A Credit Mechanism for Selecting a Unique Competitive Equilibrium*

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Abstract

This paper considers a credit mechanism for selecting a unique competitive equilibrium (CE). It is shown that in general there exists a "price-normalizing" bundle, with which the enlargement of the general-equilibrium structure to allow for default subject to appropriate penalties results in a construction of a simple credit mechanism for a credit using society to select a unique CE. With some additional conditions, there exists a common price-normalizing bundle with which any CE can be a unique selection for the credit mechanism with appropriate default penalties. The selection can be utilized to select a CE that minimizes the need for money or credit in trade.

KEYWORDS: Competitive equilibrium, credit mechanism, marginal utility of income, IOU, default penalty, welfare economics. (JEL Classification D5, C72, E4)

1 Introduction

The problem of finding the most general conditions required to guarantee a unique CE in a general-equilibrium system is complex and challenging mathematically. By enlarging the problem an approach is proposed that both offers a solution and facilitates an interesting selection.

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The general-equilibrium model does not utilize credit or financial institutions because trust is implicitly perfect. All trade is balanced at the end of the market. It is as if each individual at the start of the model has available implicitly a credit line equal to the worth of the individual's initial wealth at the final market price. When trust is imperfect and credit is introduced, a mechanism is needed to determine the worth to an individual at the end of trade. This includes the possibility of having credit left over and the cost or penalty for ending up in debt.

We consider a simple credit mechanism for a single period exchange economy, under which ending up with net credit is worthless while a penalty is levied against those who end as net debtors.[1] Consider, for specificity, that trade is in banknotes which are provided by a mutual bank. Before trading begins, traders exchange personal IOUs for banknotes with the bank charging them an interest rate of zero. Each trader may exchange personal IOUs for banknotes for up to a certain exogenously specified total amount, which consists of a credit line the bank provides to him. After all traders have received their incomes, they go to the bank to settle up all outstanding credit.[2]

We begin investigation of the credit mechanism with a useful property of general-equilibrium analysis. Namely, under some conditions, a CE for an economy corresponds to a saddle-point of a Lagrangian function for each trader. This saddle-point characterization of the CEs has useful applications for the choice of price normalization and the design of default penalties towards the selection of a unique CE, as well as for the study of the already familiar welfare properties of CE allocations.

The major part of our credit mechanism is the specification of credit lines and default penalties levied against the traders for ending as net debtors. Given prices, whether it is optimal for a trader to over spend depends on both the credit line provided to him and the size of the default penalty on him relative to his marginal utility of income. In a general-equilibrium model, a trader's marginal utility of income equals the Lagrangian multiplier associated with the trader's utility maximization problem; hence, it depends endogenously on the prices. In particular,

[1]For example, default penalties may be in the form of asset confiscation from the debtors or jail sentences or other societal punishments.

[2]One way to play the model in a classroom is at the beginning to give each student a large stack of banknotes and inform him that at the end of the game, after he has bought and received income from selling, he has to return exactly the amount he started with initially or he will have to pay a default penalty.

For discussions on various credit mechanisms for the competitive model, the reader is referred to Shubik (1999).

scaling up all the prices by some amount scales down the trader's marginal utility of income by exactly the same amount. It follows that the marginal utility of income of a trader enjoys a degree of freedom unless prices are normalized. We are interested in price normalization that calls for an equal value of some commodity bundle with a positive quantity of each good, which is referred to as a *price-normalizing bundle*. (When the total endowment bundle of the economy is used as the price-normalizing bundle, all normalized price systems yield the same value of the economy's total wealth.)

We show, by a process of iterated price normalization, that in general there exists a price-normalizing bundle, under which the enlargement of the general-equilibrium structure to allow for default subject to appropriate penalties results in a construction of a simple credit mechanism that selects a unique CE. Furthermore, with some additional conditions, there exists a common price-normalizing bundle with which any CE can be a unique selection for the credit mechanism with appropriate default penalties. This will include a CE with the "minimal cash flow" property. Such CEs are special for the reason that they minimize the need for a *substitute-for-trust* (i.e. money) in trade.

The rest of the paper is organized as follows. The next section discusses saddle-point characterization of CEs of an economy and its applications. Section 3 presents the main results. Section 4 concludes the paper.

2 Saddle-Point Characterization of Competitive Equilibria

Consider an exchange economy $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ with trader i's consumption set X^i, utility function u^i, and endowment a^i. We assume *A1:* $X^i = \Re^m_+$; *A2:* u^i *is continuous and concave*; *A3:* $a^i \in \Re^m_+$ *with* $a^i \neq 0$; and *A4: For each* $1 \leq h \leq m$, *there is a trader* i *such that* $u^i(x^i + \delta e^h) > u^i(x^i)$ *for all* $x^i \in \Re^m_+$ *and for all* $\delta > 0$, *where* $e^h \in \Re^m$ *with* $e^h_h = 1$ *and* $e^h_k = 0$ *for all* $k \neq h$.[3] These are familiar assumptions in competitive equilibrium analysis.

A CE for economy $\mathcal{E}$ is a pair $(\bar{x}, \bar{p})$ with allocation $\bar{x} = (\bar{x}^1, \cdots, \bar{x}^n)$ and price

[3] For any positive integer q, $\Re^q_+$ denotes the non-negative orthant of the q-dimensional Euclidean space and $\Re^q_{++}$ denotes the subset of $\Re^q_+$ containing vectors in $\Re^q_+$ all with positive components.

vector $\bar{p}$ such that $\bar{x}^i$ solves

$$\begin{array}{ll} \text{maximize} & u^i(x^i) \\ \text{subject to} & (i)\ \bar{p}\cdot(a^i - x^i) \geq 0 \\ & (ii)\ x^i \in \Re^m_+ \end{array} \tag{1}$$

for all i and

$$\sum_{i=1}^{n} \bar{x}^i = \sum_{i=1}^{n} a^i. \tag{2}$$

Condition (1) implies that $\bar{x}^i$ is optimal, in the sense that it generates the highest achievable utility level to consumer i, assuming that he chooses a bundle subject to the budget constraint determined by the price vector $\bar{p}$ and his endowment. Condition (2) implies that markets are cleared, in the sense that total consumption of each good equals total endowment of the good.

