Working Paper No. 243

WELFARE ANALYSIS BASED ON SYSTEMS OF PARTIAL DEMAND FUNCTIONS

by

W. Michael Hanemann

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics
November, 1982

WELFARE ANALYSIS BASED ON SYSTEMS OF PARTIAL DEMAND FUNCTIONS

W. Michael Hanemann

I. INTRODUCTION

In many areas of applied economics, the analyst sometimes finds himself in the situation of estimating a set of demand equations and calculating welfare measures with data on only a subset of the commodities purchased by a consumer. A classic example is the travel cost method of analyzing recreation demand and inferring the value of recreation sites originated by Hotelling [1949] and Clawson and Knetsch [1966]. In a typical application one has data on the prices, perhaps quality attributes, and rates of visitation of a set of recreation sites serving some population, and one estimates demand functions showing the visitation of each site as a function of the prices and quality attributes of all the sites as well as socioeconomic characteristics of the recreationists. Although recreation expenditures generally account for a small fraction of these consumers' total expenditures, the prices and attributes of other, nonrecreation goods are usually not included in these demand functions. This is because the sources of the recreation data--household or on-site surveys--typically provide no information about nonrecreation consumption activities. A similar problem arises in various other contexts; for example, one has detailed data on the prices and consumption of various foodstuffs but not on nonfood commodities, and one wishes to estimate demand functions for the different food products.

In all of these cases the question arises: using the data available for a subset of consumption activities, is it possible to formulate a demand system which is consistent with a theoretical model of utility maximization? The question is especially pertinent if one wishes to employ the fitted demand functions for this subset of goods to assess the affects of a change in their prices or quality on the consumer's welfare. This is because the standard tools of welfare analysis--the compensating and equivalent variations--are fully justified only if the demand functions are generated by a utility maximization model. How, then, is one to proceed?

Writing on the subject of recreation demand, Cicchetti, Fisher, and Smith [1976, fn. 12] appear to conclude that it is not appropriate to seek a system of demand equations compatible with utility maximization if one has data on only a subset of consumption activities. This is offered as an explanation of their decision to employ an *ad hoc* system of linear demand functions for recreation sites. However, this conclusion is unduly pessimistic. As Pollak [1971] has shown, under the assumption of separability in the consumer's preferences, there *is* a utility-theoretic justification for the formulation of a demand system for a subset of commodities in which the prices of all other commodities are omitted. In these demand functions, which are sometimes referred to as "partial demand" functions, the demand for each good in the subset is expressed as a function of the prices of all the goods in the subset and the consumer's aggregate expenditure on the subset.

Suppose that one estimates a system of partial demand functions and proceeds in the conventional manner to calculate welfare measures such as the compensating and equivalent variations. How do these welfare measures relate to the true welfare measures that would be obtained if one had estimated the *full* demand functions containing *all* commodity prices, both those for the commodities in the subset of interest and those for the other commodities? In this paper I will provide an answer. I will show that the welfare measures derived from the partial demand functions are, in general, different from the the welfare measures based on the full demand functions. An exception is the special case where some of the commodities in the subset have zero income elasticities of demand, and the price and quality changes are confined to these commodities. In that case, the two sets of welfare measures coincide. Otherwise, there is the following link between them: a compensating variation calculated from the partial demand system is a lower bound on the true compensating variation, while an equivalent variation calculated from the partial demand system is an upper bound on the true equivalent variation. These results are presented in section 3. In section 2, I set the stage by reviewing the basic theory of partial demand systems.

II. MODELING THE DEMAND FOR A SUBSET OF COMMODITIES

The theoretical set-up is as follows. An individual consumer has a strictly increasing and quasi-concave utility function defined over the commodities $x_1, \ldots, x_n$, and $z_1, \ldots, z_m$, where the x's are the particular subset of goods on which the analyst has price and consumption data and the z's are all other goods. Let $x = (x_1, \ldots, x_n)$ and $z = (z_1, \ldots, x_m)$. In addition, the consumer's utility may depend on some quality attributes of the x's, denoted by the vector b, which he takes as exogenous. The utility function will be written compactly as u(x, b, z). The consumer chooses (x, z) so as to

$$\text{maximize } u(x, b, z), \qquad [1]$$

subject to

$$\Sigma\, p_j\, x_j + \Sigma\, q_i\, z_i = y,$$

where y is his total income and $p = (p_1, \ldots, p_n)$ and $q = (q_1, \ldots, q_m)$ are vectors of commodity prices. Assuming an interior solution, [1] generates a set of ordinary demand function for the x's and z's of the form

$$x_j = h_j(p, b, q, y) \qquad j = 1, \ldots, n \qquad [2]$$

$$z_i = f_i(p, b, q, y) \qquad i = 1, \ldots, m. \qquad [3]$$

Suppose, however, that the analyst has data only on x, y, p, and b and wishes to estimate demand functions for the x's. Since he has no information on q, he cannot hope to estimate the demand functions in [2].

