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Regulating an agent with di¤erent beliefs[1]

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Abstract

There is some evidence that people have biased perceptions of risks, such as lethal or environmental risks. Hence their behavior is based on beliefs which may di¤er from the 'objective' beliefs used by a regulator. The optimal regulation then depends on this di¤erence in beliefs. We set up a general framework and study this policy change. It turns out that, in many situations, the policy change depends on the absolute 'distance' between beliefs, and not on whether agents over-estimate or under-estimate risks. We characterize the necessary and su¢cient condition for 'more distant' beliefs to always reduce the regulator's decision. We apply and extend that condition in several ways.

1   Introduction

1.1   The motivation

There is some evidence that the beliefs of people are biased. A famous example is the bias in lethal risks perception. Individuals systematically overestimate the rare causes of death such as cataclysmic storms or plane crashes and underestimate more common causes of death like cancers or automobile accidents (Lichtenstein et al., 1978).[1] Typically, individuals' beliefs on hazard risks and on environmental problems di¤er from quantitative estimates and scienti…c evidences (Slovic, 1986, Viscusi, 1998).

This paper is interested in the implications of this observation for risk regulatory policies: Should governments be concerned with the risks that people perceive they face? And, if yes, how should governments' policies account for public misperceptions?

Existing economic theory has not really answered these sort of questions: What weight should be accorded in social choices to individuals' erroneous beliefs? How will agents with di¤erent prior beliefs interact? This paper examines the e¤ect of relaxing the common prior belief assumption within a model of regulation with two agents, a 'benevolent' government and an 'irrational' individual.

1.2   Related policy debate

The following problem has been introduced by Portney (1992, p. 131). It is called 'Trouble in Happyville':

You are Director of Environmental Protection in Happyville (...). The drinking water supply in Happyville is contaminated by a naturally occurring substance that each and every resident believes may be responsible for the above-average cancer rate observed there (...).

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[1] The literature on public misperceptions and their causes is very well documented since the seminal paper by Tversky and Kahneman (1974). Individuals have di¢culties with the mathematics of probability, they use heuristics or rules of thumbs that are useful but misleading. For instance, they are subject to 'availability heuristic'. People assess the risks of heart attack by recalling such occurences among one's acquaintances. They 'anchor' their estimates to easily retrievable events in memory such as sensational stories in the medias etc..

You have asked the top ten risk assessors in the world to test the contaminant for carcinogenicity (...). These risk assessors tell you that while one could never prove that the substance is harmless, they would each stake their professional reputations on its being so.

You have repeatedly and skillfully communicated this to the Happyville citizenry, but because of the deep-seated skepticism of all government officials, they remain completely unconvinced and truly frightened.

The mirror image of Happyville is Blissville (Viscusi, 2000). In Happyville, the risk is low but perceived as large. In Blissville, the risk is important but perceived as low. The question becomes: You are the Director of Environmental Protection both in Happyville and Blissville, how do you allocate your cleanup efforts? To Viscusi, the choice is clear-cut. Efforts have to be spent in Blissville. If efforts are spent in Happyville, this is a 'statistical murder' since lives are sacri...ced to focus instead on illusory fears. Viscusi (2000)'s view is probably shared by most economists.

In Policy Sciences, many sholars have argued that the choice is not so clear-cut. If efforts are spent in Happyville, people who were worried feel protected, and so feel better. This contrasted view is particularly apparent in Europe. Indeed, European regulatory choices reflect more the differences in policy judgments and cultural values than American regulatory policies (Pollack, 1995). A recent report of the European Commission states for instance 'Decision-makers have to account of the fears generated by the perceptions and to put in place preventive measures to eliminate the risks' (CEC, 2000). Such a view is called the 'populist approach' to risk regulation by Hird (1994).

There are many arguments against a 'populist' approach to risk management. Using data on American health risk programs, Viscusi (1998) has investigated the failures of the regulatory policies based on a 'populist' approach as opposed to a cost-bene...t or say a 'rational' approach. The cost has been millions of dollars for the U.S., or said differently, it has been thousands of American lives. This is what Breyer (1993) calls the 'vicious circle' of risk regulation: Individuals' misperceptions are embodied in government regulations. There are many reasons for that. The simplest reason is because politicians are subject to the same biases in beliefs as individuals. A complex reason is because politicians respond in some way to individuals' preferences and biases. As Margolis (1996, p. 161) states 'If enough people feel worried about some risk, however remote and cautiously calculated, then

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it makes sense to say the the government ought to respond to that. How to respond is less clear'.

This moves the policy debate from a normative to a more positive question: 'How to respond?'. In Pollack (1998, p. 379)'s words, 'How do governments regulate risks when the perceptions of the public diverge from those of experts? (..) What role do risk analysis and cost-bene...t analysis play?'. Pollack argues that those questions should deserve probably much more attention than they have received so far.

1.3 Our approach to the debate

Let now present our very speci...c approach to this debate. An individual faces a risk ε, and believes that the distribution of the risk is q. Given a regulatory environment a (road safety, cigarette prices..), he makes a choice b (driving speed, smoking...). His objective is to choose b to get the best expected utility

$$E_q U(\varepsilon; a; b);$$

which yields a decision $b(a; q)$.

Importantly, this decision made by the individual is based on the subjective probability or perceived risk q for ε, not on the objective probability or the actual risk p. The decision $b(a; q)$ thus maximizes perceived expected utility $E_q U$, not actual expected utility $E_p U$.

Let now address the government regulatory policy. This policy is based on the choice of a. The objective of the government is to maximize the actual expected utility of the individual $E_p U$, i.e. the expected utility based on the objective risk, p. The government chooses a to get the best

$$E_p U(\varepsilon; a; b(a; q));$$

which yields the decision $a(p; q)$.[2] Importantly, the government thus accounts for the irrational individual's response, i.e. the response based on the perceived risk q. This is thus a sort of 'second best' choice that is selected by the government.

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[2] We are thus making a strong assumption in terms of the choice of the welfare criterion. However, as far as we know, social choice literature has not really addressed the question of which criterion to choose when people have erroneous beliefs. Interestingly, there are some papers on the e¤ect of di¤erent welfare criteria in exchange economies where people may have di¤erent subjective probabilities (see Hammond, 1981, and Marshall, 1988).

