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Publication Date

2002-10-03

Double Auctions, Ex-Post Participation Constraints, and the Hold-Up Problem

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October 3, 2002

Abstract

This paper makes two contributions in the context of seller-buyer relationships with bilateral relationship-specific investment. Firstly, we demonstrate how ex-post negotiations via double auctions can be used to alleviate and often resolve the hold-up problem. Secondly, we show that ex-post participation constraints make the hold-up problem unavoidable in environments where (i) production costs are relatively high compared to valuations and (ii) the trade price is independent of the investment levels.

Keywords: hold-up problem, double auction, participation constraints
JEL classification: C78, D21

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1 Introduction

Over the last 25 years, the hold-up problem in long-term trading relationships has attracted considerable theoretical interest (see e.g., Klein et. al., 1978, Williamson, 1985, Hart, 1995, Tirole, 2000, Segal, 1999). In a nutshell, the problem is that trading partners may hesitate to make desirable relationship-specific investments prior to trading (“ex ante”) because each firm risks to be “held up” by the partner once the investments are sunk (“ex post”).

In this paper, we firstly demonstrate how the hold-up problem can be alleviated or even resolved if the trade price is negotiated ex post via a double auction. Double auctions are widely used trading mechanisms. It is well-known that double auctions have good *ex-post* efficiency properties, in particular when the number of traders is large and traders have private information about their costs and valuations (Wilson, 1985, Satterthwaite and Williams, 2002). Our results provide a new rationale for the use of double auctions when there is just one buyer and one seller without private information about cost and valuation. In this case, the trading partners may select among the auction’s multiple equilibria in order to create prior investment incentives for themselves, and thereby promote *ex-ante* efficiency.

Secondly, our results show that in some environments, the presence of ex-post participation constraints (i.e., the requirement that both firms have non-negative ex-post continuation payoffs) makes the hold-up problem unavoidable. In these situations, none of the contractual solutions to the hold-up problem that have been proposed in the literature will work. The hold-up problem can then not be resolved. The environments where this occurs are quite simple. In particular, our results are very different from Segal’s (1999) and Hart and Moore’s (1999), who show—without imposing ex-post participation constraints—that the hold-up problem is unavoidable in certain complex environments.

Our model is based on the set-up of Hart and Moore (1988) which underlies most of the literature on the hold-up problem. Going back to this classical set-up is crucial if one wants to understand the role of double auctions and ex-post participation constraints relative to other approaches to the hold-up problem.¹ In Hart and Moore’s set-up, each of two risk-neutral firms, a seller and a buyer, simultaneously decide upon the costs spent on

¹Several authors have recently pointed out the potential role of bargaining with multiple equilibria for resolving the hold-up problem (Ellingsen and Johannesson, 2000, Carmichael and MacLeod, 2000, Tröger, 2002, Ellingsen and Robles, 2002). However, these contributions do not refer to Hart and Moore’s (1988) classical set-up.

investment in their trading relationship. Ex post, when investment costs are sunk, a single unit of a good may be produced and traded between the firms; trade with outsiders is not possible. Both the seller's production cost and the buyer's valuation for the good depend stochastically on the firms' investment levels. Formally, a cost-valuation pair (called *state*) is realized according to a probability distribution which depends on the pair of investment levels. The state is observed by both firms before the trade negotiations begin.

Now suppose that the trade price is negotiated via a double auction where the seller submits an ask price and, simultaneously, the buyer submits a bid price. If the bid is greater than or equal to the ask then the good is traded at an intermediate price; otherwise no trade occurs. Because the state is observed by *both* firms, there exist ex-post efficient equilibria. In fact, any trade price that is not smaller than production costs and does not exceed the valuation is an equilibrium price. In other words, via coordination on ex-post efficient equilibria in the double auction, the trade price may be chosen to be any function of the state—and possibly of the investment levels—that satisfies the ex-post participation constraints.

The double-auction approach contrasts typical contractual solutions to the hold-up problem, like Rogerson's (1992) mechanism-design contracts, Chung's (1991) and Aghion et al.'s (1994) specific-performance contracts, or Nöldeke and Schmidt's (1995) option contracts. These contracts must be specifically tailored to the parameters of the trading relationship in order to be effective. With a double auction in place, however, only the selection of equilibria is specific,² while the trading mechanism itself is generic.

It is easy to see that the double auction fully resolves the hold-up problem if investment levels are mutually observed. In this case, the trade price is a function of the state and the investment levels. In particular, if both firms choose the efficient investment level then prices may be chosen such that in all states the trading surplus is split in the same fixed proportion. Let this proportion be such that both firm's ex-ante expected profits (i.e., net of investment costs) are positive. Furthermore, if only one firm chooses the efficient investment level then prices may be chosen such that this firm appropriates the entire trading surplus in all states. According to this price scheme, any deviation from the efficient investment level leads to a non-positive profit and thus is not profitable. Both firms will choose the efficient

²Laboratory evidence by Binmore et al. (1998), Gantner et al. (1999), and Ellingsen and Johannesson (2000) also provides some support for the selection of equilibria that we are going to propose.

investment level.

The main part of the paper treats the case where investment levels are private. Here, results are more ambiguous. Firstly, we present an *underinvestment condition* which identifies parameter constellations such that underinvestment cannot be avoided if a double auction is in place. Secondly, we identify other parameter constellations such that double auctions may be used to provide *efficient* investment incentives (this includes parameter constellations such that the firms cannot be made residual claimants for the trading surplus). The underinvestment condition is satisfied for parameter constellations where, roughly speaking, production costs are relatively high compared to valuations, while efficient investment is obtained when production costs are relatively low.

As a core tool for our analysis, we define an investment-independent (II) *price scheme* as a function that assigns trade prices to states. An II price scheme is called individually rational (IR) if it satisfies the firms' ex-post participation constraints. Via coordination on ex-post efficient equilibria in the double auction, the firms can select any II IR price scheme. Consequently, the underinvestment condition implies that all II IR price schemes induce underinvestment. More generally, *the question whether a double auction can be used to resolve the hold-up problem is equivalent to the question whether an II IR price scheme exists which induces efficient investment.*

Ex-post participation constraints are, therefore, not just an implication of ex-post negotiations via a double auction. Rather, our results can be formulated in terms of ex-post participation constraints alone, without any reference to the ex-post interaction. In particular, for parameter constellations which satisfy the underinvestment condition, *any ex-ante contract or arrangement of any form* that resolves the hold-up problem by implementing an II price scheme *necessarily* violates an ex-post participation constraint (i.e., the implemented price scheme is not IR). This conclusion applies to virtually all the contracts suggested in the literature. Mechanism-design contracts, specific-performance contracts, and option contracts all implement II price schemes. In trading relationships where ex-post participation constraints exist, none of these solutions to the hold-up problem will work if the underinvestment condition is satisfied.

Finally, we show that second-best II IR price schemes are typically not nondecreasing in production cost and valuation. Therefore, double auctions alleviate the hold-up problem more than any contract that implements a nondecreasing II IR price scheme. In particular, double auctions are more ex-ante efficient than the renegotiated contracts analyzed in Hart and Moore (1988) and the renegotiation-proof implementation contracts proposed by

Rubinstein and Wolinsky (1992). These comparisons further emphasize our conclusion that double auctions are a powerful, yet simple, means to alleviate the hold-up problem.

Section 2 contains the description of Hart and Moore’s classical set-up of the hold-up problem and introduces key definitions. Section 3 discusses the environment with mutually observable investment (i.e., where all IR price schemes are feasible). Section 4 tackles the private investment case (i.e., where only II IR price schemes are feasible). We begin by recalling some standard results about price schemes which induce fixed-percentage shares of the trading surplus, and about constant price schemes. We also explain the comparative statics on which our main results are based. Subsection 4.1 introduces two lemmata which provide core tools and intuition for the subsequent analysis. The following two subsections contain the main results. In subsection 4.2, we formulate and analyze the underinvestment condition, while subsection 4.3 is devoted to a condition for efficient investment and to comparative statics. In subsection 4.4 we show that second-best II IR price schemes are typically not nondecreasing. In subsection 4.5 we present a simple class of parameter constellations which satisfies all our technical assumptions. Section 5 concludes, and section 6 contains proofs.

2 The Seller-Buyer Bilateral Investment Set-Up

Following Hart and Moore (1988) and the subsequent literature, we consider a risk-neutral seller firm, s , and a risk-neutral buyer firm, b , who are planning to trade one unit of a good at some future date. The buyer’s future valuation of the good, v , and the seller’s future production cost, c , are initially uncertain. We assume that $v \in V$ and $c \in C$, where C and V are finite sets. One may think of C and V as large sets (e.g., fine grids).³ Each pair $(c, v) \in C \times V$ is called a *state*. The probability of any state (c, v) is denoted $f(c, v | \beta, \sigma) > 0$. We assume that f is twice continuously differentiable with respect to (β, σ) . Any triple (C, V, f) is called a *parameter constellation*.

The firms’ interaction is as follows. First (“ex ante”), the firms simultaneously choose their investment levels, β and σ . We will consider two different cases: “mutually observable investment,” where each firm learns the investment choice of the other firm, and “private investment” where the investment levels remain private. Second (“ex post”), a state (c, v) is drawn

³It is possible to reformulate many of our results for the case where C and V are compact intervals. Such a reformulation brings hardly new insights while it is technically more demanding (cf. Tröger, 1997).

according to f , and is observed by both parties. Third, trade negotiations take place. This results in trade ($q = 1$) or no trade ($q = 0$), and a side payment p . Consequently, the realized payoffs are

$$u^b = qv - p - \beta, \quad u^s = p - qc - \sigma.$$

Any function

$$P : [0, \bar{\beta}] \times [0, \bar{\sigma}] \times C \times V \rightarrow \mathbb{R}$$

is called a *price scheme*. The *outcome* of the trading relationship can be summarized by (β, σ, P, Q) , where P is a price scheme such that $p = P(\beta, \sigma, c, v)$ denotes the payment following the path (β, σ, c, v) and, similarly,

$$Q : [0, \bar{\beta}] \times [0, \bar{\sigma}] \times C \times V \rightarrow \{0, 1\}$$

is a function such that $q = Q(\beta, \sigma, c, v)$ indicates trade ($q = 1$) or no trade ($q = 0$). An outcome (β, σ, P, Q) is called *ex-post efficient* if $Q = Q^*$, where⁴

$$\begin{aligned} Q^*(\beta, \sigma, c, v) &= 1 & \text{if } v > c, \\ Q^*(\beta, \sigma, c, v) &= 0 & \text{if } v \leq c. \end{aligned}$$

Throughout the paper, we will focus on equilibria with ex-post efficient outcomes.⁵ The *trading surplus* in state (c, v) is given by $\max\{v - c, 0\}$. Given an ex-post efficient outcome (β, σ, P, Q^*) , the *seller's share* of the trading surplus equals $P(\beta, \sigma, c, v) - c$ if $v > c$, and equals $P(\beta, \sigma, c, v)$ if $v \leq c$, while the *buyer's share* equals $v - P(\beta, \sigma, c, v)$ if $v > c$, and equals $-P(\beta, \sigma, c, v)$ if $v \leq c$. A price scheme P is called *individually rational* (IR) if both parties get nonnegative shares of the trading surplus in all states, i.e., if

$$\begin{aligned} c &\leq P(\beta, \sigma, c, v) \leq v & \text{if } v > c, \\ P(\beta, \sigma, c, v) &= 0 & \text{if } v \leq c. \end{aligned}$$

The set of IR price schemes is denoted \mathcal{IR} . A price scheme P is called *independent of the investment levels* (II) if $P(\beta, \sigma, c, v) = P(\beta', \sigma', c, v)$ for all (c, v) , $\beta, \sigma, \beta', \sigma'$. The set of II price schemes is denoted \mathcal{II} . Throughout the paper, II price schemes will be written as functions of the state, $P(\beta, \sigma, c, v) = P(c, v)$.

