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# **Safety and Capacity Analysis of Automated and Manual Highway Systems**

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Raja Sengupta**

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# Safety and Capacity Analysis of Automated and Manual Highway Systems

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## Abstract

This paper compares safety of automated and manual highway systems with respect to resulting rear-end collision frequency and severity. The results show that automated driving is safer than the most alert manual drivers, at similar speeds and capacities. We also present a detailed safety-capacity tradeoff study for four different Automated Highway System concepts that differ in their information structure and separation policy.

## 1 Introduction

This paper has two objectives. The first is to provide a basis for comparing the safety of an automated highway system (AHS) with present day limited access freeways. Secondly, it is generally accepted that the safety and capacity of an AHS are related. This paper provides one way of modeling this relationship, and uses the model to analyze different kinds of automated highway systems.

The safety of a highway system is signified by the frequency and severity of crashes that occur in the system in a given period of operation. In this paper we restrict ourselves to safety as signified by rear-end crashes (REC) of a specific type. The rear-end crashes we consider are two vehicle crashes where the two vehicles are in the same lane for a significant amount of time prior to the crash. The two vehicles in the REC are known as the leader and follower vehicles. In the crash, the front-end of the follower vehicle strikes the rear-end of the leader vehicle. It is also assumed that the REC is the first crash experienced by both the lead and follower vehicles. For example, we exclude secondary REC's, such as those that occur after the leader has hit an obstacle, or a follower has been hit by its follower. All vehicles are assumed to be light duty passenger vehicles (LDPV).

REC's may be broadly sub-divided ([23]) into crashes where the lead vehicle was not moving for a significant time prior to the crash, i.e., LVNM crashes, and crashes where the lead vehicle was decelerating for a significant time prior to the crash, i.e., LVD crashes. We restrict attention to REC's caused by LVD. This is a significant class of crashes. For example, 18 % of all 1994 crashes belong to this category [23].

The probability of a REC in a LVD situation is related to the magnitude of the lead vehicle deceleration. The higher the deceleration and the longer it lasts the greater the probability of a REC. There is very little data ([30]) available on the magnitude of lead vehicle decelerations

that cause crashes on current freeways and obviously no data available for AHS. Our approach is to estimate safety for the worst case LVD, i.e., we assume that the lead vehicle decelerates as hard as it can and continues to do so until it come to rest. We refer to this type of LVD as a hard braking emergency. We estimate REC frequency and severity for automated and manual driving in the hard braking emergency. This approach has the following advantage given the lack of data. We are able to show (section 5) that the REC probability in the hard braking emergency can be used to provide an upper bound on the number of REC's that might occur if some stretch of present day highway were to be replaced by an automated highway. It may be noted that hard braking is a rare event in the present highway environment. We expect the same in AHS, with hard braking emergencies arising only in response to rare malfunctions.

For automated driving, REC frequency and severity estimates are computed for different AHS speeds and capacities. For manual driving, the estimates are computed using data from [13], collected on a limited access freeway operating at approximately 65 mph and servicing 1600 vphpl. These AHS and manual driving estimates are compared with each other.

The safety of the follower vehicle in an LVD situation also depends on its speed and separation (range) from the leader at the onset of the lead vehicle deceleration. We know from our everyday driving experience, that for safety the separation should be an increasing function of the driving speed. We refer to this function, i.e., inter-vehicle separation as a function of speed, as separation policy. Currently, the driver decides separation policy. For automated vehicles, separation policy must be designed and programmed into the vehicle controllers. In either case, given a LVD scenario, the equations of motion can be used to relate safety, i.e., REC frequency and severity, to separation policy. This paper establishes the necessary models, populates the models with required data and analyzes the safety of several separation policies. We show how safety is an increasing function of separation at a specified speed.

Highway capacity is also related to separation policy in the following manner. Under equilibrium conditions, traffic flow on a freeway is the product of speed and density, which is the reciprocal of inter-vehicle separation, i.e.,

$$\phi = d(v) \times v = \frac{1}{L + s(v)} v, \quad (1)$$

where  $v$  is the highway speed,  $d(v)$  is the density,  $L$  is the vehicle length, and  $s(v)$  is the separation as a function of the speed  $v$ . By definition, capacity is the maximum value of  $\phi$ . For manual drivers, freeway capacity is usually between 1,800 and 2,200 vehicles per lane per hour. The above equation implies that capacity is a decreasing function of separation at a specified speed.

The dependence of both safety and capacity on separation policy is used to establish a quantitative relationship between the safety and capacity of an AHS. We show how the safety and capacity of an AHS are inversely related, i.e., increases in capacity are accompanied by decreases in safety. Four AHS concepts [28] are analyzed, namely Autonomous Individual Vehicles, Low Cooperative Individual Vehicles, High Cooperative Individual Vehicles, and Cooperative Platooned Vehicles. The high and low cooperative concepts are assumed to transmit emergency warnings from leader to follower vehicles in the hard braking emergency. Thus they presuppose some kind of communication capability. The autonomous concept on the other hand assumes that the follower vehicle must rely entirely on its range and range-rate sensors. No warnings are sent by the lead vehicle. The platooned concept differs from the others in that it assumes that there are groups (platoons) of vehicles at close spacings (2m for example), with large spacings (60m for example) separating the groups. This concept also presupposes the presence of a inter-vehicle communication

system. For each concept, we are able to analyze the sensitivity of safety to capacity. This allows us to assess the relative merits and demerits of the different AHS concepts.

We have developed a safety analysis software package that takes the input data, e.g., probability distributions of initial conditions, braking capability, reaction times, etc., and computes a collision velocity distribution. Collision velocity is the relative velocity of the follower w.r.t the leader at the moment of impact. The package incorporates a simple model of vehicle and braking dynamics and computes the collision statistics analytically. REC frequency is the total probability associated with the event that the collision velocity is greater than zero. REC severity is the expected value of the square of the collision velocity conditioned on the occurrence of a collision. This measure is proportional to the kinetic energy exchanged on impact. The measure is also a good indicator of the injury sustained by the occupants of the colliding vehicles, although that also depends on vehicle design, use of seat belts, airbags, etc. These are the two measures of safety.

The relevant literature is as follows. The safety benefits of Forward Collision Warning Systems (FCWS) are analyzed in [2, 1]. The analysis in [2] also considers Adaptive Cruise Control (ACC). The frequency and severity of collisions in an automated vehicle platoon has been analyzed in [5]. The mobility implications of the safety analysis assumptions are not explored in any of these studies. AHS capacity benefits have been analyzed in [4, 3], at a fixed level of safety. Once again, the explicit relationship between safety and capacity analyses are not explored. Here we analyze the variation of safety with capacity. Moreover, in our estimation of AHS REC mitigation benefits we account for the propagation of braking disturbances in a string of automated vehicles.

The layout of the paper is as follows. Section 2 describes the models used to compute collision velocity distributions. Section 3 describes how we assembled the data used to model manual driving and the four AHS concepts. It also describes the AHS concepts in some detail. Section 4, divided into two subsections, presents the analysis results. Section 4.1 compares manual and automated driving in the hard braking emergency. Section 4.2 presents the safety and safety/capacity analyses for the four AHS concepts. Section 5 describes how the results presented in section 4 can be used to estimate the number of rear-end crashes reduced by an automated highway. Section 6 summarizes the paper.

## 2 Modelling

This section describes the models used to analyze safety and capacity for different modes of driving. The four AHS concepts analyzed in subsequent sections are also described in more detail.

### 2.1 Analytical Process Description

The behavior of the two vehicles in the hard braking scenario is modeled by six parameters given below. The vehicles are modeled by second order systems with a pure time delay, in the same lane, with no lateral motion. No jerk constraints are imposed on either vehicle. Each set of values for the following parameters defines an initial condition from which hard braking may begin.

$\tau$	<i>Lumped Reaction Delay sec</i> This represents the total reaction delay (sensing + computing + actuating) of the follower vehicle. For manual driving this is the driver reaction time plus the braking actuation delay.
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$\Delta x$	<i>Inter-vehicle Spacing</i> $m$ This is the distance between the rear bumper of the lead vehicle and the front bumper of the follower vehicle.
$v_f$	<i>Follower Vehicle Velocity</i> $m/s$ The absolute velocity of the follower vehicle. This parameter can also model the highway operating speed.
$\Delta v$	<i>Relative Velocity</i> $m/s$ The velocity of the lead vehicle relative to the follower vehicle ( $v_f - v_l$ ).
$d_f^{\max}$	<i>Maximum Follower Vehicle Deceleration</i> $m/s^2$ The maximum braking capability of the follower vehicle.
$d_l^{\max}$	<i>Maximum Lead Vehicle Deceleration</i> $m/s^2$ The maximum braking capability of the lead vehicle.

### 2.1.1 Calculating the Collision Velocity

The hard braking Scenario is modeled by the six parameters listed above. It is assumed that the lead vehicle brakes at its maximum capability at time  $\mathbf{0}$ . The follower vehicle begins braking at its maximum at time  $\tau$ . The follower vehicle’s acceleration during the delay is assumed to be zero. The relative velocity at time  $\mathbf{0}$  is given by  $\Delta v$ . The collision velocity is calculated kinematically by determining the applicable collision case scenario from among the following:

Collision occurs...

