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PARTNERSHIPS

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Abstract

A partnership is a coalition within which output must be shared equally. We show that when partnerships can form freely, a stable or "core" partition into partnerships always exists and is generically unique. When people differ in ability, the equal-sharing constraint inefficiently limits the heterogeneity of partnerships. We give conditions under which partnerships containing abler people will be larger, and show that if the population is replicated, partnerships may become more or less homogeneous, depending on an elasticity condition. We also examine when the equal-sharing inefficiency vanishes in the limit.

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1. INTRODUCTION

Among the central features of economic life is that individuals form groups to exploit gains from cooperation. Although economists generally emphasize the possibility of side-payments to distribute those gains, very often no side payments are made and the rewards from joint efforts are equally shared, even if the participants contributed (in some sense) quite different amounts. We call such an arrangement a <u>partnership</u>: a joint activity whose returns are equally shared among the partners, even though they might provide different inputs or incur different opportunity costs.

The limits on group size and stability, due to differences between people and the inability to make side payments, have been discussed in a wide range of settings by Schelling (1978). He considers, with the aid of numerical examples, problems that arise in the composition of tennis clubs, retirement homes, housing integration, and coeducational dining halls when people care who is in their group. This paper studies a class of similar problems, for which an equilibrium outcome exists (unlike some of Schelling's examples).

When social convention or problems of information make side payments impossible and make equal-division partnerships the only available way to organize cooperation, inefficiencies arise if some agents are more able than others, and the more able are reluctant to share equally with the less able. Without side payments agents cannot resolve this conflict to achieve the gains from cooperation. As a result, there is too little cooperation in equilibrium.

In this paper, we take as given the inability to make side payments and consider the equilibrium formation of partnerships when people differ in ability. The natural equilibrium concept for coalition formation is the core: a set of coalitions such that no new coalition could form and improve the welfare of all its members.[1] In many games, the core is empty or nonunique. Section 2 shows that with equal division, the core is nonempty and is generically unique.[2]

Equilibrium is typically not efficient in the sense of maximizing industry output, as it would be with flexible sharing rules or side payments, because the coalitions that form in equilibrium will be too small. Enlarging a group is unattractive to present members if it involves admitting less-able members, thus reducing the group's average ability. Admitting less-able new members creates a social gain from increased cooperation and makes the new members better off, but without side payments this gain cannot be internalized by current members. Hence, there is a social cost to the equal-sharing rule. Section 3 characterizes equilibrium. If ability is uniformly distributed, then groups with the highest ability are the largest. Section 3 also gives an example to show the social cost imposed by the equal sharing constraint.[3]

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[1] Since we assume that coalitions cannot engage in side payments, we naturally assume the same for blocking coalitions.

[2] Equal division means equal division of physical output of the coalition. More generally, equilibrium exists and is unique, provided that when two coalitions contain the same two people, each of these people prefers the same coalition. This condition holds for equal division of physical output, and may hold for other sharing rules also.

[3] Guesnerie and Oddou (1981) study a model of the provision of local public goods when agents' wealth levels vary and the only means available to finance public goods is a linear wealth tax. In their model, as in ours, the inability to choose individualized taxes makes groups too small and reduces efficiency. A similar inefficiency arises in Greenberg and Weber (1986),

In Section 4 we replicate the distribution of abilities (and hence the population) and show that the growth in number of equilibrium partnerships depends on the rate of change of elasticity of average product with respect to group size. If this elasticity is constant, the number of equilibrium partnerships remains constant. We discuss the conditions under which the limiting equilibrium, for a large economy, is efficient.

Although we think this model of partnerships has wide applicability, our interest in the subject arises especially from two examples. The first is the Pacific salmon fishing industry. Much of the difference in skill among fishermen is not in the ability to catch fish once located, but in finding schools of fish, which involves knowledge of the underwater terrain and the habits of fish. Here, "cooperation" means telling other fishermen when one has found a school of fish. There are gains to cooperation because fishermen who would otherwise be idle (or fruitlessly searching) are catching fish from the school before it disperses. Provided that the total number of fish caught from a school is an increasing function of the number of boats fishing it, efficiency requires sharing "finds" with all other fishermen.[4] If all fishermen had the same ability

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for a similar reason. Spulber (1986) defines a restricted notion of core designed to show how price stability may arise in decreasing cost industries. Prices in a blocking coalition are anonymous and linear, and hence the core is "second-best." A similar problem was studied by Sorenson, Tschirhart and Whinston (1978a,b) who find that with nonlinear prices the first-best can be sustained.