This framework thus captures a complex channel for why the individual's perception q a¤ect the eΦcacy of regulatory choices. This channel is related to the anticipation of the irrational response of individuals, i.e. the response based on q; not on p.[3]

More generally, our approach allows us to consider three polar cases. The regulator may select:

$$
\begin{aligned}
a(p;q) &: \quad \text{the 'second-best' policy,} \\
a(p;p) &: \quad \text{the 'rationalist' policy,} \\
a(q;q) &: \quad \text{the 'populist' policy.}
\end{aligned}
$$

The 'rationalist' approach is clearly ineΦcient because it does not anticipate correctly the agent's reactions. The 'populist' approach is intuitively ineΦcient because it does not make use of the regulator's information. The 'second-best' approach is illustrated in the following example.

1.4   An example: Regulation in Happyville

Let us come back to the choice faced by the Director of Environmental Protection in Happyville. Let

$$
U(x;a;b) = u(b) \ ¡ \ (1 ¡ a)±bx ¡ c(a);
$$

where

$$
\begin{aligned}
u(:) &\quad \text{is the individual's utility from drinking water,} \\
b &\quad \text{is water consumption,} \\
a &\quad \text{is cleanup e¤ort, } 0 \cdot a \cdot 1, \\
± &\quad \text{is the desutility from getting a cancer,} \\
x &\quad \text{is the dose-response risk of carcinogenicity,} \\
c(:) &\quad \text{is the cleanup cost function.}
\end{aligned}
$$

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[3] This means that in Happyville there are complex spill-over e¤ects (Viscusi, 1998). A good example of spill-over e¤ect is related to the in‡uence of automobile seatbelts in the United States in the late 60s. Since seatbelts reduced the risk of injury to the driver, an associated e¤ect has been to make drivers driving fast, o¤setting the bene…ts of the safety regulatory policy (Peltzman, 1975). Lowering speed limit on major highways decreases the average driving speed but increases the incentive to drive on back roads (Viscusi, 1998).

Assume very simplistic functional forms

$$u(b) = -\frac{(1-b)^2}{2};\quad 0 \leq b \leq 1;$$

$$\xi = 1;$$

$$c(a) = \frac{a^2}{2}:$$

Assume also that the risk is binary with an objective probability p of $x = 1$, 0 otherwise.[4]

Given these assumptions, the agent simply chooses b to maximize

$$E_q U(a; b; x) = -\frac{(1-b)^2}{2} - (1-a)qb - \frac{a^2}{2};$$

so that we get

$$b(a; q) = 1 - (1-a)q:$$

According to the intuition, optimal water consumption b is decreasing in the perceived probability of getting a cancer q and increasing in the level of cleanup efforts a.

The regulator chooses a to maximize

$$E_p U(x; a; b(a; q)) = -\frac{((1-a)q)^2}{2} - (1-a)p(1 - (1-a)q) - \frac{a^2}{2}; \quad (1)$$

so that we get

$$a(p; q) = \frac{p - 2pq + q^2}{1 - 2pq + q^2} \in [0; 1]: \quad (2)$$

We are now in a position to examine the effect of individual beliefs on the regulator decision. This effect is represented on figure 1. This figure represents cleanup efforts as a function of individual's misperceptions q:

Let us first examine the 'rationalist' regulator decision $a(p; p)$: This decision does not internalize individual's misperceptions, so it is a straight line on the figure. The 'rationalist' decision is insensitive to public beliefs.

Then, turn to the opposite case, the 'populist' decision which is equal to

$$a(q; q) = \frac{q}{1 + q}:$$

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[4] We make a slight abuse of notation here: the probability vector reduces to a scalar, also denoted p .

This decision is too sensitive to public beliefs. In Happyville, i.e. in the city where the perceived risk is large, $q > p$; cleanup efforts are high. In Blissville, where the perceived risk is low, $q < p$; cleanup efforts are low.

Finally, let us turn to the most sophisticated decision, that is the 'second best' decision. From equation (2), decision $a(p; q)$ is decreasing in q then increasing in q. Importantly, this function takes a minimum at $q = p$. Thus we have $a(p; q) \geq a(p; p)$: This shows that the optimal decision is always larger than the 'rationalist' decision. Why is that?

In Happyville, individuals are pessimistic, they do not consume water enough. Cleaning water thus will make the risk lower. Happyville population will react to that change in risk. Hence, cleanup efforts gives an incentive for the population to consume more water, which is a source of welfare in Happyville where people overestimated the risk. In Blissville, the reason for why cleanup efforts increase is different. People are optimistic and consume too much water. Risk-exposure to cancer is thus too large in Blissville. Hence, cleaning water simply reduces risk-exposure.[5]

Finally, note that the difference between the 'second-best' policy $a(p; q)$ and the 'rationalist' policy $a(p; p)$ increases as the absolute value $|p - q|$ increases. Moreover, they increase exactly at the same rate. Indeed replace q by $p + u$ in (2) to get

$$a(p; p + u) = \frac{p(1 - p) + u^2}{(1 + p)(1 - p) + u^2}$$

so that the value of a is independent of the sign of u. This means that cleanup efforts are the same in Blissville and Happyville.[6] Hence, an important lesson from that example is that the difference between the public and the regulator beliefs is more important than the direction of the misperception. In other words, it is not so much important for the regulator to know whether he is in Blissville or Happyville. What is important is to know 'how large' is the misperception.

To summarize: because the population's response is 'irrational', regulation may depart strongly from a myopic Cost-Benefit Analysis. Yet, this

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[5] This interpretation suggests that this result is model-dependent. We will precisely examine this question in the paper.

[6] The decisions are the same in the sense that $a(p; q)$ is symmetric around p. This symmetry is due to the selection of the parameters. For a different set of parameters, the symmetry is lost. The message remains though.

example has shown that this departure displays several 'regularity' properties. For instance, policy change depends on the absolute 'distance' between beliefs, not on whether agents over-estimate or under-estimate risks. This raises the question of the e¤ect of di¤erent beliefs on regulatory policies in general.

