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THE PROSPECTS FOR HIGH OCCUPANCY TOLL (HOT) LANES: WHERE SHOULD THEY BE IMPLEMENTED?

Final Report for MOU 361

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Executive Summary

There is increasing interest in building new high occupancy toll (HOT) lanes and in converting high occupancy vehicle (HOV) lanes with unused capacity to HOT lanes. Like HOV lanes, HOT lanes provide an incentive for travelers to use HOVs, but unlike HOV lanes, they can always be well utilized by varying the toll over the congested period, thus providing more congestion relief than an HOV lane with unused capacity. This report provides guidelines regarding the circumstances in which HOT, HOV, and mixed flow lanes are most appropriate.

Using a queueing model combined with a mode choice model, the reduction in delay from adding an HOV, HOT, or mixed flow lane to an existing three-lane freeway was estimated for three levels of initial HOV use and three levels of initial delay. The findings for these particular cases were that:

- An added mixed flow lane eliminates delay if the initial maximum delay is 30 minutes or less.
- An added HOT lane reduces delay as much or more than an HOV lane in all circumstances because it is more fully utilized.
- An added HOT lane reduces delay almost as much as or more than a mixed flow lane in all circumstances. When delay remains after the new lane is added, the HOT lane results in higher vehicle occupancy and therefore fewer vehicle trips.
- Delay will be reduced if an HOV lane with unused capacity on a congested freeway is converted to either a mixed flow or HOT lane because of the higher utilization of these lanes.
- When an HOV lane with unused capacity is converted, it will perform slightly better as a mixed flow lane than as a HOT lane unless the delay is very high and the proportion of HOVs is high.

HOV lanes are often justified on the basis of future growth, even though they have substantial unused capacity when they are first built. HOT lanes can provide the same benefits when there is future growth, without sacrificing potential benefits in the early years because of low utilization.

Because the people not using the HOT lanes could travel faster if all the lanes were mixed flow, consideration should be given to using the toll revenues in a manner that benefits the non-users. The manner in which the toll revenue is used is important in ensuring that HOT lanes provide positive net benefits.

Experience with HOT lanes shows that more affluent people use them more often. But tolls on existing HOT lanes are not beyond the means of anyone who can buy a car and operate it on a freeway, and less affluent people use the HOT lanes, too. They may value the opportunity to buy a fast trip as much as more affluent people. So HOT lanes are not inherently inequitable.

In deciding between mixed flow, HOV or HOT lanes, cost should be considered as well as benefits. HOV lanes have additional costs for enforcement and may have higher capital costs for buffers, barriers, or enforcement areas. HOT lanes have these same costs

as well as additional capital and operating costs for toll collection. Therefore, if a mixed flow and HOT lane perform equally well in terms of delay, the mixed flow lane would be more cost-effective because of its lower costs.

An investigation of toll revenues when a HOT lane is added to a three-lane freeway showed that revenues on HOT lanes would be minimal if the maximum delay before construction were 15 minutes or less and if the percentage of HOVs were 20% or more. In case of conversion of an HOV lane on a four lane freeway to a HOT lane, revenues would be minimal if the percentage of HOVs were 20% or more or if the delay with the HOV lane were less than 6 minutes.

Conclusions:

- Adding an HOV lane is a good choice only if the initial proportion of HOVs and the initial maximum delay are very high.
- Adding a HOT lane is a good choice if the initial proportion of HOVs is low and the initial maximum delay is very high or is expected to increase substantially.
- Adding a mixed flow lane is a good choice if the initial maximum delay is not very high (less than 30 minutes on a three-lane freeway).
- Converting an underutilized HOV lane on a heavily congested freeway to a HOT lane will reduce delay and generate substantial revenues.
- Converting an underutilized HOV lane on an uncongested freeway to another type of lane will not reduce delay.
- Converting an underutilized HOV lane on a moderately congested freeway to a mixed flow lane will reduce delay to minimal levels.

Before any action is taken its effects should be analyzed using a simulation model with current vehicle volumes, speeds, and vehicle occupancies over the entire congested period, because these vary substantially over time. Toll revenues should also be carefully analyzed.

Table of Contents

List of Tables

List of Figures

THE PROSPECTS FOR HIGH OCCUPANCY TOLL LANES

The Context for Decisions Regarding HOT Lanes

Interest in high occupancy toll lanes is derived primarily from two sources. First, there is increasing interest in converting high occupancy vehicle (HOV) lanes with unused capacity to either mixed flow or high occupancy toll (HOT) lanes. A study by the California Legislative Analyst's Office in January 2000 recommended converting HOV lanes with unused capacity in congested corridors to HOT lanes and those in uncongested corridors to mixed flow lanes. Senate Bill 545, now before the California legislature, would require that all HOV lanes be evaluated to determine if they would be more effective as HOT or mixed flow lanes in terms of person delay, emissions and cost.

Second, regions considering highway expansion see HOT lanes as a means of generating revenue and giving travelers the choice of paying a toll and experiencing no delay or experiencing delay but paying no toll, while still maintaining the incentive for people to use high-occupancy vehicles. Santa Cruz County is studying the feasibility of constructing a HOT lane on Highway 1 near Capitola. HOT lanes have also been proposed on I-680 between Pleasanton and San Jose and US 101 between Petaluma and Novato.

On What Basis Should Decision-makers Choose HOT Lanes?

How should decision makers decide which type of lane is best—a mixed flow lane, a HOV lane, or a HOT lane? The first consideration should be which will be most effective in reducing delay, since this is the primary goal of any type of highway expansion or conversion. The first section of this report analyzes the delay reduction potential of the three types of lanes in a variety of circumstances, in order to show which is most effective in each circumstance. But decision-makers are also concerned about the revenue potential for HOT lanes. This is discussed in the second section of the report. The choice that HOT lanes provide is also a benefit, and this is discussed in the third section. The final section discusses the net benefit to society for each type of lane and identifies the most appropriate type of lane for different circumstances.

The intent of the report is to provide guidelines and an approach that decision-makers may use in deciding which types of lanes to implement.

Readers should keep in mind that any type of freeway expansion or improved freeway utilization generally will reduce delay, so none of these types of lanes is ineffective in an absolute sense. But in most circumstances one will provide greater public benefits than others, and the differences in benefits may be large.

The Relative Effects of HOT, HOV, and Mixed Flow Lanes on Delay

The advantage of an HOV lane over a mixed-flow lane is that it motivates travelers to use HOVs, thus reducing overall vehicle volumes and vehicle emissions. The advantages of a HOT lane over an HOV lane are that it can be better utilized and it generates revenue, while still motivating HOV use. But these lanes also have disadvantages. Both HOV and HOT lanes require some sort of enforcement and more complicated and expensive lane configurations. HOT lanes require expensive toll collection systems, and HOV lanes often have unused capacity. In fact, it is virtually impossible for an HOV lane to be fully utilized during the period in which it is in operation unless it is congested during some of that period. Even if occupancy requirements were changed during the congested period, as is done on the Katy freeway in Houston, occupancy can only be changed in increments of one person.

