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Publication Date
1993-07-01

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HOLDING COSTS AND EQUILIBRIUM ARBITRAGE

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First version: June, 1992

This version: July 1993

Abstract

This paper constructs a dynamic model of the equilibrium determination of relative prices when arbitragers face holding costs. The major findings are that 1) models based on riskless arbitrage arguments alone may not provide usefully tight bounds on observed prices, 2) arbitragers are often most effective in eliminating the mispricings of shorter-term assets, 3) arbitrage activity increases the mean reversion of changes in the mispricing process and reduces their conditional volatility, and 4) there may be "spillovers" in arbitrage activity, i.e. profits per arbitrager in a given market may increase with the number of arbitragers. Finally, several predictions of the model are consistent with the growing empirical literature on deviations of prices from fundamental values.

Acknowledgments: We would like to thank participants in the WFA annual meetings in Whisler, Paul Asquith, Anne Fremault Vila, John Heaton, Francis Longstaff and Jeremy Stein for helpful comments and suggestions. Also Jean-Luc Vila wishes to acknowledge financial support from the International Financial Services Research Center at the Sloan School of Management.

1. Introduction

In frictionless markets, the law of one price guarantees that securities with identical cash flows sell for the same price. In real markets, however, most agents face frictions of some sort. While the class of agents most active in enforcing the law of one price, namely arbitragers, face particularly low costs, they do face non-trivial costs. The many studies that document deviations from the law of one price[1] testify to the proposition that the existence of arbitragers does not guarantee that market prices equal their fundamental values and, consequently, does not assure an optimal allocation of capital.

Of the many frictions that inhibit arbitrage activity, trading costs have received the most attention. In the simplest of models, the difference between the prices of two securities that have identical cash flows and finite maturities may not exceed the magnitude of the trading costs. Furthermore, arbitrage activity is easy to describe. When market prices do not admit riskless profit opportunities inclusive of trading costs, arbitragers do nothing. When market prices do admit such opportunities, arbitragers stake all they have on the sure proposition that prices will come back into line by the maturity date.

Because arbitrage behavior is not well described by this simplest of models, researchers have sought to enrich that model. Hodges and Neuberger (1989) examine the impact of trading costs when the arbitrage position must be periodically rebalanced. Brennan and Schwartz (1990) impose position limits and allow arbitragers to unwind positions optimally. Fremault (1991, 1993) studies the effects of non-synchronous trading across markets for otherwise identical securities. Holden (1990) explores the impact of oligopolistic behavior on the part of arbitragers.

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[1] In the case of government bonds, see, for example, Amihud and Mendelson (1991), Cornell and Shapiro (1989), and Tuckman and Vila (1992a). In the case of stock index futures, see, for example, Bacha and Fremault (1993), Chung, Kang, and Rhee (1992), MacKinlay and Ramaswamy (1988), Strickland and Xu (1991), and Pope and Yadav (1992). In the case of closed-end funds, see, for example, Pontiff (1993) and Lee, Shleifer, and Thaler (1991). In the case of scores and primes, see, for example, Jarrow and O'Hara (1989) and Canina and Tuckman (1993).

Another recent approach to enriching models of arbitrage behavior has been the introduction of holding costs [see Tuckman and Vila (1992a).] Since total costs increase with the time that the arbitrage position is maintained, arbitrage behavior is quite different from that in the simplest of models described above. First, the no-riskless arbitrage condition requires that the difference between the prices of securities with identical cash flows be no greater than the present value of the accumulated holding costs from the time the position is established until the securities mature. Thus, arbitrage bounds widen with maturity. Second, within these bounds, mispricing presents risky, but potentially profitable, investment opportunities. An arbitrager will profit from a long position in the relatively dear security and a short position in the relatively cheap security only if prices come into line soon enough; if prices diverge for too long, the accumulated holding costs will outweigh any realized profit.

Holding costs appear in many arbitrage contexts. First, shorting any spot security or commodity will often result in unit time costs since, for as long as the position is maintained, arbitragers must usually pay a borrowing fee, sacrifice the use of at least some of the short sale proceeds, or earn less than the market rate on posted collateral. Second, futures market positions may generate unit time costs since part of margin deposits may not earn interest. Third, banks making markets in forward contracts often charge a *per annum* rate over the life of the contract. Fourth, when collateral requirements cause deviations from desired investment strategies, these requirements are essentially generating unit time costs.[2]

Recent empirical work has begun to find that market mispricings may reflect the presence of these various incarnations of holding costs. Several papers, cited below, find that deviations from

[2] See Tuckman and Vila (1992a) for references supporting these institutional details. Also see that paper for a detailed discussion of holding costs in the U.S. Treasury bond market.

fundamental values increase with a security's maturity. These findings are consistent with the previous discussion that, under holding costs, riskless arbitrage bounds widen with maturity. Other papers, e.g. Bacha and Fremault (1993) and Pontiff (1993), find that mispricings increase with the level of interest rates. When holding costs arise because of foregone interest, this result follows easily.

Tuckman and Vila (1992a), in focusing on the behavior of arbitragers facing holding costs, take the mispricing process to be exogenous. This paper allows the activity of arbitragers to feed back into the mispricing process and, therefore, allows for an examination of how arbitragers facing holding costs affect market mispricings. The major findings of the paper are as follows.

One, because arbitragers take positions even when no riskless profit opportunities are available, equilibrium prices may be kept well within the riskless arbitrage bounds. This implies that models based on riskless arbitrage arguments alone, like many in the option replication literature,[3] may not provide usefully tight bounds on observed prices.

Two, because total position costs increase with the holding period, arbitragers are most effective in eliminating mispricings that tend to disappear most quickly, e.g. those involving short-term assets. This finding serves as a microeconomic foundation for the recent literature on the connection between investor impatience and market mispricings. De Long, Shleifer, Summers, and Waldmann (1990) and Lee, Shleifer, and Thaler (1991) have hypothesized that investors' short term horizons allow for persistent deviations from fundamental values. Shleifer and Vishny (1990) argue that when mispricings increase in asset maturity, risk averse corporate managers will pursue short-term objectives.

Three, arbitrage activity increases the mean reversion of changes in the mispricing process and reduces their conditional volatility. A number of recent studies have examined the properties of

[3] See, for example, Boyle and Vorst (1992) and Leland (1985).

mispricing processes. Mackinlay and Ramaswamy (1988), Strickland and Xu (1991), and Pope and Yadov (1992) all find mean reversion when examining changes in the mispricing of stock index futures relative to the index itself. Pope and Yadov (1992), after eliminating other possible explanations for this finding, conclude that "mean reversion is being caused by trading activity linked directly or indirectly to arbitrage."

Four, profits per arbitrager may increase with the number of arbitragers. New entrants into the arbitrage industry do reduce the profits of existing arbitragers by lowering the current level of mispricing, but they increase profits by increasing the speed at which mispricings are eliminated, i.e. by reducing the accumulation of holding costs. This "positive spillover" implies that arbitragers may sometimes flock to relatively well-arbitraged markets. Recent work in different contexts has relied on short-term horizons, rather than holding costs, to generate herding behavior. Froot, Scharfstein, and Stein (1992) show that traders with short-term horizons may herd on a subset of all possible information because short-term investors cannot profit by trading on information that will not be incorporated into prices soon enough. Hirshleifer, Subrahmanyam, and Titman (1993) induce short-term horizons through undiversifiable risk and risk-aversion and then assume that some traders may become informed about a security before others do. Consequently, traders may herd on particular securities so that, if they do indeed become informed before others, and trade on that advantage, the activity of other traders will correct mispricings quickly thereafter.

Section 2 presents the model. Section 3 provides numerical examples to illustrate the model's implications. Section 4 concludes and suggests avenues for future research.

2. The Model

2.1 Preliminaries

Any model of equilibrium arbitrage must 1) posit the existence of two distinct assets which generate identical cash flows, 2) assume some forces which cause the prices of these assets to differ, and 3) restrict investor and arbitrager behavior so that mispricings do not vanish as soon as they appear.

Markets provide many examples of distinct portfolios which generate identical cash flows. Some common instances are a forward or futures contract vs. a levered position in the spot asset, a bond denominated in one currency vs. a bond denominated in another currency plus a cross-currency forward contract, and a coupon bond vs. a cash-flow matched portfolio of other coupon bonds.

This model assumes that there exist two distinct bond issues with identical coupons and maturities. For ease of exposition, one of the issues will consist of "red" bonds while the other will consist of "green" bonds. Not much effort would be required to recast the model in terms of the other mentioned examples.

The prices of the red and green bonds may tend to differ for a number of reasons. For example, the bonds might be traded by different clienteles with different valuation rules. Clienteles of this sort may have developed for historical reasons or may exist for institutional reasons. Another reason for price differences across these markets might be temporary supply and demand imbalances due to microstructure imperfections. In any case, because this paper aims at explaining the force which constrains these price differences, no serious effort has been made to model the source of price differences. Instead, the model assumes that noise traders occasionally shock the red and green bond markets with buy and sell orders. To ensure that these shocks affect market prices, it is furthermore assumed that the demand for the individual bond issues is not perfectly elastic.

Because close substitutes exist for most financial assets, the more usual assumption is that the demand for individual financial assets is perfectly elastic. But, in the presence of market frictions, considerations other than the existence of substitutes become important. For example, investors in different tax brackets value the same asset differently, thus generating a downward-sloping demand curve. Similarly, to the extent that an asset is purchased after the sale of other assets, investors with low transaction costs will buy at higher prices than investors with high transaction costs, again leading to a downward-sloping demand curve. This model employs the tax motivation because, as shown below, the functional form of the demand curve can be easily derived. But, any downward-sloping demand curve will translate liquidity shocks into price differences across markets.

