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REVISITING OKUN'S LAW: AN HYSTERETIC PERSPECTIVE

BY

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An Hysteretic Perspective

By Yann Schorderet *

August 2001

Abstract

Many developed countries have suffered from high unemployment rates during the last few decades. Beyond the economic and social consequences of this painful experience, the understanding of the mechanisms underlying unemployment still constitutes an important challenge. Focusing on one of its relevant determinants, we reexamine the link between output and unemployment. We show that the difficulty of detecting the close relationship between the two is due to a phenomenon of non-linearity. The asymmetric feature characterizing the data refers to a theory known as hysteresis. (*JEL* C22, J64)

Keywords : Hysteresis, unemployment, non-linearity, cointegration.

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Many developed countries have suffered from high unemployment rates during the last few decades. Beyond the economic and social consequences of this painful experience, these often dramatic records once more raise some important issues. In particular, what are the driving forces of these bewildering processes? Does the unemployment rate actually obey an independent dynamics, free from the influence of any other macroeconomic indicators? Focusing on one of these theoretically relevant variables, we reexamine the link between output and unemployment. As both variables are affected by the business cycle, one should expect there exists a simple relationship between the two. It sounds clear that a strong growth causes unemployment to fall while low or negative growth rates are accompanied by increases in unemployment. However, the empirical assessments of this rule have been quite disappointing in reality to understand the level of unemployment.

The first contribution on the topic goes back to Okun's paper in 1962. The two equations related to this work have been called Okun's laws in the literature. The first argues that the unemployment rate u in deviation from its natural rate u^* is a fraction of the *output gap*, the deviation of the actual output y from its potential level y^*

$$(u - u^*) = \beta\,(y - y^*) \tag{1}$$

The second law links the variables after first differencing

$$\triangle u = \beta\,(\triangle y - \triangle y^*) \tag{2}$$

where $\triangle y^*$ denotes the trend growth of output. Okun's estimates claim that in the United States an output growth of 3 percent above its potential counterpart is required to reduce the unemployment rate by one percentage point. This result rests on the assumption that the equilibrium level of unemployment is about 4 percent.

Both equations are unable to explain the unemployment dynamics over the last thirty years. As a matter of fact, a survey of the literature leads to the following conclusions. On one hand, it seems difficult to get a suitable regression connecting the level of the two variables. On the other hand, differencing the data is not a rewarding option since the idea of any long run relationship, crucial to understand the unemployment level, has to be given up. On the whole, one could be tempted to conclude that the output series is not relevant in any way or at least, does not bring out much information concerning the unemployment process.

Claiming then that the two series might take unrelated paths or move away independently from each other is definitely not satisfactory. In the spirit of a recent theory commonly known as the hysteresis paradigm, we assert that the preceding diagnosis may be the consequence of an exclusive focus on linear models. In technical terms, while it sounds clear that the variables in question cannot be linearly cointegrated, they might do so in some non-linear way. Indeed, as we discuss below, the assumption of linearity is incompatible with the hysteresis hypothesis and we show that the output series can account for the unemployment levels provided that we take care of the asymmetric response of unemployment according to the sign of the output growth. A simple econometric model based on this general rule proves to be able to explain the American data over the last three decades.

The outline of the paper is as follows. In the next section, we briefly review the unemployment hysteresis paradigm. Section II presents a basic model resulting from this theory. Section III is devoted to empirical evidence while Section IV contains conclusions.

I. The Unemployment Hysteresis Paradigm

Since the famous paper of Milton Friedman in 1968, the economic theories about unemployment are mainly based on the assumption of a natural or equilibrium rate around which the actual levels are seen as temporary deviations. However, the discrepancy between theory and reality has led the economists to consider alternative hypotheses to account for the unemployment dynamics. Subsequent research on the field gave rise to the hysteresis theory based on the idea that cyclical shocks may affect the structural part of unemployment. Thus, today's unemployment depends fundamentally on past unemployment rates.

From a historical angle, the use of the term hysteresis has its beginnings in the 19th century when the physicist James Ewing employed the term to refer to some electromagnetic phenomena which persist although their actual causes have disappeared. In the context of unemployment,[1] the origin of the hysteresis debate can be traced back to at least some thirty years ago when Edmund Phelps suggested that the natural rate of Friedman may obey the same kind of dynamics mentioned by Ewing: past influences may be responsible for the actual level of the natural rate. It is arguably in 1986 that the theory gains popularity following the paper by Olivier Blanchard and Lawrence Summers. Faced with the persistence of high unemployment in Europe during the eighties, the authors argued that one could not understand the unemployment levels without referring to the path of unemployment and explain this dynamic dependence in the framework of the model of *insiders-outsiders* wage-setting.[2] Since then, several channels through which hysteresis effects may appear have been suggested. The models of depreciation of human capital show that long unemployment spell reduces the chances to find a job;[3] the theory of capital shortage claims that an hysteresis phenomenon will work if there is not enough substitution possibilities between labor and capital (Charles Bean, 1989, Hargreaves Heap, 1980).

