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Abstract

This paper tests the hypothesis that the net of extraction cost price of a natural resource does not change with volume. The hypothesis is shown to be a consequence of Hotelling's theory. The tests are performed on equations estimated by a nonparametric regression (ACE), and we show that the usual least squares estimation techniques are not general enough to successfully perform the test. The test rejects the pure form of the Hotelling theory and shows that it is necessary to adjust sale prices for volume sold.

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INTRODUCTION

Writing a half century ago, Hotelling theorized that the present value net of marginal cost price of a nongrowing resource, such as old-growth redwood, harvested on different dates would be the same. The immediate consequences of this theory for natural resource valuation are enormous. Given a price per unit of resource from some source such as comparable sales, the theory asserts that there is no need to adjust that price for time to liquidate, size of sale, or risk. Most particularly, there is no need to perform a discounted cash flow analysis. These consequences of Hotelling's theory make testing that theory both practical and necessary.

There are at least three ways to test Hotelling's hypothesis. The most obvious is to directly test the assertion that prices go up at the rate of interest or, equivalently, that present value price is constant (Feige and Geweke [5], Heal and Barrow [7], and Slade [10]). The second is to test the equivalence of the asset or stock market value--the value implied by the net of cost price of the product times the reserves (Miller and Upton [9]). This paper gives the third, which is to test for the constancy of net, per unit, price as the size of the sale varies.

THEORY

The argument for the Hotelling valuation principle is an arbitrage argument. A small sale of material will always be made in the year in which it is the most advantageous. An owner deciding to extract in one of two years would choose to sell in the years that have the highest present value of price.

Since owners choose to sell in every year, it must be that they expect the present value of price to be the same in every year. Constant present value of price can occur only if prices are going up at the rate of interest, which is Hotelling's rule. Since the present value of price is constant, it does not matter in which year a small sale of material is made.

Now consider the owner of a large parcel. One might reason that, since a large parcel is a large supply in a single year, it would depress the market and sell for less than the price of a small parcel. The counterargument is that a large parcel can be broken up into many small parcels, each one sold in a different year if necessary. Since small parcels are worth the same regardless of which year they are sold in, it must be that a large parcel is worth no less than a small parcel on a per unit basis. At least this is true in the Hotelling theory, which is a theory of perfect markets.

Very small sales are likely to incur significant transactions and set-up costs, at least on a per unit basis. Contracting between parties, finding willing parties to a contract, etc., need to be done regardless of the size of the sale. Similarly, men and machines need to be moved to the site of the sale to perform the extracting operations. These things are all in the nature of fixed costs. Insofar as it is not possible to glue many small plots together to make a single large plot, and this is certainly the case in many private timber sales, small sales will sell at a discount. The "market failure" here is the inability to assemble a sale of minimum efficient size.

The other end of the spectrum represented by very large sales suffers (at least potentially) from a wholly different set of problems. The number of bidders for a very large sale may be few or even one. Particularly for resources for which specialized immobile capital are necessary for extraction (modern sawmills and private road networks, to name two), the amount of competition may be small. Similarly, $100+ million deals require access to major money markets and may expose the firm to substantial risk of bankruptcy. Although I have stated these as reasons why such sales are disadvantageous, the same points could be made in reverse. Large sales allow the construction of modern extraction capital and make it possible to seek financing in the central capital markets at the lowest possible rates. However one argues this, large sales could sell for more or less than small sales with the proper set of market imperfections.

What remains then is an empirical question. Does the size of the sale affect the price per unit? In the remainder of this paper, we use data on old-growth redwood stumpage sales from 1953 to 1977 to find how discounts and premiums actually vary with sale size. Our methods allow a very general functional relationship. Finally, we test the hypothesis that price first rises and then declines with sale size.

ESTIMATES

The sales price per thousand board feet (MBF) was fitted to the month of sale (expressed as an integer beginning with January, 1953, equal to zero), the percent of upper grades contained in the sale (one of 40 percent, 50 percent, or 60 percent), the volume of the sale in MBF, dummy variables for the county of sale (Mendocino, Del Norte, or Humboldt), and dummy variables for the type of seller or buyer [State of California as buyer, California Department of Forestry (CDF) as seller, U. S. Forest Service (USFS) as seller, or an all-private sale].

