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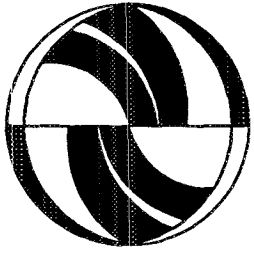https://escholarship.org/uc/item/33s027ms

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Modelling Worker Residence Distribution in the Los Angeles Region

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Summary

This paper examines the spatial pattern of worker residences with three different density functions: monocentric, polycentric, and dispersive. Analysis of the 1980 journey-to-work census data for the Los Angeles region reveals that the polycentric density function statistically explains the actual distribution better than the monocentric density function, but the dispersive density function fits best. These findings confirm a polycentric spatial pattern, and also imply that overall accessibility to employment opportunities is the primary determinant of residential location choices.

Introduction

Urban economists often use density functions to describe the spatial patterns of settlement within cities and to examine their changes over time (Muth, 1969; Small and Song, 1993). Density functions relate population (or worker residence) per unit area within a city to the distance(s) from the center(s) of the city. The economic model underlying these density functions is a static equilibrium model, in which households value access to urban center(s) and maximize their utilities by trading off transportation costs and land rents. Put differently, urban households choose locations to maximize their welfare according to commuting costs, space consumption, and their income. Access to center(s) is reflected in land rents. Hence, the closer to the center(s), the higher the rent is. In turn, higher rents reduce space consumption, leading to higher densities.

In the literature on density functions, almost all analyses assume monocentricity of urban structure. The standard monocentric model, however, has been thought to be a poor description of large metropolitan areas. Several recent studies have demonstrated the presence of multiple employment centers in such areas (McDonald, 1987; Cervero, 1989; Giuliano and Small, 1991). Studies using the monocentricity assumption thus might lead to a distorted understanding of spatial structure.

The polycentric nature of urban structure has been incorporated into empirical work on density analyses (Griffith, 1981; Gordon *et al.*, 1986; Small and Song, 1993). Griffith was unable to detect the effects of employment subcenters on population in Toronto in 1971. Gordon *et al.* have examined the distributions of population and employment with a polycentric model for the Los Angeles area. They found that the polycentric model fits better than the monocentric model for the population and employment distributions. Their

study, however, lacked statistical tests between these two models, and analyzed population based on population centers rather than employment centers. Small and Song use density functions to examine spatial patterns and their changes during 1970s for the Los Angeles region. Using small-zone data, they estimate monocentric and polycentric density functions for employment and population, for 1970 and 1980. They find that polycentric density functions fit statistically better than monocentric density functions, and there was some shift in employment distribution toward to a more polycentric pattern. Their findings statistically verify the existence of polycentricity in the Los Angeles region, both for employment and population distributions.

This paper extends Small and Song (1993) by using three density functions: monocentric, polycentric, and dispersive. A polycentric density function generalizes the standard monocentric model by assuming that urban residents value access to all employment centers in their location choices. A dispersive density function further generalizes the polycentric model by assuming that urban residents not only value access to employment centers but also value access to the overall job opportunities in their location choices. Recent studies have shown that most employment is located outside major employment centers (Giuliano and Small, 1991; McDonald and Prather, 1991). Hence, the importance of employment centers on location choices might be limited. This paper examines the effects of employment centers and accessibility to the overall employment opportunities on the spatial pattern of worker residences. It analyzes resident workers rather than population because location theory really relates the former to employment opportunities.

Density Functions

This section presents three different density functions for modelling urban structure. First, it uses the standard monocentric model. Second, it employs a polycentric model suggested by Griffith (1981), assuming that worker residences are distributed in a pattern consistent with several employment centers, not just one. Third, it formulates a more general model, called a *dispersive model*, which assumes that urban workers value accessibility to all jobs, no matter where located, in their residence choices.