There are two ways to proceed. One approach, based on Hick's composite commodity theorem, is to assume that the prices $q_1, \ldots, q_m$ always move in proportion and replace the vector z by a single composite commodity z_0 with price q_0.[1] Thus, the consumer's utility function may be written as $\bar{u}(x, b, z_0)$, which is a function of n + 1 rather than n + m consumption levels. The utility maximization problem is now to choose (x, z_0) so as to

$$\text{maximize } \bar{u}(x, b, z_0), \qquad [4]$$

subject to

$$\Sigma\, p_j\, x_j + q_0\, z_0 = y,$$

which yields the ordinary demand functions,

$$x_j = \bar{h}_j(p, b, q_0, y) \qquad j = 1, \ldots, n \qquad [5]$$

$$z_0 = \bar{h}_0(p, b, q_0, y) = \frac{(y - \Sigma\, p_j\, \bar{h}_j)}{q_0}\,.$$

In order to estimate the demand functions [5], one needs to know the price of q_0. If there is reason to believe that the underlying price vector q does not vary across the consumers in the sample--e.g., if there is a cross-section of data for a single time period--then it would be appropriate to adopt the normalization $q_0 \equiv 1$. Otherwise, one could employ some general price index, such as the Consumer Price Index, to measure q_0; this would be justified if the x's account for only a small portion of consumer expenditures so that movements in the Index mainly reflect variations in q.

I am concerned here with the alternative approach which is to assume a weakly separable utility function of the form

$$u(x, b, z) = \phi[u^*(x, b), z], \qquad [6]$$

where u^* is a scalar-valued function strictly increasing and quasi-concave in x, and ϕ is a strictly increasing function of m + 1 arguments and quasi-concave in z. Thus, the marginal rate of substitution between any pair of x's or between any elements of x and b is independent of z. Let y_x denote the total expenditure on the x's. For any given level of y_x, consider the following utility maximization problem: choose x so as to

$$\text{maximize } u^*(x, b), \qquad [7]$$

subject to

$$\Sigma\, p_j\, x_j = y_x.$$

The solution is a set of ordinary demand functions,

$$x_j = h_j^*(p, b, y_x) \qquad j = 1, \ldots, n, \qquad [8]$$

which are known as partial demand functions. These exhibit the optimal allocation of the total expenditure y_x among the individual x's as a function of their prices and qualities. It should be emphasized that, for given y_x, they possess all the standard properties of a demand system, including homogeneity of degree zero in (p, y_x), the adding-up property, and the Slutsky symmetry and negativity properties.

If the partial demand functions are substituted into the utility function in [7], one obtains the partial indirect utility function, $v^*(p, b, y) \equiv u^*[h^*(p, b, y), b]$. Now consider the utility maximization problem: choose y_x and z so as to

$$\text{maximize } \phi[v^*(p, b, y), z], \qquad [9]$$

subject to

$$y_x + \Sigma\, q_i\, z_i = y.$$

The solution is a set of demand functions for y_x and z of the form

$$y_x = H(p, b, q, y) \qquad [10]$$

$$z_i = F_i(p, b, q, y) \qquad i = 1, \ldots, m. \qquad [11]$$

Pollak [1971] shows that, under the separability assumption in [6], the demand functions for the z's in equation [11] coincide with those in [3], i.e., $F_i(p, b, q, y) \equiv f_i(p, b, q, y)$, and the demand functions for the x's in [8] and [10] are related to those in [2] by the identity

$$x_j = h_j(p, b, q, y) \equiv h_j^*[p, b, H(p, b, q, y)] \qquad j = 1, \ldots, n. \qquad [12]$$

Assuming that the analyst has data on x, p, b, and y and, therefore, on $y_x = \Sigma\, p_j\, x_j$, it is possible to estimate the partial demand functions [8]. Moreover, if one chooses functional forms for $h_j^*(\cdot)$ which possess the properties mentioned above, it is possible to derive from the fitted demand equations an estimate of the underlying utility function $u^*(\cdot)$. However, without data on q, it is not possible to estimate the demand functions in equation [10] or [11] and recover the underlying utility function $\phi(\cdot)$. One could always specify an arbitrary equation relating y_x to p, b, y and perhaps some general price index and estimate this as a crude approximation to [10]. This could be combined with the partial demand functions along the lines of equation [12] in order to predict the overall demand for the x's. But I assume that one cannot recover the utility function $\phi(\cdot)$ with sufficient accuracy to construct the welfare measures associated with the full utility model [6]. The question is: what is the relationship between these welfare measures and those which are computed from the partial demand functions based on $u^*(\cdot)$? This will be answered in the next section.