The fitting of a sale price to its characteristics is a hedonic regression (Adelman and Griliches [1]) and has been previously applied to forestry by

Berck and Bible [2], Haynes [6], and Jackson and McQuillan [8], though their interests lie elsewhere than the Hotelling theory.

The choice of stumpage sales obviates the problems faced by other authors in measuring net price. When working with minerals, the price of the output is measured and an estimate (usually poor) of cost (which should be marginal cost) is subtracted to get net price. Stumpage price is the return to the owner so the same sort of measurement problem is not encountered.

The regression method chosen was alternating conditional expectations (ACE), which is a consistent form of nonparametric regression (Breiman and Freidman [3]). ACE finds the functions ϕ and Φ that best fit the equation

$$\Phi(y) = \sum_{i=1}^{N} \phi_i(x_i) + \varepsilon \qquad (1)$$

where y is the dependent variable, price; the x_i's are the independent variables, volume, percent upper grades, etc.; and ε is an error term. The functional form is quite general. It clearly contains linear, log-linear, and log-log regressions as special cases [ϕ and/or Φ could be the function log( ) or the identity function, 1x]. In fact, it will approximate any smooth function that can be written in the strictly additive form. The functions are not limited to being monotonic, so ϕ( ) could easily be quadratic, cubic, or any other shape. The price of this generality is that the results are not parameters or parametric representations of functions, rather they are plots. Thus, the way one presents ACE results is by presenting the plots.

The first plot gives the transformation function for price (see Figure 1). The actual value of price is on the horizontal axis while Φ (price) is on the vertical axis. The plot is basically increasing until $150/MBF and then