Monocentric Density Function

The assumption of monocentricity in urban form has been used in almost all analyses of urban structure (McDonald, 1989). Households value access to the center and trade off this access with housing costs to maximize their utility. As a result, urban residents are distributed in a circularly symmetric manner with density function $f(r)$, where r is the commuting distance or time from the center.

It is easy to show that the density function $f(r)$ has a negative slope. The closer to the center, the higher the land rent is. The higher land rents are reflected in higher housing costs, indicating that housing prices are negatively related to the commuting distance or time to the center, *ceteris paribus*. But higher housing prices reduce housing consumption because housing is a normal good. Hence, residential density is higher where housing prices are higher, which in turn implies that density is negatively related to commuting distance or time from the urban center. Muth (1985) and Mills and Hamilton (1989, pp. 112-114) show that this result still holds when some employment is decentralized.

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In the monocentric city literature, the only density function form that has been derived theoretically is the negative exponential density function (Muth, 1969, chpt.4; Mills and Hamilton, 1989, appendix A; Papageorgiou and Pines, 1989). The negative exponential density function is also the most commonly used and supported by empirical studies (Muth, 1969; Mills, 1972). Straszheim (1974, p. 445) stated "all existing empirical evidence indicates that densities at different distances from the city center can be approximated by the negative exponential, and that this function provides as good as any other nonlinear form." McDonald and Bowman (1976) conducted a test on ten different density functions based on the two criteria of maximum explanatory power in standard analysis and accuracy in predicting total population in the urbanized area. They conclude that no single function will best describe population distribution for all urbanized areas, and the negative exponential density function is not surpassed by other functions.

The negative exponential density is used in this paper, and is written here as

$$D_i = D_0 e^{-g r_i} e^{u_i} , \qquad i = 1, 2, ..., M , \qquad (1)$$

where D_i is the worker residence density at commuting distance or time r_i to the single urban center, D_0 and g are parameters to be estimated from the data by ordinary least squares after taking the natural logarithm of equation (1), M is the sample size and e^{u_i} is a multiplicative error term associated with zone i.[1] Theoretically, D_0 is the density extrapolated to the urban center, and g is the density gradient measuring the percentage fall off in density for a unit increase in distance from the central business district (CBD).

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Polycentric Density Function

The assumption of monocentricity has been relaxed in a number of theoretical studies (White, 1976; Odland, 1978; Sasaki, 1990). Several recent empirical studies have also demonstrated the presence of employment subcenters in large American cities (McDonald, 1987; Cervero, 1989; Giuliano and Small, 1991). For example, Giuliano and Small (1991) have identified 32 centers for the Los Angeles region in 1980, based on simple intuitive criteria on employment density and total employment. The polycentric nature of urban structure has been also incorporated into the density analyses (Griffith, 1981; Gordon *et al.*, 1986; Small and Song, 1993).

The natural extension of the monocentric model is to assume that workers value access to all employment centers in their location decisions. Hence, a polycentric model can be formulated by assuming that worker residences are distributed in a pattern consistent with several employment centers, not just one. At a given location, densities are functions of commuting distances or times to all employment centers.

This paper uses an additive density function which has been suggested and estimated by Griffith (1981), Gordon *et al.* (1986), and Small and Song (1993):[2]

$$D_i = \sum_{n=1}^{N} a_n e^{-b_n r_{ni}} + v_i, \qquad i = 1, 2, ..., M, \qquad (2)$$

where N is the number of employment centers in an urban area, r_{ni} is the commuting distance or time from center n to zone i, v_i is an error term associated with zone i,[3] a_n and b_n are parameters to be estimated for each employment center n. This specification of the polycentric model assumes that density at any location is the sum of negative exponential

density functions, each reflecting the influence of a center on that location.

When the intercepts of all centers except one are zero, the polycentric form collapses to the monocentric form. Therefore, I can perform statistical tests on hypothesis that the polycentric model explains the actual distributions better than the monocentric model. I can also examine the impact of an individual employment center on the worker residence distribution and test the significance of each center n in explaining the overall density pattern by means of F-test on its parameters a_n and b_n.