Modeling the Effects of Adding an HOT, HOV, or Mixed Flow Lane

The Model

The model, which is described in detail in Appendix A, uses a queueing analysis combined with a mode choice model to calculate the demand for the freeway and the delay for each traveler when a three lane freeway is expanded by the addition of:

- an HOV lane,
- a HOT lane, or
- an additional mixed flow lane

The model assumes an idealized freeway segment as shown in Figure 1. There is a bottleneck at the downstream end and the congested region has no entrances or exits. The queue builds at the builds up and dissipates as shown in the lower section of the figure. The model is described in detail in the Appendix A. It assumes that:

- the maximum flow for mixed flow lanes is 2000 vehicles per lane per hour,
- the freeway initially has 3 lanes,
- the congested period before the lane is added is three hours,
- travel time increases at a constant rate until the middle of the congested period and then falls at a constant rate until the end of the congested period,
- the carpool occupancy requirement is 2
- all HOVs use the HOV or HOT lane, and
- only HOVs use the HOV lane and only HOVs and toll-paying vehicles use the HOT lane

Figure 1. Idealized Freeway Segment

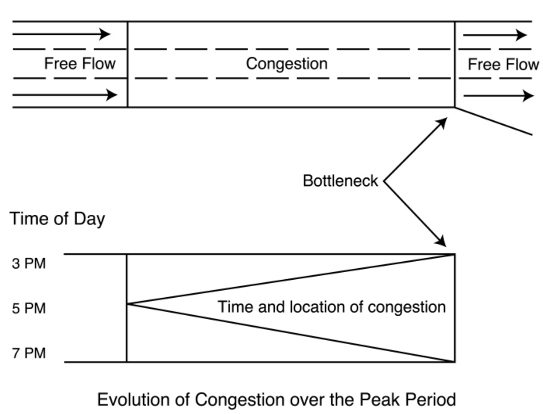
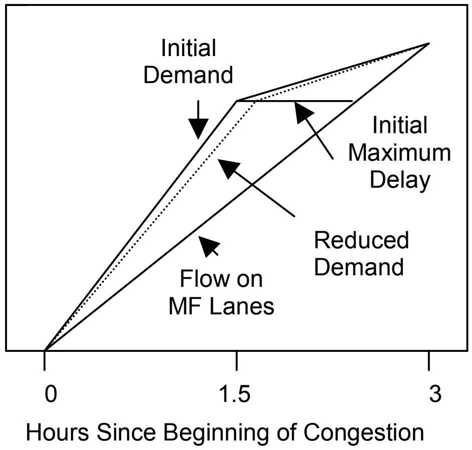


Figure 2 shows the vehicle demand and flow through a three-lane bottleneck before the lane is added. If either an HOV or HOT lane is added, the demand for the 3 initial mixed flow lanes is reduced by the number of vehicles whose drivers choose to use an HOV or pay the toll, resulting in the reduced demand for the mixed flow lanes shown by the dotted line.

Figure 2 Vehicle Demand and Flow Before and After Added HOV or HOT Lane



The model uses a logit choice model to estimate the proportion of people choosing to use HOVs. Please see Appendix A for details. A sensitivity to travel time that is at the high end of all such values found in the literature is used, and therefore shows HOV lanes in the most favorable light relative to mixed flow lanes. In addition to this parameter, there is a second parameter that captures all of the other determinants of HOV use. It is calculated from the proportion of people choosing HOVs when the time differential is zero, before the HOV lane is added.

To estimate the number of vehicles and the delay with an HOV lane, the proportion of people opting to use HOVs out of those wanting to enter the freeway during each minute of the congested period is estimated using a logit choice model. Given the proportion of travelers using the HOV lane, the travel time in each type of lane in the next minute is calculated. Then the proportion choosing each type of lane during the next minute is calculated and the resulting travel time differential during the following minute is calculated, and so on, until the end of the congested period. Clearly travelers do not have minute by minute estimates of travel time, but over the course of days, traveling at similar times or under similar delay conditions, travelers make some assessment of the freeway travel time they will save in an HOV lane and then weigh that against the other benefits and costs of HOV use.

The analysis for the HOT lane is similar, except that it is assumed that the toll can be set so that the HOT lane is fully utilized when there is congestion on the mixed flow lanes. For each minute, the proportion of people choosing to use HOVs is calculated as described above, based on the travel time differential between the HOT lane and the mixed flow lanes. The number of toll-paying people using the HOT lane is assumed to be the number that will fully utilize the remaining capacity without making the lane congested. Full utilization of the HOT lane is assumed to be 1800 vehicles per hour so that free flow speeds can be maintained in the lane. Then the travel time differential is recalculated, based on the number of people choosing the HOT lane. This calculation somewhat overstates the proportion of people who will use HOVs because their choice in this case does not take into account the toll paying option. However, in the cases modeled, an assumption that as many as 20% of the people who would be motivated to switch to HOVs would pay the toll instead, given the HOT lane option, does not change the performance ranking of the three types of lanes.

Effects of Model Assumptions

The model makes a number of simplifying assumptions. The sensitivities of the outcomes to these assumptions are discussed in Appendix A. Most assumptions have the effect of making an HOV lane look relatively better compared to mixed flow or HOT lanes than would actually be the case. The results presented in this paper also do not account for the shifts from other routes or shifts from other times of day that would actually occur if any type of lane were added. Because of such shifts there would likely be some delay during the peak even after the lane was added, so the delays after the addition of the new lanes that are shown in the following section are somewhat understated. However, these shifts do not change the relative performance of the three types of lanes, and to the extent such shifts occur, they reduce delay at other times and on other routes[1].

Circumstances Modeled

Six cases were modeled to show how circumstances affect the relative performance of the three types of lane. The circumstances that varied in these cases are:

Initial percent of HOVs:

- 5% –indicating very limited opportunities for carpooling and transit use (assuming 10.2% of people in HOVs),
- 10% –indicating moderate opportunities for carpooling and transit use (assuming 20.3% of people in HOVs), and
- 20% –typical of a dense central employment area with bus service (assuming 45% of people in HOVs).

Initial maximum delay:

- moderate delay—15 minutes,
- high delay—30 minutes, and
- very high delay—45 minutes

The initial maximum delay and the initial percentages of HOVs and people in HOVs, along with the length of the peak period and the time when the maximum delay occurs, describe the person-demand and vehicle-demand for the freeway for every minute during the congested period.

Findings Regarding Delay

Table 1 shows how sensitive delay is to initial circumstances. Table 2 shows which type of lane results in the least average delay in each circumstance modeled. Notice that in two cases only a range in delay is shown for HOT lanes. In these cases there will be some delay if only HOVs

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[1] The reduction in delay on alternate routes and the reduction in schedule delay (arriving either earlier or later than desired because of congestion) will likely result in even greater total delay reductions than estimated by the model. (Dahlgren, 1994)

use the HOT lane but none if some of the other vehicles use the HOT lane. The equilibrium delay would not be 0, but it would be very low, so it would not make a good HOT lane.

In most cases the rankings are the same in terms of both the maximum and average delays, but for the 5% HOV/45 minute initial maximum delay case the HOT lane performs best in terms of average delay but the mixed flow lane performs best in terms of maximum delay. In this case the HOT lane has higher maximum delay because the delay is not equalized across all lanes as it is with the mixed flow lane. Both maximum and average delays are shown because both are important. If 75% of people experience 1 minute of delay and 25% experience 17 minutes of delay it is considered worse than if all of the people experience 5 minutes of delay, even though the average delay is the same in both cases.