⁴Labeling trade ex-post inefficient if $c = v$ simplifies our presentation.

⁵This simplification is standard (e.g., Hart, 1995, p. 38), but not strictly necessary for our results (cf. footnote 11).

Double auctions

Now suppose that the trade negotiations take the form of a double auction where the firms make simultaneous trade price offers, p^b and p^s ; if $p^b \geq p^s$ then the good is traded ($q = 1$) at price $p = (p^b + p^s)/2$, but if $p^b < p^s$ then the good is not traded ($q = 0$), and no side payment is made ($p = 0$).⁶

We say that a price scheme P is *feasible* via a double auction if there exists a pure-strategy perfect Bayesian equilibrium with outcome (β, σ, P, Q^*) for some $\beta \in [0, \bar{\beta}]$, $\sigma \in [0, \bar{\sigma}]$. We say that a price scheme P is *selected* if the respective equilibrium is played. As a first step of the analysis, we characterize the set of feasible price schemes.

Proposition 1. *If investments are mutually observable then the set of feasible price schemes equals \mathcal{IR} . If investments are private then the set of feasible price schemes equals $\mathcal{IR} \cap \mathcal{II}$.*

Proof. The first statement follows from well-known properties of the double auction. As for the second statement, consider any equilibrium path (β, σ, P, Q^*) . A buyer deviation to some $\beta' \neq \beta$ will not be detected by the seller. Thus, for all states (c, v) with $c < v$, the seller will continue to offer the price $P(\beta, \sigma, c, v)$, and thus the buyer will also continue to offer the price $P(\beta, \sigma, c, v)$ (due to ex-post efficiency, this is true even if $P(\beta, \sigma, c, v) = v$). This implies $P(\beta', \sigma', c, v) = P(\beta, \sigma, c, v)$ for all states (c, v) with $c < v$, all β' and all σ' . From this one easily sees that only price schemes in $\mathcal{IR} \cap \mathcal{II}$ can be feasible. On the other hand, well-known properties of the double auction show that in fact *all* price schemes in $\mathcal{IR} \cap \mathcal{II}$ are feasible. \square

With Proposition 1 in mind, we do not have to refer to double auctions anywhere in the following formal analysis. Rather, we can take a set of feasible price schemes \mathcal{P} as a primitive. By assuming $\mathcal{P} = \mathcal{IR}$, we then implicitly analyze trading relationships where the ex-post interaction takes the form of a double auction and investments are mutually observable; similarly, $\mathcal{P} = \mathcal{IR} \cap \mathcal{II}$ refers to the case of private investment.

⁶ One might object against the double auction that it is not renegotiation-proof in the sense of contract theory (see, e.g., Hart, 1995) because some non-equilibrium outcomes are inefficient. A renegotiation-proof variant of the double auction is, however, easily obtained. In the spirit of Rubinstein and Wolinsky (1992), suppose that following any inefficient outcome the firms start another round of simultaneous offers (instead of implementing the inefficient outcome), and this is repeated forever or else until ex-post efficient trade is implemented. With respect to ex-post efficient equilibrium outcomes, such an iterated double auction is identical to the one-shot version that we assume. In this sense, our results are not disrupted by issues of renegotiation-proofness.

Ex-post participation constraints

The firms' *ex-post participation constraints* are defined as the requirement that the firms' ex-post continuation payoffs are non-negative. The role of ex-post participation constraints for the hold-up problem can be analyzed without reference to double auctions. Rather, one may consider any (possibly ex-ante) contractual or other arrangement of the firms which in equilibrium—possibly after renegotiations—leads to the implementation of an arbitrary price scheme and to ex-post efficient trade. In this framework, the firms' ex-post participation constraints are equivalent to the requirement that the implemented price scheme is IR. By assuming that the set of feasible price schemes equals $\mathcal{P} = \mathcal{IR} \cap \mathcal{II}$, we can ask *without reference to any specific model of contracting and/or renegotiations* whether resolving the hold-up problem with investment-independent trade prices requires violating an ex-post participation constraint.

Given a (nonempty) set of feasible price schemes \mathcal{P} , the trading relationship can be analyzed as follows. Suppose the price scheme $P \in \mathcal{P}$ is selected. Then, the ex-ante expected returns (gross of investment costs) are

$$R^b(\beta, \sigma, P) = \sum_{v>c} v f(c, v|\beta, \sigma) - \sum_{c,v} P(\beta, \sigma, c, v) f(c, v|\beta, \sigma),$$

$$R^s(\beta, \sigma, P) = - \sum_{v>c} c f(c, v|\beta, \sigma) + \sum_{c,v} P(\beta, \sigma, c, v) f(c, v|\beta, \sigma).$$

We say that P induces (β, σ) , or (β, σ) is an *investment equilibrium* for P if

$$\beta \in \arg \max_{\tilde{\beta}} R^b(\tilde{\beta}, \sigma, P) - \tilde{\beta}, \quad \sigma \in \arg \max_{\tilde{\sigma}} R^s(\beta, \tilde{\sigma}, P) - \tilde{\sigma}. \quad (1)$$

Throughout the paper, we will confine ourselves to parameter constellations such that investment equilibria exist for all relevant price schemes. The sum of the firms' ex-ante expected payoffs is

$$S(\beta, \sigma) = \sum_{v>c} (v - c) f(c, v|\beta, \sigma) - \beta - \sigma.$$

We call $S(\beta, \sigma)$ the *ex-ante expected surplus*. We assume that S has a unique maximizer (β^*, σ^*) with $(\beta^*, \sigma^*) \in (0, \bar{\beta}) \times (0, \bar{\sigma})$ which we call the *pair of efficient investments*. Note that this implies $\max V > \min C$ because otherwise $(\beta^*, \sigma^*) = (0, 0)$. Ex ante, no firm has private information. Therefore, it is the firms' common interest to maximize the ex-ante expected surplus

over all investment pairs which are induced by a feasible price scheme; i.e., to solve *problem* \mathcal{P} :

$$\max_{(\beta, \sigma, P)} S(\beta, \sigma) \quad \text{s.t. } P \in \mathcal{P}, \quad (1).$$

Implicit in this formulation is the assumption that for any given price scheme, the firms will coordinate on the most favorable investment equilibrium. If (β, σ, P) is a solution of problem \mathcal{P} then P is called a *second-best* price scheme (in \mathcal{P}). The first-best ex-ante expected surplus, $S(\beta^*, \sigma^*)$, is attainable if and only if efficient investment is induced by some feasible price scheme; i.e., if and only if there exists $P \in \mathcal{P}$ such that (β^*, σ^*, P) solves problem \mathcal{P} . If this is not the case, we say that the *hold-up problem* occurs.

Most of our results assume the “self-investment” condition which requires that the buyer’s investment has no impact on the production cost and, vice versa, the seller’s investment has no impact on the valuation.⁷ Formally, a parameter constellation (C, V, f) has the *self-investment* property if for all $c \in C$ and $\sigma \in [0, \bar{\sigma}]$ there exists a number $f^s(c|\sigma) \geq 0$, and for all $v \in V$ and $\beta \in [0, \bar{\beta}]$ there exists a number $f^b(v|\beta) \geq 0$ such that

$$\sum_{c' \in C} f^s(c'|\sigma) = 1, \quad \sum_{v' \in V} f^b(v'|\beta) = 1, \quad f(c, v|\beta, \sigma) = f^s(c|\sigma)f^b(v|\beta).$$

Self-investment implies that $f^s(c|\sigma)$ is the probability of production cost c if the seller invests σ , and, similarly, $f^b(v|\beta)$ is the probability of valuation v if the buyer invests β . Note that self-investment implies that $|V| \geq 2$ because otherwise $\beta^* = 0$; similarly, we have $|C| \geq 2$.

Some of our results will also assume a condition which models that the buyer’s investment “shifts probability” to large valuations while the seller’s investment shifts probability to small production costs. Formally, a parameter constellation (C, V, f) with the self-investment property has the (strict) *monotone likelihood ratio property (MLRP)* (s. Milgrom, 1981) if⁸ $f^b_\beta(v|\beta)/f^b(v|\beta)$ is strictly increasing in v for all $v \in V$ and $\beta \in (0, \bar{\beta}]$, and $f^s_\sigma(c|\sigma)/f^s(c|\sigma)$ is strictly decreasing in c , for all $c \in C$ and $\sigma \in (0, \bar{\sigma}]$.

3 Mutually Observable Investment

In this short section, we assume that all individually rational price schemes are feasible and ask whether the hold-up problem can be resolved under

⁷Most papers on the hold-up problem assume self-investment. For an analysis without this assumption, see Che and Hausch (1999).

⁸Throughout the paper, β - and σ -subscripts denote partial derivatives.

these circumstances. In other words, we analyze problem \mathcal{IR} :

$$\max_{(\beta, \sigma, P)} S(\beta, \sigma) \quad \text{s.t. } P \in \mathcal{IR}, \quad (1).$$

Problem \mathcal{IR} applies if investment is mutually observable and the ex-post interaction takes the form of a double auction (Proposition 1). To solve problem \mathcal{IR} , we define a price scheme P^* as follows:

$$P^*(c, v, \beta, \sigma) = \begin{cases} 0, & \text{if } v \leq c, \\ c, & \text{if } v > c, \beta = \beta^*, \sigma \neq \sigma^*, \\ v, & \text{if } v > c, \beta \neq \beta^*, \sigma = \sigma^*, \\ c + \lambda^*(v - c), & \text{otherwise,} \end{cases}$$

where

$$\lambda^* = \frac{S(\beta^*, \sigma^*) + 2\sigma^*}{2S(\beta^*, \sigma^*) + 2\sigma^* + 2\beta^*}.$$

If P^* is selected then the trading surplus is shared in proportions $\lambda^* : 1 - \lambda^*$ if both firms have invested efficiently or no firm has, and is given to one firm if it has complied to the efficient investment level while its partner has not. We have constructed λ^* such that each firm gets half the first-best ex-ante expected surplus if both invest efficiently.