Dispersive Density Function

The share of employment in majors centers is not large. For the Los Angeles region, the 1980 journey-to-work data show that 18 percent of employment is located in 6 major centers defined later in this study. Using the same data, Giuliano and Small (1991) show that only 32 percent of employment are located in 32 centers they identified. These facts may indicate that the importance of employment centers on residential location choice is limited.

A more general urban pattern can be formed if workers not only value access to employment centers but also value access to the overall job opportunities in their location choices. In this paper, I assume that worker residence densities are a function of accessibility to all job opportunities in the region, and formulate a more general density function as the following,

$$D_i = e^{\alpha_1} A_i^{\alpha_2} e^{\epsilon_i} , \qquad i = 1, 2, ..., M. \qquad (3)$$

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where D_i is the density of worker residences in zone i, A_i is the accessibility of zone i to employment opportunities in all zones, and ϵ_i is an error term. α_1 and α_2 are parameters to be estimated.

The variable A_i measures the potential of employment opportunities that can be reached from a given location. It is defined here as a negative exponential function of commuting distance or time to other locations weighted by the amount of employment at each location. This specification is recommended by Ingram (1971) as the most suitable form for determining the accessibility at a given location, and is used by Hansen (1959) and Dalvi and Martin (1976).

Denote r_{ij} to be the commuting distance or time from i to j, and E_j to be the amount of employment in zone j. Accessibility at zone i (A_i) can be expressed as

$$A_i = \frac{\sum_j E_j e^{-\alpha r_i}}{E} , \tag{4}$$

where α is a parameter to be estimated measuring the resistance of space separation and $E = \sum_j E_j$ is the total employment in an urban area.

In the model specified by equation (3), no employment centers are defined; but each zone is like a center because urban workers value the accessibility to the job opportunities in that zone. In line with the monocentric and polycentric models, I call this the *dispersive* model. In fact, a parallel can be derived. First, the dispersive form collapses to the monocentric form if employment is assumed to be located entirely in the central business district (CBD), since

$$D_i = e^{\alpha_1}\left[\frac{Ee^{-\alpha r_i}}{E}\right]^{\alpha_2} e^{\epsilon_i} = e^{\alpha_1} e^{-\alpha\alpha_2 r_i} e^{\epsilon_i} = D_0 e^{-g r_i} e^{\epsilon_i} \tag{5}$$

where $D_0 = e^{\alpha_1}$ and $g = \alpha\alpha_2$. Second, the dispersive form becomes a special case of the polycentric form when $\alpha_2 = 1$. Substituting equation (4) and $\alpha_2 = 1$, equation (3) becomes

$$D_i = e^{\alpha_1}\left[\frac{\sum_j E_j e^{-\alpha r_{ij}}}{E}\right] e^{\epsilon_i} = e^{\epsilon_i}\sum_j a_j e^{-\alpha r_{ij}}, \tag{6}$$

where $a_j = (e^{\alpha_1}/E)E_j$. Like a center in the polycentric model, each zone in the dispersive model exerts some influence on worker residence distribution. Including α_2, the dispersive model has three unknown parameters; the other two are α_1 and α. Hence, it has many fewer parameters than the polycentric model.

When α_2 is not equal to one, the relationship between the dispersive and the polycentric models becomes less intuitive. The economic behavior underpinning the dispersive model, however, is still clear. That is, urban workers value accessibility to employment both within and outside centers in their residential location choices. In this sense, the dispersive model generalizes the polycentric model.

Data

The study area consists of five counties in the greater Los Angeles region, covering the urban parts of Los Angeles, Orange, Riverside, San Bernardino, and Ventura counties. The geographical unit is traffic analysis zones (AZ), defined by the Southern California

Association of Governments (SCAG). Like census tracts, traffic analysis zones are aggregates of census blocks, and their boundaries are determined by functional traffic characteristics; but they need not have a fixed population. Hence, they reduce the "census-tract delineation bias" observed by Frankena (1978) in density estimation. The study area consists of 1124 AZs, after deleting 161 very low-density zones for simplicity.[4] These 1124 zones cover 3,401 square miles.