III. WELFARE MEASURES

If the demand functions [2] and [3] are substituted into the original utility function in [1], one obtains the indirect utility function v(p, b, q, y). Under the separability assumption [6], this takes the form

$$v(p, b, q, y) = \phi\{v^*[p, b, y_x(p, b, q, y)], f(p, b, q, y)\}. \quad [13]$$

As is well known, the indirect utility function can be employed to define monetary measures of the effect on the consumer's welfare of a change in the set of prices and quality characteristics which confronts him. Specifically, suppose that the prices and qualities of the x's change from (p^o, b^o) to (p', b') while the prices of the z's and the consumer's overall income remain constant at (q, y). Thus, the consumer's welfare changes from $u^o \equiv v(p^o, b^o, q, y)$ to $u' \equiv v(p', b', q, y)$. The compensating and equivalent variations for this change, CV and EV, are implicitly defined by

$$v(p', b', q, y - CV) = v(p^o, b^o, q, y) \quad [14a]$$

$$v(p', b', q, y) = v(p^o, b^o, q, y + EV). \quad [14b]$$

Observe that, since $v(\cdot)$ is increasing in y,

$$\text{sign } (CV) = \text{sign } (EV) = \text{sign } (u' - u^o). \quad [15]$$

Therefore, the signs of these quantities provide an indication of the direction in which the consumer's welfare changes; their magnitudes provide an indication of the size of the change in the consumer's welfare.

However, I am assuming that the data are insufficient to identify $v(\cdot)$ and permit the calculation of CV and EV. But, since $v^*(\cdot)$ is identified, one can use it to calculate some alternative welfare measures. Suppose that the analyst either knows or can estimate y_x^o and y_x', which are the expenditure allocations corresponding to (p^o, b^o, q, y) and (p', b', q, y)-- i.e., $y_x^o = y_x^o(p^o, b^o, q, y)$ and similarly for y_x'. One possible set of welfare measures based on the observed partial indirect utility function is CV* and EV* defined by

$$v^*(p', b', y_x^o - CV^*) = v^*(p^o, b^o, y_x^o) \quad [16a]$$

$$v^*(p', b', y_x^o) = v^*(p^o, b^o, y_x^o + EV^*). \quad [16b]$$

Another set is CV^+ and EV^+ defined by

$$v^*(p', b', y_x' - CV^+) = v^*(p^o, b^o, y_x') \quad [17a]$$

$$v^*(p', b', y_x') = v^*(p^o, b^o, y_x' + EV^+). \quad [17b]$$

Whereas CV* and EV* have the sign, as do CV^+ and EV*, it is <u>not</u> necessarily true that CV* and CV^+ have the same sign. Moreover, CV* and CV^+ are, in general, different from CV; for example, compare equation [16a] with the formula for CV, equation [14a], which, by virtue of [13], can be written as

$$\phi\{v^*[p', b', y_x(p', b', q, y - CV)], f(p', b', q, y - CV)\}$$
$$= \phi[v^*(p^o, b^o, y_x^o), z^o],$$

where $z_i^o = f_i(p^o, b^o, q, y)$, $i = 1, \ldots, M$. Similarly, EV* and EV^+ are, in general, different from EV. However, the following result provides a link between CV* and CV and a link between EV^+ and EV:

THEOREM. For the change from (p^o, b^o, q, y) to (p', b', q, y),

$$CV^* \leq CV \qquad [18a]$$

$$EV \leq EV^+. \qquad [18b]$$

Since the proof is rather lengthy, it is placed in the Appendix where I also offer an intuitive, diagrammatic explanation of these inequalities.

An immediate corollary of [18a] and [18b] is that, if $CV^* > 0$, then $CV > 0$ and, hence, one can safely conclude that the consumer's welfare has been improved by the change. Similarly, if $EV^+ < 0$, then $EV < 0$. In these two cases, therefore, the sign of the true welfare measures can be deduced from that of the partial welfare measures.