The next Section introduces a general framework for studying the impact of distant beliefs. Section 3 o¤ers a simple necessary and su¢cient condition for distant beliefs to reduce the regulatory e¤ort, and applies this condition to the Happyville example. Section 4 derives an equivalent condition, based only on the primitives of the model. Section 5 discusses the relationships with other problems, in particular the question of decision-making when some information is awaited for in the future. Section 6 concludes.

2   The framework

The simplest manner to introduce our framework is the following. A regulator chooses a regulatory e¤ort a. An agent reacts to a and chooses a decision b. These choices are performed under uncertainty on the true state of nature $x \in X$. The Von Neumann-Morgenstern preferences of the agent are given by the utility function $U(x; a; b)$. Because the regulator is only interested in the agent's welfare, he shares the same preferences.[7]

We assume that x takes a ...nite number of values, and by a slight abuse of notation we also denote this number by X; so that $X = fx_1; :::; x_X g$. Decision a is a real number. Decision b is a real vector of dimension $N \geq 1$. Also, U is three-times continuously di¤erentiable with respect to (a, b).

Suppose ...rst that the regulator and the agent have the same beliefs p, de...ned in the usual manner[8]:

$$8 x \quad p(x) > 0 \qquad \sum_{x \in X} p(x) = 1:$$

Once a is chosen, the agent chooses b to maximize

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[7] One could include in the regulator's preferences a cost for a: $V(x; a; b) = U(x; a; b) \; i \; c(a; x)$, without any change.

[8] In what follows we could suppose that weights belong to an open convex subset of all possible weights. The assumption that weights are strictly positive plays a role.

$$\sum_{x \in X} p(x)U(x; a; b): \tag{3}$$

We assume that for any admissible p and a, this criterion is strictly quasi-concave in b. Consequently de...ne $b(a; p)$ as the unique decision b which maximizes this criterion, and $j(a; p)$ as the value of the program:

$$j(a; p) = \max_b \sum_{x \in X} p(x)U(x; a; b) = \sum_{x \in X} p(x)U(x; a; b(a; p)): \tag{4}$$

Hence, when both agents share the same beliefs, the best decision a must maximize $j(a; p)$. Notice that $j(a; p)$ is the maximum of functions which are linear in p. Hence $j(a; p)$ must be convex in p.

Now suppose that the weight q used by the agent di¤ers from the weight p used by the regulator. Acting as a Stackelberg leader, the regulator should adjust his ...rst-period decision consequently, by maximizing

$$\sum_{x \in X} p(x)U(x; a; b(a; q)): \tag{5}$$

over a. Our ...rst objective in the following is to compare (5) to (4), i.e;, to study the impact of a change in the agent's beliefs. To do so, we need to introduce a measure for the di¤erence in beliefs.

We will de...ne a simple measure for that di¤erence. Let introduce two weights p and q and two scalars $r; s \in [0; 1]$. Then $(1_i r)p + rq$ and $(1_i s)p + sq$ are also weights, and an increase in s makes the latter more distant from the former if $s > r$, and closer otherwise. Hence the absolute value $js_i rj$ is an index for the di¤erence in beliefs. De...ne the regulator's expected payo¤ as

$$K(a; r; s) = \sum_{x \in X} [(1_i r)p(x) + rq(x)]U(x; a; b(a; (1_i s)p + sq)): \tag{6}$$

Here the regulator uses the weight $(1_i r)p + rq$, and the agent uses the weight $(1_i s)p + sq$. Such a de...nition still permits to consider our three polar cases:

² The 'second best' case, (r, s),

² The 'rationalist' case, (r, r),

² The 'populist' case, $(s; s)$.

Notice that these linear forms for the weights appear quite naturally in many cases. For example, suppose that initially agents share the same beliefs, but an experiment is performed, giving additional information on the true state of nature. Nevertheless, there is an exogenous probability that the experiment has failed, and in that case its results are uninformative. Moreover there is no way to tell whether the experiment has failed or not. If the regulator and the agent do not agree on the probability of failure, their revised beliefs take these linear forms.

In what follows, we shall make s vary in order to capture the impact of the di¤erence in beliefs. We will say that beliefs are more distant if $|s - r|$ increases, r given. Finally, the regulation problem (r, s, p, q) is de…ned as the problem of maximizing (6) with respect to a. We assume that solutions to such problems always exist.

3   The impact of more distant beliefs

Our objective in this section is to investigate the e¤ect of the di¤erence in beliefs as de…ned above. We …rst examine this e¤ect on the regulator's expected utility, then on the regulator's decisions. Note that in this section p and q are given, so that beliefs are restricted to belong to the straight interval $[p; q]$.

By de…nition of K and j, we have

$$K(a; r; r) = j(a; (1 - r)p + rq) = \max_b \sum_x [(1 - r)p(x) + rq(x)]U(x; a; b)$$

so that

$$8 r; s \qquad K(a; r; s) \cdot K(a; r; r): \tag{7}$$

This result states that the regulator's expected utility reaches its maximum when the agent and the regulator have the same beliefs, $s = r$. This raises the question of what happens when s moves progressively from r. This result is presented in the next proposition.[9]

Proposition 1 The regulator's expected utility decreases with more distant beliefs, i.e. $K(a; r; s)$ weakly increases with s for $s < r$, and weakly decreases with s for $s > r$.

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[9] The proofs are given in appendix.

The meaning of that proposition is simple. More distant beliefs reduces the regulator's expected utility.

3.1 Equivalence result

Let us now turn to the e¤ect of the di¤erence in beliefs on the regulator's decision. Recall that the regulator maximizes the value function $K(a; r; s)$ as de…ned in (6). Hence the properties of the derivative K_a are essential here.

Suppose for example that we have

$$8 \ a; r; s; \ K_a(a; r; s) \cdot \ K_a(a; r; r): \qquad (8)$$

Then it is clear that the optimal regulatory e¤ort is reduced when di¤erent beliefs are introduced. Reciprocally, if this condition does not hold, then it is possible to build a case in which a is made higher with di¤erent beliefs. Therefore (8) is equivalent to the fact that a di¤erence in beliefs reduces optimal a. Again the comparison extends to more distant beliefs.