1. ...before the follower vehicle reacts, before the lead vehicle stops.
2. ...after the follower vehicle reacts, before the lead vehicle stops.
3. ...before the follower vehicle reacts, after the lead vehicle stops.
4. ...after the follower vehicle reacts, after the lead vehicle stops.
5. ...never. The follower vehicle stops in time.

Figure 1 illustrates the way in which we use step functions to approximate the braking action of the vehicles. It is assumed that the actuator commands of both the lead and rear vehicles are step commands that saturate the brake actuator. The commands are shifted in time by the communication or sensing delay. The response of the brake actuator is a dead time in series with a first order lag response [17]. The lead and follower vehicle responses are shown as “Front car response (actual)” and “Back car response (actual).” Some typical dead times and lags are described in table 1.

	Lead car	Rear car
Dead time (ms)	40	60
Lag time (ms)	40	60
Max. decel (g)	0.8	0.5

Table 1: Brake model parameters

The step approximations of the lead and follower car brake response used in the collision velocity calculation model are labelled “Front car (approx)” and “Back car (approx)” respectively. The step functions “Front car (approx)” and “Back car (approx)” are shifted in time by  $\tau$ . In this example  $\tau$  is  $150ms$ .

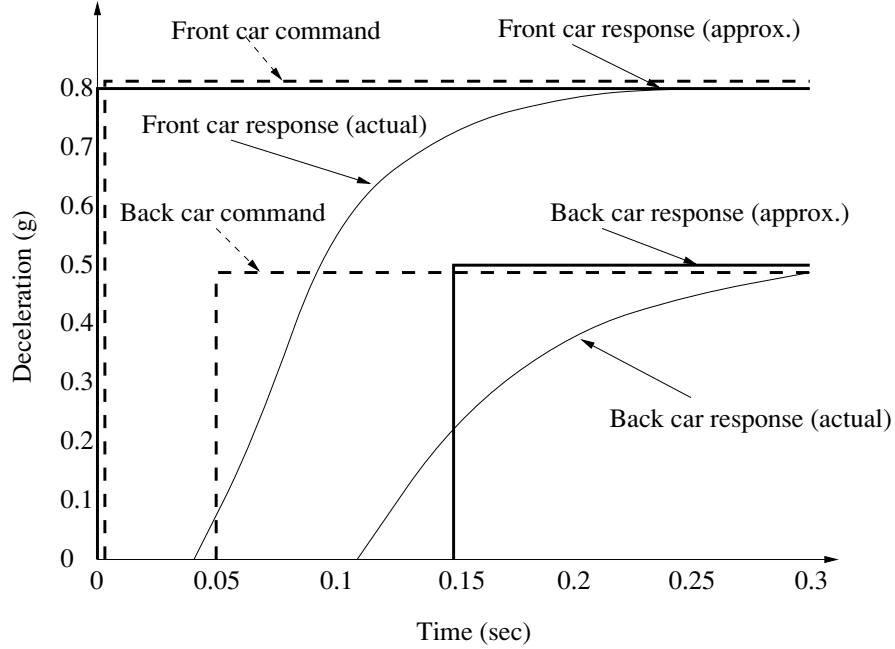


Figure 1: Brake response approximation

### 2.1.2 Calculating the Collision Velocity Distribution

If one or more of the six modeling parameters is a random variable, then the collision velocity will also be a random variable. The software package uses the equations of motion to map the deterministic and stochastic input parameters to a collision velocity distribution and the associated safety metrics. In order to calculate this collision velocity distribution, the vehicle parameter distributions are first discretized. For every combination of these discrete parameter values, a collision velocity is calculated using the equations of motion. The probability of that collision velocity is incremented by the probability of the corresponding input parameter value combination.

### 2.1.3 Calculating the Highway Capacity

If the inter-vehicle spacings are known, it is easy to calculate an upper-bound on the per-lane flow of the automated highway at a given operating speed by one of the following formulae:

Individual Vehicles:

$$C = 3600 \cdot \frac{v}{(\Delta x + L)} \quad (2)$$

Platoons:

$$C = 3600 \cdot \frac{(v \cdot N)}{((L \cdot N) + i(N - 1) + I)} \quad (3)$$

where:

$C$	Highway Capacity $vphpl$
$L$	Vehicle Length $m$
$N$	Number of Vehicles in a Platoon
$i$	Spacing Between Vehicles in a Platoon (Intra-Platoon Spacing) $m$



$I$	Spacing Between Platoons (Inter-Platoon Spacing) $m$
$v$	Speed in m/s

We refer to this upperbound as pipeline capacity. Alternatively, given capacity and speed, we can use Equation 2 to get  $\Delta x$  and Equation 3 to get  $i$  or  $N$ . Then the methods described in Sections 2.1.1 and 2.1.2 can be used to analyze safety. Highway capacity under manual driving is obtained from flow measurements on actual highways.

## 2.2 The AHS Concepts

We describe the four AHS concepts analyzed in this paper. It is assumed that no manually driven vehicle are present on the AHS. This assumption is made primarily due to the lack of data required to model mixed operation. In the individual vehicle concepts it is assumed that at any given AHS operating speed, the minimum allowable inter-vehicle spacing is identical for all vehicles. In the platooning concept we assume there are two spacings, an intra-platoon spacing for vehicles within a platoon and an inter-platoon spacing between platoons. The four AHS concepts are as follows.

- *Autonomous Individual Vehicles* - This type of AHS consists of vehicles that are self reliant. Vehicle do not send emergency messages in the hard braking scenario. The follower vehicle must use its range, range rate sensors to detect that the preceding vehicle is braking hard. This concept has relatively high detection delays.
- *Low Cooperative Individual Vehicles* - This type of AHS consists of vehicles that send emergency messages through a shared access communication channel that does not provide delivery time guarantees. This concept has lower detection delays than the autonomous concept.
- *High Cooperative Individual Vehicles* - This type of AHS consists of vehicles that send emergency information through a communication channel with delivery time guarantees to the vehicle behind. This concept has the lowest communication and detection delays.
- *Platooned Vehicles* - This type of AHS consists of platooned vehicles. Vehicles in a platoon follow the vehicle ahead at a small separation (of the order of 1-2 m) independent of speed. Different platoons are isolated from each other by large gaps. It is assumed that platoons send emergency messages to each other exactly like the low cooperative case and vehicle within a platoon send emergency messages to each other exactly like the high cooperative case. The inter-platoon separation is selected so as to make the probability of inter-platoon collision negligible in the hard braking scenario. Thus for the platooning concept we concentrate only on the intra-platoons collisions.

The technological differences between these concepts are modelled in the value of the lumped reaction delay used to analyze the concept. Section 3.1 describes how the value of this parameter is derived for each concept.

## 3 Parameter Modeling

With this framework in place, we now turn to the task of filling in the values or distributions for the six parameters. Depending on the highway system under evaluation, parameters may have different

values or distributions. For AHS vehicles, parameter values are determined by the capabilities of automated vehicle components (i.e., sensors, computers, actuators) and the automated highway parameters (i.e., operating speed, spacing policy). For manually driven vehicles, parameter values are determined by the capabilities of the vehicle and the driver. The sources of data are described below for each parameter for each highway system.

### 3.1 Parameter Values for AHS Concept Modeling

#### 3.1.1 Automated Vehicle Parameters

The physical system parameters describe the aggregate capabilities of the automated vehicle's sensors, transceivers, onboard computers, and actuators. Various longitudinal control laws may be used on a single automated vehicle, but these parameters would not change since they are determined by hardware constraints.

**Lumped Reaction Delay** For automated vehicles, this lumped value is the sum of sensing, computing, communicating, and actuating delay. Each component of the delay is described as follows:

$\tau_{act}$	100 ms This delay is the actuation time between the initial hard braking step command and the step function representing the maximum braking response. Although current production vehicles can not achieve this value, it is consistent with brake actuator tests, on prototype automated vehicles as reported in [17].
$\tau_{com,hi}$	20 ms This delay is the communication and computation time needed to discern that the vehicle ahead is applying emergency braking, assuming that the lead vehicle communicates its <i>onset of emergency braking</i> on a guaranteed delivery time channel, i.e., there is high cooperation between individual vehicles. Such communication delivery times are used for automated vehicle experiments as described in [10].
$\tau_{com,lo}$	50 ms This delay is the communication and computation time needed to discern that the lead vehicle is applying emergency braking, assuming that the lead vehicle communicates its <i>onset of emergency braking</i> over a channel without delivery time guarantees. It is assumed that the delivery time is exponentially distributed with mean delivery time 20ms. 50ms represents the time interval within which the probability of the message being received is 0.9.
$\tau_{sen,no}$	200 ms This delay is the sensing and computation time needed to discern that the lead vehicle is applying emergency braking. The lead vehicle does not communicate an emergency message during hard braking. Thus hard braking detection is by range and range-rate sensors. The value of 200 ms allows estimation of the acceleration of the lead vehicle using 10 sensor readings at the 20 ms loop time reported in [10].