2.1 *A Saddle Point Theorem in Nonlinear Programming*

Recall that a nonlinear programming problem (NP) involves maximizing an objective function $f : D \subseteq \Re^L \longrightarrow \Re$ subject to constraints of $g(d) \geq 0$, $d \in D$, where $g : D \longrightarrow \Re^K$ is a R^K-valued function on D. The constraint satisfies *the Slater condition* if $g(d) \in \Re^K_{++}$ for some $d \in D$.

The *Saddle-Point Theorem* states that if (i) D is convex, (ii) f and g are concave, and (iii) the Slater condition is satisfied, then $\bar{d}$ solves (NP) if and only if there exists a vector $\bar{\lambda} \in \Re^K_+$ such that the pair $(\bar{d}, \bar{\lambda})$ satisfies the following *saddle-point conditions:*

$$\mathcal{L}(d, \bar{\lambda}) \leq \mathcal{L}(\bar{d}, \bar{\lambda}) \leq \mathcal{L}(\bar{d}, \lambda)$$

for all $d \in D$ and for all $\lambda \in \Re^K_+$. Here $\mathcal{L}(d, \lambda) = f(d) + \lambda \cdot g(d)$ is the Lagrangian function for the problem (see, e.g., Uzawa 1958). The pair $(\bar{d}, \bar{\lambda})$ is known as a *saddle point* of Lagrangian $\mathcal{L}$ and $\bar{\lambda}$ is known as a *Lagragian multiplier.*

When f and g are concave and continuously differentiable, the above saddle-point conditions are equivalent to the *Kuhn-Tucker conditions* : $\partial\mathcal{L}(\bar{d}, \bar{\lambda})/\partial d_l \leq 0$ and $\bar{d}_l \partial\mathcal{L}(\bar{d}, \bar{\lambda})/\partial d_l = 0$, for $l = 1, 2, \cdots, L$; $\partial\mathcal{L}(\bar{d}, \bar{\lambda})/\partial\lambda_k \geq 0$ and $\bar{\lambda}_k \partial\mathcal{L}(\bar{d}, \bar{\lambda})/\partial\lambda_k = 0$, for $k = 1, 2, \cdots, K$.

2.2 *A Saddle Point Characterization*

To apply the Saddle-Point Theorem, notice first that due to A4, all prices are positive in a CE. The saddle-point characterization of CEs in Theorem 1 below may be already well-known to many economists. We provide a proof for self-completeness.

Theorem 1 (Saddle-Point Characterization) *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ be an exchange economy satisfying A1-A4. Then, a pair $(\bar{x}, \bar{p}) \in \Re_+^{mn} \times \Re_{++}^m$ is a CE if and only if $\bar{x}$ satisfies the market clearance condition (2) and there exists a vector $\bar{\lambda} \in \Re_{++}^m$ such that for all i the triplet $(\bar{x}^i, \bar{p}, \bar{\lambda}^i)$ satisfies*

$$\begin{array}{l} u^i(x^i) + \bar{\lambda}^i \bar{p} \cdot (a^i - x^i) \\ \leq \\ u^i(\bar{x}) + \bar{\lambda}^i \bar{p} \cdot (a^i - \bar{x}^i) \\ \leq \\ u^i(\bar{x}^i) + \lambda^i \bar{p} \cdot (a^i - \bar{x}^i) \end{array} \tag{3}$$

for all $x^i \in \Re_+^m$ and for all $\lambda^i \in \Re_+$. The latter condition is equivalent to $(\bar{x}^i, \bar{\lambda}^i)$ being a saddle-point for the Lagrangian of problem (1).

Proof. Let $(\bar{x}, \bar{p})$ be a CE. Then, for all i, $\bar{x}^i$ solves (1). Since $\bar{p} \in \Re_{++}^m$, A3 implies $\bar{p} \cdot a^i > 0$. Hence, trader i's utility maximization problem (1) satisfies the Slater condition. Consequently, by the Saddle-Point Theorem, there exists a Lagrangian multiplier $\bar{\lambda}^i \geq 0$ such that $(\bar{x}^i, \bar{p}, \bar{\lambda}^i)$ satisfies (3) for all i. If $\bar{\lambda}^i = 0$, then the first inequality in (3) implies $u^i(x^i) \leq u^i(\bar{x}^i)$ for all $x^i \in \Re_+^m$. This contradicts A2. Hence $\bar{\lambda}^i > 0$. That $\bar{x}$ satisfies condition (2) follows directly from the market clearance condition of competitive equilibrium allocations.

Conversely, let $(\bar{x}, \bar{p}) \in \Re_+^{mn} \times \Re_{++}^m$ be a pair such that $\bar{x}$ satisfies (2) and there exists a vector $\bar{\lambda} \in \Re_{++}^m$ such that $(\bar{x}^i, \bar{p}, \bar{\lambda}^i)$ satisfies (3). As before, trader i's utility maximization problem (1) satisfies the Slater condition. It follows from the Saddle-Point Theorem that (3) implies that $\bar{x}^i$ solves utility maximization problem (1). This together with market clearance condition (2) implies that $(\bar{x}, \bar{p})$ is a competitive equilibrium. ■

When a triplet $(\bar{x}, \bar{p}, \bar{\lambda})$ satisfies (2) and (3), we call it *a competitive triplet* and we call $\bar{x}$, $\bar{p}$, and $\bar{\lambda}$, respectively, a *competitive allocation*, a *competitive price vector*, and a *competitive multiplier vector*. We call Lagrangian multipliers in a competitive multiplier vector *competitive multipliers*.

Two applications of Theorem 1 are relevant to the rest of the paper. Corollary 1 below shows that a competitive allocation maximizes a weighted welfare function with the welfare weights equal to the reciprocals of the associated competitive multipliers. Corollary 2 shows that under some additional conditions, there is a one-to-one correspondence between competitive equilibria and competitive multiplier vectors. Corollary 1 follows easily from Theorem 1.

Corollary 1 *Assume $\mathcal{E}$ satisfies A1-A4. If $(\bar{x}, \bar{p}, \bar{\lambda}) \in \Re^{mn}_{+} \times \Re^{m}_{++} \times \Re^{n}_{++}$ is a competitive triplet for $\mathcal{E}$, then $\bar{x}$ solves the weighted welfare maximization problem:*

$$\begin{array}{rl} \textit{maximize} & \sum_{i=1}^{n} \frac{1}{\lambda^i} u^i(x^i) \\ \textit{subject to} & (i) \sum_{i=1}^{n}(a^i - x^i) \geq 0 \\ & (ii)\ x^i \in \Re^m_+, i = 1, 2, \cdots, n. \end{array} \tag{4}$$

The first fundamental theorem of welfare economics follows directly from Corollary 1.