Data are from the 1980 Journey-to-Work Census for the Los Angeles region, provided by the SCAG. The data include aggregate zone-to-zone commute flows. They are used to compute zonal worker residences and employment. For each zone, zonal worker residences is the sum of its outward commute flows to all destinations (including workers who live and work in the same zone); and zonal employment is the sum of its inward commute flows from all origins (including workers who live and work in the same zone). This research analyzes 4.53 million workers who both live and work in the 1124-AZ study area. Since location theory really only considers resident workers and since only employed individuals commute to work, this paper analyzes resident workers rather than population. Data on zone-to-zone commuting distances and times are extracted from the data created for the Urban Transportation Planning Package (UTPP), which is calibrated based on a peak-period representation of the road network.

The data used in this study are the same as those in Giuliano and Small (1991) but different from those in Small and Song (1993). Small and Song instead use data obtained from the California Department of Transportation (Caltrans) which provide information on population and employment for the same system of analysis zones. This study uses journey-

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to-work census data because it analyzes worker residences rather than population.

Empirical Results

Prior to density estimation, I identify employment centers in the region. McDonald (1987) discusses several empirical criteria for the identification of urban employment subcenters. He suggests that local peaks in gross employment density and the employment-population ratio are the best indicators of employment subcenters. Giuliano and Small (1991) present a simple systematic identification of employment centers, based on McDonald's suggestion. They define a center as a contiguous set of zones, each with density above $\bar{D}$, that together have at least $\bar{E}$ total employment. Using 1980 census journey-to-work data for the Los Angeles region, Giuliano and Small have identified 32 centers with criteria $\bar{D}=10$ employees per acre and $\bar{E}=10,000$ employees ($\bar{E}=7,000$ for outer centers).

This research uses a version of McDonald's definition suggested by Giuliano and Small (1991), because it incorporates adjacent high-density zones and restricts attention to centers large enough to exert a potentially significant influence on the urban structure in a metropolitan area. In order to have a manageable number of employment centers in density function estimation, I use criteria of $\bar{D}=15$ and $\bar{E}=35,000$. Six employment centers are identified based on these criteria. They are listed in Table 1. Among the six centers identified, five are located in Los Angeles County and one, Santa Ana, is located in Orange County.

Monocentric Density Estimates

The monocentric density function of worker residences is estimated by ordinary least squares, after taking the natural logarithm of equation (1). Seven zones with zero density of worker residence are deleted, leaving 1117 observations in the regression. Table 2 presents the estimated monocentric density gradients, with respect to center downtown Los Angeles.[5] The density gradient estimates show that worker residence distribution was quite flat in 1980 in the Los Angeles region. Based on distance, $g=0.0457$, implying that worker residence density falls off at 4.6 percent per mile increase in distance from the urban center. Based on time, $g=0.0306$, implying that worker residence density falls off at 3.1 percent per minute increase in commuting time from the urban center.

Table 2 also shows that the monocentric density function has low R^2-values. It explains only 38.6 percent of the variation in worker residence distribution based on distance and 37.2 percent based on time. These results imply that the monocentric model is poor at explaining the actual distribution in large urban areas, such as the Los Angeles region.

The monocentric estimates based on distance, $log(D_0)=8.657$, $g=0.0457$, and $R^2=0.39$ (Table 2), are comparable to those obtained by Gordon *et al.* (1986) and Small and Song (1993) for the Los Angeles region. Analyzing the population distribution in 1980, Gordon *et al.* have $D_0=10700$ (i.e., $log(D_0)=9.278$) and $g=0.0357$ (page 167, Table 6) and $R^2=0.31$ (page 168, Table 8). Small and Song have $log(D_0)=9.414$, $g=0.0411$, and $R^2=0.35$. Hence, I have a lower intercept and a higher density gradient. These results, however, are consistent with the expectation that worker residences have lower densities but are more centralized than population. The R^2-values indicate that the monocentric model

explains worker residences slightly better than it explains population.

