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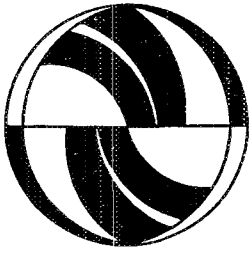
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### **Publication Date**

1992-10-01



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Shunfeng Song

Working Paper  
UCTC No. 198

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**Monocentric and Polycentric Density Functions  
and Their Required Commutes**

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*Working Paper  
October 1992*

UCTC No. 198

The University of California Transportation Center  
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## ABSTRACT

This paper examines the relationship between urban structure and commuting behavior. Analyzing the 1980 journey-to-work data for the Los Angeles region, this paper has shown that polycentric density functions fit the actual urban structure better than the conventional monocentric model. This finding indicates the preeminence of accessibility to major employment centers in location choices.

This paper also estimates commute flows implied by the polycentric and monocentric functions. It finds the monocentric model very poor at explaining commuting behavior. The empirical results show that polycentric urban structure increases the urban commute. This finding helps to preserve the assumption that urban workers economize on commuting, and suggests that efforts to promote more efficient urban form, such as the jobs-housing balance policy, have the potential to succeed.

I would like to thank Kenneth A. Small, David Brownstone, Gordon (Pete) Fielding, and Genevieve Giuliano for their helpful comments and suggestions. Financial supports from the University of California Transportation Center and the Ou Jou Yi Scholarship are greatly acknowledged. The views expressed here are the sole responsibility of the author.

## INTRODUCTION

The standard urban economic model was developed during the 1960s as economists sought to provide an analytic framework for studying urban spatial structure (Alonso [1], Mills [25] and Muth [30]). The central features of the model are monocentricity and the trade-off between housing costs and commuting costs. Using the indifference principle, the value of locational advantage (access to the city center) is capitalized in land rent. Locational equilibrium is achieved when the marginal housing savings of moving farther from the city center equal the marginal increased commuting costs. The model predicts that both land rent and residence density decline with the distance from the center (Mills [27], chpts 6-7, Mills and Hamilton [28], appendix A, Muth [31] and Wheaton [40]).

The standard urban model, however, has been thought to be a poor description of reality. Hamilton [16] shows that the actual average commute in typical U.S. metropolitan areas is eight times as large as the minimum average commute consistent with the standard monocentric model. Hamilton called this unexplained commute "wasteful commuting;" here I call it *excess commuting* to avoid any normative connotations. Since 87 percent of the actual commute is excess, Hamilton concludes that the monocentric urban model has little predictive value concerning commuting behavior, and claims that "it is not clear that the trade-off between commuting and land rent plays any significant role at all in location decisions" (Hamilton [16], p. 1050). Small and Song [36] find that the monocentric required commute is only about one-fifth of the actual commute for Los Angeles County in 1980, verifying Hamilton's finding that the monocentric model is very poor at explaining urban commuting.

One possible explanation of Hamilton's results is that employment and residences are distributed in a pattern consistent with many employment centers, not just one. Several recent studies have demonstrated the presence of employment subcenters in large American cities (Cervero [3], Giuliano and Small [10] and McDonald [23]), casting doubts on the assumption of monocentricity. The theoretical basis for urban subcenters has also received attention (Helsley and Sullivan [19], Odland [33], Sasaki [35] and White [41, 43]). A few empirical researches have examined the impacts of urban subcenters on spatial patterns (Gordon et al. [12], Griffith [15] and Small and Song [37]).

Polycentric density functions, however, have not been incorporated into the calculation of excess commute. In a polycentric urban area, some outward and circumferential commuting might be going on and excess commuting might not be as much as that found by Hamilton (Suh [38] and White [42]). Some studies, however, have argued that polycentric spatial structure reduces rather than lengthens urban commuting (e.g., Gordon et al. [11] and Gordon and Wong [13]). It is unclear what effect that polycentricity has on urban commuting.

Moreover, Hamilton calculates the excess commute assuming a monocentric form for U.S. cities. His result could be misleading if the monocentric model inadequately represents the spatial structure. Hence, we are unable to distinguish whether his result is an indictment of the monocentricity or of the more fundamental assumption on commuting behavior that households trade off between land rents and commuting costs. An indictment of the monocentricity might not be surprising; but rejecting the trade-off assumption is more drastic. The latter strikes at the heart of urban economics and hence implies an urgent need

to reformulate the analytical land-use model most commonly used by urban economists; it also indicates a need to re-assess those efforts to promote more efficient urban form, such as the jobs-housing policy that pursues a jobs-housing balance development in urban areas so that the amount of commuting and its consequent peak period congestion and air pollution can be reduced.

This paper tests the existence of polycentricity and determines its effect on the spatial structure and the estimate of excess commuting; it also examines the behavioral validity regarding the trade-off assumption. Using 1980 small-zone journey-to-work data for the Los Angeles region, I first estimate monocentric and polycentric density functions for both employment and resident workers, and perform hypothesis tests on the fit of the monocentric model in explaining the spatial distributions and the existence of polycentricity. I then determine the effect of polycentricity on the estimate of excess commuting by estimating the average minimum commutes required by the monocentric model and the polycentric model. Finally, I examine the behavioral validity that households make attempts to economize on commuting in their location choices.

