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Abstract

This paper develops a theoretical framework for studying contract and enforcement in settings with nondurable trading opportunities and complete but unverifiable information. The framework explicitly accounts for the parties' individual trade actions. The sets of implementable state-contingent payoffs, under various assumptions about renegotiation opportunities, are characterized and compared. The results indicate the benefit of modeling trade actions as individual, rather than as public, and they highlight the usefulness of a structured game-theoretic framework for applied research. *JEL Classification: C70, D74, K10.*

Economic models of contract have yielded important insights on the implications of imperfections in the contracting environment. Many of the insights derive from mechanism-design analysis—a methodology whose elegance relies on stripping away technological detail and focusing on a few fundamental strategic ingredients. To the extent that technological constraints play a critical role in the formation and performance of contracts, however, it is important to develop ways of incorporating these constraints into models.

One issue that has received a great deal of attention is the possibility that parties can *renegotiate* in the midst of a contractual relationship. Hart and Moore (1988), for example, showed how renegotiation following specific, unverifiable investments can inhibit the parties' ability to induce optimal investment. Recently, researchers have settled on a particular mechanism-design formulation for the analysis of contracting with renegotiation. In this formulation, the parties write a contract and then make unverifiable investments and/or learn the resolution of uncertainty, which determines the commonly-known state of the world. Afterward, the parties interact in the mechanism that their contract dictates. The outcome of the mechanism is a specification of a monetary transfer and trade action, such as

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"the number of units delivered by the seller" or "whether the buyer accepts delivery". The typical setting features *ex post* renegotiation, where the outcome of the mechanism can be renegotiated before the trade actions are taken. Maskin and Moore (1999) provide general characterization results, building from Maskin's (1999) work on Nash implementation.

In this paper, I study how renegotiation opportunities interact with the technology of trade in contractual relationships. I demonstrate the value of precisely modeling the technology of trade and I develop a framework to facilitate this practice in applied work. In comparison with the related literature, the key issue is whether trade actions are modelled as *individual actions* or as *public actions*. An individual action is one taken directly by one of the contracting parties, whereas a public action is one taken directly by an external enforcement authority. Much of the recent contract-theory literature deals with public-action mechanism-design models.[1] My framework treats trade actions as individual and inalienable, taking the view that this is the case in many real settings.[2] I show how, in this context, public-action models can be interpreted as restricting attention to "forcing contracts." Furthermore, limiting the parties to forcing contracts can constrain implementation.

The modeling framework presented here explicitly accounts for the timing and nature of individual trade actions and the manner in which an external enforcer compels behavior. I focus on settings with complete but unverifiable information, verifiable trade actions, and nondurable trading opportunities (where there is a fixed date at which irreversible trade actions must be made). I characterize the sets of implementable outcomes under a variety of assumptions about when renegotiation can take place and whether parties are restricted to use forcing contracts. I show that, in settings where parties can renegotiate only at the interim stage (after the state is realized but before sending messages to the external enforcer), individual- and public-action models are equivalent. However, in settings where the parties can renegotiate ex post (after sending messages), limiting attention to forcing contracts can impose a significant constraint on implementation.

To see why the modeling of trade actions matters, consider a contractual setting represented by the partial time line shown in Figure 1; a complete time line appears in the next section. Assume that at Date 6 an irreversible trade action must be taken. Also suppose that the parties can renegotiate their contract at Date 5 (ex post). If the trade action is modelled as public then it is assumed to be chosen by some external enforcer who simply executes the terms of a contract in force at Date 6. In this setting, the contracting parties direct the trade action through their contract and through messages they send to the external enforcer at Date 4. Then, at Date 5, the parties will know whether the action to be taken by the

[1] Prominent examples include the work of Che and Hausch (1999), Edlin and Reichelstein (1996), Maskin and Moore (1999), Segal (1999), and Segal and Whinston (2002). Myerson (1991) uses the term "collective-choice problem" to describe public-action models.

[2] Individual-action models have been studied previously by Hart and Moore (1988), MacLeod and Malcomson (1993), and Nöldeke and Schmidt (1995), among others. Also relevant is the work of Myerson (1982,1991), whose mechanism design analysis nicely distinguishes between inalienable individual and public actions.

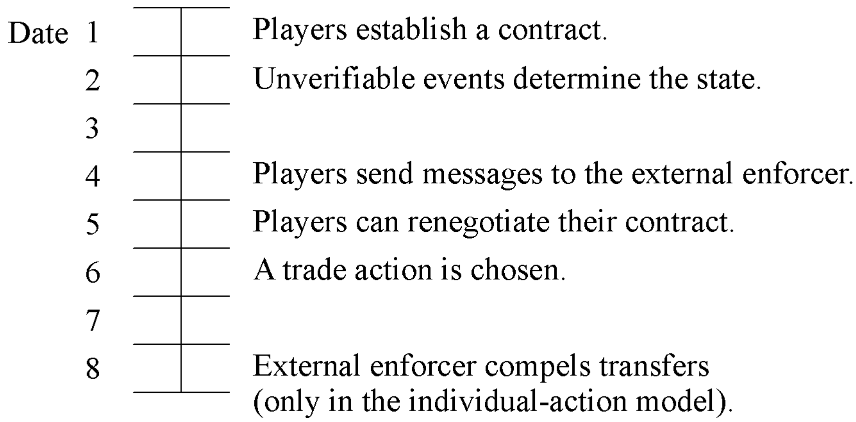

Figure 1: Partial time line of a contractual relationship.

enforcer is efficient in the current state of the world. If it is not efficient, the parties will renegotiate the contract to achieve an efficient outcome. Importantly, an efficient outcome is realized regardless of the parties' behavior at earlier dates (in or out of equilibrium).

Alternatively, suppose that the trade action is modelled as an individual action taken by one of the parties. In this case, the contract specifies monetary transfers between the parties (compelled at Date 8) as a function of the trade action and the messages sent earlier. By using forcing contracts, it is possible to duplicate the results of treating the trade action as public, because the contract can specify transfers that induce any particular action regardless of the state of the world. However, other contracts may implement outcomes that would not be implementable in the model with the public trade action. The reason is that renegotiation at Date 5 concerns only the *equilibrium* trade action at Date 6; there is no requirement that every selection that could be made at Date 6 must result in an efficient outcome, because there is no "time left" to renegotiate an inefficient trade action chosen at Date 6.

Option contracts provide a practical illustration. Observe that there are two ways of designing an option contract. In one form, the option entails a message that one of the parties sends at Date 4; this message triggers a response by the external enforcer that forces the players to choose a particular trade action. In the other form, the individual trade action *itself* serves as an option, with the external enforcer simply compelling transfers as a function of this action.[3] The latter option form is not available when trade actions are treated as public and this makes a difference if renegotiation is possible at Date 5. Thus, in

[3]Public-action mechanism-design models study options of the first type; more structured models, such as Nöldeke and Schmidt's (1995), focus on the second type. The law treats option generally, as a limit on a parties "power to revoke an offer" (Section 25, *Restatement (Second) of Contracts*; see Barnett 1999). In addition to more conventional forms, options are implicitly created by liquidated damage provisions and standard breach remedies. A party, for example, has the option of breaching and then paying the damage amount.

settings with ex post renegotiation, treating individual trade actions as public may impose an artificial restriction on the set of contracts. On the other hand, in settings with only interim renegotiation (at Date 3), public- and individual-action models are equivalent.

The general modeling framework is described in the next section. Section 2 contains definitions and analysis that are useful for representing the parties' contracting problem as a mechanism-design problem. Section 3 defines and characterizes implementation for various settings (differentiated by if and when renegotiation can take place). In Section 4, I present the analysis of the specific example introduced in Section 1. This example illustrates, and supplies intuition for, my general analysis.

In Section 5, I prove a theorem that ranks by inclusion the sets of implementable state-contingent payoffs under various assumptions about renegotiation. I also provide theorems that give conditions under which the inclusion relations are strict; under these conditions, individual-action models with ex post renegotiation yield different results than do public-action models (in both the cases of ex post and interim renegotiation). Section 6 contains a discussion of related modeling issues, comments on the related literature, and concluding remarks. Proofs of the lemmas are contained in the appendix.