In order to calculate the total lumped delay, we sum the appropriate delay components as follows:

Autonomous individual vehicles:  $\tau = \tau_{sen,no} + \tau_{act} = 300 \text{ ms}$

Low-Cooperative Individual Vehicles:	$\tau = \tau_{com,lo} + \tau_{act}$	= 150 ms
High-Cooperative Individual Vehicles:	$\tau = \tau_{com,hi} + \tau_{act}$	= 120 ms
Platoon Leader Vehicles:	$\tau = \tau_{com,lo} + \tau_{act}$	= 150 ms
Platoon Follower Vehicles:	$\tau = \tau_{com,hi} + \tau_{act}$	= 120 ms

Figure 1 indicates that the lead vehicle braking is modeled by a step function. In reality, this is impossible and the lead vehicle brake response would be lagged like that of the following vehicle though it may be faster or slower than the follower. Since our lag values have are based on extremely fast acting prototype brake systems we have no knowledge of the possible variations in brake response delays in a population of vehicles using such brake systems. Therefore our step function approximation is conservative in the sense that if the following vehicle is safe for the step function disturbance, other things being equal, it will be safe for any actual lead vehicle brake response.

**Relative Velocity** In vehicle following, the desired relative velocity between any two vehicles is zero. However, in any vehicle following control system, there will be fluctuation about the desired velocity. Since both lead and follower vehicles are trying to track the same velocity, any relative velocity is due to velocity tracking errors arising from controller performance limitations. For each automated vehicle design, a worst-case velocity tracking error of 1.5% is used. This value is consistent with test results for an actual automated vehicle longitudinal control system [7]. Note that using the worst case value makes the safety estimates conservative.

**Maximum Follower/Lead Deceleration** The braking capabilities of the AHS vehicle population are modeled by using data on the maximum deceleration on dry pavement of new light duty passenger vehicles as compiled in [8]. The proportion of each type of vehicle on the highway is derived from the North American production figures in [9]. Braking capability of a vehicle is highly sensitive to the tire and roadway surface condition. In order to account for wear, we assume a 30% derating factor applies to the vehicle’s maximum deceleration. This gives us low and high braking capability values that are consistent with data estimated in [6]. Since all the analyses in the literature [3, 2, 1] make assumptions on vehicle braking capabilities, better data on brake wear and tear is extremely desirable. This data yields the histogram shown in Figure 2<sup>1</sup>. A truncated Gaussian distribution is fitted to this data as shown in the figure. The gaussian is clipped at -10 and -4  $m/s^2$ , has a mean of -7.01  $m/s^2$ , and a standard deviation of 1.01  $m/s^2$ .

### 3.1.2 Automated Highway Parameters Value

We assume that the AHS sets the highway operating speed and separation policy. Recall, that speed and separation policy determine capacity under equilibrium conditions.

**AHS Operating Speed** Since all vehicles are assumed to be traveling at the operating speed of the AHS, the follower vehicle’s velocity is fixed at that operating speed. The lead vehicle speed is assumed to be lower by the velocity tracking error. The AHS operating speeds investigated in this paper are 20, 30, and 40  $m/s$  (45, 67, and 89  $mph$ ).

<sup>1</sup>Thus we assume that braking capabilities of AHS vehicles would be similar to current production vehicles.

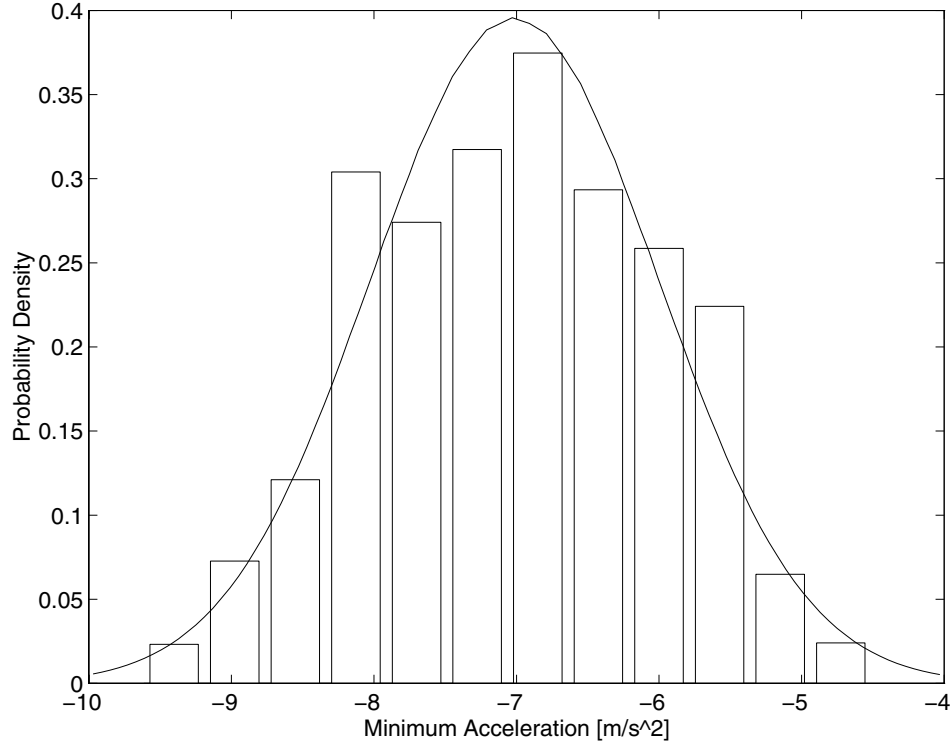


Figure 2: Estimated Distribution of Braking Capabilities for all Automated Vehicles

**Inter-Vehicle Spacing** This study evaluates the performance of AHS concepts at highway capacities between 500 and 8000 *vphpl* at the AHS operation speeds mentioned above. Thus, Individual Vehicle spacings are calculated from highway capacity and speed using equation 2 to be from 4 to 280 *m*. For vehicles traveling in platoons, the inter-platoon spacing is determined from the upper and lower limits of the distribution in figure 2. The inter-platoon spacing is selected so that if the last vehicle of the lead platoon was to brake at  $1g$  (maximum value of the distribution) and the first vehicle of the follower platoon was to brake at  $0.4g/A$  (minimum value of the distribution,  $A$  is the platoon brake amplification factor) then the follower platoon will be able to stop without colliding with the lead platoon in the hard braking scenario. The amplification factor  $A$  represents the maximum increase in vehicle braking effort that occurs as vehicles brake successively in a platoon. In [15], it is shown that in order to achieve string stability (attenuation of spacing errors in a string of vehicles) during constant space following, the following vehicles should apply higher actuation force than the platoon leader. We set  $A = 1.2$  as determined in [15]. This value ensures that the follower platoon stops without any intra-platoon collisions. All collisions occur in the lead platoon of the hard braking scenario. This stringent inter-platoon spacing design criterion is imposed to make the probability of platoon-platoon collisions in the hard braking scenario negligible <sup>2</sup>. Thus, the inter-platoon spacing varies over the above AHS speeds within the range of 42 to 163 *m*. In order to investigate changes in capacity at a fixed speed for platooned vehicles, we vary the number of vehicles in a platoon or the intra-platoon spacing, but not the inter-platoon spacing. Intra-platoon spacing is small so that even if the lead vehicle brakes hard, the follower vehicle will hopefully not

<sup>2</sup>The intra-platoon collisions in the lead platoon result in instantaneous speed reductions that may give rise to inter-platoon collisions [20]. This phenomenon has not been accounted for while designing inter-platoon spacing in this study.

be involved in a high relative velocity collisions<sup>3</sup> [16]. This study considers intra-platoon spacings between 1 and 10  $m$ .

## 3.2 Parameter Values for Manual Driving Model

Just as an AHS was decomposed into a vehicle and an AHS system, a manual highway system can be decomposed into a vehicle and the driver behavior. Once again, we evaluate each parameter in the model based upon the performance of these two components.

### 3.2.1 Vehicle Parameter Values

The maximum acceleration of the follower vehicle and the braking capabilities of the vehicle population are the same as those described above for automated vehicles. The lumped delay described above must now change since only the actuator delay applies to the physical system. All of the delays associated with computing and sensing hardware are replaced by delays associated with the driver performing these functions. The portion of the lumped reaction delay that originates from the brake actuation system is assumed to be the same as that for the automated vehicles, i.e.,  $\tau_{act} = 100 \text{ ms}$ . Thus the driver is assumed to be operating a vehicle with the same actuation capabilities.

### 3.2.2 Driver Behavior Parameter Values

The parameters which depend on the behavior of a human driver are described below. The parameters are estimated based on data collected under highway driving conditions.

**Lumped Reaction Delay: Driver Reaction Time** As mentioned above, the lumped delay is composed of a braking actuator delay in series with a driver reaction delay. An estimate of the distribution of driver reaction times for highway driving was obtained by fitting a lognormal distribution to reaction time data collected by Michael Sivak as detailed in [12]. The data was collected by measuring brake reaction times of unalerted drivers at highway speeds up to about 20  $m/s$ . The lognormal distribution is fixed with median  $\lambda = 1.07 \text{ s}$ , mean  $\mu = 1.21 \text{ s}$ , standard deviation  $\sigma = 0.63 \text{ s}$ , and dispersion parameter  $\zeta = 0.49$ . This reaction time distribution is discretized as shown in Figure 3 to calculate the collision velocity distribution.

**Inter-Vehicle Spacing/Relative Velocity** An infra-red sensor measured the inter-vehicle distance and the correlated relative velocity during vehicle following for a sample of 36 drivers in a study conducted by researchers at the University of Michigan Transportation Research Institute. This data was collected on a 55 mile sequence of highways in Ann Arbor and the Metropolitan Detroit area [13]. We used a probability distribution of the correlated range and range-rate from this data as shown in Figure 4. Note that this range and range rate data is collected using a separate experiment from the one that is used to get reaction times. Hence the driver reaction time and spacing distributions used in this study are uncorrelated.