## DENSITY FUNCTIONS

In a monocentric city, urban residents are assumed to value access to the center and trade off this access and housing costs in their location decisions. In consequence, urban residents are distributed in a circularly symmetric manner with density function  $f(r)$ , where



$r$  is the distance from the center. Assuming some employment is decentralized, urban economists have postulated the distribution of employment similar to that of resident workers (Hamilton [16], Mills [27], Muth [30] and Thurston and Yezer [39]).

The negative exponential density function is the most commonly used model in the monocentric city literature; and it is also used in this paper. For the case of resident worker distribution, this function can be derived theoretically with a maximizing model. For example, Mills and Hamilton [28] and Papageorgiou and Pines [34] have derived the negative exponential density function using a compensated demand for housing with a unitary price elasticity; Bussiere and Snickars [2] derive the same function form with an entropy maximizing method. The negative exponential density function is also supported by empirical evidence (Clark [5], Mills [27] and Muth [30]). For the case of employment distribution, this functional form has been also theoretically derived (McDonald [22] and Mills [26]) and commonly used (Kemper and Schmenner [20], Mills [27] and Mills and Ohta [29]).

The negative exponential density is written here as

$$D_m = D_0 e^{-gr_m} e^{u_m}, \quad m = 1, 2, \dots, M, \quad (1)$$

where  $D_m$  is the worker residence or employment density at distance  $r_m$  to the single urban center;  $M$  is the total number of zones in an urban area;  $e^{u_m}$  is a multiplicative error term associated with zone  $m$ ;  $D_0$  and  $g$  are parameters to be estimated from the data by

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<sup>1</sup>An additive error term has been also used in the literature on the negative exponential density function. Greene and Barnbrock [14], however, shows that a multiplicative error term is more appropriate.

ordinary least square after taking logarithm of equation (1). Theoretically,  $D_0$  is the density extrapolated to the urban center, and  $g$  is the density gradient measuring the percentage fall off in density for an unit increase in distance from the CBD.

The natural extension of the monocentric model is to assume that access to all employment centers is of primary importance in location decisions, and specify that resident workers and employment are functions of distances to all employment centers.<sup>2</sup> Such density function has been suggested and estimated by Gordon et al. [12], Griffith [15] and Small and Song [37], which is written here as

$$D_m = \sum_{n=1}^N A_n e^{-b_n r_{nm}} + v_m, \quad m = 1, 2, \dots, M, \quad (2)$$

where  $N$  is the number of employment centers in an urban area;  $r_{nm}$  is the distance from center  $n$  to zone  $m$ ;  $v_m$  is the error term associated with zone  $m$ ;  $A_n$  and  $b_n$  are parameters to be estimated for each employment center  $n$ . This specification of polycentric model assumes that the density at any location is the vertical summation of the negative exponential density functions, each reflecting the influence of a center on that location.

When the intercepts of all centers except one are zero, the polycentric form collapses to the monocentric form. Therefore, we can perform statistical tests on hypotheses that the polycentric model explains the actual distributions better than the monocentric model, and that the existence of polycentricity in the spatial structure. We can also test the significance

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<sup>2</sup>A polycentric density function could be the upper envelope of the density gradients for the centers if we assume the influences from all centers are completely substitutable; it could also be a multiplicative function of centers' influences if we assume centers are complementary (Heikkila et al. [18]). An additive function is specified by assuming that centers are in between of these two extreme cases.

of center  $n$  in explaining the overall density pattern by means of  $t$ -test on its parameters  $A_n$  and  $b_n$ .

Estimating the polycentric density functions, we are able to determine the impact that each center has on the overall distributions of employment and resident workers. The estimated aggregate impact of center  $n$  is computed from the formula

$$IMPACT_n = \sum_{m=1}^M (\hat{A}_n e^{-\hat{b}_n r_{nm}}) S_m = \hat{A}_n \sum_{m=1}^M S_m e^{-\hat{b}_n r_{nm}} , \quad (3)$$

where  $S_m$  is the area of zone  $m$ ;  $\hat{A}_n$  and  $\hat{b}_n$  are the estimated intercept and density gradient.  $IMPACT_n$  is positively related to  $\hat{A}_n$  and negatively related to  $\hat{b}_n$ .

### CALCULATION OF REQUIRED COMMUTE

The "wasteful commuting" literature has used two methods to estimate the excess commute. One uses estimated monocentric density gradients, here called *Hamilton's calculation*. The other uses a linear assignment model, called *White's calculation*. They are reviewed in turn.