A second corollary is based on the following result, which is proved in Hanemann [1980]. Suppose that all of the x's whose prices change are normal goods, and some or all of these goods are weakly complementary with respect to the elements of b that change.[2] Suppose, also, that all prices and quality characteristics which change move in the same direction from the point of view of the consumer's welfare--i.e., either all price changes are increases and all quality changes are decreases, or all price changes are decreases and all quality changes are increases. Then,

$$|CV| \leq |EV|. \qquad [19]$$

Alternatively, if all of the x's whose prices change are inferior goods but the other conditions are met, the inequality in [19] is reversed. In order to be able to apply this result here, one needs to estimate the sign of $\partial h_i/\partial y = (\partial h_i^*/\partial y_x)(\partial y_x/\partial y)$. The first term, $\partial h_i^*/\partial y_x$, is obtained directly from the fitted partial demand functions; the second term would have to be inferred from the auxiliary regression of y_x on y which approximates equation [10]. Suppose it is determined that the goods whose prices change are normal and the other conditions mentioned above are met.[3] If the change represents an improvement in welfare, combining [18] with [19] yields the following chain of inequalities:

$$CV^* \leq CV \leq EV \leq EV^+. \qquad [20]$$

As a final corollary, observe from equation [12] that, if the partial demand functions for some subset of the x's exhibit zero income effects, the same must also be true of the full ordinary demand functions--i.e., if $h_i^*(p, b, y_x) = \psi_i(p, b)$ for some function $\psi_i(\cdot)$ which is homogeneous of degree zero in p, then $h_i(p, b, q, y) = \psi_i(p, b)$.[4] In this case, therefore, the observed partial ordinary demand functions coincide not only with the partial compensated demand functions but also with the full compensated demand functions. Accordingly, as long as the price changes are confined to the goods with zero income effects and the quality changes occur in these elements of b which are weakly complementary with them, all of the welfare measures coincide:[5]

$$CV^* = EV^* = CV^+ = EV^+ = CV = EV. \qquad [21]$$

The absence of income effects in the partial ordinary demand functions ensures the equality of CV^*, EV^*, CV^+, and EV^+. Similarly, the absence of income effects in the full ordinary demand functions ensures the equality of CV and EV. The equality of all six welfare measures follows from the coincidence of the partial and full compensated demand functions since the welfare measures may be expressed as areas under these demand functions.

IV. CONCLUSIONS

Applied economics, like politics, is the art of the possible. One is frequently caught in a conflict between the limitations of the available data, on one hand, and a desire to estimate demand or supply functions that are consistent with economic theory, on the other. In the context of consumer demand where the analyst has data on the prices and consumption of only a subset of commodities, it is indeed possible to specify demand functions that require no more than the available data if weak separability is assumed. The purpose of this paper is to clarify the status of the welfare measures which might be computed from these demand functions. Ideally, one would like them to coincide with the true welfare measures that would be obtained if one could estimate the full set of demand functions for <u>all</u> goods. This turns out to be true only when some of the goods in the subset have zero income elasticities of demand, and the price and quality changes are confined to these goods. Otherwise, one has to be content with the fact that the welfare measures computed from the partial demand functions provide bounds on the true welfare measures--this is the price that one pays for being unable to estimate the full demand system.

APPENDIX

Here I will prove the inequalities in [18a] and [18b]. For this purpose it is convenient to work with the expenditure functions arising out of the minimization problem dual to [1] and [7]. Define the full and partial expenditure functions, m(p, b, q, u) and m*(p, b, u), by

$$m(p, b, q, u) = \underset{x,\ z}{\text{mimimize}}\ \Sigma\ p_j\ x_j + \Sigma\ q_i\ x_i,\ \text{s.t. } u(x, b, z) = u \qquad [A1]$$

$$m^*(p, b, u^*) = \underset{x}{\text{minimize}}\ \Sigma\ p_j\ x_j,\ \text{s.t. } u^*(x, b) = u^*. \qquad [A2]$$

Since $m(p^o, b^o, q, u^o) = y$ and $m^*(p^o, b^b, u^{*o}) = y_x^o$, where $u^{*o} = v^*(p^o, b^o, y_x^o)$, alternative definitions of CV and CV*, equivalent to [14a] and [16a], are

$$CV = y - m(p', b', q, u^o) \qquad [A3]$$

$$CV^* = y_x^o - m^*(p', b', u^{*o}). \qquad [A4]$$

Similarly, since $m(p', b', q, u') = y$ and $m^*(p', b', u^{*\prime}) = y_x'$, where $u^{*\prime} = u^*(p', b', y_x')$, alternative definitions of EV and EV^+, equivalent to [14b] and [17b], are