[4] This analysis is not about the problem of the commons, and we thus ignore the fact that individually rational fishermen may collectively overfish. From this point of view, the reduction in total catch due to equal sharing may be socially desirable. From the point of view of the theory of the core, the problem of the commons corresponds to the notion that there are externalities among coalitions. The profit opportunities of one coalition would depend on the total catch of other coalitions. But this is a different subject.

to find fish, they would all share information in equilibrium, even without side payments. The agreement would be that when one fisherman found a school of fish, he would inform the others.[5] If a flexible system of side payments were available, it would also be an equilibrium for all fishermen to share information, even if they differed in ability.

In fact, however, fishermen share information selectively. There are multiple groups of fishermen, and information is shared within groups, but not between groups.[6] The best fishermen form a group that is highly respected, and in which membership is coveted. This group appears to be larger than the groups of less skilled fishermen. Our result below, that abler groups are larger when ability is uniformly distributed, is a possible explanation of this.

Our second example is that of law firms. The gains to cooperation in a law firm arise from specialization. As with fishing, the inhibition on forming very large groups when no side payments are allowed is that skillful attorneys do not want their high ability diluted by less able attorneys. It seems that well-connected, experienced (hence, productive) attorneys tend to agglomerate in large firms, while there are many smaller firms composed of less experienced,

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5 There is, of course, the possibility that a partner will fail to inform the others. But deception is difficult because fish are unloaded in public.

6 The information-sharing discussed here occurs in the salmon trolling industry. Since communication takes place on public air frequencies, fishermen have elaborate codes in which information is passed during the course of conversation. The groups are called "coding partnerships"; one says "Alan codes with Tony" or "Alan is Tony's coding partner."

less well-connected, and perhaps less skilled, attorneys.[7] Like the similar observation in the fishing industry, this is consistent with our theory.

There are three basic systems by which profit is divided among law partners, but there are essentially no data on how many firms use each system (since artnerships have no reason to make public their compensation schemes). The first and most straightforward system is that all members with the same seniority receive the same profit share. Since junior partners eventually become senior partners, such a system would be equal division if the profitability of the firm were constant over time. Such a sharing scheme corresponds quite closely to our assumption, and is probably used mostly by two- or three-person law firms, which account for about 2/3 of all firms but less than half the lawyers.[8] A second division system is that the firm's founders or dominant partners have authority to make compensation decisions, presumably constrained by their desire not to lose partners. Such a system works well until the founders retire. If there is no obvious heir, the system breaks down, and may be replaced by the third system, which is for the partners to elect a finance committee to make compensation decisions. The formulas used by many such committees compensate according to time billed, business brought in, and (sometimes) public stature. The latter two compensation schemes are designed for

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[7] This is obviously difficult to verify by observing attorneys' abilities, but it is consistent with the fact that average earnings of partners increase with firm size. Flood (1985, p.48) reports that in 1981 partners in firms of 2 to 4 lawyers averaged $49,500, partners in firms with 5 to 9 lawyers averaged $76,300, and partners in firms with 10 or more lawyers averaged $89,400. In the few hundred law firms with more than 50 partners, earnings in six figures are common.

[8] The data reported here are taken from the Lawyer Statistical Report, 1980, and from Flood (1985).

the needs of large firms. But firms with more than 20 partners comprise less than three percent of law firms, and less than one quarter of lawyers.

There are two useful benchmarks in studying how compensation schemes might affect the equilibrium partition of lawyers into partnerships. First, if side payments (compensation rules) are unrestricted, then, as in any game of coalition formation, the partition into law firms would be efficient. But, even though the compensation schemes described above allow more flexibility than the strict equal-sharing rule discussed in this paper, they are not unrestricted. A lawyer who works efficiently for clients who pay on a retainer basis will be very profitable for the firm, but the billable-hours-plus-seniority formula will not compensate him/her for this. One possible approach to studying the formation of partnerships might be to start from the benchmark of unrestricted sidepayments and try to characterize the bias introduced to group formation by the disallowance of some particular forms of compensation. Our approach is to analyze the other extreme: equal division of profit within the partnership.

There are many other examples. For instance, members of a group medical practice find they share equally in the group's reputation, and this tends to deter the formation of heterogeneous groups, although there might well be advantages to heterogeneity. (The most able members could concentrate on the difficult cases.) Similarly, members of an academic department share equally in the reputation and to some degree in a general atmosphere. Finally, in economics (though not in the physical sciences) there is a strong convention that credit for jointly authored work is shared equally, and this plausibly makes some scholars reluctant to collaborate with those who could make a lesser, but still positive, contribution to their work.

2. A GENERAL EXISTENCE AND UNIQUENESS RESULT

Partnerships, in which a single output (such as profit, fish, or utility) must be equally divided among the partners, turn out to be a very simple case for core analysis. In this section, we show that, in a class of coalition-formation games that includes partnerships, the core cannot be empty, and generically contains just one partition. Moreover, the core allocation has a very intuitive description: first, the "best" group forms; then the "best" group from those remaining, and so on.