#### *Hamilton's Calculation*

Hamilton [16] estimates the minimum average commuting distances required by the monocentric model for 14 U.S. cities and 21 Japanese cities in the late 1970s. He shows that the minimum commute required by the monocentric is the average distance of residents

to the center minus the average distance of jobs to the center. To estimate the required monocentric commute, Hamilton first calculates the average commute ( $A$ ) if all jobs were located in the center,

$$A = \frac{2\pi}{P} \int_0^{\bar{r}} r^2 P(r) dr , \quad (4)$$

where  $P$  is the metropolitan total population and  $P(r)$  is the population density at distance  $r$  from the city's center. He then calculates the amount of reduction ( $B$ ) in average commute by job decentralization from the center,

$$B = \frac{2\pi}{J} \int_0^{\bar{r}} r^2 J(r) dr , \quad (5)$$

where  $J$  is the metropolitan total employment and  $J(r)$  is the employment density at distance  $r$  from the city's center. Finally, he calculates the required commute ( $C = A - B$ ), which is the difference of the average distance of population distribution to the center and the average distance of employment to the center.

Assuming that both employment and population densities decline exponentially from an urban center, Hamilton [16] found that the average minimum commuting distance is only 1.12 miles ( $C = 1.12$  miles), based on the Mills-type density gradients which are updated by Macauley [21]. But the average actual commute is 8.7 miles in his sample cities. Hence, 87 percent of the actual commute is excess. Based on this finding, Hamilton concludes that the monocentric model is inadequate in explaining actual commuting in urban areas.

Applying Hamilton's calculation to the same sample of U.S. cities for 1980 census data, Thurston and Yezer [39] recompute the minimum commute predicted by the monocentric model. In their calculation, heterogeneity (in term of occupational groups) in

employment and resident workers is considered, and individual density functions for each occupational group are estimated. Their results show that about 60 percent of actual commute is excess. Small and Song [36] apply Hamilton's calculation to Los Angeles County for 1980 journey-to-work census data. They find that about four-fifths of the actual commute is excess, confirming the general order of magnitude of Hamilton's original estimate and verifying his original argument that the monocentric model is very poor at explaining urban commuting.

### *White's Calculation*

White [42] uses a linear program to calculate the minimum commute for the actual distributions of resident workers and employment. A linear program is used to assign trip flows among locations (swap homes or jobs) so that the aggregate commute is minimized, knowing the exogenous numbers of jobs and resident workers at each location and the commuting costs (time or distance matrix) among locations.

Let  $n_{ij}$  be the number of commuters from zone  $i$  to zone  $j$ ,  $c_{ij}$  be the corresponding network commuting cost (travel distance or time). A linear program is used to find  $n_{ij}^*$  to

$$\min \bar{c} = \frac{1}{N} \sum_i \sum_j c_{ij} n_{ij} , \quad (6)$$

subject to the constraints

$$\sum_j n_{ij} = N_i , \quad \sum_i n_{ij} = E_j , \quad n_{ij} \geq 0 , \quad (i, j = 1, 2, \dots, I) , \quad (7)$$

where  $N = \sum_i N_i = \sum_j E_j$  is the number of commuters in the urban area, while  $N_i$  is the

number of resident workers in zone  $i$  and  $E_j$  is the number of jobs in zone  $j$ .

Applying this calculation to the actual distributions of resident workers and jobs, White [42] finds that the average minimum commuting time is 20 minutes for a sample which overlaps Hamilton's. Comparing the average actual commuting time of 22.5 minutes in these sample cities, she concludes that there is little excess commuting in American cities, with only 11 percent is excess. White therefore concludes that the urban commuting is not "wasteful" and claims "that the monocentric urban models are in better shape than Hamilton's gloomy diagnosis would imply." (p. 1109)

White's calculation has been also used by Hamilton [17], Cropper and Gordon [6] and Small and Song [36]. Using travel distance rather than travel time as by White, Hamilton [17] shows that 47 percent of urban commuting is excess in Boston area. Comparing 16 percent found by White in the same area, Hamilton concludes that this discrepancy is mainly due to the difference between using distance and using time as a measure of commuting cost. Cropper and Gordon [6] use a modified White's calculation to micro data in Baltimore, which reallocates households to houses to minimize the aggregate commuting distance subject to the constraint that no household's utility is lowered. They find that about 50 percent of the actual commute is excess. Small and Song [36] apply White's calculation to small-zone 1980 journey-to-work data for Los Angeles County. They find that about two-thirds of the actual commute is excess, with little difference between using distance and using time; they also show that White's estimate of excess commute is downward biased due to aggregation bias from using large zones in her data set.

This paper uses both Hamilton's and White's calculations to estimate the minimum

commute required by the monocentric and polycentric models. In principle, White's calculation can be used to any estimated set of density functions which serves as the basis for calculating a minimum required commute. Applying White's calculation to the predicted monocentric densities, we are able to examine the bias of Hamilton's calculation due to its assumptions that there exists a radial transportation network and commuting trips proceed along radial routes. Applying White's calculation to the predicted polycentric densities, we are able to determine the effect of polycentricity on the estimate of minimum required urban commuting.

## **STUDY AREA AND IDENTIFICATION OF EMPLOYMENT CENTERS**

This section first describes the study area and data. It then shows how employment centers are identified.