Due to the importance of the issue, the hysteresis hypothesis has led to a bulk of papers aimed at assessing its empirical relevance. The approach suggested by Robert Gordon (1989) results directly from the theory expounded by Phelps. Some augmented Phillips curves are estimated in an attempt to show that the natural rate is a function of the past unemployment rate. Focusing on the mechanism of human capital depreciation, some authors give evidence that the shifts of the Beveridge curve can be related to the long-term unemployment (Alan Budd et al., 1987). In a similar way, a Beveridge curve framework has been explored to test whether in times of high unemployment the steady-state unemployment-vacancy relation bows out from its usual location. Besides, the hysteresis paradigm has been suggested to account for the embarrassing fact that the unemployment series seem to behave like unit root processes (Jorgen Elmeskov and Maitland MacFarlan, 1993, William Mitchell, 1993). Lastly, a few studies discuss the hysteresis hypothesis in the context of different non-linear representations such as threshold autoregressive models (David Peel and Alan Speight, 1995) and asymmetric ARMA formulations (Kurt Brännäs and Henry Ohlsson, 1995).[4]

We examine another way of detecting such a phenomenon. The intuition results from the following scenario. Consider an economy characterized by some unemployment rate u_0 and output level y_0. A recessionary shock perturbing the succeeding period causes the economic activity to fall ($y_1 < y_0$) while unemployment rises to u_1 ($u_1 > u_0$). When the economy stages a recovery in the next period, the output returns to its initial level ($y_2 = y_0$) and the unemployment decreases to u_2 but the belief that some hysteresis mechanisms are acting means that a part of cyclical unemployment generated by the past depression is converted into structural unemployment involving $u_2 > u_0$. In other words, the unemployment rate is stuck at a level above the one we would expect by reasoning in a linear way.

In conclusion, the hysteresis hypothesis implies an asymmetric response of unemployment according to the state of the economy. While unemployment strongly reacts to economic downturns, it slowly decreases in case of recoveries. This feature constitutes an important clue in the research of an hysteresis effect. If the previous scenario is true in some way, this non-linear property should be taken into account in the hope of explaining the unemployment series.

II. A Basic Model

In an attempt to assess the earlier reasoning, it is natural to start with the unemployment rate as a function of the real output level Y_t and any other relevant determinants captured by [5] Z_t

$$u_t = u_t(Y_t, Z_t)\,, \qquad \frac{\partial u_t}{\partial Y_t} < 0 \tag{3}$$

Assuming the effects of the explanatory variables after possibly transformations [6] are linear and additive, the relation becomes

$$u_t = \beta\, y_t + \tau'\, z_t\,, \qquad \beta < 0 \tag{4}$$

where β is the so-called Okun's coefficient while τ is the vector containing the parameters relative to z_t.

Suppose now that unemployment reacts differently according to the sign of the output growth

$$\beta = \begin{cases} \beta^+ & \text{if} \quad \triangle y_t > 0 \\ \beta^- & \text{if} \quad \triangle y_t < 0 \end{cases} \tag{5}$$

Under the hysteresis hypothesis, we expect the coefficient relative to an economic expansion to be smaller in absolute value than its counterpart

$$|\beta^+| < |\beta^-| \tag{6}$$

Unemployment change is therefore given by

$$\triangle u_t = \beta^+ \, I(\triangle y_t > 0)\triangle y_t + \beta^- \, I(\triangle y_t < 0)\triangle y_t + \tau' \, \triangle z_t \tag{7}$$

where $I(\triangle y_t > 0)$ and $I(\triangle y_t < 0)$ denote the indicator functions

(8)

$$I(\triangle y_t > 0) = \begin{cases} 1 & \text{if} \quad \triangle y_t > 0 \\ 0 & \text{if} \quad \triangle y_t < 0 \end{cases} \qquad I(\triangle y_t < 0) = \begin{cases} 0 & \text{if} \quad \triangle y_t > 0 \\ 1 & \text{if} \quad \triangle y_t < 0 \end{cases}$$

As $u_t = u_0 + \sum_{i=0}^{t-1} \triangle u_{t-i}$, we can write

$$u_t = \alpha + \beta^+ \, y_t^+ + \beta^- \, y_t^- + \tau' \, z_t \tag{9}$$

where $\alpha = u_0 - \tau' \, z_0$, $y_t^+ = \sum_{i=0}^{t-1} I(\triangle y_{t-i} > 0)\triangle y_{t-i}$, $y_t^- = \sum_{i=0}^{t-1} I(\triangle y_{t-i} < 0)\triangle y_{t-i}$.

