

UC Berkeley

Other Recent Work

Title

Dynamic Competition with Lock-In

Permalink

<https://escholarship.org/uc/item/1d43h5sq>

Authors

Farrell, Joseph
Shapiro, Carl

Publication Date

1986-01-27

Peer reviewed

UNIVERSITY OF CALIFORNIA, BERKELEY

Department of Economics

Berkeley, California 94720

Working Paper 8727

DYNAMIC COMPETITION WITH LOCK-IN

Joseph Farrell and Carl Shapiro

January 27, 1986

Key words: Switching costs, dynamic competition, lock-in, long-term relationships.

Abstract

We analyze a duopoly model of multiperiod rivalry in the presence of consumer switching costs. Competition for locked-in buyers is continually intermingled with competition for new, uncommitted buyers. A typical equilibrium pattern is for the incumbent--the firm with locked-in customers--to exploit those customers and concede the new buyers to its rival. This pattern persists even in the presence of economies of scale, network externalities, or cost differences. Switching costs thus can lead to inefficiency in a surprising way: rather than serve as an entry barrier, they encourage entry to serve unattached customers even in circumstances where the entrant is less efficient than the incumbent.

Dynamic Competition with Lock-In

Joseph Farrell

Carl Shapiro

1. Introduction

In many markets, buyers must sink resources into their relationships with particular suppliers. For instance, they may have to spend time learning to use a vendor's product, and the skill may not be fully transferable to using a competitor's product. Or buyers may acquire stocks of complementary goods (such as computer software) that work only with one vendor's product. A worker may buy a home near his or her job and find it costly to move. A city finds it costly to replace a cable-TV franchisee. In addition to these ways in which buyers are *rationaly* locked-in to sellers, the marketing literature consistently suggests that, even without such effects, buyers tend to continue buying from the seller they first patronize.

Such *lock-in effects* evidently give the seller some monopoly power over the locked-in buyers. While this suggests that there ought to be long-term contracts in such markets, there often are not.¹ Absent such contracts, a buyer is open to exploitation by an opportunistic seller who could raise the price above competitors' by an amount almost equal to the buyer's switching cost. Of course, if competitors are making similar calculations, then,

We are grateful to Avinash Dixit and Drew Fudenberg for comments and discussion. We also thank Hal Varian for his help in implementing this paper on his VERTeX system.

¹ Long-term contracts may be unattractive due to transaction costs, cost uncertainty, demand uncertainty, or the need to maintain incentives for the seller to provide non-contractible quality. We do not explicitly explore these factors in the current paper. Clearly, with perfect information, long-term contracts would completely solve any problems associated with lock-in. In Farrell and Shapiro (1986), we explore the role of contracts in market equilibria with private information about costs and benefits in conjunction with switching costs.

some sophistication) choosing whom to patronize when first in the market and thereafter when (if at all) to switch. Sellers would have to condition their competitive choices of price on their market shares. Thus, even with an exogenous specification of buyers' behavior, we would have a dynamic game with at least one continuous state variable (market share), and more if we allow for the fact that buyers with different switch costs or different remaining lifespans are locked in to different extents. Such games have generally proven intractable in the closed-loop (perfect equilibrium) framework which, we argue, is necessary to get at the strategic questions here raised.

In the face of these difficulties, various authors have studied different simplifications of the problem. Klemperer (1985a,b,c) studied two-period models, which can fairly readily be solved using backward induction. Farrell (1986) and Summers (1985) have also pursued this approach. Sutton (1980) assumed that buyers cannot switch once they buy. Von Weizsäcker (1984) modeled buyers' choices in a sophisticated way, but considered only constant-price strategies in the text of his paper, although in an appendix he solved for open-loop equilibrium. Scotchmer (1986) and Green and Scotchmer (1986) studied static equilibrium (equivalent to allowing all agents' discount factors to approach zero, and looking for a stationary equilibrium).

We have chosen a model in which we can solve for the (generically unique) perfect Markov equilibrium in a model with infinitely many periods. The model has a natural dynamic programming structure. Competition for buyers today is more or less intense according to the future value of having buyers locked in, and that value itself depends on the intensity of competition at yet later dates. In order to make the model tractable, we restrict attention to the case of identical buyers.