**Follower Vehicle Velocity** In actuality, the velocity of the follower vehicle is correlated to the spacing and relative velocity distribution described above. However, the velocity data was recorded

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<sup>3</sup>Refer to [20] for the effect of multiple collision analysis on platoon collision dynamics.

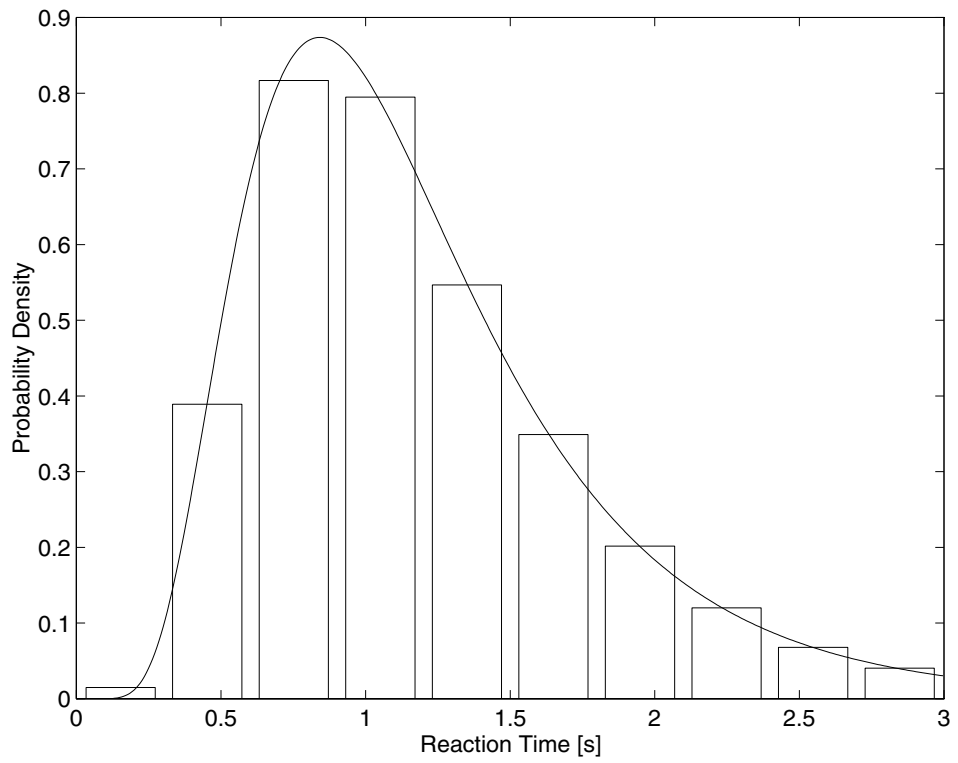


Figure 3: Reaction Time of a Typical Unalerted Driver

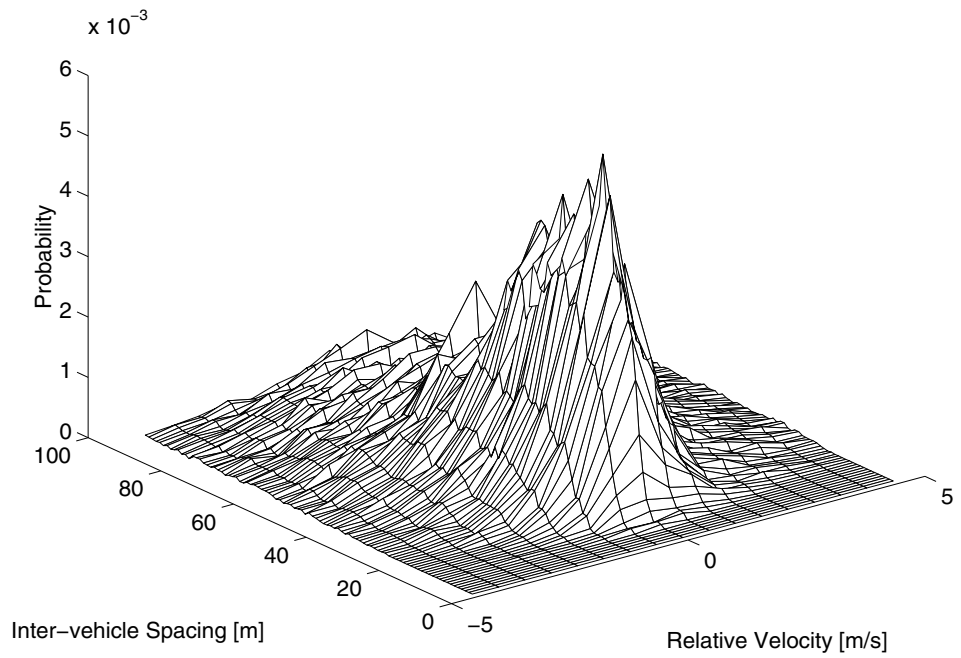
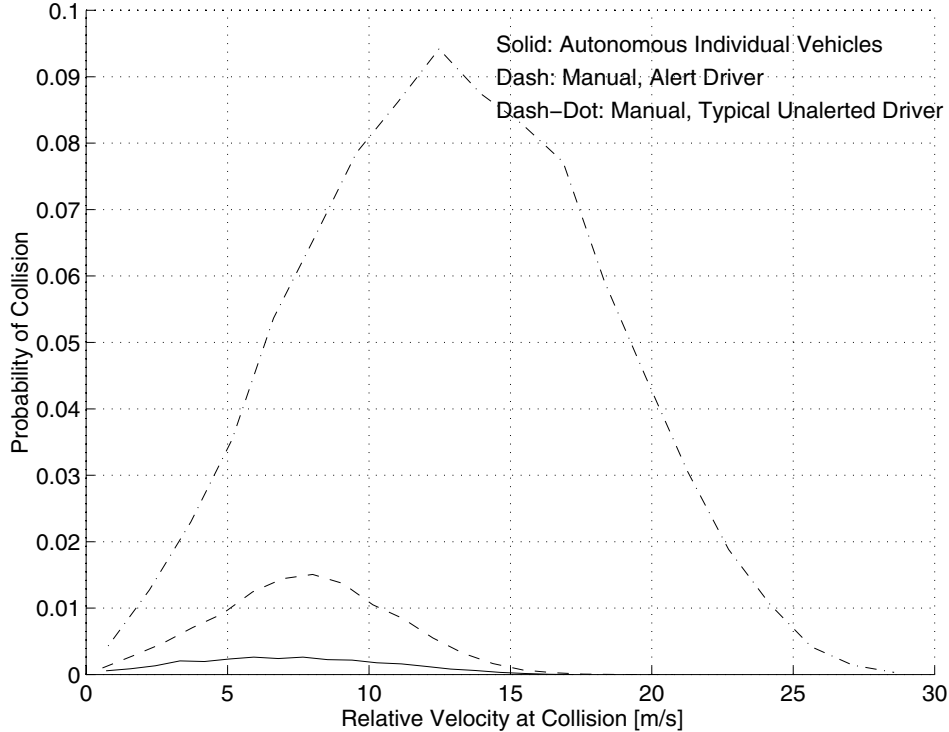


Figure 4: Highway Vehicle Following for a Typical Manual Driver



AHS Concept	Capacity	Total Prob of Collision	Expected $\Delta v_{coll}^2$
Autonomous	2500 <i>vphpl</i>	0.028	64.1 $m^2/s^2$
Manual Alert	< 1500 <i>vphpl</i>	0.11	69.5 $m^2/s^2$
Manual Typical	< 1500 <i>vphpl</i>	0.87	195 $m^2/s^2$

Figure 5: Safety Comparison between Manual and Automated Vehicles

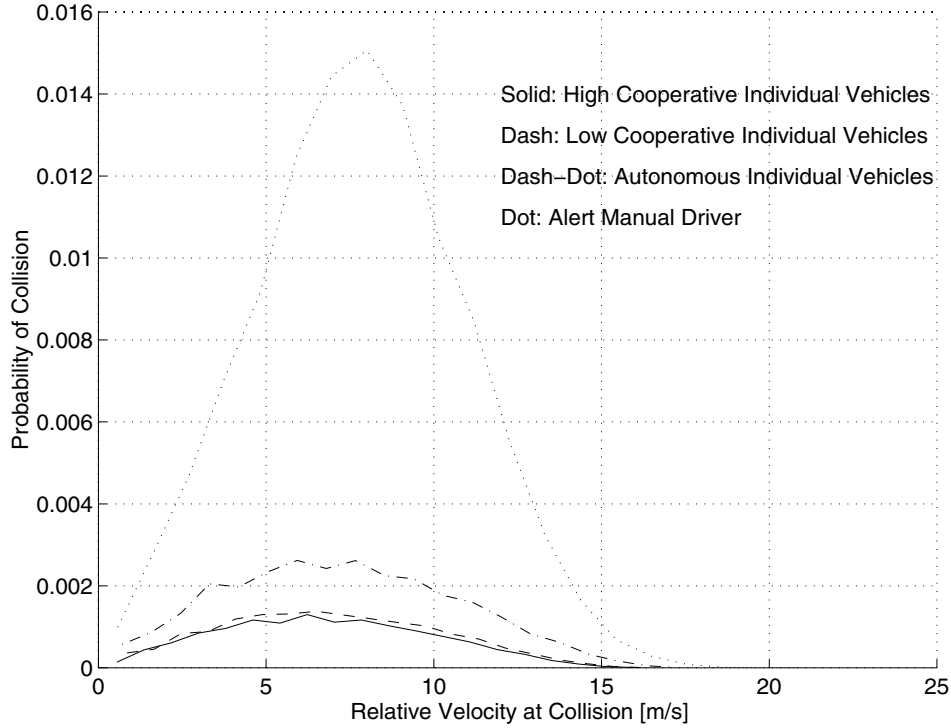
independently of range and range rate. In order to estimate the collision velocity distribution under hard braking for manual driving, we assumed that the velocity of the follower vehicle was the mean of the velocity distribution in [13]. The mean of the velocity distribution is  $v_{mean} = 29.3 \text{ m/s}$ .