1 The Theoretical Framework

Two contracting parties, whom I call "players 1 and 2," engage in a contractual relationship with external enforcement. Their relationship has the following payoff-relevant components, occurring in the order shown:

The *state of the relationship* θ. The state represents unverifiable events that are assumed to happen early in the relationship. The state may be determined by individual investment decisions and/or by random occurrences, depending on the setting. When the state is realized, it becomes commonly known by the players; however, it cannot be verified to the external enforcer. Let Θ denote the set of possible states.

The *trade actions* $a = (a_1, a_2)$. This is a profile of individual, inalienable actions that the players choose, determining whether and how the relationship is consummated. The trade actions are commonly observed by the players and are verifiable to the external enforcer. I assume that a is an element of the set $A \equiv A_1 \times A_2$, where A_1 is the feasible set of actions for player 1 and A_2 is the feasible set of actions for player 2. I assume that the players select their trade actions simultaneously and independently.

The *monetary transfers* $t = (t_1, t_2)$. Here t_i denotes the amount given to player i, for $i = 1, 2$, where a negative value represents an amount taken from this player. These transfers are compelled by the external enforcer, who is not a strategic player but,

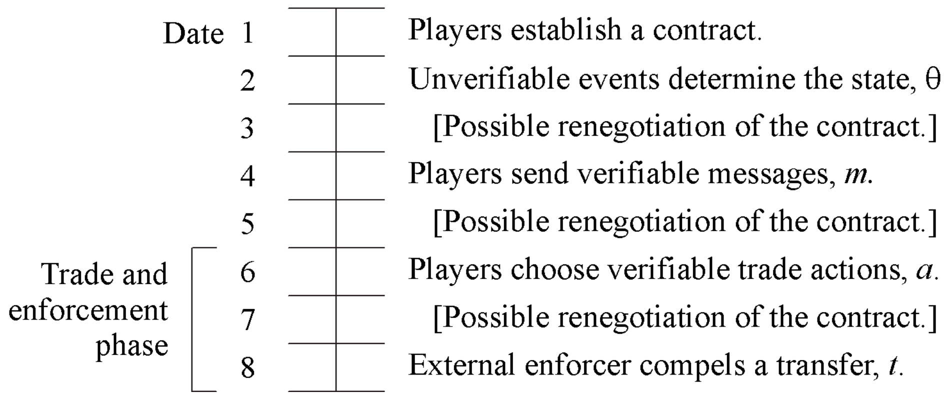

Figure 2: The contractual relationship.

rather, who behaves as directed by the contract of players 1 and 2.[4] Assume $t_1 + t_2 \leq 0$.

I assume that the players' payoffs are additive in money and are thus defined by a function $u : A \times \Theta \to \mathbf{R}^2$. In state θ, with trade action a and transfer t, the payoff vector is $u(a, \theta) + t$. I assume that u is bounded and that the maximal joint payoff, $\max_{a \in A}[u_1(a, \theta) + u_2(a, \theta)]$, exists for every θ. It will not be necessary to put any restrictions on the sets A and Θ.

In addition to the payoff-relevant components of their relationship, I assume that the players can communicate with the external enforcer using public, verifiable messages. Let $m = (m_1, m_2)$ denote the profile of messages that the players send and let M_1 and M_2 be the sets of feasible messages. The sets M_1 and M_2 will be endogenous in the sense that they are specified by the players in their contract.

I focus on nondurable trading opportunities, meaning that there is a fixed date at which the trade actions are chosen. This date is designated as "Date 6" in Figure 2, which shows the time line of the contractual relationship. At even-numbered dates through Date 6, the players make joint observations and they make individual decisions—jointly observing the state at Date 2, sending verifiable messages at Date 4, and selecting the trade actions at Date 6. At Date 8, the external enforcer compels transfers.

At odd-numbered dates, the players make joint contracting decisions—establishing a contract at Date 1 and possibly renegotiating it later. The contract has an externally-enforced component consisting of (i) feasible message spaces M_1 and M_2 and (ii) a function $y : M \times A \to \mathbf{R}^2$ specifying the transfer t as a function of the verifiable items m and a. That is, having seen m and a, the external enforcer compels transfer $t = y(m, a)$. I call y the *transfer function*. The contract also has a self-enforced component, which specifies how the players coordinate their behavior for the times at which they take individual actions. Renegotiation of the contract amounts to replacing the original transfer function y

[4]That the external enforcer's role is limited to compelling transfers is consistent with what courts do in practice.

with some new function y', in which case y' is the one submitted to the external enforcer at Date 8.

I model rational behavior in the contractual relationship as follows. The players' individual actions at Dates 4 and 6 are assumed to be consistent with sequential rationality; that is, each player maximizes his expected payoff, conditional on what occurred earlier and on what the other player does, and anticipating rational behavior in the future. The joint decisions (initial contracting and renegotiation at odd-numbered periods) are assumed to be consistent with a "black-box" cooperative bargaining solution in which the players divide surplus according to fixed bargaining weights π_1 and π_2 for players 1 and 2, respectively. The bargaining weights are nonnegative, sum to one, and are written $\pi = (\pi_1, \pi_2)$. Surplus is defined relative to a disagreement point. More details are given later in this section. The rationality conditions identify a *contractual equilibrium*; see Watson (2004) for a general definition of this concept and for a discussion of the relation between "cooperative" and "noncooperative" approaches to modeling negotiation.[5]

The effect of the renegotiation opportunity at Date 7 can be incorporated at this point by constraining transfers to be "balanced"—that is, $t_1 + t_2 = 0$. Renegotiation at Date 7 will occur in a given contingency if and only if the contract specifies unbalanced transfers ($t_1 + t_2 < 0$) for this contingency. Further, if players renegotiate at Date 7 then they will select a contract that has balanced transfers (to get the surplus $-t_1 - t_2$). Because the players anticipate that renegotiation results in a fixed split of the surplus, and therefore know that any particular unbalanced specification will become a particular balanced one, we can assume that the players choose balanced transfer functions earlier. Thus, I represent the implication of Date 7 renegotiation by assuming that the transfer function satisfies $y(m, a) \in \mathbf{R}^2_0$, where

$$\mathbf{R}^2_0 \equiv \{t \in \mathbf{R}^2 \mid t_1 + t_2 = 0\},$$

and I can disregard Date 7 interaction hereinafter. This is a common theoretical step in the related literature.

A *(state-contingent) value function* is a function $v : \Theta \to \mathbf{R}^2$ that gives the players' expected payoff vector from the start of Date 3, as a function of the state. An *implementable* value function is that which results from rational behavior for some contract selected at

[5] Because the players are risk neutral in money, the cooperative solution yields the same expected payoffs as does the following non-cooperative specification of negotiation: Nature selects one of the players to make an ultimatum offer to the other player, who either accepts or rejects it; Nature selects player i with probability π_i. We assume that (i) on the self-enforced component of contract, players behave as agreed whenever this is consistent with individual rationality; and (ii) if an offer is rejected, then the equilibrium in the continuation of the game does not depend on the identity of the offerer or on the nature of the offer. These are the Agreement and Disagreement Conditions described in Watson (2004). See also Watson (2002a) for an introduction to game-theory models of contract. Fixed bargaining weights capture the idea that renegotiation activity is non-contractible, so that the parties can exercise bargaining power and hold each other up during the relationship. This assumption is realistic for many applied settings and it is a key ingredient of most recent contract models in the literature.

Date 1. Formal definitions are in Section 3. The main theoretical exercise is to determine the set of implementable value functions. This set depends on whether renegotiation is possible at Dates 3 or 5.

Calculating the set of implementable value functions is important because the players and society have preferences over them. In particular, consider settings in which the players make unverifiable "ex ante" investments at Date 2—investments that determine the state. In such settings (generally called *hold-up problems*), a player cannot be rewarded or punished directly as a function of his investment choice or of the state, because the external enforcer does not observe the state. Instead, a player's incentive to invest at Date 2 is closely tied to the value function that is implemented, but the value function is constrained by the prospect of renegotiation in future dates. The example in the next subsection is a hold-up problem. Segal and Whinston (2002) provide a good overview of prominent hold-up models in the recent literature.

Example

Here is a simple numerical example that illustrates the model's components. The example is analyzed in detail in Section 4. Player 1 is the buyer of an intermediate good, player 2 is the seller, and the external enforcer is the court. To be concrete, imagine that the buyer is a masonry supply company that hopes to gain new customers at a regional trade show. The seller is an advertisement agency. The buyer wishes to hire the seller to develop an advertisement package for the trade show.

