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Bribery in Rank-Order Tournaments[*]

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Abstract

In many principal-agent relations, objective measures of the agents' performance are not available. In those cases, the principals have to rely on subjective performance measures for designing incentive schemes. Incentive schemes based on subjective performance measures open the possibilities for influencing activities by the agents. This paper extends Lazear and Rosen's (1981) model of rank-order tournaments by considering further competition between the agents in a bribery game after production but before selection of the winner. The paper studies how the bribery game affects the principal's design of the rank-order tournament and how the anticipation of the bribery game affects the agents' effort choices.

KEYWORDS: Principal-agent problem, relative performance, subgame-perfect equilibrium, tournaments. (JEL C72, J33)

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1 Introduction

In many firms workers are commonly rewarded by tournament incentive schemes.[1] Workers are promoted to the next levels due to their excellent performance relative to their cohorts. Sometimes bonuses are allocated to a certain number of sales managers not because their sales figures surpass predetermined target levels but because they outperform the others.

In a tournament, compensation is based on the rank order of the contestants' performance rather than their absolute performance. Prizes are fixed in advance and then allocated to the contestants according to their positions in the ranking. This type of incentive structure was first analyzed by Lazear and Rosen (1981). For a model with a risk-neutral principal and two risk-neutral agents, they show that a scheme which rewards according to the ranking of the agents' performance yields an outcome identical to what would be generated by the efficient piece-rate scheme.

There are various reasons why firms choose tournament-type contracts for their workers over other incentive contracts such as piece-rate incentive schemes. Lazear and Rosen (1981) view the rank-order tournament as a method for avoiding information costs that would be borne if compensation were to be based on the agents' absolute productivities rather than their ordinal ranks. Green and Stokey (1983) and Nalebuff and Stiglitz (1983) show that the rank-order tournament can be superior to absolute performance schemes when risk-averse agents are faced with a large common shock. The reason is because the rank-order tournament can filter out the common shock and thereby reduce the amount of risk imposed on each agent. This cannot be accomplished by absolute performance schemes. Kim (1999) show that the rank-order tournament performs better than absolute performance schemes when the principal also participates in the production process by investing his own effort which has a common effect on every agent's performance.[2]

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[1]According t o Broil (2001), 25 percent of the Fortune 500 companies use forced ranking systems for their workers. See also Krakel (2003).

[2]This type of "productive principal-agents" model was initially studied by Carmichael (1983).

One assumption that has been typically made in the literature is that agents' performance measures are perfectly verifiable to a third party such as a court. However, as emphasized by Williamson, Wachter, and Harris (1975), in many employment situations, employees' performance can be assessed only subjectively by employers.[3] Indeed, within a firm information either about a worker's effort or his contribution to the firm's profit is rarely available to the top manager. Furthermore, information on the firm's profit that can be potentially used as a variable on which each worker's pay can be based is often nonverifiable. And, even if it is verifiable, basing each worker's pay on the firm's profit (i.e., the group incentive scheme) will inevitably cause free-riding problems on the workers' side. Thus, as a way of coping with this problem, the top manager often hires supervisors to monitor and to assess the workers' effort or their individual performance for him. The problem is that the supervisors' monitoring outcomes are in many circumstances just their personal judgements about the workers' performance which are hard to verify.

When the supervisor's assessment on each worker's performance cannot be verified, the top manager will have an incentive to renege on the contract *ex post* by claiming down the worker's performance to the lowest possible level if his wage is based only on his performance assessed by the supervisor. However, as shown in Malcomson (1984), this problem will disappear if the workers are rewarded by the tournament scheme.

The subjectivity of the supervisor's assessments on the workers' performance, however, will activate the workers' desire to influence the supervisor's subjective assessments. For example, after realization of their performance the workers may try to bribe the supervisor to win his favor. The supervisor on the other hand may find it profitable to falsify assessments in order to induce the workers to bribe. In fact, bribery creates a game between the workers upon realization of their performance.[4] Of course, in reality, the supervisor's

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[3]See Baker, Gibbons, and Murphy (1994) for further discussion on the necessity of using subjective assessments of the agents' performance.

[4]See Milgrom (1988) and Milgrom and Roberts (1988) for discussions on the workers' influencing activities.

ability to extract bribes from the workers is limited by law enforcement. The limit depends on how likely bribery can be legally detected, and what penalties the supervisor and the workers must pay when detected.

To address these features of a firm we consider a model in which a risk-neutral manager hires a risk-neutral supervisor to assess two identical workers' performance. Our primary objective is to study the impacts of bribery on the top manager's optimal design for the rank-order tournament and how the existence of the bribery game affects the workers' effort choices.

Bribery modifies the incentive structure of the rank-order tournament. On the one hand, because effort is costly to the workers, each worker may be better off by simply bribing the supervisor to win the prize rather than by supplying more effort to outperform his rival. On the other hand, the larger the assessed performance gap between the workers, the less likely the supervisor will falsify the ranking of the workers, because the probability for being detected and penalized is higher. This means that law enforcement provides each worker with an incentive to supply more effort. However, it is still far from clear whether the workers will supply the same effort as they would without bribery.

We assume that the supervisor does not take any bribe from a worker whom he will not select as the winner.[5] We show that when there is no lower bound for the supervisor's basic salary (so that the top manager can charge the supervisor an "entrance fee"), it is optimal for the manager to design the rank-order tournament which will induce the socially efficient effort level from each worker, as long as the total penalty that the supervisor and the bribing worker should pay when detected is above a certain level. This lower bound on the penalty is determined explicitly by the effectiveness of law enforcement and the distribution of the random factors that affect the supervisor's monitoring process. In addition, we show

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[5]One reason for not taking a bribe from the worker who is not going to be selected as the winner is that the supervisor wishes to avoid a potential legal dispute which might be initiated by the worker whose bribe is taken but who is not selected as the winner.

that the nominal bonus becomes smaller as the total penalty increases, implying that the pay inequality between the winner and the loser reduces as the relationship between the supervisor and the workers become less corrupt.

If there is a minimum requirement for the supervisor's basic salary (so that the top manager cannot charge any entrance fee), the socially efficient outcome will not be achieved by the rank-order tournament. The top manager will set the nominal bonus at the level that is smaller than the socially efficient one, and thereby the workers will exert less effort than the socially efficient level.

Papers on issues of corruption may be broadly divided into two categories. The first one includes Rose-Ackerman (1975), Laffont and Tirole (1991), and Burguet and Che (2004), among others. These authors considered corruption within a procurement framework and focused on whether a more efficient firm (a firm with a lower cost) would be always awarded with the contract. However, the channel through which the corruption possibility would affect the worker's effort incentives was not explicitly modeled in their papers. The second category includes Prendergast and Topel (1996) and Fairburn and Malcomson (2001) studies corruption within an organization. The focus of these papers is on how the presence of bribery or favoritism affects the workers' effort incentives as well as their compensation schemes. Prendergast and Topel (1996) consider how a supervisor's personal favoritism toward some of his subordinates affects the compensation structure of an organization. However, the supervisor's favoritism toward certain subordinates is exogenously given and further competition between subordinates in a bribery game to win the supervisor's favor is not explicitly modeled.

Our paper falls in the second category. Closely related to our paper is Fairburn and Malcomson (2001). These authors studied how the existence of a bribery game affects the internal incentive structure of a firm. In absence of law enforcement, they showed that a tournament scheme with 'promotions' as the prizes can provide each worker with an effort incentive, whereas a tournament scheme with 'monetary bonuses' as the prizes cannot. In

contrast, we assume that there is law enforcement which limits bribery to a certain extent and we study how the existence of law enforcement affects the top manager's design for the rank-order tournament with monetary bonuses as the prizes.

The rest of the paper is organized as follows. In Section 2, we briefly review the basic model of the rank-order tournament as in Lazear and Rosen (1981). We then add a bribery game to the rank-order tournament to model competition between the agents to win the supervisor's favor after realization of production. In Section 3, we derive a necessary and sufficient condition under which it is optimal for the manager to design the rank-order tournament that will induce the socially efficient effort level from each worker. Concluding remarks are given in Section 4. Proofs of propositions and lemmas are all presented in an appendix.