## 4 Results

First, we compare the safety of automated and manual driving in the hard braking scenario. In Section 4.2, we investigate safety and capacity for the four AHS concepts in detail.

### 4.1 Comparison of Automated and Manual Driving

Figure 5 has three collision velocity distributions that show how the safety of autonomous individual vehicles compares to the safety of manual driving at a highway operating speed of  $30 \text{ m/s}$ . The spacing of the autonomous vehicles is chosen to correspond to an AHS capacity of 2500 *vphpl*. The manual driving data from [13] was collected under non rush hour conditions along a route whose average volume was less than 1600 *vphpl*. Thus the demonstration of the increased safety of the automated vehicles over the manual case is even stronger, since the level of provided utility of the



AHS Concept	Capacity	Total Prob of Collision	Expected $\Delta v_{coll}^2$
Autonomous	2500 <i>vphpl</i>	0.028	64.1 $m^2/s^2$
Low Cooperative	2500 <i>vphpl</i>	0.015	58.2 $m^2/s^2$
High Cooperative	2500 <i>vphpl</i>	0.013	56.9 $m^2/s^2$
Manual Alert	< 1600 <i>vphpl</i>	0.11	69.5 $m^2/s^2$

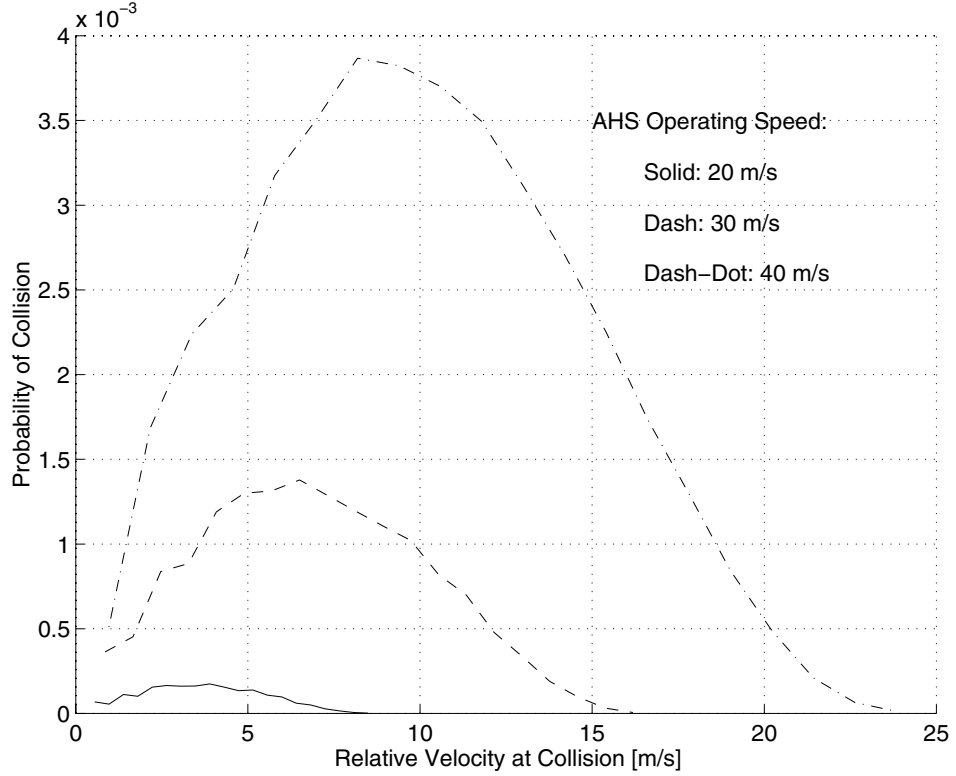
Figure 6: Safety Comparison at Different Levels of Cooperation

automated system is also higher than the manual baseline. The two collision velocity distributions shown for manual driving reflect two possible assumptions about the driver reaction delay. The curve representing an ‘alert’ driver was derived assuming a reaction time of 0.5 s, which is near the minimum delay reported in [12]. The other curve is derived using the full reaction delay distribution described above for unalerted highway drivers. Thus, these two distributions form a reasonable range for the safety of manual driving under the hard braking scenario. If driver reaction delay data correlated to vehicle following range and range-rate is used, then the true collision velocity distribution would likely fall between the two derived bounds. The table accompanying Figure 5 provides some statistics about these collision velocity distributions. Both frequency and severity safety metrics indicate the benefit of automation.<sup>4</sup>

Adding cooperation between vehicles can increase safety as shown in Figure 6. The alert manual driver baseline from Figure 5 is included for comparison. Again, the table accompanying Figure 6 quantifies the benefit of decreasing the hard braking delay by increasing cooperation between vehicles. The low cooperative vehicles are much more safe than the autonomous vehicles because

<sup>4</sup>In a multilane highway, vehicles may also use lane change to avoid collisions under hard braking scenario. This effect is not studied in this paper. It might have different effect on automated and manual highway environment.





AHS Speed	Capacity	Total Prob of Collision	Expected $\Delta v_{coll}^2$
20 m/s	2500 vphpl	0.002	16.8 m <sup>2</sup> /s <sup>2</sup>
30 m/s	2500 vphpl	0.015	58.2 m <sup>2</sup> /s <sup>2</sup>
40 m/s	2500 vphpl	0.041	121 m <sup>2</sup> /s <sup>2</sup>

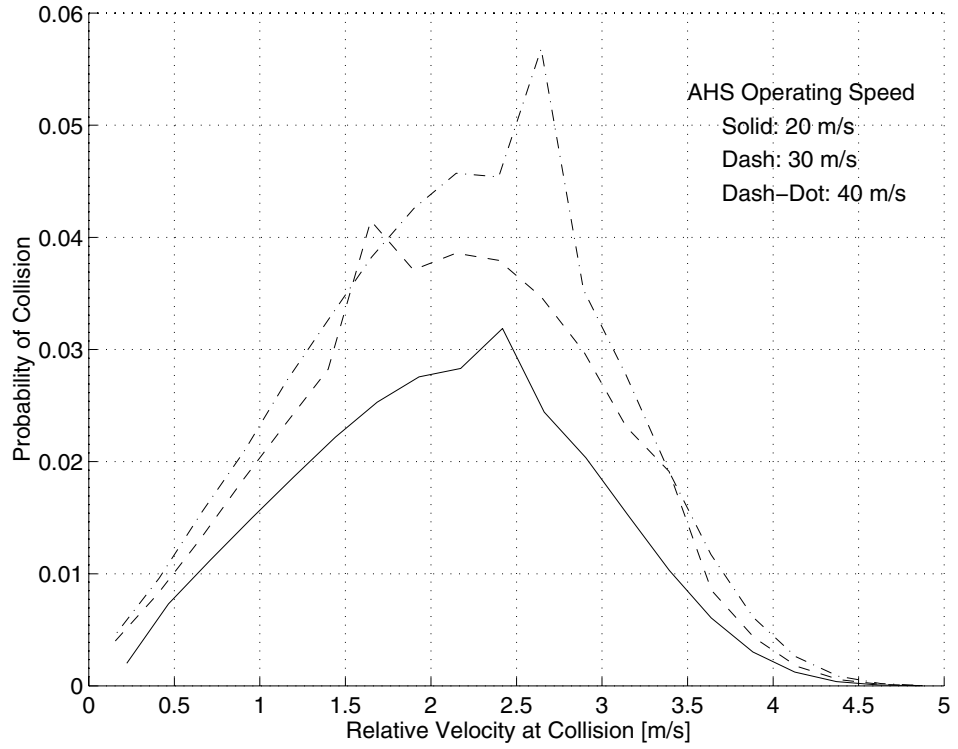
Figure 7: Low Cooperative Individual Vehicle Safety at Different AHS Speeds

of the significant shortening of the hard braking delay (from 300 ms to 150 ms). The safety gain between the high cooperative and low cooperative Individual Vehicles is relatively small. It results from the slight shortening of the hard braking delay (from 150 ms to 120 ms). As before, the automated vehicle evaluations are done at 30 m/s and 2500 vphpl, while the manual baseline is near the same speed, but at less than 1600 vphpl.

## 4.2 Safety and Capacity analysis of AHS Concepts

In this section, we analyze and compare the four AHS concepts. The safety and capacity effects of variations in intra-platoon spacing and AHS operating speed are also studied. We start by analyzing the variation of AHS safety with operating speed.