$$EV = m(p^o, b^o, q, u') - y \qquad [A5]$$

$$EV^+ = m^*(p^o, b^o, u^{*\prime}) - y_x'. \qquad [A6]$$

Let $y_z^t = \Sigma\ q_i\ z_i^t$, where $z_i^t = f_i(p^t, b^t, q, y)$, t = 0, 1, and observe that $y = y_x^o + y_z^o = y_x' + y_z'$. Then [A4] and [A6] can be rewritten as:

$$CV^* = y - y_z^o - m^*(p', b', u^{*o}) \qquad [A4']$$

$$EV^+ = m^*(p^o, b^o, u^{*\prime}) + y_z' - y. \qquad [A6']$$

By comparing [A3] with [A4'] and [A5] with [A6'], it will be seen that, if

$$y_z^o + m^*(p', b', u^{*o}) \geq m(p', b', q, u^o) \qquad [A7]$$

and

$$y_z' + m^*(p^o, b^o, u^{*\prime}) \geq m(p^o, b^o, q, u'), \qquad [A8]$$

then the inequalities in [18a] and [18b] are proved.

In order to demonstrate [A7] and [A8], it is necessary to introduce a new type of expenditure function:

$$\hat{m}(p, b, q, u, \bar{z}) = \underset{x}{\text{minimize}} \; \Sigma \, p_j \, x_j + \Sigma \, q_i \, \bar{z}_i, \text{ s.t. } u(x, b, \bar{z}) = u, \qquad [A9]$$

where $\bar{z}$ is a vector of fixed values. A comparison of [A1] and [A9] shows that, whereas $m(\cdot)$ measures the minimum cost of attaining a given level of utility, u, when x and z can be freely varied, $\hat{m}(\cdot)$ measures the minimum cost of attaining the same utility level with z fixed and only x variable. Therefore,

$$\hat{m}(p, b, q, u, \bar{z}) \geq m(p, b, q, u). \qquad [A10]$$

Under the separability assumption [6], [A9] can be rewritten as

$$\hat{m}(p, b, q, u, \bar{z}) = \underset{x}{\text{minimize}} \; \Sigma \, p_j \, x_j + \Sigma \, q_i \, \bar{z}_i, \text{ s.t. } \phi[u^*(x, b), z] = u. \qquad [A9']$$

Clearly,

$$\hat{m}(p, b, q, u, \bar{z}) = \Sigma \, q_i \, \bar{z} + m^*(p, b, u^*), \qquad [A11a]$$

where u* satisfies

$$\phi(u^*, \bar{z}) = u. \qquad [A11b]$$

In particular,

$$\hat{m}(p', b', q, u^o, z^o) = y_z^o + m^*(p', b', u^{*o}) \qquad [A12]$$

$$\hat{m}(p^o, b^o, q, u', z') = y_z' + m^*(p', b', u^{*'}), \qquad [A13]$$

since $u^o = \phi(u^{*o}, z^o)$ and $u' = \phi(u^{*'}, z')$. Combining [A10] with [A12] and [A13] yields [A7] and [A8]. Q.E.D.

In order to provide an intuitive explanation of the inequalities in [18a,b], I will focus on the special case where the change is limited to a single price, say p_1. Thus, $p^o = (p_1^o, \overline{p})$ and $p' = (p_1', \overline{p})$, where $\overline{p} = (p_2, \ldots, p_n)$, and $b^o = b' = b$. Let $x_1 = g_1(p_1, \overline{p}, b, q, u)$ be the compensated demand function for x_1 associated with the minimization problem in [A1], and let $x_1 = g_1^*(p_1', \overline{p}, b, u^*)$ be the partial compensated demand function for x_1 associated with the maximization problem in [A2]. For this change, CV is equal to the area under the compensated demand function $g_1(\cdot)$ evaluated at $(\overline{p}, b, q, u^o)$ between p_1^o and p_1', while CV* is equal to the area under the partial compensated demand function $g_1^*(\cdot)$ evaluated at $(\overline{p}, b, u^{*o})$ between p_1^o and p_1'. Similarly, EV and EV^+ are equal to areas under $g_1(\cdot)$ and $g_1^*(\cdot)$ evaluated at $(\overline{p}, b, q, u')$ and $(\overline{p}, b, u^{*'})$, respectively. Therefore, it is necessary to compare the graphs of $g_1(\cdot)$ and $g_1^*(\cdot)$ as functions of p_1.