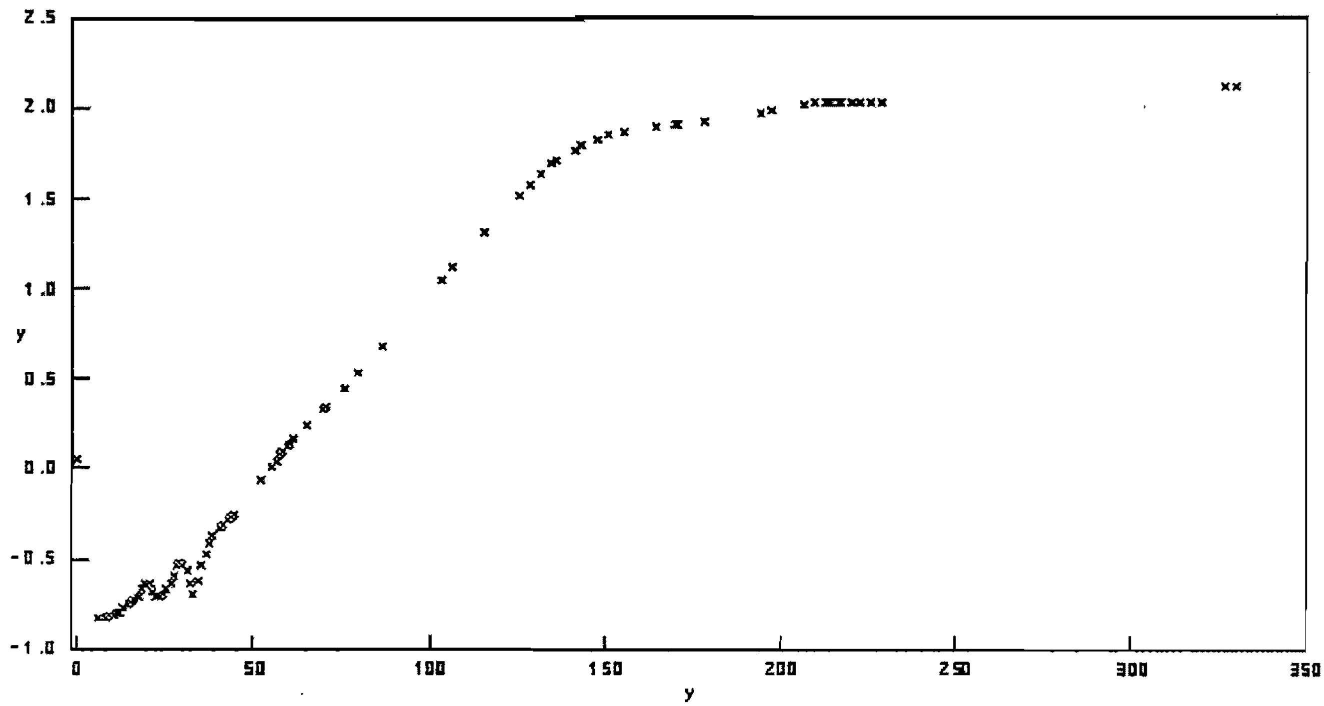

Figure 1. Price and transformed price.

increasing at a much slower rate. The second plot (Figure 2) gives month and its transformation. It is also basically increasing. Taken together, these plots elucidate the relationship of price and time, ceteris paribus. If one knew that, in month 144, price was approximately $50, one could reason as follows. From the time plot, month 144 has transformed value of approximately -0.5. A new month, say month 200, has a transformed value of about 0.0. Thus, in going forward 56 months, the transformed value of y should increase by about 0.5. The transformed value of price ($50) was originally about -0.3, so in month 200 it should be +0.2. The price that has a transformed value of 0.2 is about $80. These sorts of calculations illustrate what the plots mean, but they are not very accurate. To make accurate predictions, one uses the ACE prediction procedure, which smooths price on the transformed independent variables. This method correctly handles the error term (which was done in the above calculation by knowing the original price was $50). The result of the prediction procedure for price as a function of time is given in Table 1. The prices given are for a private sale in Humboldt County with the sample mean volume, 11,234, and the sample mean percent uppers, 4,703. The price predictions show a tremendous upward trend.

The plot for volume (Figure 3) exhibits a distinct peak at 7,000 MBF. Thus, price increases with volume for small sales and then decreases with volume for larger sales. A more careful look at the plot shows that there is also a local minimum and maximum, though Table 2, which gives price as a function of volume for 1976, reveals that the price dip is of no interesting magnitude.

There are also ACE plots for percent uppers and dummy variables for sale type and county. The percent uppers plot does not have any meaningful