2 The Basic Model

We consider a single period principal-agent model. A risk-neutral manager (the principal) hires two risk-neutral workers (the agents). It is assumed that the two workers are identical in every respect. For $i = 1, 2$, worker i performs his own task by providing effort $e_i \in [0, \infty)$. Joint profit y depends on random factors as well as workers' effort as follows:

$$y = Y(e_1, e_2) + \theta, \quad E(\theta) = 0. \tag{1}$$

We assume for $(e_1, e_2) \in [0, \infty) \times [0, \infty)$:

Assumption 1: $Y(e_1, e_2) = Y(e_2, e_1)$;

Assumption 2: $Y_1(e_1, e_2) \equiv \frac{\partial}{\partial e_1} Y(e_1, e_2) > 0$, $Y_1(0, e_2) = \infty$, $Y_1(\infty, e_2) = 0$, and $Y_{11}(e_1, e_2) \equiv \frac{\partial^2}{\partial e_1^2} Y(e_1, e_2) < 0$;

Assumption 3: $Y_{12} \equiv \frac{\partial^2}{\partial e_1 \partial e_2} Y(e_1, e_2) < 0$.

Assumptions 1-2 implies $Y_2(e_1, e_2) \equiv \frac{\partial}{\partial e_2} Y(e_1, e_2) > 0$ and $Y_{22}(e_1, e_2) \equiv \frac{\partial^2}{\partial e_2^2} Y(e_1, e_2) < 0$. Thus, $Y(e_1, e_2)$ is increasing and concave for e_1 given e_2 and also in e_2 given e_1. Assumption 3 implies that e_1 and e_2 are substitutes. These assumptions are needed to guarantee the existence of optimal effort levels of the workers.

Workers' effort level is not observable to the manager but joint profit is observable only to the manager. To provide an effort incentive for each worker, the manager needs to hire another person, a risk-neutral supervisor, who can observe each worker's effort level with an error but without incurring any monitoring cost. Specifically, given worker i's effort level e_i, the supervisor observes the following randomly perturbed effort level x_i from worker i

$$x_i = e_i + \epsilon_i, \ \epsilon_i \in (-\infty, +\infty), \tag{2}$$

where ϵ_i captures all the random factors that affect the supervisor's monitoring process. The random variables ϵ_1 and ϵ_2 are continuous and they are independently and identically distributed with mean zero.

In general, the supervisor's personal judgements on the workers' performance levels will not be fully revealed to the workers, but the workers may have rough estimates on those measures. Thus, for simplicity, we assume that x_1 and x_2 are commonly understood by the supervisor and the workers to be the supervisor's assessments of the workers' effort levels. However, since x_1 and x_2 are the supervisor's subjective assessments, they are not verifiable to any third party including the manager.[6]

The supervisor reports x_1 and x_2 to the manager and the manager pays monetary wages M and s_i to the supervisor and worker $i, i = 1, 2$, respectively. Worker i's utility function

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[6]Assuming that the workers can observe x_1 and x_2 may be a bit strong. However, even if we assume that there are some parts of the supervisor's personal judgements that cannot be observed by the workers, the main results established in this paper will not change qualitatively in the sense that each worker's incentive to work hard still exists with law enforcement.

is additively separable:

$$U(s_i, e_i) = s_i - C(e_i), \tag{3}$$

where $C(e_i)$ denotes worker i's disutility of exerting effort level e_i. We assume

Assumption 4: $C(\cdot)$ is twice continuously differentiable with $C'(0) = 0$, $C'(\infty) = \infty$, and $C''(e_i) > 0$ for $e_i \geq 0$.

The manager's and the supervisor's utility functions are linear in the monetary wage and residual profit, respectively. By working elsewhere, the highest utility level the supervisor can achieve is denoted by $\overline{U}_s$ and that each worker can achieve is denoted by $\overline{U}_w$.

2.1 The Rank-Order Tournament without Bribery

When the workers' assessed performance levels are not verifiable to any third party, the manager cannot offer each worker an absolute performance scheme that ties his pay only to his own performance level (i.e., $s_i = s_i(x_i)$). This is because under such a pay schedule the manager has an incentive to claim down each worker's performance to the lowest possible level after it has been realized, whereas each worker has an incentive to claim it up to the highest possible level.

As shown by Malcomson (1984), this problem can be avoided if the manager offers instead a relative performance scheme in which each worker's pay depends on how good his performance is compared with the other worker's performance. The reason is because the total payment that the manager makes to the workers is independent of x_1 and x_2 under the relative performance schemes, and thereby the manager has no incentive to claim down x_1 and x_2 while competition to win the prize drives the workers to work hard.

For example, under the rank-order tournament as formulated by Lazear and Rosen (1981), the manager promises at the outset to pay base wage W to each worker and monetary bonus B to the winner, the worker with a higher assessed performance level.

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The total payment the manager makes to the workers is $2W + B$ which is independent of x_1 and x_2. The timing of moves under the tournament is as follows:

- Stage 0: Nature independently draws ϵ_1 and ϵ_2. The nature's drawing is revealed to none of the parties. However, the distribution of ϵ_1 and ϵ_2 is common knowledge.

- Stage 1: The manager sets a base wage $W \geq 0$ that he commits to pay to each worker regardless of the worker's performance level and a bonus $B \geq 0$ that he commits to pay to the winner. In addition, the manager also sets a basic salary M for the supervisor who is hired to monitor and assess the workers' performance.

- Stage 2: Given the bonus-wage pair (B, W), the workers individually choose effort levels, $e_1, e_2 \geq 0$, individually. The supervisor then subjectively assesses the workers' performance levels x_1 and x_2 and reports to the manager who the winner is based on x_1 and x_2. The manager pays base wage W to each worker, bonus B to the winner, and the basic salary M to the supervisor.[7]

[Insert FIGURE 1 here]

The tournament game is solved by backward induction, using the prior beliefs on nature's drawing: We solve the tournament subgame for a given bonus-wage pair for the workers and salary for the supervisor, we then compute the manager's reduced-form expected profit as a function of the bonus-wage pair, and finally we determine the bonus-wage pair for the workers and the salary for the supervisor from the manager's expected profit maximization problem.

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[7] We will consider in the next subsection the case in which it is up to the supervisor to decide whether to have the workers compete in a bribery game before he selects the winner. In that case, there is an incentive problem associated with the supervisor's reporting x_1 and x_2 truthfully. Even in that case, however, the supervisor's pay schedule must be a fixed salary because there is no variable on which the supervisor's incentive can be based.

Due to the nonverifiability of the supervisor's subjective assessments, the workers may find in their best individual interest to win the supervisor's favor by bribing him after production but before selection of the winner. Bribery induces the workers to further compete after realization of production. In order to see the impacts of the bribery game on the structure of the rank-order tournament, it will be useful to understand how the socially efficient outcome can be obtained by the rank-order tournament without bribery.

Note that from (2), given the workers' effort levels e_1 and e_2, the probability that worker 1 performs better is given by

$$\text{Prob}(x_1 > x_2) = \text{Prob}(\epsilon_1 - \epsilon_2 > e_2 - e_1) = 1 - G_{12}(e_2 - e_1),$$

where G_{12} denotes the cumulative distribution function of random variable $\epsilon_1 - \epsilon_2$. Thus, given a bonus-wage pair (B, W) and the workers' effort levels e_1 and e_2, worker 1's expected utility is $W + [1 - G_{12}(e_2 - e_1)]B - C(e_1)$. It follows that, given (B, W) and e_2, worker 1's optimal effort choice satisfies the following first-order condition:

$$B g_{12}(e_2 - e_1) = C'(e_1),$$

where $g_{12}(\cdot)$ denotes the probability density function of random variable $\epsilon_1 - \epsilon_2$. We assume that the usual second-order conditions hold. The reader is referred to Bhattacharya and Guasch (1988) for a detailed discussion regarding the second-order conditions for the rank-order tournament. Given (B, W) and e_1, worker 2's optimal effort choice satisfies a similar first-order condition.

