UC Berkeley
CUDARE Working Papers

Title
Open Access and Extinction

Permalink
https://escholarship.org/uc/item/1np5k97n

Author
Berck, Peter

Publication Date
1977-05-01

eScholarship.org Powered by the California Digital Library
University of California

OPEN ACCESS AND EXTINCTION

by

Peter Berck

Working Paper No. 34

OPEN ACCESS AND EXTINCTION*

By Peter Berck

Open-access renewable resources are exploited too much from a private point of view and may be extinguished which may be a public bad. Government regulation is justified both by its ability to increase the present discounted value of profits and by its ability to preserve some level of population in the face of market pressure. Of concern here is the matter of extinction and the conditions under which open access will lead to extinction.

Following Smith [5], define an open-access resource (fish) bioeconomic system by (1) a population growth law, (2) a law of short-run profit maximization by firms, and (3) a law describing the entrance and exit of capital in the long run. The form these laws take depends on the theory of profit maximization, the technology for growing fish, a restricted profit function for fishing, and an ad hoc assumption.

Define S as the amount of capital or the number of firms, X as the level of the exploited population $\bar{X}$--a level at which there is critical depensation (population decline even without fishing)--and Q as the harvest per firm. The population law states

(1) $$\dot{X} = f(X - \bar{X}) - SQ$$

where $f(0) = 0$; $f(X) \leq 0$ if $X < \bar{X}$; $f' > 0$ if $X > \bar{X}$; and $f'' < 0$.

---

*Giannini Foundation Paper No. .

If $C(Q, S, X)$ is the cost of catching Q with one unit of capital when there are S firms fishing and X fish and the price is a constant P, then the law of short-run profit maximization is

(2) $$P = C_1 \text{ or } Q \text{ is } 0 \text{ or } \infty$$

and

(3) $$\dot{S} = \delta[PQ - C(Q, S, X)]$$

which says profit is proportional to entry or exit. This, though economically plausible, is an ad hoc assumption.

The system (1), (2), and (3) differs from the system used by Smith; Fullenbaum, Carlson, and Bell [2]; and Leung and Wang [4] only in equation (2). None of these authors include the possibility of $Q = 0$. For Smith, this is not a problem because he only considers fishing technologies in which $P = C_1$ is the appropriate rule. Fullenbaum, Carlson, and Bell and Leung and Wang, however, consider a Leontief or fixed coefficients model, and there the "corner" conditions enter into the analysis. Beddington, Watt, and Wright [1] set $\delta \equiv 0$ in equation (3) which accounts for their results. The cost function used here is specialized to a form that is quite simple yet broad enough to include economically sensible versions of all the aforementioned followers of Smith as special cases.

First, let T be the externality effects of the crowding of vessels and the scarcity of fish, and let

$$T = S^L X^{-R}.$$

Now define the short-run costs given one unit of capital and the externality effects as

$$C(Q, T) = AQ^{K}T + FC \qquad \text{if } Q < E$$

$$= \infty \qquad \text{if } Q > E$$

and $K \geq 1$.

Beddington, Watt, and Wright set $L = 0$, $\bar{X} = 0$, $FC = 0$, and $E \equiv \infty$. Fullenbaum, Carlson, and Bell and Leung and Wang prefer $L = 0$, $\bar{X} = 0$, $R \equiv 1$, and $V/T = E$. Instead of writing the cost function, Fullenbaum, Carlson, and Bell and Leung and Wang prefer the primal or production function.

Let Q be the catch when effort Y (possibly labor or fuel) and one unit of capital are used. If

$$Q = \frac{1}{T} \min \{V, WY\}$$

and the price of Y is π_Y, then define $A \equiv \pi_Y T/W$, and one has a generalization of the Fullenbaum, Carlson, and Bell and Leung and Wang technology (which is Leontief's fixed coefficients). Fullenbaum, Carlson, and Bell insist $W = \infty$ which is that only the capital matters; while Leung and Wang want to pay labor a percentage (Q) of the profits. Their equations go so far as to state that labor is paid a percentage of the profits even if the profits are negative. This is not very appealing. The problem with the Fullenbaum, Carlson, and Bell and Leung and Wang variants is that they do not accept the possibility of short-run shutdown. An explicit account of factors other than capital (fuel and equipment deterioration), which means $W \neq \infty$, makes $Q = 0$ or short-run shutdown a possibility.