Proposition 2 The three following statements are equivalent:
i) The regulator's decision decreases with more distant beliefs;
ii) $8 \ a; r; s; \ K_a(a; r; s)$ increases with s when s < r, and decreases with s when s > r;
iii) $K_a(a; 0; s)$ decreases with s.

This Proposition thus derives necessary and su¢cient conditions for signing the comparative statics analysis of more distant beliefs on the regulatory policy. The simplest condition is clearly iii), which asks to verify a simple property of the value function K. Let us now give some examples of the usefulness of that Proposition.

3.2 Examples

Example 1 Prudence in Happyville

Let us now consider a more general Happyville population composed by agents with a general utility function for drinking water u, increasing and concave, and a general cleanup cost function $c(a)$. Now $b(a; s)$ is de…ned by

$$u^0(b(a; s)) = (1 \ \mathrm{i} \ a)s;$$

where s stands for the perceived probability of getting cancer. Hence, K writes

$$K(a, r, s) = u(b(a, s)) - (1 - a)b(a, s)r - c(a);$$

where r is objective risk-probability level. We then obtain

$$K_a(a, 0, s) = u'(b(a, s))\frac{\partial b}{\partial a} = -s\frac{u'(b(a, s))}{u''(b(a, s))} - c'(a)$$

$$= -\frac{1}{1 - a}\frac{u'(b(a, s))^2}{u''(b(a, s))} - c'(a).$$

Since $b(a, s)$ is decreasing with s, the question that remains is whether

$$\frac{u'(b)^2}{u''(b)} \tag{9}$$

is increasing with b. Note that this property holds for

$$u(b) = -\frac{(1 - b)^2}{2};$$

but does not hold for a general concave u. This shows that this is the modeling choice of a quadratic utility function that was at the root of the result displayed in the Introduction. In fact, it can be easily shown that (9) is increasing with b if and only if the population displays a coefficient of prudence (Kimball, 1990) that is larger than twice the coefficient of absolute risk-aversion. For an iso-elastic function (with constant relative risk-aversion ρ), the condition amounts to $\rho < 1$; notice that for a logarithmic function, regulation is unaltered by different beliefs.

Example 2 Taxes in Happyville

Suppose now that there is no cleanup technology available for the regulator. Yet, suppose that the regulator may set a positive tax a on water consumption. Preferences are given by

$$U(x, a, b) = u(b) - ab - bx.$$

The agent's decision $b(a, s)$ is thus defined by

$$u'(b(a, s)) = a + s.$$

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Hence

$$K(a; r; s) = u(b(a; s)) - ab(a; s) - rb(a; s)$$

so that

$$K_a(a; 0; s) = [u'(b(a; s)) - a]\frac{@b}{@a} - b(a; s)$$

$$= \frac{s}{u''(b(a; s))} - b(a; s)$$

by using the ...rst order condition. Let us now di¤erentiate with respect to s. We get

$$K_{as}(a; 0; s) = -\frac{su'''(b(a; s))}{[u''(b(a; s))]^3};$$

so that more distant beliefs increases the optimal tax a if and only if consumers are prudent ($u''' > 0$).[10]

4   Characterization results

The previous section has identi...ed the necessary and su¢cient condition so that more distant beliefs leads to decrease the regulator's decision. This condition reduced to examining the property of the value function $K(a; r; s)$. Notice that K is the value of function of a Stackelberg game, i.e. the value function of the regulator's problem when the agent has di¤erent beliefs, s. Analyzing the properties of K is thus quite a technical problem. We solved two simple examples which displayed some linearity properties - U linear in x -. This raises a more general general question: Is it possible to solve the comparative statics analysis for any problem?

In this section, we will answer this question in two di¤erent ways. First, we will derive the equivalent property required on the value function $j(a; p)$; second, we will derive the properties in terms of restrictions of the original problem U . The next Proposition displays the easier result, that is the ...rst one.

Proposition 3 The three following statements are equivalent:
   i) The regulator's decision decreases with more distant beliefs;
   ii) $K_a(a; s; s)$ is convex in s;
   iii) $j_a(a; p)$ is convex in p.

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[10]When beliefs are the same, or when consumers are not prudent, optimal tax is thus simply equal to zero.

This proposition derives the necessary and su¢cient condition to examine the impact of the di¤erence in beliefs on the regulator's decision. Its main interest is that it shows that the exploration of the "rationalist" case - or the "populist" case - are su¢cient to solve the comparative statics analysis.[11] This is convenient since this corresponds to the value function for the problem when the regulator and the agent display the same beliefs, that is the simpler dynamic problem.

However, one still may object that any restriction on j makes necessary to solve the problem as well. This leads us now to the most complex problem, that is deriving some restrictions on the primitives of the model. This is important since this is the only way to guarantee that the comparative statics analysis leads to unambiguous results in the following sense: Does it exist some restrictions on the function $U(x; a; b)$ for any $x; a$ and b such that more distant beliefs always decrease the regulator's decision?[12]

First of all, let us introduce some new notations. Denote U_a the derivative of U with respect to a; U_b the gradient of U with respect to b, and U_{ab} its derivative with respect to a; $U_{bb}(x; a; b)$ the Hessian matrix of U with respect to b. Finally de…ne the Hessian matrix of the objective in (4):

$$H(a; p) = \sum p(x)U_{bb}(x; a; b(a; p)):$$

Recall that by assumption H is negative de…nite, and thus invertible. Finally prime ($'$) stands for transposition.

Proposition 4 The regulator's decision decreases with more distant beliefs if and only if for any a, b, there exists a $N \pounds N$ matrix $M(a; b)$ and a N-vector $d(a; b)$ such that:

i) for any x,

$$U_{ab}(x; a; b) + U_{bb}(x; a; b)d(a; b) = M(a; b)U_b(x; a; b):$$

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[11] For instance, use again the example 1 above. We have $K_a(a; s; s) = sb(a; s);$ so that $K_{ass}(a; s; s) = s(@b=@s)^2[_i \ u'''=u'' _i \ (_i \ 2u''=u'')]$, where u stands for $u(b(a; s))$. We thus …nd the same necessary and su¢cient condition as before.