### *Study Area and Data*

The study area contains most of urbanized portion of five counties in the greater Los Angeles region, namely Los Angeles, Orange, Riverside, San Bernardino, and Ventura Counties. Journey-to-work data from the 1980 Census are used, that are provided by the Southern California Association of Governments (SCAG). The data include aggregate zone-to-zone commute flows. Information on zone-to-zone travel distances is extracted from the data created for the Urban Transportation Planning Package (UTPP), which is calibrated

based on a peak-period representation of the road network.

The geographic unit is the transportation analysis zone (AZ) defined by the SCAG. Like census tracts, Azs are aggregates of census blocks but have their boundaries determined by functional traffic characteristics and need not have a fixed population, and hence reduce the "census-tract delineation bias" observed by Frankena [8] in density estimation.

The study area consists of 1124 Azs, after deleting 161 very low-density zones for simplicity.<sup>3</sup> The 1124 zones cover 3,401 square miles. This paper analyzes the 4.53 million workers who both live and work in the 1124-AZ study area. Because the standard location model only considers resident workers and only employed individuals commute to work, this paper analyzes resident workers rather than population.

#### *Identification of Employment Centers*

A number of peaks of densities may exist in an urban area. A definition of employment center, however, should capture the notion that the concentration of employment in a zone of a group of adjacent zones are large enough to have some discernible impact on the overall spatial structure. Recent empirical studies have provided a variety of criteria on defining employment centers. Gordon et al. [12] identified density peaks via visual inspection of density maps. Heikkila et al. [18] treat the centroids of the Regional Statistical Area (defined by a planning agency) as the proxies of subcenters. Cervero [3] and Dunphy [7] defined subcenters with a sequential process, using criteria such as the size of

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<sup>3</sup>All are remote from the highly developed parts of the region, with the exception of 11 largely undeveloped zones in the Santa Monica mountain that separates the densely developed West Los Angeles corridor from the more suburban San Fernando Valley.



employment and specialization of employers.

McDonald [23] discussed several empirical criteria for the identification of urban employment subcenters. He suggested that local peaks in gross employment density and the employment-population ratio are the best indicators of employment subcenters. Giuliano and Small [10] use a version of McDonald's definition and present a simple systematic identification of employment subcenters. They define a center as a contiguous set of zones, each with density above  $\bar{D}$ , that together have at least  $\bar{E}$  total employment. Using 1980 census journey-to-work data for the Los Angeles region, they have identified 32 centers with criteria  $\bar{D} = 10$  employees per acre and  $\bar{E} = 10,000$  employees.

This paper uses the approach suggested by Giuliano and Small, because it incorporates adjacent high-density zones and restricts attention to centers large enough to exert potentially significant influences on the overall urban structure in a metropolitan area. In order to have a manageable number of employment centers in the density function estimation, criteria become  $\bar{D} = 15$  and  $\bar{E} = 35,000$ . Using these criteria, seven employment centers are identified; they are listed in Table 1. Among the seven centers, five locate in Los Angeles County and two locate in Orange County.

## EMPIRICAL RESULTS

### *Monocentric Density Estimates*

The monocentric density functions of resident workers and employment are estimated

Table 1. Employment Centers Identified in 1980

Center Location	County	Total Emp.	Emp. Den. (Emp/Acre)	Dist. from CBD(miles)
Downtown LA (CBD)	L. A.	429869	42.26	0.1
UCLA/Santa Monica	L. A.	208166	25.31	15.8
LA Airport	L. A.	48510	18.77	18.8
Orange Co. Airport	Orange	47459	16.01	40.7
West Hollywood	L. A.	43761	23.01	7.3
Santa Ana	Orange	37305	17.18	32.9
Pasadena	L. A.	35675	25.14	12.6

by ordinary least square, after taking logarithm of equation (1). Table 2 presents the results. As expected, the density gradient of employment is greater than that of resident workers, implying that resident workers are more dispersed than the employment. The gradient estimates show that both resident worker and employment distributions were quite flat in 1980 in Los Angeles region; resident worker density falls off at 4.6 percent and employment density falls off at 5.0 percent by one mile increase in distance from the CBD. These two density gradients, however, are not much different. This result suggests that the degrees of employment decentralization and resident worker suburbanization are close, implying a general balance between housing and jobs over the distance from the CBD.

Table 2 shows that the monocentric density function fits the resident worker distribution better than the employment distribution, based on the criteria of maximum explanatory power in standard regression analysis and accuracy in predicting the total employment and resident workers of the urbanized area (McDonald and Bowman [24]). The monocentric function explains 38.6 percent of the variance in resident worker distribution; whereas it explains 34.3 percent in the case of employment distribution. Integrating the predicted densities over the whole region, I found that the monocentric model predicts 99.1 percent of the total resident workers but only 79 percent of the total employment.