Adding a stochastic disturbance to (9) to get an econometric model raises several issues. Assuming for simplicity that the variables Z_t remain constant through time and adopting the usual stylized facts concerning the output series, that is integrated of order one, two different interpretations can be drawn according to the characterization of the unemployment rate:

(i) regarding the latter as an integrated process,[7] the model can be seen as a cointegration relation, non-linear in the original variables u_t and y_t but linear in the specific regressors appearing in (9). This interpretation requiring y_t^+ and y_t^- to be integrated of the same order than the regressand, it can be shown that these peculiar series are intrinsically non-stationary as both a stochastic and a deterministic trend are part of their dominant properties.[8]

(ii) on the other hand, considering the unemployment rate as stationary leads to rewrite (9) as

$$u_t = \alpha + \beta^+ (y_t^+ - \gamma\, y_t^-) \tag{10}$$

where $\gamma = -\frac{\beta^-}{\beta^+}$. This specification still constitutes a balanced equation provided that $y_t^+ - \gamma\, y_t^-$ defines a stationary series. This implies in particular that the respective drift of y_t^+ and y_t^-, denoted m^+ and m^-, reflects the parameters βs in the following way [9] $\frac{m^+}{m^-} = -\frac{\beta^-}{\beta^+}$. On average, an expansionary shock should be proportional to a recessionary downturn, the factor of proportionality being the ratio of the two Okun's coefficients. Such a case may be seen as the consequence of some underlying forces such as the existence of an equilibrium of unemployment.

It is interesting to note that model (9) is able to account for some apparent 'shifts' in the plot of common macroeconomic variables such as those under consideration here. As an illustration, we reproduce in figure 1 the scatter plot of US real output in logarithm and the unemployment rate, when the latter is either the actual data or the predicted values resulting from the model fitted (see section III). The graphs clearly show that rather than moving back along the original path, the unemployment rate reacts in a different way whenever output comes to decrease. Accordingly, a greater level of output is required to restore the pre-shock unemployment level causing the mentioned 'shifts' in the relation investigated. In empirical modelling, this feature is often presumed to be the outcome of some deterministic function of time, yet such detrending methods will lead to spurious interpretations if the true structure is the phenomenon suspected.

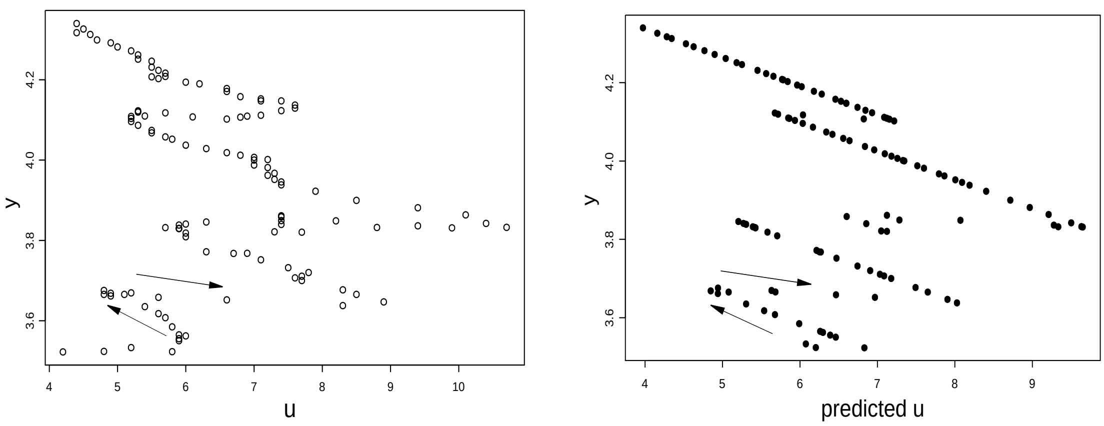

Figure 1. Actual and predicted scatter plots of US output and unemployment rate.