As in Lazear and Rosen (1981), due to symmetry we restrict our attention to a solution in which the workers choose an identical effort level for a given bonus-wage pair. Thus, given a bonus-wage pair (B, W), it follows from the above first-order condition that worker 1's optimal effort level satisfies

$$B g_{12}(0) = C'(e).$$

Since $C''(e) > 0$ for $e \geq 0$, $C'(0) = 0$, and $C'(\infty) = \infty$, the above first-order condition has a unique positive solution as long as $B > 0$ and $g_{12}(0) > 0$. Note also from the above

equation that worker 1's optimal effort level depends only on bonus B but not on base wage W. We let $e^o(B)$ denote the workers' optimal effort level at bonus B and call the mapping e^o the workers' *effort plan*.

Since $G_{12}(0) = 1/2$, it follows that given each worker's effort plan e^o, the manager's expected profit with a bonus-wage pair (B, W) is

$$Y(e^o(B), e^o(B)) - [2W + B + M]. \tag{4}$$

The participation constraint for the supervisor is

$$M \geq \overline{U}_s, \tag{5}$$

whereas the participation constraint for each worker is

$$W + \frac{1}{2}B - C(e^o(B)) \geq \overline{U}_w. \tag{6}$$

Equations (4), (5), and (6) imply that the manager's profit maximization problem is equivalent to maximizing:

$$Y(e^o(B), e^o(B)) - 2C(e^o(B)) - 2\overline{U}_w - \overline{U}_s.$$

Consequently, the manager's optimal design for the bonus, B^*, satisfies

$$Y_1(e^o(B^*), e^o(B^*)) = C'(e^o(B^*)). \tag{7}$$

The manager's optimal design for the worker's base wage, W^*, is then determined by each worker's participation constraint such as

$$W^* + \frac{1}{2}B^* - C(e^o(B^*)) = \overline{U}_w. \tag{8}$$

Likewise, the manager's optimal design for the supervisor's wage, M^*, will be determined by his participation constraint such as

$$M^* = \overline{U}_s. \tag{9}$$

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Equation (7) indeed guarantees that the resulting rank-order tournament is socially efficient in the sense that the bonus-wage pair and the workers' effort levels induced thereby maximize the expected total surplus. In other words, the rank-order tournament with (B^*, W^*) results in $e^o(B^*) = e^*$ where e^* satisfies $Y_1(e^*, e^*) = C'(e^*)$.

2.2 Rank-Order Tournament with Bribery

As reviewed in the previous subsection, when bribery is not present, it is optimal for the manager to design the rank-order tournament which will induce the socially efficient effort level from each worker. However, since the ranking of the workers' performance is largely at the supervisor's discretion, the workers may find it in their interests to win the supervisor's favor by bribing him after he has subjectively assessed their performance. On the other hand, the supervisor will accept bribes as long as they increase his expected utility. This means that bribery is more likely to occur the less perfect the law enforcement.

If law enforcement is not in place at all, the rank-order tournament provides no effort incentive for the workers. This is because the supervisor's selection of the winner will be based entirely on the workers' bribing offers (see Fairburn and Malcomson (2001)). On the other hand, if bribery is always detected as in the case of perfect law enforcement and the penalties for bribery are sufficiently high, then bribery will not occur and the results associated with rank-order tournament without bribery will still be valid. In reality, however, law enforcement against bribery is neither perfect nor nonexistant. Furthermore, the size of the penalty that can be collected by law enforcement when bribery is detected cannot be unlimited either.

To incorporate such a legal aspect of bribery into the context of the rank-order tournament, we assume that if the better performer is not selected as the winner, bribery will be detected with positive probability. This probability is positively correlated with the difference between the two workers' performance levels, $|x_1 - x_2|$. That is, the more the selected winner is outperformed by the other worker, the more likely the bribery is detected by law

enforcement. On the other hand, if the better performer is selected as the winner, bribery will be detected with probability 0. One justification for this is that no legal dispute will arise in this case because the supervisor has reported the right person as the winner.

When bribery is detected, both the supervisor and the bribing worker must pay penalties. Since in practice it is often difficult to verify the size of the bribe, we assume that the penalties do not depend on the size of the bribe. Those penalties include not only fines that will be collected by the court but also nonmonetary consequences such as imprisonment. Whether the supervisor will accept bribes depends on the extra expected cost in the form of a penalty he faces. As mentioned in the introduction, we assume that the supervisor does not take any bribe from the worker whom he is not going to select as the winner. It follows that the bribery game is just a bidding race in a sealed-bid auction (a refundable contest).[8] Accordingly, the bribery game adds the following stage to the tournament:

- Stage 3: After assessing the workers' performance levels x_1 and x_2, the supervisor decides whether to have the workers play the bribery game by accepting bribes from them. If the bribery game is presented, the workers will choose their bribe offers b_1, b_2. The supervisor then determines which worker to report as the winner.

[Insert FIGURE 2 here]

Let $p(x_i - x_j)$ be the probability that bribery will be detected when the supervisor reports worker j as the winner when worker i is the better performer (i.e., when $x_i > x_j$). In that event, the supervisor will pay a penalty equal to D_s and worker j will pay a penalty equal to D_a. Set $D \equiv D_a + D_s$. We assume

Assumption 5: $p(\cdot)$ is monotonically increasing with $p(0) = 0$ and $p(\infty) = 1$.

[8]If we assume that the supervisor takes bribes from both workers, then the bribery game will be the "war of attrition" game (i.e., a non-refundable contest). Even in this case, however, our main results will not change qualitatively due to the general "revenue equivalence theorem" discussed by Riley and Samuelson (1981).

We first have:

PROPOSITION 1: *Let (B, W) with $B > 0$ be any given bonus-wage pair and let (D_a, D_s) with D_a, $D_s > 0$ be any given penalty pair. Then, given the workers' assessed performance levels $x_i > x_j$,*

(i) it is optimal for the supervisor to report worker i as the winner whenever their bribe offers satisfy $b_j - b_i \leq p(x_i - x_j)D_s$, otherwise, it is optimal for the supervisor to report worker j as the winner.[9] The optimal bribe amounts for worker i and worker j to offer are respectively

$$b_i^* = \max\{0, B - p(x_i - x_j)D\} \quad and \quad b_j^* = \max\{0, B - p(x_1 - x_2)D_a\}; \qquad (10)$$

(ii) the supervisor is better off with the bribery game if and only if

$$p(x_i - x_j) < \frac{B}{D}. \qquad (11)$$

Proposition 1 shows that, in our framework, the better performer is always selected as the winner implying that the existence of the bribery game lead to any falsification of the ranking of the workers in equilibrium. This non-falsification outcome hinges on our assumption that the supervisor always reports the better performer as the winner when he is indifferent in selecting any worker. However, if we apply another tie-breaking rule that requires the supervisor to randomly pick the winner when he is indifferent, then

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[9] If $b_2 - b_1 = p(x_1 - x_2)D_s$, the supervisor is actually indifferent between having worker 1 and worker 2 as the winner. In this case, as a simple tie-rule, we assume that the supervisor always chooses the better performer, worker 1, as the winner. The justification for adopting this tie-rule is that, if $b_2 \leq B$, worker 1 is better off by slightly raising his bribe. If we instead adopt a tie-rule in which the supervisor randomly picks up the winner, each worker's optimal bidding strategy will be a randomized one. However, even in this case, the main results in this paper will not change qualitatively due to the "revenue equivalence theorem".

both workers will adopt random strategies for bribing and thus there is always a positive probability that the poor performing worker is selected as the winner.