Proposition 2. *A solution to problem \mathcal{IR} is given by (β^*, σ^*, P^*) .*

Proof. Observe that $P^* \in \mathcal{IR}$ because $\lambda^* \in [0, 1]$. Suppose that P^* is selected. By construction of λ^* , each firm's ex-ante expected payoff equals $S(\beta^*, \sigma^*)/2 > 0$ if both invest efficiently. A deviation to the investment $\beta \neq \beta^*$ is not profitable for the buyer because it leads to the payoff $-\beta \leq 0$; a similar argument applies to the seller. \square

The proposition shows that double auctions are an effective means to resolve the hold-up problem if investment levels are mutually observable. The multiplicity of equilibria in the double auction has a particularly strong effect here, allowing to punish any deviation from efficient investment although investment levels are not directly payoff-relevant in the double auction. Note also that the ability of the double auction to provide efficient investment incentives does not rely on any of our specific assumptions about f and even generalizes to settings with more than two firms. This may be seen as one reason for the popularity of double auctions as trading procedures.

4 Private Investment

We now assume that all investment-independent individually rational price schemes are feasible and ask how much the hold-up problem can be alleviated if not solved, depending on the underlying parameter constellation. In other words, we turn to the analysis of problem $\mathcal{IR} \cap \mathcal{II}$:

$$\max_{(\beta, \sigma, P)} S(\beta, \sigma) \quad \text{s.t. } P \in \mathcal{IR} \cap \mathcal{II}, \quad (1).$$

Due to Proposition 1, problem $\mathcal{IR} \cap \mathcal{II}$ applies if investment is private and the ex-post interaction takes the form of a double auction.

First of all, note that the investment-dependent price scheme P^* which solves the hold-up problem in the observable-investment case is not feasible anymore. Price schemes are restricted to a dependence on the state. However, because the probability distribution over states depends on the investment levels it may still be possible to provide efficient investment incentives or at least to alleviate the hold-up problem.

Now suppose that any price scheme $P \in \mathcal{II}$ is selected. The firms' marginal returns to investment are given by

$$\begin{aligned} R_\beta^b(\beta, \sigma, P) &= \sum_{v > c} v f_\beta(c, v | \beta, \sigma) - \sum_{c, v} P(c, v) f_\beta(c, v | \beta, \sigma), \\ R_\sigma^s(\beta, \sigma, P) &= - \sum_{v > c} c f_\sigma(c, v | \beta, \sigma) + \sum_{c, v} P(c, v) f_\sigma(c, v | \beta, \sigma). \end{aligned}$$

It will be useful to think of the marginal returns at the efficient investment levels as a point in the plane:

$$M(P) = (R_\beta^b(\beta^*, \sigma^*, P), R_\sigma^s(\beta^*, \sigma^*, P)) \in \mathbb{R}^2.$$

Note that a necessary condition for a price scheme P to induce efficient investment is $M(P) = (1, 1)$.

Fixed-percentage shares

As an illustration and for later reference, let us consider the investment incentives which are provided by a class of price schemes which, implicitly, occur frequently in the contracting literature on the hold-up problem at least since Grout (1984). An II price scheme P is called a *fixed-percentage-share* price scheme if there exists $\lambda \in \mathbb{R}$ such that $P = P^\lambda$, where

$$P^\lambda(c, v) = \begin{cases} 0, & \text{if } v \leq c, \\ c + \lambda(v - c), & \text{if } v > c. \end{cases}$$

The price scheme P^λ represents the Nash Bargaining Solution when the firms' relative bargaining powers are $\lambda : 1 - \lambda$. Two fixed-percentage-share price schemes of particular importance are $P^0 =: P^b$ where the buyer appropriates the entire trading surplus in all states, and $P^1 =: P^s$ where the same is true for the seller. Using these schemes, the first-order conditions for efficient investment can be written as follows:

$$(R_\beta^b(\beta^*, \sigma^*, P^b), R_\sigma^s(\beta^*, \sigma^*, P^s)) = (1, 1). \quad (2)$$

If P^λ is selected then the marginal returns to investment are

$$\begin{aligned} R_\beta^b(\beta, \sigma, P^\lambda) &= (1 - \lambda) \sum_{v > c} (v - c) f_\beta(c, v | \beta, \sigma), \\ R_\sigma^s(\beta, \sigma, P^\lambda) &= \lambda \sum_{v > c} (v - c) f_\sigma(c, v | \beta, \sigma). \end{aligned}$$

Using (2) we get $M(P^\lambda) = (1 - \lambda, \lambda) \neq (1, 1)$. Thus, no fixed-percentage-share price scheme can induce efficient investment. Put differently, because either $\lambda \leq 1/2$ or $1 - \lambda \leq 1/2$, at least one firm internalizes at most half the social returns to her investment, and this results in inefficient investment.

Constant price schemes

For later reference, let us also recall a result about the investment incentives which are provided by constant price schemes. We call an II price scheme P *essentially constant* if there exists $p \in \mathbb{R}$ such that $P = P^{\text{const}, p}$, where

$$P^{\text{const}, p}(c, v) = \begin{cases} 0, & \text{if } c \geq v, \\ p, & \text{if } c < v. \end{cases}$$

If $\max C \leq \min V$ then all essentially constant price schemes with $p \in [\max C, \min V]$ are feasible via a double auction. Alternatively, any such price scheme might be implemented via an ex-ante fixed-price contract which stipulates that the trade price is p if both firms are willing to trade. The following well-known remark shows that such price schemes are effective in solving the hold-up problem if the self-investment condition is satisfied. The proof is standard (e.g., Hart and Moore, 1988, Proposition 3(1)).

Remark 1. *Let (C, V, f) be a parameter constellation with the self-investment property.*

If $\max C \leq \min V$ and $p \in [\max C, \min V]$ then $(\beta^, \sigma^*, P^{\text{const}, p})$ solves problem $\mathcal{IR} \cap \mathcal{II}$.*

Notice the assumption that production costs cannot exceed valuations ($\max C \leq \min V$). This implies that the fixed price p satisfies the ex-post participation constraints and makes both firms residual claimants for the trading surplus. Without this assumption, problem $\mathcal{IR} \cap \mathcal{II}$ becomes more complicated because in general it will not be possible anymore to make both firms residual claimants. In particular, the following is true.

Remark 2. *Let (C, V, f) be a parameter constellation with the self-investment property and the MLRP. If $\max C > \min V$ then no essentially constant price scheme in $\mathcal{IR} \cap \mathcal{II}$ induces efficient investment.*

Our main results below refer to the case $\max C > \min V$.

Comparative Statics

Some of our results (Proposition 5 and Proposition 6) present comparative statics with respect to the relative size of production costs and valuations. We will consider two parameter constellations, (C, V, f) and $(C, V - \Delta, f_\Delta)$, which are identical up to a valuation component $\Delta \in \mathbb{R}$ that is independent of the investment levels. I.e.,

$$\begin{aligned} V - \Delta &= \{v - \Delta | v \in V\}, \\ f_\Delta(c, v | \beta, \sigma) &= f(c, v + \Delta | \beta, \sigma). \end{aligned}$$

Given an initial parameter constellation (C, V, f) , if Δ is large then $(C, V - \Delta, f_\Delta)$ represents a parameter constellation where production costs are high compared to valuations. Vice versa, a small (possibly negative) Δ represents comparatively low production costs. At the same time, the constellations (C, V, f) and $(C, V - \Delta, f_\Delta)$ are identical with respect to the marginal effect of investment on production cost and valuation. As an example, suppose that the buyer firm plans to resell the good on a downstream market and her investment represents marketing effort. Then her valuation may include a component that depends on downstream market conditions which are outside of her control. A large Δ would represent bad, a small Δ good market conditions.

All our comparative statics could be presented with respect to a production cost component rather than a valuation component. Only the relative size of production costs and valuations is relevant.

4.1 The Relative Investment Elasticity, and Two Lemmata on Marginal Returns To Investment

This subsection contains some novel terminology and auxiliary Lemmata which describe how price schemes and marginal returns to investment are related. For any state (c, v) and investment pair (β, σ) , we define the *b-elasticity* and the *s-elasticity* by

$$\begin{aligned}\epsilon^b(c, v|\beta, \sigma) &= f_\beta(c, v|\beta, \sigma) \frac{\beta}{f(c, v|\beta, \sigma)}, \\ \epsilon^s(c, v|\beta, \sigma) &= f_\sigma(c, v|\beta, \sigma) \frac{\sigma}{f(c, v|\beta, \sigma)}.\end{aligned}$$

The *b-elasticity* is the elasticity of a state's probability with respect to the buyer's investment level; similarly, the *s-elasticity* refers to the seller's investment. Wherever a state's *s-elasticity* is non-zero, the ratio of *b-elasticity* and *s-elasticity* is defined,

$$\epsilon(c, v|\beta, \sigma) = \frac{f_\beta(c, v|\beta, \sigma)\beta}{f_\sigma(c, v|\beta, \sigma)\sigma}.$$

We call $\epsilon(c, v|\beta, \sigma)$ the *relative investment elasticity*. It can be interpreted as a measure of the relative importance of buyer versus seller investment for increasing the probability of (c, v) .

If the self-investment condition is satisfied then b-elasticities are independent of production cost and the seller's investment level, and vice versa for s-elasticities. We can then use the shortcuts

$$\epsilon^b(v|\beta) = \frac{f_\beta^b(v|\beta)\beta}{f^b(v|\beta)}, \quad \epsilon^s(c|\sigma) = \frac{f_\sigma^s(c|\sigma)\sigma}{f^s(c|\sigma)}.$$

Note that the MLRP is satisfied if and only if $\epsilon^b(v|\beta)$ is strictly increasing in v for all $v \in V$ and $\beta \in (0, \bar{\beta}]$ and $\epsilon^s(c|\sigma)$ is strictly decreasing in c for all $c \in C$ and $\sigma \in (0, \bar{\sigma}]$. I.e., the MLRP requires that b-elasticities are strictly increasing in the valuation level and s-elasticities are strictly decreasing in the production cost level. In particular, b-elasticities are positive for large valuations, negative for small valuations, and zero for at most one intermediate valuation.

To understand the relevance of Lemma 1 below, recall from the previous section that no fixed-percentage-share price scheme P^λ can induce efficient investment. The reason is a trade-off between buyer and seller in the provision of marginal returns to investment: if λ is changed such that the seller's

marginal return increases, the buyer's inevitably decreases. Lemma 1 deals with the problem how, starting from any II price scheme, prices may be changed such that *both* firms' marginal returns to investment are increased. This is not difficult as such: decreasing the price in a state with a positive b -elasticity and a negative s -elasticity obviously increases both firms' marginal returns to investment. However, this trick does not work in a state where both elasticities are positive (or where both are negative). Lemma 1 shows that it is nevertheless possible to increase both firms' marginal returns to investment by changing prices in such states. The trick is a simultaneous price change in two different states; the only requirement is that the two states' relative investment elasticities are not identical.