In Section 2 we present a very simple overlapping-generations duopoly model, and show that the unique perfect equilibrium involves alternation: the seller without a customer base always attracts the new, unattached buyers. We also calculate the equilibrium prices and profit margins. In Section 3 we extend the basic model by allowing for economies of scale or network externalities, either of which benefit the firm with a larger customer base. Clearly, a sufficient advantage of either kind should change the alternating equilibrium result; and indeed it does. But we show that a firm's cost advantage must be large relative to the

of the firms sells to all of the youngsters. In any given period, we call the firm with the locked-in buyers from the previous period the *incumbent*, and the other firm the *entrant*.

Each oldster is locked-in to the seller from whom he bought when a youngster, *i.e.* (in equilibrium) the incumbent. This does not mean that he *must* buy again from the same firm he frequented as a youngster. But if he switches and meets his second-period requirements from the other seller, he incurs a switch cost c , whose value is the same for all buyers and is common knowledge. Each buyer requires one unit of the good in each of the two periods he is in the market (a completely inelastic demand), and tries to minimize his discounted outlays, including switching costs.⁸ The discount factor is $\beta < 1$.

We must now indicate how sellers set their prices in each period. This technical problem has plagued research in this area: If the sellers were to set prices noncooperatively and simultaneously, there would (often) be no pure-strategy equilibrium. The entrant would typically want to price just below $q - c$ if the incumbent's price is q , in order to induce the oldsters to switch. The incumbent, however, would want to price just below $p + c$ if the entrant's price is p , so as to prevent the oldsters from switching. These wishes are incompatible. One solution would be to examine mixed-strategy equilibria, but it is difficult even to solve for such equilibria here.⁹ The other approach, used for instance by Klemperer, is to smooth the payoff functions by allowing switch costs to differ across buyers. This is undoubtedly the realistic resolution, but again presents problems of tractability in our multiperiod model.

Our resolution of these difficulties is to consider a different within-period pricing game. We suppose that prices are set in a Stackelberg fashion. In each period, the entrant first sets a price. Next, the incumbent sets his price.¹⁰ The buyers then choose whom to patronize. The incumbent can exploit his locked-in oldsters, but if the entrant sets a very low price he may prefer to withdraw, while if the entrant sets a high price the incumbent

⁸ For simplicity, we suppose that buyers' reservation prices are so high as never to bind. In the basic model, the qualitative results are little affected if the reservation price binds.

⁹ Doing so requires solving a set of mixed difference-differential equations for the distribution functions that constitute the mixed strategies.

¹⁰ In Section 5 below we report our findings for the alternative model in which the incumbent must choose a price first. We do not consider it obvious which model is "right," and find the comparisons between the two models to be of some interest themselves.

Since the right-hand side of (2) is smaller than that in (1), in equilibrium the entrant will be able to win the auction to sell to the youngsters (and in fact he will select p so that (1) is satisfied with equality).

To establish our primary result in this section, that the entrant always sells to the youngsters but the oldsters do not switch, we need only check that the entrant will not choose to induce the oldsters to switch. To do so, the entrant would have to set a price p such that the incumbent preferred strategy (i) to strategy (ii). This would require $(p + c) + \beta W_0 \leq \beta W_0$, i.e., $p \leq -c$. But then $W_0 = 2p + \beta W_1 \leq -2c + \beta W_1$. We would also have $W_1 = \beta W_0$, since the incumbent makes no sales today and earns profits only as tomorrow's entrant. Therefore both W_0 and W_1 would be strictly negative, which cannot be: either firm would prefer to drop out altogether.

We have shown, then, that in equilibrium the entrant always sells to the youngsters and the incumbent sells to the oldsters. Intuitively, the incumbent is unsuccessful in selling to unattached buyers because his locked-in buyers tempt him to keep his price high. The reason is that each seller knows that a small reduction in price would bring in the youngsters. For the entrant, this would be its only effect. But for the incumbent, it would also reduce his revenues from the oldsters. Therefore, the incumbent is less willing to make the reduction. We call this a *common-margin* phenomenon: the youngsters are a "common margin" over whom the sellers compete, but one of them has other considerations (the inframarginal oldsters) in setting his price. For related common-margin results, see Fudenberg and Tirole (1984) and Farrell (1986).