For technical reasons, we work here with finitely many agents. This assumption is true, as well as making the analysis easier. Later (in Sections 3 and 4) we will find it convenient to analyze a continuum of agents. Our proof that the core is nonempty does not strictly apply to that case, although a similar proof could be constructed.[9]

Our theorem concerns finite coalition-formation games in which, whenever two agents are both in each of two potential coalitions, they agree on how to rank them.[10] This obviously holds if the coalition's output is equally divided among

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[9] In the continuous case, the core will be nonempty if we require that blocking coalitions must differ from equilibrium coalitions by a set of positive measure. Without this requirement, the core would be empty, because the most-able coalition could always expand by one lower-ability individual (who has measure zero) without reducing the group's average ability, but increasing that individual's payoff.

[10] Stated thus, our condition already assumes two things: (i) that there is a sharing rule that determines how the payoff is split within the coalition, and (ii) that payoffs to members of coalition S do not depend on what happens outside S. This latter assumption rules out externalities among coalitions; it holds, for instance in the classic case of exchange economies,

the members. It also holds with certain kinds of "separable" departures from equal division. As we will discuss in the conclusion, a fisherman may have a "first-finder" advantage: he may be able to fish for a while before his partners arrive. Then his payoff differs from his partners', but in a way that may not interact with his choice of partners. Likewise, in law partnerships, our theorem may apply even when partners are paid a bonus for bringing in business.

Theorem: Let N be a finite set of agents. Suppose that each member i of a coalition S contained in N gets a payoff $u^i(S)$ that is independent of the other coalitions that form. Suppose further that if i,j are two agents and S,T are two coalitions containing both i and j, then $u^i(S) > u^i(T)$ if and only if $u^j(S) > u^j(T)$. Then (i) there is a partition of N into coalitions, such that no new coalition could form and make all its members strictly better-off; (ii) that partition is generically unique.

Proof: We can assume without loss of generality that there is a single utility function u(S) such that $u^i(S) = u(S)$ for all $i \in S$.[11] Then, generically, there is a unique S^*_1 that maximizes $u(\bullet)$. Call this S^*_1 the "best coalition." Now apply the same argument to the remaining agents, $(N-S^*_1)$, to produce the "second-best coalition," S^*_2, and so on. Eventually, all the remaining agents

as well as in our case of production partnerships, when the price of output is fixed.

[11] Agent 1 ranks only the coalitions of which he is a member. Those coalitions which contain agents 1 and 2 are ranked the same by both. Choose 2's utility numbers to be the same as 1's utility numbers for those coalitions. Choose the remaining utilities for agent 2 such that they reflect his ranking. Since agent 3 agrees with both 1 and 2 on the coalitions they jointly belong to, the same construction can be applied to agent 3. Etc. If the coalition produces a real-valued output such as profit, u(S) is per-capita payoff.

will unanimously want to join together.[12] We claim that $\{S^*_i\}$ is a core partition, and that no other is.

To see this, simply observe that no blocking coalition could contain any member of S^*_1, the best coalition, for it is impossible to give any member of S^*_i more than he is currently getting. But, by the same argument, given that no member of the best coalition would join the defectors, they could not attract any member of the second-best coalition S^*_2 either; and so on.

Uniqueness follows immediately from the fact that any core partition must make S^*_1 one of its coalitions (else S^*_1 could defect), and then must make S^*_2 another, and so on. What if this construction yields two possible coalitions that give the same maximum utility payoff and contain some of the same individuals? Then those coalitions cannot both be in the same core partition. (This would require that the individuals they have in common belong to two different coalitions!) But there is a core partition containing one of the coalitions, and another core partition containing the other: The core partition is not unique. However, this kind of coincidence is unlikely or nongeneric in the following sense. If the set of abilities is perturbed slightly so that each

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[12] This last coalition is made up from the leftovers, those who were not previously invited to join a coalition. Typically they will wish their coalition were larger. In many coalition-formation models (e.g., Pauly (1967)) such leftover groups cause existence problems, sometimes called "integer problems." No such problems arise here because the leftovers cannot bribe others to join them. They have no power to disrupt the coalitions with higher payoffs.

coalition's product is perturbed,[13] there will no longer be two coalitions with the same payoffs. Thus, the core is generically unique. Q.E.D.

Because this theorem not only guarantees existence and generic uniqueness, but specifies how to construct the core partition, it is very useful in applications. One can say exactly what partnerships will form. In Section 3, we apply this constructive technique to a simple model of partnerships when there are gains to cooperation but when agents differ in ability.