The set of states is $\{H, L\}$, where H indicates the "high" state in which the advertisement package will be successful and L denotes the "low" state in which the advertisement will not be successful. The state is determined by an investment that one of the players makes at Date 2. I will consider two versions of the example. In the first version, it is the buyer who makes this investment; think of it as effort that player 1 exerts to evaluate his downstream market and to provide information to player 2. In the second version of the example, the seller makes the investment; imagine it as player 2's effort to learn about player 1's business and downstream market. I assume that the effort choice is binary (either "exert" effort or "not"), that "exert" entails an immediate cost of c in monetary units, and that the high state is realized if and only if "exert" is chosen.

Suppose that the trade action is the buyer's choice of whether to adopt the advertisement package. Specifically, let $A_1 = \{1, 0\}$ and $A_2 = \emptyset$, where $a = 1$ indicates that the buyer adopts the advertisement and $a = 0$ indicates that he does not adopt it. The buyer's decision to adopt the advertisement can also be described as "the buyer consummates the trade" or "the buyer accepts delivery." The trading opportunity is nondurable; in other words, the buyer's action of whether to adopt the advertisement cannot be reversed or delayed.[6]

[6]The buyer must choose his trade action just before the trade show begins, at Date 6. After the trade show, there is no use for the advertisement and there is no way to undo the advertisement if it was adopted.

Above any effort costs, the payoffs are defined as follows. In state H, if the buyer adopts the advertisement package and is compelled to make a monetary transfer p to the seller, then the buyer gets $5 - p$ and the seller gets $3 + p$. The buyer's value of 5 is the profit generated by a successful advertisement. The seller's value of 3 reflects the extra profit the advertising agency will receive from future clients due to its public success with the masonry firm. In state H, if the buyer decides not to adopt the advertisement package yet transfers p to the seller, then the buyer gets $-p$ and the seller gets p. In state L, the advertisement package is worthless to both the buyer and the seller; in this case, regardless of whether the buyer adopts the advertisement, the payoffs are simply the players' monetary transfers. In the notation of the general model, we thus have $u(1, \mathrm{H}) = (5, 3)$ and $u(0, \mathrm{H}) = u(1, \mathrm{L}) = u(0, \mathrm{L}) = (0, 0)$. Note that the effort cost c is not included in these expressions.

For this example, I assume that the bargaining weights are $\pi_1 = \pi_2 = 1/2$, so the players share equally any gains from renegotiation. Assume that $c \in (0, 8)$, which implies that "exert" is the efficient effort level at Date 2 (leading to the high state in which the players obtain a joint value of 8 when the buyer accepts delivery). To have a successful relationship, the parties must design a contract at Date 1 that will align their incentives to invest at Date 2. This critically depends on the set of implementable value functions. In the case in which the buyer makes the effort choice at Date 2, he will exert effort only if the implemented value function satisfies $v_1(\mathrm{H}) - c \geq v_1(\mathrm{L})$. This requires $v_1(\mathrm{H}) - v_1(\mathrm{L})$ to be as large as 8, depending on c. In the other case, where the seller makes the effort choice, the seller has the incentive to exert effort only if $v_2(\mathrm{H}) - c \geq v_2(\mathrm{L})$, requiring $v_2(\mathrm{H}) - v_2(\mathrm{L})$ to be as large as 8.

2 Preliminaries for Mechanism-Design Analysis

This section describes how the contractual relationship can be represented in terms of a standard mechanism-design problem. The form of the mechanism-design problem depends on when renegotiation can occur and on whether one treats trade actions as public actions. I first express outcomes of rational behavior from Date 6 as state-contingent payoffs and I use these to write the players' contracting problem. I then discuss the notion of a "forcing contract" and its relation to the treatment of trade actions as public. Finally, I define some notation for describing renegotiation.

Outcomes of the Trade and Enforcement Phase

It is useful to consider the state-contingent payoff vectors that can be achieved from the beginning of Date 6 (the "trade and enforcement phase" shown in Figure 2), for a fixed message profile m. The set of achievable state-contingent payoff vectors is clearly independent of m, because the message profile is not payoff-relevant. Thus, for the sake of calculating feasible state-contingent payoffs from Date 6, I can ignore m and write the externally enforced transfer function as $\hat{y} : A \to \mathbf{R}^2$, where $\hat{y} \equiv y(m, \cdot)$.

Given the state θ, $\hat{y}$ defines a *trading game*, in which the space of action profiles is A and the payoffs are given by $u(\cdot, \theta) + \hat{y}(\cdot)$. I focus on pure-strategy Nash equilibria of the trading game.[7] Let $\hat{a}(\theta)$ denote the equilibrium action profile that is chosen by the players in state θ. The state-contingent payoff vector from Date 6 is thus given by the function $w : \Theta \to \mathbf{R}^2$ defined by

$$w(\theta) \equiv u(\hat{a}(\theta), \theta) + \hat{y}(\hat{a}(\theta)). \tag{1}$$

I use the term *outcome* for any such function from Θ to $\mathbf{R}^2$. Think of an outcome, therefore, as a state-contingent payoff that results from interaction in the trade and enforcement phase.[8] The set of outcomes is:

$$W \equiv \Big\{ w : \Theta \to \mathbf{R}^2 \;\Big|\; \text{Functions } \hat{y} \text{ and } \hat{a} \text{ exist such that, for every } \theta \in \Theta, \text{ Equation 1 holds and } \hat{a}(\theta) \text{ is a Nash equilibrium of the game } \langle A, u(\cdot, \theta) + \hat{y}(\cdot) \rangle \Big\}. \tag{2}$$

Forcing Contracts and the Alienability Issue

Because trade actions are verifiable, the external enforcer can effectively force the players to choose any particular trade action independent of the state. This can be done using a *forcing contract*, which specifies (i) a large transfer from player 1 to player 2 in the event that player 1 does not take his contractually-specified action, and (ii) a large transfer in the other direction if player 2 does not take her contractually-specified action. For instance, in the example described in the previous section, the buyer can be forced to adopt the advertisement by a contract that specifies the transfer vector $(-p, p)$ if the buyer adopts and $(-p - 6, p + 6)$ if the buyer does not adopt, for any given number p. Regardless of the state, the buyer then has a strict incentive to adopt the advertisement. With this contract for a given message profile, the physical outcome from Date 6 will be adoption of the advertisement and a transfer of p from the buyer to the seller.

[7]Allowing mixed strategy Nash equilibria may expand the set of implementable value functions, but it is not an issue for the example presented here and, more generally, it would only reinforce the main point of this paper.

[8]This should be differentiated from the "trade outcome," which describes the physical trade action and monetary transfer.

In general, suppose the players want to force themselves to play action profile a^* and have transfer t^*, regardless of the state.[9] This can be accomplished by specifying $\hat{y}$ as follows. Let L be such that $L > \sup_{a,\theta} u_i(a,\theta) - \inf_{a,\theta} u_i(a,\theta)$ for $i = 1, 2$. Then, for $i = 1, 2$ and every $a_i \neq a_i^*$, set $\hat{y}_i(a_i, a_j^*) \equiv t_i^* - L$ and $\hat{y}_j(a_i, a_j^*) \equiv t_j^* + L$; for every other action profile a, set $\hat{y}(a) \equiv t^*$. Then a^* is the only Nash equilibrium of the trading game in every state.

Definition 1: *The transfer function $\hat{y}$ is called* **forcing** *if there is a unique Nash equilibrium of the trading game $\langle A, u(\cdot,\theta) + \hat{y}(\cdot)\rangle$ and this equilibrium is independent of the state.*

Let W^{F} be the subset of outcomes that can be supported using forcing contracts. It is easy to see that

$$W^{\mathrm{F}} \equiv \left\{ w : \Theta \to \mathbf{R}^2 \,\middle|\, \text{There exist } a^* \in A \text{ and } t^* \in \mathbf{R}^2_0 \text{ such that } w(\theta) = u(a^*,\theta) + t^* \text{ for all } \theta \in \Theta \right\}.$$

Forcing contracts lie implicitly behind the common treatment of verifiable actions as public in the related literature. The traditional view is that, because the trade actions are verifiable and can therefore be forced, we might as well assume—for modeling simplicity and elegance—that these actions can be taken out of the players' hands (they are alienable). In models that take this approach, both a and t are chosen directly by the external enforcer and, thus, the contracted physical outcomes simply are elements of $A \times \mathbf{R}^2_0$. One can then perform standard mechanism-design analysis, which formally conditions the physical trade outcome on the messages sent at Date 4.[10] Treating trade actions as public is, in the notation of my model, equivalent to restricting attention to the subset W^{F} of the outcome set W.