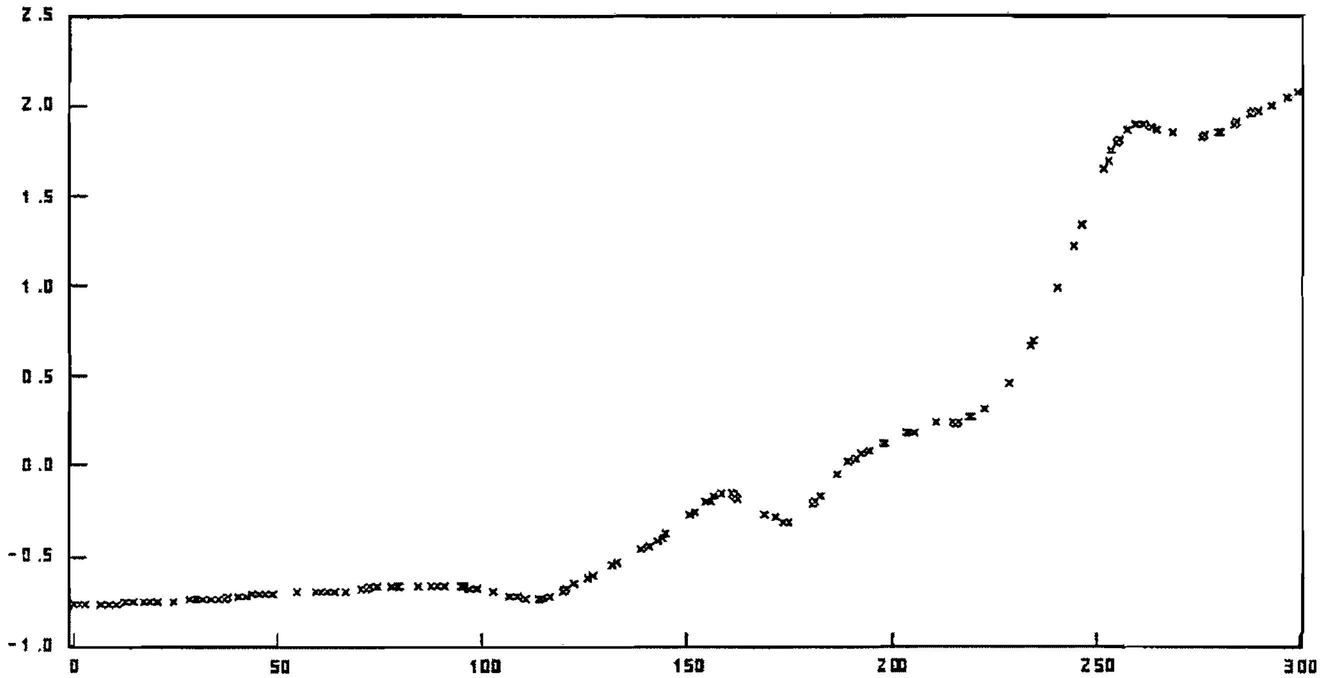

Figure 2. Time and transformed time.

Table I

Price by Year

Year	Price U.S. dollars	Year	Price U.S. dollars
1953	17.8	1966	46.2
1954	18.4	1967	41.2
1955	19.0	1968	44.2
1956	19.7	1969	59.8
1957	20.7	1970	64.6
1958	21.2	1971	67.8
1959	22.6	1972	80.9
1960	23.3	1973	118.6
1961	22.5	1974	169.0
1962	19.8	1975	189.8
1963	22.1	1976	186.2
1964	27.3	1977	198.0
1965	35.7		

Source: Computed.