To examine a possible crater at the CBD in the case of resident worker distribution, the monocentric density function is also fitted with a quadratic distance term added to  $gr_m$ ,

$$D_m = D_0 e^{-gr_m + g_2 r_m^2} e^{u_m}, \quad m = 1, 2, \dots, M, \quad (8)$$

where  $g_2$  is the coefficient of the quadratic term. When both  $\hat{g}$  and  $\hat{g}_2$  are negative, a crest of density is predicted at location of  $\hat{g}/(2\hat{g}_2)$  miles from the CBD. The results,

Table 2. Estimates on Monocentric Density Functions

	Intercept ( $\log D_0$ )	Gradient ( $g$ )	$R^2$	Integration (1000s)
Employment	8.5321* (0.0688)	0.0498* (0.0021)	0.343	3,577
Resident Worker	8.6574* (0.0586)	0.0457* (0.0017)	0.386	4,487

Note: Standard errors are in parentheses.

There are 1117 and 1102 observations for resident worker and employment equations. The data set has 4,528 thousand actual jobs (and resident workers).

\* Estimate is statistically significant at 0.05 level, 1-sided test.

however, show that  $\hat{\beta}_1=0.0222$  and  $\hat{\beta}_2=-0.0003$ , implying that the (predicted) resident worker density declining monotonically from the center. Hence, no crater is predicted at the center.

### *Polycentric Density Estimates*

The polycentric density functions are estimated by non-linear least squares. Tables 3 and 4 show the results, with the Orange County Airport center excluded. Regressions on the polycentric density functions reveal that the Orange County Airport center has very low  $t$ -values on its intercept and gradient estimates. For example,  $t$ -values are less than 0.52 for the employment equation. Moreover, this center has a negative intercept in the resident worker equation;  $F$ -test (discussed below) shows that the six-center model cannot be rejected even at a 25 percent significance level in the case of employment distribution. For these reasons, the Orange County Airport center is eliminated.

Tables 3 and 4 suggest the existence of polycentricity in the Los Angeles region. Consider the number of centers with both intercept and gradient statistically significant at a 5 percent level (1-sided test, i.e., with  $t > 1.64$ ). Five centers pass this test in the case of resident worker distribution (Table 3); four centers satisfied this criterion in the case of employment distribution, with Pasadena close to the margin (Table 4).

The estimates of total impact,  $IMPACT_n$ , that each center  $n$  has on the regionwide distributions of resident workers and employment also reveal the polycentricity in the overall spatial structure. They are shown in the last column of Tables 3 and 4, with  $t$ -values in

Table 3. Estimates on Polycentric Density Function: Resident Workers

Center Location	Intercept	Gradient	Impact (1000s)
Downtown LA (CBD)	2125* (2.76)	0.3361* (1.87)	58.1 (1.09)
UCLA/Santa Monica	2673* (3.53)	0.0665* (1.69)	937.4 (1.46)
LA Airport	2517* (2.80)	0.0270* (2.15)	2789.6* (2.69)
West Hollywood	6936* (7.63)	0.2484* (4.86)	311.1* (2.63)
Santa Ana	2493* (6.49)	0.0614* (2.83)	1269.0* (2.24)
Pasadena	701 (1.08)	0.0344 (0.55)	707.9 (0.71)

$R^2 = 0.481$

Note: *t*-values are in parentheses.

There are 1124 observations.

\* Estimate is statistically significant at 0.05 level, 1-sided test.

Table 4. Estimates on Polycentric Density Function: Employment

Center Location	Intercept	Gradient	Impact (1000s)
Downtown LA (CBD)	254990* (35.00)	1.2199* (38.98)	422.5* (21.19)
UCLA/Santa Monica	76020* (1.69)	1.5472* (2.22)	39.5* (2.07)
LA Airport	3925* (5.09)	0.0234* (3.30)	4957.6* (5.92)
West Hollywood	12539* (3.16)	0.2910* (3.15)	400.8* (2.08)
Santa Ana	6467 (1.19)	0.3114 (1.31)	211.0 (1.02)
Pasadena	33516 (1.51)	1.4203* (1.67)	33.2 (1.28)

$R^2 = 0.705$

Note: *t*-values are in parentheses.

There are 1124 observations.

\* Estimate is statistically significant at 0.05 level, 1-sided test.

parentheses.<sup>4</sup> The results show that three centers have statistically significant impacts on resident worker distribution and four centers on the employment distribution.

One might think that distance to the ocean has large impact at least on resident worker distribution, and that the large impact of the Los Angeles Airport center is due to its location on the ocean shore. Estimation on the resident worker equation with inclusion of distance to the ocean, however, shows that the ocean has a very localized impact; its density gradient equals -6.7979 per mile ( $t=4.37$ ), implying that its influence on resident worker distribution falls off 99.9 percent for one mile increase from the ocean shore. Moreover, the Los Angeles Airport center still has the largest impact, although its intercept decreases from 2517 to 2205 and its gradient increases from 0.0270 to 0.0335.