The following lemma identifies a useful property of the sets W and W^{F}.

Lemma 1: *W and W^{F} are closed under constant transfers. For example, if $w \in W$ and $\bar{t} \in \mathbf{R}^2_0$, then $w + \bar{t} \in W$ as well.*

[9] To achieve public randomization over trade actions using forcing contracts, a public randomization device must be included in the model. This is done in the working paper Watson (2002b). In fact, allowing such randomization does not expand the set of implementable value functions here, except in the case of no renegotiation. We could also assume A is a mixture space, but this implies that the external enforcer can observe how the players randomize.

[10] Many models in the mechanism-design and contract-theory literature implicitly associate verifiability with forcing contracts. Some game-theory models, such as that of Bernheim and Whinston (1998), also take this view.

Contracted Mechanisms

Holding the issue of renegotiation aside for now, the players' contracting problem can be stated as a standard mechanism-design problem. The players' contract specifies a mechanism, which maps messages sent at Date 4 to outcomes induced in the trade and enforcement phase. The revelation principle applies in the following sense. We can restrict attention to direct-revelation mechanisms, each of which is defined by a message space $M \equiv \Theta^2$ and a function $f : \Theta^2 \rightarrow W$. With such a mechanism, at Date 4 the parties simultaneously and independently report the state. For any report profile m, the mechanism specifies an element $f(m) \in W$, which then determines the payoffs conditional on the state. We can concentrate on equilibria of the mechanism in which the parties report truthfully.[11] If we wish to treat trade actions as public, and so focus on forcing contracts, we constrain attention to the subset of mechanisms in which f maps Θ^2 to W^{F}.

Any mechanism (Θ^2, f) can be translated back into the notation of contract in the basic model, with y specified appropriately. For each message profile m, we define $y(m, \cdot) \equiv \hat{y}(\cdot)$, where $\hat{y}$ is a transfer function that supports $w = f(m)$ in Expression 2.

Renegotiation

Contract renegotiation at Dates 3 and 5 can be viewed as an opportunity for the players to discard their originally specified f mapping and replace it with another mapping f'. I assume the players divide the renegotiation surplus according to fixed bargaining weights π_1 and π_2. The generalized Nash bargaining solution has this representation.

To state the bargaining solution more precisely, I let $\gamma(\theta)$ denote the maximal joint payoff that can be obtained in state θ:

$$\gamma(\theta) \equiv \max_{a \in A} [u_1(a, \theta) + u_2(a, \theta)]. \tag{3}$$

Clearly, we have

$$\gamma(\theta) = \max_{w \in W^{\mathrm{F}}} [w_1(\theta) + w_2(\theta)] \tag{4}$$

because the trade action that solves the maximization problem in Equation 3 can be specified in a forcing contract to yield the outcome that solves the problem in Equation 4. An outcome w is called *efficient in state* θ if $w_1(\theta) + w_2(\theta) = \gamma(\theta)$.

Suppose the original mechanism (M, f) would lead to outcome w in state θ. If w is inefficient in state θ, then the players have a joint incentive to renegotiate the mechanism.

[11] The revelation principle usually requires a public randomization device to create lotteries over outcomes (or that the outcome set is a mixture space), but it is not needed here. To elaborate on Footnote 9, randomization will not be needed for implementation with renegotiation because it is neither required to achieve the ex post efficient outcome on the equilibrium path nor for the construction of the most severe punishments following out-of-equilibrium message profiles. To take care of the no-renegotiation case, I focus on pure strategy equilibria of the message phase, so the revelation principle applies without need for public randomization.

The *renegotiation surplus* is

$$r(w, \theta) \equiv \gamma(\theta) - w_1(\theta) - w_2(\theta).$$

The players will select a new mapping f' that induces an efficient outcome. Further, the surplus will be divided according to the players' bargaining weights, so that player i obtains $w_i(\theta) + \pi_i r(w, \theta)$. In practical terms, when the players renegotiate in state θ, they replace the transfer function with one that achieves an outcome w' satisfying $w'(\theta) = w(\theta) + \pi r(w, \theta)$. Equation 4 and Lemma 1 imply that such an outcome w' exists and is supported by a forcing contract.

3 Implementation Conditions

In this section, I define and characterize the set of implementable value functions.[12] I group the analysis into three categories, distinguished by whether the players have the opportunity to renegotiate at Dates 3 and/or 5. The characterization lemmas in this section are all straightforward variations of well-known theorems from the contract theory literature—in particular, due to Maskin (1999), Maskin and Moore (1999), and Moore and Repullo (1988).

No Renegotiation

First consider the setting in which the players cannot renegotiate. A mechanism (M, f) implies, for each state θ, a *message game* in which the players engage at Date 4. The message game has action profiles given by M and payoffs specified by $f(\cdot)(\theta)$. As discussed in the previous section, using the revelation principle we can focus on truthful reporting in direct-revelation mechanisms so that in state θ the players will send message profiles (θ, θ) in equilibrium. Thus, hereinafter I refer to a mechanism as (Θ^2, f). With no renegotiation, implementability is defined as follows.

Definition 2: *Mechanism* (Θ^2, f) *is said to* **implement** *value function* v *if* $f : \Theta^2 \to W$ *and, for each state* θ*,* (θ, θ) *is a Nash equilibrium of the message game and it leads to the payoff vector* $v(\theta)$*. Value function* v *is said to be* **implementable** *if there is a mechanism that implements it.*

Let V^{N} be the set of implementable value functions for the setting in which the players cannot renegotiate.

[12] I focus on implementation in the weak sense of not requiring uniqueness of equilibrium in each state. In fact, though, equilibrium utilities are always unique in the setting in which the players can renegotiate at Date 5, because ex post renegotiation implies a constant-sum message game in every state (as noted by Maskin and Moore 1999). Thus, strong implementation is implied for the case of renegotiation at Date 5.

Considering the equilibrium conditions for implementation, it is essential that the outcome specified for message profile (θ', θ) be sufficient to simultaneously (i) dissuade player 1 from declaring the state to be θ' when the state is actually θ and (ii) discourage player 2 from declaring "θ" in state θ'. Thus, letting w and w' denote the outcomes specified for message profiles (θ, θ) and (θ', θ'), respectively, implementation relies on the existence of an outcome $\hat{w}$ satisfying $w_1(\theta) \geq \hat{w}_1(\theta)$ and $w'_2(\theta') \geq \hat{w}_2(\theta')$. Combining this with Lemma 1 yields the following characterization of V^{N}.

Lemma 2: *Value function v is an element of V^{N} if and only if (i) for every $\theta \in \Theta$, there is an outcome $w \in W$ such that $w(\theta) = v(\theta)$; and (ii) for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $\hat{w} \in W$ such that $v_1(\theta) + v_2(\theta') \geq \hat{w}_1(\theta) + \hat{w}_2(\theta')$. Also, the set V^{N} is closed under constant transfers.*

In reference to this characterization, I call $\hat{w}_1(\theta) + \hat{w}_2(\theta')$ the "punishment value for the state pair (θ, θ')."

When players cannot renegotiate, there is no loss of generality in modeling trade actions as public, as the next lemma confirms.

Lemma 3: *If value function v is implementable, then there is a mechanism (Θ^2, f) that implements v and has the property that $f(m) \in W^{\mathrm{F}}$ for every $m \in \Theta^2$.*

The intuition behind this lemma is standard. Any strategic elements in the actual trading game can be mimicked through the use of messages. The mechanism can be designed so that the players announce what trade actions they want to select and then the external enforcer forces them to take these actions.

Interim Renegotiation

Next consider the setting in which renegotiation is possible at Date 3 but not at Date 5. In other words, the players can renegotiate between the time that they jointly learn the state and when the message game is played. I call this the *interim renegotiation* setting.[13] The players will renegotiate if, in the realized state, their anticipated equilibrium of the message game would yield an inefficient outcome. Thus, if the players' original contract would implement v' without renegotiation, then it leads to payoff vector $v'(\theta) + \pi r(v', \theta)$ in state θ with interim renegotiation.