It is now a simple matter of algebra to calculate the equilibrium prices and profits. Let p denote the entrant's equilibrium price; then the incumbent charges $(p + c)$, although given an infinitesimally higher entrant's price p' he would instead match p' and sell to both buyers. We therefore have three equations for W_0 , W_1 and p :

$$\begin{cases} W_0 = p + \beta W_1 \\ W_1 = p + c + \beta W_0 \\ W_1 = 2p + \beta W_1. \end{cases}$$

design decisions. This finding is in contrast to Klemperer's (1985c) finding (see also Katz and Shapiro (1986)) that increases in c may so greatly exacerbate first-period competition that firms would prefer to keep c low. The price, and the incumbent's per-period profits, $(1 - \beta)W_1$, decrease as β increases; when the future is more important, competition for the youngsters is keener and so prices are lower. In any event, W_1 exceeds W_0 : having the locked-in oldsters has positive value, even though the incumbent faces fiercer competition from the entrant than the entrant faces from the incumbent.

In equilibrium, there is no switching. Since our model assumed inelastic demand, this means that the equilibrium is efficient. In a more realistic model, however, the fact that prices are above marginal cost would lead to an allocative inefficiency of the usual kind. It is particularly notable that even the entrant finds it unnecessary to price below marginal cost: the reason is, of course, that he need only compete with the incumbent, who is not interested in intense price competition for the youngsters.

Many Firms

It is easy to modify our model by supposing that at each date there are *many* entrants competing with the incumbent (and with one another). Many firms would be able to compete if production requires no significant specialized assets. With many firms, competition among entrants drives W_0 down to zero. Entrants set a price that yields zero overall profits, accounting for the fact that the price they will charge from their locked-in oldsters will exceed the entry price by exactly c . The zero-profit condition is $p + \beta(p + c) = 0$, or $p = -\beta c / (1 + \beta)$. The incumbent charges c more than this, $c / (1 + \beta)$, to retain his locked-in customers.¹³

In contrast to the duopoly model, competition for youngsters results in below-cost pricing. Each period a new firm loses money on the youngsters that it captures, while the established firm recoups this investment with above-cost pricing. The value of incumbency is $W_1 = c / (1 + \beta)$. Comparing these prices with equation (3), we see that the presence of many firms reduces both the introductory price and the locked-in price by c . Competition

¹³ Although the entrant prices below cost, he need not worry about losing money by selling to oldsters: the incumbent will certainly set his price low enough to retain these consumers.

of age k is exactly c for any age! Of course, for infinitely-lived consumers, there is no such thing as age. For $n = \infty$, new buyers pay $p = -\beta c$, and locked-in buyers pay $p + c = (1 - \beta)c$.

We can summarize the lesson of this section as follows. With identical firms, whether consumers live for two or more periods, and whether there are two or many firms, we have identified a strong tendency for incumbent firms to specialize in serving their locked-in consumers, leaving the "new" market to firms without a customer base. In the duopoly case, this tendency is manifest in the alternating equilibrium. In the competitive case, again we find that incumbent firms make no sales to unattached consumers. The remainder of this paper discusses some efficiency consequences of this tendency, and explores offsetting factors that may cause an incumbent firm to make such sales.

3. Economies of Scale or Network Externalities

We have identified a reason for firms to share the market that had nothing to do with increasing marginal costs of any kind. Since sharing of the market is not driven by efficiency factors, we can reasonably expect that, if we relax our assumption of constant and equal marginal costs, the market sharing may persist despite its causing efficiency losses. In this and the next section, we show how this intuition applies to two cases: where there are economies of scale (so that efficiency calls for a single firm to serve the whole market), and where there are cost differences between firms (so that production should be undertaken exclusively by the low-cost firm). We show that *moderate* efficiency incentives to concentrate production are effectively ignored by the market: the alternating tendency persists. Of course, if the incentives to concentrate production are large enough, then they prevail and overcome the alternating tendency.

In this section, we consider *economies of scale*: there is a cost saving if the same seller supplies both cohorts in a given period. An alternative interpretation of this analysis is that there are economies of scale on the demand side: each buyer is better-off if the other current buyers also patronize the same seller. With our simple demand structure, such network externalities are equivalent to conventional economies of scale.

It is perhaps worth discussing here what it means for prices to be "independent of m ." Suppose temporarily that the first unit's marginal cost is x and the second unit's marginal cost is y . This can be rephrased: each unit costs y ; and in addition there is a "further" cost of $x - y$ for the first unit. So $x - y$ plays the role of m , and we must simply add the marginal cost y to prices. The entrant's price, therefore, is $p = c/(1 + \beta) + y$; x is irrelevant. This is quite intuitive, given the forces that determine pricing in our alternating equilibrium. The entrant's price is determined by the incumbent's willingness to lower his price to match the entrant's, and therefore to sell to all buyers. His *additional* costs from doing that are just y ; his *inframarginal* costs of selling to the oldsters are irrelevant.