As with the other followers of Smith, a logistic function, possibly displaced by $\bar{X}$, is used to describe the dynamics of the annual population:

$$f(X) = g(X - \bar{X}) \left[1 - \frac{(X - \bar{X})}{K}\right].$$

If K > 1, the assumptions on cost curves and profit maximization imply:

$$P = K\, Q^{K-1}\, TA$$

and

$$Q = \left(\frac{P}{KTA}\right)^{1/K-1};$$

if k = 1, they imply

$$Q = \frac{V}{T} \qquad \text{if } P \geq AT = \frac{\pi_Y}{W}\, T$$

and

$$Q = 0 \qquad \text{if } P < AT = \frac{\pi_Y}{W}\, T.$$

By direct substitution, the system with K = 1 becomes

$$\dot{X} = f(X) - S\,\frac{V}{T} \qquad \text{if } P \geq AT$$

$$= f(X) \qquad \text{otherwise;}$$

$$\dot{S} = \delta\left[\left(\frac{P}{T} - A\right)\, V - FC\right] \qquad \text{if } P > AT$$

$$= -\delta FC \qquad \text{otherwise;}$$

while the system with K > 1 becomes

$$\dot{X} = f(X) - S\left(\frac{P}{KTA}\right)^{1/K-1}$$

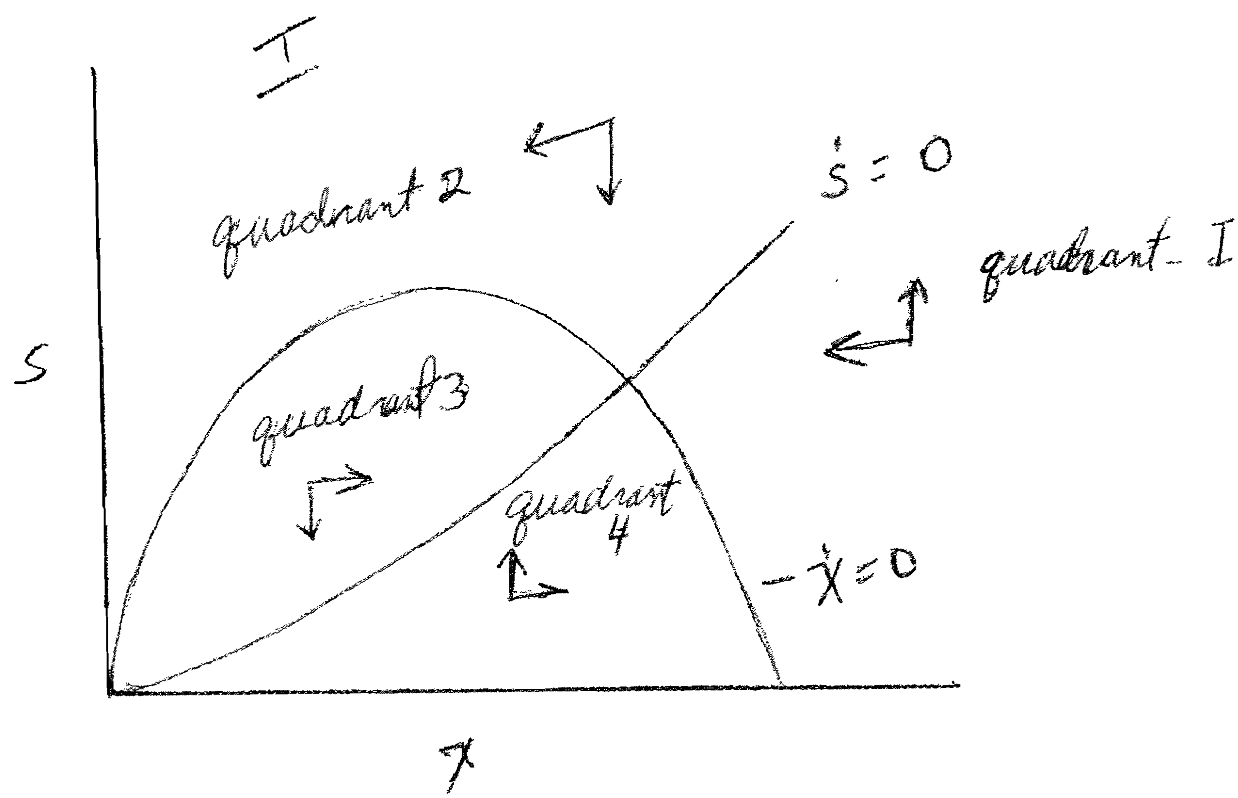
$$\dot{S} = \delta\left[P\left(\frac{P}{KTA}\right)^{1/K-1} - FC - AT\left(\frac{P}{KTA}\right)^{\bar{K}}\right].$$