Table 1 Delay with Alternate Types of Added Lane

	Initially 5% HOVs and 10.2% of people in HOVs			Initially 10% HOVs and 20.3% of people in HOVs			Initially 20% HOVs and 45% of people in HOVs		
	Initial Maximum Delay (Minutes)								
	15	30	45	15	30	45	15	30	45
Maximum delay (minutes)									
Added mixed flow lane	0	0	11.2	0	0	11.2	0	0	11.2
Added HOT lane	<2.2	2.2	11.5	<1.7	1.7	8.7	0	.9	5.8
Added HOV lane	5.7	11.6	15.7	1.9	6.5	9.8	0	1.4	5.8
Average delay for all lanes (minutes)									
Added mixed flow lane	0	0	4.5	0	0	4.5	0	0	4.5
Added HOT lane	<.6	.6	3.7	<.5	.5	2.7	0	.2	1.8
Added HOV lane	2.0	4.4	6.3	.6	2.3	3.6	0	.4	1.8

Table 2 Lane Types Resulting in the Least Average Delay in Various Circumstances

Initially 5% HOVs and 10.2% of people in HOVs			Initially 10% HOVs and 20.3% of people in HOVs			Initially 20% HOVs and 45% of people in HOVs		
Initial Maximum Delay (Minutes)								
15	30	45	15	30	45	15	30	45
MF	MF	HOT	MF	MF	HOT	MF, HOT, or HOV	MF	HOT or HOV

- Adding a mixed flow lane to a three lane freeway eliminates delay if the initial maximum delay is 30 minutes or less

This suggests that if the initial maximum delay is 30 minutes or less *and* significant traffic growth is not expected, a mixed flow lane is the best choice. Not only does it reduce delay as much or more than the other lane types, it also costs less to construct and operate.

- An added HOT lane reduces delay as much as or more than an HOV lane in all circumstances

Only if there is a high initial proportion of HOVs and high initial maximum delay does an HOV lane result in as little delay as a HOT lane. In this case, the high initial proportion of HOVs and the large travel time advantage combine to ensure high utilization of the HOT lane by HOVs throughout the congested period. But in some such cases, such as the 20% initial HOV/45-minute initial maximum delay case shown above, there will be delay in the HOT lane during some of the peak period because of the large number of HOVs. In this case HOT lane delay averages 0.9 minutes, with an maximum of 2.9 minutes, and there is no excess HOT lane capacity for toll-paying vehicles until HOV demand drops and the HOV queue dissipates, and there are only 3 minutes after this in which there is still delay on the mixed flow lane. So in this case the HOT lane performs essentially the same as an HOV lane, and an HOV lane would be the better choice because it does not require toll collection facilities and the toll revenues would be minimal.

- An added HOT lane reduces delay almost as much as or more than a mixed flow lane in all circumstances

Table 1 shows that average delay with an added HOT lane is almost as low as with a mixed flow lane when the initial delay is 30 minutes or less. When the initial delay is 45 minutes, the HOT lane performs better than a mixed flow lane because the delay motivates some people to shift to HOVs. In terms of delay reduction, the HOT lane performs relatively well over a broad range of conditions.

Converting HOV Lanes with Unused Capacity

Figure 3 shows the conditions under which delay could be reduced by converting an HOV lane on a 4-lane freeway to either a HOT lane or a mixed flow lane.

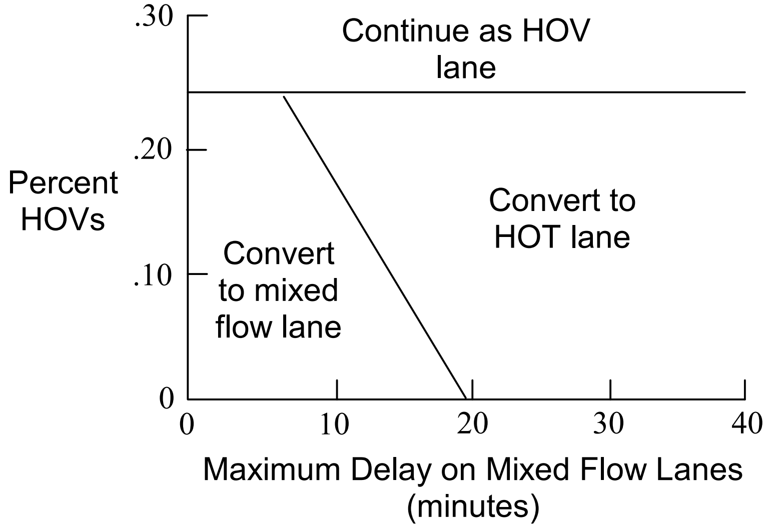
- Delay will be reduced if an HOV lane with unused capacity on a congested freeway is converted to either a mixed flow or HOT lane.

As the percent of HOVs approaches the proportion of capacity allocated to HOVs, 25% in this case, the HOV lane is well utilized and nothing is gained by conversion (for a 3-lane freeway this line would be close to 33% HOV, for a 5-lane freeway it would be close to 20%). But if the percent of HOVs is lower, there is unused capacity on the HOV lane, and delay can be reduced by converting it to a lane that can be fully utilized.

- When an underutilized HOV lane on a congested freeway is converted it will perform better as a mixed flow lane than a HOT lane when the delay is low or the proportion of HOVs is low

Figure 3 shows that the type of lane that minimizes delay depends on the combination of the proportion of HOVs and the maximum delay on the mixed flow lanes. Greater maximum delay and a higher percent of HOVs favor conversion to a HOT lane, both because there are more people in HOVs to benefit from it and because the greater delay provides a greater incentive to use an HOV. However, in making the decision to convert to a HOT lane, the cost of the toll collection must be taken into consideration.

Figure 3 Conditions Under Which to Convert HOV Lanes



Latent Demand and Growth

The model does not assume a growth rate or elasticity of travel demand with respect to travel time, because these vary widely depending on economic conditions and the availability of land for development. Any reduction in travel time will induce people to shift from other routes and other departure times, resulting in delay during the peak of the peak being more than shown in the model, but this is offset by the added convenience of departing at a more convenient time. The effect varies directly with the reduction in travel time, so it will not affect the ranking of the three types of lanes in terms of delay. If substantial growth is expected, then the maximum delay will be higher when this growth is realized, and converting an HOV lane with unused capacity to a HOT lane might be a better choice than converting it to a mixed flow lane. However, until the growth occurs, travelers will experience more delay than if the lane were converted to a mixed flow lane.

Findings Regarding Tolls and Revenues

The model assumes that the toll can be set so as to achieve full utilization of the HOT lane when the mixed flow lanes are fully utilized. Although it would be difficult to predict what this toll might be in advance, with good monitoring of toll lane volumes and densities, it should be possible to develop heuristics to allow modification of tolls in real time to approach full utilization. The toll required to minimize delay at any particular time will depend on several factors:

- total demand,
- number of HOVs using the lane,

- difference in delay between the mixed flow and HOT lanes,
- variation in travel time at the same time of day from day to day,
- highway users' value of time and ability to pay for time savings,
- other factors, such as the status value of using the HOT lane, or better maintenance or greater security on the HOT lane.