Corollary 2 *Assume $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ satisfies A1 and A4. Assume further*

A2′: u^i is continuously differentiable and strictly concave;

A5: CE allocations are all interior allocations.[4]

Then, there is a one-to-one correspondence between competitive equilibria and competitive multiplier vectors.

Proof. Let $(\bar{x}, \bar{p})$ be a CE. Then, by A5, $\bar{x}^i \in \Re^m_{++}$ for all i. Consequently, by A2′ and by the Kuhn-Tucker conditions, competitive multiplier vectors $\bar{\lambda} \in \Re^n_+$ that correspond to $(\bar{x}, \bar{p})$ all satisfy [5]

$$\bigtriangledown u^i(\bar{x}^i) = \bar{\lambda}^i \bar{p}, \ \ i = 1, 2, \cdots, n.$$

The uniqueness of competitive multiplier vector corresponding to $(\bar{x}, \bar{p})$ follows from the above equation.

With strict concavity of the utility functions, it follows from Corollary 1 that, corresponding to a competitive multiplier vector $\bar{\lambda}$, there is a unique competitive allocation $\bar{x}$. Since, by A5, $\bar{x}^i \in \Re^m_{++}$ for all i and since prices are Lagrangian

[4] A sufficient condition to guarantee the interiority of the CE allocations is for all i, $u^i(x^i) > u^i(y^i)$ whenever x^i is an interior bundle and y^i is a corner bundle. The reason is that all CE prices are strictly positive under A4 and hence the value of each trader's endowment at these prices are positive.

[5] Here $\bigtriangledown u^i(\bar{x}^i)$ denotes the gradient of u^i at x^i.

multipliers for the welfare maximization problem (4), it follows from the Kuhn-Tucker conditions that competitive price vectors that correspond to $\bar{\lambda}$ all satisfy

$$\bar{p} = \sum_{i=1}^{n} \bar{\lambda}^i \bigtriangledown u^i(\bar{x}^i).$$

The uniqueness of the competitive price vector corresponding to $\bar{\lambda}$ follows from the above equation. ■

The one-to-one correspondence in Corollary 2 implies that the products $\bar{\lambda}^i \bar{p}$, $i = 1, 2, \cdots, n$, of the price vector and the Lagrangian multipliers are uniquely determined in every CE under the conditions in the corollary.

3 Price Normalization and Selection of CEs

Competitive equilibrium prices are homogeneous of degree zero. Consequently, prices can be normalized without changing competitive allocations. When normalizing the prices, however, economists rarely consider the effects of price normalization on competitive multipliers, hence on marginal utilities of income of the traders.

3.1 *A Credit Mechanism*

Competitive multipliers are known as marginal utilities of income. By Corollary 2, normalizing the prices would change these marginal utilities accordingly. To consider the possibility of selecting a unique CE using a credit mechanism, the sizes of the per unit default penalties relative to the marginal utilities of income at CEs turn out to be essential.

Consider now a credit mechanism under which trade is in banknotes: traders use banknotes to buy or sell goods. A mutual bank provides all traders with banknotes with zero interest. Before trading begins, trader i exchanges personal IOUs for banknotes with the bank. Let $C^i_{m+1} \subseteq \Re_+$ denote the set of quantities of the banknotes that trader i can obtain with his personal IOUs. This interval may be considered as the zero-interest credit line the bank offers to trader i. After he has bought and received income from selling, trader i settles up all outstanding credit with the bank. It is of no value to him to end as a net creditor, while he will be penalized for ending as a net debtor.

Let $P \subseteq \Re^m_+$ denote the compact set of normalized price vectors. Define value V_P by

$$V_P = \max\{p \cdot \sum_{i=1}^{n} a^i \mid p \in P\}.$$

Now let $\bar{p} \in P$ be any competitive price vector. By A4, $\bar{p} \in \Re^m_{++}$. Since $a^i \in \Re^m_+$ and $a^i \neq 0$ by A1 and A3, we have $\bar{p} \cdot a^i < \bar{p} \cdot \sum_{j=1}^{n} a^j$. This inequality together with the above definition of V_P implies that $\bar{p} \cdot a^i < V_P$ for all i.

Set $C^i_{m+1} = [0, V_P]$ for all i. This means that each trader i can obtain credit more than he could repay with the worth of his endowment at any normalized price vector in P. However, trader i pays a penalty of $\mu^i > 0$ for each unit of his debt he is unable to repay . Let $\mu = (\mu^i)$ denote the vector of these per unit penalties.

The credit mechanism transforms economy $\mathcal{E}$ into $\mathcal{E}_\mu$ in which trader i has utility function

$$u^i(x^i) + \mu^i \min[x^i_{m+1}, 0],$$

where x^i_{m+1} denotes the amount of excess credit left over (the difference between the worth of i's endowment and the expenditure on bundle x^i). Hence, x^i_{m+1} denotes the amount of debt he is unable to repay when $x^i_{m+1} < 0$ or credit left over when $x^i_{m+1} > 0$.

Definition 1 *The credit mechanism selects a CE, $(\hat{x}, \hat{p})$, of $\mathcal{E}$ if $(\hat{x}, \hat{x}_{m+1}, \hat{p})$ is a CE for $\mathcal{E}_\mu$ where $\hat{x}_{m+1} = (\hat{x}^1_{m+1}, \cdots, \hat{x}^n_{m+1})$ with $\hat{x}^i_{m+1} = 0$ for all i.*

The following theorem provides a connection of the CEs of $\mathcal{E}_\mu$ with those of $\mathcal{E}$.