[12] For example, Gollier, Jullien and Treich (2000) have shown for a di¤erent problem that the property $j_a(a; p)$ convex in p always leads to ambiguous results when the primitive model is $U(x; a; b) = u(a) + v(b_i \ x(a + b))$, for any u and v increasing and concave.

ii) For any (p, q), if $b = b(a; q)$, then

$$(\sum_x p(x)U_b(x; a; b))'(M(a; b) + d_b(a; b))'[H(a; q)]^{i\,-1}(\sum_x p(x)U_b(x; a; b)) \cdot 0:$$

Let us comment this result. First, it asks to ...nd M and d independent from x such that i) holds, for any x (this equality would not change if one wants the di¤erence in beliefs to increase a). Such an equality is clearly non-generic, and requires speci...c functional forms for U.

In particular, if x represents beliefs as in the Happyville example, then U is linear with respect to x, and so is condition i). Hence i) reduces to two conditions specifying that both the constant and the x-factor are equal to zero (which gives $N(N + 1)$ unknown for $2N$ equations).

Second, choose $b = b(a; p)$ and sum i) over x; then one gets

$$\sum_x p(x)U_{ab}(x; a; b) + H(a; p)d(a; b) = M(a; b)[\sum_x p(x)U_b(x; a; b)] = 0$$

so that one must have

$$d(a; b(a; p)) = \frac{@b}{@a}(a; p):$$

Hence $d(a; b)$ characterizes how $b(a; p)$ varies with a. This also means that this partial derivative cannot depend directly upon p. This remark may help ...nding the vector d.

Third, ii) basically expresses that a matrix is positive[13] semi-de...nite (note that d_b is the di¤erential of the vector d with respect to b, and is thus a matrix). In fact, a su¢cient condition for ii) is that $M + d_b$ itself be positive semi-de...nite.

Fourth, these conditions only depend on the properties of U_b. This vindicates our view that any cost function $c(a; x)$ can be added to U, without any change.

Finally, these conditions become simpler in the case when b is uni-dimensional, as the following example illustrates.

Example 3 Back to Happyville

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[13] Negative if one wants a to be increased by a di¤erence in beliefs.

Let us generalize our Happyville model, by proceeding to a change in variables. Consider the case when

$$U(x; a; b) = v(a; b) - bx - c(a)$$

where, as before, a is water quality and $c(a)$ is the cleanup cost function. The change is that b is now an equivalent quantity, computed from the actual quantity consumed and water quality; a higher b indicates a higher exposure to the risk x. $v(a; b)$ is the surplus associated with the consumption of the equivalent quantity b, when water quality is a. Assume that $v_b > 0$, $v_{bb} < 0$, $v_{ab} > 0$; so that a better quality increases the equivalent consumption.

Let us apply Proposition 4. Note that U is linear with respect to x. Therefore condition i) splits into two conditions:

$$v_{ab} + dv_{bb} = mv_b \qquad 0 = -mx:$$

Hence we must have $m = 0$ and $d = -v_{ab}=v_{bb}$. Condition ii) then reduces to $d_b \geq 0$. In other words, $@b=@a$ must be higher when b is higher. Since a higher b corresponds to a lower x, one would intuitively expect the opposite to be true; that is, an increase in water quality should have more effect on 'equivalent water' consumption when the damage x is high, compared to when the damage x is low.

As a result, we obtain that cleanup efforts are always increased by a difference in beliefs, if and only if the sensitivity of equivalent-water consumption to cleanup efforts is highest when the damage is highest.

5   Heterogeneous beliefs in the population

Until now, we have assumed that society was composed by one agent. It is trivial that the results extend to a population of homogeneous agents. In this section, we will prove that the results also generalized to a population of agents with heterogeneous beliefs.

Denote q_i for the beliefs of group i in the population. Assume that the regulator now maximizes

$$L(a; r; s) = E_i \sum_{x \in X} [(1 - r)p(x) + rq_i(x)]U(x; a; b(a; (1 - s)p + sq_i));$$

where E_i denotes the expectation operator over group i's beliefs. In such a framework, we will say that the population of heterogeneous agents display "more different beliefs" if, as before, s moves away from r.

The concept of "more di¤erent beliefs" is depicted on ...gure 1; There are three states of the world and two subgroups $i = 1; 2$. Point r corresponds to the distribution probability selected by the regulator. Points q_1 and q_2 correspond to the distributions respectively selected by subgroups 1 and 2. When s drifts progressively away from r, beliefs are said to become "more di¤erent". This concept clearly generalizes the previous de...nition of "more distant" beliefs to the case of heterogeneous agents.

However, notice that this concept takes the heterogeneity in beliefs in the society i as given. Let us introduce another de...nition related to the degree of heterogeneity in the society. Beliefs are said to be "more heterogeneous" in population i^0 than in population i if and only if

$$E_{i^0}{}' (q_{i^0}) \, E_i{}' (q_i); \text{ for any } ' \text{ convex:} \qquad (10)$$

In other words, beliefs are "more heterogeneous" in society i^0 than in society i when beliefs q_{i^0} are a mean-preserving spread of beliefs q_i. This leads to the following Proposition.

Proposition 5 The three following statements are equivalent:

i) The regulator's decision decreases with more distant beliefs;

ii) The regulator's decision decreases with "more di¤erent" beliefs (heterogeneous beliefs case);

iii) At $r = s$, the regulator's decision decreases with "less heterogeneous" beliefs.

The meaning of the equivalence between i) and ii) is simple. Suppose that the regulator's decision is known to decrease with more distant beliefs. Then, under ii), no additional condition is required for the regulator's decision to decrease with "more di¤erent" beliefs. In other words, all previous qualitative results directly extend to an heterogeneous population.

Furthermore, property iii) states that when introducing heterogeneous beliefs another e¤ect enters into the picture. At $r = s$, "more heterogeneity" has actually an exact opposite e¤ect as "more di¤erent", or equivalently "more distant" beliefs. One potential intuition relies on the presence of a "wealth e¤ect". Remember that more distant beliefs reduces expected utility (Proposition 1). As a result, one could view the regulator's decision as a response to this reduction in welfare. Yet, "more heterogeneity" has exactly an opposite e¤ect on welfare. Indeed, it increases and not decreases expected

utility.[14] The regulator's response thus would go the opposite direction since the "wealth e¤ect" goes precisely the opposite direction.