Lemma 1. *Let $(\beta, \sigma) \in (0, \bar{\beta}] \times (0, \bar{\sigma}]$. For $j = 1, 2$, suppose $(c_j, v_j) \in C \times V$,*

$$\epsilon^b(c_j, v_j | \beta, \sigma) > 0, \quad \epsilon^s(c_j, v_j | \beta, \sigma) > 0, \quad (3)$$

and

$$\epsilon(c_2, v_2 | \beta, \sigma) > \epsilon(c_1, v_1 | \beta, \sigma). \quad (4)$$

Then, for all $P \in \mathcal{II}$, there exist $\eta_1 > 0$ and $\eta_2 > 0$ such that for all $\delta > 0$ we have

$$R_\beta^b(\beta, \sigma, P_\delta) > R_\beta^b(\beta, \sigma, P), \quad R_\sigma^s(\beta, \sigma, P_\delta) > R_\sigma^s(\beta, \sigma, P),$$

where $P_\delta = P + 1_{(c_1, v_1)} \eta_1 \delta - 1_{(c_2, v_2)} \eta_2 \delta$.⁹

To get an intuition for this result, note that (4) says that the buyer's investment is relatively more important for state 2, whereas the seller's investment is relatively more important for state 1. Therefore, the desired positive effect on marginal returns to investment is obtained by a price increase in state 1 (favoring the seller), and a proportional price decrease in state 2 (favoring the buyer). The lemma states the existence of a whole ray of price changes, including arbitrarily small changes.

The following variant of Lemma 1 shows that it is possible to increase both firms' marginal returns to investment even under weaker assumptions than Lemma 1, but then nothing can be said about the signs of the required price changes.

⁹The "indicator" function $1_{(c_j, v_j)}$ is defined by $1_{(c_j, v_j)}(c, v) = 1$ if $(c, v) = (c_j, v_j)$ and $1_{(c_j, v_j)}(c, v) = 0$ otherwise.

Corollary 1. Let $(\beta, \sigma) \in (0, \bar{\beta}] \times (0, \bar{\sigma}]$. For $j = 1, 2$, suppose $(c_j, v_j) \in C \times V$, and

$$\begin{aligned} \exists j : f_\sigma(c_j, v_j | \beta, \sigma) &\neq 0, \\ \exists j : f_\beta(c_j, v_j | \beta, \sigma) &\neq 0, \\ \forall \alpha > 0 \exists j : \alpha f_\sigma(c_j, v_j | \beta, \sigma) &\neq f_\beta(c_j, v_j | \beta, \sigma). \end{aligned} \tag{5}$$

Then, for all $P \in \mathcal{II}$, there exist η_1 and η_2 such that for all $\delta > 0$ we have

$$R_\beta^b(\beta, \sigma, P_\delta) > R_\beta^b(\beta, \sigma, P), \quad R_\sigma^s(\beta, \sigma, P_\delta) > R_\sigma^s(\beta, \sigma, P),$$

where $P_\delta = P + 1_{(c_1, v_1)} \eta_1 \delta - 1_{(c_2, v_2)} \eta_2 \delta$.

Note that (5) is satisfied whenever at (β, σ) the relative investment elasticities of (c_1, v_1) and (c_2, v_2) are defined, positive, and unequal.

The next Lemma confirms the intuitive idea that there can never be a problem of inefficiently *high* marginal returns to investment: such high returns can always be eliminated by an appropriate change of the price scheme. More precisely, the Lemma starts with an II price scheme such that, at the efficient investment levels, the marginal returns to investment (weakly) exceed the marginal investment costs (which are 1); we then show that there exists another II price scheme which equalizes marginal returns and marginal costs for both firms. Moreover, it is possible to keep the IR property if one starts with it.¹⁰

Lemma 2. *If there exists a price scheme $P^u \in \mathcal{II}$ such that $M(P^u) \geq (1, 1)$, then there also exists a price scheme $P^{\text{eff}} \in \mathcal{II}$ such that $M(P^{\text{eff}}) = (1, 1)$. Moreover, if $P^u \in \mathcal{IR}$ then we can choose P^{eff} such that $P^{\text{eff}} \in \mathcal{IR}$.*

The proof of Lemma 2 uses the fixed-percentage-share price schemes P^b and P^s that have been defined earlier. If P^b is selected, the buyer's marginal return to investment at the efficient investment levels equals her marginal investment cost (which is 1), and the seller's marginal return to investment is 0; P^s provides opposite incentives. If we imagine marginal returns as points in a plane, P^b and P^s correspond to the points $M(P^b) = (1, 0)$ and $M(P^s) = (0, 1)$, while P^u corresponds to a point to the upper right of $(1, 1)$. The convex hull of these three points includes the point $(1, 1)$. Thus, there exists a convex combination P^{eff} of P^u , P^b and P^s , such that marginal returns are "levelled down" to marginal costs, corresponding to the point $(1, 1)$.

¹⁰From now on, we will apply the operators \leq and \ll to two-dimensional variables, with the interpretation $(x, y) \leq (x', y')$ if and only if $x \leq x'$ and $y \leq y'$, and $(x, y) \ll (x', y')$ if and only if $x < x'$ and $y < y'$.

4.2 The Underinvestment Condition

In this section, we introduce and discuss a condition on parameter constellations which implies underinvestment. After defining this underinvestment condition, we first show that it indeed implies underinvestment, and we provide a partial characterization of second-best II IR price schemes under the condition (Proposition 3). To get a deeper understanding of its nature, we then characterize the condition in terms of the restrictions it imposes on the state space, first under no further assumptions (Remark 3), and then under the assumptions of self-investment, MLRP, and a concavity property (Proposition 4).

Underinvestment condition

For all states $(c, v) \in C \times V$ with $v > c$, and all investment pairs $(\beta, \sigma) \in (0, \bar{\beta}] \times (0, \bar{\sigma}]$, we have

- (r) $f_\beta(c, v|\beta, \sigma) > 0$ and $f_\sigma(c, v|\beta, \sigma) > 0$,
- (rr) $f_{\beta\beta}(c, v|\beta, \sigma) < 0$ and $f_{\sigma\sigma}(c, v|\beta, \sigma) < 0$,
- (rrr) $f_{\beta\sigma}(c, v|\beta, \sigma) > 0$.

Let us call a state (c, v) a *trade state* if $v > c$, and call a state (c, v) *regular* if (r), (rr), and (rrr) are satisfied for all positive investment pairs (β, σ) . If a state is regular then its probability is (r) strictly increasing in both investments (i.e., its s- and b-elasticities are positive) and (rr) strictly concave in both investments, and (rrr) investments are complements at the margin. The underinvestment condition requires the regularity of all trade states, but does not require anything for any other state. I.e., the condition refers to a subset of the support of the underlying probability distributions only. Because this is an unusual type of condition, we will spend some time relating it to more common conditions: self-investment and the monotone likelihood ratio property.

Below is our first main proposition. It states that if the underinvestment condition is satisfied then all II IR price schemes induce underinvestment and therefore the hold-up problem occurs. The proposition moreover provides a partial characterization of second-best II IR price schemes.

Proposition 3. *Assume the underinvestment condition is satisfied. If $\tilde{P} \in \mathcal{IR} \cap \mathcal{IL}$, and $(\tilde{\beta}, \tilde{\sigma})$ is an investment equilibrium for \tilde{P} then*

$$(\tilde{\beta}, \tilde{\sigma}) \ll (\beta^*, \sigma^*);$$

if in addition $(\tilde{\beta}, \tilde{\sigma}, \tilde{P})$ solves problem $\mathcal{IR} \cap \mathcal{II}$ and $(\tilde{\beta}, \tilde{\sigma}) \gg (0, 0)$, then

$$\begin{aligned} \exists \epsilon^* > 0 \forall (c, v) \in C \times V, c < v : \quad & \epsilon(c, v | \tilde{\beta}, \tilde{\sigma}) > \epsilon^* \implies \tilde{P}(c, v) = c, \\ & \epsilon(c, v | \tilde{\beta}, \tilde{\sigma}) < \epsilon^* \implies \tilde{P}(c, v) = v. \end{aligned}$$

An immediate conclusion from this proposition is that double auctions cannot generally resolve the hold-up problem if investment is private. This contrasts the observable-investment case where the hold-up problem is no problem (Proposition 2). Maybe more importantly, the proposition also implies that no contract or arrangement of any form that resolves the hold-up problem by implementing an II price scheme can generally avoid violating an ex-post participation constraint.

The second part of the proposition provides, with the exception of the extreme cases where one firm does not invest at all, a partial characterization of second-best II IR price schemes, as follows. Except for “knife edge” states with $\epsilon = \epsilon^*$, each state’s trading surplus is fully appropriated by one firm; which one is determined by the relative investment elasticity of the state in question. Intuitively, a firm appropriates the trading surplus in those states where her investment is relatively more important (in terms of elasticities) than the partner firm’s investment for increasing the state’s probability.

The logic behind Proposition 3 is as follows. Suppose, any II IR price scheme is selected. Then, firstly, by (rr) each firm’s expected payoff is concave in her own investment level. Consequently, first order conditions are sufficient for investment equilibria, and best-response investments are unique. Secondly, by (rrr) and (rr) each firm’s best-response investment is a nondecreasing function of the partner firm’s investment (i.e., the investments are “strategic complements”). Thirdly, by (r) and (rr) lowering the price in any trade state shifts the buyer’s best-response curve upwards, and shifts the seller’s best-response curve downwards (vice versa for price increases). This implies that for any II IR price scheme, firm i ’s ($i = b, s$) best response curve lies below that induced by P^i (as defined on p. 12, where i appropriates the entire trading surplus), and at least one firm’s curve lies *strictly* below in the relevant range. Hence, if investment pairs are drawn as points in a plane, the best-response curves intersect to the lower left of the efficient investment point — underinvestment occurs.

To understand the characterization of second-best price schemes in Proposition 3, observe that, due to strategic complementarity, any change of prices of \tilde{P} such that both firms’ marginal returns to investment are increased would induce higher investments than \tilde{P} (idea: the change of prices increases both firms’ best responses at the intersection point of the best response curves, hence there exists a new intersection point to the upper right

of the original intersection point). Hence, if \tilde{P} is a second-best II IR price scheme, any such change of prices must violate the IR restriction. This line of argument can be combined with Lemma 1 (which provides the desired price changes of \tilde{P}) to prove the characterization result.¹¹

Because the underinvestment condition is unusual, it is important to get a deeper understanding of its nature. The condition makes strong requirements for all trade states, but none for other states. This suggests that the condition might be easier to satisfy if only a few states are trade states than if most states are trade states. In order to make this idea more precise, we will now characterize the underinvestment condition in terms of the restrictions it imposes on the state space.

First of all, without any more assumptions on parameter constellations, the underinvestment condition only implies that not all states are trade states; i.e., the condition requires ex-ante uncertainty about whether gains from trade will exist, as shown in the following remark.