As shown in Burguet and Che (2004), the falsification outcome which is due to bribing parties' adopting random strategies for bribing incurs a social cost in a procurement game. In contrast, social efficiency in this paper is achieved with provision of proper incentives for the workers and hence, the supervisor's falsification itself which may occur when the workers are playing random strategies will not have any consequence on the social efficiency as long as the same effort incentives are given to the workers.

Proposition 1 shows that the rank-order tournament even under imperfect law enforcement can provide some effort incentives for the workers. This is because the expected total penalty the supervisor and the poor performing worker have to pay when falsification is detected plays an incentive role that induces each worker to be the better performer. In fact, the better performing worker can win the race without having to bribe as much as the poor performing worker has to.

To be more precise, let $\bar{z}_p \equiv \inf\{z > 0 \mid p(z) = 1\}$. Thus, $\bar{z}_p$ stands for the assessed performance gap at and beyond which the supervisor's falsification of the winner due to bribery can surely be detected. As a standard convention, the infimum is taken to be ∞ when the underlying set is empty. Set $p^{-1}(\frac{B}{D}) \equiv \bar{z}_p$ if $\frac{B}{D} \geq 1$.

When $B < D$, condition (11) determines a finite range of the workers' performance gap given by $|x_1 - x_2| < p^{-1}(B/D)$ over which it is profitable for the supervisor to have the bribery game. Thus, by (10), worker 1 receives W when $x_1 - x_2 < 0$; he receives W and bonus B but pays bribe $B - p(x_1 - x_2)D$ when $0 \leq x_1 - x_2 < p^{-1}(B/D)$; and he receives W and B when $x_1 - x_2 \geq p^{-1}(B/D)$. On the other hand, when $B \geq D$, the supervisor is always better off with the bribery game. Thus, in that case, worker 1 receives W when $x_1 - x_2 < 0$; he receives W and bonus B but pays bribe $B - p(x_1 - x_2)D$ when $0 \leq x_1 - x_2$. Note also that $B - p(x_1 - x_2)D = B - D$ when $x_1 \geq x_2 + \bar{z}_p$. Worker 1's payoff schedule can be depicted as in Figure 3:

REMARK 1: *With $B \geq D$, the net bonus that the winner gets is determined by D but not by B. That is, if the total penalty is not large enough relative to the bonus, then no matter how significant the performance gap may turn out to be, the supervisor is always better off with the bribery game and the fraction of the bonus that the better performer retains is exactly the expected total penalty the supervisor and the poor performer face.*

Another thing to note is that each worker's effort incentive depends only on the total penalty $D \equiv D_a + D_s$ rather than on the individual penalties D_s and D_a. Given a penalty pair, (D_a, D_s) and given performance levels $x_1 > x_2$, the supervisor pays expected penalty $p(x_1 - x_2)D_s$ if he reports worker 2 as the winner. This expected penalty would be avoided if he reports worker 1 as the winner instead. Thus, in order for worker 2 to be selected as the winner, he has to offer a bribe that exceeds worker 1's offer at least by $p(x_1 - x_2)D_s$. Furthermore, as the poor performer worker 2 incurs an expected cost equal to expected penalty $p(x_1 - x_2)D_a$ for becoming the winner. From (2), the more effort a worker supplies, the more likely he will have a better performance assessment. Hence, the winning bribe difference between the better and the poor performers that provides each worker with an incentive to supply more effort is $p(x_1 - x_2)D$ which depends only on the total penalty. We have:

PROPOSITION 2: *With imperfect law enforcement, the rank-order tournament with bribery can provide some effort incentives for the workers to supply more effort that only depend on the total penalty $D \equiv D_a + D_s$.*

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3 Analysis

3.1 The Workers' Effort Choice Problem

Having analyzed the supervisor's and the workers' decision problems in stage 3, we now consider each worker's effort choice problem in stage 2. Due to symmetry, we only consider worker 1's effort choice problem and drop the subscripts in G_{12} and g_{12}, which are respectively the cdf and pdf of random variable $\epsilon_1 - \epsilon_2$.

By (2), the cdf and the pdf of random variable $z \equiv x_1 - x_2$ are $G(e_2 - e_1 + z)$ and $g(e_2 - e_1 + z)$, respectively.[10] Thus, given bonus-wage pair (B, W) and total penalty D, and given both workers' effort levels e_1 and e_2, worker 1 receives expected payment $W + E(B, D, e_1, e_2)$ where $E(B, D, e_1, e_2)$ denotes the expected bonus of worker 1. From Figure 3,

$$
\begin{aligned}
E(B, D, e_1, e_2) = \ & B[1 - G(e_2 - e_1 + p^{-1}(\tfrac{B}{D}))] \\
& + D \int_0^{p^{-1}(\frac{B}{D})} p(z) g(e_2 - e_1 + z) dz \quad \text{for} \quad B < D,
\end{aligned}
\tag{12}
$$

and

$$
\begin{aligned}
E(B, D, e_1, e_2) = \ & D[1 - G(e_2 - e_1 + \overline{z}_p))] \\
& + D \int_0^{\overline{z}_p} p(z) g(e_2 - e_1 + z) dz \quad \text{for} \quad B \geq D.
\end{aligned}
\tag{13}
$$

Given any bonus-penalty pair and the other worker's effort level, each worker chooses an effort level to maximize the difference between his expected payment and his cost of effort. Since we consider solutions in which the workers choose identical effort levels, (12) and (13) imply that the workers' optimal effort level satisfies the following first-order condition:[11]

$$
Bg(p^{-1}(\tfrac{B}{D})) - D \int_0^{p^{-1}(\frac{B}{D})} p(z) g'(z) dz = C'(e) \quad \text{when } B < D,
\tag{14}
$$

---

[10] Note that we use z both as random variable $x_1 - x_2$ and a realization of the random variable.

[11] We continue to assume that the usual second-order condition holds.

and

$$Dg(\overline{z}_p) - D \int_0^{\overline{z}_p} p(z)g'(z)dz = C'(e) \text{ when } B \geq D. \tag{15}$$

Since $p(0) = 0$ and $p(\overline{z}_p) = 1$, by integration by parts we can simplify both (14) and (15) to

$$D \int_0^{p^{-1}(\frac{B}{D})} g(z)p'(z)dz = C'(e), \tag{16}$$

where $p^{-1}(\frac{B}{D}) = \overline{z}_p$ for $B \geq D$. Note that $g(z) > 0$ for $z > 0$ and $p'(z) > 0$ for $z \in [0, p^{-1}(\frac{B}{D}))$. Thus, the assumptions on cost function $C(\cdot)$ guarantee that equation (16) determines a unique positive effort level. We denote this unique effort level at bonus-penalty pair (B, D) by $e^b(B, D)$ and call it the workers' *effort plan*. Note also that, since $C(\cdot)$ is twice continuously differentiable, $e^b(B, D)$ is continuously differentiable with respect to (B, D) by the implicit function theorem.

<u>LEMMA 1</u>: *The effort plan $e^b(B, D)$ satisfies the following properties:*

$$\frac{\partial e^b(B, D)}{\partial B} > 0 \quad when \quad B < D,$$

$$\frac{\partial e^b(B, D)}{\partial B} = 0 \quad when \quad B \geq D.$$

Lemma 1 shows that more effort can be induced from the workers by increasing the bonus as long as the existing bonus is smaller than total penalty D. This is sensible given Remark 1. Accordingly, any bonus bigger than D is not sensible because it provides no further effort incentive to the workers than bonus $B = D$ does.

