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# STRICTLY SPEAKING, THE LAW OF ONE PRICE WORKS IN COMMODITY MARKETS\*

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## ABSTRACT

Pippenger and Phillips (forthcoming) show how four common pitfalls cause cointegration tests to reject the law of one price when in fact it holds. They conclude that there is *no* reliable evidence that rejects the LOP. We consider a stronger test, half lives. The literature suggests that half lives for differentials in spot prices last several quarters. We show that, when we avoid the four common pitfalls, half lives for differentials for spot grain prices range from 3 to 8 weeks. Such short half lives have potentially important implications for the Borders literature and real exchange rates.

JEL: D41, F15, F3, G1.

Keywords: law of one price, exchange rates, half lives

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The term “law of one price” has been widely used to refer to the convergence of any two similar prices or price indexes. When we refer to the law of one price or LOP, we mean the strict definition found in encyclopedias and dictionaries for economics. As pointed out below, encyclopedias and dictionaries make it clear that arbitrage is the mechanism behind the LOP.

Commodity prices can converge without arbitrage being possible, but for the strict version of the LOP to hold, arbitrage must be possible. This distinction is important. The absence of price convergence when arbitrage is not possible does not conflict with any basic assumptions of economics. The absence of price convergence when arbitrage is possible indirectly rejects the assumption of utility maximization and directly rejects the assumption of wealth maximization.

Pippenger and Phillips (forthcoming) identify four practical implications of arbitrage that are widely ignored. The “pitfalls” are: (1) Using retail prices. Arbitrage is normally impossible between retail markets because resale is normally impossible. (2) Ignoring transportation costs. For arbitrage to operate, price differentials must cover the relevant transaction costs. (3) Ignoring time. Unlike arbitrage between financial assets, commodity arbitrage takes time. (4) Using prices for products that are not identical. If the products are not identical, arbitrage is impossible. All articles that fail to support the law of one price fall into at least two of these pitfalls and some fall into all four. See for example Richardson (1978), Ardeni (1989), Haskel (2001) and Lutz (2004).

Avoiding these pitfalls as much as is possible with spot data, Pippenger and Phillips (forthcoming) show that monthly prices between the United States and Japan for three different varieties of wheat are cointegrated. They then show how each of the last three pitfalls weakens the test results. With all of the last three pitfalls, there is no evidence of cointegration. Given the prevalence of these pitfalls in the literature, they conclude that there is *no* evidence from tests for cointegration that would lead them to reject the law of one price as that law is defined in encyclopedias and dictionaries.

But cointegration is a minimal test for the law of one price. We consider a stronger test, half lives. The literature suggests that half lives last several quarters. If true, such long half lives would raise serious questions about the effectiveness of arbitrage and the usefulness of the law of one price in empirical and theoretical models. We show that, if one avoids the pitfalls pointed out above, the evidence suggests that half lives for the LOP should be measured in weeks or perhaps even days. When combined with the earlier literature on the law of one price, our results have potentially important implications for the Borders literature and real exchange rates.

Section I briefly discusses the meaning of the term “law of one price” and illustrates the importance of arbitrage for that law. Section II reviews some of the literature from the perspective of Section I. Section III uses Section I to develop our model of the LOP. Section IV describes the data from international grain markets. Like all previous research, we are forced to use spot prices. Unlike most of the previous research, particularly research that fails to support the LOP, our spot prices are from markets where arbitrage is possible. Section V reports the results of tests for unit roots in the relevant series. Section VI reports the results of tests for cointegration between the relevant series. Section VII reports our estimates of the half lives for spot price differentials in international grain markets. Because we avoid the common pitfalls pointed out above, our half lives are much shorter than those reported earlier. In addition, unlike earlier estimates of half lives for price differentials, we provide confidence intervals and correct for the bias in least squares. Section VIII summarizes our results and presents our conclusion. Our primary conclusion is straightforward. The law of one price, as it is defined in dictionaries and encyclopedias, works in commodity markets.

## I. ARBITRAGE AND THE LOP

Dictionaries and encyclopedias for economics make it clear that arbitrage is the mechanism behind the LOP.<sup>1</sup> For example, *The Penguin Dictionary of Economics* defines the LOP as follows:

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<sup>1</sup> If anyone doubts this very important point, we suggest that they spend some time perusing the dictionaries and encyclopedias in their university or college library.

The law, articulated by Jevons, stating that ‘In the same open market, at any moment, there cannot be two prices for the same kind of article.’ The reason is that, if they did exist, arbitrage should occur until the prices converge.

Other dictionaries and encyclopedias that we have perused that include a reference to the LOP make a similar reference to arbitrage.

By mentioning or appealing to arbitrage, most articles trying to test the LOP in commodity markets at least implicitly appear to attribute the LOP to arbitrage. Some articles make the link between the LOP and arbitrage explicit. For example, in the first paragraph of his seminal article, Isard (1997) says the following:

In the assumed absence of transport costs and trade restrictions, perfect commodity arbitrage insures that each good is uniformly priced (in common currency units) throughout the world—the “law of one price” prevails.

Of course arbitrage is not the only mechanism that causes prices to converge. Even without arbitrage, competition normally limits the divergence between prices of even similar goods. But the persistence in price differentials and the low power of tests for cointegration often combine to make it impossible to reject the null of no cointegration when arbitrage is impossible.

To illustrate the importance of arbitrage for the law of one price, consider the following mental experiment: Following David Hume, suppose four-fifths of all the wheat in Europe disappeared over night. Spot prices in Rotterdam for wheat would rise far above the spot price in Gulf ports plus conventional transportation and transaction costs. This differential in spot prices would *not* create an opportunity for arbitrage because it would be impossible to move wheat from Gulf ports to Rotterdam within a couple of days. With arbitrage impossible, this price differential would not reject the LOP.

Arbitrage would be possible using forward contracts. Forward prices in Rotterdam would also rise. Higher forward prices in Rotterdam than at Gulf ports would create an opportunity for wheat dealers to engage in profitable arbitrage. To eliminate risk, within as short a period of time as is possible, arbitrageurs would enter

into at least three forward contracts:<sup>2</sup> One contract would be for wheat to be delivered on board ship in a Gulf port in say two weeks. Another contract would secure the ship to load the wheat in two weeks and would fix the freight rate. A third contract would sell the wheat in Rotterdam when the ship is scheduled to arrive. All these contracts are forward contracts and all prices are forward prices. This combination of forward contracts meets all the conditions for arbitrage including identical products, resale, and no risk. If these forward contracts are omitted, the transaction is no longer arbitrage because it involves risk.

Arbitrage raises forward prices in Gulf ports relative to what they would have been without arbitrage. Arbitrage lowers forward prices in Rotterdam relative to what they would have been. Arbitrage also bids up forward freight rates relative to what they would have been. Arbitrage does not directly reduce the differential in current spot prices, but it indirectly reduces that differential. Higher forward prices in Gulf ports encourage grain elevators there to hold over wheat for future delivery. The reduced supply of wheat for sale in spot markets raises spot prices in Gulf ports. Lower forward prices in Rotterdam encourage grain elevators there to offer more wheat in spot markets, reducing current spot prices in Rotterdam.

Arbitrage does *not* provide a direct link between spot prices in Gulf ports and Rotterdam. But arbitrage provides an important indirect link between the spot prices for wheat. Competition limits the spread between spot and forward prices for identical products in each location. After accounting for transaction costs, effective arbitrage equalizes the appropriate forward prices for identical products in different locations. With the LOP holding for forward prices, the link between spot and forward prices in each individual location limits the divergence in spot prices between locations.

