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**Publication Date**

1979-08-01

THE IMPACT OF ENHANCED RISK ON CAPITAL  
BUDGETING DECISIONS

25-79

by

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\* A paper prepared for presentation at the Conference on Financial Management of Corporate Resource Allocation, The Netherlands School of Business, Breaklem, Netherlands, August 1979.



The Impact of Enhanced Risk on Capital Budgeting Decisions\*

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Marshall Sarnat

1. INTRODUCTION

At best, the allocation of real corporate resources has always been a rather risky endeavor. However, forces in recent events have combined to increase the degree of uncertainty attaching to firms' capital budgeting decisions. This paper examines some of the implications of the rapidly changing economic environment for the corporate decision-making process.

Section II Briefly reviews the major landmarks in the development of the theory of capital-budgeting; and critically examines some of the problems which have arisen when applying the standard canons of finance theory to the specific question of corporate resource allocation. The significance for capital budgeting of the enhanced uncertainty of the business environment in the 1970's is examined in Section III. In the following Section, the use of "distribution free" methods of risk analysis for capital budgeting is examined in some detail. Section V concludes the paper with a brief discussion of some of the salient characteristics of these methods and their applicability to capital budgeting problems.

## II. RISK AND THE DEVELOPMENT OF CAPITAL BUDGETING THEORY

The modern theory of capital budgeting emerged only recently. The appearance in 1951 of two remarkable books<sup>1</sup>: Joel Dean's exposition of the principles of capital budgeting and Friedrich and Vera Lutzs' analysis of the theory of corporate investment, marks the effective date of the rediscovery of the underlying principles of capital budgeting theory. Dean's slim volume, which laid the foundation for the use of time discounting in the evaluation of capital investment proposals, was a rediscovery of the principle of the time-discounted rate of return.<sup>2</sup> The second book provided the logical foundation for the use of net present value (NPV) as the decision rule in capital budgeting decisions. Once again, the theory of investment advocated by the Lutzes represents a rediscovery; writing in the 1930's, Samuelson [24] had earlier demonstrated the theoretical superiority of the NPV rule.

These two books literally opened the flood-gates, and by the second half of the 1950's a series of insightful articles by Lorie and Savage [17a], Hirshleifer [10] and Bailey [2] had clarified most, if not all, of the outstanding theoretical questions. Thus, by the beginning of the decade of the sixties the feasibility analysis of capital investment under certainty, i.e., given a project's cash flow, was well defined, and appropriate decisions rules had been devised.<sup>3</sup> Investment opportunities were to be evaluated by the present values of their cash flows, and only those projects having a positive NPV were to be accepted.

The early capital budgeting literature either ignored risk or handled it in an ad hoc manner. Thus, Dean [3] advocated the use of higher hurdle rates for projects which introduced new products than for those which represented the expansion of existing lines of activity, presumably because of the greater risk of the former. But this comparative neglect of the riskiness of capital investments did not persist for long. Parallel to the development of the theory of capital budgeting under certainty, there emerged a formal theory of risk which was to have a profound impact upon capital-budgeting theory and practice.

In their monumental work on the theory of games, von Neumann and Morgenstern [29] provided a rigorous axiomatic justification for the use of expected utility to explain rational choice under conditions of uncertainty, by demonstrating that if a number of apparently reasonable consistency requirements are satisfied, utility can be introduced in a way which ensures that rational choices among risky alternatives will be made solely on the basis of their expected utilities. The von Neumann-Morgenstern analysis led directly to the formulation of efficiency criteria based on expected utility, such a criterion being defined as a decision rule for partitioning all potential investment options into two mutually exclusive sets: an efficient set of potentially acceptable options, and an inefficient set. No expected utility maximizer will willingly choose an option out of the inefficient set.

The modern theory of investment choice uses such criteria to dichotomize the decision process into two steps: first, the number of options is reduced by constructing the efficient set of alternatives using an efficiency criterion which is appropriate for a given class of utility functions (for example, the class of all risk averters). Second, the individual is assumed to make his final choice out of the reduced set in accordance with the shape of his specific utility function.

