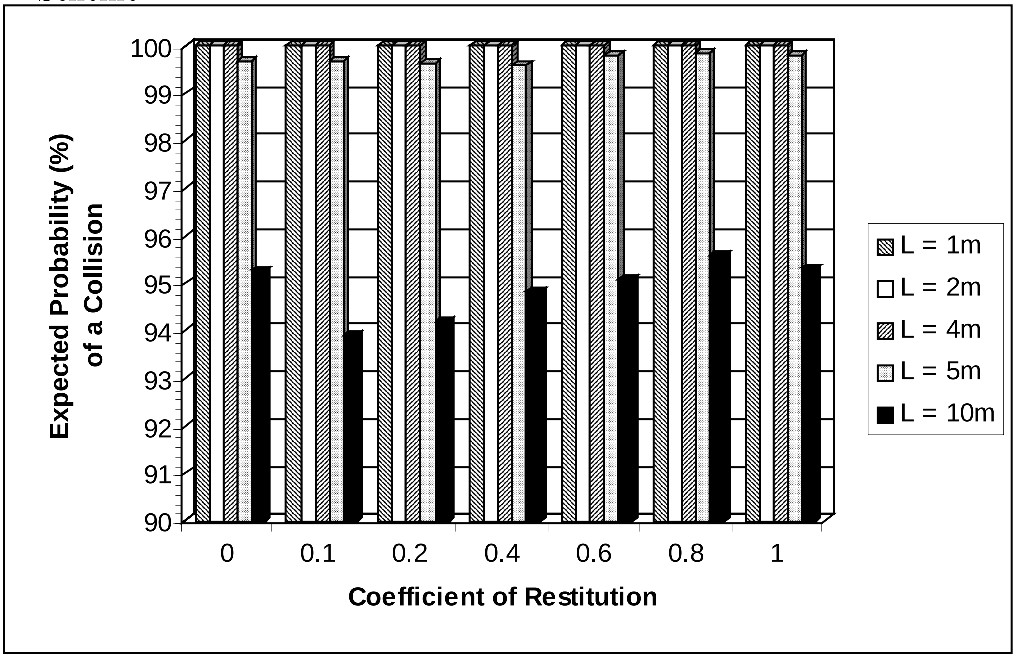


Fig. 15. Expected Probability of a Collision with the Coordinated Braking Scheme I