Finally, price differences must not vanish as they appear. This certainly requires some segmentation of the markets; if investors in one bond market can easily purchase bonds in the other, price differences will result in migrations from the relatively dear market to the relatively cheap market. And these migrations will, in turn, equalize the red and green bond prices. While market segmentation seems a reasonable assumption from the point of view of many investors, arbitragers can usually trade across markets. Nevertheless, arbitrager activity might not be sufficient to force prices back into line. To this end, arbitragers are assumed to be risk-averse and to face holding costs when shorting bonds. As in Tuckman and Vila (1992a), these assumptions imply that optimal arbitrage positions are not necessarily large enough to eliminate price differences across markets.

2.2. Equilibrium pricing without arbitragers

Turning to the model of this paper, begin by assuming that there are two distinct bond issue, one red and one green, trading in two distinct markets. Both bond issues mature n periods from now and entitle holders to \$d at the end of each period and to \$1+d at maturity. Finally, let r denote the discount rate which is assumed constant through the maturity date.

The appendix derives demand curves for each of the bond issues by assuming that investors face different tax rates on coupon income and that those who buy bonds plan to hold their investments untill maturity. The resulting demand curve for each issue can be written in the following functional form

$$D_n(P_n) = \eta_n - \frac{P_n}{\theta_n} \qquad n \geq 1$$

where

$$\theta_n = \frac{d}{\alpha r}\left(1 - \frac{1}{(1+r)^n}\right)$$

and α denotes the number of investors in each market. At maturity, i.e. at $n = 0$, the demand curve is perfectly elastic since a security that immediately pays \$1 and nothing thereafter must sell for \$1.

Assume that the quantity of each bond issue available for investor trading is Q. In the absence of noise traders, the market price in each bond market would be the P_n that solves $D_n(P_n) = Q$. The presence of noise traders, however, can cause the prices of the red and green bonds to differ.

Noise traders enter the markets to sell or to buy. Let $L_{R,n}$ and $L_{G,n}$ be the *cumulative* amount of noise trading in the red and green bond markets, respectively. By convention, positive quantities denote supply shocks while negative quantities denote demand shocks. If, for instance, $L_{R,n} > L_{R,n+1}$ there are $L_{R,n} - L_{R,n+1}$ sellers in the market for red bonds. If, on the other hand, $L_{R,n} < L_{R,n+1}$, there are $L_{R,n+1} - L_{R,n}$ buyers. As in other papers,[4] no explicit model of the sources of noise trading will be presented. Finally, note that noise trading shocks affect the quantity of bonds available for investor trading: the supply of red bonds after the shock is $Q + L_{R,n}$ while the supply of green bonds after the shock is $Q + L_{G,n}$

In the absence of arbitragers, the equilibrium prices in the red and green market, $P_{R,n}$ and $P_{G,n}$,

[4]See for instance, Kyle (1985), DeLong *et. al.* (1990), Fremault (1991 and 1993) and Holden (1990).

respectively, are determined by the following equations:

$$\eta_n - \frac{P_{R,n}}{\theta_n} = Q + L_{R,n} \tag{2}$$

and

$$\eta_n - \frac{P_{G,n}}{\theta_n} = Q + L_{G,n} \tag{3}$$

Solving for the prices gives,

$$P_{R,n} = \frac{\eta_n - Q - L_{R,n}}{\theta_n} \tag{4}$$

and

$$P_{G,n} = \frac{\eta_n - Q - L_{G,n}}{\theta_n} \tag{5}$$

Of particular interest is the difference between the price of red bonds and the price of green bonds. Letting $\Delta_n = P_{R,n} - P_{G,n}$ and using the pricing equations (4) and (5), the relative mispricing equals

$$\Delta_n = \theta_n L_n \tag{6}$$

with

$$L_n = L_{G,n} - L_{R,n}.$$

2.3. Equilibrium pricing with arbitragers

Arbitragers can now be introduced into the model. Let x_n be the number of green bonds bought by arbitragers and the number of red bonds sold by arbitragers. The optimal choice of x_n will be discussed below. For now, consider the effect of arbitrage on the mispricing Δ_n. If $L_n > 0$ and $\Delta_n > 0$, i.e. if $P_{R,n} > P_{G,n}$, arbitragers will want to sell red bonds and buy green bonds, so x_n will be positive. Furthermore, x_n will be added to the supply of red bonds and added to the demand of green bonds. Adjusting equations (4) and (5) accordingly and subtracting (5) from (4) to obtain Δ_n for this case, $\Delta_n = \theta_n (L_n - 2x_n)$. Notice that arbitrage activity lowers the relative mispricing.

If $L_n < 0$ and $\Delta_n < 0$, i.e. $P_{R,n} < P_{G,n}$, arbitragers will want to buy red bonds and sell green bonds, so x_n will be negative. Furthermore, x_n will be added to the demand for red bonds and to the supply of green bonds. In this case also $\Delta_n = \theta_n(L_n - 2x_n)$ and arbitrage activity reduces the relative mispricing.

Summarizing this discussion, adjusting supply and demand in the bond markets to account for arbitrager activity changes the mispricing Δ_n from (6) to

$$\Delta_n = \theta_n(L_n - 2x_n), \tag{7}$$

By definition, x_n is the sum of positions across arbitragers. If, there are I arbitragers, i=1 ..I, an equilibrium in this model consists of strategies x_n^i and a process Δ_n such that 1) each arbitrager chooses an optimal strategy given the evolution of Δ_n, and 2) the resulting $x_n = \Sigma_i x_n^i$, in turn, generates the process Δ_n given by (7).

Assume that all arbitragers have negative exponential utility functions with risk tolerance τ_i. Each arbitrager maximizes his expected utility of wealth as of the date the bonds mature so, letting W_n denote wealth with n periods to maturity, each arbitrager maximizes:

$$-\exp\frac{W_0^i}{\tau_i}. \tag{8}$$

Assuming that all arbitragers face same holding cost, the price process in the multi-arbitrager economy is the same as when there is one representative arbitrager with risk tolerance

$$\tau = \sum_{i=1}^{I} \tau^i .$$ [5]

Consider therefore the trading decision of the representative arbitrager and begin with the value of an arbitrage position from one period to the next. Assume for the moment that $\Delta_n > 0$, i.e. $P_{R,n} > P_{G,n}$. As in Tuckman and Vila (1992), the arbitrager will buy x_n green bonds financed by borrowing $x_n P_{G,n}$

[5] The extension to several groups of arbitragers facing different holding costs does not present any conceptual difficulties (see section 3.4).

dollars, short x_n red bonds, and lend $x_n P_{R,n}$ dollars, $x_n > 0$. Denote the cost of maintaining a unit short position over one period by c, the position next period is worth

$$x_n P_{G,n-1} - x_n P_{R,n-1} + x_n P_{R,n}(1+r) - x_n P_{G,n}(1+r) - c|x_n| = x_n [\Delta_n(1+r) - \Delta_{n-1}] - c|x_n|. \quad (9)$$

If $\Delta_n < 0$, the expression does not change, but x_n will be negative: the arbitrage position entails buying green bonds and shorting red bonds.

From the above discussion, the evolution of can be described by the following equation

$$W_{n-1} = W_n(1+r) + x_n [\Delta_n(1+r) - \Delta_{n-1}] - c|x_n|. \quad (10)$$

Defining $w_n = W_n(1+r)^n$, $\delta_n = \Delta_n(1+r)^n$, and $c_n = c(1+r)^n$, (10) can be rewritten as

$$w_{n-1} = w_n + x_n [\delta_n - \delta_{n-1}] - c_{n-1}|x_n|. \quad (11)$$

Equation (11) and the objective to maximize (8) completes the specification of arbitrager i's investment problem given Δ_n.

The model is completed by specifying an exogenous stochastic process for the net liquidity shocks, L_n. For simplicity it will be assumed that L_n evolves as a binomial process: if its value with n periods to maturity is L_n, then it will take on the value $L_{n-1} = L_n + u$ with probability $\pi(n,L_n)$ and a value $L_{n-1} = L_n - u$ with probability $1-\pi(n,L_n)$.

2.4. Model Solution

This section begins by solving arbitrager i's investment problem. Let:

$$V(w_n, L_n, n) = \max E_n[-\exp \frac{w_0}{\tau}]$$

where E_n denotes the expectation when there are n periods to maturity and the maximum is over the strategy x_n. By the principle of optimality in dynamic programming,

$$V(w_n,L_n,n) = \max_{x_n} \{ \pi(n,L_n)V(w_n+x_n[\delta_n(L_n)-\delta_{n-1}(L_n+u)]-c_{n-1}|x_n|,L_n+u,n-1) + (1-\pi(n,L_n))V(w_n+x_n[\delta_n(L_n)-\delta_{n-1}(L_n-u)]-c_{n-1}|x_n|,L_n-u,n-1)\} \quad (12)$$

Also, because the mispricing must vanish at the maturity date, the initial condition of the problem is $V(w_0,L_0,0) = -\exp(-Aw_0)$.