The toll that minimizes delay by maximizing HOT lane utilization is generally not the same as the toll that maximizes revenue. In this report only the former is considered because it is assumed that the primary purpose of adding a lane is to reduce delay. Of course, the revenue depends not only on the tolls, but also on the amount of capacity available for toll-paying vehicles. To investigate these interactions, the revenue generated in the above cases was estimated using the following demand for travel time savings:

$$R(t) = \alpha \left[\frac{s(t)}{w_{MF}(t) - w_H(t)} \right]^{\varepsilon}$$

where R(t) represents the proportion of travelers entering the road at a particular time that will choose to pay a toll to use the HOT lane, s(t) is the toll at time t, the expression inside the brackets is the cost per minute saved, ε is the elasticity of demand with respect to the cost per minute saved, and α is a constant that is related to other factors that influence how much a traveler is willing to pay to use the HOT lane. This constant elasticity of demand formula represents a type of demand curve often found in empirical studies. It was also found to provide the best fit in an analysis of SR 91 demand. (Dahlgren, 1999). The elasticity of demand with respect to cost per minute saved, ε, is first assumed to be -0.5. This is a rough estimate derived from the proportion of people using the express lanes on Rt 91 at different times of day when the tolls and travel time savings, and thus cost per minute saved, ranged from $0.23 to $1.60 (Sullivan 1998). A -0.5 elasticity means that if the toll increases 10% while the time saving remains constant, the number of people wanting to use the toll lane will decrease 5%. Since this elasticity is not necessarily transferable to another site, the relative revenues given a toll-elasticity of –0.9 are also computed to show how sensitive revenues are to elasticity. Revenues are calculated by normalizing α to 1. Then, because α is unknown, the toll revenues are normalized so that the revenue in the 5% HOV/30 minute initial delay case equals 1 and other toll revenues are expressed relative to the revenue for this case.

Estimating the relative revenues yields some interesting results. First, if the initial maximum delay is only 15 minutes and there are only 5 or 10% HOVs, delay can be eliminated if enough people pay tolls to use the HOT lane. But if there were no delay, why would they pay a toll? An equilibrium would be reached where both the average delay and toll were quite low. (Of course, delay could be eliminated if the toll was 0, in which case the lane would become a mixed flow lane.) Table 3 shows that in these cases there is no significant revenue. Second, if the proportion of HOVs is too high, there will be little space for toll-paying vehicles, and very high tolls will be required to limit use by toll-paying vehicles. In this case there will be little revenue also. But in this case there is the option to increase the required occupancy for HOVs as was done on the Katy freeway in Houston, where occupancy was raised to 3+ from 2+ during the peak hour. This provides a means to maintain revenues to cover the costs of the toll collection system. If the occupancy requirement were raised to 3+ for the cases modeled, the toll revenues for the 20% 2+ HOV case would be closer to the 5% 2+ HOV case because the number of 3+ HOVs is much less than the number of 2+ HOVs.

Table 3 Relative Revenues in Various Circumstances with Full HOT Lane Utilization and Toll-elasticities of -0.5 (-0.9)

		Maximum Delay Before Adding New Lane (minutes)		
		15	30	45
Percent HOVs Before Adding New Lane	5%		1 (1)	8.8 (6.0)
	10%		1.1 (.8)	19.4 (4.9)
	20%			

Tolls are feasible if the delay is substantial and the proportion of HOVs is not too large. But there is wide variance in revenues. Table 3 shows the relative revenues assuming a toll-elasticity of -0.5 and (-0.9) and an occupancy requirement of 2 for HOVs. With the higher elasticity, revenues vary less over different circumstances. However, regardless of the elasticity, revenue is much greater if the initial maximum delay is greater. The effect of a higher initial proportion of HOVs is to reduce the capacity available for toll-paying vehicles. In this case higher tolls must be charged. If demand is quite inelastic (-0.5) then the tolls are much higher, leading to higher revenues. If demand is less inelastic (-0.9) tolls are lower, leading to lower revenue.

There are two unfortunate paradoxes. In order for HOT lanes to be successful at generating revenues or motivating a shift to HOVs, they must be unsuccessful at eliminating delay on the mixed flow lanes. And in order for HOT lanes to generate substantial revenues, they must be unsuccessful at motivating a shift to HOVs.

The Benefits of a Choice of Levels of Service

The final consideration in making a decision regarding HOT lanes is the value to be gained by travelers from having the choice of level of service that the HOT lane provides. Such a choice is offered by the post-office, airlines, and a host of other service providers. Generally people are better off having a choice of levels of service. In fact, they may benefit just by knowing a higher level of service is available even if they never need to use it.

For simplicity, consider first a comparison of a freeway with all mixed flow lanes to the same freeway with one of the lanes operated as a toll lane. The basic idea is that when there is congestion with all mixed flow lanes, the total delay will be the same whether the lanes are all mixed flow or one is a toll lane operating at capacity but not above capacity. The difference is that with one uncongested toll lane there will be no delay for the vehicles whose passengers have the highest value of time, and the delay will be experienced by vehicles with lower values of time, so that the cost of the delay will be less.

In extending this analysis to the case of a HOT lane, there are two main differences. First, to the extent that people who previously drove alone now travel via HOV, overall delay, and thus the delay differential is less, tending to reduce the benefit of choice. Second, the average value of time for vehicles in the mixed flow lanes in the HOT lane case will be higher than for these vehicles in the simple toll lane case because in the HOT lane case, some HOVs with lower values of time will shift to the HOT lane. The detailed analysis is shown in Appendix B.

Equity for People in the Free Lanes

Unlike postal service or air transport service, where additional resources can be used to provide higher levels of service, the number of lanes in the freeway is fixed. The higher level of service provided to users of the HOT lane, comes at the expense of lower levels of service to people in the mixed flow lanes because capacity, and thus delay, is fixed. So, although the people paying the tolls are better off because the value of their time saving is greater than the toll, the people in the mixed flow lanes experience more delay than they would with all mixed flow lanes. As a group they would certainly be better off if all of the toll revenues collected could be paid to them, because the money they would receive would have a higher value to them than the time they lose. In this case they would actually be selling their time to the toll payers, making everyone better off. This has in fact been advocated by DeCorla-Souza (2000). His FAIR Lane concept allows vehicles in the free lanes that have toll tags to receive a credit commensurate with the excess delay they experience due to the toll lane. But, of course, some of the toll revenues must be used to collect the tolls, so not all revenues are available to compensate users of the mixed flow lanes.

Alternatively, the net toll revenue could be used for something that directly benefited mixed flow lane users, such as further expansion or more efficient operation of the freeway. If the revenues were used to provide bus service, the people using the bus service who would otherwise use the free lane would be better off than if they were using the free lanes—if not, they would not use the buses. But they still might be worse off than with all mixed lanes. The people who did not use the buses would certainly be worse off.

It is clear that if the people who use the free lanes always used the free lanes, it would be difficult to devise a system other than the FAIR system that adequately compensated them for the additional delay they experience because of the toll lane. However, a study of travelers on SR 91 in Orange County found that only 58% of travelers in 1999 never used the toll lane, compared to 72% in 1996. (Sullivan, 2000) In 1996 half of the toll lane users reported using the lane once a week or less. (Sullivan, 1998) These statistics indicates that many people often have different values of time for different trips. This mitigates the inequities between the people in the toll lane and those in the free lanes. To see how, consider the example shown in Table 4. A traveler's time on most trips is worth 10 cents a minute, but 20% of the time it is worth 50 cents a minute, so that his average value of time is 18 cents. With all mixed flow lanes, the delay on each trip is 15 minutes; with one toll lane the delay on the mixed flow lanes is 20 minutes. So with all mixed flow lanes, the average value of this traveler's delay is $2.70. With one toll lane, which the traveler uses when his time is worth 50 cents a minute, the average value of his delay is $1.60. So if he uses the toll lane 20% of the time, he is better off with the toll lane as long as the toll is less than $5.50.

In contrast, if his average value of time were always 18 cents a minute, the cost of his delay would be $2.70 with all mixed flow lanes and $3.60 on the free lanes if there were one toll lane as shown in Table 5. If the toll were less than $3.60 the traveler would use the toll lane all of the time, otherwise he would use the free lanes. But only if the toll were less than $2.70 would he be better off than with all mixed flow lanes.