3. CHARACTERIZATION OF EQUILIBRIUM

In the previous section, we showed existence and generic uniqueness of equilibrium in a model which realistically had a finite number of individuals. Characterizing equilibrium is easiest with a continuum of individuals, with the distribution of abilities, e, described by a distribution function $F(e)$ on support $[0,M]$. The variable e, ability, represents the expected number of schools of fish found in the fishing example, or the expected number of clients brought in by an attorney. We let $t(n)$ represent the total profitability (output) of each find for a group of size n, and $a(n)=t(n)/n$ is the average. We assume that $t'(n)>0$ to represent gains from cooperation, but (at least beyond some point) $a'(n)<0$. (If $a(n)$ is increasing, then any new member is always welcome, no matter how few finds he brings in.) In the case of fishermen, $t(n)$

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[13] When the payoff to members is generated by splitting a real-valued output, perturbation of abilities causes outputs of different coalitions to change slightly. When the payoff to members is generated by nonanonymous crowding effects, as in clubs where people care about the identity of the other members, the perturbation must be on preferences, such that a member of both coalitions is no longer indifferent between them.

is the total number of fish taken from a school before it disperses, and in the case of a law firm, it is the profitability of each client brought in.

The first feature of equilibrium in this model is that groups are "consecutive"--they contain all the agents with abilities in some interval.[14] The payoff to each member of a group consisting of an interval S in [0,M] is $a(|S|) \int_S e\,dF(e)$ where $|S|$ is the size of the group, $|S|=\int_S dF(e)$. A group containing a "gap" could clearly improve average payoffs by filling the gap and eliminating an equal number of agents with lower abilities. However, for such a coalition to block, the agents who fill the gap must also increase their payoffs, so the result is not totally obvious. Before we show this, we summarize the other features of equilibrium.

Since each group in equilibrium is consecutive, groups can be unambiguously ordered by ability. Our second discovery is that the size of equilibrium partnerships increases with ability, at least when the distribution of abilities is uniform. To see the reasons for this, consider the incentive for the more able members to form a new partnership excluding the least-able member. This increases the average ability in the group. When the distribution of abilities is uniform, eliminating the least-able member <u>increases</u> average ability by the same amount for every size-n group, irrespective of the group's <u>average</u> ability.

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[14] "Consecutive" groups are discussed by Greenberg and Weber (1986). Their model, which examines jurisdiction formation and production of local public goods, has a prespecified division of taxes, but utility is not shared in a prespecified way, since preferences depend differently on the amount of local public goods provided. That their equilibrium is fundamentally different from ours is evident in the fact that no individual in their equilibrium would want to move, irrespective of whether the other coalition wants to accept him. In our model, every individual would like to join the best partnership.

On the other hand, the loss to eliminating the marginal member does depend on how able the group is. The contribution of the least-able member is higher in a high-ability group, and this provides the incentive for larger groups. Another way to see this is to note that, in the uniform case, while the absolute difference in ability between the group's mean and its least able member is independent of its mean (among same-sized groups), this difference becomes proportionally less important as the mean ability grows.

We now state these results formally.

Proposition 1: Equilibrium groups have the "consecutive" property that all individuals between the lowest and highest ability in the group are also in the group.

Proof: This follows from the construction of equilibrium given in the proof of existence. The most-able equilibrium group, S^*_1, is the coalition that maximizes per-capita payoff. This group is composed of the most-able individuals, down to some cutoff. The next equilibrium group maximizes per-capita payoff among coalitions of people not in the most-able group. This group again includes the most-able individuals in the remainder, down to some cutoff. Thus, each group is composed of the individuals whose abilities lie in some interval. Q.E.D.

Proposition 2: When ability is uniformly distributed, the sizes of equilibrium groups increase with ability: If one equilibrium group contains abler individuals than another, it will be larger.

Before proving Proposition 2, we introduce some notation. Let $n(\ell,m)$ be the number of people between abilities ℓ and m. That is, $n(\ell,m)={}_{\ell}\int^{m}dF(e)$. By Proposition 1, the group with top ability m that maximizes average payoff is an interval $[\ell(m),m]$ of abilities, containing $n[\ell(m),m]$ people. Proposition 2 says that $n[\ell(m),m]$ increases with m if F is uniform.

<u>Proof of Proposition 2:</u> $\ell(m)$ maximizes $a[n(\ell,m)]\ {}_{\ell}\int^{m}edF(e)$, which is the product of average payoff per find and total number of finds in the group. Taking logarithms before differentiating, and multiplying by $n(\ell,m)/f$, where f is the density, we can write the first-order condition as[15]

$$(1)\qquad \gamma[n(\ell,m)] - \frac{\ell}{\mu(\ell,m)} = 0 \quad ,$$

where $\mu(\ell,m)$ is the mean ability in the interval and $\gamma(n)$ is the elasticity of average product:

$$\mu(\ell,m) \equiv \frac{1}{n(\ell,m)} \int_{\ell}^{m} edF(e) \qquad \text{and}$$

$$\gamma(n) \equiv -\,na'(n)/a(n).$$

Suppose that, when m increases, ℓ were to increase just enough to maintain n. Then the first term in (1) would be unchanged; and the second term ℓ/μ would

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[15] The logarithm of the objective function is concave, provided that $\gamma(n)$ is increasing, or, if the $\gamma(n)$ is decreasing, it is not decreasing too fast.

increase, given that F is uniform.[16] So (1) would now be negative, and it would pay to reduce ℓ to a little, increasing n. Q.E.D.