Remark 3. *Let $C \times V$ be any state space. A family of probability distributions f such that the underinvestment condition is satisfied for (C, V, f) exists if and only if*

$$\max C \geq \min V. \tag{6}$$

In the following, we will make additional assumptions on parameter constellations. Mainly, we will assume self-investment and the MLRP. It turns out that under these assumptions, the underinvestment condition implies something stronger than (6). A precise characterization of what is implied can be obtained if a certain concavity condition, C1, is satisfied. To prepare for the definition of C1, note first that for parameter constellations with the MLRP, the b -elasticity of the smallest valuation level, $\min V$, is non-positive for some positive investment level. Let v^* be the largest valuation with this property,

$$v^* = \max\{v \in V \mid \exists \beta > 0 : f_\beta^b(v|\beta) \leq 0\}.$$

Similarly, we define

$$c^* = \min\{c \in C \mid \exists \sigma > 0 : f_\sigma^s(c|\sigma) \leq 0\}.$$

¹¹ Proposition 3 continues to hold even if the firms are free to destroy some or all of the trading surplus. In the second-best solution, no trading surplus will be destroyed. This also implies that private information about production cost and valuation, as in Myerson and Satterthwaite (1983), cannot alleviate the hold-up problem.

The MLRP implies that the probability of any valuation level $v > v^*$ is strictly increasing in the buyer’s investment, and the probability of any cost level $c < c^*$ is strictly increasing in the seller’s investment. Property C1 adds concavity:

$$\begin{aligned} \text{C1} \quad & \forall c < c^*, \sigma > 0 : f_{\sigma\sigma}^s(c|\sigma) < 0, \\ & \forall v > v^*, \beta > 0 : f_{\beta\beta}^b(v|\beta) < 0. \end{aligned}$$

The following result is immediate from the definitions.

Proposition 4. *Let (C, V, f) be a parameter constellation such that self-investment, the MLRP, and C1 are satisfied. Then (C, V, f) satisfies the underinvestment condition if and only if*

$$v^* \leq \min C \text{ and } c^* \geq \max V. \tag{7}$$

First of all, this shows that the underinvestment condition is not totally inconsistent with self-investment and the MLRP. On the other hand, these two assumptions, together with the concavity assumption C1, imply that only certain states can be trade states if the underinvestment condition is satisfied. Specifically, (7) says that gains from trade can only exist in states with “large” valuations ($v > v^*$) and “small” production costs ($c < c^*$).¹² Note that (7) is stronger than condition (6).

4.3 Comparative Statics and the Efficiency Condition

In this section, we present two complementary comparative statics results which show that underinvestment occurs for parameter constellations where production costs are relatively high compared to valuations, while efficient investment can be obtained for parameter constellations with relatively low production costs.

The proof of the first comparative statics result below is immediate from Proposition 4.¹³

¹² Using first-order conditions one can show that the hold-up problem still occurs even if one of the conditions, $v^* \leq \min C$ or $c^* \geq \max V$, is violated. This demonstrates that the underinvestment condition is not a necessary condition for the hold-up problem.

¹³Note that, besides the lower bound on Δ that is established in Proposition 5, there is also an implicit *upper* bound. We must have $\Delta < \max V - \min C$ because otherwise no trading surplus would exist in constellation $(C, V - \Delta, f_\Delta)$ and thus—contrary to our earlier assumption—the investment pair $(0, 0)$ would be efficient. Hence, the proposition requires Δ to be in the interval $(\max\{v^* - \min C, \max V - c^*\}, \max V - \min C)$. This interval is nonempty under the conditions $v^* < \max V$ and $c^* > \min C$ —weak conditions if V and C are fine grids.

Proposition 5. *Let (C, V, f) be a parameter constellation such that self-investment, the MLRP, and C1 are satisfied. Let $\Delta \in \mathbb{R}$. Moreover, let $\underline{\Delta} = \max\{v^* - \min C, \max V - c^*\}$. Then, for all $(C, V - \Delta, f_\Delta)$ with*

$$\Delta \geq \underline{\Delta},$$

underinvestment is obtained in any solution to problem $\mathcal{IR} \cap \mathcal{II}$.

To state the second comparative statics result, another concavity property, C2, needs to be introduced. Note that the MLRP implies that the expected production cost is strictly decreasing in the seller's investment, and the expected valuation is strictly increasing in the buyer's investment,

$$\forall \beta \in [0, \bar{\beta}], \sigma \in [0, \bar{\sigma}] : \sum_{c \in C} cf_{\sigma}(c|\sigma) > 0, \quad \sum_{v \in V} vf_{\beta}^b(v|\beta) < 0.$$

Concavity property C2 adds convexity of production costs, and concavity of valuations,

$$\begin{aligned} \text{C2} \quad \forall \beta \in [0, \bar{\beta}] : \sum_{v \in V} vf_{\beta\beta}^b(v|\beta) < 0, \\ \forall \sigma \in [0, \bar{\sigma}] : \sum_{c \in C} cf_{\sigma\sigma}(c|\sigma) > 0. \end{aligned}$$

Neither follows C2 from C1 nor vice versa. However, there are parameter constellations such that both C1 and C2 are satisfied (cf. Section 4.5).

We can now state a comparative statics result parallel to Proposition 5. Note that the assumptions $|C| \geq 3$ or $|V| \geq 3$ are weak if we think of C and V as fine grids.

Proposition 6. *Let (C, V, f) be a parameter constellation such that self-investment, the MLRP, and C2 are satisfied, and assume $|C| \geq 3$ and $|V| \geq 3$. Let $\Delta \in \mathbb{R}$. Then there exists $\bar{\Delta} > \min V - \max C$ such that for all $(C, V - \Delta, f_\Delta)$ with*

$$\Delta < \bar{\Delta},$$

efficient investment is obtained in any solution to problem $\mathcal{IR} \cap \mathcal{II}$.

We already know from Remark 1 that the conclusion of this proposition holds for all $\Delta \leq \min V - \max C$ because an essentially constant price scheme induces efficient investment. The strength of the proposition is that even for some $\Delta > \min V - \max C$ efficient investment obtains although, by Remark 2, in these cases no essentially constant price scheme induces efficient investment.

The proof of the Proposition is based on the following ideas. Suppose first that $\Delta = \min V - \max C$ and the essentially constant price scheme that induces efficient investment is selected for $(C, V - \Delta, f_\Delta)$. Then there exist at least two different states in which both firms get strictly positive shares of the trading surplus. One state is given by the second-lowest feasible production cost level and the highest feasible valuation level, the other state is given by the lowest production cost level and the second-highest valuation level. If the trade prices in these states are slightly changed, the price scheme is still IR. Therefore, we can apply Corollary 1 to obtain an II IR price scheme with marginal returns that exceed $(1, 1)$ at the efficient investment levels (note that state 1 has a larger relative investment elasticity than state 2, due to the MLRP and self-investment). A continuity argument now shows that even for all sufficiently small $\Delta > \min V - \max C$, marginal returns exceeding $(1, 1)$ can be provided by some II IR price schemes at the efficient investment levels. By Lemma 2, we can change this price scheme such that the first-order conditions for an efficient investment equilibrium are satisfied. C2 implies that first-order conditions are sufficient if $\Delta = \min V - \max C$; again, a continuity argument shows sufficiency for all sufficiently small $\Delta > \min V - \max C$.

4.4 Properties of Second-best II IR Price Schemes

This section contains results about the shape of second-best II IR price schemes. The main conclusion is that such price schemes are typically not nondecreasing. Therefore, the double auction outperforms all contracts or arrangements which implement nondecreasing II IR price schemes, like the renegotiated contracts of Hart and Moore (1988) and the renegotiation-proof implementation contracts of Rubinstein and Wolinsky (1992).

Under the assumptions of Proposition 5 and the underinvestment condition, the shape of second-best II IR price can be characterized beyond what we have obtained in Proposition 3, as shown in the following result.¹⁴

Remark 4. *Let (C, V, f) be a parameter constellation such that self-investment, the MLRP, and C1 are satisfied. If*

$$v^* \leq \min C, \quad c^* \geq \max V,$$

and $(\tilde{\beta}, \tilde{\sigma}, \tilde{P})$ solves problem $\mathcal{IR} \cap \mathcal{II}$, and $(\tilde{\beta}, \tilde{\sigma}) \gg (0, 0)$, then then there

¹⁴Innes (1990) obtains a similar shape in the context of unilateral investment problems, Dewatripont-Tirole (1994) in the context of the optimal financial structure of a firm.

exists a strictly decreasing function $t : C \rightarrow \mathbb{R}$ such that

$$\begin{aligned} \forall (c, v) \in C \times V, c < v : \quad & v > t(c) \implies \tilde{P}(c, v) = c, \\ & v < t(c) \implies \tilde{P}(c, v) = v, \end{aligned}$$

and $v = t(c)$ only if $\epsilon(c, v | \tilde{\beta}, \tilde{\sigma}) = \epsilon^*$.

The shape of second-best II IR price schemes is intuitive. Due to the MLRP, the buyer's investment shifts probability to large valuation levels. Hence, to alleviate her underinvestment problem, the buyer is rewarded for valuations v above a threshold $t(c)$ by getting the trading surplus in the respective state (c, v) , but she is punished for valuations below the threshold by losing the trading surplus to the seller. Note that the threshold $t(c)$ is strictly decreasing in the production cost c : the smaller the production cost the less valuation levels qualify for the buyer's reward. This alleviates the seller's underinvestment problem because, due to the MLRP, the seller's investment shifts probability to small cost levels.¹⁵

The states (c, v) with $v = t(c)$ might be called “knife-edge” states. Due to the MLRP, there exists at most one knife-edge state for each production cost level. Therefore, the fraction of trade states which are “knife-edge” becomes arbitrarily small if C and V are sufficiently fine grids, implying that \tilde{P} is then almost completely characterized by the function t . On the other extreme, if there exists only a single trade state, this one will be a “knife-edge” state unless one firm does not invest at all.

We call an II IR price scheme P *nondecreasing* if $c < c' < v$ implies $P(c, v) \leq P(c', v)$, and $c < v < v'$ implies $P(c, v) \leq P(c, v')$. (Note that this definition refers to the trade states only.) For any V with $|V| \geq 2$, denote the second-largest valuation in V by v^2 , and for any C with $|C| \geq 2$, denote by c^2 the second-smallest production cost level in C . Then we have the following.¹⁶

Remark 5. *Under the assumptions of Remark 4, if $c^2 < v^2$ then \tilde{P} is not nondecreasing.*

The conclusion of Remark 5 also holds under our sufficient condition for efficient investment. We only have to exclude the case where production

¹⁵This intuition reads as if seller and buyer had asymmetric roles. This is, however, not the case. We could replace t by a function $t^{-1} : V \rightarrow \mathbb{R}$ such that $c < t^{-1}(v)$ implies $\tilde{P}(c, v) = v$, and $c > t^{-1}(v)$ implies $\tilde{P}(c, v) = c$. This would swap the roles of buyer and seller.

¹⁶Note that the assumption $c^2 < v^2$ is weak if C and V are fine grids.

costs cannot exceed the valuation (in this case, the essentially constant price scheme from Remark 1 is nondecreasing).