Theorem 2 *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i \in N}$ be an exchange economy. Assume $\mathcal{E}$ satisfies A1-A4. Assume further prices are normalized so that the resulting normalized price vectors consist of the compact set P. If $((x^*, x^*_{m+1}), p^*)$ is a CE for $\mathcal{E}_\mu$, then $x^{*i}_{m+1} = 0$ for all i and (x^*, p^*, λ^*) is a competitive triplet for $\mathcal{E}$ for some competitive multiplier vector $\lambda^* \in \Re^n_{++}$ with $\lambda^* \leq \mu$.*

Proof. Let $((x^*, x^*_{m+1}), p^*)$ be a CE for $\mathcal{E}_\mu$. Since being a net creditor is worthless and since traders are price-taking, we conclude that for all $i \in N$, $x^{*i}_{m+1} \leq 0$. This implies that (x^{*i}, x^{*i}_{m+1}) solves

$$\begin{array}{ll} \text{Maximize} & u^i(x^i) + \mu^i x^i_{m+1} \\ \text{Subject to} & p^* \cdot a^i - p^* \cdot x^i \geq x^i_{m+1}, \\ & x^i \in \Re^m_+,\ p^* \cdot a^i - V_P \leq x^i_{m+1} \leq 0. \end{array}$$

By the Saddle-Point Theorem, there exists a number $\lambda^{*i} \geq 0$ such that the triplet $(x^{*i}, x^{*i}_{m+1}, \lambda^{*i})$ satisfies

$$\begin{aligned} & u^i(x^i) + \mu^i x^i_{m+1} + \lambda^{*i}(p^* \cdot a^i - p^* \cdot x^i - x^i_{m+1}) \\ & \leq \\ & u^i(x^{*i}) + \mu^i x^{*i}_{m+1} + \lambda^{*i}(p^* \cdot a^i - p^* \cdot x^{*i} - x^{*i}_{m+1}) \\ & \leq \\ & u^i(x^{*i}) + \mu^i x^{*i}_{m+1} + \lambda^i(p^* \cdot a^i - p^* \cdot x^{*i} - x^{*i}_{m+1}) \end{aligned} \tag{5}$$

for all $x^i \in \Re^m_+$, all $p^* \cdot a^i - V_P \leq x^i_{m+1} \leq 0$, and for all $\lambda^i \in \Re_+$.

The non-satiation of u^i together with the first inequality in (5) implies $\lambda^{*i} > 0$. This in turn with the second inequality in (5) implies

$$p^* \cdot x^{*i} + x^{*i}_{m+1} = p^* \cdot a^i. \tag{6}$$

Since $\sum_{i \in N} x^{*i} = \sum_{i \in N} a^i$ and since $x^{*i}_{m+1} \leq 0$ for all i, it follows from (6) that $x^{*i}_{m+1} = 0$ for all i.

Since $x^{*i}_{m+1} = 0$, by taking $x^i_{m+1} = 0$ it follows from (5) that $(x^{*i}, p^*, \lambda^{*i})$ satisfies (3). Thus, by the market clearance condition of $\sum_{i \in N} x^{*i} = \sum_{i \in N} a^i$, (x^*, p^*, λ^*) is a competitive triplet for $\mathcal{E}$. Next, by taking x^i to be x^{*i} in (5), the first inequality implies

$$(\mu^i - \lambda^{*i}) x^i_{m+1} \leq 0, \ \forall x^i_{m+1} \in [p^* \cdot a^i - V_P, 0].$$

Since $p^* \cdot a^i < V_P$, the condition $\lambda^{*i} \leq \mu^i$ follows from the above inequality. ■

When a competitive multiplier vector $\bar{\lambda}$ at a CE is such that $\bar{\lambda}_i > \mu_i$ for some i, the per unit penalty on trader i unable to repay his debt is not severe enough, in the sense that on the margin he gains from being in debt. When this occurs,the budget constraint will be violated. Since no one ends as a net creditor, the market for commodities will be imbalanced. Hence, such a CE cannot survive the credit mechanism. A direct application of Theorem 2 implies:

Corollary 3 *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i \in N}$ be an exchange economy. Assume $\mathcal{E}$ satisfies A1-A4. Assume further prices are normalized so that the resulting normalized price vectors consist of the compact set P. Then, only those CEs of $\mathcal{E}$ with multiplier vectors $\lambda \leq \mu$ are selected by the credit mechanism.*

In the sequel we consider price normalization of the form

$$P = \{p \in \Re^m_+ \mid p \cdot r \equiv \text{constant}\}$$

for some price-normalizing bundle $r \in \Re^m_{++}$. Without loss of generality, we may take the constant to be 1. Given a CE, Corollary 3 shows that a sufficient condition for the CE to be uniquely selected is that there exits a price-normalizing bundle, under which the competitive multiplier vector of the CE does not dominate the competitive multiplier vector of every other CE. With such a price-normalizing bundle, corollary 3 implies that the CE is a unique selection for the credit mechanism with a *non-discriminatory default penalty* equal to the maximum multiplier of the competitive multiplier vector.

3.2 *Price Normalization for Unique Selection of a CE*

In this subsection, we show that in general a price-normalizing bundle exists with which default penalties can be specified for the credit mechanism to select a unique CE. To this end, we confine attention to exchange economies with *finitely many* CEs. This is not too restrictive because the property of having finitely many CEs is *generic*.

3.2.1 *Selection with Heterogenous Price-Normalizing Bundles*

We will apply the following theorem of the alternative for matrices. This theorem was presented and proved as a lemma in Owen (1982).

Theorem 3 (Theorem of the Alternative for Matrices) *Let $A = (a_{ij})$ be a $m \times k$ matrix. Then, either (i) or (ii) must hold:*

(i) The origin $0 \in \Re^k$ is contained in the convex hull of the m column vectors

$$a_j = (a_{1j}, a_{2j}, \cdots, a_{mj})$$

of A and the m unit vectors $e_i \in \Re^m$ with

$$e_{ij} = \begin{cases} 1 & if\ i = j; \\ 0 & if\ i \neq j. \end{cases}$$

(ii) There exists a vector $r \in \Re^m_{++}$ such that $a_j \cdot r > 0$ for $j = 1, 2, \cdots, k$.

Proof. See Lemma II.4.3 in Owen (1982, pp. 17-8) for a proof. ■

Let $(\hat{x}, \hat{p})$ be a CE for economy $\mathcal{E}$. We now consider a process of iterated price normalization that eventually transforms the competitive multiplier vector associated with $(\hat{x}, \hat{p})$ into the smallest of all competitive multiplier vectors, provided that the price vector $\hat{p}$ is not equal to any non-negative linear combination of competitive price vectors of the other CEs.[6]

The iteration begins from the basic normalization given by the price-normalizing bundle $e = (1, 1, \cdots, 1) \in \Re^m$. The is the familiar price normalization that results in normalized price vectors that consists of the price simplex $\Delta^m = \{p \in \Re^m_+ | \sum_{h=1}^m p_h = 1\}$. Let the other CEs of $\mathcal{E}$ be indexed as $(\hat{x}(j), \hat{p}(j))$, $j = 1, 2, \cdots, k$.