With effective arbitrage, after accounting for the transaction costs, differentials between the relevant forward prices should be relatively small and not highly autocorrelated. Half lives should be very short. With no direct market mechanism to reduce differentials in spot prices, spot differentials will be larger and more persistent. As a result, even tests that support the LOP using spot prices from markets like ours will find half lives that are

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<sup>2</sup> At the same time arbitragers must also arrange for financing, insurance and drawing up the appropriate documents.

relatively long as compared to what we would expect with effective arbitrage. When interpreting our half lives for spot prices, remember that they are only pale reflections of the true half lives relevant for the LOP.

## II. Literature Review

Most attempts to test the LOP ignore the practical restrictions implied by the strict definition of the law of one price. For example, products must be identical, there must be no risk and resale must be possible. As a result of these restrictions, one cannot use price indexes, narrow classes of commodities like bread, or retail prices even when the retail prices are for identical goods.

Early attempts to test the LOP relied almost exclusively on sub-indexes of general price indexes like the CPI. Isard (1997) and Richardson (1978) are early examples. Later articles using sub-indexes include Fraser, Taylor and Webster (1991) and Engel and Rogers (2001). No matter how refined an index, it is impossible to arbitrage between price indexes.

Concerns about the reliability of tests based on price indexes led to the use of classes of commodities like “wool” or “bearings” in different countries. Using these classes of commodities, the results become more mixed. Ardeni (1989) finds no support for the LOP. But Baffes (1991) and Vataja (2000) find some evidence of cointegration.

Concerns about the reliability of tests based on classes of commodities led to the use of identical products. When prices for identical products are from markets where arbitrage is not possible, the results are at best mixed. See for example Asplund and Friberg (2001), Haskel and Wolf (2001), Goldberg and Verboven (2005) and Lutz (2004). When prices for identical products are from markets where arbitrage is possible, there is consistent support for the LOP. See for example Goodwin (1992), Michael, Nobay and Peel (1994) and Pippenger and Phillips (forthcoming).

Most modern tests of the LOP test for linear cointegration. A few test for non-linear cointegration. Obstfeld and Taylor (1997) and Lo and Zivot (2001) apply non-linear techniques to sub-indexes. Sarno, Taylor and

Chowdhury (2004) use classes of commodities including “rubber” and “wood and wood products”. They all find more evidence of cointegration than one normally finds using similar data and testing for linear cointegration. Obstfeld and Taylor (1997) and Sarno, Taylor and Chowdhury (2004) interpret their results as evidence of the importance of transportation costs when testing the law of one price. But the results in Pippenger and Phillips (forthcoming) suggest that the thresholds are probably more the result of using price indexes or prices from markets where arbitrage is not possible. They find that omitting freight rates has only a small effect on tests for cointegration. However using prices for different varieties of wheat has a substantial effect. The results in Lo and Zivot (2001) are consistent with that interpretation. They conclude that the type of thresholds they find are not consistent with transportation costs.

The title of Isard’s seminal article in 1977 asks the following question: How Far Can We Push the “Law of One Price”? Given the literature reviewed here and the results presented below, we think we can answer that question. The answer is not very far. For there to be systematic and strong support for the LOP, prices must be for identical products and they must come from markets where arbitrage is possible.

If further research supports our results, then our answer to Isard’s question has important implications. Our answer helps explain why Borders appear to be so wide and why half lives for real exchange rates are so long. Real exchange rates have long half lives because the prices in the price indexes are not for identical products or are from markets where arbitrage is not possible. Even when prices are for identical products, the absence of arbitrage causes many price differentials underlying real exchange rates to have very long half lives. Based on the earlier literature for the LOP, for many other price differentials tests for cointegration would fail. In those cases, the implied half life for the underlying price differential would be infinite. If tests for real exchange rates used identically weighted indexes constructed from market prices where arbitrage is possible, our results suggest that half lives for real exchange rates would be very short.



Our answer to Isard’s question also helps explain how we can find such short half lives in international grain markets while the Borders literature finds such wide borders. We find short half lives because our prices are from markets where arbitrage is possible. Engle and Rogers (1996) and Parsley and Wei (2001) find wide borders because they mix sticky retail prices, where arbitrage is not possible, with exchange rates, which are highly volatile. As Alchian (1969) and (1977) points out, the information and transaction costs that produce money, vacant apartments, shopping centers and brand names also produce ‘sticky’ retail prices. Sticky retail prices within a country produce low volatility in the Borders literature.<sup>3</sup> With arbitrage impossible, mixing those sticky prices with exchange rates produces high volatility. Although the relatively high volatility produces wide ‘borders’, it does not imply any market failure. That high volatility is what we should expect in a world with the kind of information and transaction costs that produce money, vacant apartments, shopping centers and brand names. If the Borders literature used prices like our grain prices, our results suggest that borders would hardly matter.<sup>4</sup>

### III. Modeling the LOP

Our model of the law of one price begins with what looks like a standard statement for the LOP. Let  $P_t^J$  or  $P_t^R$  be the price in dollars of a metric ton of a particular variety of grain in Japan or Rotterdam in month  $t$ . Let  $P_t^P$  or  $P_t^G$  be the price of that same variety of grain in dollars at Pacific or Gulf ports in month  $t$ . Let  $F_t$  represent the relevant freight rate and  $C_t$  represent all the other relevant transaction costs. Equation 1 is our basic model for the LOP.

$$P_t^J / (P_t^P + F_t + C_t) = 1.0 + u_t \quad (1)$$

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<sup>3</sup> The Borders literature uses borders, distance and other variables to explain the volatility in the change in  $P_{jk}^i$  where  $P_{jk}^i$  is the log of the price of good  $i$  in locations  $j$  and  $k$ . All prices converted into U.S. dollars before taking relative prices.

<sup>4</sup> If international markets have half lives of only 4 to 8 weeks, then national markets would have to have almost zero half lives for borders to have any substantial effect.

Our interpretation of equation 1 is not standard because commodity arbitrage takes time. As illustrated in the earlier mental experiment, prices and the relevant transaction costs should be from appropriate forward contracts written in month  $t$ . Under this interpretation, the error term  $u_t$  should be relatively small and not highly correlated. Large and highly persistent errors in equation 1 would be inconsistent with effective arbitrage.

With prices, freight rates and other transaction costs from spot markets, equation 1 becomes equation 2.

$$p_t^J / (p_t^P + f_t + c_t) = 1.0 + u_t + z_t \quad (2)$$

In equation 2, lower case indicates spot rather than forward prices and  $z_t$  is the additional error due to using spot prices. In effect, the spot information in equation 2 acts as a proxy for the forward information in equation 1. What most earlier tests of the LOP actually test is a version of equation 2 where freight rates and other transaction costs are on the right-hand side.<sup>5</sup> While  $u_t$  should be small and not highly autocorrelated,  $z_t$  can be relatively large and highly autocorrelated. When the forward markets that lie behind equation 2 do not exist, we would expect the error term to be much larger and much more persistent. It is that additional persistence that causes many tests for cointegration to fail.

Our test equation differs from equation 2 in two ways. First we account for the fact that commodity arbitrage takes time by lagging export prices by one month. Goodwin, Grennes and Wohlgenant (1990) search for the optimum delivery lag over a variety of agricultural products between the United States and Rotterdam and conclude that the optimal lag is one month. Michael, Nobay and Peel (1994) using data similar to ours also choose a one month delivery lag. We have adopted their delivery lag of one month.

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<sup>5</sup> A few articles such as Goodwin, Grennes and Wohlgenant (1990) and Michael, Nobay and Peel (1994) use freight rates.