Polycentric Density Estimates

The polycentric density function is estimated by nonlinear least squares. An issue which arises here is the problem of spatial multicollinearity in regression models because the polycentric density function includes several distance variables (Heikkila, 1988). Regression results in this research, however, do not indicate a severe collinear problem. First, by examining correlation matrix (Table 4), I find there is little correlation among centers except between the centers of L.A. Airport and Pasadena. Second, I obtain a moderate R^2-value, with most centers having statistically significant estimates for both the distance and time measures (Table 3). A common symptom of a multicollinearity problem is a high R^2-value with insignificant estimates.

The estimates presented in Table 3 support the existence of polycentricity; both intercept and gradient are statistically significant at a 5 percent level (1-sided test, i.e., with $t > 1.64$) for five of the six centers. The formal test for the polycentricity, however, is based on the statistic

$$F = \frac{(SSR^r - SSR^U)/q}{SSR^u/(M-p)} \tag{7}$$

where SSR^r and SSR^u are the restricted (monocentric) and unrestricted (polycentric) sums of squared residuals, M is the sample size, p is the number of parameters being estimated in the unrestricted estimate, and q is the number of restrictions on these parameters in the restricted estimate.[6]

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Performing such a test to the unrestricted (polycentric) model with six centers, I have F-statistic values of 27.68 based on distance and 23.19 based on time. With (10, 1112) degrees of freedom, these F-values indicate that the null hypothesis (i.e., the model is monocentric) is soundly rejected at a significance level of 0.0001. Therefore, the polycentric model statistically explains the distribution of worker residences much better than the monocentric model.

I cannot make a center-to-center comparison between my polycentric estimates for worker residence distribution and those by Small and Song (1993) for population distribution, due to the different centers included in the models.[7] The general conclusion, however, is the same; the polycentric model explains the actual distribution statistically better than the monocentric model. In addition, I observe that these two studies have similar R^2-values. Using a distance measure, I have $R^2 = 0.481$ for a six-center model; Small and Song have $R^2 = 0.498$ for an eight-center model.

Dispersive Density Estimates

The dispersive density function (equation 3) is a nonlinear regression model with respect to parameters (α_0, α_1, α), because α is an unknown parameter in the definition of accessibility (A). It is extremely difficult to directly estimate this nonlinear regression model, because the independent variable A is the sum over 1124 zonal terms and each of those terms contains parameter α in its exponent.

A grid search is a feasible strategy, since α is the single parameter that causes the nonlinearity in the regression model (Greene, 1990, p. 364). The search seeks and

minimizes the sum of squared residuals (*SSR*) for all of the parameters by scanning over values of α for the one that gives the lowest *SSR*. For a given value of α, the dispersive density function becomes a linear regression model after taking the natural logarithm of equation (3); for a given α, A_i can be computed, and remaining parameters can be estimated by least squares. In turn, the sums of squared residuals and coefficients of determination (R^2) can be obtained. Scanning over a range of α, the best value of α is chosen as that with the lowest value of *SSR* and the highest value of R^2, based on some desired precision. The associated least squares estimates of parameters (α_0, α_1) and their standard errors are then estimated.

The grid search proceeds with $\alpha = 0$, 1, 2, ... K, where K = 10, then $\hat{\alpha}_{max}$-1 to $\hat{\alpha}_{max}$+1 in increments of 0.1, and so on, until the desired precision is achieved. The parameter $\hat{\alpha}_{max}$ is the value of α that has a highest R^2-value for each round of grid research. The grid search continues until the coefficient of determination (R^2) does not change in its third decimal point. Table 5 shows the results: with a distance measure, $\alpha = 0.499$ per mile and $R^2 = 0.560$; with a time measure, $\alpha = 0.211$ per minute and $R^2 = 0.557$.