Lemma 1 *Assume A1, A2', and A3-A5 are satisfied. Assume further* $\hat{p} \not= \sum_{j=1}^k \alpha_j \hat{p}(j)$ *for any* $\alpha_j \geq 0$ *for all* j. *Then,* $0 \in \Re^m$ *is not in the convex hull of vectors* $\{\frac{\hat{p}(j)}{\hat{p}(j)\cdot e} - \frac{\hat{p}}{\hat{p}\cdot e}\}_{j=1}^k \cup \{e_i\}_{i=1}^m$.

Proof. Suppose on the contrary that 0 is in the convex hull. Then, there exist non-negative weights $\alpha_1, \cdots, \alpha_k$ and $\beta_1, \beta_2, \cdots, \beta_m$ such that

$$\sum_{j=1}^k \alpha_j + \sum_{i=1}^m \beta_i = 1 \text{ and } \sum_{j=1}^k \alpha_j [\frac{\hat{p}(j)}{\hat{p}(j)\cdot e} - \frac{\hat{p}}{\hat{p}\cdot e}] + \sum_{i=1}^m \beta_i e_i = 0. \tag{7}$$

Taking the inner product with e on both sides of (7) yields $\sum_i \beta_i = 0$. We have $\beta_i = 0$ for all i. This shows $\hat{p} = \sum_j \alpha_j \frac{\hat{p}\cdot e}{\hat{p}(j)\cdot e} \hat{p}(j)$, which contradicts the assumption that $\hat{p}$ is not equal to any non-linear combination of price vectors $\hat{p}(j)$ for all j. ∎

Set

$$\hat{p}^1 = \frac{\hat{p}}{\hat{p}\cdot e} \text{ and } \hat{p}^1(j) = \frac{\hat{p}(j)}{\hat{p}(j)\cdot e}.$$

Lemma 1 and Theorem 3 together imply that under the assumptions in Lemma 1, there exists a strictly positive bundle in $\Re^m_{++}$ which we denote by r^1 such that

$$\hat{p}^1 \cdot r^1 < \hat{p}^1(j) \cdot r^1, \ j = 1, \cdots, k. \tag{8}$$

Taking r^1 to be a price-normalizing bundle, we can normalize $\hat{p}^1$ and $\hat{p}^1(j)$ to

$$\hat{p}^2 = \frac{\hat{p}^1}{\hat{p}^1 \cdot r^1} \text{ and } \hat{p}^2(j) = \frac{\hat{p}^1(j)}{\hat{p}^1(j)\cdot r^1}. \tag{9}$$

[6] This is equivalent to the condition that when normalized to be in the price simplex, $\frac{\hat{p}}{\hat{p}\cdot e}$ is not in the convex hull of $\frac{\hat{p}(j)}{\hat{p}(j)\cdot e}$, $j = 1, 2, \cdots, k$.

Lemma 2 *Assume conditions in Lemma 1 are satisfied. Then, there exists* $r^1 \in \Re^m_{++}$ *such that (8) is satisfied and* $0 \in \Re^m$ *is not in the convex hull of the vectors* $\{\frac{\hat{p}^1 \cdot r^1}{\hat{p}^1(j) \cdot r^1}\hat{p}^2(j) - \hat{p}^2\}_{j=1}^k \cup \{e_i\}_{i=1}^m$.

Proof. Suppose that Lemma 2 is not valid. Then, for any r satisfying (8), there exist non-negative weights $\alpha_1, \cdots, \alpha_k$ and $\beta_1, \beta_2, \cdots, \beta_m$ such that

$$\sum_{j=1}^k \alpha_j + \sum_{i=1}^m \beta_i = 1 \text{ and } \sum_{j=1}^k \alpha_j[\frac{\hat{p}^1 \cdot r^1}{\hat{p}^1(j) \cdot r^1}\hat{p}^2(j) - \hat{p}^2] + \sum_{i=1}^m \beta_i e_i = 0. \tag{10}$$

By (9), $\hat{p}^2 \cdot r^1 = 1$ and $\hat{p}^2(j) \cdot r^1 = 1$ for all j. By taking inner product with r^1 on both sides of the second equation in (10), we get

$$\sum_{j=1}^k \alpha_j[\frac{\hat{p}^1 \cdot r^1}{\hat{p}^1(j) \cdot r^1} - 1] + \sum_{i=1}^m \beta_i r_i^1 = 0. \tag{11}$$

Since scaling down r^1 does not change the ratios $\frac{\hat{p}^1 \cdot r^1}{\hat{p}(j) \cdot r^1}$ nor inequalities in (8), the weights α_j, β_i in (10) remain unchanged as r^1 is scaled down. However, as r^1 is scaled down, the second term on the left-hand-side of (11) approaches to zero while the first sum stays constant. By (8), $\frac{\hat{p}^1 \cdot r^1}{\hat{p}^1(j) \cdot r^1} - 1 < 0$ for all j and by (10), $\alpha_j > 0$ for at least one j. It follows that the first sum on the left-hand-side of (11) remains constant and negative as r^1 is scaled down. This establishes the desired contradiction. ■

By Theorem 3 and Lemma 2, there exists a strictly positive bundle $r^2 \in \Re^m_{++}$ such that

$$\hat{p}^2 \cdot r^2 < \frac{\hat{p}^1 \cdot r^1}{\hat{p}^1(j) \cdot r^1}\hat{p}^2(j) \cdot r^2, \; j = 1, \cdots, k. \tag{12}$$

Taking r^2 to be a normalizing bundle, $\hat{p}^2$ and $\hat{p}^2(j)$ can be normalized to

$$\hat{p}^3 = \frac{\hat{p}^2}{\hat{p}^2 \cdot r^2} \text{ and } \hat{p}^3(j) = \frac{\hat{p}^2(j)}{\hat{p}^2(j) \cdot r^2}. \tag{13}$$

By (8) and (12),

$$\frac{\hat{p}^2 \cdot r^2}{\hat{p}^2(j) \cdot r^2} < \frac{\hat{p}^1 \cdot r^1}{\hat{p}^1(j) \cdot r^1} < 1.$$