It is of interest to ask how "robust" this conclusion is to the distribution of abilities. The basic insight is that partnerships will be larger if groups can be expanded without introducing too much heterogeneity. This is because the gains from cooperation can be exploited without the countervailing problem that high-ability partners do not want to share with low-ability partners. It follows that an equilibrium with equal-size partnerships (that is, $n[\ell(m),m]$ constant) would require that high ability groups have high variance in ability, which discourages expansion. Otherwise stated, the density of abilities would have to be declining.

This is exactly correct. The distribution of abilities that generates partnerships of equal size has density k/e, where k is a constant. Intuitively, distributions of ability that have flatter slope than k/e will generate equilibria in which the more-able partnerships are larger than the less-able, and vice versa for distributions that are more steeply sloped (more skewed) than k/e.

We now solve a simple example to illustrate the social loss in output due to the able peoples' unwillingness to be partners with the less able.

Example: Suppose that the per-capita payoff from each find, a(n), has constant elasticity, γ. Fish-finding ability is uniformly distributed on [0,M].

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[16] $\mu = \frac{1}{2}(\ell+m)$, so $\ell/\mu = 2/(1+m/\ell)$. If one adds the same amount to ℓ and m, so as to hold n fixed, then ℓ/μ rises.

The per-capita payoff in a coalition S is $|S|^{-\gamma} \int_S e de$, where $|S|$ is the size of the coalition S. Provided $\gamma>0$, total catch is maximized by having one grand partnership, but that is not an equilibrium. Instead, the unique core partition is given by

$$S^*_1 = (m_1, m_o]$$
$$S^*_2 = (m_2, m_1] \quad , \text{etc.}$$

where $m_o = M$ and $m_{i+1} = [\gamma/(2-\gamma)]^i M$. Thus, the population $[0,M]$ splits into infinitely many partnerships, the higher-ability ones being also larger (as in Proposition 2). Calculation shows that total output is

$$\sum_{i=0}^{\infty} (m_{i+1}-m_i)^{-\gamma} \int_{m_{i+1}}^{m_i} e \, de$$

$$= M^{3-\gamma} \left[\frac{2(1-\gamma)}{2-\gamma}\right]^{2-\gamma} \frac{1}{2-\gamma} \frac{1}{1-\left(\frac{\gamma}{2-\gamma}\right)^{3-\gamma}}$$

as compared with the maximum possible output, which is

$$M^{1-\gamma} \int_0^M e \, de = 1/2 \quad M^{3-\gamma}$$

It is easy to check that at $\gamma=1$, when there is neither social nor private gain to cooperation (that is, $t(n)$ is constant), the equilibrium outcome is efficient. At $\gamma=0$ (that is, $a(n)$ is constant), we again find efficiency, because equilibrium will have one group. The proportional loss is greatest at about $\gamma=.7$, when almost 25 percent of total possible output is lost as a result of the inability to keep the whole group together.

4. (IN)EFFICIENCY IN THE LIMIT

In this section we ask how heterogeneity of equilibrium groups depends on population size. We study this by replicating the set of agents. Intuitively, such replication allows partnerships to expand without having to increase heterogeneity, or to reduce heterogeneity without having to shrink. Size and homogeneity are "goods" for a partnership, and replication expands the feasible set. As in the ordinary theory of demand, agents may exploit their improved opportunities by taking more of both goods (size and homogeneity) or by taking more of only one. We find that replication may cause the top group to become more homogeneous, or (depending on the elasticity of the average-catch function $a(\bullet)$) may make it more heterogeneous.

We multiply the original distribution function by a parameter $v>1$, so that the replicated population is distributed according to $vF(e)$ and the population size is v. Let $\ell(m,v)$ denote the bottom ability cutoff of the equilibrium group that would form if m were the top ability. Then we can describe how ℓ changes as v increases, in terms of the elasticity of average payoff (per find) with respect to group size, $\gamma(n) = -na'(n)/a(n)$:

<u>Proposition 3:</u> As the population is replicated, groups become more or less homogeneous according as $\gamma(n)$ is increasing or decreasing. More precisely,

(i) If $\gamma(n)$ is decreasing, then $\ell(m,v)$ is decreasing in v (for any m). Therefore, as v grows, partnerships become less homogeneous, and grow faster than v.

(ii) If $\gamma(n)$ is constant, then $\ell(m,v)$ is independent of v (for any m). As v grows, each group grows in proportion, but its composition does not change.