6   Conclusion

This paper has introduced a model of risk regulation with two agents. The novelty has been to assume that they have di¤erent prior beliefs on the risk they face. Although there is some empirical evidence that support that assumption, this is an unusual assumption in economics that is often considered as inconsistent (Thaler, 2000).

For economic situations where that critique applies, this paper reduces to a mathematical object.[15] This raises another question: Can this object be useful? The answer is yes. The idea is to reinterpret the model so that agents' beliefs are the same but their preferences di¤er. In fact, there are several 'classical' models where our mathematical object could be applied.[16] To understand how that could be done, a simple way is to consider the 'trendy' economic model of self-control.

Take the following three-periods consumption model

$$K(a; r; s) = u(a) + \bar{} u(b(a; s)) + \bar{}^2 r u(w_¡ a_¡ b(a; s));$$

where $b(a; s)$ is de…ned by

$$u^0(b(a; s)) = \bar{} s u^0(w_¡ a_¡ b(a; s)):$$

This latter equation characterizes the optimal level of consumption of self-2 while the former characterizes the value function of self-1, given self-2 decision. Note that preferences between self-1 and self-2 di¤er in that framework. Indeed, self-1's discount factor between period 2 and period 3 is $\bar{} r$. Self-2's discount factor is $\bar{} s$. Hence the marginal rate of substitution between period 2 and 3 changes depending on whether this rate is computed from self-1 or

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[14] Formally, the function $L(a; s; s)$ is convex in q_i by construction. Then apply inequality (10).

[15] For a more standard but quite exhaustive second-best approach based on externalities and principal-agent relations with an informed principal, see recently Barigozzi and Villeneuve (2002).

[16] The Rotten-Kid Theorem is one example (Bergstrom, 1989). In this situation, the comparative statics analysis is about 'how much' the kid is rotten. Another example is the e¤ect of better future information on early decisions (Epstein, 1980).

self-2's perspectives. This introduces a problem of time-inconsistency. Self-1 cannot perfectly control self-2 consumption decision.

In such a model, the most natural measure of lack of self-control is captured by the distance $|r - s|$. The difference between the marginal rates between period 2 and 3 increases with that distance. In this framework, our results state that self-1 expected utility decreases with the lack of self-control (Proposition 1), that self-1 consumption decreases or increases depending on the sign of the derivative $K_a(a; 0; s)$ in s (Proposition 2), or depends on whether $K_a(a; s; s)$ is convex in s (Proposition 3), and, finally, that this problem belongs to the set of problems that yield unambiguous results (Proposition 4). Thus a necessary and sufficient condition exists for signing the comparative statics analysis of less self-control.

This exact condition[17] and several extensions are presented in a companion paper (Salanié and Treich, 2002). A first insight from that paper is that this condition is such that it is perfectly plausible that the lack of self-control increases, and not decreases, self-1 savings. A more general insight is that the qualitative effect of self-control is independent of the structure of discount rates. It is the same no matter whether preferences are present-biased $r < s$ or future-biased $r > s$. In other words, self-1 does not care so much about whether self-2 saves too much or too little from his viewpoint. He does care about 'how much' self-2 is different. Also, this result is qualitatively equivalent to analyzing the effect of uncertainty on future discount rates, but in a model without self-control, $r = s$ (Proposition 5). These results convey quite different messages from the ones which are generally delivered in the economic literature on self-control.

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[17]The condition is $-\frac{u'''}{u''} > 2 - \frac{u''}{u'}$.

References:

Barigozzi, F. and B. Villeneuve, 2002, In‡uencing the misinformed misbe-haver: An analysis of public policy towards uncertainty and external e¤ects, mimeo, University of Toulouse.

Bergstrom, T., 1989, A fresh look at the Rotten Kid Theorem - and other household mysteries, Journal of Political Economy, 97-5, 1138-1159.

Breyer, S. G., 1993, Breaking the Vicious Circle: Toward E¤ective Risk Regulation, Harvard University Press.

CEC (Commission of the European Communities), 2000, Communication from the commission on the Precautionary Principle, available at www.europa.eu.int.

Esptein, L.G., 1980, Decision-making and the temporal resolution of un-certainty, International Economic Review, 21, 269-284.

Gollier, C., B. Jullien and N. Treich, 2000, Scienti…c progress and irre-versibility: An economic interpretation of the Precautionary Principle, Jour-nal of Public Economics, 75, 229-53.

Hammond, P., 1981, Ex ante and ex post welfare optimality under un-certainty, Economica, 48, 235-50.

Hird, J., 1994, Superfund: The Political Economy of Environmental Risk, John Hopkins University Press.

Kimball, M., 1990, Precautionary savings in the small and in the large, Econometrica 58:53-73.

Lichtenstein, S., Slovic P., Fischhoo¤ B., Layman M. and B. Combs, 1978, Judged Frequency of Lethal Events, Journal of Experimental Psychology: Human Learning and Memory, 4.

Margolis, H., 1996, Dealing with Risks: Why the Public and the Experts Disagree on Environmental Issues, Chicago Press.

Marshall, J.M., 1989, Welfare analysis under uncertainty, Journal of Risk and Uncertainty, 2, 385-403.

Peltzman, S., 1975, The e¤ects of automobile safety regulation, Journal of Political Economy, 83.

Pollack, R.A., 1995, Regulating risks, Journal of Economic Literature, 33.

Pollack, R.A., 1998, Imagined risks and cost-bene…t analysis, American Economic Review, 88.

Portney, P.R., 1992, Trouble in Happyville, Journal of Policy Analysis and Management, 11.

Salanié, F. and N. Treich, 2002, Savings and the economics of self-control, mimeo, University of Toulouse.

Slovic, P., 1987, Perception of risk, Science, 236.

Thaler, R.H., 2000, From Homo Economicus to Homo Sapiens, Journal of Economic Perspectives, 14.

Tversky, A. and D. Kahneman, 1974, Judgment under uncertainty: Heuristics and biases, Science, 185.

Viscusi, K.P., 1998, Rational Risk Policy, Oxford University Press.

Viscusi, K.P, 2000, Risk Equity, Journal of Legal Studies, forthcoming.