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REMARK 2: *Since $G(a) = \int_{-\infty}^{a} g(z)dz = (1/2) + \int_{0}^{a} g(z)dz$ for any $a > 0$, we have $B[1 - G(p^{-1}(\frac{B}{D}))] = B/2 - B \int_{0}^{p^{-1}(B/D)} g(z)dz$. Thus from (12), with $e_1 = e_2$,*

$$E(B, D, e_1, e_2) = \frac{B}{2} - \int_{0}^{p^{-1}(B/D)} [B - p(z)D]g(z)dz \quad \text{when} \quad B < D. \tag{17}$$

Note also that $D[1 - G(\bar{z}_p)] = D/2 - D \int_{0}^{\bar{z}_p} g(z)dz$, $p(z) = 1$ for $z \geq \bar{z}_p$, and $\int_{0}^{\infty} g(z)dz = 1/2$. Thus from (13), with $e_1 = e_2$,

$$
\begin{aligned}
E(B, D, e_1, e_2) =\ & D/2 - \int_{0}^{\bar{z}_p} [D - p(z)D]g(z)dz \\
=\ & D/2 - \int_{0}^{\infty} [D - p(z)D]g(z)dz \\
=\ & D \int_{0}^{\infty} p(z)g(z)dz \\
=\ & \frac{B}{2} - \int_{0}^{\infty} [B - p(z)D]g(z)dz \quad \text{when} \quad B \geq D.
\end{aligned}
\tag{18}
$$

When the workers supply an identical effort level, equations (17) and (18) imply that the expected bonus for each worker is simply the difference between the expected bonus he would get without the bribery game and the expected bribe he would pay to the supervisor in the bribery game. Also note that when $e_1 = e_2$, the workers' expected bonuses are identical and independent of the workers' effort levels. Thus, we will shorten the notation by suppressing e_1 and e_2 from $E(B, D, e_1, e_2)$ and write it simply as $E(B, D)$.

3.2 The Manager's Optimization Problem

In this subsection, we study when it is optimal for the manager to set the bonus that will induce the workers to supply the socially efficient effort level. To this end, note first that given each worker's effort plan with bribery $e^b(B, D)$ and given total penalty D, the manager's expected profit is $Y(e^b(B, D), e^b(B, D)) - 2W - B - M$. Note also that the supervisor's expected payoff in equilibrium is

$$\pi_s = M + 2 \int_{0}^{p^{-1}(\frac{B}{D})} [B - p(z)D]g(z)dz \quad \text{when} \quad B < D, \tag{19}$$

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and
$$\pi_s = M + 2\int_0^\infty [B - p(z)D]g(z)dz \quad \text{when} \quad B \geq D. \tag{20}$$

In each of the above two equations, as before M stands for the supervisor's official fixed salary from the manager whereas the second terms denote his expected illegal payoffs from bribery. Using equations (17) and (18), we can simplify equations (19) and (20) as

$$\pi_s = M + B - 2E(B, D),$$

where $B - 2E(B, D)$ indicates the portion of the bonus that would belong to the supervisor through bribery. Accordingly, the manager's optimization problem given total penalty D is:

$$\begin{aligned} &Maximize && Y(e^b(B, D), e^b(B, D)) - 2W - B - M \\ &W, B, M \end{aligned}$$

$$\text{Subject to} \tag{21}$$

$$(i) \ M + B - 2E(B, D) \geq \overline{U}_s$$
$$(ii) \ W + E(B, D) - C(e^b(B, D)) \geq \overline{U}_w,$$

where the constraints denote the supervisor's and each worker's participation constraints, respectively.

Let $(W^b(D), B^b(D), M^b(D))$ denote the solution for the above optimization problem. It is easy to see that the both participation constraints must be binding at the optimum. Thus, by substituting both participation constraints into the manager's expected profit function, we can reduce the manager's optimization problem to

$$Maximize_B Y(e^b(B, D), e^b(B, D)) - 2C(e^b(B, D)) - 2\overline{U}_w - \overline{U}_s. \tag{22}$$

In other words, given total penalty D, the manager would set the bonus so as to maximize expected total surplus $Y(e, e) - 2C(e)$ subject to nonnegativity constraint on the bonus.

PROPOSITION 3: *Let $\underline{D}$ be given by*

$$\underline{D} = \frac{Y_1(e^b(\underline{D}, \underline{D}), e^b(\underline{D}, \underline{D}))}{\int_0^{\bar{z}_p} g(z)p'(z)dz}.$$ (23)

Then, $e^b(B^b(D), D) \begin{pmatrix} = \\ < \end{pmatrix} e^*$ *whenever* $D \begin{pmatrix} \geq \\ < \end{pmatrix} \underline{D}$; $B^b(D) \begin{pmatrix} < \\ = \end{pmatrix} D$ *whenever* $D \begin{pmatrix} > \\ \leq \end{pmatrix} \underline{D}$.

Proposition 3 elicits the condition under which the socially efficient outcome can be obtained from the rank-order tournament with bribery. It states that if the total penalty for bribery D is larger than a certain minimal level, it is still in the manager's best interest to design the rank-order tournament that induces the socially efficient effort. However, if the total penalty is smaller than that minimal level, then the effort that will be induced from the workers is lower than the socially efficient level. In that case, the manager's optimal design for the bonus is equal to the given total penalty. As shown in (23), the lower bound of the total penalty which guarantees the social efficiency is determined by the effectiveness of law enforcement and the distribution of random factors affecting the supervisor's monitoring process.[12]

As depicted in Figure 3, given (B, D), the fraction of the bonus that the winner receives at assessed performance gap z is $\min\{B, p(z)D\}$. Therefore, if D is not big enough $(D < \underline{D})$, then the fraction will be limited by D but not by B and it is too small to induce the socially efficient effort level from the workers. On the other hand, $W^b(D)$ and $M^b(D)$ will be determined simply from the participation constraints:

$$M^b(D) + B^b(D) - 2E(B^b(D), D) = \overline{U}_s,$$

and

$$W^b(D) + E(B^b(D), D) - C(e^b(B^b(D), D)) = \bar{U}_w.$$

We now focus on the case where the total penalty D is greater than $\underline{D}$ in (23) so that the socially efficient outcome can be obtained through the rank-order tournament.

---

[12]The effectiveness of law enforcement is measured by the probability with which the supervisor's falsification of the ranking of the workers is detected.

PROPOSITION 4: *The triplet $(B^b(D), W^b(D), M^b(D))$ converges to (B^*, W^*, M^*) as D increases, where B^*, W^*, and M^* are respectively defined in (7), (8), and (9).*

Proposition 4 shows that as total penalty D goes to infinity, the manager's optimal design for (B, W, M) with bribery converges to those without bribery. It follows that considering the triplet (B^*, M^*, W^*) as corresponding to the case without corruption, the size of the total penalty may be considered as a measure of how clean the supervisor-worker relationship is. Namely, the bigger the total penalty the law enforcement can implement to penalize bribery, the less corrupt the supervisor-worker relationship would become.

PROPOSITION 5: *Assume g is differentiable with $g' < 0$ in $(0, \infty)$. Then,*

$$dB^b(D)/dD < 0, \ D > \underline{D}.$$

Note that each worker's effort incentive depends on the expected bonus, $E(B, D)$ rather than the nominal bonus B itself. By Remark 2, given nominal bonus B, the expected bonus increases as the total penalty increases. Thus, to maintain each worker's incentive at the socially efficient level, the manager must decrease the nominal bonus B as the total penalty D increases.

Lazear (1989) shows that with the tournament without bribery the pay inequality between the winner and the loser as measured by the nominal bonus becomes smaller the more damage a worker can inflict on the other worker's production by sabotage. Proposition 5 implies that the pay inequality is reduced as the total penalty D increases with bribery. In other words, the larger the total penalty is and thereby the less corrupt the supervisor-worker relationship is, the smaller the nominal pay inequality will be.

However, Proposition 5 does not imply that the pay inequality as measured by the expected bonus $E(B^b(D), D)$ also decreases as the total penalty D increases. In fact, as we

show in the following proposition, the opposite is true under the condition that $g(e_2 - e_1 + z)$ satisfies MLRP (Monotone Likelihood Ratio Property) with respect to e_1.[13]

PROPOSITION 6: *Assume g is differentiable with $g' < 0$ in $(0, \infty)$. Assume further $g(e_2 - e_1 + z)$ satisfies MLRP with respect to e_1. Then,*

    (i) $dW^b(D)/dD < 0$,

    (ii) $dE(B, D)/dD > 0$, and

    (iii) $dM^b(D)/dD > 0$ for $D \geq \underline{D}$.