(iii) If $\gamma(n)$ is increasing, then $\ell(m,v)$ is increasing in v. As v grows, equilibrium partnerships become more homogeneous.

Proof: For a group with top ability m, define the group size as

$$(2) \quad n(\ell,m,v) = \int_{\ell}^{m} v e dF(e) \quad .$$

The equilibrium choice $\ell(m,v)$ maximizes $\pi(\ell,m,v) \equiv a[n(\ell,m,v)] \int_{\ell}^{m} v e dF(e)$ and therefore also maximizes its logarithm. The first-order condition, multiplied by $n(\ell,m,v)/f(\ell)$, is[17]

$$(3) \qquad 0 = \gamma[n(\ell,m,v)] - \frac{\ell}{\mu(\ell,m)} \quad \equiv \quad \frac{\partial \log(\pi)}{\partial \ell} \frac{n(\ell,m,v)}{f(\ell)}$$

To see how the equilibrium choice of lower cutoff, $\ell(m,v)$, varies with v, we must sign the cross-derivative $\partial^2 \log[\pi(\bullet)]/\partial v \partial \ell$ at the optimum. From (3), we see that this is simply $\gamma'[n(\ell,m,v)]\partial n(\ell,m,v)/\partial v$, evaluated at $\ell=\ell(m,v)$. But clearly $\partial n(\ell,m,v)/\partial v > 0$, so $\partial \ell(m,v)/\partial v$ has the same sign as $\gamma'[n(\ell,m,v)]$. This establishes Proposition 3.

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[17] The average ability in the interval $[\ell,m]$ is independent of v.

The inefficiency described above results from the fact that economies of scale are underexploited because of able agents' intolerance of heterogeneity. As we replicate the economy, scale economies can be exploited with less heterogeneity. One might therefore hope that, at least on a per-capita (or per-find) basis, these inefficiencies would vanish in the limit. In general, however, they do not. Although equilibrium groups become large in the limit as $v \to \infty$, we cannot generally argue that in the limit, all economies of scale are exhausted in equilibrium groups. The fact that a representative group-size $n[\ell(m,v),m,v]$ is unbounded as v gets large does not imply that the limit of $t[n(\ell(m,v),m,v)]/t(v)$ is one. The problem is that $n[\ell(m,v),m,v]$ may grow too slowly, and the function $t(\bullet)$ may be such that that matters in the limit.

In Proposition 4, we show that if t(n) is bounded above (and thus approaches a finite limit) as n grows large, then we do have efficiency in the limit, even though the number of groups becomes large. The limiting equilibrium is very similar to core allocations in "club" economies with anonymous crowding, in which coalitions will usually be homogeneous and no side payments will occur because identical agents get equal utility in the core. (It is generally assumed in club economies, however, that economies of scale are exhausted at a finite group size.)

However, it is not the case that efficiency results in the limit whenever $t'(n) \to 0$ as n becomes large. For instance, $t'(n) \to 0$ in the constant-elasticity case $t(n) \equiv n^{1-\gamma}$, $0<\gamma<1$, but we show in Proposition 5 that if $\gamma(n)$ converges to a limit strictly between 0 and 1 then inefficiency persists even in the limit.

However, Proposition 6 shows that, if $\gamma(n)$ converges to zero, then we do have efficiency in the limit because all agents are in one group, which exploits the economies of scale.

Proposition 4: If $t(n)$ approaches a finite limit as n grows large, then groups in the limiting equilibrium as v becomes large will become homogeneous and the social loss per find will approach zero.

Proof: First, we show that if $t(n)$ is bounded and thus converges to a finite limit as $n \to \infty$, then $\gamma(n)$ converges to 1. Since $\gamma(n) = 1-nt'(n)/t(n)$, it is enough to show that $nt'(n) \to 0$. It is obvious that $t'(n) \to 0$, but not that $nt'(n) \to 0$. Suppose that there exists $\varepsilon > 0$ and $N > 0$ such that $nt'(n) \geq \varepsilon$ for all $n > N$. Then for such n, $t(n)-t(N) =$

$$\int_N^n t'(x)\,dx \geq \varepsilon \int_N^n dx/x = \varepsilon \log(n/N),$$ which implies that $t(n)$ is unbounded as n grows, contrary to assumption. This proves that $nt'(n)$ must come arbitrarily close to zero infinitely often. We simply assume away oscillatory behavior, and conclude that $nt'(n) \to 0$ and $\gamma(n) \to 1$.

Because $\gamma(n) \to 1$, the first-order condition (3) implies that groups become very homogeneous in the limit if they grow large (and certainly become homogeneous in the limit if they do not!) Thus, the economy becomes very finely divided ($n(v)/v \to 0$), and, recalling that efficiency requires that everyone be in one group, one might well expect that this means inefficency in the limit. But when $n(v) \to \infty$ and $t(n)$ is bounded above, $t(n(v))/t(v) \to 1$. It does not matter that group sizes grow more slowly than the economy. Q.E.D.