Appendix

Proof of proposition 1: We want to show that $K_s(a; r; s)$ has the sign of $(r - s)$: For any r and s, we have

$$K(a; r; s) = K(a; s; s) - (s - r) K_r(a; .; s);$$

where $K_r(a; .; s)$ denotes the slope of $K(a; r; s)$ in r (by linearity, it is independent of r). By the Envelope Theorem, i.e. $K_s(a; s; s) = 0$; we have

$$
\begin{aligned}
K_s(a; r; s) &= K_r(a; .; s) - K_r(a; .; s) - (s - r) K_{rs}(a; .; s) \\
&= (r - s) K_{rs}(a; .; s):
\end{aligned}
\tag{11}
$$

But note that

$$\frac{d^2 K(a; s; s)}{ds^2} = K_{rs}(a; .; s);
\tag{12}$$

which is positive since $K(a; s; s)$ is a convex function in s by construction (it is the maximum of linear functions in s).¥

Proof of Proposition 2: As shown in the text, statements i) and ii) are equivalent. We need to show that $K_{as}(a; r; s)$ has the sign of $(r - s)$ if and only if $K_a(a; 0; s)$ decreases with s, or equivalently that $(s - r) K_{as}(a; r; s)$ has the sign of $K_{as}(a; 0; s)$. For any r and s, we have

$$K(a; r; s) = K(a; 0; s) + r K_r(a; .; s):$$

Hence

$$
\begin{aligned}
K_s(a; r; s) &= K_s(a; 0; s) + r K_{rs}(a; .; s) \\
&= K_s(a; 0; s) + \frac{r}{r - s} K_s(a; r; s), \text{ by equality (11)} \\
&= \frac{s}{s - r} K_s(a; 0; s):
\end{aligned}
$$

By di¤erentiating the last equality with respect to a we get the result.¥

Proof of Proposition 3: Since $K_a(a; s; s) = j_a(a; sp + (1 - s)q)$; it is immediate that $j_a(a; p)$ is convex in p if and only if $K_a(a; s; s)$ is convex in s. From (11) and (12), we have

$$(r - s) \frac{d^2 K(a; s; s)}{ds^2} = K_s(a; r; s)$$

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By differentiating this last equality with respect to a we get that $K_a(a; s; s)$ is convex in s if and only if $(s - r)K_{as}(a; r; s)$ is negative. Proposition 2 concludes.¥

Proof of Proposition 4: First recall that $b(a; (1 - s)p + sq)$ is the unique maximizer of

$$\sum [(1 - s)p(x) + sq(x)]U(x; a; b)$$

so that it is characterized by

$$\sum [(1 - s)p(x) + sq(x)]U_b(x; a; b(a; (1 - s)p + sq)) = 0: \qquad (13)$$

Differentiating with respect to s yields

$$\sum [q(x) - p(x)]U_b(x; a; b(a; (1 - s)p + sq)) + H(a; (1 - s)p + sq)\frac{@}{@s}b(a; (1 - s)p + sq) = 0: \qquad (14)$$

Using (13) once more we get

$$\sum [q(x) - p(x)]U_b(x; a; b(a; (1 - s)p + sq)) = -\frac{1}{s}\sum p(x)U_b(x; a; b(a; (1 - s)p + sq)):$$

Therefore

$$K_s(a; 0; s) = [\sum p(x)U_b(x; a; b(a; (1 - s)p + sq))]:\frac{@}{@s}b(a; (1 - s)p + sq)$$

$$= \frac{1}{s}[\sum p(x)U_b(x; a; b(a; (1 - s)p + sq))]'[H(a; (1 - s)p + sq)]^{-1}$$
$$[\sum p(x)U_b(x; a; b(a; (1 - s)p + sq))]:$$

Now saying that this quantity decreases with a, for any a, s, p, q, is equivalent to saying that

$$f(a; p; q) \;´\; [\sum p(x)U_b(x; a; b(a; q))]'[H(a; q)]^{-1}[\sum p(x)U_b(x; a; b(a; q))]$$

is decreasing with a, for any a, p, q.

Let us ...rst suppose that a is reduced by a difference in beliefs, so that iii) in Proposition 2 holds, and $f(a; p; q)$ is decreasing with a. Now suppose that $b(a; p) = b(a; q)$ at some (a, p, q). Then not only $f(a; p; q) = 0$, but also $f_a(a; p; q) = 0$ because all terms in the derivative vanish. Since anyway

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$f_a \cdot 0$ by assumption, then it must be that f_a is at its maximum value, so that $f_{aa} = 0$. Computing this second derivative, all terms vanish but

$$[\frac{@}{@a} \sum p(x) U_b(x; a; b(a; q))]^0 [H(a; q)]^{i\ 1} [\frac{@}{@a} \sum p(x) U_b(x; a; b(a; q))]$$

so that this term must be zero. Since $H^{i\ 1}$ is negative de...nite, we get

$$\frac{@}{@a} \sum p(x) U_b(x; a; b(a; q)) = 0: \tag{15}$$

So we have proven that $b(a; p) = b(a; q)$ implies

$$\frac{@b}{@a}(a; q) = \frac{@b}{@a}(a; p):$$

Then there exists a vector $d(a; b)$ such that

$$b(a; p) = b_0 \ ) \quad \frac{@b}{@a}(a; p) = d(a; b):$$

Another manner to rephrase our result is the following. $b(a; p) = b(a; q)$ means that

$$\sum p(x) U_b(x; a; b(a; q)) = 0:$$

De...ne $G(a; q)$ as the $N \ \pounds \ X$ matrix with typical line $U_b(x; a; b(a; q))^0$. Then we have

$$G(a; q)p = 0:$$

Now, from (15) we have shown that this implies

$$G_a(a; q)p = 0:$$

This implication is valid for any p. Therefore this means[18] that there exists a matrix $M(a; b)$ such that

$$G_a(a; q) = M(a; b)G(a; q):$$

This shows i).

---

[18] Lemma 1 below provides a formal proof of this point.