Comparing the monocentric results (Table 2), the dispersive density function has considerably higher coefficients of determination (R^2). Hence, the dispersive model explains the actual distribution of worker residences much better than the monocentric model. To test whether the dispersive model is statistically superior to the monocentric model, a likelihood-ratio test for non-nested hypotheses developed by Vuong (1989) was performed. The test is described in the Appendix. The null hypothesis is that the dispersive model is equivalent to

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the monocentric model. Test results give Vuong's value of 6.05 based on distance and 6.46 based on time, rejecting the null hypothesis at a significance level of 0.0001 in favor of the dispersive model.

The dispersive model not only fits better but also has many fewer unknown parameters than the polycentric model. The dispersive model has greater accuracy in predicting total worker residences than the polycentric model, although this cannot be tested statistically due to the different dependent variables in each model. The dispersive model underpredicts the actual total worker residences by 8.69 percent based on distance and 6.82 based on time; whereas the polycentric model overpredicts the actual total by 34.12 percent and 34.45 percent respectively. Hence, the dispersive model is superior to the polycentric model based on the criterion of accuracy on predicting total workers, which is one of the two criteria used by McDonald and Bowman (1976).

With regard to unknown parameters, the dispersive model has only three (α, α_1, α_2); while the polycentric model has twelve for a six-center model. Therefore, it is much easier to obtain reliable estimates for the dispersive density functions than for the polycentric density functions. In fact, it becomes difficult to obtain convergence when a polycentric density function includes more employment centers, because the number of unknowns in the function increases by twice the number of centers, and the function itself is nonlinear. Moreover, the polycentric density function has a potential problem of spatial multicollinearity, as discussed earlier.

The empirical results for parameters α and α_2 have two implications. First, the estimated α-values of 0.499 per mile and 0.211 per minute imply that accessibility to job

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opportunities (or attractiveness of job opportunities) declines 39 percent per mile and 19 percent per minute increase from the workplace.[8] Result using the time measure (i.e., $\alpha = 0.211$) is comparable to those of Dalvi and Martin (1976), who report $\alpha = 0.17$ when total employment is used as the measure of attractiveness, and $\alpha = 0.225$ when only retail employment is used. No comparable studies have been conducted using a distance measure, although some studies have specified a negative exponential distance decay function where the exponent is determined by experimentation (Brigham, 1965; Ihlanfeldt and Raper, 1990).

The second implication relates to the parameter α_2, which measures the elasticity of worker residence density with respect to accessibility. Table 5 shows that α_2 has an estimated value less than one, implying that worker residence density responds to accessibility less than proportionately. This result also implies that the dispersive density function is not a direct extension of the polycentric density function. As shown earlier, the former is a "generalization" of the latter when $\alpha_2 = 1$. Table 5, however, shows that α_2 is statistically different from 1 even though its standard error is downward-biased by the grid search procedure (Fomby et al., 1984, pp. 426-431).

Conclusions

I have examined the spatial pattern of worker residences with three urban density functions: monocentric, polycentric, and dispersive. Analyzing the 1980 journey-to-work census data in the Los Angeles region, I have shown that the polycentric density function fits the actual distribution much better than the monocentric model, thus confirming polycentric form in the Los Angeles region.

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I also formulated a more general density function, the dispersive function, by .

assuming that urban workers not only value access to employment centers but also value

access to the overall job opportunities in their location choices. Results show that this model

best fits the actual worker residence distribution. The dispersive model is also superior to

the polycentric model based on the criterion of accuracy on predicting total workers. These

findings imply that overall accessibility to employment opportunities is the primary

determinant of residential location choices.