Just as the maximization problem [1] can be decomposed under the separability assumption [6] into the maximization problems [7] and [9], so, too, the minimization problem [A1] can be decomposed into the minimization problem [A2] and the following: choose u* and z so as to

$$\text{minimize } m^*(p_1, \overline{p}, b, u^*) + \Sigma\, q_i z_i, \text{ s.t. } \phi(u^*, z) = u. \qquad [A14]$$

The solution is a set of compensated demand functions for u* and z; in particular, the function for u* takes the form

$$u^* = G(p_1, \overline{p}, b, q, u), \qquad [A15]$$

which is dual to the demand function for y_x in [10]. It follows that the partial and full compensated demand fuctions for the x's are related by the identity

$$x_j = g_j(p_1, \overline{p}, b, q, u) \equiv g_j^*[p_1, \overline{p}, b, G(p_1, \overline{p}, b, q, u)] \quad j = 1, \ldots, N, \qquad [A16]$$

which parallels the identity linking the ordinary demand functions in [12]. An implication of [A16] is that

$$x_1^o = g_1(p_1^o, \overline{p}, b, q, u^o) = g_1^*(p_1^o, \overline{p}, b, u^{*o}) \qquad [A17]$$

$$x_1' = g_1(p_1', \overline{p}, b, q, u') = g_1^*(p_1', \overline{p}, b, u^{*\prime}). \qquad [A18]$$

Another implication concerns the slopes of $g_1(\cdot)$ and $g_1^*(\cdot)$ graphed as functions of p_1:

$$\frac{\partial g_1}{\partial p_1} = \frac{\partial g_1^*}{\partial p_1} + \frac{\partial g_1^*}{\partial u^*} \frac{\partial G}{\partial p_1} . \qquad [A19]$$

It follows from the concavity of the expenditure functions $m(\cdot)$ and $m^*(\cdot)$ that $\partial g_1/\partial p_1 < 0$ and $\partial g_1^*/\partial p_1 < 0$--i.e., the compensated demand functions have a negative slope. It is necessary, however, to determine the sign of the second term on the right-hand side of [A19].

I will now show that this term is negative and, therefore, $\partial g_1/\partial p_1 \leq \partial g_1^*/\partial p_i$. For this purpose, it is convenient to reformulate the minimization problem [A14] and then examine the resulting first-order condition for u*. Since $\phi(u^*, z)$

is strictly increasing in its arguments, one can invert the constraint $\phi(u^*, z) = u$ for one of the z's--say, z_1--to obtain $z_1 = \Theta(u^*, u, z_2, \ldots, z_m)$. Thus, an unconstrained minimization problem equivalent to [A14] is: choose u* and $z_2, \ldots, z_m$ so as to

$$\text{minimize } m^*(p_1, \bar{p}, b, u^*) + q_1 \Theta(u^*, u, z_2, \ldots, z_m) + \sum_2^m q_i z_i. \quad \text{[A14']}$$

The first-order condition for the choice of u* is

$$T(u^*, p_1) \equiv m^*_{u^*} + q_1 \Theta_{u^*} = 0, \quad \text{[A20]}$$

and the second-order conditions include

$$T_{u^*}(u^*, p_1) \equiv m^*_{u^*u^*} + q_1 \Theta_{u^*u^*} \geq 0, \quad \text{[A21]}$$

where subscripts denote first- and second-order partial derivatives. By implicitly differentiating [A20], one obtains

$$\frac{\partial u^*}{\partial p_1} = -\frac{m^*_{u^*p_1}}{T_{u^*}}. \quad \text{[A22]}$$

Observe also that, by the continuity of m*(·) and Shepherd's Lemma,

$$\frac{\partial g_1^*}{\partial u^*} = m^*_{u^*p_1}. \quad \text{[A23]}$$

Combining [A22] and [A23] and applying [A21],

$$\frac{\partial g_1^*}{\partial u^*} \cdot \frac{\partial G}{\partial p_1} = -\frac{\left(m^*_{u^*p_1}\right)^2}{T_{u^*}} \leq 0. \quad \text{[A24]}$$

It follows that $\partial g_1/\partial p_1 \leq \partial g_1^*/\partial p_1$ or, in terms of the conventional diagram where price is plotted on the vertical axis, the compensated demand curve $g_1(\cdot)$ is flatter than the partial compensated demand curve $g_1^*(\cdot)$. This is illustrated in Figures 1 and 2. The first diagram exhibits the relationship between CV and CV*. I consider two different cases: (a) p_1 decreases from p_1^o to p_1^a, and (b) p_1 rises from p_1^o to p_1^b. The quantity CV is represented by the shaded area under $g_1(p_1, \overline{p}, b, q, u^o)$ while the quantity CV* is represented by the cross-hatched area under $g_1^*(p_1, \overline{p}, b, u^{*o})$. It can be seen that, when price falls and CV and CV* are both positive, the area CV* is smaller while, when price rises and CV and CV* are both negative, the absolute value of the area CV* is larger--which corresponds to the inequality in [18a]. Similarly, Figure 2 exhibits the relationship between EV and EV^+ for both a price decrease and a price increase. The quantity EV is represented by the shaded area under $g_1(p_1, \overline{p}, b, q, u')$ while the quantity EV^+ is represented by the cross-hatched area under $g_1^*(p_1, \overline{p}, b, u^{*\prime})$. The absolute value of the area EV^+ is larger for a price increase and smaller for a price decrease--which corresponds to the inequality in [18b]. This argument, therefore, provides an intuitive justification for the inequalities in [18a,b]. It should, however be emphasized that these inequalities remain valid for more general changes in (p, b) than the single price change depicted in figures 1 and 2.