In this alternating equilibrium, unlike that in the previous section, there is an inefficiency caused by the switching costs, *even though* there is no switching in equilibrium. It is the switching costs that make the incumbent decide to sell only to the oldsters. This results in a productive inefficiency: output is inefficiently divided between the firms.

Dominant Equilibrium

If economies of scale, as represented by m , are important enough, then we should expect that production in each period will be concentrated in one firm. Moreover, because of lock-in, the incumbent always sells something - it is never an equilibrium for the entrant to sell to all the buyers. Therefore, in a *dominant equilibrium*, the same firm keeps the entire market forever, although if the entrant were to capture the youngsters *he* would thereafter sell to all buyers. The equations for W_1 and W_0 in such an equilibrium are therefore

$$\begin{cases} W_1 = 2p - m + \beta W_1 \\ W_0 = 0, \end{cases} \quad (5)$$

where p denotes the price offered by the entrant and matched by the incumbent.

What price does the entrant (unsuccessfully) offer? We assume that the entrant offers as low a price as possible consistent with non-negative profits, were the incumbent, contrary to equilibrium, to fail to match p in order to retain his incumbency. But the entrant's out-of-equilibrium profits depend upon the precise way in which the incumbent deviates from his equilibrium strategy of matching p . In the spirit of proper equilibrium, we assume in

For the sudden-retreat dominant equilibrium to be consistent again requires that the incumbent's second-best strategy (his best deviation) involve conceding the oldsters as well as the youngsters to the entrant. In other words, the incumbent would have to lose money by keeping the oldsters, since sudden retreat gives zero profits. This requires that $p + c - m \leq 0$. Since $p = m/2$, this condition is $m/c \geq 2$. Observe that in the range $2 \leq m/c \leq (1 + \beta)/\beta$, both types of dominant equilibria exist.

In the sudden retreat dominant equilibrium, even the incumbent earns no profits. Economies of scale are much more important than are switching costs, and the entrant has an effective entry threat, even for the locked-in oldsters: by capturing the youngsters, the entrant can undermine the incumbent's scale economies and threaten to enjoy his own. By pricing at average cost, $m/2$, the entrant drives the incumbents profits down to zero. When scale economies are so strong, the incumbent cannot earn rents on his locked-in customers, since he also needs to sell to the new buyers to take advantage of the scale economies.

Interpretation

We can summarize our findings in this section with

Proposition 2. *With economies of scale, there may be either an alternating equilibrium or a dominant equilibrium. Alternation occurs if and only if $c \geq m$; dominance arises if and only if $m \geq c$.*

As above, we can extend our findings to markets with many competitors. With many entrants, an alternating equilibrium again arises if and only if $m \leq c$, with a dominant equilibrium otherwise. In the alternating equilibrium, potential entrants offer a price of $p = m - \beta c / (1 + \beta)$, while the incumbent prices at c above this, $m + c / (1 + \beta)$. In the dominant equilibrium, entrants set prices of $p = m / (1 + \beta)$, which the incumbent matches. In either case, the presence of many potential entrants leads to more aggressive competition for youngsters, to the benefit of youngsters and oldsters alike. With many entrants, youngsters enjoy the benefits of below-cost pricing by entrants, but in a dominating equilibrium these prices remain above the incumbent's marginal cost of serving them.

4. Efficiency Differences

We now consider equilibrium when one seller (A) is simply more efficient than the other: his costs are lower (or, equivalently, his product is superior). For simplicity, we suppose that A has zero marginal costs, while B 's marginal cost is m . All the other assumptions are as in our basic model, including the assumption that young buyers buy from the seller with the lower current price.¹⁷ Denote by W_0^i the value of firm i when it is the entrant, and by W_1^i its value as an incumbent, $i = A, B$.

The analysis is simplified by the following fact:

Lemma. Whoever is the incumbent, the low-cost firm (A) sells to at least one consumer cohort in any equilibrium. If A is the incumbent, he sells to the oldsters (and perhaps also to the youngsters). If B is the incumbent, A sells to the youngsters (and perhaps also to the oldsters).