Definition 3: *Value function v is* **implementable with interim renegotiation** *if there is a value function $v' \in V^{\mathrm{N}}$ such that $v(\theta) = v'(\theta) + \pi r(v', \theta)$ for every state θ.*

Let V^{I} denote the set of implementable value functions when there is interim renegotiation.

Lemma 4: *$v \in V^{\mathrm{I}}$ if and only if $v \in V^{\mathrm{N}}$ and $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ for every $\theta \in \Theta$. Also, V^{I} is closed under constant transfers.*

[13] It is called "ex ante" renegotiation in some articles in the related literature.

This result is simply the "renegotiation-proofness principle" (Dewatripont 1989, Hart and Tirole 1988, Laffont and Tirole 1990) for the case of interim renegotiation, which says that a value function that can be implemented with interim renegotiation can also be implemented in a way in which renegotiation does not actually occur in each state.[14]

With interim renegotiation, as with the case of no renegotiation, there is no loss of generality in modeling trade actions as public because Lemma 3 applies to V^{I}. That is, public-action and individual-action models are equivalent in the setting of interim renegotiation.

Ex Post Renegotiation

Finally, consider the case in which renegotiation is possible at Date 5—between the time the players send messages and the beginning of the trade and enforcement phase. The idea is that the players interact in the contracted mechanism, which leads to an outcome w. But then, just before the outcome would be induced, the players can renegotiate to obtain a different outcome. This is the setting of *ex post renegotiation.* Here, renegotiation implies efficient outcomes in every state and after every message profile in the mechanism.[15]

To characterize implementability for this setting, I incorporate renegotiation into the definition of an outcome. The set of *ex post renegotiation outcomes* is defined as

$$Z \equiv \Big\{ z : \Theta \to \mathbf{R}^2 \;\Big|\; \text{There is an outcome } w \in W \text{ such that } z(\theta) = w(\theta) + \pi r(w, \theta) \text{ for every } \theta \in \Theta \Big\}.$$

An ex post renegotiation outcome is a state-contingent payoff vector that results when, in every state, the players renegotiate from a fixed outcome in W. Note that all elements of Z are efficient in every state; also, Z and W are generally not ranked by inclusion. One can analyze mechanism design in the setting of ex post renegotiation by simply replacing W with Z in Definition 2.

Definition 4: *Value function v is* **implementable with ex post renegotiation** *if there is a mechanism (Θ^2, f) with $f : \Theta^2 \to Z$ and, for each state θ, (θ, θ) is a Nash equilibrium of the message game and it leads to the payoff vector $v(\theta)$.*

Letting V^{EP} denote the set of implementable value functions when there is ex post renegotiation, we have:

Lemma 5: *$v \in V^{\mathrm{EP}}$ if and only if (i) $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ for every $\theta \in \Theta$; and (ii) for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $\hat{z} \in Z$ such that $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$. Also, V^{EP} is closed under constant transfers.*

[14] See also Brennan and Watson (2001).

[15] With renegotiation possible at Date 5, implementability is not affected by whether renegotiation can also occur at Date 3. Renegotiation at Date 5 implies ex post efficiency in all states and with all message profiles, which means there is no surplus to be obtained from earlier renegotiation.

If trade actions are treated as public, and so attention is limited to forcing contracts, then the set of ex post renegotiation outcomes is calculated as

$$Z^{\mathrm{F}} \equiv \Big\{ z : \Theta \to \mathbf{R}^2 \;\Big|\; \text{There is an outcome } w \in W^{\mathrm{F}} \text{ such that } z(\theta) = w(\theta) + \pi r(w, \theta) \text{ for every } \theta \in \Theta \Big\}.$$

Definition 5: *Value function v is* **implementable with ex post renegotiation and a forcing contract** *if it is implemented by a mechanism* (Θ^2, f) *with* $f : \Theta^2 \to Z^{\mathrm{F}}$.

Let V^{EPF} be the value functions that are implementable with ex post renegotiation and forcing contracts. Public-action mechanism-design models study precisely the set V^{EPF}.

Lemma 6: $v \in V^{\mathrm{EPF}}$ *if and only if (i)* $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ *for every* $\theta \in \Theta$*; and (ii) for every pair of states* $\theta, \theta' \in \Theta$*, there is an outcome* $\hat{z} \in Z^{\mathrm{F}}$ *such that* $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$*. Also,* V^{EPF} *is closed under constant transfers.*

4 Analysis of the Example

The example described in Section 1 demonstrates the usefulness of accounting for the technology of trade. In this section, I calculate the sets V^{I}, V^{EP}, and V^{EPF} for the example, which shows the difference between what individual-action and public-action models identify as implementable.

Interim Renegotiation

I first perform the analysis of the example for the setting of interim renegotiation. Without loss, I constrain attention to direct-revelation mechanisms and truthful reporting in equilibrium (by the revelation principle) and forcing contracts (by Lemma 3). The calculations are a simple application of Lemmas 2 and 4; for illustrative purposes, I will provide a more direct construction.

Note that, because renegotiation only occurs before the message phase, the contracted mechanism may lead to an ex post inefficient outcome in some state, for some message profiles. However, using Lemma 4, an ex post efficient outcome occurs in equilibrium. Thus, to incorporate renegotiation, we must specify "adoption of the advertisement package" when the message profile is (H, H). Because it does not matter in state L whether the advertisement package is adopted, let us also specify "adoption of the advertisement package" when the message profile is (L, L). We can further limit attention to mechanisms that specify no adoption when message profiles (H, L) and (L, H) are sent, because this makes for the most relaxed incentive constraints. Let $p^{\theta\theta'}$ denote the monetary transfer from player 1 to player 2 that is specified by the mechanism for message profile (θ, θ'), for $\theta, \theta' \in \Theta$.

The game form implies a message game for each state, as pictured below.

B \ S	H	L
H	$-p^{HH}, \; p^{HH}$	$-p^{HL}, \; p^{HL}$
L	$-p^{LH}, \; p^{LH}$	$-p^{LL}, \; p^{LL}$

Message game in state L

B \ S	H	L
H	$5-p^{HH}, \; 3+p^{HH}$	$-p^{HL}, \; p^{HL}$
L	$-p^{LH}, \; p^{LH}$	$5-p^{LL}, \; 3+p^{LL}$

Message game in state H

We look for equilibria with truthful reporting. For truthful reporting to be a Nash equilibrium in each state, it must be that $p^{\mathrm{LH}} \leq p^{\mathrm{LL}} \leq p^{\mathrm{HL}}$, $5-p^{\mathrm{HH}} \geq -p^{\mathrm{LH}}$, and $3+p^{\mathrm{HH}} \geq p^{\mathrm{HL}}$. Combining these inequalities yields

$$p^{\mathrm{LL}} + 5 \geq p^{\mathrm{HH}} \geq p^{\mathrm{LL}} - 3,$$

which implies that the set of implementable value functions in the interim-renegotiation setting is:

$$V^{\mathrm{I}} = \Big\{ v : \{\mathrm{H},\mathrm{L}\} \to \mathbf{R}^2 \;\Big|\; v(\mathrm{L}) = (\alpha, -\alpha), \;\; v(\mathrm{H}) = (5 + \alpha - \beta, 3 - \alpha + \beta), \\ \text{for any } \alpha \in \mathbf{R} \text{ and } \beta \in [-3, 5] \Big\}. \tag{5}$$

Note that this can also be easily derived from Lemmas 2 and 4, which require $v_1(\mathrm{H}) + v_2(\mathrm{H}) = 8$, $v_1(\mathrm{L}) + v_2(\mathrm{L}) = 0$, $v_1(\mathrm{H}) + v_2(\mathrm{L}) \geq 0$, and $v_1(\mathrm{L}) + v_2(\mathrm{H}) \geq 0$, where the latter inequalities relate to the punishment value of compelling "no adoption" for message profiles (H, L) and (L, H).

Recall that, in this example, the effort cost c is not included in the value function. The maximal difference $v_1(\mathrm{H}) - v_1(\mathrm{L})$ is 8, which is sufficient to give the buyer the incentive to exert effort in the version of the example in which he makes the effort choice. Likewise, the maximal difference $v_2(\mathrm{H}) - v_2(\mathrm{L})$ is also 8, which means the seller can be motivated to exert effort in the version in which she makes the effort choice.