Hence, it follows from (8), (12), and (13) that the proof of Lemma 2 can be adapted to show the existence of a strictly positive bundle $r^3 \in \Re^m_{++}$ such that

$$\hat{p}^3 \cdot r^3 < \frac{\hat{p}^2 \cdot r^2}{\hat{p}^2(j) \cdot r^2} \hat{p}^3(j) \cdot r^3.$$

To iterate this process inductively, suppose that for $t \geq 2$, $\hat{p}^0, \hat{p}^1, \cdots, \hat{p}^{t-1}, \hat{p}^t$ and $\hat{p}^0(j), \hat{p}^1(j), \cdots, \hat{p}^{t-1}(j), \hat{p}^t(j)$ for all j have been determined such that for some strictly positive bundles $r^0, r^1, \cdots, r^t \in \Re^m_{++}$

$$\frac{\hat{p}^{\tau-1} \cdot r^{\tau-1}}{\hat{p}^{\tau-1}(j) \cdot r^{\tau-1}} < 1 \text{ and } \hat{p}^\tau \cdot r^\tau < \frac{\hat{p}^{\tau-1} \cdot r^{\tau-1}}{\hat{p}^{\tau-1}(j) \cdot r^{\tau-1}} \hat{p}^\tau(j) \cdot r^\tau, \; j = 1, \cdots, k, \tag{14}$$

where $\hat{p}^0 = \hat{p}, \hat{p}^0(j) = \hat{p}(j)$, and $r^0 = e$. Using r^t as a normalizing bundle, we can normalize $\hat{p}^t$ and $\hat{p}^t(j)$ to

$$\hat{p}^{t+1} = \frac{\hat{p}^t}{\hat{p}^t \cdot r^t} \text{ and } \hat{p}^{t+1}(j) = \frac{\hat{p}^t(j)}{\hat{p}^t(j) \cdot r^t}, \; j = 1, 2, \cdots, k. \tag{15}$$

By (14),

$$\frac{\hat{p}^t \cdot r^t}{\hat{p}^t(j) \cdot r^t} < 1$$

for all j. It follows that the proof of Lemma 2 can be adapted to prove Lemma 3 below. For this reason, the proof of Lemma 3 is omitted.

Lemma 3 *Assume conditions in Lemma 1 are satisfied. Assume further for $t \geq 2$, $\hat{p}^0, \hat{p}^1, \cdots, \hat{p}^{t-1}, \hat{p}^t$ and $\hat{p}^0(j), \hat{p}^1(j), \cdots, \hat{p}^{t-1}(j), \hat{p}^t(j)$ for all j have been determined such that (14) and (15) are satisfied. Then, there exists a bundle $r^t \in \Re^m_{++}$ such that (14) is satisfied and $0 \in \Re^m$ is not in the convex hull of the vectors $\{\frac{\hat{p}^t \cdot r^t}{\hat{p}^t(j) \cdot r^t} \hat{p}^{t+1}(j) - \hat{p}^{t+1}\}_{j=1}^k \cup \{e_i\}_{i=1}^m$.*

By Lemma 3 and Theorem 3, there exists a strictly positive bundle $r^{t+1} \in \Re^m_{++}$ such that

$$\hat{p}^{t+1} \cdot r^{t+1} < \frac{\hat{p}^t \cdot r^t}{\hat{p}^t(j) \cdot r^t} \hat{p}^{t+1}(j) \cdot r^{t+1}, \; j = 1, 2, \cdots, k.$$

Thus, by induction, the process can be iterated to any finite times. We now show that the process leads to the existence of a price-normalizing bundle $r \in \Re^m_{++}$ with which the competitive multiplier vector associated with $(\hat{x}, \hat{p})$ is the smallest.

Theorem 4 *Assume A1, A2', and A3-A5 are satisfied. Let* $(\hat{x}, \hat{p})$ *and* $(\hat{x}(j), \hat{p}(j))$*,* $j = 1, 2, \cdots, k$ *be the CEs for* $\mathcal{E}$*. If* $\hat{p} \neq \sum_{j=1}^{k} \alpha_j \hat{p}(j)$ *for any* $\alpha_j \geq 0$ *for all* j*, then there exists a price-normalizing bundle* $r \in \Re^m_{++}$ *with price normalization that results in the set of normalized price vectors*

$$P = \{p \in \Re^m_+ \mid p \cdot r = 1\},$$

the competitive multiplier vector associated with $(\hat{x}, \frac{\hat{p}}{\hat{p} \cdot r})$ *is strictly dominated by the competitive multiplier vector associated with* $(\hat{x}(j), \frac{\hat{p}(j)}{\hat{p}(j) \cdot r})$*, for* $j = 1, 2, \cdots, k$*.*

Proof. Denote the competitive multiplier associated with $(\hat{x}, \hat{p})$ by $\hat{\lambda}$ and that with $(\hat{x}(j), \hat{p}(j))$ by $\hat{\lambda}(j)$ for $j = 1, 2, \cdots, k$. The process of iterated price normalization generates the sequences $\{r^t\}$, $\{\hat{p}^t\}$, and $\{\hat{p}^t(j)\}$ for all j that satisfy (14) and (15) with $r^0 = e$, $\hat{p}^0 = \hat{p}$, and $\hat{p}^0(j) = \hat{p}(j)$. Since CE allocations remain unchanged throughout the iteration process, it follows from Corollary 2 and (15) that

$$\hat{\lambda}^t = (\hat{p}^{t-1} \cdot r^{t-1})\hat{\lambda}^{t-1} \text{ and } \hat{\lambda}^t(j) = (\hat{p}^{t-1}(j) \cdot r^{t-1})\hat{\lambda}^{t-1}(j), \ \forall t, \forall j \tag{16}$$

with $\hat{\lambda}^0 = \hat{\lambda}$ and $\hat{\lambda}^0(j) = \hat{\lambda}(j)$ are the Lagrangian multiplier vectors associated with $(\hat{x}, \hat{p}^t)$ and $(\hat{x}(j), \hat{p}^t(j))$, respectively.