The formal test for the existence of polycentricity, however, is based on the statistic

$$F = \frac{(SSR^r - SSR^u) / q}{SSR^u / (M-p)} \quad (9)$$

where  $SSR^r$  and  $SSR^u$  are the restricted (monocentric) and unrestricted (polycentric) sums of squared residuals, with same dependent variable (density  $D$ );  $M$  is the sample size;  $p$  is the number of parameters being estimated in the unrestricted estimate; and  $q$  is the number of restrictions on these parameters in the restricted estimate. Under the null hypothesis,  $F$  is approximately distributed according to a central  $F$ -distribution with

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<sup>4</sup> $IMPACT_n$  is a nonlinear function of two random variables,  $\hat{A}_n$  and  $\hat{b}_n$ . Knowing their estimated variances and covariance we can compute an approximate variance for  $IMPACT_n$  from the formula

$$\sigma_{IMPACT}^2 = d' \Sigma d ,$$

where  $\Sigma$  is the variance-covariance matrix of  $(A_n, b_n)$  and  $d$  is the vector of derivatives of equation (3) with respect to  $A_n$  and  $b_n$ . See Chow [4] pp. 182-183.



degrees of freedom ( $q, M-p$ ),<sup>5</sup> where  $p=2N$ ,  $q=2(N-1)$ , and  $N$  is the (unrestricted) number of centers (Small and Song [37]).

Performing  $F$ -tests to the unrestricted (polycentric) model with six centers, I find that the  $F$ -statistic has values of 27.68 for resident worker distribution and 25.11 for employment distribution. These resulting tests, with (10,1112) degrees of freedom, indicate that the null hypothesis (monocentric model) is soundly rejected at a significance level of 0.0001 in both cases.<sup>6</sup> The polycentric model, therefore, explains statistically better the distributions of resident workers and employment. Rejecting the monocentric model indicates that the overall access to major employment centers is much more important in location choices than the access to the central business district.

#### *Monocentric Required Commute*

Using Hamilton's computation and taking these monocentric functions to represent smoothly varying distributions, I find that the average worker lives at 25.02 miles ( $A=25.02$ , equation 4) from the center and the average job locates 24.00 miles ( $B=24.00$ , equation 5) from the center. The difference,  $C=1.02$  miles, is the average minimum commute required by the monocentric model. It accounts for only about one-tenth of the average actual commute of 10.81 miles; i.e., 90.56 percent of actual commute is excess. The first row in Table 5 shows the result, verifying Hamilton's original finding that the standard monocentric

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<sup>5</sup>See Chow [4], pp. 229-230 and Gallant [9], pp. 78-79.

<sup>6</sup>With (10,1112) degrees of freedom, the  $F$ -value for rejecting the null hypothesis at 0.0001 significance level is 3.5564.

model greatly underpredicts the actual commuting observed in urban areas.

Hamilton's computation assumes that there exists a radial transportation network which is everywhere dense. The actual road network, however, is not ubiquitous; commuting trips do not always proceed along radial routes. The calculation of minimum required commute with equations (4) and (5) might therefore be downward biased. To determine the effect of actual road network on the estimate of excess commuting, I also calculate the minimum commute required by the monocentric model by applying White's calculation to the predicted densities of employment and resident workers instead of actual density pattern in previous studies (Hamilton [17], Small and Song [36] and White [42]). In this case, a linear program is used to assign trip flows among zones so that the aggregate commuting distance is minimized, given the predicted numbers of jobs and resident workers at each zone.<sup>7</sup>

The second row in Table 5 presents the results, showing that the minimum required commute is 1.99 miles and about 81 percent of the average actual commute is excess. The actual road network, therefore, results in a minimum commute that is about twice as large as that obtained by Hamilton's computation, increasing from 1.02 miles to 1.99 miles. This finding indicates that the assumption of radial transportation network considerably underpredicts the minimum commute required by the monocentric model.

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<sup>7</sup>The monocentric density functions have different accuracies in predicting the total employment and resident workers. A linear program, however, requires same aggregation for employment and resident workers. Here I scale the predicted aggregation down or up to match the actual total (which is same for both employment and resident workers).

Table 5. Estimates of Average Required and Excess Commute

	Required (miles)	Excess (%)
Monocentric Model <sup>a</sup>	1.02	90.56
Monocentric Model <sup>b</sup>	1.99	81.59
Polycentric Model <sup>b</sup>	4.35	59.76

Note: The average actual commuting distance is 10.81 miles.

<sup>a</sup> Estimates with Hamilton's calculation.

<sup>b</sup> Estimates with White's calculation.

### *Polycentric Required Commute*

The polycentric required commute is estimated by applying White's calculation to the predicted densities of the polycentric functions. The third row in Table 5 presents the results, showing that the polycentric model has a minimum average required commute of 4.35 miles. This result indicates that the polycentric required commute is considerably larger than that required by the monocentric model. The latter requires 1.02 miles by Hamilton's calculation and 1.99 miles by White's calculation. Comparing with the monocentric excess commutes of 90.6 percent and 81.6 percent, the polycentric model has a much smaller amount of excess commute, with less than 60 percent of the actual commute being excess. Hence, it explains the observed commuting patterns much better than the monocentric model.