Remark 6. *Under the assumptions of Proposition 6, second-best II IR price schemes for parameter constellations $(C, V - \Delta, f_\Delta)$ with $\Delta \in (\min V - \max C, \bar{\Delta})$ are not nondecreasing.*

4.5 An Example

In this section, we report on a class of parameter constellations which satisfy all our technical assumptions. These constellations have the property that each state's probability is bilinear in the investment levels if investment is measured in appropriate units. Formally, we call a parameter constellation (C, V, f) *rescaled bilinear*¹⁷ if there exist functions $g^b(v)$, $h^b(v)$, $g^s(c)$, $h^s(c)$, $k^b(\beta)$, and $k^s(\sigma)$, such that

$$k_\beta^b(\beta) > 0, \quad k_{\beta\beta}^b(\beta) < 0, \quad k_\sigma^s(\sigma) > 0, \quad k_{\sigma\sigma}^s(\sigma) < 0,$$

$$f(c, v | \beta, \sigma) = \underbrace{(g^b(v) + h^b(v)k^b(\beta))}_{\in(0,1)} \underbrace{(g^s(c) + h^s(c)k^s(\sigma))}_{\in(0,1)},$$

$$\frac{h^b(v)}{g^b(v) + h^b(v)k^b(\beta)} \text{ strictly increasing in } v,$$

$$\frac{h^s(c)}{g^s(c) + h^s(c)k^s(\sigma)} \text{ strictly decreasing in } c.$$

These formulae imply that if one rescales the investment levels such that they are measured in units of k^b and k^s , respectively, then each state's probability is bilinear in these rescaled investment levels. In particular, for any II price scheme, a firm's marginal return to her rescaled investment is independent of her investment level. The actual investment costs β and σ are strictly increasing and convex in k^b and k^s , respectively. Notice that rescaled bilinearity is not as strong as to imply that each firm's expected payoff is concave in her investment for all II IR price schemes. However, we have the following:

Remark 7. *Any rescaled bilinear parameter constellation has the self-investment property, the MLRP, and the concavity properties C1 and C2.*

¹⁷Hart and Moore (1988, Proposition 4) have used this class. Translated to an unilateral investment set-up ($\bar{\beta} = 0$ or $\bar{\sigma} = 0$), such parameter constellations have the *linear distribution function property* which is used in the principal-agent literature.

5 Concluding Remarks

The approach of this paper is conceptually different from most of the literature on “incomplete contracting” which has formulated conditions for the occurrence of the hold-up problem in terms of non-verifiability to courts, freedom to renegotiate earlier contracts, and non-feasibility of certain mechanisms (cf. Tirole, 2000). Our contributions are much simpler. We advertise double auctions as ex-post trading mechanisms which alleviate the hold-up problem, and we show that ex-post participation constraints make the hold-up problem unavoidable in some environments. We make no contribution to the question whether or how ex-post participation constraints can be explained in terms of incomplete contracting. However, as observed by Innes (1990), in many applications ex-post participation constraints arise naturally from limited liability of the firms.

Our approach also bears on Segal’s (1999) and Hart and Moore’s (1999) “complex environment.” In Segal’s and Hart and Moore’s world, each state consists of many cost-valuation pairs which belong to different goods (“widgets”). These multidimensional states will generally contain much more stochastic information about investment levels than our 2-dimensional states. Thus, in the presence of a double auction, resolving the hold-up problem is *easier* in Segal’s and Hart and Moore’s complex environment than in the simpler classical set-up. This demonstrates the strength of the standard assumption made by Segal and Hart and Moore that in the absence of an ex-ante contract the ex-post trading surplus is split in fixed proportions.

6 Proofs

Proof of Remark 2. First of all, note that the MLRP implies $f_{\beta}^b(\min V|\beta^*) < 0$ and $f_{\beta}^b(\max V|\beta^*) > 0$, and there exists $\hat{v} \in V$ such that, for all $v \in V$, we have $f_{\beta}^b(v|\beta^*) < 0$ if $v < \hat{v}$ and $f_{\beta}^b(v|\beta^*) > 0$ if $v > \hat{v}$. Therefore,¹⁸

$$\forall v' \in \mathbb{R} : \sum_{v \in V, v \geq v'} f_{\beta}^b(v|\beta^*) \geq 0. \quad (8)$$

and

$$\forall v' \in \mathbb{R}, v' > \min V : \sum_{v \in V, v \geq v'} f_{\beta}^b(v|\beta^*) > 0. \quad (9)$$

¹⁸By convention, empty sums are 0.

Similarly,

$$\forall c' \in \mathbb{R} : \sum_{c \in C, c \leq c'} f_{\sigma}^s(c|\sigma^*) \geq 0. \quad (10)$$

and

$$\forall c' \in \mathbb{R}, c' < \max C : \sum_{c \in C, c \leq c'} f_{\sigma}^s(c|\sigma^*) > 0. \quad (11)$$

An indirect proof of the remark follows. Suppose that for some $p \in \mathbb{R}$, the price scheme $P = P^{\text{const}, p}$ induces efficient investment, and $P \in \mathcal{II} \cap \mathcal{IR}$.

Define $\underline{v} = \min\{v \in V \mid v > \min C\}$. First we show

$$\max C > \underline{v}. \quad (12)$$

Suppose not. Subtract one of the first-order conditions for equilibrium, $R_{\beta}^b(\beta^*, \sigma^*, P) = 1$, from one of the first-order conditions for efficient investment, $R_{\beta}^b(\beta^*, \sigma^*, P^b) = 1$. This yields

$$\sum_{c < v} (P(c, v) - c) f_{\beta}^b(v|\beta^*) f_{\sigma}^s(c|\sigma^*) = 0,$$

or

$$\sum_{v \geq \underline{v}} f_{\beta}^b(v|\beta^*) \sum_{c < v} (p - c) f_{\sigma}^s(c|\sigma^*) = 0.$$

Combining this with (8), $\underline{v} \geq \max C > \min V$, (9), and $p \geq \min C$ implies $p = \min C$, which is not possible because $\sigma^* > 0$. This shows (12).

Now subtract the first-order condition $R_{\sigma}^s(\beta^*, \sigma^*, P) = 1$ from the first-order condition $R_{\sigma}^s(\beta^*, \sigma^*, P^s) = 1$. This yields

$$\sum_{c < v} (v - P(c, v)) f_{\beta}^b(v|\beta^*) f_{\sigma}^s(c|\sigma^*) = 0,$$

or

$$\sum_{v \geq \underline{v}} (v - p) f_{\beta}^b(v|\beta^*) \sum_{c \in C \text{ such that } c < v} f_{\sigma}^s(c|\sigma^*) = 0. \quad (13)$$

Combining this with (11), (12), (10), and $p \leq \underline{v}$ implies $p = \underline{v}$.

A similar argument shows that $p = \bar{c} = \max\{c \in C \mid c < \max V\}$. Therefore, $\max C \geq \max V$ (because otherwise $\bar{c} = \max C > \underline{v}$ by (12)). Hence, (10), (11), (13), and $p \leq \underline{v}$ imply $p = \max V$. This contradicts $p = \bar{c}$. \square

Proof of Lemma 1. Due to (4), there exists $\eta_2 > 0$ such that

$$\frac{f_\beta(c_1, v_1|\beta, \sigma)}{f_\beta(c_2, v_2|\beta, \sigma)} < \eta_2 < \frac{f_\sigma(c_1, v_1|\beta, \sigma)}{f_\sigma(c_2, v_2|\beta, \sigma)}.$$

Defining $\eta_1 = 1$, we get

$$\begin{aligned} R_\beta^b(\beta, \sigma, P_\delta) - R_\beta^b(\beta, \sigma, P) \\ = -\eta_1 \delta f_\beta(c_1, v_1|\beta, \sigma) + \eta_2 \delta f_\beta(c_2, v_2|\beta, \sigma) > 0, \end{aligned} \quad (14)$$

$$\begin{aligned} R_\sigma^s(\beta, \sigma, P_\delta) - R_\sigma^s(\beta, \sigma, P) \\ = \eta_1 \delta f_\sigma(c_1, v_1|\beta, \sigma) - \eta_2 \delta f_\sigma(c_2, v_2|\beta, \sigma) > 0. \end{aligned} \quad (15)$$

□

Proof of Corollary 1. Assumption (5) says that the vectors

$$\begin{aligned} x^b &= (-f_\beta(c_1, v_1|\beta, \sigma), f_\beta(c_2, v_2|\beta, \sigma)) \neq 0, \\ x^s &= (f_\sigma(c_1, v_1|\beta, \sigma), -f_\sigma(c_2, v_2|\beta, \sigma)) \neq 0, \end{aligned}$$

do not point in exactly opposite directions in \mathbb{R}^2 . Hence, there exists $(\eta_1, \eta_2) \in \mathbb{R}^2$ such that (by standard properties of inner products)

$$(\eta_1, \eta_2) \cdot x^b > 0, \quad (\eta_1, \eta_2) \cdot x^s > 0.$$

Thus, (14) and (15) hold. □

Proof of Lemma 2. At the efficient investments (β^*, σ^*) , the first-order conditions $S_\beta(\beta^*, \sigma^*) = 0$ and $S_\sigma(\beta^*, \sigma^*) = 0$ are satisfied. Therefore, $M(P^b) = (1, 0)$ and $M(P^s) = (0, 1)$. We define x and y by $M(P^u) = (x, y)$. Because $x \geq 1$ and $y \geq 1$, we can define

$$\lambda^u = \frac{1}{x + y - 1}, \quad \lambda^b = \frac{y - 1}{x + y - 1}, \quad \lambda^s = \frac{x - 1}{x + y - 1},$$

and $P^{\text{eff}} = \lambda^u P^u + \lambda^b P^b + \lambda^s P^s$. Due to $\sum_k \lambda^k = 1$ and the linearity of $M(P)$, we have

$$M(P^{\text{eff}}) = \lambda^u M(P^u) + \lambda^b M(P^b) + \lambda^s M(P^s) = (1, 1).$$

Because $\lambda^k \geq 0$ ($k = u, b, s$), P^{eff} is an IR price scheme if P^u is. □

Proof of Proposition 3. For all investment pairs (β, σ) , we define the gross surplus

$$G(\beta, \sigma) = S(\beta, \sigma) + \beta + \sigma.$$

Regularity condition (rr) implies that $G(\beta, \sigma)$ is strictly concave in both arguments. Moreover, for any fixed $\sigma \in [0, \bar{\sigma}]$ and $P \in \mathcal{II} \cap \mathcal{IR}$, the function $\beta \mapsto R^b(\beta, \sigma, P)$ is strictly concave or identically 0. Similarly, for any fixed β , the function $\sigma \mapsto R^s(\beta, \sigma, P)$ is strictly concave or identically 0. Therefore, the following *BR-functions* are well-defined for all $P \in \mathcal{II} \cap \mathcal{IR}$:

$$\text{BR}^b(\sigma, P) = \arg \max_{\beta} R^b(\beta, \sigma, P) - \beta,$$

$$\text{BR}^s(\beta, P) = \arg \max_{\sigma} R^s(\beta, \sigma, P) - \sigma,$$

$$\text{BR}^{b*}(\sigma) = \arg \max_{\beta} G(\beta, \sigma) - \beta,$$

$$\text{BR}^{s*}(\beta) = \arg \max_{\sigma} G(\beta, \sigma) - \sigma.$$

(In the following, it will be instructive to imagine the graphs of these functions in a coordinate system with β on one axis and σ on the other axis.) Using (r) to (rrr) from the underinvestment condition, and the fact that

$$G(\beta, \sigma) = R^b(\beta, \sigma, P^b) = R^s(\beta, \sigma, P^s),$$

the following facts can be obtained:

(j) All BR-functions are continuous and nondecreasing in investment.