Because of the special form of the utility function, $V(w_n,L_n,n)$ can be written as $\exp\{-Aw_n\}J(L_n,n)$. Using this fact, (12) becomes

$$J(L_n,n) = \max_{x_n} \{ \pi(n,L_n)\, e^{-A[x_n[\delta_n(L_n)-\delta_{n-1}(L_n+u)]-c_{n-1}|x_n|]}J(L_n+u,n-1) + (1-\pi(n,L_n))\, e^{-A[x_n[\delta_n(L_n)-\delta_{n-1}(L_n-u)]-c_{n-1}|x_n|]}J(L_n-u,n-1)\} \quad (13)$$

with initial condition $J(L_0,0) = -1$.

To solve for the optimal strategy as a function of δ_n, begin as follows. If $\delta_n > 0$, the mispricing increases with a move to $\delta_{n-1}(L_n+u)$ while the mispricing decreases with a move to $\delta_{n-1}(L_n-u)$. Since the per period arbitrage profits is $x_n(\delta_n-\delta_{n-1}-c_{n-1})$, the position will never be profitable if $\delta_n < \delta_{n-1}(L_n-u) + c_{n-1}$, i.e. if the position is not profitable even when the relative mispricing falls. So, for these value of δ_n, $x_n = 0$. On the other hand, $\delta_n < \delta_{n-1}(L_n+u) + c_{n-1}$ is inconsistent with equilibrium; if the position is profitable even when the relative mispricing rises, prices furnish a riskless arbitrage inclusive of holding costs and the optimal x_n would equal $+\infty$. In the intermediate range,

$$\delta_{n-1}(L_n+u) + c_{n-1} > \delta_n > \delta_{n-1}(L_n-u) + c_{n-1}$$

x_n can be found by solving the optimization problem (13) to obtain

$$x_n = \left[\frac{1}{A[\delta_{n-1}(L_n+u)-\delta_{n-1}(L_n-u)]} \mathrm{Ln}\left\{ \frac{1-\pi(n,L_n)}{\pi(n,L_n)} \frac{\delta_n(L_n)-\delta_{n-1}(L_n-u)-c_{n-1}}{c_{n-1}+\delta_{n-1}(L_n+u)-\delta_n(L_n)} \frac{J(L_n-u,n-1)}{J(L_n+u,n-1)} \right\} \right]^+ \quad (14)$$

where $(H)^+ = \mathrm{Max}\{H, 0\}$.

If $\delta_n < 0$, similar arguments reveal that $x_n = 0$ when $\delta_n > \delta_{n-1}(L_n+u) - c_{n-1}$ while $x_n = -\infty$ when $\delta_n < \delta_{n-1}(L_{n-1}-u) - c_{n-1}$. In the intermediate range,

$$\delta_{n-1}(L_n+u) - c_{n-1} > \delta_n > \delta_{n-1}(L_n-u) - c_{n-1}$$

the optimal x_n is given by

$$x_n = \left[\frac{1}{A[\delta_{n-1}(L_n+u) - \delta_{n-1}(L_n-u)]} \text{Ln} \{ \frac{1-\pi(n,L_n)}{\pi(n,L_n)} \frac{\delta_n(L_n) - \delta_{n-1}(L_n-u) + c_{n-1}}{-c_{n-1} + \delta_{n-1}(L_n+u) - \delta_n(L_n)} \frac{J(L_n-u,n-1)}{J(L_n+u,n-1)} \} \right]^- \quad (15)$$

where $(H)^- = \text{Min}\{H;0\}$

To solve for the equilibrium values of x_n and δ_n, rewrite the equilibrium condition (7) in terms of δ_n:

$$\delta_n = \theta_n(1+r)^n(L_n - 2x_n) . \quad (16)$$

Then, solve (16) simultaneously with (14) or (15), as appropriate. For the case $\delta_n > 0$, figure 1 illustrates the optimal position size and the equilibrium condition as a function of the mispricing δ_n. The dotted line represents the mispricing which generates infinite arbitrage activity. The simultaneous solution is given by the intersection of the two functions.

Given the solution technique for any n given the values at n-1, backward induction will provide the solution for all n. The initial condition of the problem gives $J(\cdot,0)$. This allows for the solution of x_1 and δ_1 along the lines described above. Then, substituting these values into (13) yields $J(\cdot,1)$. Proceeding in this fashion produces the entire mispricing process and the accompanying arbitrage strategy.

3. Numerical Examples

This section explores the implications of the model presented in section 2 through several numerical examples. These examples require the specification of particular parameter values and of a particular binomial process governing the evolution of the net liquidity shock, L. The following choices constitute the "base case" scenario.

As mentioned in the previous section, the parameter α is related to the depth of the market. Since a $10 billion Treasury coupon issue is not uncommon [First Boston (1990), pp. 49-54], set α = 10 billion. Relying on an often cited estimate of the reverse spread in the special issues market, set the holding cost parameter, c, equal to .5% [Stigum (1983), p. 414].

The other parameters are initially set at reasonable, but more arbitrarily chosen values. Let the aggregate risk tolerance, τ, be fixed at 1 million, let the bond's maturity at the time of issue be 5 years, and let the liquidity imbalance at that time equal 0. Finally, let the coupon rate and discount rate be 12%.

The net liquidity shock is assumed to follow a discretized version of the Ornstein-Uhlenbeck proeess. The continuous time version can be written as

$$dL_t = - \rho L_t dt + \sigma db_t$$

where ρ and σ are non-negative constants and db_t is the increment of a Brownian motion. To define the discrete version, let h be the length of the trading interval and set u and $\pi(n,L_n)$ of the previous section such that

$$u_h = \sigma \sqrt{h} \quad \text{and}$$

and

$$\pi(n,L_n) = \begin{cases} \frac{1}{2}\left(1 - \frac{\rho L_n}{\sigma}\sqrt{h}\right) & \text{if } \frac{1}{2}\left(1 - \frac{\rho L_n}{\sigma}\sqrt{h}\right) \in [0,1] \\ 0 & \text{if } \frac{1}{2}\left(1 - \frac{\rho L_n}{\sigma}\sqrt{h}\right) < 0 \\ 1 & \text{if } \frac{1}{2}\left(1 - \frac{\rho L_n}{\sigma}\sqrt{h}\right) > 1 \end{cases}$$

[See Nelson and Ramaswamy (1990), pp. 399-400, for the relevant convergence results.]

Since the net liquidity shock process is not directly observable, numerical values for ρ and σ were chosen in the following manner. As shown below, given ρ and σ one can compute the average mean reversion and the average conditional volatility of the relative mispricing process. Furthermore, estimates of the relative mispricing process in the Treasury market are reported in Tuckman and Vila (1992a). Therefore, select ρ and σ so that the mispricing process of the model in section 3 matches those reported estimates. This procedure gives values of approximately $\rho = 2$ and $\sigma/\alpha = 8\%$. Note that σ is quoted as a percentage of market size.

The trading interval h was set so that there are 800 steps per year, i.e. $h = .00125$. This choice provided satisfactory convergence of the discrete process to its continuous time limit.

The first subsection of this section discusses the optimal position size of the arbitrager. The second illustrates how arbitrage activity affects equilibrium mispricing. The third subsection demonstrates how arbitrage activity changes the mean reversion and conditional volatility of mispricing changes. The fourth subsection draws lessons about the arbitrage industry.

3.1 Arbitrage Positions

Figure 2a graphs the optimal position size of the arbitrager as a function of the equilibrium mispricing when the bond is two years from maturity. All parameters are set as in the base case except for σ/α which is set at 2%. The domain of the graph, as of all others to follow, captures approximately 95% of the probability distribution of mispricings.

For relatively small levels of mispricing, arbitragers do not take any position: the potential profits are not large enough relative to the potential accumulation of holding costs. For larger levels of mispricing, optimal position size increases with the mispricing.

Figure 2b shows the same graph with $\sigma/\alpha = 8\%$, the base case. The increased volatility substantially increases position size. Furthermore, the region in which arbitragers do not take positions becomes imperceptible.

3.2 Equilibrium Mispricing

Figure 3a graphs the equilibrium mispricing with and without arbitragers as a function of the net liquidity shock when $\sigma/\alpha = 2\%$. Once again, the bonds mature in two years.

As expected, larger net liquidity shocks increase mispricing.[6] Without arbitragers the mispricing increases linearly, as given by equation (6). For relatively small shocks, when arbitragers do not take positions, the mispricing is the same with or without arbitragers. For larger shocks, arbitragers reduce mispricings below what they would otherwise be. The equilibrium mispricing with arbitragers

[6]It is worth noting that mispricing does not go to zero as the discretization step goes to zero. Indeed, *a priori*, the following 'efficient' equilibrium might have been proposed as a candidate equilibrium in continuous time: mispricing is always zero and when mispricing is not zero (off equilibrium) arbitragers take an aggregate position which immediately eliminates mispricings. Since a non-zero mispricing vanishes immediately, the arbitragers' strategy is optimal. Note that the discrete time equilibrium does not converge to the 'efficient' equilibrium. The solution of the model in continuous time is beyond the scope of the present paper.

eventually flattens out at the maximum mispricing consistent with no riskless arbitrage opportunities, i.e. at the present value of the holding costs incurred by maintaining a unit arbitrage position until maturity.

Figure 3a graphs the equilibrium mispricing when $\sigma/\alpha = 8\%$. Because the inactive region is imperceptible in this case, arbitragers always strictly decrease the level of mispricing from what it would have been in their absence.

Two lessons emerge from figures 3a and 3b. First, when the equilibrium mispricing without arbitragers is not zero, the equilibrium mispricing with arbitragers is also not zero. Due to holding costs and risk aversion, arbitragers bring prices closer into line but never eliminate mispricing altogether. Second, arbitrage activity reduces mispricing to a level below that which would trigger riskless arbitrage transactions. Therefore, market prices may commonly reflect these smaller mispricings so that the no-riskless arbitrage bound may rarely be binding. Stated another way, mispricing in the presence of arbitragers does not equal the minimum of the mispricing without arbitragers and the no-riskless arbitrage bound: mispricing in the presence of arbitragers is less than or equal to that minimum.