These findings have two implications. First, the polycentric model requires more urban commuting than the monocentric model. Put differently, polycentricity has a positive effect on the estimate of minimum required urban commute. This implication is consistent with the argument of Suh [38] and White [42] but conflicts with that of Gordon et al. [11] and Gordon and Wong [13].<sup>8</sup> As shown earlier, the monocentric density gradients are quite similar for employment and resident worker distributions, indicating that the degrees of employment decentralization and resident worker suburbanization are close. This general balance between housing and jobs, in turn, requires a smaller amount of minimum

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<sup>8</sup>Without question, Gordon et al. [11] and Gordon and Wong [13] are correct within the "strong" monocentric model that assumes all jobs are located at the CBD. The standard monocentric model, however, has incorporated the assumption that some employment is decentralized. Mills and Hamilton [28] and Muth [32] show that the standard monocentric predictions remain regarding the rent function and density function, assuming that firms pay wages compensating the commuting costs.

commuting than the polycentric model. Hence, it is the decentralization of employment rather than the polycentric structure that makes it possible to reduce the urban commuting.

The second implication suggests that households do make attempts to economize on commuting. Like the standard monocentric model, the polycentric model assumes that accessibility to workplace is the primary determinant of the residential location choices. Since it is shown earlier that the polycentric model is superior to the monocentric model in explaining the spatial structure, the trade-off assumption expects that the polycentric model is also superior to the monocentric model in explaining the actual urban commute. The findings that the polycentric model explains much better both the actual commuting distance and the actual distribution patterns, therefore, supports the assumption in the standard urban model regarding the commuting behavior.

It is worth noting that Hamilton [16] also presents evidence which supports the assumption that urban households economize on their commuting. Supposing that households are indifferent to commuting and select their homes and job sites at random, Hamilton found that the mean commute is 12.09 miles. Comparing with the average actual commuting distance of 8.7 miles, the actual commute is less than 72 percent of the commute that would emerge if households chose their homes and job sites at random. Clearly, urban households make attempts to economize on commuting.

## CONCLUSIONS AND IMPLICATIONS

Analyzing of the 1980 journey-to-work Census data in the Los Angeles region, this paper has shown that the polycentric density functions fit the actual spatial structure statistically better than the monocentric model. For both resident worker and employment distributions, the monocentric model is rejected at a significance level of 0.0001 in favor of the polycentric model. This result suggests that polycentricity clearly exists in large urban areas and indicates that the overall access to major employment centers is much more important in location choices than the access to the central business district.

This paper has determined a positive effect of polycentricity on the estimate of minimum required commute, by comparing the monocentric and polycentric required commutes. The results show that the monocentric model requires a small amount of commute: 1.02 miles by Hamilton's calculation and 1.99 miles by White's calculation; whereas the polycentric model requires 4.35 miles. Comparing with the actual average commute of 10.81 miles, they account for 9.4 percent, 18.4 percent, and 40 percent of the actual average commuting distance. These results indicate that the polycentric structure increases the estimate of minimum required urban commuting, and that the polycentric model explains better the observed commuting pattern than the monocentric model.

Hamilton's original finding, that the monocentric model does a very bad job in predicting the actual commuting behavior, is more an indictment of the monocentricity assumption than a rejection of the assumption on commuting behavior in location choices. The monocentric model greatly underpredicts the actual commute because it inadequately

represents the spatial structure in large urban areas. Hamilton's calculation, which assumes there exists a radial transportation network and commuting trips proceed along radial routes, also contributes to this bias. Once polycentricity is substituted for monocentricity and the actual road network is used, actual commuting behavior is much better explained.

The findings that the polycentric model explains much better both the observed commuting distance and the actual distribution patterns help to preserve the assumption that urban households make strong attempts to economize on commuting. In turn, they suggest the policy implication that efforts to promote more efficient urban form, such as the jobs-housing policy may have the potential to succeed.

## REFERENCES

- [1] W. Alonso. *Location and Land Use*. Harvard University Press, 1964.
- [2] R. Bussiere and F. Snickars. Derivation of the Negative Exponential Model By an Entropy Maximizing Method. *Environment and Planning A* 2: 295-301, 1970.
- [3] R. Cervero. *America's Suburban Centers: The Land Use Transportation Link*. Unwin Hyman, 1989.
- [4] G. C. Chow. *Econometrics*. McGraw-Hill Book Company, 1983.
- [5] C. Clark. Urban Population Densities. *Journal of Royal Statistical Society, Serial A*, 114: 490-494, 1951.
- [6] M. L. Cropper and P. L. Gordon. Wasteful Commuting: A Re-Examination. *Journal of Urban Economics* 29(1): 2-13, 1991.
- [7] R. T. Dunphy. Defining Regional Employment Centers in an Urban Area. *Transportation Research Record* 861: 13-15, 1982.
- [8] M. W. Frankena. A Bias in Estimating Urban Population Density Functions. *Journal of Urban Economics* 5: 35-45, 1978.
- [9] A. R. Gallant. Nonlinear Regression. *The American Statistician* 29(2): 73-81, 1975.
- [10] G. Giuliano and K. A. Small. Subcenters in the Los Angeles Region. *Regional Science and Urban Economics* 21: 163-182, 1991.
- [11] P. Gordon, A. Kumar, and H. Richardson. The Influence of Metropolitan Spatial Structure on Commuting Time. *Journal of Urban Economics* 26: 138- 151, 1989.
- [12] \_\_\_\_\_, H. Richardson and H. L. Wong. The Distribution of Population and Employment in a Polycentric City: The Case of Los Angeles. *Environment and Planning A* 18: 161-173, 1986.
- [13] \_\_\_\_\_ and H.L. Wong. The Costs of Urban Sprawl: Some New Evidence. *Environment and Planning A* 17: 661-666, 1985.
- [14] D. L. Greene and J. Barnbrock. A Note on Problem in Estimating Exponential Urban Density Models. *Journal of Urban Economics* 5: 285-290, 1978.
- [15] D. A. Griffith. Modelling Urban Population Density in a Multi-Centered City.