Table 4 Average Cost of Delay for a Person with Different Values of Time for Different Trips

Varying value of time	**Percent of Trips**		**Average**
	80%	20%	
Value of time ($/minute)	$0.10	$0.50	$0.18
Delay with all mixed flow lanes (min)	15	15	15
Average cost of delay with all mixed flow lanes ($)	$1.50	$7.50	$2.70
Delay with one toll lane (min)	20	0	16
Average cost of delay with one toll lane ($)	$2.00	0	$1.60
Breakeven toll cost		$5.50	
Delay plus toll cost with one toll lane	$2.00	$5.50	$2.70

Table 5 Average Cost of Delay for a Person with the Same Value of Time for All Trips

Constant value of time	**Percent of Trips**
	100%
Value of time ($/minute)	$0.18
Delay with all mixed flow lanes (min)	15
Average cost of delay with all mixed flow lanes ($)	$2.70
Delay on free lane with one toll lane (min)	20
Average cost of delay with one toll lane ($)	$3.60

Equity for People with Lower Incomes

HOT lanes have been criticized as "Lexus Lanes" because more affluent people are assumed to be the primary beneficiaries of HOT lanes. However, the tolls currently charged on Rt 91 and I-15 in San Diego, are not beyond the means of anyone who can afford to buy a car and use it on the highway. People with less wealth may have less ability to pay to save time, but they may value their time more, and thus be willing to pay as much as a wealthier person. Studies of users of the "value priced" lanes on Rt 91 have shown that they are not used exclusively by the rich and that most users do not use them every day. Although a higher percentage of higher income people are frequent users, a significant proportion of people in the lowest income bracket also are frequent users. Roughly 33% of travelers with household incomes over $100,000 never use the toll lane, whereas roughly 66% of travelers with household incomes under $40,000 never use the toll lane. About 18% of the latter are frequent users.

Net benefits

HOT lanes will serve the public well if they have higher net benefits than other types of lanes.

A Freeway with a HOT Lane Compared to One with an HOV Lane

The net benefit (B_{HOT}) of a freeway with a HOT lane compared to one with an HOV lane is the difference in the delay (D) multiplied by the average value of time (μ) for the two types of lanes plus the net benefit from the use of the toll revenue, which is the benefit from the tolls (B_R) less the tolls collected (R) less the cost of the toll collection, C. In mathematical terms the net benefit is:

$$B_{HOT} = D_{HOV}\mu_{HOV} - D_{HOT}\mu_{HOV} + B_R - R - C$$

In cases in which the difference in delay is positive, the average value of time for vehicles in the HOT lane will be greater than for vehicles in HOV lane, so the saving in the cost of delay will also be positive. In this case there will be overall benefits as long as the savings in the cost of delay plus the net benefit from the tolls is positive. There could be a net loss from the use of the toll revenue if the cost of toll collection was high or if the toll revenue was used for something with little value to the public.

A Freeway with a HOT Lane Compared to One with All Mixed Flow Lanes

The net benefit (B'_{HOT}) of a freeway with a HOT lane compared to one with an HOV lane is the difference in the cost of delay ($D_{MF}\mu_{MF}$-$D_{HOT}\mu_{HOT}$) plus the net benefit from the use of the toll revenue. In mathematical terms the net benefit is:

$$B'_{HOT} = D_{MF}\mu_{MF} - D_{HOT}\mu_{HOT} + B_R - R - C$$

The net benefit is greatest if there are large differences in the value of time for travelers at a particular time, the cost of toll collection is low, and the net benefit from the use of the toll revenues is high.

The way in which the toll is used is of key importance in assuring positive net benefits and generating political support. The most straigtforward use is to directly compensate people on the free lanes for the additional delay they experience because one lane is reserved for high speed travel. Using the revenue to reduce delay in the corridor is another possibility—but the delay reduction must be worth the expense. Providing a poorly utilized bus service is not likely to achieve this. Given the institutional structures related to transportation, the toll revenues are likely to be used for some transportation program, but the existance of a net public benefit only requires that it be used for some highly beneficial public purpose, whether related to transportation or not.

Air Quality Benefits

An earlier analysis of HOV lanes (Dahlgren , 1994) found that emissions of hydrocarbons and carbon monoxide are roughly in proportion to vehicle hours, and that the alternative with the least delay also provided the greatest reduction in these emissions. Nitrogen oxide emissions are roughly proportion to vehicle miles, and so are reduced by an HOV or HOT lane to the extent that such lanes motivate a shift to HOVs.

Conclusions

Adding a New Lane

An HOV lane is a good choice for an additional lane on a three-lane freeway only if the initial proportion of HOVs is over 10% and the initial maximum delay is very high. A smaller proportion of HOVs is needed when adding the lane to a four-lane freeway, an higher proportion is needed when adding to a two-lane freeway.

A HOT lane is a good choice if the initial proportion of HOVs is 10% or less and the initial maximum delay is very high or is expected to increase substantially. If delay is not initially very high, the toll collection costs and revenue potential should be analyzed to make sure that the revenue would cover the costs of toll collection.

If the initial delay is not very high and significant growth is not expected, the best choice is a mixed flow lane.

Converting an Underutilized HOV Lane

Converting an underutilized HOV lane on a heavily congested freeway to a HOT lane will reduce delay and generate substantial revenues. It is likely to provide net benefits if the toll revenues are put to beneficial use and the toll collection costs are not too high.

Converting an underutilized HOV lane on an uncongested freeway to another type of lane will not reduce delay.

As shown in Figure 3, if either the maximum delay on the mixed flow lanes or the percent of HOVs is low, the HOV lane should be converted to a mixed flow lane, unless traffic is expected to increase substantially Otherwise, it should be converted to a HOT lane, unless the cost of toll collection is too high.

Further Analysis is Needed Before Action is Taken

The above are rough guidelines to help decision-makers decide which course of action is likely to provide the greatest benefit. Before a decision is actually made, data on vehicle occupancy and traffic volumes throughout the congested period should be gathered and the effects of the candidate actions should be modeled using FREQ or some other simulation model or a model similar to that described in Appendix A. In addition, the cost of toll collection, potential revenues, and the potential uses of the toll revenue should be analyzed.

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APPENDIX A

A Model for Comparing the Effects of HOT, HOV, and Mixed Flow Lanes

This appendix first describes the HOV model, which was originally developed for an analysis of the effectiveness of HOV in various circumstances (Dahlgren, 1994). It then describes modifications to accommodate modeling of HOT lanes.

HOV Lane Model

The model used in this research bases estimates of the proportion of people using HOVs on the time differential between the HOV lane and other lanes, which is constantly changing. The model is easy to use and transparent so that the effects of uncertain inputs can be easily examined. A key feature of the model is that, while it is very simple and does not include all of the effects of adding a lane, it will be shown later in Table A1 that not including these effects does not change the delay ranking of the HOV lane relative to a mixed flow lane.

Estimating Delay

Consider an idealized freeway segment as shown in Figure A1. There is a bottleneck at the downstream end and the neck is long and uniform, contains no entry or exit points, and extends beyond the area subject to congestion. The queue builds up and dissipates during the peak period as shown in the lower section of Figure A1. Vehicles arrive at a constant rate until the time of the maximum queue and then arrive at a lower constant rate until the queue is dissipated. An idealized queue can be constructed from the following information:

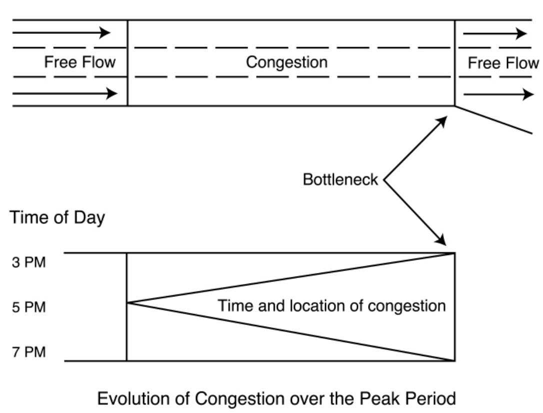
(a) the length of the congested period

(b) the maximum delay (maximum travel time minus free flow travel time)

(c) the time at which the maximum delay occurs

(d) the freeway capacity.