III. Some Empirical Evidence : The American Case

The sample consists on quarterly data [10] on the US unemployment rate and the logarithm of real GDP over the period 1970:I to 1998:IV. OLS estimates of model (9) are

$$u_t = \underset{(53.24)}{6.22} - \underset{(-22.96)}{13.65}\ y_t^+ - \underset{(-22.67)}{75.79}\ y_t^- \tag{11}$$

$$\bar{R}^2 = 0.82 \qquad DW = 0.23$$

where the t-statistics are found in parentheses and DW denotes the Durbin-Watson statistic. In order to test the cointegration hypothesis, critical values have been computed by simulations.[11] The results are summarized in table 1.[12] Hence, when testing the residuals for a unit root using the ADF tests, the latter commencing with four lags and testing down, we reject the null of no cointegration at the level of significance of 5 percent as the ADF(2) statistic is 3.81.

The error correction representation supports the cointegration hypothesis as the coefficient relative to the equilibrium deviation is significant at the 1 percent critical level. Below are reported the results where one lag of each series is included in the regression whereas only the lagged unemployment change appears to enter significantly beside the error correction term e_{t-1}

$$
(12) \qquad \underset{}{\triangle u_t =} \underset{(-2.6)}{-0.13\ e_{t-1}} + \underset{(5.43)}{0.56\ \triangle u_{t-1}} \underset{(-0.43)}{-1.17\ \triangle y^{+}_{t-1}} \underset{(-0.74)}{-6.35\ \triangle y^{-}_{t-1}}
$$

$$
\bar{R}^2 = 0.41 \qquad DW = 1.94
$$

In conclusion, the discrepancy in the Okun's coefficients gives strong evidence to the hysteresis theory. An economic expansion of more than 7 percent is required in the United States to cut unemployment by one percentage point in the long run while a downturn of only 1.3 percent induces an increase of one percent.

The preceding analysis considers the unemployment rate as an integrated process. This characterization, despite the outcome of empirical evidence, may not be acceptable on theoretical grounds. As mentioned above, the model does not require a prior classification of u_t in respect of I(0) or I(1) process and claiming that this variable is actually stationary does not disrupt the procedure provided that y_t^+ and y_t^- are cointegrated.[13] In order to assess such a condition, one may think of regressing each series on the other and test the null of no cointegration. Substituting the residuals into (10) will yield estimates of the Okun's coefficients. This reasoning provides the following results

$$y_t^+ = \underset{(0.07)}{0.001} - \underset{(-30.86)}{5.32} \; y_t^- \tag{13}$$

$$\bar{R}^2 = 0.89 \qquad DW = 0.07$$

$$u_t = \underset{(119)}{6.51} - \underset{(-22.21)}{13.65} \; w_t \tag{14}$$

$$\bar{R}^2 = 0.81 \qquad DW = 0.21$$

where w_t denotes the residual issued from (13). There is little support for cointegration. However, looking further into the series w_t and u_t leads to conclude that if we do admit that u_t is truly stationary, the same property should apply to w_t since the features of the two variables are very similar (see figure 2).[14] In particular, the autocorrelation functions shown in the appendix are fairly identical.

Figure 2. Residuals of y_t^+ on y_t^- and actual US unemployment rate.

IV. Conclusion

The present analysis tries to restore the idea of a clear interdependence between two macroeconomic indicators, output and unemployment. In this respect, this work can be seen as reviewing Okun's law in the light of a new theory and the resulting model as a possible reformulation of the equation suggested by the author. Obviously, any macroeconomic approach cannot identify one of the specific microeconomic foundations of the hysteresis paradigm. However, arguing that the non-linear feature underlying the data might not be the expression of an hysteresis effect implies an alternative explanation of this phenomenon. It is conceivable that some determinants neglected in the empirical specification may be responsible for the asymmetry detected. In particular, productivity shocks to aggregate supply could affect output without affecting unemployment, or output and unemployment could even move in the same direction. But dealing with gains in productivity usually means introducing some proxy in the modelling and may lead to spurious detrending. By contrast, the approach investigated does not rely on prior assumptions on the cyclical and trend components of the two series. Nevertheless, while output seems to be sufficient in itself to explain the evolution of the unemployment rate, a more general framework could include other theoretical determinants as explanatory variables. In this case, the model should be reinterpreted according to the statistical properties of the additional regressors. This would not alter the main conclusion of the paper.[15]

APPENDIX

A. *Statistical properties of y_t^+ and y_t^-*

We shall consider here the case of a random walk process for y_t. For convenience, we begin by assuming normality, that is $\triangle y_t = \varepsilon_t \sim IN(0;\sigma_\varepsilon^2)$. By construction $y_t = y_0 + y_t^+ + y_t^-$ where

$$y_t^+ = \sum_{i=0}^{t-1} I(\triangle y_{t-i} > 0)\triangle y_{t-i} = \sum_{i=0}^{t-1} \varepsilon_{t-i}^+ \tag{A1}$$

$$y_t^- = \sum_{i=0}^{t-1} I(\triangle y_{t-i} < 0)\triangle y_{t-i} = \sum_{i=0}^{t-1} \varepsilon_{t-i}^- \tag{A2}$$