Ex Post Renegotiation and Forcing Contracts

I next turn to the setting of ex post renegotiation and forcing contracts—that is, the public-action modeling framework. We can assume that the mechanism specifies "adoption of the advertisement package" when the message profile is (H, H) and when it is (L, L).[16]

[16] Here we have the renegotiation-proofness principle again: Any incentive-compatible mechanism that specifies "no adoption" when the report profile is (H, H) will be renegotiated in the H state. One can alter the mechanism so that the renegotiated outcome is specified for (H, H), without affecting the incentive conditions. Recall also that adoption of the advertisement package is efficient in state L.

Furthermore, it is easy to verify that the incentive constraints are most relaxed if "no adoption" is specified for report profile (L, H) and "adoption" is specified for profile (H, L). Note that the mechanism would be renegotiated in state H in the off-equilibrium case in which the buyer reports L while the seller reports H. Incorporating the renegotiation activity, a game form implies the following message games in the two states.

B \ S	H	L
H	$-p^{HH},\ p^{HH}$	$-p^{HL},\ p^{HL}$
L	$-p^{LH},\ p^{LH}$	$-p^{LL},\ p^{LL}$

Message game in state L

B \ S	H	L
H	$5-p^{HH},\ 3+p^{HH}$	$5-p^{HL},\ 3+p^{HL}$
L	$4-p^{LH},\ 4+p^{LH}$	$5-p^{LL},\ 3+p^{LL}$

Message game in state H

As in the previous subsection, $p^{\theta\theta'}$ denotes the transfer from the buyer to the seller that is specified for message profile (θ, θ').

For truthful reporting to be a Nash equilibrium in each state, it must be that $p^{LH} \leq p^{LL} \leq p^{HL}$, $5 - p^{HH} \geq 4 - p^{LH}$, and $3 + p^{HH} \geq 3 + p^{HL}$. Combining these inequalities yields

$$p^{LL} + 1 \geq p^{HH} \geq p^{LL}.$$

The set of implementable value functions for the setting of ex post renegotiation and forcing contracts is thus:

$$V^{EPF} = \left\{ v : \{H, L\} \to \mathbf{R}^2 \;\middle|\; v(L) = (\alpha, -\alpha),\ v(H) = (5 + \alpha - \beta, 3 - \alpha + \beta), \right.$$
$$\left. \text{for any } \alpha \in \mathbf{R} \text{ and } \beta \in [0, 1] \right\}. \qquad (6)$$

Note that the opportunity to renegotiate at Date 5, specifically following out-of-equilibrium message profiles, causes a refinement in the set of implementable values relative to the case of interim renegotiation. In both the seller-effort and buyer-effort versions of the example, there are values of c for which efficiency requires that effort be exerted, yet there is no mechanism that reaches this goal.

Ex Post Renegotiation and Trade Actions as Options

I next show that, with ex post renegotiation, the set of implementable value functions significantly expands when parties depart from forcing contracts and, instead, use trade actions as options. Suppose that at Date 1 the parties write the following contract: If the buyer adopts the advertisement, then he must pay $p' + \beta$ to the seller; if the buyer does not adopt, then he pays p'; further, the external enforcer is instructed to ignore messages sent at Date 4. For $\beta \in (0, 5)$, this is not a forcing contract—that is, it neither compels the buyer to adopt the advertisement in both states, nor compels the buyer to *not* adopt the advertisement in both states. Instead, this is an option contract, but one that uses the buyer's *trade action*, rather than the buyer's message, as the way to exercise the option. With $\beta \in [0, 5]$, the buyer has the incentive to adopt the advertisement in state H and not to adopt in state L.

From Date 6, this contract yields a payoff vector of $(5 - p' - \beta, 3 + p' + \beta)$ in state H and $(-p', p')$ in state L. Because the contract leads to the efficient trade action in each state, it would not be renegotiated at either Date 5 or Date 3. The contract thus implements value $(5 - p' - \beta, 3 + p' + \beta)$ in state H and $(-p', p')$ in state L.

By using the trade action as an option, the parties are able to reduce the detrimental effect of renegotiation at Date 5. Because the trading opportunity is nondurable, there is no way for the parties to reverse it through renegotiation after Date 6. The parties could use a more complicated contract that involves transfers contingent on both trade actions and messages. However, in this example, more complicated contracts cannot improve on the scope of the simple option scheme described above.[17] Thus, the set of implementable value functions in the case of ex post renegotiation is:

$$V^{\mathrm{EP}} = \left\{ v : \{\mathrm{H}, \mathrm{L}\} \to \mathbf{R}^2 \;\middle|\; v(\mathrm{L}) = (\alpha, -\alpha),\ v(\mathrm{H}) = (5 + \alpha - \beta, 3 - \alpha + \beta), \text{ for any } \alpha \in \mathbf{R} \text{ and } \beta \in [0, 5] \right\}. \tag{7}$$

With ex post renegotiation, the supported range of β is sufficient to give the seller the incentive to exert effort when it is efficient to do so. To see this, note that $v_2(\mathrm{H}) - v_2(\mathrm{L}) = 8$ when one selects $\beta = 5$. On the other hand, in the version of the model in which the buyer makes the investment, there are still values of c under which the buyer cannot be given the incentive to exert effort.

Insights from the Example

The example shows that public-action models can fail to characterize the set of implementable value functions in settings with individual trade actions. Considering ex post renegotiation, a comparison of Expressions 6 and 7 indicates that $V^{\mathrm{EPF}} \subset V^{\mathrm{EP}}$ and $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}}$. Thus, not all implementable value functions can be implemented with forcing contracts, so the public-action model does not identify all of the implementable value functions when there is ex post renegotiation.

There then arises the question of whether the example is a proper application of a model with ex post renegotiation. It might seem that, in settings with nondurable trading opportunities (where the trade action can be used as an option without being reversed by renegotiation), the set of implementable value functions is actually characterized by a model with

[17] One can easily verify this by considering Date-4 messages, calculating the most severe punishment values $\hat{z}_1(\theta) + \hat{z}_2(\theta')$ to use for the message profiles (H, L) and (L, H), and using Lemma 5. For the message profile (H, L), in particular, we have $\min_{z \in Z} z_1(\mathrm{L}) + z_2(\mathrm{H}) = 3$ (forcing trade achieves this) and so implementation is constrained by $v_1(\mathrm{L}) + v_2(\mathrm{H}) \geq 3$, implying $v_1(\mathrm{H}) - v_1(\mathrm{L}) \leq 5$ (because $v_1(\mathrm{H}) + v_2(\mathrm{H}) = 8$).

interim renegotiation. That is, in general, perhaps the individual-action model with ex post renegotiation is equivalent to the public-action model with interim renegotiation. However, it can be seen by a comparison of Expressions 5 and 7 that this is not the case, for in the example we have $V^{\mathrm{EP}} \neq V^{\mathrm{I}}$.

In summary, the example has the property that $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}} \neq V^{\mathrm{I}}$, so characterizing the set of implementable value functions with ex post renegotiation requires non-forcing contracts (an individual-action model).

5 General Inclusion Results

The following result generalizes the weak inclusion relations that the example exhibits.

Theorem 1: $V^{\mathrm{EPF}} \subseteq V^{\mathrm{EP}} \subseteq V^{\mathrm{I}} \subseteq V^{\mathrm{N}}$.

Proof: The relation $V^{\mathrm{EPF}} \subseteq V^{\mathrm{EP}}$ follows from Lemmas 5 and 6 and that $Z^{\mathrm{F}} \subseteq Z$. By the definition of Z, we see that for every $\hat{z} \in Z$ there exists $\hat{w} \in W$ such that $\hat{z}(\theta'') \geq \hat{w}(\theta'')$ for all $\theta'' \in \Theta$. Thus, condition (ii) in Lemma 5 implies condition (ii) in Lemma 2. Further, condition (i) in Lemma 5 implies condition (i) in Lemma 2, because the maximum joint value exists in every state. Thus, the conditions of Lemma 5 imply those of Lemma 4 and, as a result, $V^{\mathrm{EP}} \subseteq V^{\mathrm{I}}$. Finally, $V^{\mathrm{I}} \subseteq V^{\mathrm{N}}$ is clear from Lemma 4. *Q.E.D.*

As noted in the previous section, applicability of the public-action model for settings with ex post renegotiation turns on whether $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}} \neq V^{\mathrm{I}}$. I close the analysis of this paper with two straightforward results that give conditions under which the inclusion relations are strict. These results are intended as a bridge to future work on the properties of specific trading technologies.