Table 1 explores the reduction of mispricing by arbitragers as a function of the bond's maturity. In constructing this table, the original maturity of the bond was set at 25 years, i.e. the liquidity imbalance with 25 years to maturity was set equal to 0.

Panel I of the table uses the rest of the base case parameters, in particular, $\rho = 2$. As expected, the average absolute mispricing with arbitragers is always below that average in the absence of arbitragers. Furthermore the expected absolute mispricing decreases as the bond matures. As mentioned in the introduction, holding costs make arbitrage activity particularly difficult for longer term assets.

Panel I also shows that the percentage reduction of the mispricing due to arbitrage activity decreases

with maturity. In other words, under these parameter values, the mispricing of long-term assets is more resistant to the forces of arbitrage than is the mispricing of short-term assets. The intuition behind this result is that the total holding costs of arbitrage positions in long-term assets will usually exceed that of arbitrage positions in short-term assets.

It is not always the case, however, that the percentage reduction in mispricing decreases monotonically in maturity. When $\rho = 4$, panel II shows that the reduction increases in maturity before it decreases. An explanation for this effect is that higher levels of mean reversion reduce the disadvantage of long-term assets: faster reversion to fundamental values means that long-term assets will, on average, accumulate lower total holding costs. Nevertheless, panel II and other numerical work reveal that, for sufficiently long maturities, the percentage reduction in mispricing decreases with maturity.

The maturity effects discussed here are consistent with a number of recent empirical studies. In the government bond context, Amihud and Mendelson (1991) and Tuckman and Vila (1992a) find that mispricing increases with maturity. Chung, Kang, and Rhee (1993), and MacKinlay and Ramaswamy (1988) find the same in the context of index futures.

3.3 Mean Reversion and Conditional Volatility

This subsection analyzes how arbitrage activity changes the evolution of the mispricing process. Begin by defining the process Z_n to be the liquidity imbalance between the two markets after arbitragers take their positions, i.e.

$$Z_n = L_n - 2x_n .$$

Recall from equations (6) and (7) that $\Delta_n = \theta_n L_n$ without arbitragers and that $\Delta_n = \theta_n Z_n$ with arbitragers. Therefore, comparing the processes L_n and Z_n is equivalent to comparing the mispricing

process with and without arbitragers. This comparison is easier than studying the mispricing directly because L_n has very simple conditional moments:

$$E[dL_n|L_n] = -\rho L_n dt$$

and

$$Var[dL_n|L_n] = \sigma^2 dt.$$

These moments reveal that the effect of arbitrage activity can be seen by comparing

$$\rho' = E\left[-\frac{dZ_n}{Z_n dt} \mid L_n \right]$$

and

$$\sigma' = \sqrt{\frac{Var\left[dZ_n \mid L_n \right]}{dt}}$$

with ρ and σ, respectively.

Figure 4a graphs the mean reversion coefficient of the mispricing process with and without arbitragers as a function of the net liquidity shock when $\sigma/\alpha = 2\%$. Original bond maturity is, once again, five years, and the graph depicts the situation when there are two years to maturity. The horizontal line gives the mean reversion of the mispricing process with no arbitragers, namely $\rho = 2$. For large net shocks, the mispricing, and hence Z_n, approach bounds derived from the no-riskless arbitrage condition. Consequently, Z_n is almost constant in these regions and mean reversion is very small. As the net liquidity shock falls in absolute value, arbitragers induce more and more mean reversion into the mispricing process. But, for small liquidity imbalances, arbitragers do not take a position and the mean reversion coefficient with and without arbitragers is the same.

Figure 4b represents the base case, i.e. the case of $\sigma/\alpha = 8\%$. Because the inactive region is imperceptible here, there is no region around 0 where the mean reversion is the same with and without arbitragers.

Figure 5a graphs the conditional volatility of the mispricing process with and without arbitragers when $\sigma/\alpha = 2\%$. The horizontal line represents the volatility without arbitragers, namely $\sigma/800^{1/2}$. The conditional volatility with arbitragers is the same as that without arbitragers when the net liquidity shock is small and arbitragers do not take positions. Otherwise, arbitragers reduce the conditional volatility of the mispricing process. This reduction increases with the liquidity imbalance in the two markets.

Figure 5b explore the effect of arbitrage on conditional volatility when $\sigma/\alpha = 8\%$. Since the arbitragers are almost always active in this case, arbitragers reduce the conditional volatility of the mispricing process for all levels of the net liquidity shock.

Figures 4a-5b examined the conditional moments as a function of L_n. To further study the effects of arbitrage on the dynamics of the mispricing process, define ρ'' and σ'' as the corresponding unconditional moments, i.e.

$$\rho'' = E_N [\, \rho' | \, L_N = 0 \,]$$

and

$$\sigma'' = E_N [\, \sigma' | \, L_N = 0 \,] .$$

Tables 2 and 3 report these unconditional moments for a bond with an initial maturity of five years that has two years left to maturity. Table 2 takes $\sigma/\alpha = 8\%$ and computes ρ'' and σ''/α for different values of ρ. Note that arbitrage activity substantially increases the average mean reversion of the mispricing process and substantially lowers its average conditional volatility. Table 3 takes $\rho = 2$ and computes ρ'' and σ''/α for different values of σ/α. Once again, arbitragers increase mean reversion and reduce conditional volatility.

In the context of index futures, Pope and Yadov (1992) find that mean reversion of changes in

mispricing decreases with maturity. Holding costs may explain this finding since, as mentioned above, they make it relatively more difficult for arbitragers to profit from the mispricing of long-term assets. Table 4 reports ρ'' for a bond with an original maturity of 25 years that has 1 to 20 years left to maturity. All other parameters are as in the base case. Arbitragers increase the mean reversion parameter from 2 to about 2.5 when the bond has 20 years to maturity, but from 2 to 5.5 when the bond has 1 year to maturity. This maturity effect is similar to the maturity effect on the percentage reduction in mispricing due to arbitrager activity in that the effect is not robust to increases in ρ. While not shown in the table, higher levels of mean reversion in the liquidity shock process bring prices back into line sooner and thus reduce the holding cost disadvantage of trading long-term assets.

3.4. The Arbitrage Industry

This subsection will refer to the certainty equivalent of arbitrage profits, CE. It is defined such that the value of being an arbitrager with present wealth W_n is equal to the utility of W_n + CE. Recalling that $w_n = W_n(1+r)^n$, this condition becomes

$$V(w_n, L_n, n) = \exp\left[-\frac{w_n + CE(1+r)^n}{\tau} \right].$$

Then, using the separability of V into J and an exponential function of wealth,

$$CE = -\tau \frac{Ln[-J(L_n, n)]}{(1+r)^n}.$$

As noted in section 2, when all arbitragers face the same holding cost the model with many arbitragers can be viewed as an economy with a representative arbitrager that has a risk tolerance $\tau = \sum_i \tau_i$. Therefore, increases in τ can be thought of as increases in the number of arbitragers. In fact, if CE^i is the certainty equivalent of arbitrager i,

$$CE^i = \frac{\tau_i}{\tau} CE.$$

Table 5 analyzes various profitability indicators. The first column gives the aggregate risk tolerance or the number of arbitragers. The second column gives the aggregate certainty equivalent which can be thought of as industry profits. The third column gives the aggregate certainty equivalent per unit risk tolerance or profit per arbitrager.

The first lesson to be drawn from the table is that industry profits first rise and then fall in the number of arbitragers. There are three effects at work here. One, more arbitragers mean larger aggregate arbitrage positions and greater industry profits for a given mispricing. Two, more arbitragers mean that a current mispricing will be eliminated more quickly thus lowering realized holding costs and raising industry profits. Three, more arbitragers mean that mispricings will be lower than they would otherwise have been thus reducing profits. The first two effects dominate when the number of arbitragers is relatively small while the third effect dominates when the number of arbitragers is relatively large.

The second lesson of table 4 is that profits per arbitrager are also non-monotonic in the number of arbitragers. For relatively small numbers of arbitragers, entry into the industry raises profits for all. Since the first effect mentioned above is linear in the number of arbitragers, the second effect is responsible for this positive externality; an arbitrager makes more money when his competitors help him to drive prices back toward fundamental values.

The non-monotonicity of profits per arbitrager raises the possibility of multiple equilibria in a model of arbitrage with costly entry. For a given entry cost, there are two levels of entry that equate the profit per arbitrager to the cost of entry. The one with the lower number of arbitragers is unstable. On one hand, any entrant will earn more than the cost of entry. On the other hand, with one less arbitrager no one will find it worthwhile to enter. By contrast the equilibrium with the larger number of arbitrager is stable. In addition to this stable equilibrium, no arbitrage activity is also a stable

equilibrium since no small arbitrager can profitably enter.

Table 6 reports industry profits as a function of the holding cost. For very small holding costs mispricings are quickly eliminated and profits are very low. For very large holding costs arbitrage is extremely costly and profits are, once again, very low. Between these two extremes profits form an inverted-U as a function of the holding cost.