$\varepsilon_t^+ = \max(\varepsilon_t, 0)$, $\varepsilon_t^- = \min(\varepsilon_t, 0)$. Well-known results on the censored normal distribution imply that $E(\varepsilon_t^+) = \frac{\sigma_\varepsilon}{\sqrt{2\pi}}$, $V(\varepsilon_t^+) = \frac{\sigma_\varepsilon^2}{2}\frac{\pi-1}{\pi}$. Therefore $\varepsilon_t^+ \sim iid\left(\frac{\sigma_\varepsilon}{\sqrt{2\pi}}; \frac{\sigma_\varepsilon^2}{2}\frac{\pi-1}{\pi}\right)$. Define now $v_t^+ = \varepsilon_t^+ - \dfrac{\sigma_\varepsilon}{\sqrt{2\pi}}$. We can write

$$y_t^+ = \sum_{i=0}^{t-1}\left(v_{t-i}^+ + \frac{\sigma_\varepsilon}{\sqrt{2\pi}}\right) = \frac{\sigma_\varepsilon}{\sqrt{2\pi}} \times t + \sum_{i=0}^{t-1} v_{t-i}^+ \tag{A3}$$

where $v_t^+ \sim iid\left(0\,;\, \frac{\sigma_\varepsilon^2}{2}\frac{\pi-1}{\pi}\right)$ stating that y_t^+ is a random walk process with drift. Obviously, a similar argument stands for y_t^- but in this case the drift coefficient is given by $-\frac{\sigma_\varepsilon}{\sqrt{2\pi}}$.

The preceding result does not require a specific distribution. More generally, the density function of ε_t^+ is given by

(A4) $$f(\varepsilon_t^+) = \begin{cases} P(\varepsilon_t \leq 0) & \text{if} \quad \varepsilon_t^+ = 0 \\ f(\varepsilon_t) & \text{if} \quad \varepsilon_t^+ > 0 \end{cases}$$

where $f(\varepsilon_t)$ denotes the density function of ε_t. Provided that the first two moments truncated in zero exist

(A5) $$E(\varepsilon_t^+) = P(\varepsilon_t > 0)\, E(\varepsilon_t|\varepsilon_t > 0)$$

(A6) $$V(\varepsilon_t^+) = P(\varepsilon_t > 0)\, \left[E(\varepsilon_t^2|\varepsilon_t > 0) - P(\varepsilon_t > 0)\, E^2(\varepsilon_t|\varepsilon_t > 0)\right]$$

we get $\varepsilon_t^+ \sim iid\left(E(\varepsilon_t^+)\,;V(\varepsilon_t^+)\right)$ with $E(\varepsilon_t^+) > E(\varepsilon_t) = 0$. y_t^+ is a random walk process with positive drift $E(\varepsilon_t^+)$.

Generalizing further, suppose now that y_t is a random walk with drift parameter [16] m. In this case

(A7) $$\varepsilon_t^+ = \begin{cases} m + \varepsilon_t & \text{if } m + \varepsilon_t > 0 \\ 0 & \text{otherwise} \end{cases}$$

y_t^+ is a random walk process with drift $m^+ = E(\varepsilon_t^+) > m$. Indeed

$$\begin{aligned} \text{(A8)} \qquad E(\varepsilon_t^+) &= P(m + \varepsilon_t > 0)\, E(m + \varepsilon_t|m + \varepsilon_t > 0) \\ \text{(A9)} \qquad E(m + \varepsilon_t) &= P(m + \varepsilon_t < 0)\, E(m + \varepsilon_t|m + \varepsilon_t < 0) + E(\varepsilon_t^+) \end{aligned}$$

hence

$$\text{(A10)} \qquad E(\varepsilon_t^+) - m = -P(m + \varepsilon_t < 0)\, E(m + \varepsilon_t | m + \varepsilon_t < 0) > 0$$

We conclude that when y_t is a random walk process, both y_t^+ and y_t^- can be considered as integrated of order one. The latter series contain a drift even if the former does not. The sum of the drift parameters of y_t^+ and y_t^- respectively is equal to the one of the original series y_t, that is zero if y_t is a pure random walk or m in the more general case.