As the implications of the risk analysis based upon the von Neumann-Morgenstern expected utility hypothesis permeated the academic community, uncertainty was explicitly incorporated into the existing framework of time-adjusted measures of investment worth:

- (a) Utility considerations of risk-aversion served as the basis for ad hoc rule of thumb risk adjustments of cash flows and/or discount rates.
- (b) The Markowitz-Tobin mean-variance analysis,<sup>4</sup> itself an outgrowth of the expected utility hypothesis, was applied to single projects as well as to "portfolios" of capital investment projects.
- (c) Following the development of a formal apparatus for the pricing of risky assets by Sharpe [26], Lintner [17], and Mossin [22], various insights derived from the capital-asset pricing model (CAPM) were applied to the analysis of capital investment decisions.<sup>5</sup>

The formal framework for risk analysis was developed largely with financial investments in mind. Its later application to capital investments has been plagued by many difficulties:

- (a) In their initial forms, both the CAPM, and the mean-variance analysis on which it is based, were single-period models; capital budgeting decisions, however, are inherently multi-period decision problems. Moreover, multi-period extensions of these models, for example, those which base their analysis on the concept of continuous time, depend on a set of restrictive assumptions which greatly reduces their applicability to corporate (physical) investments.<sup>6</sup>
- (b) The risk of ruin (bankruptcy), which unfortunately is often so germane to capital budgeting decisions, has been neglected by much (but not all) of the CAPM literature.
- (c) Finally, both the CAPM and mean-variance risk analysis rest on restrictive assumptions regarding investors' utility functions or the distribution of returns. An appreciation of the latter restriction is crucial for what follows.

The efficiency analysis of choice under conditions of uncertainty which emerged following the seminal works of Markowitz [19] [20] and Tobin [27] has demonstrated that in a number of significant cases risk-averse individuals who maximize their expected utility invariably choose an investment option which is efficient in terms of the first two moments of the distribution of returns. The popular Markowitz E-V criterion for



screening inefficient (dominated) options states that an option A will dominate an option B, thereby eliminating B from the efficient set, if the following inequalities hold:<sup>7</sup>

$$E(x_a) \geq E(x_b) \text{ and } \text{Var}(x_a) \leq \text{Var}(x_b).$$

Tobin has shown that this criterion provides an appropriate screening device if we assume quadratic utility, or alternatively, if we assume concave utility (i.e., risk aversion) and normal distributions of returns.<sup>8</sup>

The major advantage of the E-V criterion lies in its simplicity: it permits us to focus attention solely on the first two moments of the distribution of outcomes (expected mean and the variance). However, two serious objections have been raised regarding its use. First, the assumption of quadratic utility raises difficulties. Such utility functions are only valid for a bounded range; Arrow [1] and Pratt [23] have shown that the unrestricted use of quadratic utility implies the economically unacceptable assumption of ever-increasing absolute risk aversion. Second, for many problems the alternative assumption of normal, or near normal, frequency distributions of returns often does not hold.

### III IMPACT OF ENVIRONMENTAL CHANGES ON DECISION ANALYSIS

Drastic changes in the economic environment during the 1970's have refocussed attention on the shortcomings of traditional capital budgeting criteria under certainty. In particular, the enhanced riskiness of capital investments during the second half of the seventies

cannot be captured adequately by efficiency criteria which depend on the assumption of normal distributions. The source of much of this enhanced risk reflects the interaction of several environmental changes during the 1970's.

Consider the impact on the decision process of the following changes in the underlying economic environment:

(a) Exchange Rate Risk. The degree of instability of foreign exchange rates which emerged following the floating of the dollar in the 1970's, has added an important new dimension of risk to capital budgeting decisions, whenever costs, revenues and/or financing are multinational in scope.