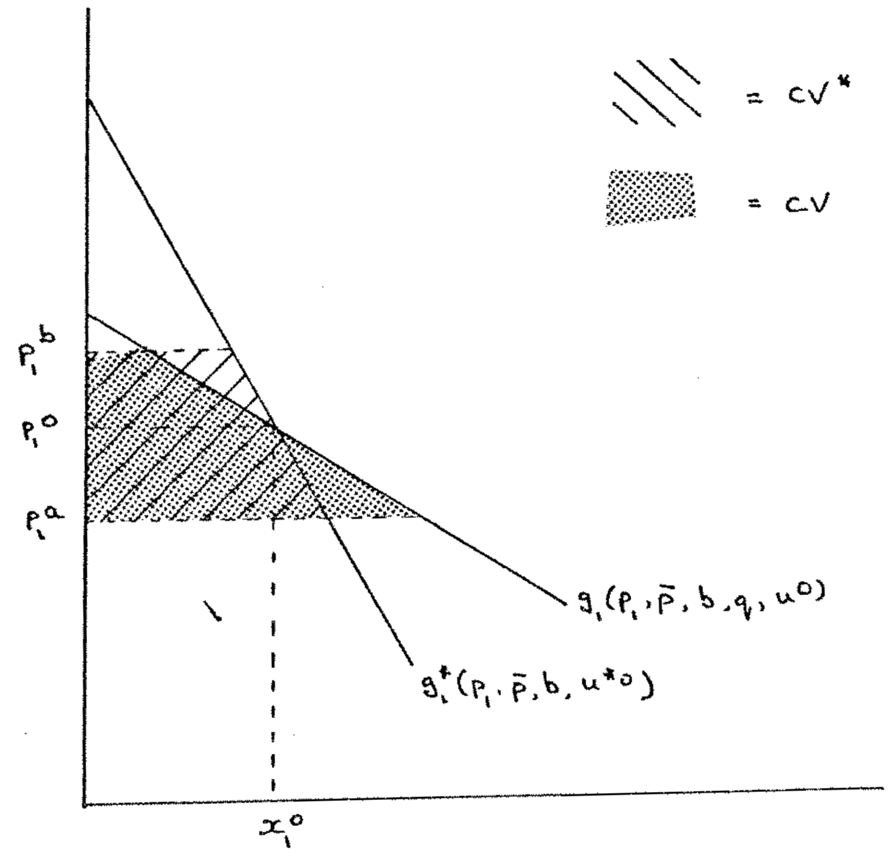

FIGURE 1. The Relationship Between CV and CV*.