B. *Autocorrelation functions of the US unemployment rate and the residuals resulting from regressing y_t^+ on y_t^-*

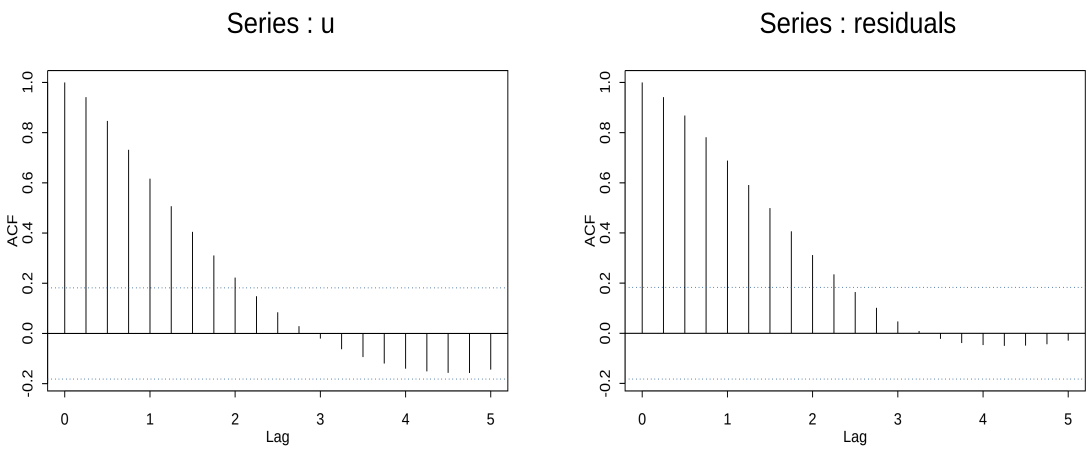

Figure 3. Autocorrelation functions.

The first ten sample autocorrelations are respectively 0.946, 0.853, 0.742, 0.625, 0.512, 0.408, 0.310, 0.218, 0.141, 0.074 for the unemployment rate and 0.942, 0.868, 0.781, 0.689, 0.591, 0.499, 0.406, 0.311, 0.234, 0.163 for the resid-

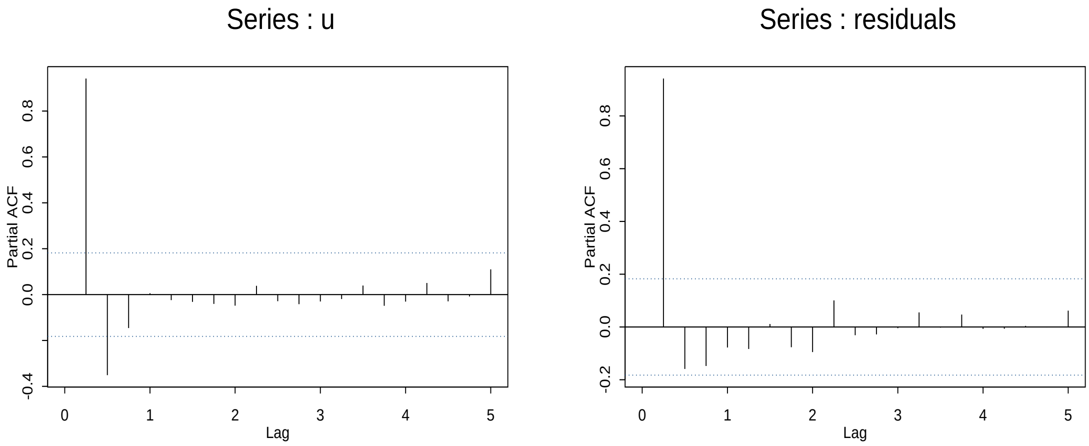

Figure 4. Partial autocorrelation functions.

uals.

C. *Data source*

The data come from the national government series, key indicators and have been downloaded from Datastream. The names of the variables with their corresponding mnemonics are

US GDP (AR) CURA (code: USGDP...B)
US IMPLICIT PRICE DEFLATOR FOR GDP SADJ (code: USIPDGDPE)
US TOTAL UNEMPLOYMENT RATE SADJ (code: USUNRATEE)

All the series are seasonally adjusted. The real GDP data used in this study are obtained by dividing the GDP series (USGDP...B) by its implicit price deflator (USIPDGDPE).

REFERENCES

Baldwin, Richard. "Hysteresis in Import Prices: The Beachhead Effect." *American Economic Review*, September 1988, 78(4), pp. 773-85.

Baldwin, Richard and Krugman, Paul. "Persistent Trade Effects of Large Exchange Rate Shocks." *Quarterly Journal of Economics*, November 1989, 104(4), pp. 635-54.