Until now, it has been assumed that all arbitragers face the same holding cost. The model, however, can easily be generalized to allow for different holding costs. Assume that most arbitragers, with an aggregate risk tolerance of 999,000, face a holding cost of .5%. A small group of arbitragers, with an aggregate risk tolerance of 1,000, face a different holding cost. Table 7 examines the profits of both these groups as a function of the holding cost of the smaller group. When both groups face a cost of .5%, the profit per unit risk tolerance is about 1.1 for each group. If the smaller group has a cost disadvantage, i.e. a holding cost of .6%, its profit per unit falls to .71 while that of the larger group stays about the same. The smaller group cannot compete because of its higher cost but its inability to do so has little effect on the broader market. On the other hand, if the smaller group has a cost advantage, i.e. a holding cost of .4%, its profit per unit jumps to 2.63 while that of the larger group falls to .58. Not only does the smaller group profit by being able to take advantage of smaller mispricings, but it tightens the no-riskless arbitrage bounds and reduces mispricings to such a great extent that the vast majority of arbitragers are left with little more than half of their previous profit levels.

The analysis of this subsection reveals that the interaction between arbitragers resembles Bertrand competition in that only the lowest cost competitors earn profits. Consequently, there are huge incentives to cost reduction and the arbitrage industry may very well be dominated by a few technologically superior firms.

4. Concluding remarks

The dynamic model of equilibrium arbitrage developed here assumes that noise trading in two segmented markets causes price deviations from fundamental values. Risk averse arbitragers facing holding costs reduce these deviations, but do not eliminate them completely. In contrast to models with an exogenously specified mispricing process, the equilibrium component of this model allows for conclusions to be drawn about the impact of arbitrage activity on market mispricing. In contrast to static models of arbitrage activity, the dynamic setting of this model allows for conclusions to be drawn about the evolution of the mispricing process.

This paper contributes not only to the arbitrage pricing literature, but also to the growing literature on imperfect financial markets. While the impact of frictions such as trading costs,[7] holding costs,[8] borrowing constraints,[9] or market incompleteness[10] on dynamic investment strategy is relatively well understood, little is known about the impact of these frictions on equilibrium price processes. While some recent papers have addressed this question, they focus on trading costs rather than holding costs.[11]

While academic research has focused on riskless arbitrage activity, the process by which market prices are kept close to fundamental values can often be characterized as a risky investment activity. This paper has focused on a risk that arises when an arbitrager facing holding costs trade on price differentials: he looses if the market becomes 'efficient' too slowly.

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[7]See Constantinides (1986), Davis and Norman (1990), Davis, Panas and Zariphopoulou (1991), Duffie and Sun (1989), Delgado and Dumas (1993), Dumas and Luciano (1990), Fleming et. al. (1989).

[8]See Tuckman and Vila (1992).

[9]See Grossman and Vila (1992), Vila and Zariphopoulou (1990), He and Pearson (1991 a and b).

[10]See He and Pearson (1991 a and b).

[11]See Ayagari and Gertler (1991), Heaton and Lucas (1992) and Vayanos and Vila (1992).

Delgado and Dumas (1993) focus on a risk that arises when an arbitrager facing trading costs[12] attempts to exploit rate differentials. In their work, the arbitrager looses if they face trading costs and if the market becomes 'efficient' too quickly, that is if the accumulated rate differential does not compensate for the trading cost. It is no accident that Delgado and Dumas (1993) consider rate arbitrage and trading costs while this paper considers price arbitrage and holding costs. As pointed out in the introduction, the problem of price arbitrage and trading costs has an extremely simple solution. Similarly, the problem of rate arbitrage and holding costs has an extremely simple solution: when the rate differential exceeds the holding cost, take an infinite position. Otherwise, do nothing.

From a theoretical perspective, future research can most improve on the present work by allowing investors and noise traders to behave more rationally than they do here. For example, one might require that traders who buy bonds choose to buy those in the cheaper market while traders who buy sell bonds sell the very bond they own, cheaper or not.[13] In this scenario, mispricings are reduced by buyers as well as arbitragers and are, therefore likely to lower and more short lived than in the present model.

From an empirical perspective, the present model can be used by investigators to frame hypotheses about the time series behavior of mispricing processes and about differences in mispricing processes across markets that differ in structural parameters such as size, level of holding costs, etc.

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[12]In the case of rate arbitrage, trading costs are the costs of moving capital into a different market.

[13]See Vayanos and Vila (1993) for a similar situation in the case of trading costs.◆

Appendix

This appendix derives the demand curve for a bond issue when investors face different taxes. As in the text, assume that the bond matures n periods from now and entitles holders to \$d at the end of each period before maturity and to \$1+d at maturity. Coupon income is taxed at a rate β, the after-tax discount rate is constant at r, and investors assume that they will hold the bond until maturity.[14]

Under these assumptions, the value of the bond to an investor with tax rate β is

$$V_n(\beta) = \frac{d(1-\beta)}{r}\left[1 - \frac{1}{(1+r)^n}\right] + \frac{1}{(1+r)^n} = V_n(0) - \frac{\beta d}{r}\left[1 - \frac{1}{(1+r)^n}\right]. \qquad \text{(A1)}$$

For a given bond price, P_n, an investor with tax rate β will be willing to buy bonds if $P_n \le V_n(\beta)$. When n=0, this condition is $P_n \le 1$. For $n \ge 1$, solving (A1) for β shows than investors with tax rate β will be willing to buy bonds so long as

$$\beta \le \frac{r}{d}\,\frac{V_n(0) - P_n}{1 - \frac{1}{(1+r)^n}}. \qquad \text{(A2)}$$

To derive a demand curve, assume that i) each investor buys at most 1 bond and ii) the number of investors with tax rates below some β is given by $\alpha\beta$.

From these assumptions and (A2), the demand function, $D_n(P_n)$, can be written as

$$D_n(P_n) = \frac{\alpha r}{d}\,\frac{V_n(0) - P_n}{1 - \frac{1}{(1+r)^n}}. \qquad (3)$$

Defining $\theta_n = (d/\alpha r)(1-1/[1+r]^n)$ and $\eta_n = V_n(0)\theta_n$ gives the demand function reported in the text.

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[14]This assumes that investors are myopic in the following sense. When deciding to buy or sell a bond, they compare the market price to their own valuation under a buy and hold strategy. But this strategy is not necessarily optimal, since investors may prefer to delay a sale in the expectation that prices will rise. While one might be tempted to think of the decision to sell in terms of exercising an option, the analogy is misleading: an investor who sells, i.e. exercises, can repurchase the bond later and sell yet again. In fact, in a related context, Tuckman and Vila (1992b) show that the myopic strategy is sometimes optimal. In any case, careful modeling of investor decisions in the context of the present model will be left as a subject for future research.

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Table 1

Table 1: The effects of maturity on the reduction of relative mispricing due to arbitrage activity. All bonds were issued with 25 years to maturity, at which time there was no relative mispricing.

Panel I $\rho = 2$

Years to maturity	Expected absolute mispricing without arbitragers	Expected absolute mispricing with arbitragers	Percentage reduction because of arbitragers
1	.0034	.0015	57%
5	.0138	.0078	43%
10	.0217	.0145	33%
15	.0261	.0193	26%
20	.0286	.0221	23%

Panel II $\rho = 4$

Years to maturity	Expected absolute mispricing without arbitragers	Expected absolute mispricing with arbitragers	Percentage reduction because of arbitragers
1	.0024	.0008	65%
5	.0098	.0018	85%
10	.0153	.0086	44%
15	.0185	.0139	25%
20	.0202	.0171	15%

Table 2

Table 2: Average mean reversion and conditional volatility for different values of ρ. Issued 3 years ago with no relative mispricing, the bonds now have 2 years left to maturity. The conditional volatility of the liquidity imbalance is given as a percentage of the market size and $\sigma / \alpha = 8\%$.

ρ	ρ'	σ'/α
0	.84	1.76%
1	3.06	2.64%
2	5.19	2.18%
4	9.83	1.44%

Table 3

Table 3: Average mean reversion and conditional volatility for different values of σ. Issued 3 years ago with no realtive mispricing, the bonds now have 2 years left to maturity. The conditional volatility of the liquidity imbalance is given as a percentage of the market size and $\rho = 2$.

σ/α	ρ'	σ'/α	$(\sigma-\sigma')/\sigma$
1%	2.01	.97%	2.5%
4%	5.40	1.83%	54%
8%	5.19	2.18%	72.8%
10%	5.15	2.37%	76.3%

Table 4

Table 4: The interaction of arbitrage activity and maturity on the average mean reversion of mispricing changes. All bonds were issued with 25 years to maturity, at which time there was no relative mispricing.

Years to maturity	ρ'
1	5.51
5	5.31
10	3.00
15	2.68
20	2.53

Table 5

Table 5: Aggregate arbitrage profits and arbitrage profits per arbitrager as a function of the aggregate risk tolerance a proxy for the number of arbitragers.

Aggregate risk tolerance	Aggregate arbitrage profits	Profits per arbitrager
.001	0	0
100	239	2.39
10,000	123,802	12.38
100,000	880,832	8.81
500,000	1,590,584	3.18
1 million	1,098,121	1.10
1 billion	348,75	.00
10 trillion	.02	.00

Table 6

Table 6: Aggregate arbitrage profits as a function of the holding cost c

Holding cost	Aggregate arbitrage profits
.1%	28,958
.3%	546,882
.5%	1,098,122
.8%	1,410,804
1%	1,359,353
3%	266,183
5%	39,140
8%	1,718
10%	173

Table 7

Arbitrage profit per arbitragers when arbitragers have different holding costs. For this table the risk tolerance of the larger group is 999,000 while the risk tolerance of the smaller group is 1,000. The holding cost of the larger group is fixed at .5%.