Second, as explained below, our freight rates are estimates of forward freight rates. To align spot export prices with forward freight rates, we lag the freight rates an additional month. After moving  $c_t$  to the right-hand side, these adjustments produce equation 3.<sup>6</sup> Equation 3 is our implicit test equation.<sup>7</sup>

$$\ln(p_t^J) - \ln(p_{t-1}^P + F_{t-2}) = u_t + e_t + [c_{t-1}/(p_{t-1}^P + F_{t-2})] \quad (3)$$

#### IV. DATA

We use monthly freight rates with spot prices for one variety of corn and several varieties of wheat. Similar data for wheat have been used in several articles including Goodwin, Grennes and Wohlgenant (1990), Goodwin (1992) and Michael, Nobay and Peel (1994). Goodwin, Grennes and Wohlgenant (1990) use a similar, but much shorter, data set for corn. These data have several advantages over the data used in most attempts to evaluate the LOP: (1) They are for identical varieties of wheat or corn. (2) Domestic and foreign prices are both in dollars. Both prices being in dollars eliminates potential errors caused by mismatches between prices and exchange rates. (3) Matching freight rates are available. (4) For the Rotterdam data, our wheat prices cover an unusually large number of years, 1974:01 to 2001:12. (5) Export prices are free on board (FOB) and import prices include certificates, insurance and freight (CIF).<sup>8</sup> FOB and CIF prices avoid the distortions caused by different domestic costs of distribution. (6) Although we use spot prices, arbitrage is possible in all these markets.

Table 1 describes the data, sources and the acronyms used later to identify each variety of grain. A ‘J’, ‘P’, ‘G’ or ‘R’ attached to an acronym indicates the port. For example, DNSR is Dark Northern Spring wheat at Rotterdam. An F attached to an acronym indicates that the price includes the relevant freight rate. For example, DNSGF is the FOB price for a metric ton of Dark Northern Spring wheat at Gulf ports plus the relevant freight rate.

<sup>6</sup>  $\ln(p_{t-1}^P + F_{t-2} + c_{t-1})$  can be written as  $\ln(p_{t-1}^P + F_{t-2}) + \ln\{1.0 + [c_{t-1}/(p_{t-1}^P + F_{t-2})]\}$ . When  $x$  is small,  $\ln(1.0+x) \approx x$ . Therefore  $\ln[p_t^J/(p_{t-1}^P + F_{t-2} + c_{t-1})] = \ln[1.0 + u_t + e_t]$  can be written as  $\ln(p_t^J) - \ln(p_{t-1}^P + F_{t-2}) = u_t + e_t + [c_{t-1}/(p_{t-1}^P + F_{t-2})]$ .

<sup>7</sup> Because forward freight rates replace spot freight rates,  $e_t$  in equation 3 is not identical to  $z_t$  in equation 2.

<sup>8</sup> Import prices for Japan include only certificates and freight.

Table 1  
Varieties of Grains, Interval and Source\*

Variety	Acronym/Ports	Period and Source
No. 2 Dark Northern Spring Wheat, 14% Protein	DNS Japan, Pacific and Gulf Gulf Rotterdam Rotterdam  Rotterdam	1975:09-1981:10 <i>World Wheat Statistics</i> 1974:01-2001:12 <i>World Wheat (or Grain) Statistics</i> 1974:01-1990:12 <i>World Wheat (or Grain) Statistics</i> 1989:06-1995:10 U.S. Department of Agriculture <i>Wheat Situation &amp; Outlook Report</i> 1994:07-2001:12 <i>World Grain Statistics</i>
No. 2 Western White Wheat	WW Japan and Pacific	1975:09-1981:10 <i>World Wheat Statistics</i>
No. 2 Hard Winter Wheat, 13% Protein	HW Japan and Gulf	1976:01-1981:07 <i>World Wheat Statistics</i>
No. 2 Yellow Corn	YC Rotterdam and Gulf	United Nations Conference on Trade and Development, <i>Handbook of Statistics</i> 1987:01-1998:06

\* All prices and freight rates are for metric tons.

Freight rates are from *World Wheat Statistics* or *World Grain Statistics*. Freight rates vary with the size of the ship and at times more than one rate is published. We use the same freight rates as Michael, Nobay and Peel (1994). Unlike the prices, freight rates are not spot. Footnotes in *World Wheat Statistics* describe the freight rates as follows: “Estimated mid-month rates based on current chartering practices for vessels to load six weeks ahead.”<sup>9</sup> Freight rates are forward, not spot, prices.

Wheat is a “heavy” grain. Corn is lighter. For wheat, one metric ton equals 36.7437 bushels. For corn, one metric ton equals 39.3682 bushels. Since a metric ton of corn takes up more space in the hold of a ship than a metric ton of wheat, we would expect freight rates for corn to be slightly higher than for wheat. Unfortunately we do not have separate freight rates for corn. We apply our rates for wheat to corn. Since the difference should be small and the two freight rates should move together, applying freight rates for wheat to shipments of corn should not be a serious problem.

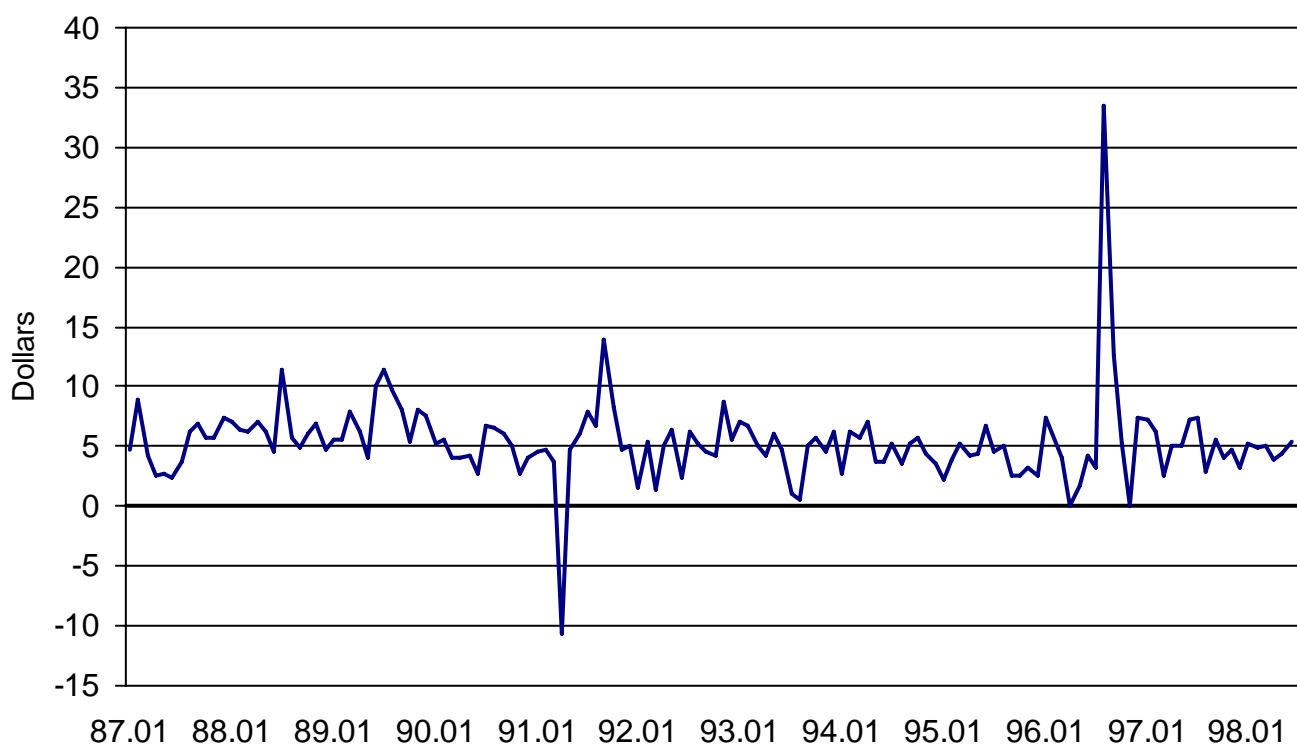
For trade with Japan, we use the same intervals used earlier by Michael, Nobay and Peel (1994) and Pippenger and Phillips (forthcoming). For a visual inspection of that data, see Michael, Nobay and Peel. We do