(jj) Let $\sigma_1 < \sigma_2$ with $\text{BR}^{b*}(\sigma_1) < \bar{\beta}$ and $\text{BR}^{b*}(\sigma_2) > 0$. Then

$$\text{BR}^{b*}(\sigma_1) < \text{BR}^{b*}(\sigma_2).$$

Similarly let $\beta_1 < \beta_2$ with $\text{BR}^{s*}(\beta_1) < \bar{\sigma}$ and $\text{BR}^{s*}(\beta_2) > 0$. Then

$$\text{BR}^{s*}(\beta_1) < \text{BR}^{s*}(\beta_2).$$

(jjj) We have $\text{BR}^b(\sigma, P) \leq \text{BR}^{b*}(\sigma)$ for all σ and all $P \in \mathcal{II} \cap \mathcal{IR}$. We have $\text{BR}^s(\beta, P) \leq \text{BR}^{s*}(\beta)$ for all β and all $P \in \mathcal{II} \cap \mathcal{IR}$.

(jv) If $P \in \mathcal{II} \cap \mathcal{IR}$ and $P \neq P^b$, then $\text{BR}^b(\sigma, P) < \text{BR}^{b*}(\sigma)$ for all σ with $\text{BR}^{b*}(\sigma) > 0$.

Now let $\tilde{P} \in \mathcal{IR} \cap \mathcal{II}$, and let $(\tilde{\beta}, \tilde{\sigma})$ be induced by \tilde{P} .

The function BR^{b^*} maps $[\tilde{\sigma}, \bar{\sigma}]$ into $[\tilde{\beta}, \bar{\beta}]$. This is because, for all $\sigma \geq \tilde{\sigma}$, (j) and (jjj) imply

$$\text{BR}^{b^*}(\sigma) \geq \text{BR}^{b^*}(\tilde{\sigma}) \geq \text{BR}^b(\tilde{\sigma}, \tilde{P}) = \tilde{\beta}.$$

Similarly, one can show that BR^{s^*} maps $[\tilde{\beta}, \bar{\beta}]$ into $[\tilde{\sigma}, \bar{\sigma}]$. By continuity, the graphs of BR^{b^*} and BR^{s^*} intersect somewhere in $[\tilde{\sigma}, \bar{\sigma}] \times [\tilde{\beta}, \bar{\beta}]$. In other words,

$$(\beta^*, \sigma^*) \geq (\tilde{\beta}, \tilde{\sigma}).$$

Now suppose $\beta^* = \tilde{\beta}$. This implies

$$\text{BR}^{b^*}(\sigma^*) = \beta^* = \tilde{\beta} = \text{BR}^b(\tilde{\sigma}, \tilde{P}) \leq \text{BR}^{b^*}(\tilde{\sigma}).$$

Therefore, $\sigma^* = \tilde{\sigma}$ by (jj) and thus

$$\text{BR}^{b^*}(\sigma^*) = \text{BR}^b(\sigma^*, \tilde{P}).$$

This implies $\tilde{P} = P^b$ by (jv). Hence, $\tilde{\sigma} = 0 \neq \sigma^*$. Contradiction! Hence, $\beta^* > \tilde{\beta}$. An analogous argument shows that $\sigma^* > \tilde{\sigma}$.

To prove the remaining part of the Proposition, it is sufficient to show the following: for any trade state (c_1, v_1) with $\tilde{P}(c_1, v_1) < v_1$ and trade state (c_2, v_2) with $\epsilon(c_2, v_2 | \tilde{\beta}, \tilde{\sigma}) > \epsilon(c_1, v_1 | \tilde{\beta}, \tilde{\sigma})$, we have $\tilde{P}(c_2, v_2) = c_2$.

Suppose instead $\tilde{P}(c_2, v_2) > c_2$. Then Lemma 1 and the conditions (rr) and (rrr) show that there exists a $\delta > 0$ such that $P_\delta \in \mathcal{II} \cap \mathcal{IR}$, and

$$\text{BR}^b(\tilde{\sigma}, P_\delta) > \text{BR}^b(\tilde{\sigma}, \tilde{P}), \quad \text{BR}^s(\tilde{\beta}, P_\delta) > \text{BR}^s(\tilde{\beta}, \tilde{P}).$$

Therefore, $\text{BR}^b(\cdot, P_\delta)$ maps $[\tilde{\sigma}, \bar{\sigma}]$ into $]\tilde{\beta}, \bar{\beta}]$, and $\text{BR}^s(\cdot, P_\delta)$ maps $[\tilde{\beta}, \bar{\beta}]$ into $]\tilde{\sigma}, \bar{\sigma}]$. Hence, P_δ induces an investment equilibrium $(\hat{\beta}, \hat{\sigma})$ such that

$$(\tilde{\beta}, \tilde{\sigma}) \ll (\hat{\beta}, \hat{\sigma}) \ll (\beta^*, \sigma^*).$$

Using that $S_\beta(\beta, \sigma) > 0$ and $S_\sigma(\beta, \sigma) > 0$ for all (β, σ) with $\beta < \text{BR}^{b^*}(\sigma)$ and $\sigma < \text{BR}^{s^*}(\beta)$, one now sees that $S(\hat{\beta}, \hat{\sigma}) > S(\tilde{\beta}, \tilde{\sigma})$. Contradiction! \square

Proof of Remark 3. “If.” Let $(c_1, v_1), \dots, (c_n, v_n)$ be the states with $c_j \geq v_j$, and let $(c_{n+1}, v_{n+1}), \dots, (c_{n+m}, v_{n+m})$ be the states with $c_j < v_j$. Let

$$f(c_j, v_j | \beta, \sigma) = \begin{cases} 1/(2n) - k/n \sqrt{(1+\beta)(1+\sigma)} & \text{if } j \leq n, \\ 1/(2m) + k/m \sqrt{(1+\beta)(1+\sigma)} & \text{if } j > n, \end{cases}$$

with a sufficiently small $k > 0$.

“Only if.” Because the sum of all states’ probabilities is 1, (r) cannot be satisfied for all states, so not all states can be regular. Hence, the underinvestment condition implies that not all states are trade states; i.e., (6). \square

Proof of Proposition 6. W.l.o.g., we assume $\max C = \min V$ and show that there exists $\bar{\Delta} > 0$ with the desired properties. We adopt the notational convention that variables with Δ -subscripts belong to the parameter constellation $(C, V - \Delta, f_\Delta)$.

We first prove an auxiliary technical result:

$$\psi(\Delta) = (\beta_\Delta^*, \sigma_\Delta^*) \text{ is continuous at } \Delta = 0.$$

By C2 we have for $\Delta = 0$:

$$\forall (\beta, \sigma) \in [0, \bar{\beta}] \times [0, \bar{\sigma}] : S_{\Delta\beta\beta}(\beta, \sigma) < 0, \quad S_{\Delta\sigma\sigma}(\beta, \sigma) < 0. \quad (16)$$

Because $S_{\Delta\beta\beta}$ and $S_{\Delta\sigma\sigma}$ are continuous in Δ , β , and σ , and the investment space $[0, \bar{\beta}] \times [0, \bar{\sigma}]$ is compact, (16) holds even for all Δ in a neighborhood of 0. In this neighborhood, the efficient investments $(\beta_\Delta^*, \sigma_\Delta^*)$ are characterized by the first-order conditions $S_{\Delta\beta}(\beta_\Delta^*, \sigma_\Delta^*) = 0$ and $S_{\Delta\sigma}(\beta_\Delta^*, \sigma_\Delta^*) = 0$. Hence, by the implicit functions theorem, $\psi(\Delta)$ is differentiable at $\Delta = 0$ and thus also continuous.

For all $\Delta \in \mathbb{R}$, all II price schemes P on $C \times (V - \Delta)$, and all (β, σ) , we define

$$M_\Delta(\beta, \sigma, P) = (R_{\Delta\beta}^b(\beta, \sigma, P), R_{\Delta\sigma}^s(\beta, \sigma, P)).$$

For $\Delta = 0$, Remark 1 implies

$$M_0(\beta_0^*, \sigma_0^*, P^{\text{const}, \max C}) = (1, 1).$$

Now denote by c^2 the second lowest production cost level, and by v^2 the second largest valuation level. Next we show that (5) holds with $(c_1, v_1) := (c^2, \max V)$ and $(c_2, v_2) := (\min C, v^2)$ at $(\beta, \sigma) = (\beta^*, \sigma^*) = (\beta_0^*, \sigma_0^*)$. The first line of (5) holds because

$$\frac{f_\sigma(c_1, v_1 | \beta^*, \sigma^*)}{f(c_1, v_1 | \beta^*, \sigma^*)} = \frac{\epsilon^s(c_1 | \sigma^*)}{\sigma^*} \stackrel{\text{MLRP}}{<} \frac{\epsilon^s(c_2 | \sigma^*)}{\sigma^*} \frac{f_\sigma(c_2, v_2 | \beta^*, \sigma^*)}{f(c_2, v_2 | \beta^*, \sigma^*)}.$$

The second line follows from a similar calculation. To check the third line, suppose there exists $\alpha > 0$ such that $\alpha f_\sigma(c_1, v_1) = f_\beta(c_1, v_1)$, and

$\alpha f_\sigma(c_2, v_2) = f_\beta(c_2, v_2)$. Then,

$$\begin{aligned} \frac{\epsilon^s(c_1|\sigma^*)}{\sigma^*} &= \frac{f_\sigma(c_1, v_1|\beta^*, \sigma^*)}{f(c_1, v_1|\beta^*, \sigma^*)} = \alpha \frac{f_\beta(c_1, v_1|\beta^*, \sigma^*)}{f(c_1, v_1|\beta^*, \sigma^*)} = \alpha \frac{\epsilon^b(v_1|\beta^*)}{\beta^*} \\ \text{MLRP} & > \alpha \frac{\epsilon^b(v_2|\beta^*)}{\beta^*} = \alpha \frac{f_\beta(c_2, v_2|\beta^*, \sigma^*)}{f(c_2, v_2|\beta^*, \sigma^*)} = \frac{f_\sigma(c_2, v_2|\beta^*, \sigma^*)}{f(c_2, v_2|\beta^*, \sigma^*)} = \frac{\epsilon^s(c_2|\sigma^*)}{\sigma^*}, \end{aligned}$$

which implies $c_1 < c_2$ by the MLRP. Contradiction!