Proof. First, we show that firm A will retain the oldsters when it is the incumbent. Suppose not. Then $W_1^A = \beta W_0^A$. As incumbent, firm A would price as low as its own cost, zero, to retain its oldsters. If B is to capture A 's oldsters, his price p must be $-c$ (or lower). But then B is losing (at least) $c + m$ on these customers. Firm B would be better off if it could select a higher price that would attract the youngsters and avoid losing money on A 's oldsters. Clearly, $p = -c + \epsilon$ for small ϵ would cause A to retain his oldsters. To show that B indeed prefers such a price to $p = -c$, we must establish that the higher price would still allow B to attract the youngsters: that is, that A would not be tempted to go after the youngsters as well. From firm A 's viewpoint, seeking the oldsters gives $\epsilon + \beta W_0^A$, while going after the youngsters as well gives $2(-c + \epsilon) + \beta W_1^A$. Since $W_1^A = \beta W_0^A < W_0^A$, it is immediate that firm A prefers to concede the youngsters to B in response to $p = -c + \epsilon$. This establishes that it cannot be optimal for B to sell to A 's oldsters when A is the incumbent.

Next, we show that firm A will sell to the youngsters when B is the incumbent. Since A will price at least as low as its own cost, zero, to attract the youngsters, B must set a price no higher than zero if it is to capture them. But this cannot be equilibrium behavior:

¹⁷ This is again optimal for each buyer, when sellers follow their equilibrium pricing strategies.

Since A sells only to the oldsters when he is the incumbent, we have

$$W_1^A = (p^B + c) + \beta W_0^A, \quad (9)$$

and

$$W_0^B = p^B - m + \beta W_1^B. \quad (10)$$

B 's choice of price, p^B , is such that any higher price would make A undercut and sell to all consumers (otherwise, B would gain by raising his price slightly). So, A is in fact indifferent to undercutting:

$$W_1^A = 2p^B + \beta W_1^A. \quad (11)$$

Analogously, when B is the incumbent, we have

$$W_1^B = p^A + c - m + \beta W_0^B, \quad (12)$$

$$W_0^A = p^A + \beta W_1^A, \quad (13)$$

and

$$W_1^B = 2(p^A - m) + \beta W_1^B. \quad (14)$$

These six equations in the unknowns p^A , p^B , W_0^A , W_1^A , W_0^B , and W_1^B imply prices of

$$p^A = \frac{c}{1 + \beta} + \frac{m(1 + 2\beta)}{1 + 3\beta} \quad (15)$$

and

$$p^B = \frac{c}{1 + \beta} + \frac{m\beta}{1 + 3\beta}. \quad (16)$$

Both of the firms' entry prices increase in m , but for somewhat different reasons. When $m > 0$, B as incumbent is less willing (given entrant A 's price) to undercut, because it means bearing the extra cost m . Therefore A as entrant can get away with a higher price p^A . And the fact that A can succeed with a higher entry price when m is large implies that A is relatively happier to let B become the incumbent. This in turn implies that A is less willing to undercut entrant B 's price, so p^B also increases with m .

Notice also that $p^B < p^A$ when $m > 0$: the higher-cost firm charges a *lower* price when it enters than does the lower-cost firm. Firm B must keep its entry price low for fear

and

$$W_0^A = 2p^A + \beta W_1^A. \quad (20)$$

Firm B 's offer as an entrant, p^B , determines equilibrium pricing. As above, we suppose that p^B is set at the lowest level that, if it were taken by the youngsters, would not cause B to lose money. Since $W_1^B = 0$, this gives

$$p^B = m. \quad (21)$$

In equilibrium, firm A is never the entrant. But if it were, we require (for a dominant equilibrium) that it would set so low a price as to make B cede even his locked-in oldsters. Since B would price as low as his cost, m , to retain his oldsters, we must have

$$p^A = m - c. \quad (22)$$

Using equations (18)-(22), we can solve for W_0^A and W_1^A , yielding

$$W_1^A = \frac{2m}{1-\beta}. \quad (23)$$

and

$$W_0^A = \frac{2m}{1-\beta} - 2c. \quad (24)$$

Again we must check some further inequality conditions that are necessary for a dominating equilibrium to exist. The binding constraint turns out to be the one requiring that firm A as entrant indeed find it optimal to capture B 's oldsters. Alternatively, A could choose the highest price that would allow it to capture the youngsters but concede B 's oldsters. Such a price would make B indifferent to matching it, and hence satisfies $p + c - m + \beta W_0^B = 2(p - m) + \beta W_1^B$, or $p = c + m$. If he set this price, A would get, instead of $W_0^A = 2p^A + \beta W_1^A$, $(c + m) + \beta W_1^A$. Thus we require that $c + m \leq 2p^A = 2(m - c)$ or

$$m \geq 3c. \quad (25)$$

Proposition 4. *If $m \geq 3c$, then there is a dominant equilibrium. Prices are given by (21) and (22). In equilibrium, $p^B = m$ is charged to all buyers.*