- Journal of Urban Economics* 9: 298-310, 1981.
- [16] B. W. Hamilton. Wasteful Commuting. *Journal of Political Economy* 90: 1035-1053, 1982.
- [17] \_\_\_\_\_. Wasteful Commuting Again. *Journal of Political Economy* 96: 1497-1504, 1989.
- [18] E. Heikkila, P. Gordon, J. I. Kim, R. B. Peiser, H. Richardson and D. Dale-Johnson. What Happened to the CBD-Distance Gradient?: Land Values in a Policentric City. *Environment and Planning A* 21: 221-232, 1989.
- [19] R. W. Helsley and A. M. Sullivan. Urban Subcenter Formation. *Regional Science and Urban Economics* 21: 255-275, 1991.
- [20] P. Kemper and R. Schmenner. The Density Gradient for Manufacturing Industry. *Journal of Urban Economics* 1: 410-427, 1974.
- [21] M. K. Macauley. Estimation and Recent Behavior of Urban Population and Employment Density Gradients. *Journal of Urban Economics* 18: 251-260, 1985.
- [22] J. F. McDonald. The Intensity of Land Use in Urban Employment Sectors: Chicago 1956-1970. *Journal of Urban Economics* 18: 261-277, 1985.
- [23] \_\_\_\_\_. The Identification of Urban Employment Subcenters. *Journal of Urban Economics* 21: 242-258, 1987.
- [24] \_\_\_\_\_ and H.W. Bowman. Some Tests of Alternative Urban Population Density Functions. *Journal of Urban Economics* 3: 242-252, 1976.
- [25] E. S. Mills. An Aggregative Model of Resources Allocation in a Metropolitan Area. *American Economic Review* 57: 192-210, 1967.
- [26] \_\_\_\_\_. The Value of Urban Land. In H. S. Perloff, editor, *The Quality of the Urban Environment*. Johns Hopkins Press, 1969.
- [27] \_\_\_\_\_. *Studies in the Structure of the Urban Economy*. Johns Hopkins Press, 1972.
- [28] \_\_\_\_\_ and B. W. Hamilton. *Urban Economics*. Scott, Foresman and Co., fourth edition, 1989.
- [29] \_\_\_\_\_ and K. Ohta. Urbanization and Urban Problems. In H. Patrick and H. Rosovsky, editors, *Asia's New Giant: How the Japanese Economy Works*. The Brookings Institution, 1976.

- [30] R. F. Muth. *Cities and Housing*. University of Chicago Press, 1969.
- [31] \_\_\_\_\_. Numerical Solution of Urban Residential Land-Use Models. *Journal of Urban Economics* 2: 307-332, 1975.
- [32] \_\_\_\_\_. Models of Land-Use, Housing, and Rent: An Evaluation. *Journal of Regional Science* 25(4): 593-606, 1985.
- [33] J. Odland. The Conditions for Multi-Centered Cities. *Economic Geography* 54: 234-245, 1978.
- [34] Y. Y. Papageorgiou and D. Pines. The Exponential Density Function: First Principles, Comparative Static, and Empirical Evidence. *Journal of Urban Economics* 26: 264-268, 1989.
- [35] K. Sasaki. The Establishment of a Subcenter and Urban Spatial Structure. *Environment and Planning A* 22: 369-383, 1990.
- [36] K. A. Small and S. Song. Wasteful Commuting: A Resolution. *Journal of Political Economy* 100: 888-898, 1992.
- [37] \_\_\_\_\_. Population and Employment Densities: Structure and Change. Working Paper, Department of Economics, University of California, Irvine, 1992.
- [38] S. H. Suh. Wasteful Commuting: An Alternative Approach. *Journal of Urban Economics* 28: 277-286, 1990.
- [39] L. Thurston and A. M. Yezer. Testing the Monocentric Urban Model: Evidence Based on Wasteful Commuting. *AREUEA Journal* 19: 41-51, 1991.
- [40] W. C. Wheaton. A Comparative Static Analysis of Urban Spatial Structure. *Journal of Economic Theory* 9: 223-237, 1974.
- [41] M. J. White. Firm Suburbanization and Urban Subcenters. *Journal of Urban Economics* 3: 323-343, 1976.
- [42] \_\_\_\_\_. Urban Journey Are Not Wasteful. *Journal of Political Economy* 96: 1097-1110, 1988.
- [43] \_\_\_\_\_. Commuting and Congestion: A Simulation Model of a Decentralized Metropolitan Area. *AREUEA Journal* 18: 335-368, 1990.