Proposition 6 shows that under the rank-order tournament with bribery, the workers' base wage deceases but the expected bonus increases as the total penalty D increases, implying that the manager relies more on the bonus part to give the workers effort incentives as the supervisor-worker relationship becomes less corrupt. This is because, as the total penalty increases, the nominal bonus B becomes more effective as an incentive device. On the other hand, the supervisor's base salary increases as D increases. This implies that, as the supervisor-worker relationship becomes less corrupt, the manager in compensating the supervisor can rely more on the official salary rather than the illegal benefit from bribery. Furthermore, the results in (i) and (iii) in the above proposition indicate that $M^b(D) - W^b(D)$ (assuming that $M^b(D) > W^b(D)$) increases as D increases. Thus, it predicts that, as the supervisor-worker relationship becomes less corrupt, the wage gap between the bribe-taking party and the bribe-offering party increases.

As a major result, Proposition 3 states that if the total penalty for bribery is bigger than a certain minimum level, then the socially efficient effort can be induced from the workers by the rank-order tournament even when law enforcement is imperfect. However, this result hinges on our assumption that the supervisor's basic salary M can be lowered unlimitedly. This needs a more careful consideration.

---

[13]The density function $g(e_2 - e_1 + z)$ is said to satisfy MLRP with respect to e_1 if g_{e_1}/g is increasing in z where g_{e_1} denotes the partial derivative of g with respect to e_1. For details of MLRP, see Milgrom (1981, pp 383) and Innes (1990, pp 48).

Recall that the supervisor's participation constraint at the optimum satisfies

$$M^b(D) + 2 \int_0^{p^{-1}(\frac{B^b(D)}{D})} [B^b(D) - p(z)D]g(z)dz = \overline{U}_s.$$

The above equation indicates that the manager can fully extract the extra benefit that the supervisor can enjoy from illegal bribery by lowering the supervisor's basic salary M. This is why the existence of the bribery game does not in fact reduce the manager's expected payoff and thereby does not affect the manager's optimal decision on (W, B).

It is possible that the supervisor's expected benefit from illegal bribery is so large that it may even exceed his reservation utility. That is, it may well be the case that

$$2 \int_0^{p^{-1}(\frac{B^b(D)}{D})} [B^b(D) - p(z)D]g(z)dz > \overline{U}_s.$$

When this happens, the manager must be able to offer a negative salary to the supervisor $(M^b(D) < 0)$ to completely extract the supervisor's extra benefit from illegal bribery. In a sense, this is equivalent to charging the supervisor an entrance fee which is not quite realistic in many cases.

To be more realistic, we consider the case where there is a minimum level $\overline{M}$ that the supervisor's basic salary cannot go below. Note that the existence of the additional constraint affects neither the supervisor's decision as to whether to allow the workers to bribe him nor the workers' decisions in the bribery game if presented and their effort choices. Consequently, the manager's optimization problem given total penalty D now becomes:

$$
\begin{aligned}
&Maximize && Y(e^b(B, D), e^b(B, D)) - 2W - B - M \\
&W, B, M \\
&\quad Subject\ to \\
&&& (i)\ M + B - 2E(B, D) \geq \overline{U}_s \\
&&& (ii)\ W + E(B, D) - C(e^b(B, D)) \geq \overline{U}_w, \\
&&& (iii)\ M \geq \overline{M}.
\end{aligned}
\tag{24}
$$

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By (19), the supervisor's participation constraint can be written as

$$M + 2 \int_0^{p^{-1}(\frac{B}{D})} [B - p(z)D]g(z)dz \geq \overline{U}_s. \tag{25}$$

Let $(W^o(D), B^o(D), M^o(D))$ be the solution for optimization problem (24). If $M^b(D) \geq \overline{M}$, then the solution $(W^b(D), B^b(D), M^b(D))$ for problem (21) satisfies all the constraints in (24). Thus, $(W^o(D), B^o(D), M^o(D)) = (W^b(D), B^b(D), M^b(D))$ and $e^b(B^o(D), D) = e^*$. We now focus on the case where $M^b(D) < \overline{M}$.

PROPOSITION 7: *If $M^b(D) < \overline{M}$, then*

(i) $B^o(D) < B^b(D)$,

(ii) $e^b(B^o(D), D) < e^*$, *and*

(iii) $M^o(D) + B^o(D) - 2E(B^o(D), D) > \overline{U}_s$.

With $M^b(D) < \overline{M}$, to induce the workers to exert the socially efficient effort e^*, the manager must set $(W, B, M) = (W^b(D), B^b(D), \overline{M})$. Notice that $M^b(D) < \overline{M}$ implies

$$\overline{M} + 2 \int_0^{p^{-1}(\frac{B^b(D)}{D})} [B^b(D) - p(z)D]g(z)dz > \overline{U}_s. \tag{26}$$

Equation (26) implies that the supervisor enjoys extra rent from illegal bribery which is a cost to the manager. Thus, it may be in the manager's interests to reduce the supervisor's rent at the expense of deviating from the socially efficient outcome.

As shown in Lemma 1, lowering the nominal bonus B lowers the effort level that would be induced from each worker. this is a cost to the manager. On the other hand, from (26) lowering B also reduces the supervisor's extra rent from illegal bribery. This is a benefit to the manager. Proposition 7 shows that if the minimum salary constraint for the supervisor is binding (i.e., $M^b(D) < \overline{M}$), then the nominal bonus for the workers $B^o(D)$ will be lowered to reduce the supervisor's extra rent from illegal bribery, and thereby the workers are induced to exert a lower effort level than e^* but the supervisor still enjoys a positive extra rent from illegal bribery (Proposition 7 (iii)).

4 Conclusion

In many principal-agent relations, objective measures of the agents' performance are not available. In these cases, the principals may have to condition the pay to the agents on subjective measures of the agents' performance, such as subjective performance assessments by a supervisor. When the pay to the agents are conditioned on subjective measures, it opens the possibility for influencing activities by the agents. This paper extends the standard model of the rank-order tournament of Lazear and Rosen (1981) by considering further competition between the agents in a bribery game after production but before the selection of the winner.

We have shown that the principal as in the case of no bribery sets the bonus at the level that induces the socially efficient outcome, provided the penalty for falsifying the ranking of the agents due to bribery is greater than a particular minimum level. This minimum level is explicitly determined by the effectiveness of law enforcement and the distribution of the random factors affecting the agents' performance. We have also shown that as the penalty for bribery increases, the principal-agent relationship becomes less corrupt. Consequently, as the penalty for bribery increases, the nominal pay inequality between the winner and the loser becomes smaller.

APPENDIX

PROOF OF PROPOSITION 1: Without loss of generality, we only consider the case where $i = 1$ and $j = 2$. Let b_1 and b_2 be the bribe amounts that workers 1 and 2 respectively make to the supervisor. Then, conditional on reporting worker 1 as the winner, the supervisor's payoff is $M + b_1$, whereas his payoff becomes $M + b_2 - p(x_1 - x_2)D_s$ if he reports worker 2 as the winner. Thus, given bonus-wage pair (B, W), penalty D_s, and given the assessed performance levels $x_1 > x_2$ of the workers,

- it is optimal for the supervisor to report worker 1 as the winner whenever $b_2 - b_1 \leq p(x_1 - x_2)D_s$; and

- it is optimal for the supervisor to report worker 2 as the winner whenever $b_2 - b_1 > p(x_1 - x_2)D_s$.