Barro, Robert J. "The Persistence of Unemployment." *American Economic Review*, May 1988, 78(2), pp. 32-37.

Bean, Charles R. "Capital Shortages and Persistent Unemployment." *Economic Policy: A European Forum*, April 1989, 0(8), pp. 11-53.

Blanchard, Olivier J. and Summers, Lawrence H. "Hysteresis and The European Unemployment Problem." National Bureau of Economic Research Working Paper No. 1950, June 1986.

Brännäs, Kurt and Ohlsson, Henry. "Asymmetric Cycles and Temporal Aggregation." Working paper, University of Umea, 1995.

Budd, Alan; Levine, Paul and Smith, Peter. "Long-term Unemployment and the Shifting U-V Curve: A Multi-country Study." *European Economic Review*, Feb.-Mar. 1987, 31(1/2), pp. 296-305.

Elmeskov, Jorgen and MacFarlan, Maitland. "Unemployment Persistence." *OECD Economic Studies*, Winter 1993, 0(21), pp. 59-88.

Engle, Robert F. and Granger, Clive W.J. "Co-integration and Error

Correction : Representation, Estimation and Testing." *Econometrica*, March 1987, 55(2), pp. 251-76.

Engle, Robert F. and Yoo, Byung S. "Forecasting and Testing in Co-integrated Systems." *Journal of Econometrics*, May 1987, 35(1), pp. 143-59.

Franz, Wolfgang. "Hysteresis in Economic Relationships : An Overview." *Empirical Economics*, 1990, 15(2), pp. 109-25.

Friedman, Milton. "The Role of Monetary Policy." *American Economic Review*, March 1968, 58(1), pp. 1-17.

Gordon, Robert J. "Hysteresis in History : Was There Ever a Phillips Curve?" *American Economic Review*, May 1989, 79(2), pp. 220-25.

Granger, Clive W.J. "Modelling Nonlinear Relationships Between Extended-Memory Variables." *Econometrica*, March 1995, 63(2), pp. 265-79.

Hargreaves Heap, S. P. "Choosing the Wrong 'Natural' Rate: Accelerating Inflation or Decelerating Employment and Growth?" *Economic Journal*, September 1980, 90(359), pp. 611-620.

Jaeger, Albert and Parkinson, Martin. "Some Evidence on Hysteresis in Unemployment Rates." *European Economic Review*, February 1994, 38(2), pp. 329-42.

Mitchell, William F. "Testing for Unit Roots and Persistence in OECD Unemployment Rates." *Applied Economics*, December 1993, 25(12), pp. 1489-1501.

Neftçi, Salih N. "Are Economic Time Series Asymmetric over the Business Cycle?" *Journal of Political Economy*, April 1984, 92(2), pp. 307-28.

Nelson, Charles R. and Plosser, Charles D. "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications" *Journal of Monetary Economics*, September 1982, 10(2), pp. 139-62.

Okun, Arthur. "Potential GNP : Its Measurement and Significance." *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*, 1962, pp. 98-104.

Pissarides, Christopher. "Unemployment and Vacancies in Britain." *Economic Policy: A European Forum*, October 1986, 3, pp. 500-48.

Peel, David and Speight, Alan. "Non-Linear Dependence in Unemployment, Output and Inflation : Empirical Evidence for the UK," in Rod Cross, ed., *The Natural Rate of Unemployment: Reflections on 25 Years of the Hypothesis*, Cambridge; New York and Melbourne: Cambridge University Press, 1995, pp. 231-55.

Phelps, Edmund S. *Inflation policy and unemployment theory.* London; Basingstoke: Macmillan, 1972.

Schorderet, Yann. "Econometric Modelling of Hysteresis in Unemployment." Ph.D. dissertation, University of Geneva, 2000.

Sims, Christopher A.; Stock, James H. and Watson, Mark W. "Inference in Linear Time Series Models with Some Unit Roots." *Econometrica*, January 1990, 58(1), pp. 113-44.

Notes

[1]In other fields of economics, hysteresis effects have been pointed out in international trade. Richard Baldwin (1988), Baldwin and Paul Krugman (1989) discuss this theory in the case of the exchange rate, when a change in the exchange rate that is later exactly reversed nonetheless leaves a long-term impact on the trade account.

[2]It is assumed that employed union members, the *insiders*, set wages so as to maintain just their own employment rather than so as to care for the unemployed, the *outsiders*. Therefore, any adverse shocks which reduce the number of insiders will raise unemployment in a permanent way as no tendency to return to its previous level.