Notice that c does not affect the prices in this equilibrium. This tells us that when switching costs are small relative to cost differences, they do not affect the equilibrium at

Firm A sets its entry price p^A as high as it can subject to not inciting B to sell to both cohorts. Thus B would be equally well-off selling to both cohorts:

$$W_1^B = 2(p^A - m) + \beta W_1^B. \quad (31)$$

These six equations in six unknowns can be solved to give:

$$\left. \begin{aligned} p^A &= m + \frac{1-\beta}{1+\beta}c \\ p^B &= m - \frac{2\beta}{1+\beta}c \end{aligned} \right\} \quad (32)$$

Two binding inequality conditions limit the range of parameters over which the quasi-dominating equilibrium exists. First, A must not prefer to sell only to his oldsters when he is incumbent. This condition can be shown to be equivalent to

$$\frac{m}{c} \geq \frac{1+3\beta}{1+\beta}. \quad (33)$$

Notice that equation (33) is the opposite of equation (17), so that (except when (33) holds with equality) there can never be both an alternating and a quasi-dominant equilibrium.

The other relevant deviation would be for A , as an entrant, to price so low as to make B drop out. Again we simply report that firm A will not seek B 's oldsters so long as

$$\frac{m}{c} \leq \frac{3+\beta}{1+\beta}. \quad (34)$$

Proposition 5. *If $(1+3\beta)/(1+\beta) \leq m/c \leq (3+\beta)/(1+\beta)$, then there is a quasi-dominant equilibrium, given by equations (32). Otherwise there is not.*

In a quasi-dominant equilibrium, in contrast to dominant equilibrium, the level of switching costs does affect prices and profits. The reason is that B knows (as entrant) that, if he succeeds in selling to the youngsters, he will be able to extract some profit from them in the next period, since A will sell only to youngsters when he is the entrant. This future profit does depend on c , and so B 's willingness to price below his cost m as entrant depends on c ; therefore the equilibrium price depends on c . Here B as entrant is willing to price below cost, so A 's limit pricing is more aggressive and sensitive to c .

The unique perfect Markov equilibrium involves mixed strategies. These strategies require that if B were (out of equilibrium) to become the incumbent, A would randomize between selling only to the youngsters this period (thereafter selling to all buyers) and selling to both buyers this period as well as in future. We now calculate the details of such an equilibrium, and show that it neatly bridges the gap between dominant and quasi-dominant equilibria.

Suppose that, if B is the incumbent, A cedes the oldsters with probability ψ . If he does so, then he can charge the youngsters a price p_h^A (the higher of A 's two possible entry offers) such that the incumbent B is indifferent between selling to his oldsters only and selling to both buyers:

$$p_h^A + c - m + \beta W_0^B = 2(p_h^A - m) + \beta W_1^B. \quad (35)$$

Since in equilibrium firm B never sells, $W_0^B = 0$, and (35) implies

$$p_h^A = m + c - \beta W_1^B. \quad (36)$$

Furthermore, since incumbency is valueless to B if A decides to go after both buyers,

$$W_1^B = \psi(p_h^A + c - m). \quad (37)$$

To sell to all buyers, A must price low enough that B is indifferent between selling to his oldsters (at A 's price plus c) and leaving immediately. Thus A 's "low" entry price p_l^A is simply $m - c$. For A to be indifferent (as he must be if mixing) between these two strategies, we require $p_h^A + \beta W_1^A = 2(m - c) + \beta W_1^A$, or

$$p_h^A = 2(m - c). \quad (38)$$

From (38) and (37), we have

$$W_1^B = \psi(m - c). \quad (39)$$

Equating (38) and (36), and using (39), gives $2(m - c) = m + c - \beta\psi(m - c)$ or

$$\psi = \frac{3c - m}{\beta(m - c)}. \quad (40)$$

The Basic Model

With no cost differences, alternation is again the unique stationary Markov equilibrium. If the incumbent sets a price p , the entrant can either match this price, sell to the youngsters, and earn $p + \beta W_1$, or seek the oldsters, set a price of $p - c$, and earn $2(p - c) + \beta W_1$. In order to make the former strategy preferable, the incumbent's price cannot exceed $2c$. We therefore have

$$p = 2c. \quad (42)$$

The equations for W_0 and W_1 are

$$\begin{cases} W_0 = p + \beta W_0 \\ W_1 = p + \beta W_1 \end{cases}$$

which yield

$$W_0 = W_1 = \frac{2c}{1 - \beta}. \quad (43)$$

It is easy to check that this is the unique stationary Markov equilibrium. In particular, it cannot be optimal for an incumbent to price so low that the entrant stays out: for to do so he would have to price at least as low as zero (lower, if the entrant foresees profits in being the incumbent), and that cannot be optimal for the incumbent.