There remains to show ii). From iv) in Proposition 2, we know that $K_a(a;0;s)$ must decrease with s. Notice that

$$K_a(a;0;s) = \sum p(x)U_a(x;a;b(a;(1-s)p+sq))$$

$$+[\sum p(x)U_b(x;a;b(a;(1-s)p+sq))] \cdot \frac{@b(a;(1-s)p+sq)}{@a}$$

$$= \left[\sum p(x)[U_a + U_b \cdot d]\right](a;b(a;(1-s)p+sq)):$$

Di¤erentiating with respect to s yields

$$\left[\sum p(x)[U_{ab} + U_{bb}d + d_b U_b]\right] \cdot \frac{@}{@s}b(a;(1-s)p+sq) \cdot 0:$$

From i) the bracketted term is equal to $(M + d_b)U_b$; and the last term was computed in (14). Replacing we get ii).

Finally i) and ii) are clearly su¢cient, as the last paragraph has shown.¥

Lemma 1 Suppose two matrices A and B are such that
   *) for all weight p, Ap = 0 $\Rightarrow$ Bp = 0;
   **) there exists a weight q such that Aq = 0.
Then there exists a matrix M such that B = MA.

Proof of Lemma 1: given the weight q, any vector v can be written $v = \text{®}q + w$, with $\sum_x w(x) = 0$. Suppose that Av = 0. From **) we know that Aq = 0, so that we get Aw = 0. Now, for ¯ small enough $p ´ q + \bar{\ } w$ is a weight, and we have Ap = 0. From **), this implies that $Bp = 0 = Bq + \bar{\ }Bw$. From *) and **), we know that Bq = 0. Therefore we must have Bw = 0, and ...nally $Bv = \text{®}Bq + Bw = 0$. Therefore we have shown that for any vector v, Av = 0 implies that Bv = 0. This shows the Lemma, from a standard property of matrixes.¥

Proof of Proposition 5: The regulator's decision decreases with more distant beliefs if and only if $(s-r)L_{as}(a;r;s)$ is negative. We easily get, as in the proof of Proposition 1, that

$$L(a;r;s) = L(a;s;s) - (s-r)L_r(a;:;s);$$

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so that

$$L(a; r; s) = (r - s)L_{rs}(a; \cdot; s)$$
$$= (r - s)\frac{d^2L(a; s; s)}{ds^2}:$$

Hence, by differentiating with respect to a, we obtain that $(s - r)L_{as}(a; r; s)$ is negative if and only if $L_a(a; s; s)$ is convex in s. Notice that

$$L_a(a; s; s) = E_i j_a(a; (1 - s)p + sq_i):$$

It is then direct that $j_a(a; p)$ convex in p is a sufficient condition for $L_a(a; s; s)$ to be convex in s. It is also necessary at $s = 0$. By Proposition 3, we get the equivalence between i) and ii).

Now, we will prove the equivalence between i) and iii). Take $r = s$. Then for such a given r, the regulator's decision is lower in society i than in society i^0 if and only if

$$E_{i^0} j_a(a; (1 - s)p + sq_{i^0}) \geq E_i j_a(a; (1 - s)p + sq_i):$$

From inequality (10), a necessary and sufficient condition for the regulator's decision to decrease with more heterogeneous beliefs is thus, again, $j_a(a; p)$ is convex in p.∎
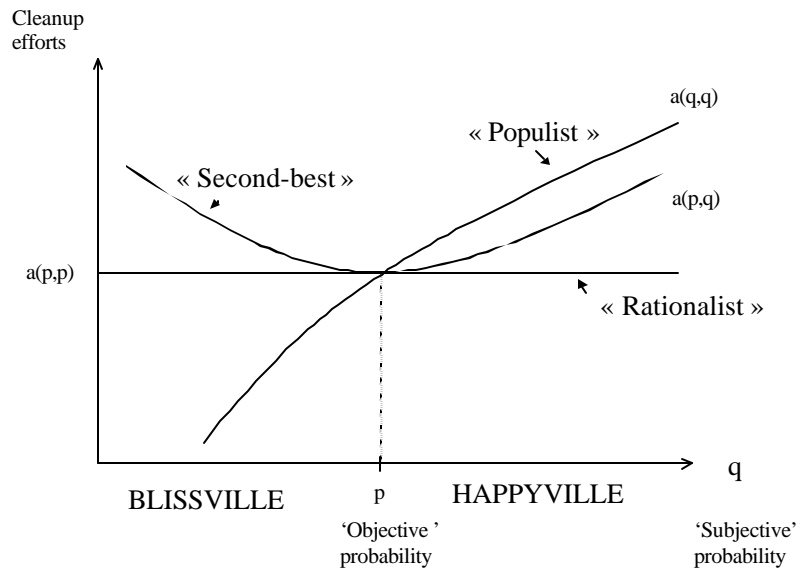
Figure 1:

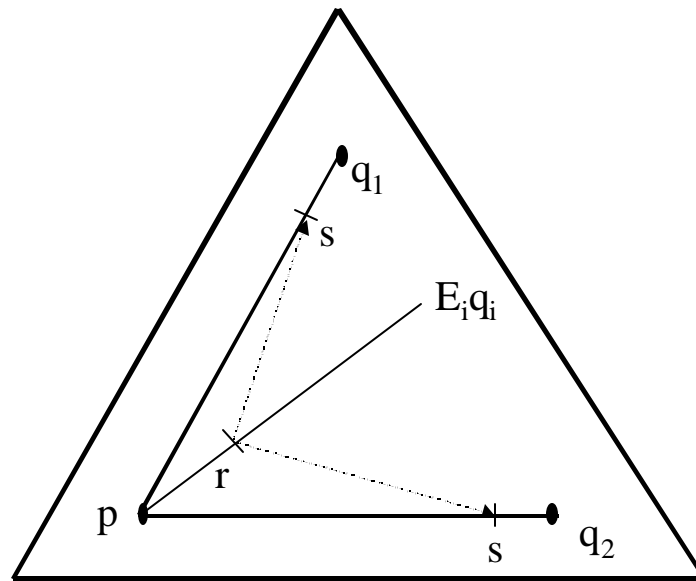Cleanup efforts as a function of beliefs for the three polar cases: functional forms given in the Introduction.

Figure 2:

"More di¤erent" beliefs for the three state of the world case: The regulator
has beliefs r; The two subgroups i = 1; 2 have beliefs s (upper s for group 1
and lower s for group 2).

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