Observe that worker 1 is better off winning the bonus with bribe b_1 such that $b_1 < B$ and is indifferent when $b_1 = B$. On the other hand, as the poor performer, worker 2 pays an expected penalty equal to $p(x_1 - x_2)D_a$ if he wins as a result of bribery. Consequently, worker 2 is better off winning the bonus with bribe b_2 only when $b_2 < B - p(x_1 - x_2)D_a$ and is indifferent when $b_2 = B - p(x_1 - x_2)D_a$. Thus, given our tie-rule (footnote 9), the workers' bribe amounts must satisfy

$$b_2 - p(x_1 - x_2)D_s = b_1 \qquad (A1)$$

and

$$b_2 = B - p(x_1 - x_2)D_a. \qquad (A2)$$

Together, (A1) and (A2) imply that worker 1 offers $b_1^* = \max\{0, B - p(x_1 - x_2)D\}$ which enables him to win the bonus and worker 2 offers $b_2^* = B - p(x_1 - x_2)D_a$. Therefore, given (B, W) and given (D_a, D_s), the supervisor receives the basic salary by not allowing the workers to bribe him while he receives the basic salary plus b_1^* by allowing the workers to

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bribe him. We conclude that the supervisor is better off allowing bribery if and only if $p(x_1 - x_2) < B/D$. ∎

PROOF OF LEMMA 1: Suppose first $B < D$. Then, we have $p^{-1}(\frac{B}{D}) < \bar{z}_p$. By Assumption 5, $p'(z) > 0$ for $z \in [0, \bar{z}_p)$ and $g(z) > 0$ for $z > 0$, $\int_0^{p^{-1}(\frac{B}{D})} g(z)p'(z)dz$ in (16) is increasing in B for $B < D$. By Assumption 4, $C''(e) > 0$ for $e \geq 0$, it follows from (16) that $\partial e^b(B,D)/\partial B > 0$ when $B < D$. Suppose now $B \geq D$. Then, (16) becomes $D \int_0^{\bar{z}_p} g(z)p'(z)dz = C'(e)$, which does not depend on B. Consequently, $\partial e^b(B,D)/\partial B = 0$ when $B \geq D$. ∎

PROOF OF PROPOSITION 3: By Lemma 1, in order for the manager's optimal bonus $B^b(D)$ to induce effort level e^* at total penalty D, it is necessary and sufficient that the effort plan e^b satisfies $e^b(D,D) \geq e^*$. The reason is because, with $e^b(D,D) \geq e^*$, the first order condition for the maximization problem in (22) is $C'(e^b(B,D)) = Y_1(e^b(B,D), e^b(B,D))$, which implies $e^b(B^b(D), D) = e^*$.

Since $C''(e) > 0$ and $Y_{11}(e,e) < 0$ for $e \geq 0$ by Assumptions 2 and 4 and since $C'(e^*) = Y_1(e^*, e^*)$ by (7), the condition $e^b(D,D) \geq e^*$ in turn is equivalent to $C'(e^b(D,D)) \geq Y_1(e^b(D,D), e^b(D,D))$. Since $p^{-1}(\frac{B}{D}) = \bar{z}_p$ at $B = D$, (16) implies

$$D \int_0^{\bar{z}_p} g(z)p'(z)dz = C'(e^b(D,D)). \tag{A3}$$

Consequently,

$$C'(e^b(D,D)) \begin{pmatrix} > \\ = \end{pmatrix} Y_1(e^b(D,D), e^b(D,D)) \text{ if and only if } D \begin{pmatrix} > \\ = \end{pmatrix} \frac{Y_1(e^b(D,D), e^b(D,D))}{\int_0^{\bar{z}_p} g(z)p'(z)dz}. \tag{A4}$$

By (A3) and by Assumption 4, $e^b(D,D)$ increases as D increases. This together with Assumptions 2 and 3 implies that $Y_1(e^b(D,D), e^b(D,D))$ decreases as $e^b(D,D)$ increases. Thus, there exists a unique total penalty level $\underline{D} > 0$ such that

$$\underline{D} = \frac{Y_1(e^b(\underline{D}, \underline{D}), e^b(\underline{D}, \underline{D}))}{\int_0^{\bar{z}_p} g(z)p'(z)dz}, \tag{A5}$$

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$$D > \frac{Y_1(e^b(D,D), e^b(D,D))}{\int_0^{\bar{z}_p} g(z)p'(z)dz}, \ D > \underline{D}, \tag{A6}$$

and

$$D < \frac{Y_1(e^b(D,D), e^b(D,D))}{\int_0^{\bar{z}_p} g(z)p'(z)dz}, \ D < \underline{D}. \tag{A7}$$

Now together (A4)-(A7) imply

$$C'(e^b(D,D)) \begin{pmatrix} > \\ = \\ < \end{pmatrix} Y_1(e^b(D,D), e^b(D,D)) \ \text{ whenever } \ D \begin{pmatrix} > \\ = \\ < \end{pmatrix} \underline{D}.$$

Thus,

$$e^b(D,D) \begin{pmatrix} = \\ < \end{pmatrix} e^* \ \text{ whenever } \ D \begin{pmatrix} \geq \\ < \end{pmatrix} \underline{D}. \tag{A8}$$

Since by Lemma 1 $e^b(D,D)$ is the maximum effort level that can be induced at total penalty D, (A8) implies

$$B^b(D) \begin{pmatrix} < \\ = \end{pmatrix} D \ \text{ whenever } \ D \begin{pmatrix} > \\ \leq \end{pmatrix} \underline{D}.$$

$\blacksquare$

PROOF OF PROPOSITION 4: By Proposition 3, $B^b(D) \leq D$ for $D \geq \underline{D}$. It thus follows from (14) that the manager's optimal bonus $B^b(D)$ for $D \geq \underline{D}$ solves

$$Bg(p^{-1}(\frac{B}{D})) - D \int_0^{p^{-1}(\frac{B}{D})} p(z)g'(z)dz = C'(e^*).$$

By Assumption 5 and by L'Hôpital's rule,

$$\lim_{D \longrightarrow \infty} D \int_0^{p^{-1}(\frac{B}{D})} p(z)g'(z)dz = 0.$$

The above two equalities together with Assumption 5 imply that the limit of $B^b(D)$ as $D \longrightarrow \infty$ satisfies the equation $Bg(0) = C'(e^*)$, implying $\lim_{D \to \infty} B^b(D) = B^*$. Since $(B^b(D), W^b(D))$ solves $W + \frac{1}{2}B - \int_0^{p^{-1}(B/D)}[B - p(z)D]g(z)dz - C(e^*) = \overline{U}_w$, the limit of $\int_0^{p^{-1}(B/D)}[B - p(z)D]g(z)dz = 0$ as $D \longrightarrow \infty$ by L'Hôpital's rule, and since

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$\lim_{D \to \infty} B^b(D) = B^*$, the limit of $W^b(D)$ as $D \to \infty$ satisfies $W^* + \frac{1}{2}B^* - C(e^*) = \overline{U}_w$, from which it follows $\lim_{D \to \infty} W^b(D) = W^*$. Finally, since $(B^b(D), M^b(D))$ solves $M + 2\int_0^{p^{-1}(B/D)}[B - p(z)D]g(z)dz = \overline{U}_s$ and since $\lim_{D \to \infty} \int_0^{p^{-1}(B/D)}[B - p(z)D]g(z)dz = 0$ by L'Hôpital's rule, the limit of $M^b(D)$ as $D \to \infty$ satisfies $M^* = \overline{U}_s$, implying $\lim_{D \to \infty} M^b(D) = M^*$. ∎

PROOF OF PROPOSITION 5: By Lemma 1, $\partial e^b(B,D)/\partial B > 0$ for $B < D$. Since $\partial e^b(B,D)/\partial B$ is continuous for $B < D$, $B^b(D)$ is differentiable over $(\underline{D}, \infty)$. By Proposition 3, $C'(e^b(B^b(D), D)) = C'(e^*)$ and $B^b(D) < D$ for $D > \underline{D}$. Thus since $g'(z) < 0$ over $(0, \infty)$, (14) implies

$$\frac{dB^b(D)}{dD} = \frac{\int_0^{p^{-1}(\frac{B}{D})} p(z)g'(z)dz}{g(p^{-1}(\frac{B}{D}))} < 0.$$