Notice that in this equilibrium oldsters and youngsters pay the same price, $2c$. However, it would be wrong to conclude that oldsters' switching costs do not lead to exploitation. Rather, youngsters as well as oldsters are being exploited: indeed, each vintage of buyer pays more in this equilibrium than in the corresponding equilibrium when the entrant moves first. The incumbent prices just low enough that the entrant will not want to *undercut* his price by c in order to sell to both cohorts; in the previous model, the entrant priced just low enough that the incumbent would not *match* his price in order to take the whole market.

Scale Economies

With economies of scale or network externalities, we again may have an alternating equilibrium or a dominating equilibrium, depending upon the ratio m/c . When $m \leq c$,

First- vs. Second-Mover Advantages

It is of interest to compare the properties of the two models with efficiency differences. Both firms are better-off when the incumbent moves first than when the entrant does so. But while the incumbent is better-off than the entrant when the entrant moves first, the entrant is better-off than the incumbent when the incumbent moves first. In other words, there is a second-mover advantage in this game.

We can identify two effects that combine here. The first is essentially static: it is clearest when $\beta = 0$. In equilibrium, the first mover sets his price so that the second is "just" willing to sell to one cohort only. So when the entrant moves first, his price p maximizes p subject to $p + c \geq 2p$. Similarly, when the incumbent moves first, his price q maximizes q subject to $q \geq 2(q - c)$. The solutions p^* and q^* to these problems satisfy $p^* + c = q^*$. Therefore, at $\beta = 0$, the first-mover's price is higher by c when the first mover is the incumbent than when the first mover is the entrant. And this is borne out by comparing equations (3) and (42).²⁰

Our second effect is inherently dynamic: it applies only when $\beta > 0$. Because $W_1 > W_0$ when the entrant moves first and the youngsters are the subject of competition, the price that actually emerges will be less than the price $p^* = c$ that solves the simple static maximization problem above. Because the oldsters do not survive to affect the following period, such a dynamic consideration does not affect the corresponding problem when the incumbent moves first (as we see from the fact that in equation (42) above, β does not affect the price). Thus this effect also works to make prices lower when the entrant moves first.

It would be simple enough to extend the model to allow some other interesting comparisons. For example, we could ask what happens if each cohort of buyers were larger by a factor γ than its predecessor (thus $\gamma > 1$ corresponds to growth of the market). This makes the youngsters more valuable relative to the oldsters than they would be if $\gamma = 1$, and so prices become *lower* if the youngsters are on the margin (entrant moves first) as γ

²⁰ Since the second mover matches when he is the entrant, and prices c above the first mover's when he is the entrant, the second-mover's price would be the same in the two models when $\beta = 0$.