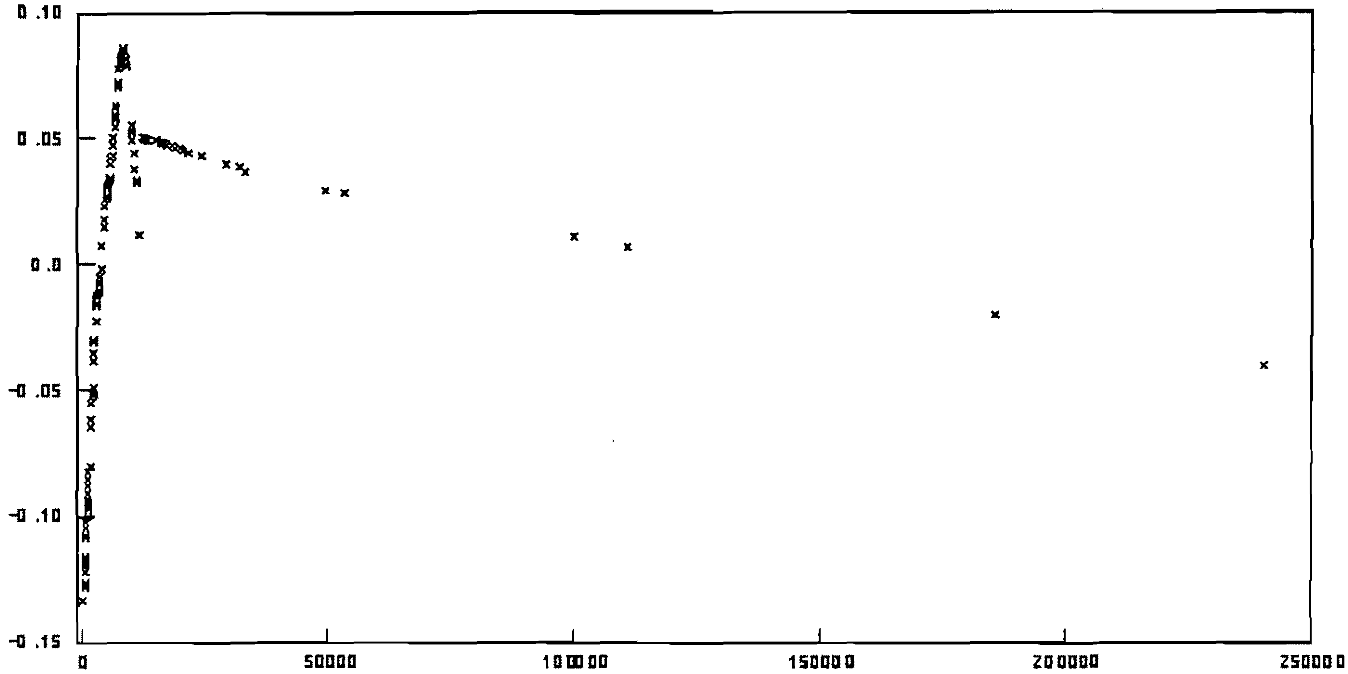

Figure 3. Volume and transformed volume.

Table II

Volume by Price for 1976

Volume MBF	Price U.S. dollars	Volume MBF	Price U.S. dollars
500	159.1	40,000	167.2
1,000	164.5	45,000	166.9
1,500	162.7	50,000	166.5
2,000	160.6	55,000	166.3
2,500	162.6	60,000	166.3
3,000	168.1	65,000	166.2
3,500	168.1	70,000	166.1
4,000	170.0	75,000	166.1
4,500	169.3	80,000	166.0
5,000	170.9	85,000	165.9
5,500	171.7	90,000	165.8
6,000	171.4	95,000	165.8
6,500	176.1	100,000	165.7
7,000	176.8	105,000	165.7
7,500	175.3	110,000	165.7
8,000	174.5	115,000	165.7
8,500	174.6	120,000	165.8
9,000	172.9	125,000	165.8
9,500	170.7	130,000	165.8
10,000	170.9	135,000	165.8
15,000	168.6	140,000	165.9
20,000	168.9	145,000	165.9
25,000	168.5	150,000	165.9
30,000	168.3	155,000	166.0
35,000	167.7		

Source: Computed.

variation in it and, of the other plots, the only seller type that is of interest is the CDF which gets a 20 percent premium over a private sale. In the interest of space, these plots are omitted.

Since the ACE plots are only consistent estimates of the true transformation functions, they are subject to statistical error in the same way that parameter estimates are. The problem is to find a way to test the hypothesis that price is first increasing and then decreasing in volume. The method chosen here is the bootstrap (Efron [4]).

Each of the sales can be viewed as the realization of a random process, so the data set is the empirical distribution of the data. One could then sample from this data set with replacement as a way of examining this random process. A bootstrap replicate is such a sample from the data set. Three hundred and fifty replicates were created and, for each replicate, ACE was run. In all but 2 of the 350 replicates, price was increasing in volume for low volumes. Thus, with over 99 percent confidence, we accept the hypothesis that very small sales sell at a discount relative to larger sales. Very large sales, however, were worth less per MBF than middle-sized sales in only 88 percent of the replicates. This provides less than conclusive evidence of a discount for large size (so it seems likely) yet, at the 95 percent significance level, one cannot reject the alternate hypothesis that very large size has no penalty.

COMPARISON WITH LINEAR REGRESSION

Compared with linear regression, ACE and the bootstrap are a considerable bother. The new technique would be valuable if it revealed evidence that the sample technique did not. Although the handling of the dummy variables, volume, and percent uppers easily carries over to linear regression, the handling of time does not. Hotelling's theory predicts price to increase exponentially with time, not linearly, but even that formulation would unduly burden ordinary least squares (OLS) relative to ACE--it still could not account for a jump in price, for instance, at the 1968 park take. By using least squares with a dummy variable for time, the least squares estimation had more flexibility than the competing ACE estimate, limited to a data smooth on time.

To allow least squares to find a nonmonotone response to volume, volume and its square and cubic were entered in the regression. Table 3 gives the regression results, and Table 4 gives the predicted prices. Comparing Tables 1 and 4 clearly shows the effects of the data smooth versus the dummy variables. In 1965, the prices predicted by ACE and OLS are nearly the same, but the smoothed (ACE) predictions do not drop nearly as far in 1967, nor rise as high as in 1970, nor drop as far in 1971, etc. The two methods clearly lead to very different views of the behavior of price.