First consider the issue of whether forcing contracts are sufficient for the analysis of settings with ex post renegotiation—that is, whether public-action and individual-action models are equivalent in the context of ex post renegotiation. There is no loss in limiting attention to forcing contracts if $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$.

Theorem 2: $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ *if and only if, for every pair of states* $\theta, \theta' \in \Theta$ *and every* $\hat{z} \in Z$, *there is an ex post renegotiation outcome* $\tilde{z} \in Z^{\mathrm{F}}$ *such that* $\tilde{z}_1(\theta) + \tilde{z}_2(\theta') \leq \hat{z}_1(\theta) + \hat{z}_2(\theta')$.

Proof: Under the hypothesis of the theorem, condition (ii) of Lemma 6 implies condition (ii) of Lemma 5, proving $V^{\mathrm{EP}} \subseteq V^{\mathrm{EPF}}$. This and Theorem 1 yield the result. *Q.E.D.*

The intuition behind this result concerns the punishment outcome $\hat{z}$ that is specified for a particular message profile (θ', θ) with $\theta' \neq \theta$. This outcome must deter player 1 from declaring "θ'" in state θ and it must deter player 2 from declaring "θ" in state θ'. The punishment value is $\hat{z}_1(\theta) + \hat{z}_2(\theta')$. Lower punishment values support a greater range of value functions.

Next consider conditions under which the setting of ex post renegotiation and the setting of interim renegotiation imply the same set of implementable value functions, so that the individual-action model with ex post renegotiation is equivalent to the public-action model with interim renegotiation.

Theorem 3: $V^{\mathrm{EP}} = V^{\mathrm{I}}$ *if and only if, for all* $\theta, \theta' \in \Theta$ *and every* $\hat{w} \in W^{\mathrm{F}}$*, there is an ex post renegotiation outcome* $\hat{z} \in Z$ *such that* $\hat{z}_1(\theta) + \hat{z}_2(\theta') \leq \hat{w}_1(\theta) + \hat{w}_2(\theta')$.

Proof: By Lemmas 3 and 4, we can assume that $\hat{w} \in W^{\mathrm{F}}$ in the interim-renegotiation implementation condition for any given message profile (θ', θ) with $\theta' \neq \theta$. Then, under the hypothesis of the theorem, the conditions for implementation with ex post renegotiation (Lemma 5) imply the conditions for implementation with interim renegotiation (Lemma 4), proving $V^{\mathrm{I}} \subseteq V^{\mathrm{EP}}$. This and Theorem 1 yield the result. *Q.E.D.*

6 Conclusion

The modeling exercise presented here demonstrates the usefulness of explicitly accounting for the technology of trade (the timing and nature of individual productive actions, communication, renegotiation opportunities, and external enforcement) in models of contractual relations. The benefit of incorporating such realistic features is that, without them, we may obtain distorted conclusions about how parties deal with contractual imperfections. With the framework developed here, I argue that the analytic cost of incorporating individual actions is small. The results do not challenge the legitimacy of mechanism-design theory for the study of contract. However, they show that, in the *application* of mechanism-design theory, one should be careful to incorporate the important technological constraints.[18]

In this paper, I have focused on one aspect of a contractual relationship (the nature of trade actions) while maintaining the basic time line of existing models in the literature and assuming that the trading opportunity is nondurable. The following two notes suggest directions for broadening the research program.

1. Staying with the layout of the model presented here, it seems important to analyze specific classes of trade technologies to more deeply explore conditions under which the inclusion relations are strict (that is, where $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}} \neq V^{\mathrm{I}}$). A previous version of this paper reported on settings with one-sided trade actions, where only one player takes an action at Date 6; the next step is to examine (perhaps more realistic) settings in which both players are required to act in making a trade. In addition, for situations in which

[18] Some authors have argued for the kind of research reported herein. Hurwicz (1994) speaks of the importance of incorporating institutional constraints into design problems. He suggests that institutional constraints should be represented as limiting design to a class of game forms, whereby the "'desired' game form [is embedded in what he calls] the 'natural' game form" (p.12). My framework may be interpreted as a model of this natural game form. Anderlini, Felli, and Postlewaite (2001), Segal and Whinston (2002), and others also recognize the need to study technological and institutional constraints in contracting environments.

the inclusion relations are strict, it will be instructive to explore whether the difference between public-action and individual-action models really matters for the players' investment incentives at Date 2. For instance, in the example analyzed here, if the seller makes the investment at Date 2 then the difference between V^{EP} and V^{I} is inconsequential.

It may also be interesting to examine settings with partially verifiable trade actions. For example, a court may observe whether a particular trade was made but have trouble identifying which party disrupted trade (in the event that trade did not occur). Hart and Moore's (1988) model has this feature. It is straightforward to incorporate partial verifiability into the modeling framework developed here. One can represent the external enforcer's information about the trading game as a partition of the space of action profiles. One can then simply assume that the contracted transfers y must be measurable with respect to this partition.

2. Expanding beyond the model presented here, there is much to learn about settings with durable trading opportunities or where trade requires a sequence of productive actions. Watson (2005) takes a step in this direction by analyzing a model in which a finite number of verifiable productive actions are taken over time and, between successive productive actions, players can renegotiate their contract and communicate with the external enforcer. It is shown that the implementable set in this multi-trading-periods model equals that of a related one-trading-period model and, further, that renegotiation between trading periods can be counteracted.

Perhaps future theoretical endeavors should examine (i) infinite-horizon models of contractual relations and (ii) different ways in which trade actions, renegotiation, and opportunities to communicate with the external enforcer intermingle over time. On item (i), Watson (2002b) has some preliminary results along the lines of Watson (2005). On item (ii), note that the modeling exercise herein maintains a rigid separation between times at which the players renegotiate, send messages, and take trade actions. This discrete separation helps make precise that the messages and trade actions are verifiable whereas renegotiation activity is not. Future models could study settings in which renegotiation activity and trade actions overlap. For example, each player may be able to postpone a trading opportunity to prolong the renegotiation session, but the court may not be able to determine which of the players is causing the delay. This type of environment features neither fully verifiable trade actions nor fully unverifiable renegotiation activity.

In the process of improving our understanding of the implications of specific technologies of trade, subsequent work can address a few related technical issues. For example, some models (in particular ones with continuous time, two-sided trade actions, or an infinite horizon) may exhibit multiple equilibria. Consideration may be given to how parties deal with multiple equilibria in contractual environments and whether multiple equilibria pose a problem for contractual performance (more on this below). Another issue to keep an eye on is a possible connection between models with durable trading opportunities and simpler models. For example, it seems natural to ask whether implementation in a model with an infinite-horizon trading opportunity is characterized by a static public-action model with ex post renegotiation.

The contract-theory literature has seen several debates regarding renegotiation and its relation to messages and productive actions.[19] One reason that some contrasting viewpoints have not been reconciled is that much of the related literature abstracts from explicitly accounting for the technology of trade, in particular regarding the nature of trade actions and the arrangement in time of trade actions, renegotiation opportunities, and messages. Perhaps more precise treatment of the trade technology can serve to elucidate modeling assumptions and improve the basis for comparing different models and theoretical perspectives.

A more recent addition to the literature's debate on the effect of renegotiation is the paper of Serrano (2004), who comments on the work reported here. Serrano defends public-action models by suggesting a way of thinking about the scope of contracts so that (using my notation) $V^{\mathrm{EP}} = V^{\mathrm{I}}$ always holds. This claim appears to be based on the assumption that one can "design" the trading game; in the example here, for instance, we can have the seller take the individual trade action rather than the buyer doing so. On one hand, this violates the premise of my modeling framework—that the trading game is a fixed component of the trade technology. On the other hand, if, in some real settings, trade actions are alienable and the trading game can be arbitrarily structured, then the public-action model with interim renegotiation applies. More generally, it may be worth studying settings in which alienable actions can be associated with individual parties, but that there are technological constraints in such assignments. For instance, in a contractual relationship between a manager and two workers, perhaps either of the two workers can be assigned a particular productive task.[20]

Serrano (2004) also makes the claim that one should expect $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ if one requires the strong/full version of implementation. This claim is not made in reference to, and does not apply to, the framework developed here. Note that, in the example that I have presented, all of the value functions that are implemented with non-forcing contracts—except those with $\beta = 0$ and $\beta = 1$—induce unique equilibrium outcomes from every date of the relationship (in particular, from the trade and enforcement phase and from the message phase) and so are strongly implemented. Serrano's claim is actually made in the context

[19] For example, Edlin and Hermalin (2000) have engaged Nöldeke and Schmidt (1995,1998), and less directly Bernheim and Whinston (1998), in a debate about the scope of options in the context of verifiable trade actions. Edlin and Hermalin argue that, in ongoing negotiation, a party could effectively let an option expire and then renegotiate from scratch, so renegotiation is forceful. On the other hand, Lyon and Rasmusen (2004) argue that parties should, in reality, be able to rescind and change option orders after an opportunity for renegotiation expires. See also MacLeod (2001) on renegotiation and the timing of the resolution of uncertainty.