Notes

1. The literature on the negative exponential density function have used two specifications of the error term in estimation. One assumes a multiplicative error term and estimates the two coefficients of the model by ordinary least squares after taking logarithm of the model; the other assumes an additive error term and estimates the density function by nonlinear least squares. Greene and Barnbrock (1978) show that a multiplicative error term is more appropriate with respect to the criterion of homoscedasticity of the error term in regression models.

2. As pointed out by Heikkila et al. (1989), a polycentric density function could be postulated under several alternative assumptions regarding the characteristics of centers and their relationship. For the Los Angeles region, Small and Song (1993) suggest that the *sum* of center-specific functions is a plausible specification.

3. When a multiplicative form of the error term is used, convergence on the parameter estimation cannot be obtained due to the high degree of nonlinearity in the density functions.

4. All the deleted zones are remote from the highly developed parts of the region, with the exception of 11 zones which have both zero worker residence and employment and 11 largely undeveloped zones in the Santa Monica mountains which separate the densely developed West Los Angeles corridor (roughly, Hollywood to Santa Monica) from the more suburban San Fernardo Valley.

5. Using the likelihood-ratio test for non-nested hypotheses developed by Vuong (1989), Small and Song (1993) have statistically identified downtown Los Angeles as the monocentric center to the region. Following their approach, the same monocentric center is identified in this research, although these two studies use different data sets.

6. Under the null hypothesis, F is approximately distributed according to a central F-distribution with degrees of freedom $(q, M\text{-}p)$ (Gallant, 1975). There are two restrictions for each center, $a_n = b_n = 0$, for centers other than the downtown Los Angeles. If only $a_n = 0$ is imposed, the moment matrix (F'F in Gallant's notation) becomes singular, and b_n is unidentified (Gallant, 1975, p. 75). Hence, $p = 2N$ and $q = 2(N\text{-}1)$, where N is the (unrestricted) number of centers.

7. I also cannot compare my polycentric estimates with those by Gordon et al. (1986, Table 3, p. 165) because they estimate a population density function for Los Angeles County only, and the estimates are based on population centers rather than employment centers.

8. That is because $1\text{-}e^{-0.499} = 0.39$, and $1\text{-}e^{-0.211} = 0.19$.

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Table 1. Employment Centers Identified in 1980

Center Location	Total Emp.	Emp. Den. (Emp/Acre)	Dist. from CBD(miles)
Downtown LA (CBD)	429869	42.26	0.1
UCLA/Santa Monica	208166	25.31	15.8
LA Airport	48510	18.77	18.8
West Hollywood	43761	23.01	7.3
Santa Ana	37305	17.18	32.9
Pasadena	35675	25.14	12.1

Table 2. Monocentric Density Estimates

	$log(D_0)$	g	R^2
Based on distance	8.6574* (0.0586)	0.0457* (0.0017)	0.386
Based on time	8.9152* (0.0686)	0.0306* (0.0012)	0.372

Note: There are 1117 observations.
Standard errors are in parentheses.
*Estimate is statistically significant at 0.05 level, 1-sided test.

Table 3. Polycentric Density Estimates

Center Location	Based on Distance		Based on Time	
	Intercept	Gradient	Intercept	Gradient
Downtown L.A. (CBD)	2128*	0.3376*	3131*	0.1010*
	(771)	(0.1804)	(972)	(0.0534)
UCLA/Santa Monica	2671*	0.0665*	2997*	0.0463*
	(757)	(0.0293)	(827)	(0.0278)
L.A. Airport	2518*	0.0270*	3074*	0.0182*
	(897)	(0.0126)	(1038)	(0.0090)
West Hollywood	6936*	0.2482*	8038*	0.1020*
	(908)	(0.0511)	(1124)	(0.0254)
Santa Ana	2493*	0.0614*	2994*	0.0342*
	(384)	(0.0217)	(577)	(0.0119)
Pasadena	701	0.0344	350	0.0176
	(649)	(0.0627)	(703)	(0.0597)

$$R^2 = 0.481 \qquad R^2 = 0.464$$

Note: Standard errors are in parentheses.
There are 1124 observations.
*Estimate is statistically significant at 0.05 level, 1-sided test.