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<sup>9</sup> After about 1982, the freight rates are for 3 to 4 weeks ahead.

not extend the Japanese data for two reasons: (1) Starting in 1982 several months of Japanese prices are missing. (2) In the 1990s Japan erected non-tariff barriers to wheat imports that created artificial price differentials.<sup>10</sup>

Figure 1  
Price Differential Including Freight Rate for Yellow Corn



We begin the series for corn as soon as both prices are available from UNCTAD's *Handbook of Statistics*. We end that data in mid 1998 because around that time Europe began to impose restrictions on importing genetically modified foods and most corn grown in the United States is genetically modified. Figure 1 shows

<sup>10</sup> For a discussion of Japanese protectionist policies in the wheat market see Fukuda, Dyck and Stout (2004).

the price differential in dollars for corn where the export price includes the contemporaneous freight rate. That is the price differential is measured as  $p_t^R - (p_t^G + F_t)$ .<sup>11</sup>

Some data are missing. Except for DNSR during the early 1990s, there are never more than two missing months in a row. As in Pippenger and Phillips (forthcoming), when only one month is missing, we replace it with the previous month. When two months are missing, we replace the first month with the preceding month and the second month with the following month.

In the early 1990s, first about 18 months of data are missing for DNSR and then a few months later about another six months of data are missing. As Table 1 indicates, we were able to find a second source for the missing data, but where they overlapped we found that the two sources did not always agree. We tried using the USDA data to fill in the missing data from World Grain Statistics, but the replacement produced unusual error terms that required long lags in tests for unit roots and cointegration. Those long lags reduced the significance of the tests. As an alternative, where we have USDA prices for DNSR, we report separate results using those prices for Rotterdam with freight rates and Gulf prices from World Grain Statistics. This experience with the data for Rotterdam suggests that there may be another potential pitfall in testing for price convergence, mixing different sources for the same series.

We have extended the data for DNS between Rotterdam and Gulf ports both backward and forward.<sup>12</sup> We extend Michael, Nobay and Peel's data back from 1975:09 to 1974:01. We do not go back farther because we want to allow time for the effects of switching to flexible exchange rates to work themselves out. Even though wheat prices in Rotterdam are apparently quoted in dollars, exchange rates can affect those prices. Other things equal, the lower the French franc or German mark price of the dollar, the greater the demand for U.S. wheat in Rotterdam. Using data from the USDA and World Wheat Statistics or World Grain Statistics, we are able to

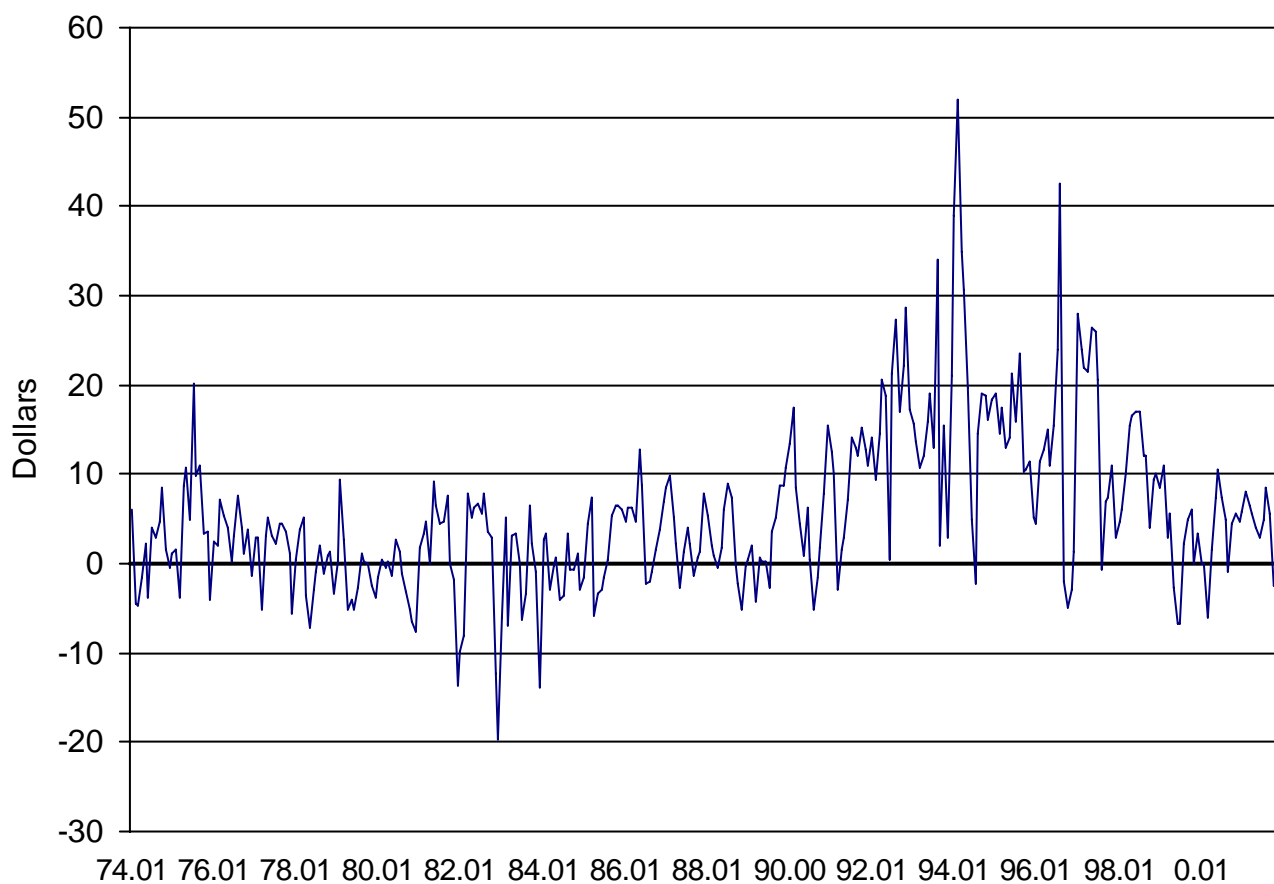
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<sup>11</sup> The large positive spike in late 1996 appears to be the result of a large price shift in one market that was not immediately reflected in the other market. The large negative spike in early 1991 may be a typo. Correcting for that possible typo has only a small effect on the estimated half life.

<sup>12</sup> DNS is the only variety of wheat for which there is a long continuous series between Rotterdam and the United States.

extend the Rotterdam data forward from 1981:10 to 2001:12. The net result is that we extend the prices for DNSR and DNSG used earlier by Michael, Nobay and Peel (1994), Pippenger and Phillips (forthcoming) and others from about 7 years to about 28 years. Using Dark Northern Spring wheat rather than yellow corn, Figure 2 shows the same price differential as Figure 1.