Holding cost of the smaller group	Arbitrage profits per arbitrager in the larger group	Arbitrage profits per arbitrager in the smaller group
.4%	.5778	2.63
.5%	1.0985	1.08
.6%	1.1034	.71

Figure 1

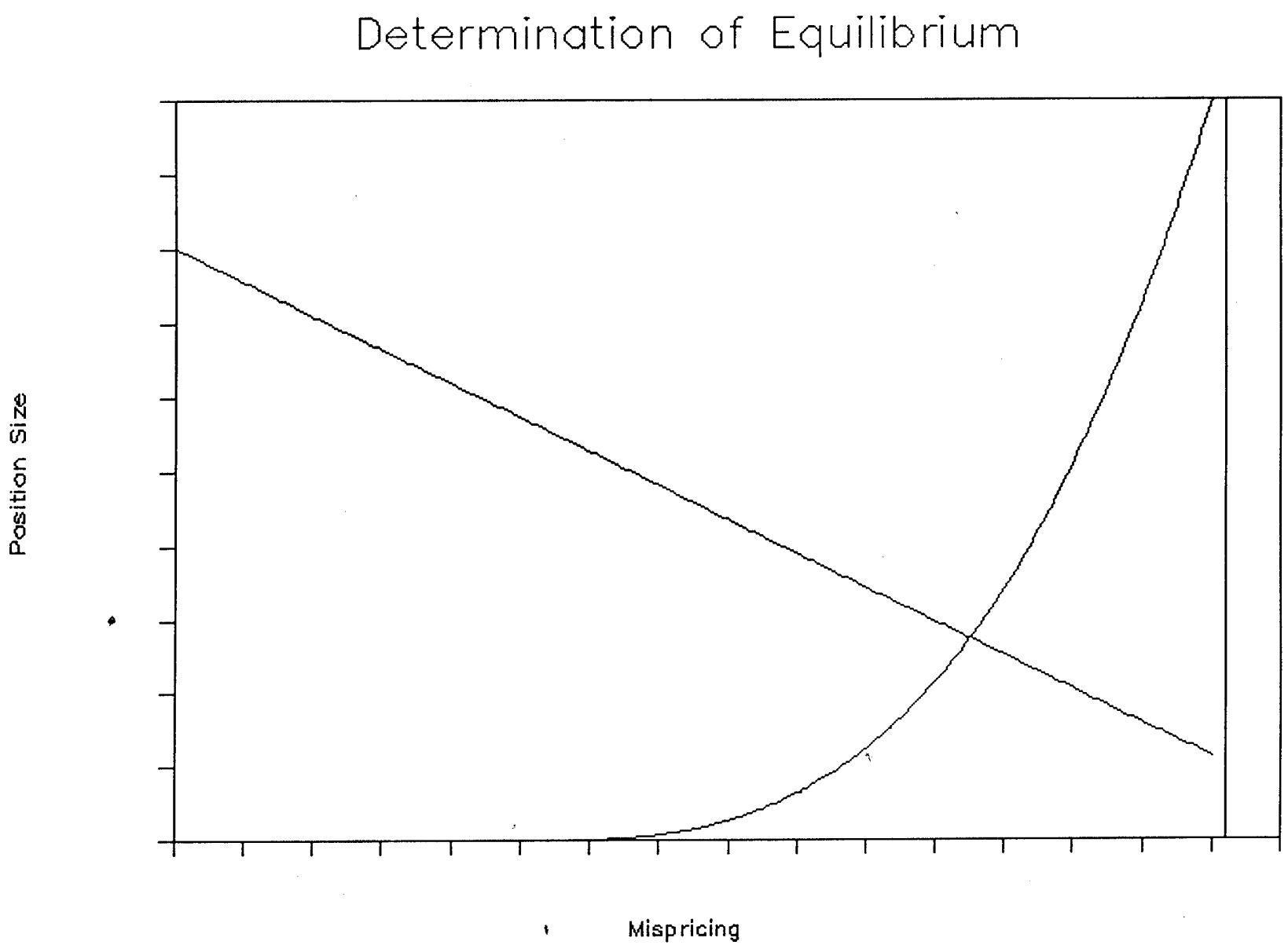