(b) Inflation. But this phenomenon cannot and should not be restricted to international operations. The instability of the foreign exchange markets reflects the greatly enhanced inflationary pressures which has characterized the world economy during the decade of the seventies. This can, and often does, create a serious difficulty because the translation of nominal returns into their real counterparts, again adds an additional dimension of risk to the analysis.<sup>9</sup>

(c) Government Intervention. Again on the purely domestic scene, the 1970's have been characterized by increasing degrees of governmental intervention in business affairs. Environmental regulations and controls and legal complications such as unlimited warranties (or the Ralph Nader syndrome as it is sometimes called) have combined to increase the uncertainty attaching to expected returns.

Clearly each of the above factors can potentially increase the uncertainty of corporate investment decisions directly via their own underlying skewness, but an important technical relationship should not be overlooked. Even in the unlikely event that the underlying distributions of nominal returns, as well as the distribution of inflation and exchange rate changes, are normally distributed, the conversion of the nominal returns to a real, or exchange-rate adjusted, basis will in itself impart skewness to the compound distribution. This reflects the fact that the product of two normal distributions is not itself normal. Thus, in general, it is not possible to specify a priori the form of a multiperiod "compound" distribution (Levy [13]).

#### IV DISTRIBUTION-FREE ANALYSIS

The above considerations, taken together, strongly suggest the need for a reassessment of the traditional two parameter approach to the evaluation of risky capital investments. A way out of the dilemma posed by the enhanced risk now confronting corporate decision-makers can be found by following up a suggestion originally made in the mid 1960's by Hillier [9] and Hertz [8] who explored the possibility of basing capital budgeting decisions on the comparison of cumulative probability distributions. Clearly, such an approach frees the analysis of any dependence on the assumption of a particular shape for the probability distribution of returns, since the entire distribution is considered by the decision-maker. However, their approach was not followed up because Hillier and Hertz, writing in the mid-1960's, lacked the tools

to discriminate between alternative investments except in cases of extreme dominance.

Fortunately, for our purposes the decade of the seventies also witnessed a significant break through in our ability to handle and evaluate cumulative probability distributions. And as was true of capital budgeting under certainty, once again we are dealing with the rediscovery of analytical techniques which had been developed earlier and then forgotten. In this instance the technique is that of "distribution-free" efficiency analysis, now commonly referred to as "stochastic dominance",

At the end of 1969, Hadar and Russell [5] and Hancoch and Levy [7] independently applied the notion of stochastic dominance to the problem of achieving a partial ordering of uncertain investment prospects.<sup>10</sup> The concepts of First Degree Stochastic Dominance (FSD) and Second Degree Stochastic Dominance (SSD) which they developed provide a rigorous method for evaluating risky investments without recourse to restrictive assumptions on the shape of the probability distributions of returns.

The papers by Hadar and Russell, and Hanoch and Levy, provided an important catalyst for further development, and stochastic dominance criteria have multiplied like amoeba. Fortunately, for the purposes of capital budgeting we can limit ourselves to three basic criteria: FSD, SSD and TSD (Third Degree Stochastic Dominance).

The importance of these criteria lies in their ability to evaluate risky options solely on the basis of limited information regarding investors' utility functions.

First Degree Stochastic Dominance (FSD)

In the most general case, in which no restrictions are placed on investors' utility functions beyond the reasonable assumption that they be non-decreasing with respect to returns,  $x$ , (i.e., the analysis is appropriate for both "risk lovers" and "risk averters"), an optimal efficiency criterion can be defined as follows: Given two cumulative probability distributions  $F$  and  $G$ , option  $F$  will dominate a second option  $G$  (thereby eliminating the latter from the efficient set), independent of the concavity or convexity of the utility function, if and only if:

$$F(x) \leq G(x) \quad (1)$$

for all values of  $x$ , with a strong inequality  $F(x_0) < G(x_0)$  holding for at least one value of  $x$ .<sup>11</sup> This is tantamount to assuming that the two cumulative probability distributions do not intersect, or in other words, option  $F$  stochastically dominates option  $G$ .

This criterion can be illustrated by a simple graphical device. Figure 1 plots the cumulative probability functions of two investment options,  $A$  and  $B$ . As  $B$  lies to the right of  $A$ , the FSD criterion is satisfied and option  $A$  is eliminated from the efficient set, independent of the shape of individuals' utility functions.