Figure 2  
Price Differential Including Freight Rate for DNS



## V. TESTS FOR UNIT ROOTS

In this section we report the results of unit root tests between Rotterdam and Gulf ports. Pippenger and Phillips (forthcoming) report the results for unit root tests for all the Japanese data. To save space, we do not repeat those results here. All tests are applied to the logarithm of the price or freight rate.

Table 2 shows the results of unit root tests for yellow corn and for freight rates between Gulf ports and Rotterdam.<sup>13</sup> In Table 2 and elsewhere we use both the augmented Dicky-Fuller (ADF) and Phillips-Perron (PP) tests for unit roots. We include the PP test because it appears to be more robust to the choice of lag length. In tests for unit roots and later cointegration, we use the Akaike Information Criterion for 1 to 12 lags from the ADF test to choose the number of lags for that and the other tests.<sup>14</sup> Again using 12 lags, we also report the results for various tests for structure in the error terms from the ADF tests. In each case, to save space, we report only the result with the lowest probability. If that test result is not significant, then the result at all other lags will not be significant either. For completeness, we also include the Jarque-Bera test for normality of the residuals.

Table 2  
Unit Root Tests for YC and Freight Rates†

Series	YCG 87:1-98:6	YCGF 87:1-98:6	YCR 87:1-98:6	Freight Rates 74:01-01:12
ADF (Lag) [Probability]	-3.318 (1) [0.016]	-3.536 (7) [0.008]	-2.991 (1) [0.038]	-3.103 (12) [0.027]
PP (Lag) [Probability]	-2.705 (1) [0.076]	-2.839 (7) [0.056]	-2.731 (1) [0.071]	-3.933 (12) [0.002]
Q (Lag) [Probability]	2.799 (3) [0.424]	0.130 (1) [0.718]	6.037 (5) [0.303]	0.073 (1) [0.787]
LM (Lag) [Probability]	4.324 (3) [0.229]	1.782 (1) [0.182]	2.382 (2) [0.304]	26.798 (12) [0.008]
Arch (Lag) [Probability]	0.229 (1) [0.632]	0.372 (1) [0.542]	1.028 (2) [0.598]	3.061 (4) [0.548]
Jarque-Bera [Probability]	182.526 [0.000]	91.802 [0.000]	105.626 [0.000]	6.503 [0.039]
Squared Residuals				
Q (Lag) [Probability]	0.236 (1) [0.627]	0.383 (1) [0.536]	1.115 (2) [0.573]	3.172 (4) [0.529]

† Here and in all other tables, LM is the Breusch-Godfrey  $obs \cdot R^2$  serial correlation LM test.

<sup>13</sup> All of our tests use EViews4.

<sup>14</sup> EViews does not report an Akaike statistic for the Johansen test for cointegration and that statistic does not change with the number of lags in the PP test for a unit root.



ADF tests for corn in Table 2 reject the null of a unit root at the 5 percent level. Most PP tests fail to reject at the 5 percent level, but reject at the 10 percent level.<sup>15</sup> For freight rates, both tests reject at the 5 percent level.

Table 3 shows the results of unit root tests for DNSR and DNSG for the three subintervals used later: 1974:01 to 1990:12, 1989:06 to 1994:10 and 1994:07 to 2001:12. We have broken the data up into these subperiods to avoid mixing the USDA prices for Rotterdam with the Rotterdam prices from *World Wheat or Grain Statistics*. None of the tests reject the null of a unit root at 5 percent or better.

Table 3  
Unit Root Tests for DNS: Subperiods †

Series	DNSG 74:1-90:12	DNSR 74:1-90:12	DNSG 89:6-94:10	DNSR 89:6-94:10	DNSG 94:7-01:12	DNSR 94:7-01:12
ADF (Lag) [Probability]	-2.138 (6) [0.230]	-2.406 (5) [0.142]	-1.495 (0) [0.530]	-1.785 (1) [0.385]	-1.690 (1) [0.433]	-0.665 (5) [0.849]
PP (Lag) [Probability]	-2.493 (6) [0.119]	-2.797 (5) [0.060]	-1.617 (1) [0.468]	-1.417 (1) [0.568]	-1.420 (1) [0.569]	-1.402 (5) [0.578]
Q (Lag) [Probability]	7.592 (12) [0.816]	0.049 (1) [0.824]	4.591 (2) [0.101]	7.670 (12) [0.810]	12.461 (12) [0.409]	0.024 (1) [0.876]
LM (Lag) [Probability]	11.064 (7) [0.136]	13.839 (11) [0.242]	5.040 (2) [0.080]	11.073 (12) [0.523]	3.858 (2) [0.145]	0.536 (1) [0.464]
Arch (Lag) [Probability]	18.439 (12) [0.103]	0.413 (1) [0.520]	11.657 (2) [0.003]	4.843 (2) [0.089]	4.515 (2) [0.105]	13.594 (5) [0.018]
Jarque-Bera\ [Probability]	20.039 [0.000]	31.966 [0.000]	1.811 [0.404]	5.171 [0.075]	8.644 [0.013]	159.169 [0.000]
Squared Residuals						
Q (Lag) [Probability]	21.392 (12) [0.045]	0.421 (1) [0.516]	12.745 (2) [0.002]	5.675 (2) [0.059]	4.786 (2) [0.091]	15.773 (5) [0.008]

† Here and in all other tables, LM is the Breusch-Godfrey obs\*R<sup>2</sup> serial correlation LM test.

Two series in Table 3 show clear evidence of conditional heteroscedasticity. Table 4 shows the results of re-estimating those tests using GARCH. The LM test is not shown in Table 4 because that test is not available in EViews for the ARCH procedure. The PP test does not appear in Table 4 because ARCH is not an option for the PP test in EViews. Correcting for conditional heteroscedasticity does not change the results.

<sup>15</sup> Corn prices were remarkably stable from 1980 to 2001. For a discussion of world grain markets see McGarry and Schmitz (1992) especially pp. 421-426. For an econometric analysis of the relation between grain prices, see Yang and Leatham (1998).

Table 4  
Unit Root Tests Using GARCH

Series	DNSG 1989:06-1994:10	DNSR 1994:07-2001:12
ADF (Lag)	-1.490 (0)	-2.842 (5)
[Significance]	[NS]	[NS]
GARCH(p,q)	(2,2)	(1,1)
Q (Lag)	0.202 (1)	0.465 (1)
[Probability]	[0.099]	[0.495]
Arch (Lag)	15.222 (12)	5.636 (5)
[Probability]	[0.229]	[0.343]
Jarque-Bera	0.501	4.454
[Probability]	[0.778]	[0.108]
Squared Residuals		
Q (Lag)	18.271 (12)	5.766 (5)
[Probability]	[0.108]	[0.330]

NS indicates not significant; \* significant at 5 percent; \*\* significant at 1 percent.

The important issue for our estimates of half lives is the relationship between the export price plus the freight rate, *e.g.*, YCGF, and the import price *e.g.*, YCR. If neither series contains a unit root, estimating a half life for the differential is legitimate. If both series have a unit root and they are cointegrated, then estimating a half life is legitimate. But if one series has a unit root and the other does not, then they cannot be cointegrated and estimating a half life would not be legitimate. As the next section shows, either both series are stationary or they are cointegrated.

## VI. TESTS FOR COINTEGRATION

This section reports the results of our tests for cointegration. First we place no restrictions on the cointegrating vector. In that case, we use three tests for cointegration: ADF, PP and the Johansen trace statistic. The ADF and PP tests are based on the residuals from regressing the logarithm of the import price on a constant and the logarithm of the export price plus the freight rate as implied by equation 3. Later we impose the restriction that the cointegrating vector is unity and apply the ADF and PP tests to the price differential on the left-hand side of equation 3.