Table 4. Correlation Coefficients for Polycentric Model: Based on Distance

Downtown LA a	b	UCLA/S.M. a	b	LA Airport a	b	W. Hollywood a	b	Santa Ana a	b	Pasadena a	b
1.00	-0.61	0.10	-0.07	0.04	-0.01	0.18	-0.20	-0.01	-0.03	0.11	-0.03
	1.00	0.21	0.02	-0.16	0.08	-0.15	-0.30	0.06	-0.05	-0.06	0.04
		1.00	0.27	-0.53	0.07	-0.02	-0.59	0.20	0.39	-0.08	-0.15
			1.00	-0.64	-0.16	-0.01	-0.08	0.19	0.43	-0.36	0.01
				1.00	0.59	0.07	0.20	-0.37	-0.37	-0.26	-0.52
					1.00	-0.02	-0.06	-0.37	0.03	-0.69	-0.90
						1.00	-0.53	-0.03	-0.08	0.11	-0.01
							1.00	-0.07	0.01	-0.16	0.08
								1.00	-0.29	0.30	0.28
									1.00	-0.41	-0.26
										1.00	0.69
											1.00

Table 5. Dispersive Density Estimates

	α_1	α_2	R^2	α
Based on Distance	11.772* (0.1200)	0.7866* (0.0209)	0.560	0.499
Based on Time	12.853* (0.1489)	0.9416* (0.0251)	0.557	0.211

Note: α is obtained by grid search.
Standard errors are in parentheses.
There are 1117 observations.
*Estimate is statistically significant at 0.05 level, 1-sided test.

Appendix

Vuong (1989) develops a simple test for model selection between a pair of competing non-nested models F_θ and G_γ. Let $LR_M(\hat{\theta}_M, \hat{\gamma}_M)$ be the likelihood ratio statistic for model F_θ against model G_γ. That is:

$$LR_M(\hat{\theta}_M, \hat{\gamma}_M) = \sum_{m=1}^{M} \log \frac{f(Y_m \mid Z_m; \hat{\theta}_M)}{g(Y_m \mid Z_m; \hat{\gamma}_M)} \; ,$$

where M is the sample size; $\hat{\theta}_M$ and $\hat{\gamma}_M$ are the ML estimates; f and g are the values taken by the corresponding probability densities for observation m, evaluated in each case at the corresponding maximum-likelihood parameter estimate. Under the null hypothesis (H_o: F_θ and G_γ are equivalent), Vuong's value is asymptotically distributed according to a central normal distribution.

To test the null hypothesis, a critical value c is chosen from the standard normal distribution for some significance level. If the value of the statistic $M^{1/2}LR_M(\hat{\theta}_M, \hat{\gamma}_M)/\omega_M$ is higher than c, the null hypothesis is rejected in favor of F_θ being better than G_γ. If $M^{1/2}LR_M(\hat{\theta}_M, \hat{\gamma}_M)/\omega_M$ is smaller than $-c$, the null hypothesis is rejected in favor of G_γ being better than F_θ. If $\mid M^{1/2}LR_M(\hat{\theta}_M, \hat{\gamma}_M)/\omega_M \mid \leq c$, the null hypothesis cannot be rejected. ω_M is the square root of the variance of $\log[f(Y_m \mid Z_m; \hat{\theta}_M)/g(Y_m \mid Z_m; \hat{\gamma}_M)]$, defined by

$$\omega_M^2 = \frac{1}{M} \sum_{m=1}^{M} \left[\log\left(\frac{f_m}{g_m}\right)\right]^2 - \left[\frac{1}{M}\sum_{m=1}^{M}\log\left(\frac{f_m}{g_m}\right)\right]^2 \; .$$