Upon reflection, the stochastic dominance analysis is almost intuitively obvious. The FSD criterion, which has been defined in terms of cumulative probability distributions, stipulates that any option F dominates another option G if  $F(x) \leq G(x)$ . But this is equivalent to the requirement that the probability of achieving a return lower than some amount, say k, will always be smaller for option F than for option G. Since  $F(k) \leq G(k)$  by definition, it follows that

$$\Pr_F (x \leq k) \leq \Pr_G (x \leq k) \quad (2)$$

where Pr denotes probability.<sup>12</sup>

#### Second Degree Stochastic Dominance (SSD)

The FSD criterion is very general; it applies very weak restrictions on utility functions, which are only required to have non-negative first derivatives, and, therefore, the criterion is appropriate for all individuals independent of the pattern of their risk-return tradeoffs. In almost all economic analysis, however, universal risk-aversion is assumed to prevail, i.e., individuals' utility functions are assumed to be concave throughout the relevant range.

Given this realistic assumption,<sup>13</sup> an efficiency criterion for the class of all risk averters, i.e., Second Degree Stochastic Dominance (SSD) criterion, can be defined. Again denoting the cumulative probability distributions of two options by F and G, a necessary and sufficient condition for F to dominate G. for all risk averters (concave utility functions) is that

$$\int_{-\infty}^x [G(t) - F(t)] dt \geq 0 \quad (3)$$

with a strong inequality holding for at least one value of x.<sup>14</sup> Thus, unlike FSD, SSD permits the cumulative probability distributions to

intersect, on the condition that the cumulative difference between G and F remains non-negative over the entire domain of x.

Once again, the efficiency criterion can be illustrated graphically. Figure 2 plots the cumulative probability distributions of two alternatives, F and G. Although the two functions intersect several times, it is clear by inspection that the cumulative first differences between the two functions remain positive for all values of return (x), that is, the cumulative shaded areas for which  $G(x) > F(x)$  always exceed the cumulative unshaded areas for which  $G(x) < F(x)$  over the entire domain of x. Hence, option F dominates G according to the SSD criterion, and G is eliminated from the efficient set for all risk averters.

#### Third Degree Stochastic Dominance (TSD)

The TSD criterion places additional restrictions on the form of the utility function, and therefore, provides an appropriate decision rule for a subset of risk averters. The particular group singled out in the TSD approach is the class of risk averters with utility functions having non-negative third derivatives. Given this assumption the following optimal decision rule can be defined. A necessary and sufficient condition for option F to dominate option G is that

$$\int_{-\infty}^x \int_{-\infty}^y [G(t) - F(t)] dt dv \geq 0 \quad (4)$$

and

$$E_F(x) \geq E_G(x) \quad (5)$$

for all  $x$ , with at least one strong inequality holding in each case.<sup>15</sup>

Defined as they are in terms of the entire cumulative distribution of returns, all three stochastic dominance criteria, FSD, SSD and TSD, permit an analysis of the risk-return relationship without recourse to restrictions on the frequency distributions of monetary returns. Thus, unlike the mean-variance criterion, the distributions of returns need not be normal, or for that matter even symmetrical. The SD criteria are essentially complements rather than substitutes. In most cases, it is desirable to apply both the stronger, (SSD and TSD) as well as the weaker, FSD, criterion. Finally, the SSD and TSD efficient sets are subsets of the FSD efficient set. The stronger assumptions of the former permit a more sensitive screening of options; hence the empirically derived SSD and TSD efficient sets can be expected to be smaller, and in many cases substantially smaller, than their FSD counterpart.

#### V CONCLUDING REMARKS

The enhanced riskiness of investment during the second half of the 1970's raises serious questions regarding the appropriateness of traditional capital budgeting models in the decade of the eighties. Since the assumption of normally distributed returns is suspect, considerable importance attaches to the comparatively recent theoretical development of distribution-free methods of efficiency analysis. However, several problems arise when an attempt is made to apply stochastic dominance techniques to the screening of investment proposals at the level of the firm.