Figure A1 Idealized Freeway Segment



The queue can be represented as in Figure A2. The congested period extends from 0 to t_E, with the maximum delay occurring at t^{max}. The cumulative number of vehicles attempting to pass through the bottleneck at time t is A(t) and the number actually passing through is D(t)=ct, where c is the capacity of the bottleneck per unit of time. The number waiting to pass through at time t is

$$Q(t) = A(t) - D(t) \qquad (1)$$

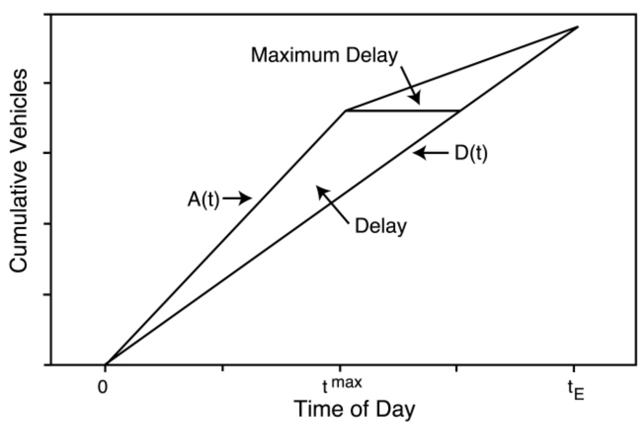
The delay for a vehicle arriving at time, t is

$$w(t) = \frac{Q(t)}{c} = \frac{A(t) - D(t)}{c} = \frac{A(t)}{c} - t \qquad (2)$$

The total delay to all travelers over the peak period is the area between A(t) and D(t), which equals

$$\int_0^{t_E} [(A(t) - D(t)] dt \qquad (3)$$

Figure A2 Idealized Queue



This idealized queue, combined with vehicle occupancies for HOVs and low occupant vehicles (LOVs) and the changes in freeway capacity for LOVs and HOVs, can be used to estimate the changes in person-delay and emissions from adding an HOV lane, adding an additional mixed flow lane, or converting an existing lane to an HOV lane.

Assumptions Regarding Delay

Recent research, some of it undertaken to inform revisions to the speed-flow relationships in the Highway Capacity Manual, has suggested that speed remains relatively constant until a freeway approaches capacity, at which point a queue forms and flow remains at capacity regardless of the queue length. The research supports a model in which 1) all delay is caused by queueing and none by increasing density per se, and 2) once the freeway reaches capacity, flow remains constant. In other words, the speed flow curve is a horizontal line at free flow speed until capacity is approached, at which point it begins to turn into a vertical line indicating constant capacity regardless of speed.

Estimating the Shift to HOVs

The probability of making a trip via HOV is a function of the attributes of 1) the HOV trip, 2) the trip via non-HOV (a single occupant vehicle in most cases), and 3) the person making the trip. HOV attributes include: waiting time, travel time, time and inconvenience arranging the carpool, ambiance in the waiting area and the HOV, and cost. Single occupant vehicle attributes include travel time, parking availability and cost, vehicle ambiance, driving conditions, and vehicle operating cost. Traveler attributes include regularity and flexibility of working hours, work and home location, child care requirements, income, and availability of an automobile.

The probability that a particular individual will use an HOV can be represented by a logit model:

$$p_{HOV} = \frac{e^{\Sigma\beta_i H_i}}{e^{\Sigma\beta_i H_i} + e^{\Sigma\beta_i L_i}} = \frac{1}{1 + e^{\Sigma\beta_i L_i - \Sigma\beta_i H_i}} = \frac{1}{1 + \Gamma\, e^{\beta_t (L_t - H_t)}} \quad (4)$$

where the $ß_i$ are the coefficients of the attributes and the H_i and the L_i are the traveler and modal attributes related to the HOV and LOV trip, respectively. When an HOV or mixed flow lane is added, the only attributes that change are the travel times for the two modes. Therefore, all other attributes and their coefficients can be represented by a constant, Γ. As a result, the exponent of e is reduced to $ß_t(L_t-H_t)$, where $ß_t$ is the coefficient of the travel time and L_t and H_t are the travel times via mixed flow lanes and the HOV lane respectively. The same coefficient for travel time is assumed for both HOVs and LOVs.

Each region has different travel patterns and opportunities for HOV travel. The extent of the shift depends on these factors as well as the travel time advantage resulting from the HOV lane. An area with a strong urban center, high congestion in the center, and good bus service has considerable opportunity for HOV use. Opportunities may be more limited in an area with low housing and job density. But in either case there will be some people using HOVs when the travel time differential is zero. When the freeway travel time for HOVs is reduced, increasing the differential between LOV and HOV travel time, the proportion of people using HOVs increases in both cases, but the increases in the proportion of people using HOVs will be quite different. These differences are reflected in the value of Γ.

Similarly, each individual has different personal and modal attributes, and consequently different probabilities of using each mode, represented by a different Γ. Some people can not shift to an HOV. They may have irregular or unpredictable trip starting times, they may have an unusual trip origin or destination, they may need their vehicle at their destination, or they may need to transport equipment, materials, or children.

Despite differences in each person's probability of using an HOV, for simplicity, the model used in this research assumes that all travelers have the same probability of using an HOV. This gives the upper limit to the number of people who might shift mode.

Given this assumption, the expected proportion of people using HOVs is equal to the individual probability of using an HOV:

$$P_{HOV} = \frac{1}{1+\Gamma\, e^{\beta_t(L_t\text{-}H_t)}} \quad (5)$$

Because the travel time differential, L_t-H_t, is initially 0, Γ can be calculated from the proportion of people initially using HOVs. Estimation of $ß_t$ is another matter. Published HOV lane evaluations do not include data that link the proportion of people using HOVs to the changing travel time differential or to shifts from other times and routes, so it has not been possible to estimate travel time coefficients from experience with real HOV lanes. Therefore, a value for sensitivity to travel time of -.05 per minute of round trip in-vehicle travel time is used. This is at the high end of all such values found in the literature : -0.012 and -0.016 (Kollo, 1986); -0.0082 (Koppelman, 1983); -0.02, -0.03, -0.04, and -0.06 (McFadden and Talvitie, 1977); and -0.02 (Small, 1977), and therefore shows HOV lanes in the most favorable light.

Interaction of the Travel Time Differential and Mode Shift with an HOV Lane

Travel times on the mixed flow lane will change over the course of the peak period. The proportion of people entering the freeway at a particular time who will use HOVs depends on the travel time differential, L_t-H_t, at that particular time, but the travel time differential, in turn, depends on the proportion of people who, up to that time, have used HOVs. This travel time differential is the difference between the delay for the HOVs and the delay for LOVs. To calculate these delays we modify Equation 2, letting A(t) represent cumulative *person* arrivals at the freeway, P(t) represent cumulative person arrivals in HOVs, L and H represent LOV and HOV average occupancies, and C_L and C_H represent capacities on the mixed flow and HOV lanes, respectively. The congested period begins at time t=0; congestion on the HOV lane begins at time t_H.