Figure 2a

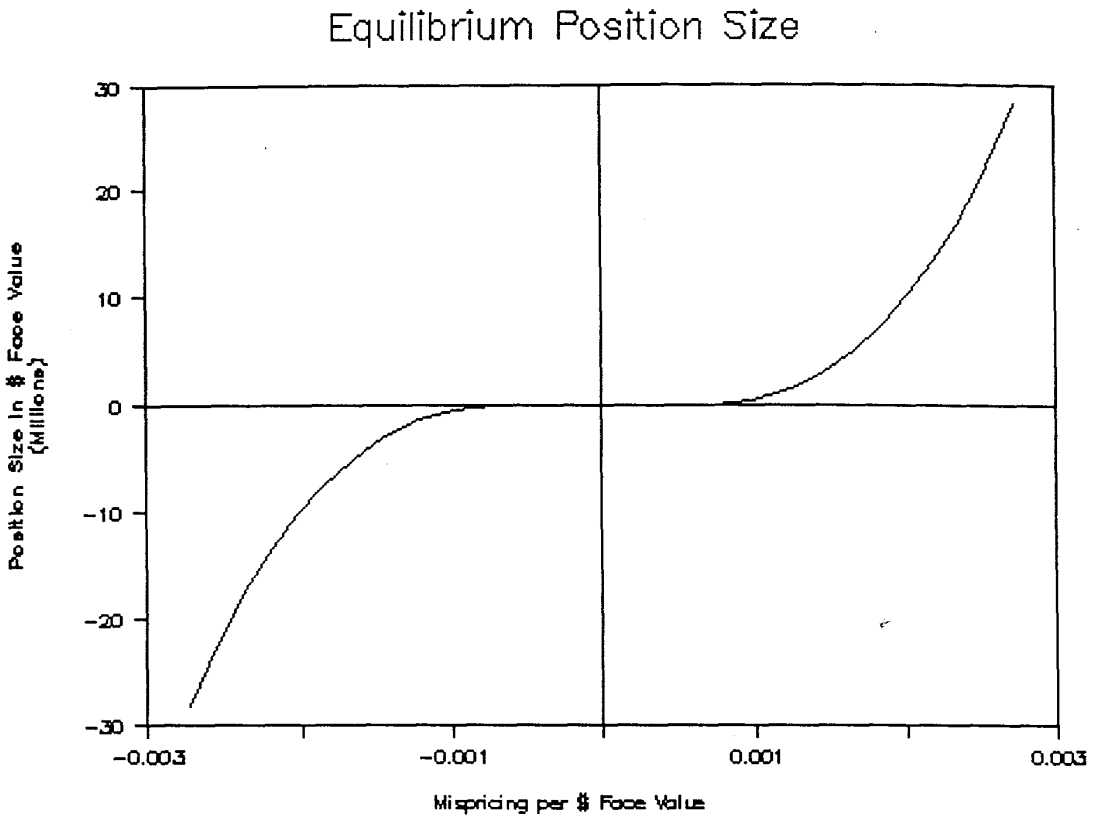

Figure 2b

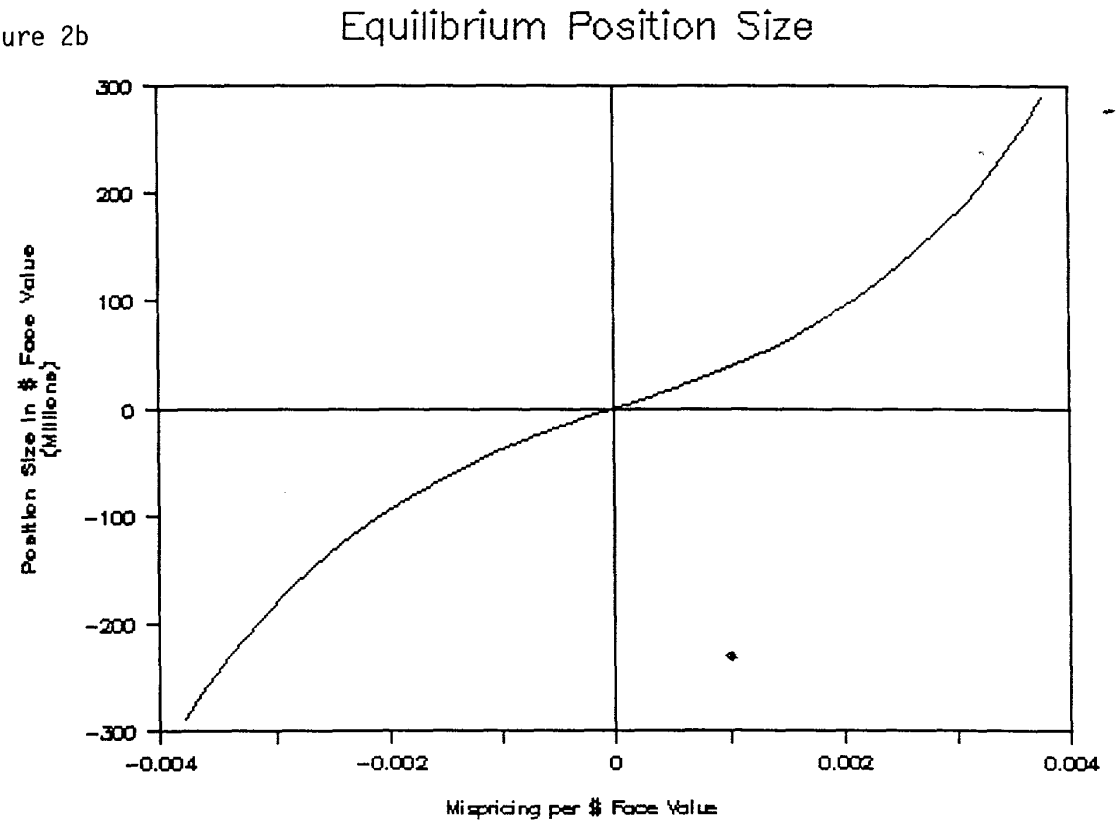

Figure 3a

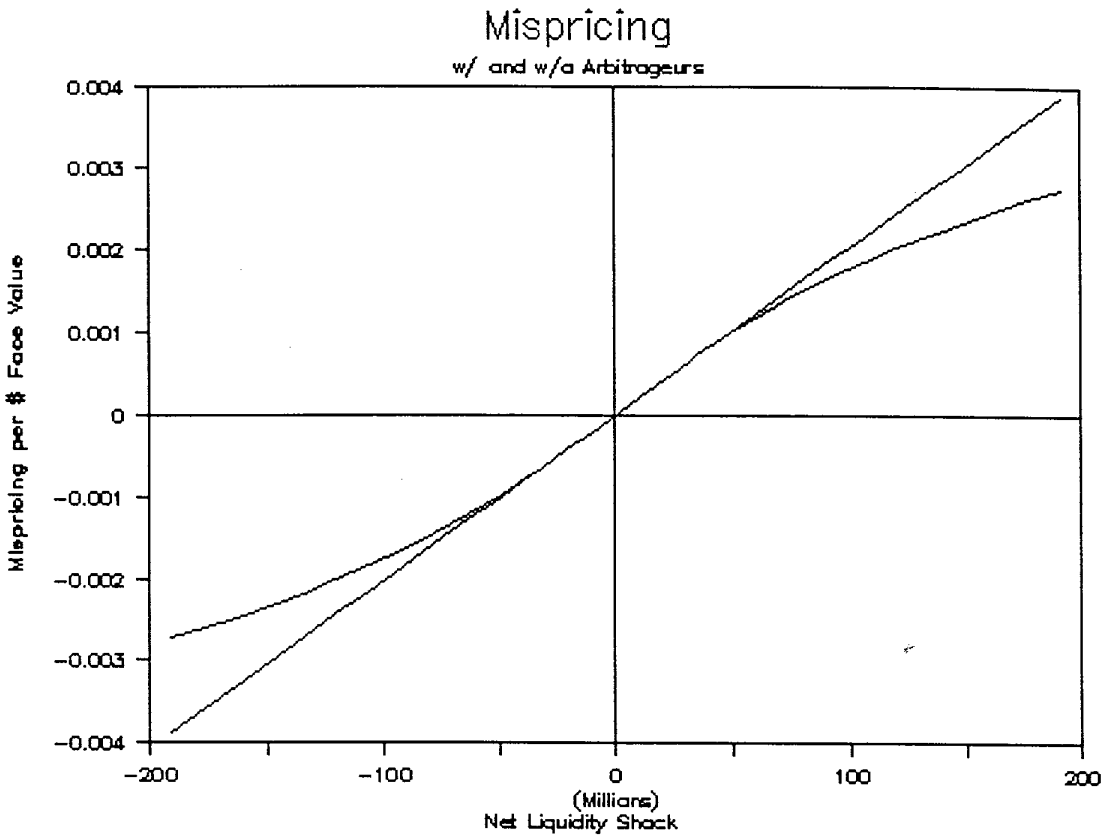

Figure 3b

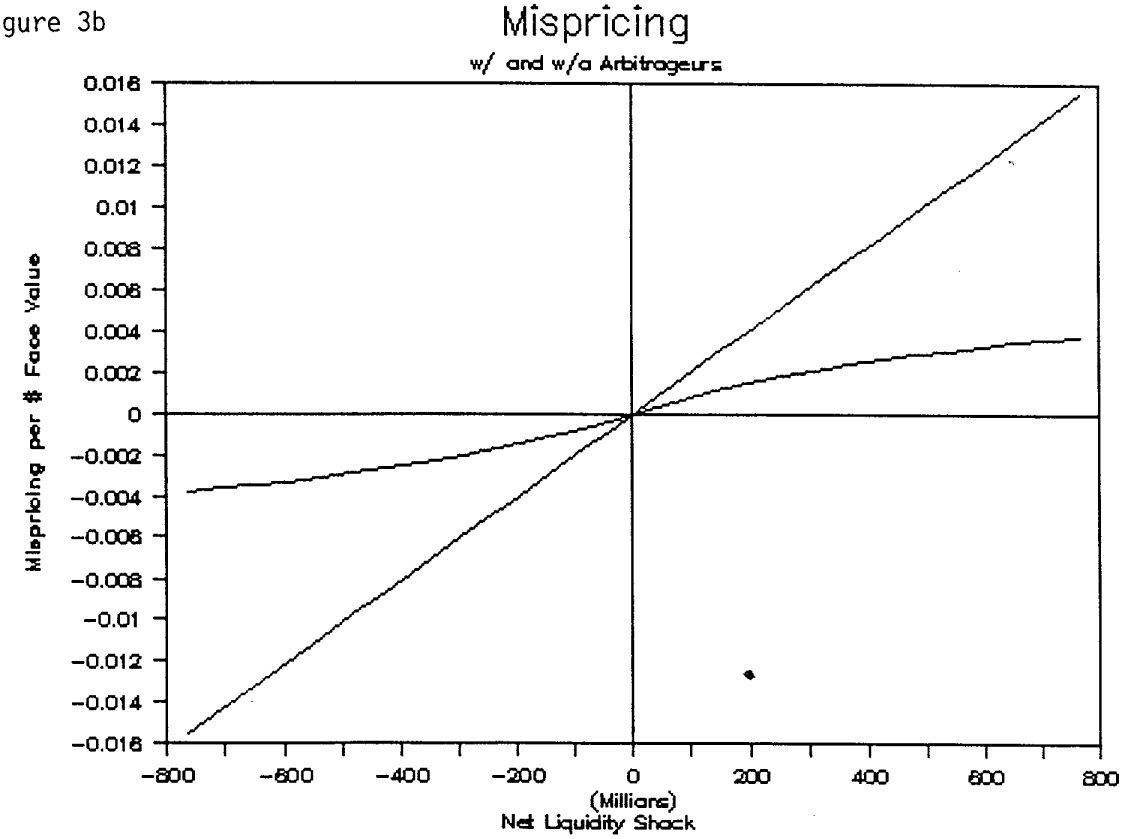