If the elasticity of average product is constant and less than one, then equilibrium partnerships are replicated in size without being replicated in number when the economy is replicated. Both terms of (3) remain constant. Since economies of scale have not been exhausted, this is inefficient. In fact, if the elasticity of $a(n)$ converges to any number strictly between 0 and 1, the limiting equilibrium will be inefficient, as we show in the appendix. This may occur even though $t'(n)$ converges to zero. Convergence of $t'(n)$ to zero is not enough to get efficiency in the limit, even though groups become large and economies of scale are exhausted for large enough groups. In order to get efficiency in the limit, the economies of scale must be exhausted fast relative to how fast equilibrium partnerships become large.

Proposition 5: (Appendix). If the elasticity of average product, $\gamma(n)$, converges to any number strictly between 0 and 1 for large n, then the limiting equilibrium is inefficient.

At the opposite extreme, economies of scale may persist for all n, rather than being exhausted. If average payoff per find is constant (and positive) for all n, then members will not object to enlarging the group, no matter how heterogeneous the group is, and there will be one grand coalition. If average payoff becomes constant in the limit, then there will be one grand partnership in the limit, which is efficient. Again, in the limit, equal sharing has no bite. We state this as a Proposition:

Proposition 6: If the elasticity of average product with respect to n, $\gamma(n)$, approaches zero, then the limiting equilibrium, as v becomes large, has one grand partnership consisting of all the individuals, and is efficient.

Proof: The top group in the limiting equilibrium becomes either infinitely large or finite and homogeneous. Suppose that $\ell[M,v]$ does not converge to the average ability in the group, $\mu[\ell(M,v),M]$, as v becomes large, so that in the limit the top group is heterogeneous and its size n is unbounded. Then in the limit the value of the derivative (3) is negative, since $\gamma(n)$ converges to zero, and it pays to expand the group, until $\ell[M,v]$ is the lowest ability in the support of F, zero. Likewise, if the top group is finite and homogeneous, it pays to expand, since $\gamma(n)$ can never equal one if t(n) is strictly increasing, and hence the value of the derivative (3) is negative whenever n is finite and v becomes very large. Q.E.D.

5. CONCLUSION

When cooperation is organized through equal-division partnership, equilibrium partnerships are smaller and more homogeneous than in the social optimum. The reason is that the most-able people will not tolerate being in as heterogeneous a group as they should for social efficiency. Unless economies of scale either become almost completely exhausted or else remain so attractive as the group gets large that increasing membership is always desirable despite heterogeneity, this inefficiency persists even in the limit as the economy grows large.

If the distribution of abilities is uniform or not "too" skewed, then size of group will increase with ability.

In our characterization of equilibrium, we assumed that the partnership's profit is equally divided. In fact, this is a stronger assumption than we need for our conclusions. For instance, in the fishing industry, the partner who actually finds the school can fish it alone for a while before his partners arrive. It is natural to model this by specifying that the payoff to a fisherman of ability e* who belongs to a partnership S is $a(|S|) \int_S e dF(e) + e^* g$, where g is the amount he can catch before the others arrive. The last term, e^*g, will not affect which partnership he joins because it is independent of the size or composition of the group. Our results are unaffected if g is unequal to zero or differs across fishermen. By letting g be negative, we can allow for costs of finding (or searching for) opportunities, but only when such costs never deter effort: we do not deal with free-rider and related moral hazard problems, which themselves have implications for group size. (See, for instance, Holmstrom (1979).)

A less ambitious generalization is also possible in the law-partnership example. There, it is not reasonable to suppose that the profitability of a client depends on which partner brings him in, as is required above if g differs among fishermen. However, our characterization applies if all partnerships pay the same bonus to the partner who brings in the business, and this bonus may even depend on the size of the partnership, although not on the identity or on the talents of the "finder."[18]

Although our model is highly simplified, we believe that the basic problem is widespread and important: inefficient limits on group heterogeneity are imposed by people's reluctance to treat partners unequally. One question we have not addressed is why rigid rules such as equal sharing are so common. A

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[18] The profitability of a client brought into a size-n firm is t(n). The part which is split among the partners would then be [t(n)-g(n)]. Thus, unlike in the generalization just given for fishing, the equilibrium does depend on g(•), but our existence and characterization proofs still apply.

possible answer is suggested by Milgrom (1986), who argues that, if rigid rules are not applied, effort may be diverted to trying to influence the division of profit: this is also the spirit of the literature on "rent-seeking". Precommitment to rigid rules therefore has social value, as well as social costs. This paper has studied the social costs.