**AHS Operating Speed:** We investigate AHS safety as a function of speed at a fixed capacity. Figure 7 shows this relationship for low cooperative individual vehicles, and Figure 8 shows the same result for platooned vehicles. The other individual vehicle cases are similar to the low cooperative



AHS Speed	Capacity	Total Prob of Collision	Expected $\Delta v_{coll}^2$
20 m/s	2500 vphpl	0.27	5.03 m <sup>2</sup> /s <sup>2</sup>
30 m/s	2500 vphpl	0.37	5.13 m <sup>2</sup> /s <sup>2</sup>
40 m/s	2500 vphpl	0.44	5.29 m <sup>2</sup> /s <sup>2</sup>

Figure 8: Platoon Safety at Different AHS Speeds with intra-platoon spacings of 2m

individual vehicle. Both individual vehicles and platoons are safer at lower speeds, though the safety of platoons is less sensitive to changes in speed at a fixed capacity (2500 *vphpl*). To keep a constant capacity at higher speeds, the individual vehicles travel at larger spacings. This larger spacing allows for a larger relative velocity build up during the time until collision in the hard braking scenario. Therefore, the severity of individual vehicle collisions increases significantly with speed. To keep a constant capacity for platoons at a fixed intra-platoon spacing while increasing AHS operating speed, each platoon must contain fewer vehicles. But since the intra-platoon spacing is always small, the collision severity remains low for all three speeds. This seeming benefit of platooning should be re-evaluated in future to account for the effects of secondary collisions that are not studied here.

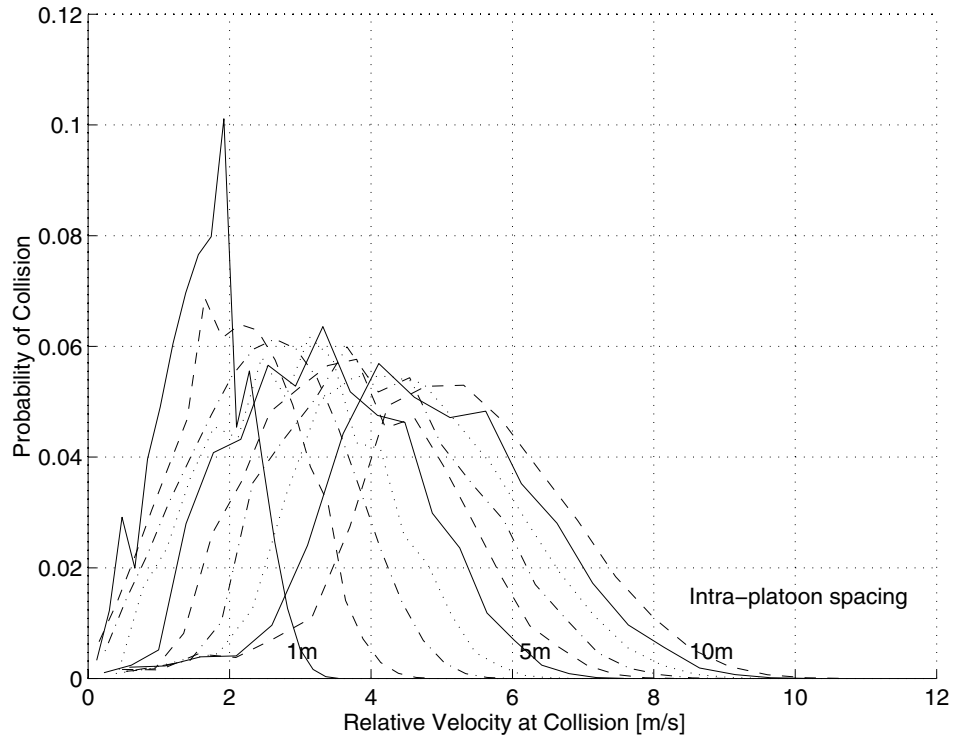
**Intra-platoon Spacing:** The intra-platoon spacing affects safety as shown in Figure 9. As the spacing increases from 1 to 10 *m*, the probability of collision decreases. This is because there are more combinations of favorable lead and follower braking rates for which collisions can be avoided. In many cases however, the vehicle pair still collides, and at larger intra-platoon spacings the severity of the collisions steadily increases. This result demonstrates the advantage of minimizing intra-platoon spacing to reduce the risk of high severity collisions.

**Platoons vs Individual Vehicles** The next result quantifies the differences in frequency and severity of collisions for low cooperative individual vehicles and platooned vehicles. Specifically, these two AHS concepts are shown to achieve safety by reducing a different one of the two safety metrics. In Figure 10, platoons have a high total probability of collision for any speed, but the expected severity of the collisions is low. For low cooperative individual vehicles, the probability of collisions are lower, but those collisions that do occur are much more severe. Again, this result is derived at a capacity of 2500 *vphpl*.

**Safety-Capacity Tradeoff:** The remaining set of results show the relationship between safety and capacity for each of the AHS concepts studied. Figures 11 and 12 plot the two safety metrics defined earlier against AHS capacity at 30 *m/s*. The y-axis of each plot is defined such that increased safety is in the +y direction. Thus, in Figure 11, the y-axis represents the probability that a collision is avoided, and in Figure 12, it represents the negative of the expected square of the collision velocity. Curves are shown for Individual Vehicles, platoons at constant intra-platoon spacing, and platoons of constant size with variation in intra-platoon spacing. In order to best understand the safety-capacity relationship for each AHS concept, we examine the frequency and severity safety metrics in conjunction.

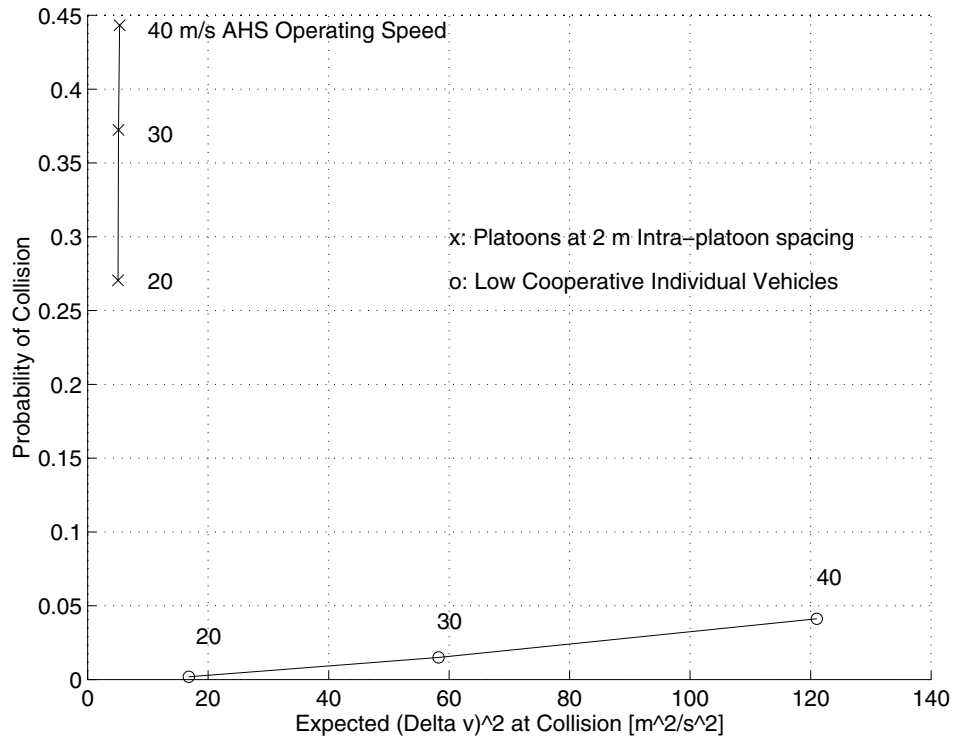
For platooned vehicles, we see in Figure 11 that collisions are avoided entirely at capacities below about 1200 *vphpl*. At these low capacity levels, the demand can be satisfied by 1 car platoons traveling at very large spacings. As capacity increases at a constant intra-platoon spacing however, more followers are added to the platoons. They are not able to completely avoid hard braking collisions, and the probability of no collision drops off sharply at 1200 *vphpl* as the first followers are added, and then flattens out at high capacities. This flattening is a result of the proportion of followers within a platoon. To find the probability of collision for platoons of *N* vehicles in which the leader maintains a safe spacing from the platoon in lead, the probability of collision for a follower is multiplied by  $\frac{N-1}{N}$ . For large platoons,  $\frac{N-1}{N}$  approaches one asymptotically, and hence the probability of collision within a platoon asymptotically approaches the probability of collision of a single follower.