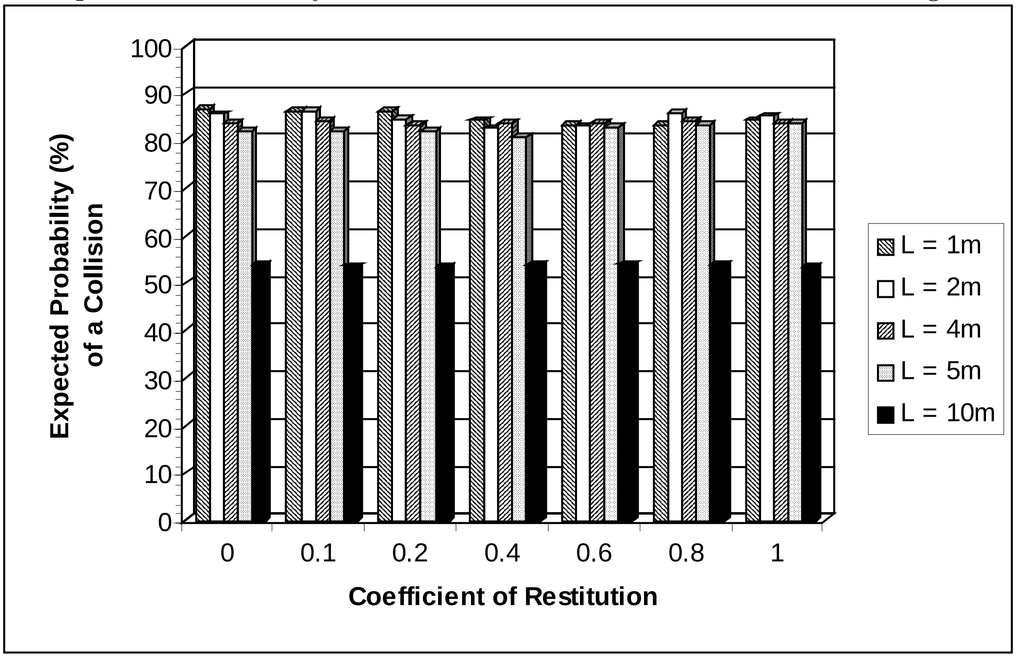
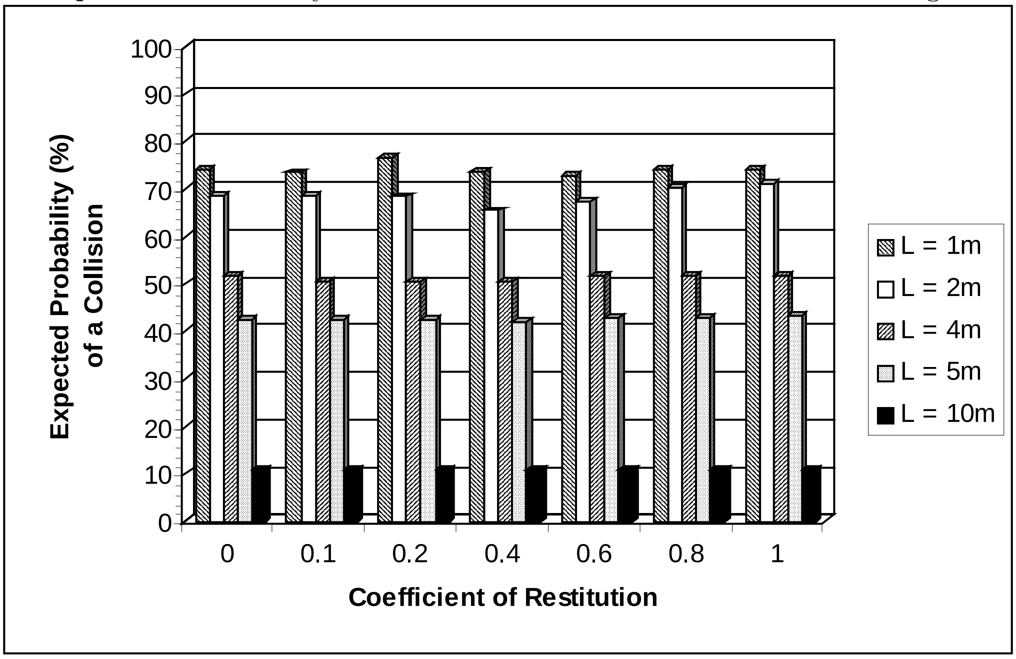


Fig. 16. Expected Probability of a Collision with the Coordinated Braking Scheme II



CHAPTER V

CONCLUSIONS AND FUTURE WORK

In this report, we have presented a methodology for assessing the benefits of coordination amongst vehicles during emergency braking via the modeling of effective braking. The proposed work is the first step towards providing a systematic, analytical procedure for assessing the benefits of coordination, namely, probability of an intervehicular collision, the expected relative velocity at impact, and expected number of collisions.

An important objective of the proposed analytical methodology for assessing the safety benefits of coordination in a vehicle platoon is to circumvent the need for extensive Monte Carlo simulations. However, to do so, one must validate the hypothesis that the total number of collisions are monotonically increasing and continuous function of the primary collisions. Work is underway to examine this hypothesis.

The proposed methodology is applied to constant spacing vehicle following control systems in this report. The extension to variable spacing vehicle following systems is challenging and is of practical relevance; this is especially important since automotive companies are currently developing adaptive cruise control systems that employ variable spacing policies. We plan to undertake this challenge in the near future.

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