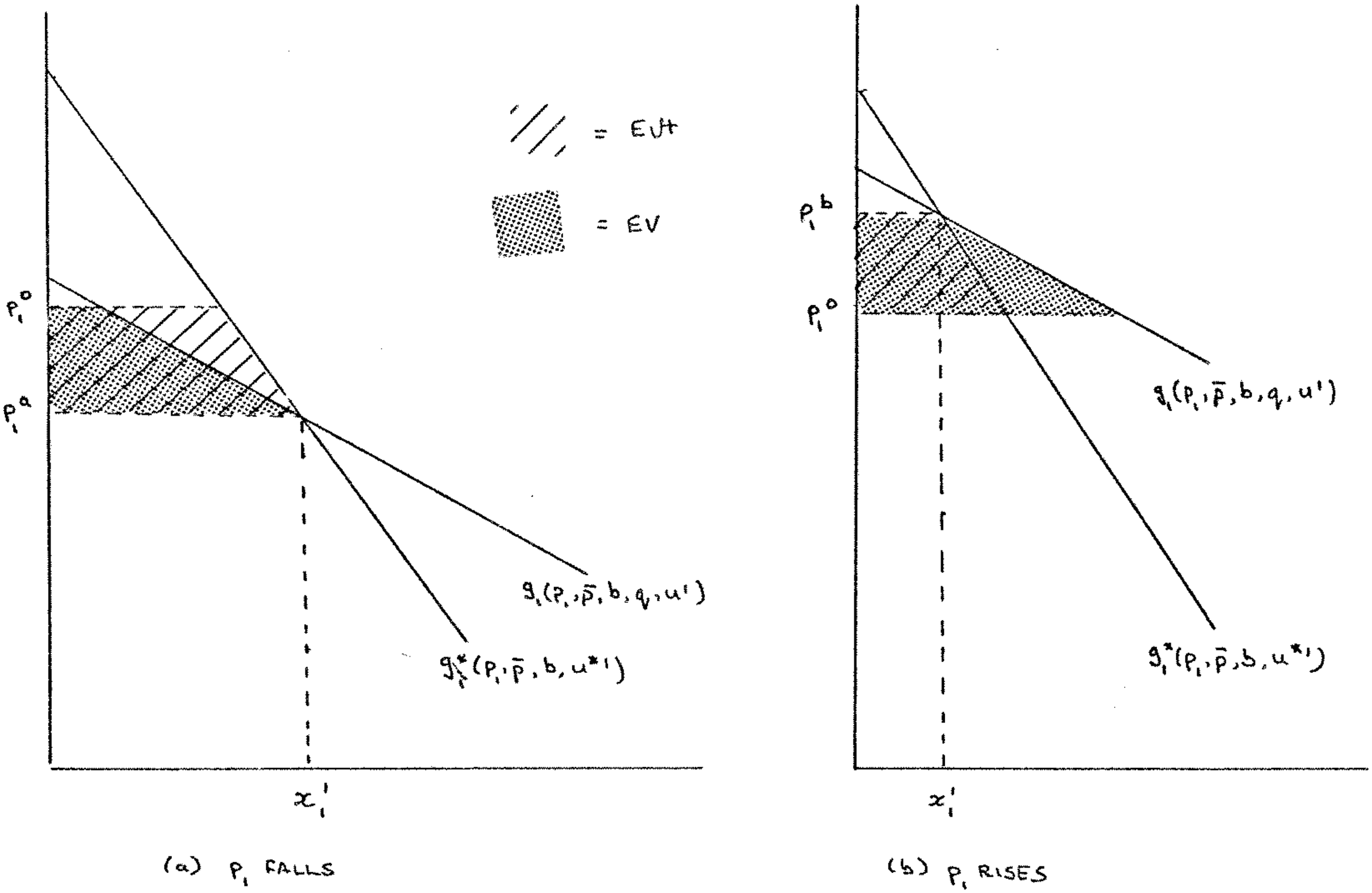

FIGURE 2. The Relationship Between EV and EV^+.

FOOTNOTES

The author is an Assistant Professor in the Department of Agricultural and Resource Economics, University of California, Berkeley.

[1] z_0 and q_0 are scalars.

[2] A good, x_j, is weakly complementary with respect to an element b_r if, when $x_j = 0$, $\partial u/\partial b_r = 0$.

[3] Weak complementarity can be checked directly from the partial utility function $u^*(\cdot)$ since, with $\phi_{u^*} > 0$, $\partial u/\partial b_r = \phi_{u^*} \cdot \partial u^*/\partial b_r = 0$ implies $\partial u^*/\partial b_r = 0$.

[4] Note that, with the utility function [6], at most (N - 1) of the x's can have partial demand functions with zero income effects.

[5] Weak complementarity is required because then the compensating and equivalent variations associated with changes in b can be identified with areas under compensated demand functions; see Maler [1974].

REFERENCES

Cicchetti, Charles J.; Fisher, Anthony C.; and Smith, V. Kerry. 1976. "An Econometric Evaluation of a Generalized Consumer Surplus Measure: The Mineral King Controversy." Econometrica 44 (Nov.):1259-1276.

Clawson, Marion; and Knetsch, Jack. 1966. Economics of Outdoor Recreation. Baltimore, Md.: Johns Hopkins University Press.

Hanemann, W. Michael. 1980. "Quality Changes, Consumer's Surplus, and Hedonic Price Indices." Working Paper No. 116 (Nov.), Department of Agricultural and Resource Economics, University of California, Berkeley.

Hotelling, Harold (1949). "The Economics of Public Recreation." In U. S. Department of the Interior, National Park Service, An Economic Study of the Monetary Evaluation of Recreation in the National Parks.

Maler, Karl-Goran. 1974. Environmental Economics: A Theoretical Inquiry. Baltimore, Md.: Johns Hopkins University Press.

Pollak, Robert A. 1971. "Conditional Demand Functions and the Implications of Separable Utility." Southern Economic Journal 37:432-433.