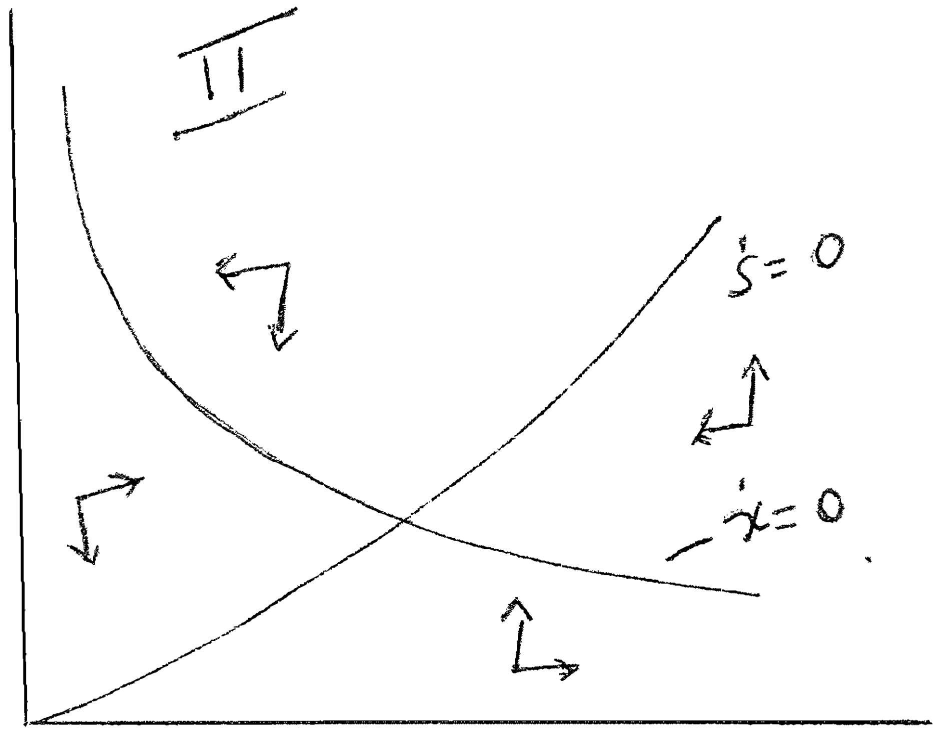
One additional assumption is necessary. Assume that more firms always increase the catch, i.e., $\partial(EQ)/\partial S > 0$. This assumption implies $1 - K + L < 0$. In the case of $K = 1$, the implication is that $L = 0$.

Analysis of extinction requires a phase diagram. First, by straightforward differentiation, one can verify that

$$\left.\frac{dS}{dX}\right|_{\dot{S}=0} = \frac{R}{L}\,\frac{S}{X} > 0;$$

so $\dot{S} = 0$ always slopes upward in the S - X plane and is a vertical line if $L = 0$. With $\bar{X} = 0$, the phase plane for $L > 0$ is





The modification for L = 0 is straightforward and uninteresting and which diagram prevails depends--as in Beddington, Watt, and Wright--on the relationship of R/K - 1 to 1. If R > K - 1, then for n small $\dot{N} \cong aN > 0$, and the picture is II. If R < K - 1, then for n small $\dot{N} \cong -DN^{R/K-1}$ where D = a positive constant and the picture is I.

THEOREM 1: In picture II, extinction is *a priori* impossible.

THEOREM 2: In picture I, extinction is also impossible if $\delta \neq 0$ (Beddington, Watt, and Wright set $\delta = 0$).

To see the assertion, look at $\dot{N} = -DN^{R/K-1}$ and assume an initial condition in the dangerous (second) quadrant.

Since S declines unless the characteristic passes into the safety of quadrant III, an upper bound on the rate of decline of N is given implicitly by

$$N_0^{R-K-1} = \frac{\dot{N}}{N}$$

and N will not get to zero in finite time.

There are two ways for extinction to happen. Either a characteristic hits the axis, X = 0, or it enters the origin. First, it is shown that a characteristic can never intersect the axis at a point other than the origin.

Let P be a point on the X = 0 axis at which extinction happens. Let $B_\varepsilon(P)$ be a closed ball of radius ε about P, and let ε be chosen small enough so $\dot{S} < 0$ everywhere in $B_\varepsilon(P)$. Let M = max of $\dot{S}$ on $B_\varepsilon(P)$. By construction, M < 0. It has already been shown that a characteristic may not hit the axis in finite time. Since $\dot{S} \leq -M$ quite clearly, S will change by more than 2ε in finite time, and the characteristic will leave $B_\varepsilon(P)$ without hitting the point P or approaching it in the limit.