Therefore, we can apply Corollary 1 with $P = P^{\text{const, max } C}$ to get an II price scheme $P^u := P_\delta$ such that

$$M_0(\beta_0^*, \sigma_0^*, P^u) \gg (1, 1). \quad (17)$$

Moreover, because

$$c_j < P^{\text{const, max } C}(c_j, v_j) < v_j, \quad (j = 1, 2).$$

we can choose δ so small that $P^u \in \mathcal{IR}$.

To make the continuity arguments in the rest of the proof more transparent, we introduce a new notion. For any $\Delta \in \mathbb{R}$, a (*buyer's*) *percentage-share function* is a function

$$b_\Delta : C \times (V - \Delta) \longrightarrow [0, 1].$$

To any such b_Δ we assign an II IR price scheme $P(b_\Delta)$ via

$$P(b_\Delta)(c, v) = \begin{cases} v - b_\Delta(c, v)(v - c) & \text{if } c < v, \\ 0 & \text{otherwise.} \end{cases}$$

Let b_0^o be any percentage-share function with $P(b_0^o) = P^u$. For all Δ , define a percentage-share function b_Δ^o on $C \times (V - \Delta)$ by

$$b_\Delta^o(c, v) = b_0^o(c, v + \Delta).$$

For all $\Delta \in \mathbb{R}$ and all (β, σ) we define

$$\Phi(\Delta, \beta, \sigma) = M_\Delta(\beta, \sigma, P(b_\Delta^o)).$$

By construction of b_Δ^o , Φ is continuous in Δ . Moreover, Φ is continuous in (β, σ) . Together with the continuity of ψ at $\Delta = 0$ we can conclude that

$$\Delta \mapsto M_\Delta(\beta_\Delta^*, \sigma_\Delta^*, P(b_\Delta^o)) = \Phi(\psi(\Delta), \Delta)$$

is continuous at $\Delta = 0$. Together with (17) this implies that there exists a neighborhood N of 0 such that for all $\Delta \in N$:

$$M_{\Delta}(\beta_{\Delta}^*, \sigma_{\Delta}^*, P(b_{\Delta}^o)) \gg (1, 1).$$

Now Lemma 2, applied with $(C, V, f) = (C, V - \Delta, f_{\Delta})$, $P = P(b_{\Delta}^o)$, and $(\beta, \sigma) = (\beta_{\Delta}^*, \sigma_{\Delta}^*)$, shows that for all $\Delta \in N$ an II IR price scheme P_{Δ}^{eff} on $C \times (V - \Delta)$ exists such that

$$M_{\Delta}(\beta_{\Delta}^*, \sigma_{\Delta}^*, P_{\Delta}^{\text{eff}}) = (1, 1). \quad (18)$$

We can assume that P_{Δ}^{eff} is constructed as in the proof of Lemma 2. We denote the coefficients occurring in the proof of Lemma 2 by

$$\lambda^u(\Delta), \lambda^b(\Delta), \lambda^s(\Delta).$$

This implies that $P_{\Delta}^{\text{eff}} = P(b_{\Delta}^{\text{eff}})$, where b_{Δ}^{eff} is defined by

$$b_{\Delta}^{\text{eff}}(c, v) = \lambda^u(\Delta)b_{\Delta}^o(c, v) + \lambda^b(\Delta) \max\{v - c, 0\}.$$

Because $\Phi(\psi(\Delta), \Delta)$ is continuous at $\Delta = 0$, the same is true for $\lambda^u(\Delta)$, $\lambda^b(\Delta)$, and $\lambda^s(\Delta)$. Therefore, the functions

$$\begin{aligned} \phi^b(\Delta, \beta, \sigma) &= R_{\Delta\beta\beta}^b(\beta, \sigma, P(b_{\Delta}^{\text{eff}})), \\ \phi^s(\Delta, \beta, \sigma) &= R_{\Delta\sigma\sigma}^s(\beta, \sigma, P(b_{\Delta}^{\text{eff}})), \end{aligned}$$

are continuous in Δ at $(0, \beta, \sigma)$, for all (β, σ) . By C2, we have for $\Delta = 0$:

$$\forall (\beta, \sigma) \in [0, \bar{\beta}] \times [0, \bar{\sigma}]: \phi^b(\Delta, \beta, \sigma) < 0, \quad \phi^s(\Delta, \beta, \sigma) < 0. \quad (19)$$

Because ϕ^b and ϕ^s are also continuous in (β, σ) , and the investment space $[0, \bar{\beta}] \times [0, \bar{\sigma}]$ is compact, (19) holds even for all Δ in a neighborhood of 0. Therefore, for these Δ the first-order conditions (18) are sufficient to assure that $(\beta_{\Delta}^*, \sigma_{\Delta}^*)$ is an investment equilibrium for P_{Δ}^{eff} . Thus, $(\beta_{\Delta}^*, \sigma_{\Delta}^*, P_{\Delta}^{\text{eff}})$ is a solution of $\mathcal{IR} \cap \mathcal{II}$. \square

Proof of Remark 4. By the MLRP, $\epsilon^b(\cdot|\tilde{\beta})$ is a strictly increasing function on V ; let $z: \mathbb{R} \rightarrow \mathbb{R}$ be a strictly increasing extension which maps onto \mathbb{R} ; i.e., $z(v) = \epsilon^b(v|\tilde{\beta})$ for all $v \in V$, and $z(\mathbb{R}) = \mathbb{R}$ (e.g., z might be piecewise linear). Now define

$$t(c) = z^{-1}(\epsilon^* \epsilon^s(c|\tilde{\sigma})),$$

where ϵ^* is from Proposition 3. \square

Proof of Remark 5. We also use the notation $c^1 = \min C$ and $v^1 = \max V$. We have $\tilde{P}(c^1, v^1) > c^2$ or $\tilde{P}(c^1, v^1) < v^2$ because otherwise $v^2 \leq c^2$. Now suppose \tilde{P} is increasing and $\tilde{P}(c^1, v^1) > c^2$ (the remaining analogous case $\tilde{P}(c^1, v^1) < v^2$ is omitted). Because $\tilde{P}(c^2, v^1) \geq \tilde{P}(c^1, v^1) > c^2$, Remark 4 implies $v^1 \leq t(c^2)$. Hence, $v^1 < t(c^1)$, and thus $\tilde{P}(c^1, v^1) = v_1$ by Remark 4. Thus, $\tilde{P}(c, v^1) = v_1$ for all trade states (c, v^1) because \tilde{P} is nondecreasing. In other words, even if the lowest cost level arises the buyer appropriates the trading surplus. Hence, $\tilde{P} = P^b$ by Remark 4 which contradicts the assumption $\tilde{\sigma} > 0$. \square

Proof of Remark 6. Consider a parameter constellation (C, V, f) with the self-investment property, MLRP, and $\min V < \max C$. By Proposition 6, it is sufficient to show that there exists no nondecreasing II IR price scheme which induces (β^*, σ^*) .

Now suppose some nondecreasing II IR price scheme P induces (β^*, σ^*) . Subtracting one of the first-order conditions for equilibrium, $R_\sigma^s(\beta^*, \sigma^*, P) = 1$, from one of the first-order conditions for efficient investment, $R_\sigma^s(\beta^*, \sigma^*, P^s) = 1$, yields

$$\sum_{c < v} (v - P(c, v)) f^b(v | \beta^*) f_\sigma^s(c | \sigma^*) = 0. \quad (20)$$

Let c^{**} be the largest production cost level $c \in C$ with $f_\sigma^s(c | \sigma^*) > 0$. Now fix some $v \in V$, and let

$$C^> = \{c \in C \mid c^{**} < c < v\}, \quad C^\leq = \{c \in C \mid c^{**} \geq c < v\}.$$

Next we show that prices are independent of production costs, i.e.,

$$\forall c, c' \in C^\leq \cup C^> : P(c, v) = P(c', v). \quad (21)$$

If $C^\leq = \emptyset$ then also $C^> = \emptyset$ and there is nothing to prove. Now consider the case $C^\leq \neq \emptyset$ and $C^> \neq \emptyset$. Using the MLRP we find

$$\begin{aligned} & \sum_{c \in C \text{ such that } c < v} (v - P(c, v)) f_\sigma^s(c | \sigma^*) \\ & \geq \sum_{c \in C^\leq} (v - P(\max C^\leq, v)) f_\sigma^s(c | \sigma^*) + \sum_{c \in C^>} (v - P(\min C^>, v)) f_\sigma^s(c | \sigma^*) \\ & \geq \sum_{c \in C^\leq} (v - P(\min C^>, v)) f_\sigma^s(c | \sigma^*) + \sum_{c \in C^>} (v - P(\min C^>, v)) f_\sigma^s(c | \sigma^*) \end{aligned}$$

$$\begin{aligned}
&\geq \sum_{c \in C \text{ such that } c < v} (v - P(\min C^>, v)) f_{\sigma}^s(c | \sigma^*) \\
&\geq 0.
\end{aligned} \tag{22}$$

Together with (20), this implies that

$$\sum_{c \in C \text{ such that } c < v} (v - P(c, v)) f_{\sigma}^s(c | \sigma^*) = 0. \tag{23}$$

Therefore, all inequalities in (22) are in fact equalities. Hence, $P(c, v) = P(\min C^>, v)$ for all $c \in C^{\leq} \cup C^>$, implying (21).

In the remaining case, $C^{\leq} \neq \emptyset$ and $C^> = \emptyset$, a similar calculation shows $P(c, v) = P(\max C^{\leq}, v)$ for all $c \in C^{\leq}$, again implying (21).

Similar reasoning, using the other first-order conditions, shows that prices are also independent of valuations. Hence, there exists a number p^* such that $P(c, v) = p^*$ for all (c, v) with $c < v$. But Remark 2 shows that P does not induce efficient investment. Contradiction! \square

Proof of Remark 7. Proof of C1. Let $v \geq v^*$. By definition of v^* we get $0 < f_{\beta}^b(v | \beta) = k_{\beta}^b(\beta) h^b(v)$, for any $\beta \in]0, \bar{\beta}]$. Hence, $h^b(v) > 0$, implying

$$f_{\beta\beta}^b(v | \beta) = k_{\beta\beta}^b(\beta) h^b(v) < 0.$$

The proof of the seller part of C1 is analog.

Proof of C2. The MLRP implies for all $\beta \in]0, \bar{\beta}]$ that

$$0 < \sum_v v f_{\beta}^b(v | \beta) = k_{\beta}^b(\beta) \sum_v v h^b(v),$$

hence $0 < \sum_v v h^b(v)$, and therefore

$$\sum_{v \in V} v f_{\beta\beta}^b(v | \beta) = k_{\beta\beta}^b(\beta) \sum_v v h^b(v) < 0.$$

The proof of the seller part of C2 is analog. \square

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