Pippenger and Phillips (forthcoming) report unrestricted cointegration tests between the United States and Japan. To save space, we do not repeat those results here. All those tests reject the null of no cointegration.

Table 5 reports our results for the unrestricted tests between DNSR and DNSGF and between YCR and YCGF. All tests reject the null of no cointegration at well beyond the 1 percent level.

Table 5  
Tests for Cointegration between Rotterdam and Gulf Ports: No Restrictions

Series	DNSRGF 74:01-90:12	DNSRGF 89:06-94:10	DNSRGF 94:07-01:12	YCRGF 87:01-98:06
ADF (Lag) [Significance]	-4.779 (9) [**]	-4.588 (1) [**]	-4.143 (1) [**]	-4.216 (5) [**]
PP (Lag) [Significance]	-7.889 (9) [**]	-4.002 (1) [**]	-4.741 (1) [**]	-7.799 (5) [**]
Johansen (Lag) [Significance]	27.246 (9) [**]	27.131 (1) [**]	25.264 (1) [**]	35.071 (5) [**]
Q (Lag) [Probability]	0.013 (1) [0.908]	8.792 (5) [0.118]	0.179 (1) [0.672]	0.174 (1) [0.677]
LM (Lag) [Probability]	2.572 (3) [0.462]	1.582 (1) [0.208]	2.330 (1) [0.127]	5.581 (2) [0.061]
Arch (Lag) [Probability]	2.418 (4) [0.659]	17.685 (11) [0.089]	8.772 (3) [0.033]	0.582 (1) [0.445]
Jarque-Bera [Probability]	27.730 [0.000]	2.842 [0.319]	8.728 [0.013]	133.390 [0.000]
Squared Residuals				
Q (Lag) [Probability]	2.391 (4) [0.664]	12.923 (11) [0.298]	14.364 (5) [0.013]	0.600 (1) [0.439]

\* Significant at 5%. \*\* Significant at 1%.

Critical values for residual based tests, (3.17) and (3.77), from Engle and Granger (1987).

Our later estimates of the half lives for price differentials assume that arbitrage is effective in the underlying forward markets and that, as a result, the cointegrating vector for our spot prices is approximately unity. Using the same series between Gulf ports and Rotterdam as in Table 5, Table 6 shows the results of our tests for cointegration when we impose the restriction that the cointegrating vector is unity. In Table 6 we test for a unit root in left-hand side of equation 3. All but one test in Table 6 is significant at well beyond the 1 percent level

and that test is significant at the 2 percent level.<sup>16</sup> Between Rotterdam and Gulf ports the left-hand side of equation 3 appears to be stationary.

Table 6  
Tests for Cointegration between Rotterdam and Gulf Ports: Cointegrating Vector Restricted to Unity

Series	DNSRGF 74:01-90:12	DNSRGF 89:06-94:10	DNSRGF 94:07-01:12	YCRGF 87:01-98:06
ADF (Lag) [Probability]	-7.326 (5) [0.000]	-4.741 (1) [0.000]	-3.887 (1) [0.003]	-3.316 (7) [0.016]
PP (Lag) [Probability]	-8.315 (5) [0.000]	-4.119 (1) [0.002]	-4.492 (1) [0.000]	-7.595 (7) [0.000]
Q (Lag) [Probability]	4.157 (8) [0.843]	9.898 (5) [0.079]	1.610 (3) [0.657]	7.434 (12) [0.828]
LM (Lag) [Probability]	12.212 (8) [0.142]	2.079 (1) [0.149]	2.437 (1) [0.115]	22.414 (11) [0.033]
Arch (Lag) [Probability]	2.379 (5) [0.795]	17.330 (11) [0.098]	14.398 (7) [0.044]	0.416 (1) [0.519]
Jarque-Bera [Probability]	18.496 [0.000]	2.232 [0.328]	10.718 [0.005]	127.272 [0.000]
Squared Residuals				
Q (Lag) [Probability]	0.244 (1) [0.622]	12.489 (11) [0.328]	14.281 (5) [0.014]	0.429 (1) [0.513]

Table 7  
Tests for Cointegration between Japan and the U.S.: Cointegrating Vector Restricted to Unity

Series	DNSJGF 1975:09-1981:10	DNSJPF 1975:09-1981:10	WWJPF 1975:09-1981:10	HWJGF 1976:01-1981:07
ADF (Lag) [Probability]	-3.915 (0) [0.003]	-4.629 (0) [0.000]	-4.827 (0) [0.000]	-2.871 (7) [0.055]
PP (Lag) [Probability]	-3.992 (1) [0.002]	-4.675 (1) [0.000]	-4.840 (1) [0.000]	-5.124 (7) [0.000]
Q (Lag) [Probability]	0.257 (1) [0.612]	0.117 (1) [0.732]	5.619 (6) [0.468]	0.160 (1) [0.689]
LM (Lag) [Probability]	2.388 (2) [0.303]	0.351 (1) [0.553]	1.143 (2) [0.565]	10.872 (7) [0.144]
Arch (Lag) [Probability]	15.248 (11) [0.171]	2.791 (1) [0.095]	4.691 (2) [0.096]	5.611 (2) [0.061]
Jarque-Bera [Probability]	6.850 [0.032]	5.967 [0.051]	1.513 [0.469]	1.499 [0.472]
Squared Residuals				
Q (Lag) [Probability]	19.261 (11) [0.057]	2.939 (1) [0.086]	0.669 (2) [0.068]	2.947 (1) [0.086]

<sup>16</sup> In Table 6 the interval 1994:07-2001:12 shows significant evidence of conditional heteroscedasticity. Eliminating that structure with ARCH(1,1) produces an ADF statistic of -8.516

Pippenger and Phillips (forthcoming) do not test price differentials between Japan and the United States for unit roots. Table 7 shows the results of applying unit root tests to those price differentials. One test rejects the null of a unit root at almost the 5 percent level. All other tests reject that null at well beyond the 1 percent level.

Given the results of these tests for cointegration, we are confident that it is legitimate to estimate half lives for all our varieties of grains.

## VII. HALF LIVES

### A. Earlier Estimates

At least three articles that test for price convergence also report half lives. Using retail prices between cities within the United States, Parsley and Wei (1996) report half lives for tradable goods of between four and five quarters. Using international prices for products like wheat and wool from *International Financial Statistics*, Vataja (2000) reports that on average about two thirds of the deviations are eliminated within one year.<sup>17</sup> Using retail prices for cars in European countries, Goldberg and Verboven (2005) report half lives of between 1.3 and 1.6 years.

Although far shorter than the half lives of three to five years reported for real exchange rates, these half lives are so long that, if they held for markets where arbitrage is possible, they would raise serious questions about arbitrage and the relevance of the law of one price. Because Parsley and Wei (1996), and Goldberg and Verboven (2005) use retail prices, arbitrage is not generally possible in their markets. For Parsley and Wei, in most cases arbitrage is probably absent even at the underlying wholesale level.<sup>18</sup> Although arbitrage is possible

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<sup>17</sup> In the closely related work on Borders, Engel and Rogers (1996) report that the border between the United States and Canada is “2,500 miles wide”. That is crossing the border increases the volatility of price differentials as much as adding that mileage to the distance between cities within the same country. Parsley and Wei (2001) estimate that the border between the United States and Japan is 43,000 trillion miles. Both articles suggest very long half lives for price deviations between countries. Both articles use retail prices

<sup>18</sup> That is not true for their perishables. Most perishables like fresh fish, lettuce, and fresh flowers have very active wholesale markets where arbitrage, or something close to arbitrage, is at work. In addition, for perishables inventories do not act as a buffer between wholesale and retail prices. We believe that it is not an accident that Parsley and Wei (1996) report half lives for perishables of only 3

for the individual varieties in the product groups like wheat and wool used by Vataja, arbitrage is not possible for the groups as a whole. In addition to these problems, their half lives suffer from some econometric problems.