Another reason partners may agree to equal sharing is that it satisfies some primitive notion of social justice. Symmetry as a social choice criterion has been studied by authors discussed in Thomson and Varian (1985). An efficient allocation is "fair" if no agent prefers (or envies) another agent's consumption bundle. In exchange economies, fair allocations can be achieved by the competitive mechanism with equal division of initial endowment. Varian (1974) extends the notion of envy-free allocations to economies with production and cooperative behavior (as here), and asks whether redistributions of initial endowment lead to equilibria that satisfy various notions of fairness. The equity restrictions imposed in this literature are on the initial distribution of endowment, after which standard concepts of core and competitive equilibrium apply. The restriction in our treatment is not on the distributions of initial endowment, but rather on the manner in which a blocking coalition can divide its output. An allocation in the equal-sharing core, as defined here, is neither efficient (in a finite economy) nor envy-free.

Another view of our work is simply that equal-sharing rules exist, and they therefore deserve analysis.

APPENDIX

Proof of Proposition 5: First we show that for large enough values of v, the grand partnership is not an equilibrium. (This result depends on the assumption that the minimum ability in the support of F is zero. Otherwise, the grand partnership may be the equilibrium outcome.) Recall that, given v, the first-order condition on the top group's choice of cutoff ℓ is

$$\gamma[n(\ell,M,v)] - \frac{\ell}{\mu(\ell,M)} = 0$$

where $\gamma(n) \equiv - na'(n)/a(n)$, $n(\ell,M,v) = \int_{\ell}^{M} v dF(e)$, and $\mu(\ell,M) = \int_{\ell}^{M} e dF(e)/n(\ell,M,1)$.

This shows that, provided $\gamma(n) > 0$ for all finite n, the grand partnership ($\ell = 0$) is never an equilibrium: a small increase in ℓ away from zero would make remaining partners better off.

This shows that $\ell(M,v) > 0$ for all v. Moreover, $\ell(M,v)$ cannot converge to zero even as $v\to\infty$. In fact, $\ell(M,v)$ must converge to a value ℓ^* such that $\ell^*/\mu(\ell^*,M) = \gamma^*$, where $\gamma^* \in (0,1)$ is the limit of $\gamma(n)$ as $n\to\infty$. This is because $n[\ell(M,v),v,M]$ grows without limit as $v\to\infty$. (If n remained bounded, then ℓ and hence μ would have to converge to M, so that by the first-order condition $\gamma(n)$ would have to converge to 1, although n is bounded. But this can occur only if $t'(n)=0$ at finite n, which we have assumed cannot occur.)

We now show that the ratio of average payoff per find in equilibrium to the payoff per find for the coalition of the whole converges to something less than one as v becomes large. Hence we have inefficiency in the limit.

Since F is strictly increasing on [0,M], $0<F(\ell^*)<1$. For large enough v, the top group contains no more than a fraction $1-F(\ell^*)/2$ of all agents, and no other group contains more than a fraction $[F(\ell^*)+1]/2$. Therefore each find generates no more than $\tau(v) \equiv \max\{ t[v(1-F(\ell^*)/2)] , t[v(F(\ell^*)+1)/2] \}$ of output, as compared with the output t(v) generated from each find if agents all cooperate in the grand partnership. It remains to show that $\tau(v)/t(v)$ is bounded strictly below one if $\tau(v) \le \max\{ t[v(1-F(\ell^*)/2)] , t[v(F(\ell^*)+1)/2] \}$. Let $\theta = \max\{ 1 - F(\ell^*)/2 , [F(\ell^*)+1]/2 \}$. Then

$$\log \{ \frac{\tau(v)}{t(v)} \} \le \log \{ \frac{t(\theta v)}{t(v)} \} =$$

$$= - \int_{\theta v}^{v} \frac{d}{dx} \log(t(x))\, dx = - \int_{\theta v}^{v} \frac{t'(x)}{t(x)}\, dx$$

$$= - \int_{\theta v}^{v} \frac{(1-\gamma(x))}{x}\, dx.$$

For large enough v, $1-\gamma(x) > (1-\gamma^*)/2$ for all $x>\theta v$. (We use $\theta>0$ here.) For such v, then

$$\log \{ \frac{\tau(v)}{t(v)} \} < - \int_{\theta v}^{v} \frac{1}{2} (1-\gamma^*) \frac{dx}{x}$$

$$= - \frac{1}{2} (1-\gamma^*) \log \frac{v}{\theta v} = - \frac{1}{2} (1-\gamma^*) \log \frac{1}{\theta} < 0$$

since $0<\theta<1$ and $0<\gamma^*<1$. Since $\log[\tau(v)/t(v)]$ is bounded strictly below zero, $\tau(v)/t(v)$ is bounded strictly below one, as claimed. Q.E.D.

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