To be operational, the decision criteria must be general, that is they must be independent of individuals specific parameters, e.g., their initial wealth. In this context, Hadar and Russell [6] and Levy and Sarnat [14] have shown that where investors' initial wealth is independent of the returns, the dominance of one project over another is independent of initial wealth.

Similarly the application of stochastic dominance methods to capital budgeting decisions requires the discounting of returns which raises a question regarding the impact (if any) of the discounting process on the partial ordering of prospects. However, Levy and Sarnat [15] have shown that the dominance relationship is also independent of the discount rate.

Finally it should be noted that the separation property of the mean-variance analysis which leads to the CAPM when riskless borrowing and lending are permitted does not hold for the stochastic dominance criteria. However, some recent work by Kroll and Levy [11] suggests that the size of the SD efficient sets can be reduced dramatically (often to a single option) by the introduction of riskless borrowing and lending.

Thus, while much more work admittedly remains, the current state of the art strongly suggests the desirability of applying SD techniques to capital budgeting problems, especially in those cases where the distributions of returns are known to be highly skewed.<sup>16</sup>

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FOOTNOTES

- \* An earlier version of this paper was presented at the Conference on Financial Management of Corporate Resource Allocation, The Netherlands School of Business, Breukelen, The Netherlands, August 1979.
1. See J. Dean [3] and F. Lutz and V. Lutz [18].
  2. This is the method which has been used in financial circles for centuries to compute the yield to maturity on a bond. Applied to the investment in real assets the method is known under a variety of names, "marginal efficiency of capital", "discounted cash flow", etc.
  3. This holds true for the case of perfect capital markets. Devising optimal decision rules for the case of capital rationing remains a difficult problem to this day.
  4. See Markowitz [19] [20] and Tobin [27].
  5. In fact the model was already applied explicitly to capital budgeting problems in the classic paper by Lintner [17].
  6. See, for example Merton [21].
  7. On the additional condition that at least one of the strong inequalities holds.
  8. Levy and Sarnat [16] have shown that the E-V criterion also provides an appropriate decision-rule in the more general case of all independent two-parameter distributions, but on the further condition that non-intersecting cumulative frequency distributions are eliminated.
  9. The translation of nominal into real returns also has serious implications for the use of the CAPM, see Sarnat [25].
  10. The concept of FSD long had been familiar to statisticians while the concept of SSD had been applied in a number of instances during the early 1950's and 1960's. Significantly, the SSD criterion was not applied to investment choice or portfolio selection. See Kroll and Levy [11] and the references cited.

11. For a formal proof see Hadar and Russell [5] and Hanoch and Levy [7].
12. Multiplying both sides of (2) by -1 and adding unity, we obtain  $1 - F(x) > 1 - G(x)$ . This expression is equivalent to the probability of receiving a return which is greater than or equal to a given return,  $x$ . Since the FSD criterion must hold for all levels of return, this implies that the probability of receiving a return greater or equal to some level,  $k$ , must always be higher for option F than for option G.
13. For empirical evidence on investors' behavior in support of this statement see Levy and Sarnat [16], ch. 9.
14. This means that given the stipulation that the first two derivatives of the utility function be non-negative, and not positive, respectively (i.e.,  $U'(x) > 0$  and  $U''(x) < 0$ ), no criterion can be found to further reduce the SSD efficient set without placing additional restrictions on investors' utility functions and/or on the probability distribution of returns. For formal proof of the optimality of the SSD criterion, see Hadar and Russell [5] and Hanoch and Levy [7].
15. For a formal proof of the optimality of the TSD criterion, see Whitmore [30].  
  
An additional SD criterion, for utility functions which have the Arrow-Pratt property of decreasing absolute risk aversion, has been developed by Vickson [28].
16. Dunbar and Sarnat [4] have applied the SSD technique to the skewed returns of U.S. railroads.