On testing the predictive power of the two methods, least squares was found to be very slightly better. The method of testing was a jackknife (Efron). Observations were deleted one at a time and predicted using both methods. The bias, standard error, and root mean square error (RMSE) of predictions were computed.

Neither method had any appreciable bias, and least squares had a $1.00 better RMSE of prediction, $28.3 versus $29.2. As far as prediction is concerned, the methods are not distinguishable.

In the matter of response to volume, ACE shows that small sales are burdened but OLS does not. Four Wald tests of the hypothesis d price/d volume were run, each at a different volume (1,000; 2,000; 5,000; and 12,513 MBF).

Table III

Ordinary Least Squares

Variable	Estimated coefficient	Standard error	T-Statistic
PUPP	0.7	0.4	1.8
VOL	-3.96E-06	8.22E-05	-4.82E-02
MENDO	7.2	6.0	1.2
DELNO	5.2	7.5	0.7
STATE	23.5	19.6	1.2
CDF	2.2	7.1	0.3
USFS	7.3	8.8	0.8
D53[a]	-25.7	20.6	-1.2
D54	-28.3	21.3	-1.3
D55	-19.8	21.3	-0.9
D56	-18.9	22.1	-0.9
D57	-14.0	24.7	-0.6
D58	-17.7	22.6	-0.8
D59	-14.3	23.4	-0.6
D60	-12.6	23.0	-0.5
D61	-13.1	24.3	-0.5
D62	-18.3	22.8	-0.8
D63	- 9.9	22.4	-0.4
D64	- 4.6	23.3	-0.2
D65	1.1	22.7	0.0
D66	8.4	24.2	0.3
D67	- 6.7	22.3	-0.3
D68	17.3	20.6	0.8
D69	21.6	20.3	1.1
D70	41.0	20.7	2.0
D71	14.7	23.3	0.6
D72	17.8	25.0	0.7
D73	106.1	22.5	4.7
D74	147.0	20.0	7.3
D75	111.0	26.0	4.3
D76	131.5	19.6	6.7
D77	238.6	21.5	11.1

[a]D53 is a variable with value one only in 1953. The other D variables are also yearly dummies.

Source: Computed.

Table IV

Price by Volume from Linear Regression

Year	Price U.S. dollars	Year	Price U.S. dollars
1953	6.8	1966	40.9
1954	4.2	1967	25.9
1955	12.7	1968	49.8
1956	13.6	1969	54.1
1957	18.6	1970	73.5
1958	14.8	1971	47.2
1959	18.2	1972	50.3
1960	19.9	1973	138.6
1961	19.4	1974	179.5
1962	14.3	1975	143.5
1963	22.6	1976	164.0
1964	27.9	1977	271.1
1965	33.6		

Source: Computed.

Each of the test statistics $x^2(1)$ was about .3 while the critical value $x^2_{.95}(1)$ is 3.84. Thus, the OLS conclusion is that price is invariant to volume while the more flexible ACE results prove low volume sales are burdened.

CONCLUSION

Hotelling's theory, in its purest form, predicts that sale size has no effect on unit price. Miller and Upton included reserves--analogous to volume--in one of their linear regressions and concluded it had no effect, as Hotelling predicted. Our linear regressions--including third order polynomials-- agreed with the earlier findings. But, when ACE was used on the data, the picture changed. There very definitely is a small sale effect and there probably is a large sale effect.

The small sale effect is expected because there truly are large fixed costs associated with harvesting trees. The large sale effect is harder to explain. Financial discrepancies and lack of competition are possible explanations that allow one to maintain the Hotelling theory. Alternately, one could maintain the perfect markets hypothesis and discard the Hotelling theory. These data simply reject the joint perfect markets and Hotelling hypotheses. By rejecting one or the other of these hypotheses, we also reject the conclusion that one need not adjust comparable sales data for size.

LIST OF SYMBOLS

ϕ = phi

Φ = capital phi

Σ = capital sigma

ϵ = epsilon

1 = one

FIGURE LEGENDS

Figure 1. Price and transformed price.

Figure 2. Time and transformed time.

Figure 3. Volume and transformed volume.

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