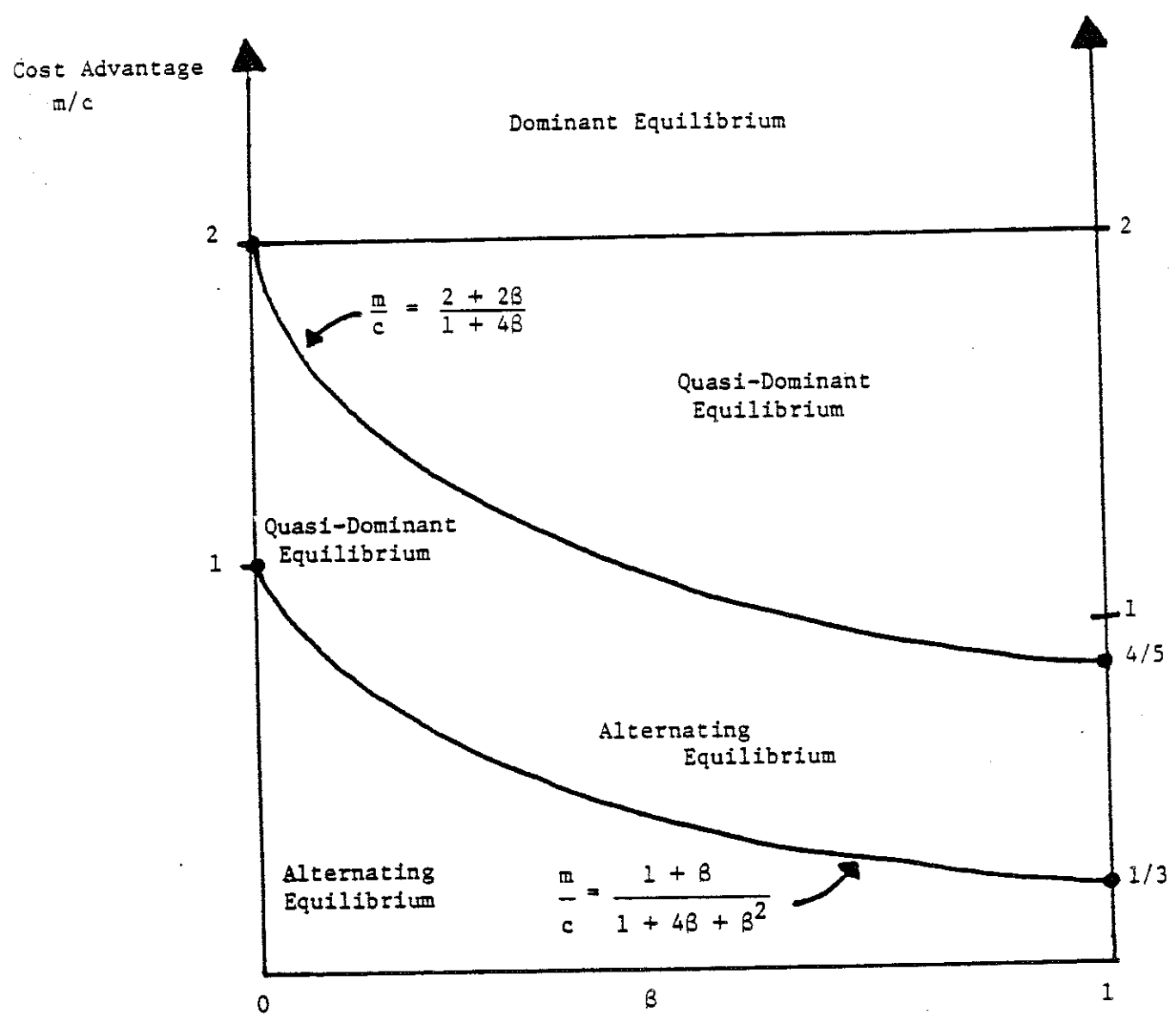
of oldsters. Further progress along these lines appears, however, to require finding ways to overcome the substantial mathematical difficulties inherent in differential games with continuous state variables.

When B is incumbent, A sells to:

		<u>Youngster only</u>	<u>Youngster & oldster</u>
<u>When A is</u> <u>incumbent, A sells to:</u>	<u>Oldster only</u>	Alternating	Quasi-Alternating
	<u>Oldster and</u> <u>Youngster</u>	Quasi-dominant	Dominant

TABLE 1

Classification of Equilibria



Equilibrium with Efficiency Differences

(Incumbent Moves First)

Figure 2

Jan-87

RECENT ISSUES OF THE WORKING PAPER SERIES
OF THE DEPARTMENT OF ECONOMICS
UNIVERSITY OF CALIFORNIA, BERKELEY

Copies may be obtained from the Institute of Business and Economic Research. See the inside cover for further details.

8620 Joseph Farrell and Robert Gibbons
CHEAP TALK IN BARGAINING GAMES

Dec-86.

8621 Robert M. Anderson
CORE ALLOCATIONS AND SMALL INCOME TRANSFERS

Dec-86.

8722 Joseph Farrell
COMPETITION WITH LOCK-IN

Jan-87.

8723 Pranab Bardhan and Nirvikar Singh
MULTINATIONAL RIVALRY AND NATIONAL ARBITRAGE: SOME THEORETICAL CONSIDERATIONS

Jan-87.

8724 Albert Fishlow
LESSONS OF THE 1890S FOR THE 1980S

Jan-87.

8725 Jeffrey A. Frankel
THE SOURCES OF DISAGREEMENT AMONG INTERNATIONAL MACRO MODELS,
AND IMPLICATIONS FOR POLICY COORDINATION

Jan-87.

8726 Jeffrey A. Frankel and James H. Stock
REGRESSION VS. VOLATILITY TESTS OF THE EFFICIENCY
OF FOREIGN EXCHANGE MARKETS

Jan-87.