By iterated substitutions, it follows from (16)

$$\hat{\lambda}^t = (\prod_{\tau=1}^{t} \hat{p}^{\tau-1} \cdot r^{\tau-1})\hat{\lambda} \text{ and } \hat{\lambda}^t(j) = (\prod_{\tau=1}^{t} \hat{p}^{\tau-1}(j) \cdot r^{\tau-1})\hat{\lambda}(j), \ j = 1, 2, \cdots k.$$

Consequently,

$$\hat{\lambda}^t << \hat{\lambda}^t(j) \text{ if and only if } (\prod_{\tau=1}^{t} \frac{\hat{p}^{\tau-1} \cdot r^{\tau-1}}{\hat{p}^{\tau-1}(j) \cdot r^{\tau-1}})\hat{\lambda} << \hat{\lambda}(j). \tag{17}$$

By (14),

$$\prod_{\tau=1}^{t} \frac{\hat{p}^{\tau-1} \cdot r^{\tau-1}}{\hat{p}^{\tau-1}(j) \cdot r^{\tau-1}} < (\frac{\hat{p} \cdot e}{\hat{p}(j) \cdot e})(\frac{\hat{p} \cdot r^1}{\hat{p}^1(j) \cdot r^1})^{t-1}. \tag{18}$$

Since, by (8), $\frac{\hat{p} \cdot r^1}{\hat{p}^1(j) \cdot r^1} < 1$, the right-hand-side of (18) approaches to 0 as the number of iterations gets large. Consequently, since $\hat{\lambda}(j) >> 0$ for all j, there exists a positive integer $\hat{t}$ such that (17) holds for $t \geq \hat{t}$. Set $r = r^{\hat{t}}$. The proof is completed by noticing from (15)

$$\hat{p}^t = \frac{\hat{p}}{\hat{p} \cdot r^t} \text{ and } \hat{p}^t(j) = \frac{\hat{p}(j)}{\hat{p}(j) \cdot r^t}$$

for all $t \geq 0$. ■

Corollary 3 and Theorem 4 together imply the following selection of a unique CE:

Corollary 4 (Selection of a Unique CE) *Assume A1, A2', and A3-A5 are satisfied. Then, there exists a price-normalizing bundle r such that with price normalization that results in the set of normalized price vectors*

$$P = \{p \in \Re^m_+ \mid p \cdot \hat{r} = 1\},$$

there exists a non-discriminatory default penalty with which the credit mechanism selects a unique CE.

Proof. The set of normalized competitive price vectors in Δ^m is compact. Thus, by the Krein-Milman Theorem, it has an extreme point.[7] Let $(\hat{x}, \hat{p})$ be a CE such that $\frac{\hat{p}}{\hat{p} \cdot e}$ is an extreme point of the set of normalized competitive price vectors in Δ^m. It follows that $\hat{p}$ cannot be equal to any non-negative linear combination of other CE price vectors. By Theorem 4, there exists a price-normalizing bundle $r \in \Re^m_{++}$ such that under the price normalization that yields the set of normalized price vectors

$$P = \{p \in \Re^m_+ \mid p \cdot r = 1\},$$

the competitive multiplier vector associated with $(\hat{x}, \frac{\hat{p}}{\hat{p} \cdot r})$ is strictly dominated by the competitive multiplier vector associated with every other CE. Consequently, by Corollary 3, $(\hat{x}, \frac{\hat{p}}{\hat{p} \cdot r})$ is a unique selection for the credit mechanism with default penalties all equal to the maximum multiplier of the competitive multiplier vector associated with it. ■

3.2.2 *Selection with Common Price-Normalizing Bundles*

In this subsection we consider price-normalizing bundles, with which the credit mechanism uniquely selects any CE with appropriate default penalties. Such unique selections require that the resulting competitive multiplier vectors do not dominate each other. We assume:

[7] See Royden (1968, p. 207).

A6: There exists a bundle $r \in \Re^m_{++}$ such that for any two interior Pareto optimal allocations $\bar{x}$ and $\hat{x}$,

$$\begin{aligned}&\bigtriangledown u^i(\bar{x}^i)\cdot r > \bigtriangledown u^i(\hat{x}^i)\cdot r \text{ for some } i\\ &\Longrightarrow\\ &\bigtriangledown u^j(\bar{x}^j)\cdot r < \bigtriangledown u^j(\hat{x}^j)\cdot r \text{ for some } j.\end{aligned}$$

The inner product of the gradient $\bigtriangledown u^i(x^i)$ with bundle r is the *directional derivative* of u^i at x^i in direction r. It measures the instantaneous rate of change in trader i's utility caused by a change from bundle x^i to bundle x^i+tr for some small number $\delta > 0$, that is, $\bigtriangledown u^i(x^i)\cdot r$ is the limit of

$$\frac{u^i(x^i+\delta r)-u^i(x^i)}{\delta}$$

as $\delta > 0$ approaches to zero. The following two examples all have three interior CEs and all satisfy A6.

Example 1 (Shapley and Shubik 1977): There are two goods and two traders with endowments $a^1 = (40, 0)$, $a^2 = (0, 50)$ and utility functions $u^1(x^1) = x^1_1 + 100(1-e^{-x^1_2/10})$, $u^2(x^2) = 110(1-e^{x^2_1/10})+x^2_2$ on $\Re^2_+$. Traders 1 and 2 are respectively named Ivan and John in Shapley and Shubik (1977); goods 1 and 2 are respectively called rubles and dollars in their paper. There are three interior CEs in this economy. Furthermore, the interior Pareto optimal allocations satisfy

$$x^2_2 = x^2_1 + 50 - 10\ln 110. \tag{19}$$

Notice $\bigtriangledown u^1(x^1) = (1, 10e^{-x^1_2/10})$ and $\bigtriangledown u^2(x^2) = (11e^{-x^2_1/10}, 1)$. Equation (19) implies that to be Pareto optimal, trader 2's consumption of good 2 increases with his consumption of good 1. Since trader 1's marginal utility of good 1 is constant and his marginal utility of good 2 is decreasing while trader 2's marginal utility of good 2 is constant and his marginal utility of good 1 is decreasing, this example satisfies A6 for any bundle $r \in \Re^2_{++}$.

Example 2 (Mas-Colell, Whinston, and Green 1995, pp. 521-522): There are two goods and two traders with endowments $a^1 = (2, r)$, $a^2 = (r, 2)$, $r > 0$, and utility functions $u^1(x^1) = x^1_1 - \frac{1}{8}(x^1_2)^{-8}$, $u^2(x^2) = -\frac{1}{8}(x^2_1)^{-8} + x^2_2$ on $\Re^2_{++}$. There are three interior CEs in this economy. Furthermore, the interior Pareto optimal allocations satisfy

$$x^2_1 x^1_2 = 1. \tag{20}$$

Notice $\bigtriangledown u^1(x^1) = (1, (x_2^1)^{-9})$ and $\bigtriangledown u^2(x^2) = ((x_1^2)^{-9}, 1)$. Equation (20) implies that to be Pareto optimal, any increase in trader 1's consumption of good 2 leads to a decrease in trader 2's consumption of good 1. Since trader 1's marginal utility of good 1 is constant and his marginal utility of good 2 is decreasing while trader 2's marginal utility of good 2 is constant and his marginal utility of good 1 is decreasing, this example satisfies A6 for any bundle $r \in \Re^2_{++}$.