CUMULATIVE  
PROBABILITY

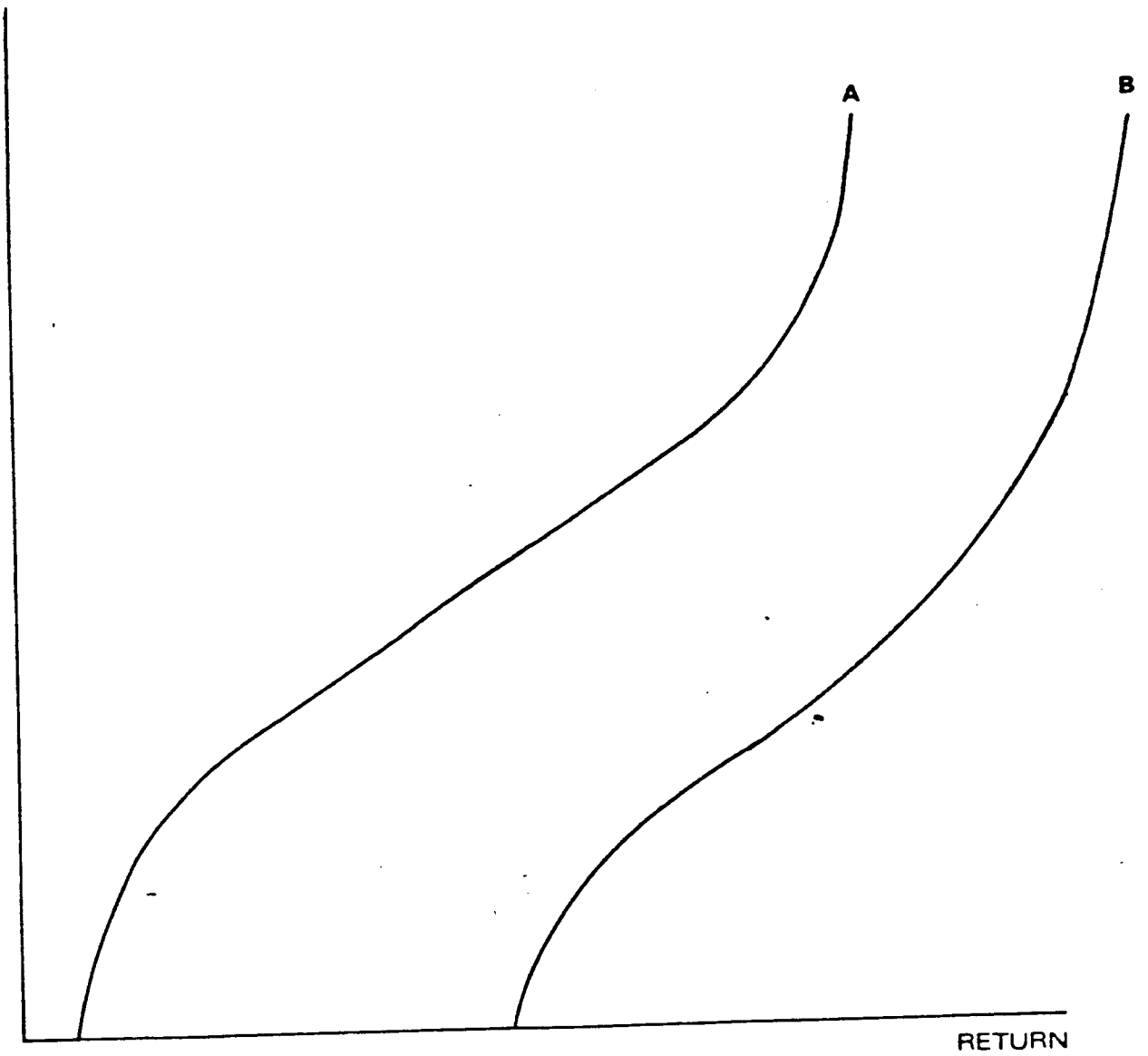


FIGURE 1

First Degree Stochastic Dominance: A Graphical Illustration