Looking at Figure 12, we see that the collision severity at constant speed and intra-platoon spacing



Intra-platoon spc	Total Prob of Collision	Expected $\Delta v_{coll}^2$
1 m	0.73	2.94 $m^2/s^2$
2 m	0.62	5.13 $m^2/s^2$
3 m	0.58	7.38 $m^2/s^2$
4 m	0.54	9.87 $m^2/s^2$
5 m	0.51	12.6 $m^2/s^2$
6 m	0.48	15.6 $m^2/s^2$
7 m	0.45	18.9 $m^2/s^2$
8 m	0.42	22.4 $m^2/s^2$
9 m	0.39	26.2 $m^2/s^2$
10 m	0.36	30.2 $m^2/s^2$

Figure 9: Platoon Safety at Different Intra-platoon Spacings



Low Cooperative Individual Vehicles

AHS Speed	Capacity	Total Prob of Collision	Expected $\Delta v_{coll}^2$
20 m/s	2500 vphpl	0.002	16.8 m <sup>2</sup> /s <sup>2</sup>
30 m/s	2500 vphpl	0.015	58.2 m <sup>2</sup> /s <sup>2</sup>
40 m/s	2500 vphpl	0.041	121 m <sup>2</sup> /s <sup>2</sup>

	AHS Speed	Capacity	Total Prob of Collision	Expected $\Delta v_{coll}^2$
platoon vehicles	20 m/s	2500 vphpl	0.27	5.03 m <sup>2</sup> /s <sup>2</sup>
	30 m/s	2500 vphpl	0.37	5.13 m <sup>2</sup> /s <sup>2</sup>
	40 m/s	2500 vphpl	0.44	5.29 m <sup>2</sup> /s <sup>2</sup>

Figure 10: The Probability/Severity Safety Trade-Off

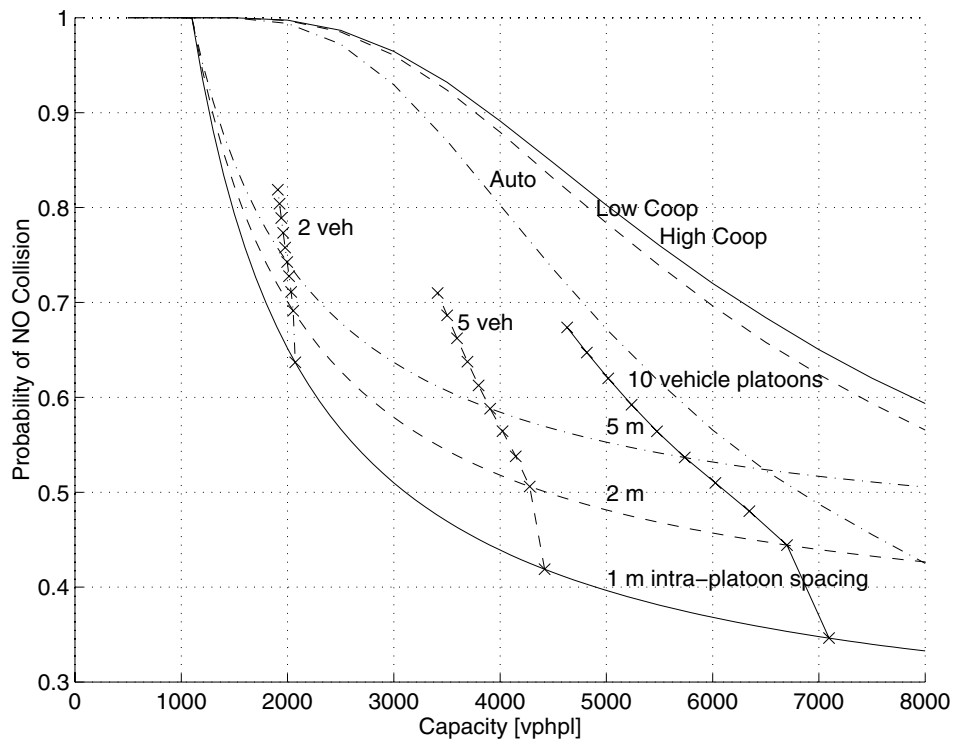


Figure 11: The Safety/Capacity Relationship for all Separation Policies: Frequency Metric

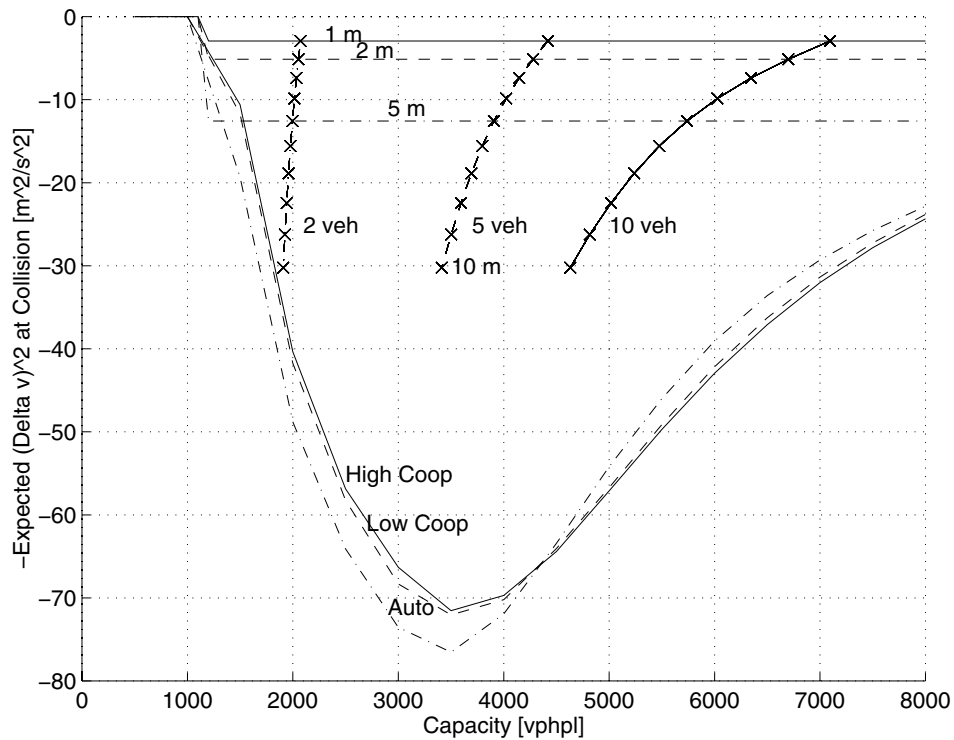


Figure 12: The Safety/Capacity Relationship for all Separation Policies: Severity Metric

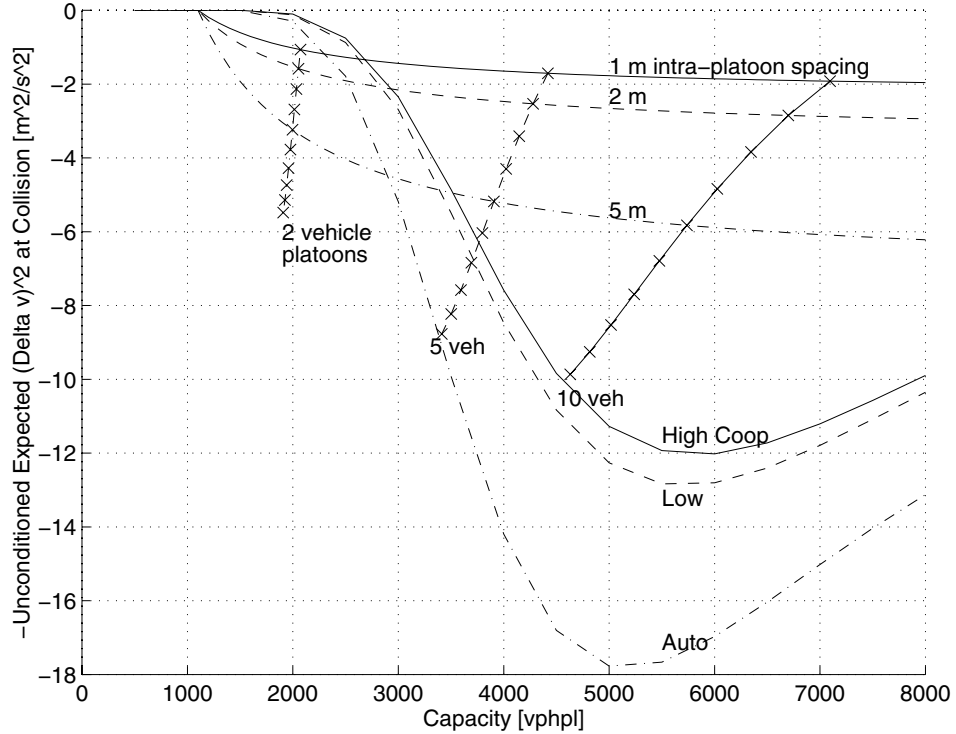


Figure 13: The Safety/Capacity Relationship for all Separation Policies: Composite Metric

shows no dependence on capacity. This is consistent with our restriction of studying only the first longitudinal collision in the hard braking scenario. The severity of the first longitudinal collision depends only on the two vehicles involved, and not the other potential followers needed to reach the given capacity.

Turning to the individual vehicle results, we see in Figure 11 that individual vehicles at all levels of cooperation are less likely to collide than platoons at all but at very high capacity levels. Increasing cooperation between vehicles reduces the probability of collision at any capacity. The advantage of individual vehicles over platoons is reversed in Figure 12. There we see that individual vehicle collisions at moderate capacities are more severe than any of the platooning plots. See Appendix A for safety-capacity relationships at other AHS operating speeds.

It is interesting to note in Figure 12 that at capacities between 4500 *vphpl* and 9000 *vphpl*, the expected severity of collisions is actually *worse* for individual vehicles with more cooperation. It is important to remember that this expected severity is for the entire vehicle *population*. For a single vehicle pair colliding in the hard braking scenario, decreasing the delay  $\tau$  will always *reduce* the severity of that collision. However, the expected severity of collision for the vehicle population is influenced by the shape of the hard braking parameter distributions as well as the kinematic calculations. The crossover region in Figure 12 can be explained by looking at the parameter distributions used in this study in more detail, as follows. We will see that the maximum deceleration (braking) parameter distributions are key in understanding this phenomenon.