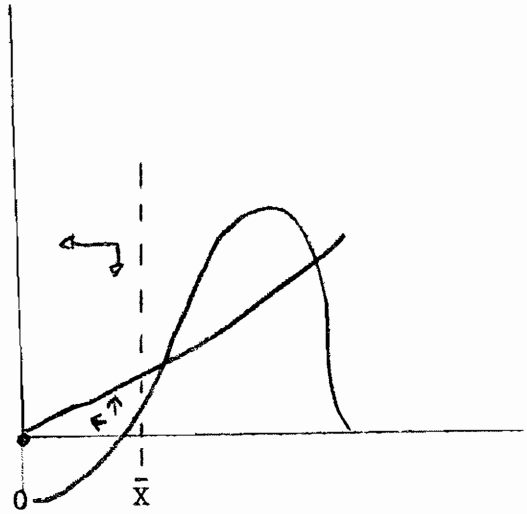
A characteristic cannot enter the origin unless it comes down the X axis. This is because one can demonstrate from the phase diagram that, for every $\varepsilon > 0$, there is a characteristic in $B_\varepsilon(0, 0)$ that does not enter the origin, so the origin is a saddle point (Hurewicz [3]).

As nice as the no-extinction conclusion is, the wooly mammoth and other exploited, extinct beasts make the conclusion unrealistic. A closer look at the equations reveals the answer. Characteristics may pass very close to the axis and still recover. In fact, the system generates a recovered population after the population has been reduced to a fraction of an individual. With the more realistic assumption of critical depensation of the growth curve, $\bar{X} > 0$, extinction becomes possible.

If $\bar{X}$ is positive, then $\dot{\bar{X}} = 0$ is described by

$$C(X) = X^{-\beta} g(X - \bar{X}) \left[1 - \frac{(X - \bar{X})}{K}\right]$$

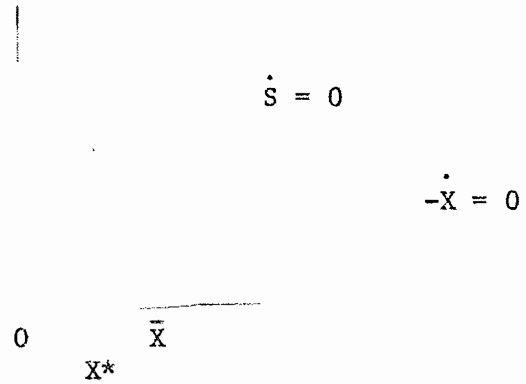
where $\beta = R/K - 1$ and, for all values of R, K, L, the diagram is



The possibility of extinction (or asymptotic extinction) is evident in this phase diagram. Any characteristic that passes the line, $X = \bar{X}$, leads to extinction.

THEOREM 3: If the growth curve yields critical depensation in a neighborhood of zero, then there are characteristics that enter the origin and, in the long run, extinction is a possibility.

The case of K = 1 differs from K > 1 in one important aspect. If K > 1, then, for any X, the $\lim_{S\to\infty} \pi = -FC$, and $\pi > -FC$. If K = 1, then $\pi = -FC$ if $T \geq P_W/\pi_Y$ which, using L = 0, implies $X^* = (\pi_Y/P_W)^{Y_R}$. The phase diagram becomes



and extinction is possible only if $X^* < \bar{X}$. For K > 1, one can introduce X* by $(X - X^*)^{-R} S^L = T'$ and obtain a similar result.

What has been shown is that a linear transformation of the growth or cost laws is sufficient to change the results of an extinction model. Allowing critical depensation ($\bar{X} > 0$) or making it infinitely costly to catch fish at X*, render the simple comparison of growth rates or the like misleading. In fact, if one estimates a system including $\bar{X}$ and/or X*, then the relation of R − K − 1 to 1 is only important with probability zero.

THEOREM 4: Unless there is *a priori* knowledge of X* and $\bar{X}$, the possibility of extinction depends upon R and K (within the limits of the model) with probability zero.

University of California, Berkeley

REFERENCES

[1] BEDDINGTON, JOHN R., C. M. K. WATTS, and W. D. C. WRIGHT: "Optimal Cropping of Self-Reproducible Natural Resources," *Econometrica*, 43 (1975), 789-802.

[2] FULLENBAUM, RICHARD F., ERNEST W. CARLSON, AND FREDERICK W. BELL: "Economics of Production from Natural Resources: Comment," *American Economic Review*, 61 (1971), 483-487.

[3] HUREWICZ, WITOLD: *Lectures on Ordinary Differential Equations*. Cambridge: MIT Press, 1958.

[4] LEUNG, ANTHONY, and AR-YOUNG WANG: "Analysis of Models for Commercial Fishing: Mathematical and Economical Aspects," *Econometrica*, 44 (1976), 295-303.

[5] SMITH, VERNON L.: "Economics of Production from Natural Resources," *American Economic Review*, 58 (1968), 409-431.