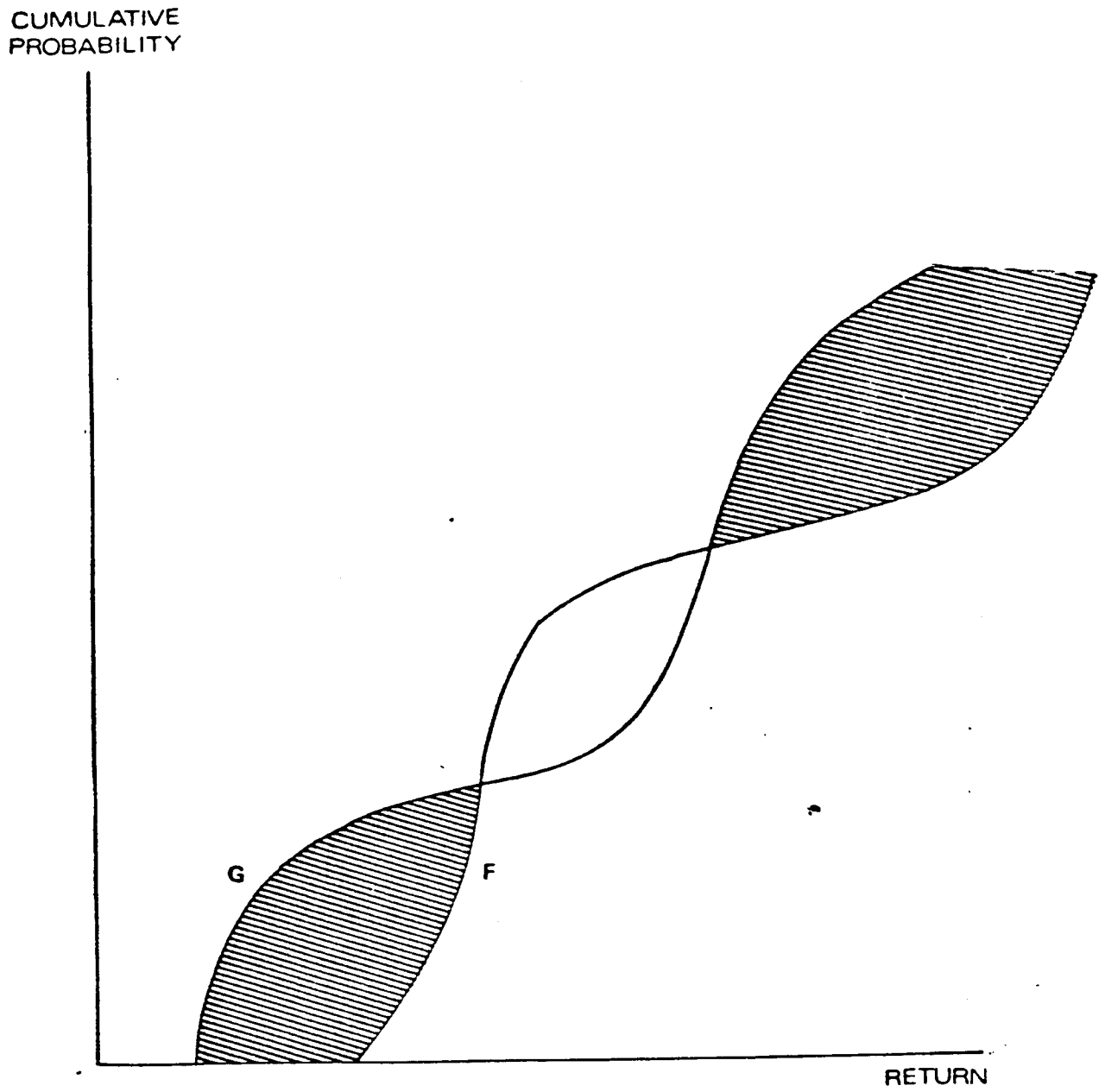


FIGURE 2

Second Degree Stochastic Dominance: A Graphical Illustration