[3]If firms use unemployment experience itself as a screening device, then unemployed persons with a long duration of unemployment are viewed as the less promising candidates. On the other side, the search process long-term unemployed may reduce their search intensity due to their discouragement as a consequence of being so often rejected by employers.

[4]Note that the suspicion of non-linearities in the unemployment process has a long history in the literature. The foundation stone is certainly due to Salih Neftçi (1984) who suggests the same kind of asymmetric feature we shall discuss in the paper, although the methodology as well as the purpose of the analysis are different.

[5]This definition may be seen as resulting from inverting a macro-production function. In this respect, one may think of the capital stock, the capacity utilization rate and the level of productivity as some of the variables appearing in Z_t.

[6]The transformed variables will be noted in small caps. In that way, y_t is usually the logarithm of real GDP.

[7]While Charles Nelson and Charles Plosser (1982) found that the unemployment rate was the only time series to reject the unit root hypothesis, this characterization seems not to be valid any more in the light of more recent work.

[8]See the appendix for the case when y_t is a random walk.

[9]As stated in (i), both series contain a deterministic trend. Therefore, $E(y_t^+ - \gamma\, y_t^-) = (m^+ + \frac{\beta^-}{\beta^+}\, m^-)\, t$.

[10]See the appendix for the explanation of the data and the data source.

[11]Even though the model may be interpreted in some way as an usual cointegration relation, two of the three series making up the system result from the decomposition of the same variable. Consequently, they are strongly dependent and it would be misleading into relying on critical values assuming three independent processes as tabulated in Robert Engle and Byung Yoo (1987).

[12]As we might expect, the values are slightly different from those tabulated in Engle and Clive Granger (1987), Engle and Yoo (1987). In a similar manner,

the CRDW and DF statistics are very sensitive to the underlying data generating process while the critical values for the ADF statistics remain relatively unchanged.

[13]Regressing an I(0) on two I(1) variables does not yield an unbalanced regression in this case. To clarify this point, consider the ARI(1,1) model $\triangle x_t = \psi \triangle x_{t-1} + \zeta_t$ rewritten as $\triangle x_t = \psi_1 x_{t-1} + \psi_2 x_{t-2} + \zeta_t$ where $\psi_1 = -\psi_2 = \psi$. The existence of a cointegration relation amongst the regressors guarantees the usual asymptotic properties of the OLS estimates (see Christopher Sims et al., 1990).

[14]To make the comparison easier, figure 2 depicts the inverse residuals, that is $-(y_t^+ - 0.001 + 5.32\, y_t^-)$.

[15]Note for instance that the estimate of the model augmented by a deterministic time trend as a proxy of productivity provides similar conclusions about the discrepancy in the Okun's coefficients.

[16]As y_t represents the GDP series, we will consider $m > 0$.

Table 1. Critical values for the no cointegration tests

	$\psi = 0$			$\psi = 0.4$			$\psi = 0.8$		
	Significance level [a]			Significance level			Significance level		
Statistic	1	5	10	1	5	10	1	5	10
CRDW	0.64	0.50	0.43	0.33	0.25	0.21	0.14	0.10	0.08
DF	4.51	3.91	3.58	3.55	2.89	2.65	3.98	2.84	2.34
ADF(1)	4.49	3.86	3.54	4.48	3.85	3.55	4.43	3.83	3.52
ADF(2)	4.37	3.79	3.47	4.36	3.77	3.47	4.38	3.76	3.45
ADF(4)	4.23	3.65	3.36	4.26	3.66	3.37	4.19	3.66	3.39

Notes : The critical values are computed by generating 100 observations of $\triangle y_t = \psi \triangle y_{t-1} + \varepsilon_{yt}$, $\triangle u_t = \psi \triangle u_{t-1} + \varepsilon_{ut}$ where ε_{yt}, ε_{ut} are independent standard normal, 10,000 replications. CRDW denotes the DW statistic of the regression of u_t on y_t^+, y_t^- and a constant. Noting $\hat{\epsilon}_t$ the OLS residual, the DF statistic is the absolute t-ratio of the OLS estimate of ρ in $\triangle \hat{\epsilon}_t = \rho\, \hat{\epsilon}_{t-1} + v_t$ while ADF(k) refers to the augmented regression $\triangle \hat{\epsilon}_t = \rho\, \hat{\epsilon}_{t-1} + \sum_{i=1}^{k} \phi_i \triangle \hat{\epsilon}_{t-i} + v_t$, $k = 1, 2, 4$. Calculations were undertaken using Splus on Sun Microsystems.

[a] In percent.