Delay for the LOVs entering the freeway at time t is

$$w_L(t) = \max\{\frac{\frac{A(t) - P(t)}{L} - t\,C_L}{C_L}, 0\} = \max\{\frac{A(t) - P(t)}{L\,C_L} - t, 0\} \qquad (6)$$

and for the HOVs is

$$w_H(t) = \max\{\frac{P(t) - P(t_H)}{H\,C_H} - (t - t_H), 0\} \quad (7)$$

P(t), the cumulative person arrivals in HOVs by time t, in turn depends on the travel time differential L_t-H_t at time t, which equals $w_L(t)$-$w_H(t)$.

$$P(t) = \int_0^t [a(x)\,P_{HOV}(x)]\,dx = \int_0^t a(x)\frac{1}{1+\Gamma\, e^{\beta_t[w_L(x)-w_H(x)]}}\,dx \quad (8)$$

where

$$a(x) = \frac{dA(x)}{dx} \quad (9)$$

Equation 8 is not solved analytically, but is the basis for calculating P(t) numerically over one minute intervals. Using this method, P(t) equals the value of the expression inside the integral evaluated at t plus the sum of this expression for all previous values of t. The travel time differential, $w_L(t)$-$w_H(t)$, is also calculated for each minute and used to calculate P(t) for the subsequent minute interval. For people entering the freeway during each interval, total person-delay, vehicle-delay, and vehicle-trips are calculated. These are summed to obtain total person-delay, vehicle-delay, and vehicle-trips for the entire peak period. Any standard spreadsheet software can be used for the calculations. These calculations are made for 1) an added HOV lane and 2) an added mixed flow lane.

Effects of Model Assumptions

The model contains a number of assumptions. They are summarized in Table A1 The assumptions in the first group make an HOV lane appear to have greater benefits relative to a mixed flow lane than would actually be the case. The assumptions in the second group would not change the ranking of the alternatives in terms of individual benefits. The effects of the assumptions in the third group would depend upon the situation. The effects of these last two assumptions are not as strong as the overall effects of the assumptions that lead to an overstatement of the benefits of an HOV lane relative to a mixed flow lane. Therefore, it is assumed that on balance the model overstates the benefits of HOV lanes relative to those of mixed flow lanes.

Modifications to Accommodate Analysis of HOT Lanes

The analysis regarding the HOT lanes does not estimate the number of people in the HOT lanes—it assumes that tolls are set so that the lanes are optimally utilized. This makes the model much simpler but it means that the results must be interpreted as dependent upon being able to set a toll that fully utilizes the HOT lanes when the mixed flow lanes are congested. While this is theoretically possible, the feasibility of doing this in all situations remains to be demonstrated.

For each minute interval the model estimates the delay on the mixed flow lanes and uses it to estimate how many people will use HOVs. Clearly this is not a decision that is made on the spur of the moment each day, but is the result of experiences over past days. Then the model calculates the number of additional vehicles that can be accommodated without causing delay on the HOT lane. This number of vehicles is assigned to the HOT lane and the travel time for vehicles entering during the next one minute interval is calculated.

Table A1 Effects of Assumptions

Assumptions That Lead to an Overstatement of the Benefits of an HOV Lane Relative to a Mixed flow Lane	
Identical probabilities of using an HOV	The mode shift with identical probabilities is always greater than with different probabilities (Please see Dahlgren 1994 for an explanation and example)
No downstream entries	Downstream entries cause measured delay to be more than actual average delay--more delay favors an HOV lane
No reduction in convenience due to shift to HOV	Only the time saving beyond that necessary to induce a shift is a benefit
All HOVs use the HOV lane	Benefits of HOV lane are less if fewer vehicles use it
People do not drive to meet the carpool or bus	Driving to meet the carpool or bus would increase emissions substantially
Assumptions That Do Not Change the Ranking of an Added HOV Lane Versus an Added Mixed flow Lane	
No route shifts	Benefits are larger with larger route shifts, and larger delay reductions result in larger route shifts
No shifts in trip start time	Larger delay reductions allow larger shifts in trip start times
No induced trips	Benefits from new trips are greater and costs of these trips are less with larger reductions in delay. Air quality benefits of reduced delay are likely to be greater than air quality costs of induced trips
No vehicles entering and exiting the queue before the bottleneck	Benefits to these vehicles are greater with larger reductions in delay
Assumptions Whose Effects Depend on the Situation	
Vehicles arrive at a constant rate until the time of maximum delay and at a lower constant rate thereafter	If the arrival rate is linearly increasing and the time of maximum delay is less than 2/3 through the peak period, the relative benefits of an HOV lane will be understated; otherwise they will be overstated
Only HOVs use the HOV lane	Allowing cheating increases utilization of the HOV lane but reduces the incentive to use an HOV

APPENDIX B

An Economic Analysis of the Difference in the Cost of Delay on A Freeway with One Toll Lane versus that on a Freeway with All Mixed Flow Lanes

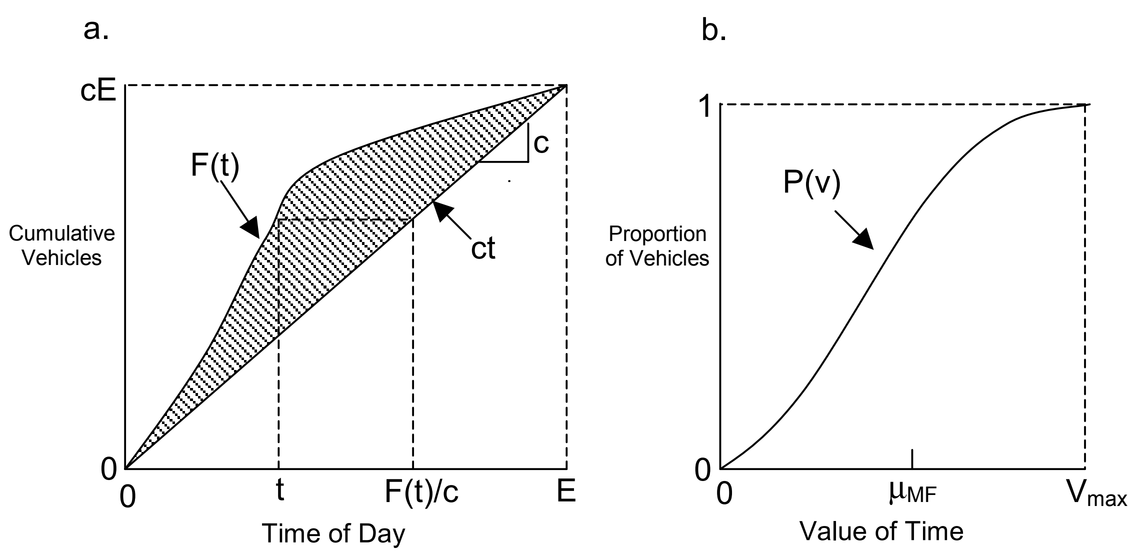
The purpose of this analysis is to show that the total travel time costs are less if people can be given the option to purchase a delay-free trip because those who exercise this option have a higher value of time than those who do not.