Figure 4a

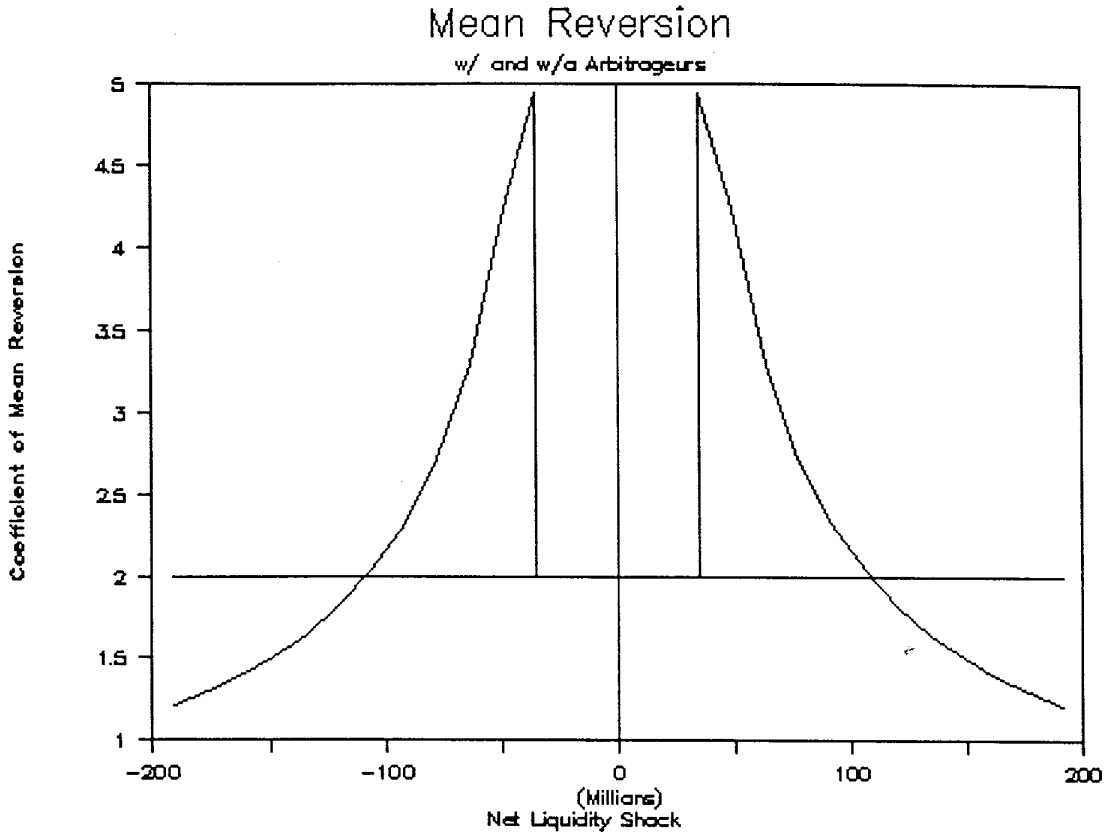

Figure 4b

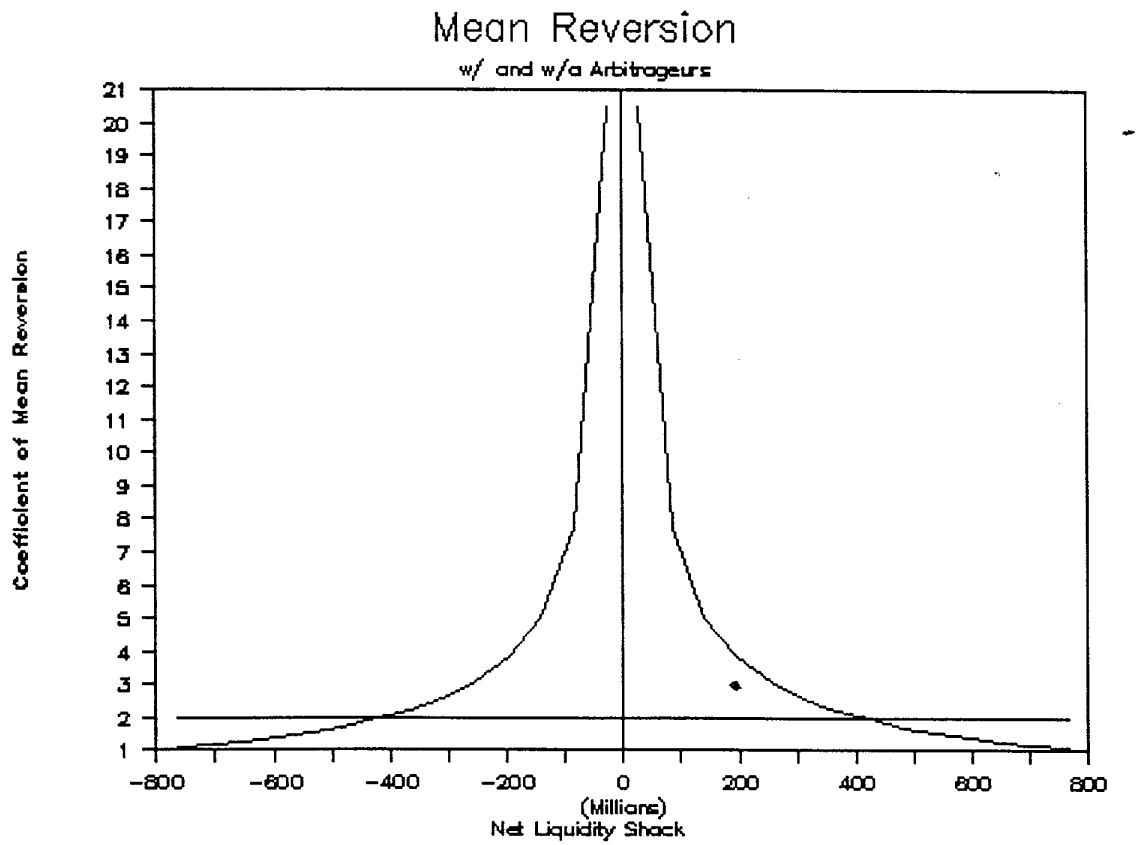

Figure 5a

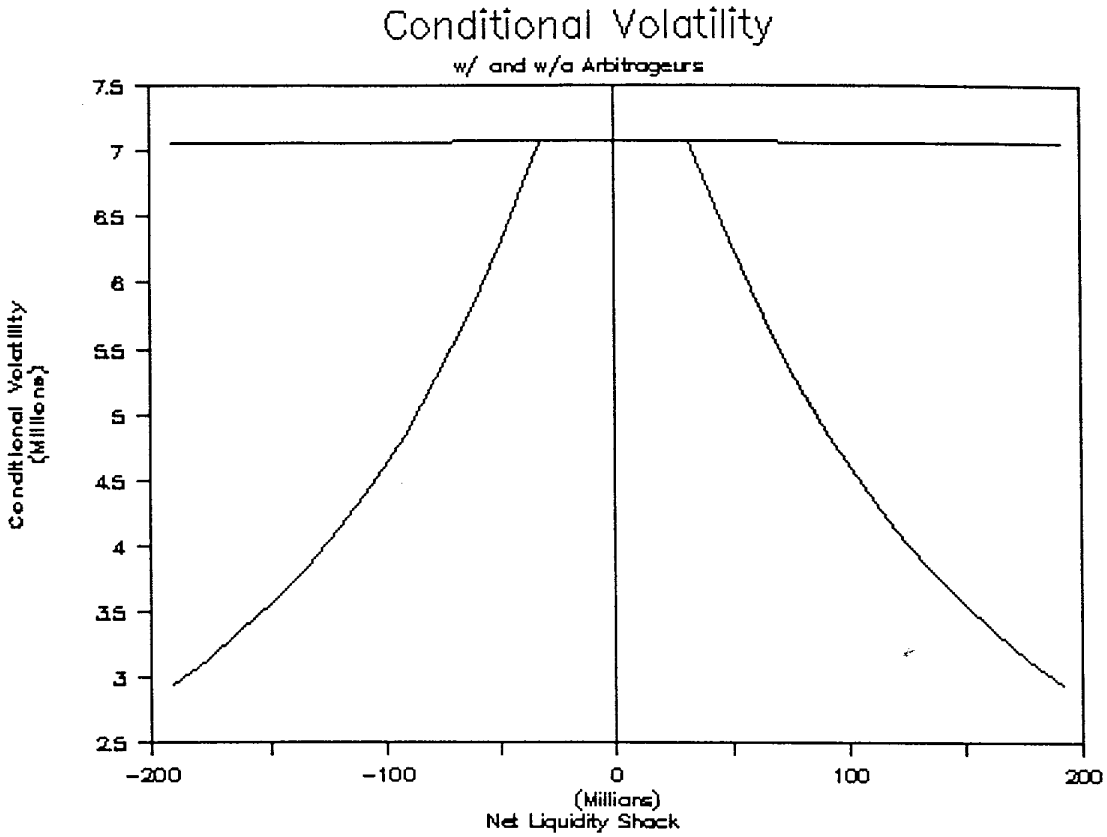
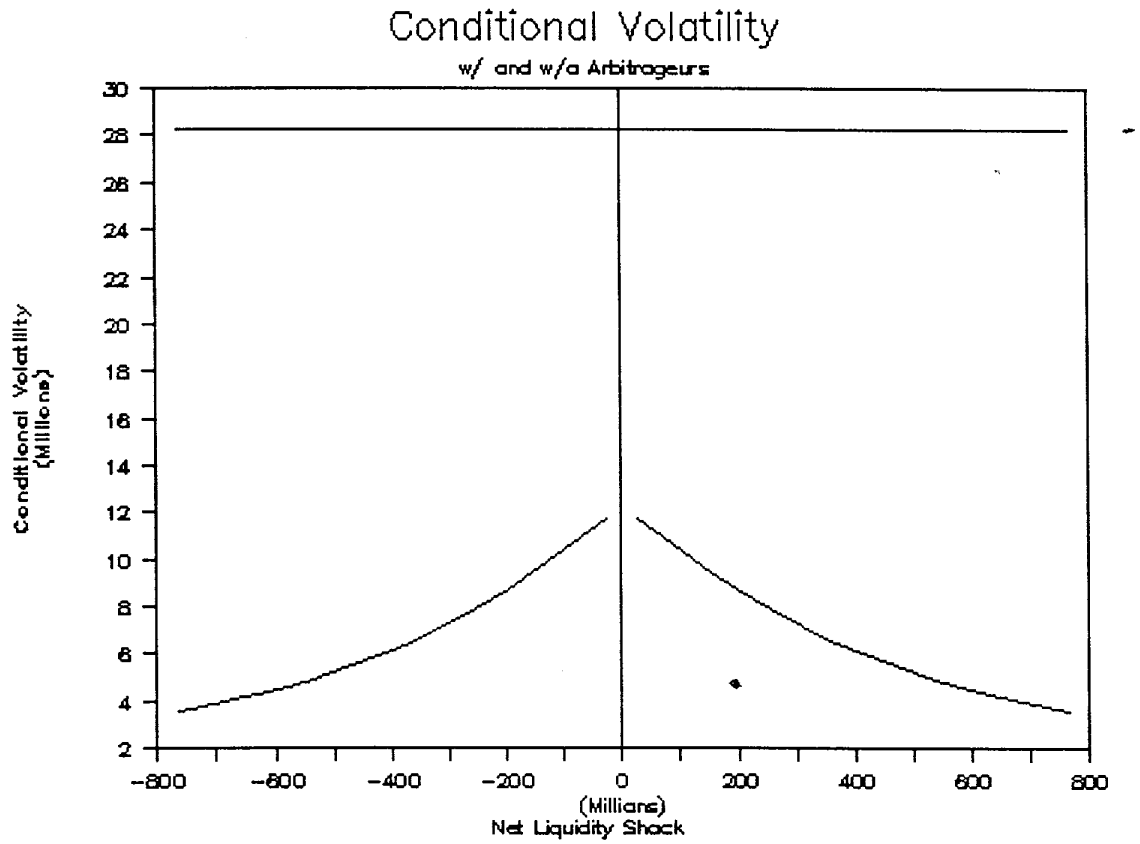
Figure 5b