Murray and Papell (2002) have criticized estimates of half lives in the closely related work on purchasing power parity on three grounds: (1) Estimates lack confidence intervals. (2) If the process is not a first order autoregressive process or AR(1) and the estimation involves lagged terms, calculating the half life using  $\ln(0.5)/\ln(\beta)$  from equation 4 below is not appropriate. (3) Least squares estimates of half lives are generally biased downward in small samples and the extent of the bias is usually greater the smaller the sample and the closer the root is to unity. Because of these problems, for real exchange rates Murray and Papell (2002) conclude that these “univariate methods provide virtually no information regarding the size of half lives.”

Neither Parsley and Wei (1996), Vataja (2000) or Goldberg and Verboven (2005) provide confidence intervals. They all use  $\ln(0.5)/\ln(\beta)$  to calculate half lives. Most of Vataja’s estimates appear to be AR(1). But most of Parsley and Wei’s estimates have long lags, some as long as 16 quarters. Most of Goldberg and Verboven’s estimates also are not AR(1). With about 140 observations, Vataja’s sample size is not small. With prices for 150 models of cars in five markets over 30 years, Goldberg and Verboven have a large data set. Parsley and Wei have quarterly data for only about 18 years, but they have that data for 48 cities. In most cases the roots are not as close to unity as for real exchange rates.

## B. Our Estimated Half Lives

Following Andrews (1993) we use the following autoregressive model to estimate our half lives:

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quarters while they report half lives for nonperishables of 6 quarters. For services, where there is no underlying wholesale market and no opportunity for arbitrage, they report half lives of 14 quarters.

$$X_t = \alpha + \beta X_t + \sum_{i=1}^N \gamma_i \Delta X_{t-i} \quad (4)$$

Where  $X_t$  equals  $\ln(p_t^R) - \ln(p_{t-1}^G + F_{t-2})$ . If the process is AR(1), then we can use  $\ln(0.5)/\ln(\beta)$  to calculate half lives. When  $X_t$  is AR(2) or larger we use the impulse response function to calculate the half lives.

Our approach here differs from our approach to estimating ADF equations. The major issue there is the choice of the number of lagged terms. For that we follow the widely used Akaike Information Criteria. Our major concern here is to find a parsimonious model with white noise errors.

Table 8 shows the results of estimating equation 4 with wheat prices between the U.S. and Japan. All errors in Table 8 are white noise and normally distributed. All estimates of  $\beta$  are far from unity. The largest estimate is 0.638. The smallest is 0.408

Table 8  
Estimated AR Model: US-Japan†

Series	DNSJGF 75:09-81:10	DNSJPF 75:09-81:10	HWJGF 76:01-81:07	WWJPF 75:09-81:10
$\alpha$ (Std. Error)	-0.007 (0.005)	0.000 (0.005)	0.178 (0.006)	0.002 (0.005)
$\beta$ (Std. Error)	0.638 (0.123)	0.481 (0.118)	0.408 (0.114)	0.472 (0.126)
$\gamma$ (Lag) (Std. Error)	0.114 (1) (0.123)	NA	NA	NA
GARCH	2,2	1,1	0,0	1,1
Q (Lag) [Probability]	19.793 (12) [0.071]	0.162 (1) [0.687]	8.127 (7) [0.322]	6.821 (6) [0.338]
LM (Lag) [Probability]	NA	NA	9.898 (7) [0.194]	NA
Arch (Lag) [Probability]	0.421 (1) [0.516]	0.254 (1) [0.614]	11.821 (9) [0.224]	0.842 (2) [0.656]
Jarque-Bera [Probability]	2.418 [0.298]	2.908 [0.234]	0.891 [0.641]	0.124 [0.940]
Squared Residuals				
Q (Lag) [Probability]	0.443 (1) [0.506]	0.268 (1) [0.605]	2.388 (3) [0.496]	0.891 (2) [0.641]

† Here and in all other tables, LM is the Breusch-Godfrey obs\*R<sup>2</sup> serial correlation LM test.

Table 9  
Estimated AR Model: US-Rotterdam†

Series	DNSRGF 74:01-90:12	DNSRGF 89:06-94:10	DNSRGF 94:07-01:12	DNSRGF 74:01-90:12 94:07-01:12	DNSRGF 90:01-94:06	YCRGF 87:01- 98:06
$\alpha$ (Std. Error)	0.000 (0.002)	0.043 (0.011)	0.015 (0.005)	0.005 (0.003)	0.039 (0.017)	0.028 (0.004)
$\beta$ (Std. Error)	0.496 (0.064)	0.482 (0.109)	0.553 (0.081)	0.573 (0.054)	0.504 (0.151)	0.331 (0.065)
$\gamma$ (Lag) (Std. Error)	NA	0.273 (1) (0.123)	NA	NA	NA	NA
September (Std. Error)	0.030 (0.012)	NA	NA	0.030 (0.009)	0.070 (0.042)	NA
GARCH (p,q)	(1,1)	(0,0)	(1,1)	(1,1)		(1,1)
Q (Lag) [Probability]	17.559 (12) [0.130]	9.898 (5) [0.078]	2.903 (3) [0.407]	6.293 (5) [0.279]		9.100 (5) [0.105]
LM (Lag) [Probability]	NA	2.079 (1) [0.149]	NA	NA		NA
Arch (Lag) [Probability]	2.563 (2) [0.277]	17.330 (11) [0.098]	0.941 (1) [0.332]	2.889 (2) [0.236]		1.019 (1) [0.313]
Jarque-Bera [Probability]	3.742 [0.154]	2.232 [0.327]	2.104 [0.349]	49.734 [0.000]		2.080 [0.353]
Squared Residuals						
Q (Lag) [Probability]	2.175 (2) [0.337]	12.489 (11) [0.328]	0.951 (1) [0.330]	3.038 (2) [0.219]		1.048 (1) [0.306]

† Here and in all other tables, LM is the Breusch-Godfrey obs\*R<sup>2</sup> serial correlation LM test.

Table 9 shows the results of estimating equation 4 using prices between Rotterdam and Gulf ports. Although grain markets are highly seasonal, we were able to ignore seasonality in our estimates for the United States and Japan. To obtain parsimonious estimates with white noise errors for Gulf ports and Rotterdam, in some cases we need to recognize the seasonality in grain markets. Three intervals in Table 9 include dummies for September.

In addition to the intervals used earlier for DNS between Gulf ports and Rotterdam, Table 9 also reports results for the entire period from 1974:01 to 2001:12. To obtain estimates for the entire period we have to use two different sources for prices in Rotterdam: USDA and *World Wheat Statistics* or *World Grain Statistics*. To



use as much of the data from *World Wheat Statistics* or *World Grain Statistics* as possible with a minimum of switching from one source to another, for the entire period we use the USDA data only from 1991:01 to 1994:06. As a result, for the entire period we estimate the following equation:

$$X_t = \alpha_1 + \beta(\alpha_1 * X_{t-1}) + \alpha_1 \sum_{i=1}^N \gamma_i \Delta X_{t-i} + \kappa(\alpha_1 * SEP) + \alpha_2 + \beta(\alpha_2 * X_{t-1}) + \alpha_2 \sum_{i=1}^N \gamma_i \Delta X_{t-i} + \kappa(\alpha_2 * SEP) \quad (5)$$

where SEP is the dummy for September and  $\alpha_1$  is 1.0 from 1974:01 to 1990:12 and 1994:07 to 2001:12 and 0.0 otherwise, while  $\alpha_2$  is 1.0 from 1991:01 to 1994:06 and 0.0 otherwise.