∎

PROOF OF PROPOSITION 6: By Proposition 3, $B^b(D) < D$ for $D > \underline{D}$. Thus, from Figure 3, worker 1's payoff schedule $s_1(z, D)$ where $z \equiv x_1 - x_2$ is

$$s_1(z, D) = \begin{cases} W^b(D) & \text{if} \quad z \leq 0, \\ W^b(D) + p(z)D & \text{if} \quad 0 < z < p^{-1}(\frac{B^b(D)}{D}), \\ W^b(D) + B^b(D) & \text{if} \quad p^{-1}(\frac{B^b(D)}{D}) \leq z. \end{cases} \tag{A9}$$

Consider any two different total penalties D^0 and D^1 such that $\underline{D} < D^0 < D^1$. Then, proving (i) is equivalent to proving $W^b(D^0) > W^b(D^1)$. Suppose on the contrary $W^b(D^0) \leq W^b(D^1)$. By Proposition 5, $B^b(D^0) > B^b(D^1)$. We have

$$B^b(D^0)/D^0 > B^b(D^1)/D^1 \text{ and } p^{-1}(B^b(D^0)/D^0) > p^{-1}(B^b(D^1)/D^1).$$

Thus, by (A9),

$$s_1(z, D^0) < s_1(z, D^1), \quad \text{for} \quad 0 < z < p^{-1}(\frac{B^b(D^1)}{D^1}). \tag{A10}$$

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If in addition $W^b(D^0) + B^b(D^0) \leq W^b(D^1) + B^b(D^1)$, then

$$s_1(z, D^0) \leq s_1(z, D^1), \quad \text{for} \ \ z \geq p^{-1}(\frac{B^b(D^1)}{D^1}). \tag{A11}$$

Thus, in this case, the supposed inequality $W^b(D^0) \leq W^b(D^1)$ together with $W^b(D^0) + B^b(D^0) \leq W^b(D^1) + B^b(D^1)$, (A10), and (A11) implies $s_1(z, D^0) \leq s_1(z, D^1)$ for all z, with strict inequality for $0 < z < p^{-1}(B^b(D^1)/D^1)$. Thus

$$E[s_1(z, D^0)] - C(e^b(B^b(D^0), D^0)) < E[s_1(z, D^1)] - C(e^b(B^b(D^1), D^1)),$$

which implies that worker 1's participation constraint must be violated at either D^0 or D^1. It must therefore be $W^b(D^0) + B^b(D^0) > W^b(D^1) + B^b(D^1)$. Consequently, there is an assessed performance gap $z_D \in (p^{-1}(\frac{B^b(D^1)}{D^1}), p^{-1}(\frac{B^b(D^0)}{D^0}))$ such that

$$s_1(z, D^0) \leq s_1(z, D^1) \ \ \text{for} \ \ p^{-1}(\frac{B^b(D^1)}{D^1}) < z \leq z_D, \tag{A12}$$

and

$$s_1(z, D^0) > s_1(z, D^1) \ \ \text{for} \ \ z > z_D.^{14} \tag{A13}$$

Together, (A12) and (A13) imply that $s_1(z, D^0)$ crosses $s_1(z, D^1)$ only once from below to above. Since $g(e_2 - e_1 + z)$ satisfies MLRP, Lemmas 1 and 2 of Innes (1990) imply

$$e^b(B^b(D^0), D^0) > e^b(B^b(D^1), D^1),$$

which contradicts the fact that $e^b(B^b(D_0), D_0) = e^b(B^b(D_1), D_1) = e^*$. Hence it must be $W^b(D^0) > W^b(D^1)$.

To prove (ii), note that from each worker's participation constraint it follows

$$W^b(D^0) + E(B^b(D^0), D^0) - C(e^b(B^b(D^0), D^0))$$
$$= W^b(D^1) + E(B^b(D^1), D^1) - C(e^b(B^b(D^1), D^1)).$$

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[14]Note that, with $W^b(D^0) \leq W^b(D^1)$, $s_1(z, D^0)$ and $s_1(z, D^1)$ must cross each other only one time because of the characteristics of the rank-order tournament scheme.

Since $W^b(D^0) > W^b(D^1)$ by (i) and $C(e^b(B^b(D^0), D^0)) = C(e^b(B^b(D^1), D^1))$ because $e^b(B^b(D^0), D^0) = e^b(B^b(D^1), D^1) = e^*$, the above equality implies $E(B, D_0) < E(B, D_1)$.

Finally, to prove (iii), note that from the supervisor's participation constraint it follows

$$M^b(D^0) + B^b(D^0) - 2E[B^b(D^0), D^0] = M^b(D^1) + B^b(D^1) - 2E[B^b(D^1), D^1].$$

Since $B^b(D^0) > B^b(D^1)$ by Proposition 5 and $E[B^b(D^0), D^0] < E[B^b(D^1), D^1]$ by (ii), it must be $M^b(D^0) < M^b(D^1)$. ■

PROOF OF PROPOSITION 7: We begin by establishing the following properties for the solution $(W^o(D), B^o(D), M^o(D))$:

$$M^o(D) = \overline{M}, \tag{A14}$$

$$W^o(D) + E[B^o(D), D] - C(e^b(B^o(D), D)) = \overline{U}_w, \tag{A15}$$

and

$$M^o(D) + B^o(D) - 2E[B^o(D), D] > \overline{U}_s. \tag{A16}$$

To show (A14), notice that if $M^o(D) > \overline{M}$, then the supervisor's minimum salary constraint (iii) in (24) is not binding. In this case, the manager's optimization problem (24) is equivalent to optimization problem (21). It follows $(W^o(D), B^o(D), M^o(D)) = (W^b(D), B^b(D), M^b(D))$. Since $M^o(D) > \overline{M}$, $M^o(D) = M^b(D)$ contradicts the assumption that $M^b(D) < \overline{M}$. Hence it must be $M^o(D) \leq \overline{M}$. This together with constraint (iii) in (24) implies $M^b(D) = \overline{M}$.

Property (A15) follows directly from the fact that it is suboptimal to set $W^o(D)$ such that $W^o(D) + E[B^o(D), D] - C(e^b(B^o(D), D)) > \overline{U}_w$. To prove property (A16), notice that the equality $M^o(D) + B^o(D) - 2E[B^o(D), D] = \overline{U}_s$ together with (A14) and (A15) implies that $(W^o(D), B^o(D), M^o(D))$ satisfies constraints (i) and (ii) in (21) with equalities. Hence, $(W^o(D), B^o(D), M^o(D)) = (W^b(D), B^b(D), M^b(D))$ which as shown above contradicts the assumption that $M^b(D) < \overline{M}$. This proves property (A15).

Properties (A14)-(A16) implies that the manager's optimization problem (24) reduces to

$$Maximize \quad Y(e^b(B,D), e^b(B,D)) - 2\overline{U}_w - \overline{M} - 2\int_0^{p^{-1}(\frac{B}{D})}[B - p(z)D]g(z)dz$$
$$B \qquad\qquad -2C(e^b(B,D)).$$

It follows from the first-order condition for the above maximization problem that

$$[Y_1(e^b(B,D), e^b(B,D)) - C'(e^b(B,D))]\frac{\partial e^b}{\partial B} = \int_0^{p^{-1}(B/D)} g(z)dz > 0.$$

Since $\frac{\partial e^b}{\partial B} > 0$ for $D > \underline{D}$ by Lemma 1, the above first order condition implies that the optimal bonus $B^o(D)$ for the manager to set satisfies

$$Y_1(e^b(B^o(D), D), e^b(B^o(D), D)) > C'(e^b(B^o(D), D)), \ D > \underline{D}.$$

Since $Y(\cdot)$ is concave and $C(\cdot)$ is convex, the above inequality in turn implies

$$e^b(B^o(D), D) < e^*, \ D > \underline{D}. \tag{A17}$$

To complete the proof, notice that properties (i) and (ii) in Proposition 7 follow from (A17) while property (iii) follows from (A16). ∎

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