[20] Such an element is present in the work of Aghion, Dewatripont, and Rey (1994), who study a model in which aspects of the renegotiation process can be designed. It may be appropriate to consider, in addition to trade actions and bargaining weights, the design of other productive actions. For example, if it is possible to design who has the trade action, perhaps it is also possible to design who has the ex ante investment action.

of a game that Serrano describes in which renegotiation activity, trade actions, and the external enforcer's actions are all combined into one abstract normal form. On one hand, Serrano's game has the problem of possibly obscuring the technology of trade, especially the presumption that trade actions are verifiable and renegotiation activity is not verifiable. On the other hand, he raises an issue that may play an important role in the analysis of other models of trading relationships, including the ones described earlier in this section. For example, one might find that multiple equilibria arise in some models with continuous time and/or infinite-horizon trading opportunities. To the extent that these technological features are present in real contractual settings (cases where my assumptions of discrete time and nondurable trading opportunities are unrealistic), the issue of multiple equilibria will be important to address in future work.

The theoretical work reported herein has some implications for how to organize applied research. Some theorists, including Segal and Whinston (2002), suggest working to discover which of the public-action models (with either ex ante or ex post renegotiation) better fits the data for any particular setting. I suggest possibly organizing empirical tests on variations in actual trade technology. For example, how does the optimal contract depend on the technical process by which the players determine whether to consummate trade? I emphasize that the mechanism-design methodology is still applicable and useful; adopting an individual-action model means simply defining the outcome set differently than is common with public-action models.

A Appendix: Proofs of the Lemmas

Proof of Lemma 1: The result follows from the fact that one can add a constant transfer $\bar{t} \in \mathbf{R}_0^2$ to any given function $\hat{y}$ without altering the players' incentives in the trading game in any state. *Q.E.D.*

Proof of Lemma 2: For any direct-revelation mechanism (Θ^2, f), define $w^{(\theta_1,\theta_2)} \equiv f(\theta_1, \theta_2)$ for all $\theta_1, \theta_2 \in \Theta$. With truthful reporting, the implemented value function v satisfies $v(\theta) = w^{(\theta,\theta)}(\theta)$ for every $\theta \in \Theta$, which is condition (i) of the lemma. Observe that truthful reporting is a Nash equilibrium if and only if $v_1(\theta) \geq w_1^{(\theta_1,\theta)}(\theta)$ and $v_2(\theta) \geq w_2^{(\theta,\theta_2)}(\theta)$, for every $\theta \in \Theta$ and all $\theta_1, \theta_2 \in \Theta$. As noted in the text, these equilibrium conditions are equivalent to the condition that, for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $\tilde{w} \in W$ such that $v_1(\theta) \geq \tilde{w}_1(\theta)$ and $v_2(\theta') \geq \tilde{w}_2(\theta')$; further, Lemma 1 implies the existence of such an outcome $\tilde{w}$ if and only if condition (ii) of the lemma is true. That V^{N} is closed under constant transfers also follows from Lemma 1. *Q.E.D.*

Proof of Lemma 3: Suppose that (Θ^2, g) implements v. For every message profile m, write $\hat{a}^m$ and $\hat{y}^m$ as the functions that support $g(m)$ as described in Equation 1. That is, $\hat{y}^m(a)$ is the transfer specified for message m and trade action a, and $\hat{a}^m(\theta)$ is the equilibrium action profile in state θ following message m.

Define mechanism (Θ^2, f) as follows. For every message profile (θ_1, θ_2), the external enforcer is directed to *force* the action profile $(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2))$ and the transfer $\hat{y}^{(\theta_1,\theta_2)}(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2))$. Players are given the incentive to select the assigned trade action by the threat of severe punishment for any deviation; recall the construction discussed in the text. This forcing contract yields outcome $w^{(\theta_1,\theta_2)}$ that, written in terms of the functions supporting (Θ^2, g), has the following payoff vector in state θ:

$$w^{(\theta_1,\theta_2)}(\theta) = u(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2), \theta) + \hat{y}^{(\theta_1,\theta_2)}(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2)).$$

Define $f(m) \equiv w^m$ for every $m \in \Theta^2$.

Note that $f(\theta, \theta)(\theta)$, which we can write as $w^{(\theta,\theta)}(\theta)$, is equal to $g(\theta, \theta)(\theta)$. To complete the proof of the lemma, we must show that truthful reporting is a Nash equilibrium of the message game in every state. Suppose that, in state θ, player 1 deviates by reporting θ_1 while player 2 reports θ. Then player 1 gets a payoff of

$$w_1^{(\theta_1,\theta)}(\theta) = u_1(\hat{a}_1^{(\theta_1,\theta)}(\theta_1), \hat{a}_2^{(\theta_1,\theta)}(\theta), \theta) + \hat{y}^{(\theta_1,\theta)}(\hat{a}_1^{(\theta_1,\theta)}(\theta_1), \hat{a}_2^{(\theta_1,\theta)}(\theta)),$$

which is weakly less than

$$u_1(\hat{a}_1^{(\theta_1,\theta)}(\theta), \hat{a}_2^{(\theta_1,\theta)}(\theta), \theta) + \hat{y}^{(\theta_1,\theta)}(\hat{a}_1^{(\theta_1,\theta)}(\theta), \hat{a}_2^{(\theta_1,\theta)}(\theta))$$

because $\hat{a}^{(\theta_1,\theta)}(\theta)$ is a Nash equilibrium of the trading game $\langle A, u(\cdot, \theta) + \hat{y}^{(\theta_1,\theta)}(\cdot)\rangle$. But this last value is exactly player 1's expected payoff conditional on message profile (θ_1, θ) in state θ, under mechanism (Θ^2, g). Write this payoff as $g(\theta_1, \theta)_1(\theta)$. It, in turn, is weakly less than $g(\theta, \theta)_1(\theta)$, the payoff for player 1 when both players report truthfully in state θ. Thus, for mechanism (Θ^2, f), we have $f(\theta_1, \theta)_1(\theta) \leq f(\theta, \theta)_1(\theta)$. The analogous calculation holds for player 2, which means truthful reporting is an equilibrium in every state. *Q.E.D.*

Proof of Lemma 4: Suppose $v \in V^{\mathrm{I}}$ and let $v' \in V^{\mathrm{N}}$ be a value function that satisfies the expression in Definition 3. Obviously, v is efficient in every state, so $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ for every $\theta \in \Theta$. By Equation 4, v satisfies condition (i) of Lemma 2. Furthermore, $v(\theta) \geq v'(\theta)$ in the vector sense, for every θ; thus, condition (ii) of Lemma 2 is also satisfied, implying $v \in V^{\mathrm{N}}$. For the other direction of the lemma, start with $v \in V^{\mathrm{N}}$ and note that $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ implies $r(v, \theta) = 0$, so, by Definition 3 and Lemma 1, we have $v \in V^{\mathrm{I}}$. *Q.E.D.*

Proof of Lemma 5: Lemma 1 implies that Z is closed under constant transfers. Since the maximum joint value exists in every state, condition (i) holds if and only if, for every $\theta \in \Theta$, there is an outcome $z \in Z$ such that $z(\theta) = v(\theta)$ for every $\theta \in \Theta$. The rest of the proof follows the proof of Lemma 2 with Z in place of W. *Q.E.D.*

Proof of Lemma 6: Recognizing that $Z^{\mathrm{F}} \subseteq Z$ and Z^{F} is closed under constant transfers, this lemma is proved in the same manner as was Lemma 5. *Q.E.D.*

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