All the errors in Table 9 are white noise. Except for the entire period that mixes data from different sources, all errors are normally distributed. Again all estimates of  $\beta$  are far from unity. The largest estimate is 0.573 and the smallest is 0.331.<sup>19</sup>

Table 10 reports the half lives implied by the estimates in Tables 8 and 9. Half lives are measured in weeks<sup>20</sup>. Table 10 reports point estimates using  $\ln(0.5)/\ln(\beta)$  and 90 percent confidence intervals for those point estimates. Confidence intervals are calculated using  $\ln(0.5)/\ln(\beta \pm 1.66 * SE)$  where SE is the standard error for  $\beta$ . In two cases estimates are AR(2). For those estimates, Table 10 reports the half life using the impulse response function. When the estimates are AR(2), no attempt is made to calculate a confidence interval.

Point estimates between the United States and Japan using  $\ln(0.5)/\ln(\beta)$  vary between 3.4 and 4.3 weeks. Those estimates are reasonably precise. The maximum range in all the 90 percent confidence intervals is about 5 weeks.

<sup>19</sup> The smallest estimate is for YCRGF, which may contain a typo. Replacing the suspect Rotterdam price of 114.8 with the more likely price of 124.8 produces a  $\beta$  of 0.348, which is still the smallest.

<sup>20</sup> The number of weeks is obtained through interpolation.

Point estimates for half lives between Gulf ports and Rotterdam are similar to those between the United States and Japan. Point estimates using  $\text{Ln}(0.5)/\text{Ln}(\beta)$  vary between 2.7 and 5.4 weeks. Again these estimates are fairly precise. The maximum range in all the 90 percent confidence intervals is about 8 weeks.

Table 10  
Estimated Half Lives Measured in Weeks Using Least Squares

Series (Obs.) Interval	Point Estimates Using $\text{Ln}(0.5)/\text{Ln}(\beta)$	95 Percent Confidence Interval	Point Estimates from Impulse Response Function
DNSJGF (79) 75:09-81:10			8.3
DNSJPF (79) 75:09-81:10	4.1	2.4 to 7.7	
HWJGF (79) 76:01-81:07	3.4	2.0 to 5.8	
WWJPF (67) 75:09-81:10	4.0	2.2 to 7.8	
DNSRGF (204) 74:01-90:12	4.3	3.2 to 5.9	
DNSRGF (64) 89:06-94:10			7.5
DNSRGF (90) 94:07-01:12	5.1	3.4 to 8.0	
DNSRGF (294) 74:01-90:12 94:07-01:12	5.4	4.1 to 7.3	
DNSRGF (42) 91:01-94:06	4.4	2.2 to 10.7	
YCRGF (138) 87:01-98:06	2.7	2.0 to 3.7	

Two estimates in Table 10 are AR(2). Accounting for the effects of that single lagged term using the impulse response function produces a half life of 7.5 and 8.3 weeks for DNSJGF and DNSRGF respectively.

Because the half lives in Table 10 use least squares, they are biased downward. For AR(1) processes this bias is greater the closer the root is to unity and the smaller the sample. For AR(1) process Andrews (1993) provides tables that allow one to convert biased estimates from least squares to exactly median-unbiased estimates. He also provides confidence intervals for those median-unbiased estimates.

For AR(p) process where  $p > 1$ , the bias is more complex. Andrews and Chen (1994) shows how to obtain approximately median-unbiased estimates for those processes. Unfortunately such estimates require complicated simulation that is well beyond the scope of this paper. As a result, Table 11 only reports the half lives and confidence intervals based on the exactly median-unbiased estimates.

Table 11  
Weekly Half Lives and Confidence Intervals Based on Exactly Median-Unbiased Estimates of  $\beta$

Series (Obs.)	Point Estimates	90 Percent Confidence Interval
DNSJPF (79) 75:09-81:10	4.5	3.6 to 8.4
HWJGF (79) 76:01-81:07	3.6	2.0 to 6.1
WWJPF (67) 75:09-81:10	5.9	2.6 to 8.1
DNSRGF (204) 74:01-90:12	4.4	3.3 to 6.1
DNSRGF (90) 94:07-01:12	5.5	3.6 to 9.5
DNSRGF (294) 74:01-90:12 94:07-01:12	5.7	4.2 to 8.1
DNSRGF (42) 91:01-94:06	5.2	2.6 to 15.1
YCRGF (138) 87:01-98:06	2.9	1.9 to 4.2

For our AR(1) estimates, the bias due to least squares is small. For the point estimates, the largest difference between the least squares and the exactly median-unbiased estimates is only 1.9 weeks.

Using spot prices from markets where arbitrage is possible produces much smaller estimates of half lives for price differentials in commodity markets than reported earlier. These shorter estimates are also more reliable because we provide confidence intervals and correct for the bias in least squares estimates.

Since it is impossible to test the law of one price directly with spot prices, our estimates of half lives should be viewed as upper limits. We conjecture that, with prices from forward contracts, half lives for the LOP would be much shorter than the 3 to 8 weeks that we find using spot prices from international grain markets.

## VIII. SUMMARY AND CONCLUSION

Dictionaries and Encyclopedias make it clear that arbitrage is the mechanism that produces the LOP. Pippenger and Phillips (forthcoming) show how ignoring the practical implications of arbitrage can seriously weaken purported tests of the LOP based on cointegration. That article concludes that there is no reliable evidence regarding cointegration that would lead them to reject the law of one price as that law is described in dictionaries and encyclopedias for economics.

But cointegration is only a minimal tests of the LOP. Here we use a stronger test. We estimate half lives for deviations between spot prices for identical varieties of grain in highly organized grain markets between the United States and Rotterdam or Japan. A crucial characteristic of our spot prices is that arbitrage is possible in the underlying markets.

Earlier work, which has not used prices from markets where arbitrage is possible, has reported half lives for price differentials of several quarters. If valid for the LOP, such long half lives would raise serious questions about the effectiveness of arbitrage and the usefulness of the LOP in empirical and theoretical models. We find half lives for differentials in spot prices that range between about three and six weeks. Both earlier and our estimates of half lives use least squares, which are biased. Where we can convert our least squares estimates to exactly median-unbiased estimates we do so. Those median-unbiased estimates range between four and eight weeks. Unlike previous work we also report confidence intervals for most of our half lives. The *largest* upper limit for all of our 90 percent confidence intervals is only 15 weeks. That upper limit is far below the point estimates of several quarters reported previously in the literature. We find much shorter half lives because we avoid the pitfalls pointed out in Pippenger and Phillips (forthcoming). In particular, we use spot prices from markets where arbitrage is possible.

If further research supports our short half lives, it will have important implications for Borders and the half lives for real exchange rates. When combined with the earlier literature on the LOP, our results suggest that

Borders appear to be wide because the Borders literature mixes sticky retail prices, where arbitrage is not possible, with exchange rates, which are highly volatile. Those results also suggest that real exchange rates have long half lives because the prices in the price indexes either are not for identical products or are from markets where arbitrage is not possible.

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