Delay with all mixed flow lanes

Consider the case in which an HOV lane is converted to a mixed flow lane and some congestion remains after the conversion. Figure 1a shows the cumulative number of vehicles, F(t) that have entered the congested area, and the cumulative number of vehicles, ct, that have passed through the congested area during the congested period, which begins at time 0 and lasts E hours. The freeway capacity when it is congested is c vehicles per hour[2]. The delay for a vehicle entering the congested area at time t is the horizontal distance between F(t) and ct:

$$d(t) = \left[\frac{F(t) - tc}{c} \right]$$

FigureB1 Delay for All Vehicles and Their Value of Time with All Mixed Flow Lanes



The delay that all vehicles experience over the course of the congested period is D, the shaded area in Figure B1a.

$$D = \int_0^E \left[F(t) - ct \right] dt$$

Cost of delay with all mixed flow lanes

If we assume that the cumulative distribution of the value of time to vehicles throughout the congested period is P(v) as shown in Figure 1b, then the mean value of time during the congested period when all lanes are mixed flow is

$$\mu_{MF} = \int_0^{v_{\max}} v \cdot p(v) dv$$

where

$$p(v) = \frac{dP(v)}{dv}$$

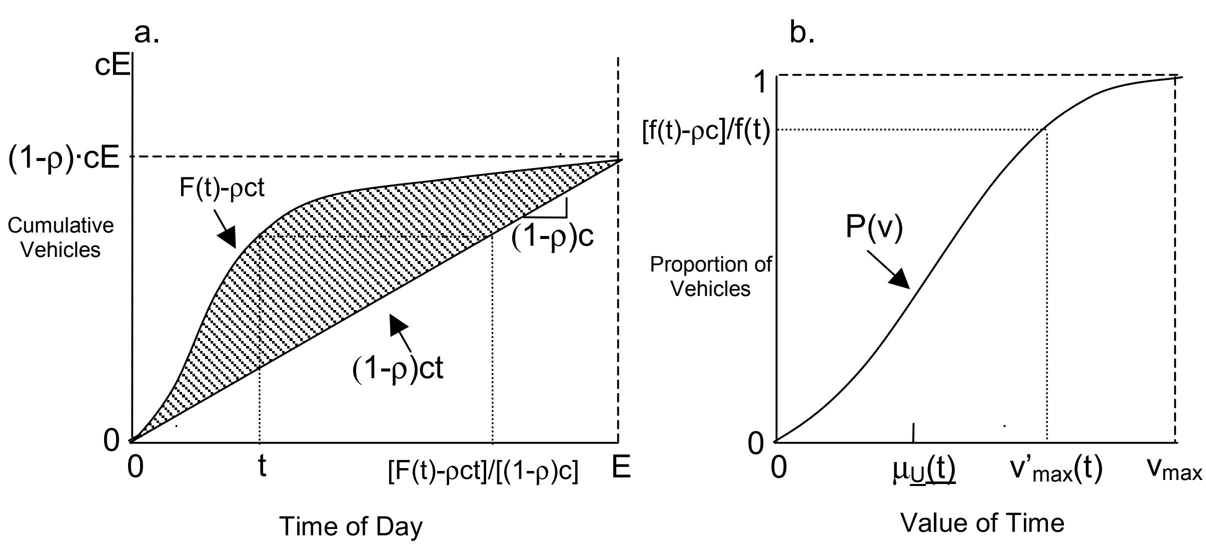
and the total cost of delay is

$$C_{MF} = \mu_{MF} \int_0^E [F(t) - ct] dt$$

Delay with one toll lane

Now consider the case in which the HOV lane is converted to a toll lane as illustrated in Figure B2. The tolls vary with demand and are set so that the capacity of the toll lane is always fully utilized but demand does not exceed capacity. If the toll lane accounts for ρ proportion of total capacity, then both the cumulative number of vehicles that have used the toll lane by time t and the cumulative capacity is ρct. The cumulative capacity for the free lanes at time t is (1-ρ)ct, and the cumulative number of vehicles entering the free lanes is F(t) - ρct.

Figure B2 Delay for Untolled Vehicles and Their Value of Time with One Toll Lane



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[2] It is assumed that there are no downstream bottlenecks and queues to diminish capacity when there is

The delay for a vehicle entering the congested area at time t and using the free lanes is

$$d_F(t) = \left[\frac{F(t) - tc}{(1-\rho)c} \right]$$

The total delay, D_F is the area between the F(t)-ρct curve and the (1-ρ)ct line in Figure B2a.

$$D_F = \int_0^E [F(t) - \rho \cdot ct - (1-\rho) \cdot ct] dt = \int_0^E [F(t) - ct] dt$$

This is the same as the delay when all lanes are mixed flow lanes.

Cost of delay with one toll lane

Because the vehicles using the toll lane are those whose passengers have the highest total value of time during a particular trip, and those using the free lanes have a lower -total value of time, the overall cost of the delay is less than with all mixed flow lanes. The distribution of the cost of delay for vehicles using the free lanes is as shown in Figure 2b. Note that for all of the vehicles in the free lanes the delay multiplied by the value of time is less than the toll, r(t), and the toll is equal to the delay multiplied by the maximum value of time of the people using the free lane, so that

$$r(t) = d_F(t) \times v'_{\max}(t)$$

Over the course of the congested period the proportion of vehicles using the free lanes will change, and therefore their average value of time will change with t. At any time, t, the proportion of vehicles using the free lanes is

$$\frac{f(t) - \rho c}{f(t)}$$

where

$$f(t) = \frac{dF(t)}{dt}$$

To calculate the average value of their time, a new cumulative probability function P'(v) is defined, representing the distribution of values of time up to $v'_{max}(t)$.

$$P'(v) = \frac{f(t)}{f(t) - \rho c} \times P(v) \text{ and}$$

$$P'(v'_{\max}(t)) = 1$$

because $P(v'_{\max}(t)) = \dfrac{f(t) - \rho c}{f(t)}$

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congestion on this freeway.

Therefore the mean cost of the time of the travelers in vehicles in the free lanes at time t is $\mu_F(t) = \int_0^{v'_{\max}(t)} v \cdot p'(v)dv$

and the total cost of delay with one toll lane is

$$C_T = \int_0^E \mu_F(t)[F(t) - ct]dt$$

Because $\mu_F(t)$ is less than $\mu_{MF}(t)$ for all t, the total cost of the delay to all vehicles is less with a toll lane than with all mixed flow lanes. The difference in the cost of delay between a highway with one tolled lane and one with all mixed flow lanes is

$$C_{MF} - C_T = \int_0^E \{[\mu_{MF} - \mu_F(t)] \times [F(t) - ct]\} dt$$

Cost of Delay with One HOT Lane

There will be more HOVs in the HOT lane than there would have been in the toll lane, because, although a vehicle with more occupants is likely to have a higher total value of time than a vehicle with fewer occupants, this would not always be the case. Therefore the average value of time for vehicles in HOT lane will be less than the average value of time for vehicles in the toll lane, and the average value for vehicles in the mixed lanes will be greater. To the extent that the HOT lane motivates a shift to HOVs, there will be fewer people in the mixed flow lanes than in the toll lane case and there will be less delay. The cost of delay with a HOT lane is

$$C_{HOT} = \int_0^E \mu_{MFH}(t)[F'(t) - ct]dt$$

where F'(t) is the new demand, which is less than F(t), and $\mu_{MFH}(t)$ is greater than $\mu_F(t)$.

The cost differential for a HOT lane versus a mixed flow lane is

$$C_{MF} - C_{HOT} = \int_0^E \{\mu_{MF}[F(t) - ct] - \mu_F(t)[F'(t) - ct]\} dt$$

The first term is larger than the second, so the cost of delay is less for a freeway with a HOT lane than for one with all mixed flow lanes.