We now establish the non-dominance of the traders' marginal utilities of income at CEs.

Theorem 5 (Non-Dominance) *Assume $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ satisfies A1, A2′, and A4-A6. Let r be a bundle as in A6. Then, for any two competitive triplets $(\bar{x}, \bar{p}, \bar{\lambda})$ and $(\hat{x}, \hat{p}, \hat{\lambda})$, $\bar{p} \cdot r = \hat{p} \cdot r$ implies that $\bar{\lambda} \geq \hat{\lambda}$ and $\bar{\lambda} \neq \hat{\lambda}$ cannot hold.*

Proof. Suppose on the contrary that there are two competitive triplets $(\bar{x}, \bar{p}, \bar{\lambda})$ and $(\hat{x}, \hat{p}, \hat{\lambda})$ with $\bar{p} \cdot r = \hat{p} \cdot r$ and

$$\bar{\lambda} \geq \hat{\lambda} \text{ and } \bar{\lambda} \neq \hat{\lambda}. \tag{21}$$

Since $r \in \Re^m_{++}$ and since $\bar{p} \cdot r = \hat{p} \cdot r > 0$, (21) implies

$$\bar{\lambda}^i \bar{p} \cdot r \geq \hat{\lambda}^i \hat{p} \cdot r$$

for all i and

$$\bar{\lambda}^j \bar{p} \cdot r > \hat{\lambda}^j \hat{p} \cdot r$$

for at least one j. However, by A2′, A5, and the Kuhn-Tucker conditions, $\bar{\lambda}^k \bar{p} = \bigtriangledown u^k(\bar{x}^k)$ and $\hat{\lambda}^k \hat{p} = \bigtriangledown u^k(\hat{x}^k)$ for all k. The above dominance of vector $(\bar{\lambda}^i \bar{p} \cdot r)$ over vector $(\hat{\lambda}^i \hat{p} \cdot r)$ then contradicts A6. ■

Combining Theorem 5 with Corollary 3, we can now establish:

Corollary 5 (Selection of a Unique CE) *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i \in N}$ be an exchange economy. Assume $\mathcal{E}$ satisfies A1, A3, A2′, and A4-A6. Then, under price normalization that results in the set of normalized price vectors*

$$P = \{p \in \Re^m_+ \mid p \cdot r \equiv 1\}$$

with bundle r as in A6, any CE, $(\bar{x}, \bar{p})$, of $\mathcal{E}$ can be a unique selection by the credit mechanism with default penalties $\mu = \bar{\lambda}$, where $\bar{\lambda}$ is the competitive multiplier associated with $(\bar{x}, \bar{p})$.

We end this section with an example to demonstrate the total cash flows and the traders' Lagrangian multipliers in CEs.

Example 3: Consider the 2-person economy of Shapley and Shubik (1977). As we showed in Example 1, for this economy A6 is satisfied even when we replace "for some bundle $r \in \Re^2_{++}$" with "for all bundles $r \in \Re^2_{++}$". Thus, we can choose $r = a^1 + a^2$ to be a price-normalizing bundle. If we normalize the prices to be such that $p \cdot r = 1,000$, so that the economy's total wealth is always 1000, then in the CEs we have competitive price vectors, competitive multiplier vectors, total cash flows, all with a two-digit decimal rounding off, as in the following table:

	x^*	p^*	λ^*	TW	TCF
CE1	((32.26, 39.26), (7.74, 10.74))	(3.4, 17.27)	(0.29, 0.06)	1000	704.34
CE2	((13.17, 20.18), (26.83, 29.82))	(12.9, 9.68)	(0.08, 0.1)	1000	541.45
CE3	((3.22, 10.23), (36.78, 39.77))	(18.5, 5.19)	(0.05, 1.9)	1000	733.52

In this table, TW stands for the total wealth of the economy and TCF for the total cash flow. The cash flow required from trader i at prices p_1, p_2 and bundle x^i is given by $p_1 \max\{0, x^i_1 - a^i_1\} + p_2 \max\{0, x^i_2 - a^i_2\}$ and the total cash flow required in a CE is the sum of the cash flows required from both traders at their respective equilibrium bundles and the equilibrium prices. Notice that the middle CE (CE2) is the only minimum cash flow CE. To uniquely select this CE, we can set the per unit default penalties equal to the traders' competitive multipliers 0.08 and 0.1. Alternatively, we can also choose a non-discriminatory per unit default penalty equal to 0.1. In fact, for the unique selection of this CE, it follows from the proof of Theorem 3 that any non-discriminatory per unit default penalties between 0.1 and the next highest maximum competitive multiplier which is equal to 0.19 would work.

4 Conclusion

In this paper we investigated the possibilities to enlarge the general-equilibrium structure by allowing default subject to appropriate penalties, so as to yield a construction of a simple credit mechanism for a credit using society to select a unique CE. The implementation of the credit mechanism involves a competitive bank providing banknotes that traders use as a direct and anonymous means of payment. The traders exchange personal IOUs for banknotes with exogenously specified credit lines at the beginning, and they settle up all outstanding credits with the bank at the end of the market. Under the credit mechanism, ending as a net debtor is penalized while ending as a net creditor is worthless.

Given price normalization and default penalties, we characterized the CEs that will be selected by the credit mechanism. They are those CEs with traders' marginal utilities of income dominated by the corresponding default penalties. Applying this result, we showed that in general price normalization exists under which the credit mechanism selects a unique CE with some non-discriminatory default penalty. Furthermore, with the additional condition that for some bundle with a positive quantity of each good, the derivatives of the traders' utility functions at each Pareto optimal allocation along the direction represented by the bundle do not dominate those at every other Pareto optimal allocation, price normalization calling for an equal value of the bundle guarantees that any CE can be a unique selection for the credit mechanism with appropriate default penalties.

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