In the stated range of capacities for which the individual vehicle collision severity plots cross over, the probability that collision occurs after the lead vehicle stops is near the probability that collision occurs while the lead vehicle is in motion. At capacities below the crossover region, nearly all collisions

occur after the lead vehicle has stopped. At capacities above the crossover region, nearly all collisions occur while the lead vehicle is in motion. Thus, outside the crossover region, the kinematically derived severity for each combination of vehicle braking rates is a smooth function, where the most and least severe collisions are experienced only by vehicles whose braking rates are on the tails of their respective distributions. Expected severity increases with delay, since the severity of collisions between pairs of vehicles with the most probable braking rates all change according to the same kinematic collision scenario. By contrast, consider the case when both collision scenarios happen with approximately equal probability. If the lead vehicle stops before a collision occurs, its deceleration becomes zero. This discontinuity in the lead vehicle’s deceleration goes into the equations of motion to calculate collision velocity. Thus, this discontinuity is propagated to the calculated severity for all combinations of vehicle braking rates. The result of the discontinuity is that the lowest severity collisions can only happen when the braking rates of the two vehicles are in a narrow band such that the lead vehicle stops before the collision occurs. At larger values of the hard braking delay, this narrow band includes vehicles of similar braking capabilities. For smaller values of delay, vehicle pairs that were in this band now avoid collision altogether. The lowest velocity collisions now occur when the follower vehicle has brakes that are somewhat less capable than the lead vehicle. However, due to the shape of the braking rate distributions, these braking rate combinations at small delays are less probable. Thus, the corresponding low velocity collisions are disproportionately less probable, and the expected severity of collision is higher.

The last result shown in Figure 13 demonstrates the safety-capacity relationship by combining the measure of frequency and severity to evaluate overall safety. The composite safety metric is the expected square of the collision velocity, not conditioned on the occurrence of the collision. It can be derived from the previously defined safety metrics by simply multiplying the expected square of the collision velocity (severity metric) by the total probability of collision (frequency metric). Therefore, if vehicles have rare severe collisions or frequent light collisions, the composite safety metric will indicate that the system is more safe than an AHS with frequent severe collisions. Using this composite safety metric, we continue to see the advantage of increasing inter-vehicle cooperation for improving the safety of the individual vehicle AHS. For the platooning concept, it is advantageous to minimize the intra-platoon spacing. The safety of a platooning system does not seem to be sensitive to increasing the number of vehicles within a platoon. This result may be modified however, if secondary collisions are considered. Finally, if this safety metric is used, we see that the safety-capacity curves for individual vehicles and platoons intersect. For instance, high cooperative individual vehicles and platoon with 2  $m$  intra-platoon spacing are equally safe at just under 3000 *vphpl*. At higher capacities, platoons appear to be much safer, but this depends on the composite measure of safety used. A different composite measure could produce different results by changing the weighting of the two safety metrics. See Appendix A for the composite safety-capacity relationship at other AHS operating speeds.

## 5 AHS Rear-end Crash Mitigation Benefits

One important way of evaluating the safety benefits of an automated highway is to estimate the number of crashes that might occur on an automated highway per year of operation. Such an estimate may be quantified in financial terms by multiplying it by the cost per crash. For example, we know from [23] that there were 1.66 million rear-end crashes in the year 1994 that cost a total of 35.4 billion dollars. We also know from [29] that these 1.66 million rear-end crashes corresponds to 2,347,295,000,000 vehicle miles travelled. We hypothesize a baseline highway with a corresponding



operating environment (weather, obstacles, etc.) and demand level (vehicle miles travelled per year of operation, average vehicles per hour, average trip length, etc.) Statistics such as those cited may be used to estimate the number of crashes expected to occur on the baseline highway per year of operation. Alternatively, if the baseline is some actual highway then the number of crashes per year of operation could be obtained from the corresponding crash statistics. We are interested in estimating the expected number of crashes per year of operation if the baseline highway were to be replaced by an automated highway servicing the baseline demand in the baseline operating environment. The subsequent development shows how the probability of a collision in the hard braking emergency scenario may be used to obtain such an estimate.

We restrict our crash mitigation estimation to rear-end crashes occurring on limited access freeways in LVD scenarios. A LVD event is usually a response to another event. Moreover, LVD can be a string phenomenon, i.e., a vehicle in a string decelerates because the one in front of it decelerates. We assume that any string of LVD's is caused by

1. an obstacle appearing on the highway and the first vehicle approaching the obstacle decelerating in response to it, or
2. a vehicle failing (no fuel, tire-blowout, etc.) on the highway and the first vehicle approaching the failed vehicle decelerating in response to it, or
3. an inattentive or careless drivers on the highway braking harder than usual in response to normal highway conditions, or causing cut-in disturbances during lane changes,
4. string instabilities, wherein comfortable braking in response to normal traffic is amplified by following vehicles to cause LVD's as the disturbance propagates through the vehicle string.

Obstacles, vehicle failures, inattentive drivers, and string instabilities are hazards. Note that one occurrence of a hazard may cause more than one LVD.

Assume the existence of a baseline highway and a baseline year for it. Let  $H_b$  denote the set of LVD causing hazards that occurred in the baseline highway in the baseline year.  $H_b$  may be partitioned as  $H_b = H_b^c \uplus H_b^{nc}$ , where  $H_b^c$  and  $H_b^{nc}$  are the sets of LVD causing hazards that did and did not cause rear-end crashes. The set  $H_b^c$  for the baseline highway may be known by collecting and analyzing accident data for the baseline highway. The set  $H_b^{nc}$ , possibly considerably more numerous, is very difficult to know. It represents the LVD occurrences in which drivers successfully evaded crashes by timely and intelligent action.

It is expected that the number of crashes on an automated highway will be directly proportional to the number of LVD causing hazards that occur. Since the baseline and automated highways are to be compared under the same environmental conditions and demand levels, the following assumptions are made. It is assumed that the set of LVD causing obstacle occurrences is the same for baseline and automated highways. It is also assumed that the number of LVD causing vehicle failure occurrences is the same on the baseline and automated highways. This assumption is reasonable for failures such as tire blowouts, no fuel etc., that are shared by manual and automated vehicles, given identical demand levels. For the time being we ignore crashes caused by failures unique to automated vehicles. We will comment on this again at the end of the section. Let  $H_{of} \subseteq H_b$  denote the set of LVD causing obstacle or failure hazard occurrences on the baseline and automated highways.

The other two LVD causing hazards on the baseline highway are driver inattention and string instabilities. The former set of hazard occurrences are assumed to be absent on the automated

highway for obvious reasons. With regard to the latter, we assume that the stable operation of automated vehicle strings is an AHS design requirement. For the technological feasibility of designing vehicle following systems that attenuate longitudinal disturbances as they propagate through a vehicle string (i.e., string stability) refer to [15]. For these reasons, it is assumed that LVD's caused by string instabilities are negligible on the automated highway. Thus  $H_{of}$  represents the set of LVD causing hazard occurrences on the automated highway. Accordingly, the expected number of automated highway crashes  $n_a$  is given by

$$n_a = \sum_{h \in H_{of}} p_a(REC||h),$$

where  $p_a(REC||h)$  is the probability that there is a rear-end crash on the automated highway given the hazard occurrence  $h$ . It is assumed a hazard occurrence  $h$  is associated with a sequence  $\langle v_{mk}^h, r_{mk}^h, \dot{r}_{mk}^h, d_{mk}^h, d_{mk}^{h,max}, \tau_{mk}^h \rangle_{k=1}^{k=n_b^h}$ , where the respective terms are the speed, range, range-rate, deceleration, deceleration capability, and reaction delay of the  $k$ -th follower of the obstacle or failed vehicle.  $n_b^h$  is the number of LVD's caused by the hazard occurrence  $h$ .

Let  $H_{of} = H_{of}^c \uplus H_{of}^{nc}$ , where  $H_{of}^c$  and  $H_{of}^{nc}$  are the set of hazards in  $H_{of}$  that respectively did and did not cause crashes on the baseline highway. We assume for all  $h \in H_{of}^{nc}$  that  $p_a(REC||h) = 0$ . This assumption imposes a challenging set of design requirements on the AHS which may be understood in the following manner. For each  $h \in H_{of}^{nc}$  the vehicle and roadway capabilities are such that for each LVD caused by the hazard occurrence  $h$ , there exists a set of crash avoiding vehicle trajectories that may be realized by braking or lane changing. Furthermore, the driver is able to find and execute one such feasible crash avoiding trajectory. Since the automated vehicle reacts much faster, has better speed regulation, never follows at closer than the set headway, and never goes faster than the set speed, it may be argued that if there exists a crash avoiding feasible trajectory for the manual vehicle, there exists one for the automated vehicle. This argument is similar to that used in [22, 23] for estimating the crash mitigation properties of adaptive cruise control and longitudinal crash warning systems. Benefit assessment relies on a thought experiment that replaces the manual vehicle in every LVD occurrence by an automated vehicle, and assumes (amongst other assumptions) that at the time the LVD starts

- if the manual vehicle was at a speed less than the set automated speed, then the automated vehicle would also be at the same speed, and
- if the manual vehicle was at a range greater than the set automated headway, the automated vehicle would also be at the same range.

Further research and design is required to ensure that partially or fully automated vehicles conform to these behavioral assumptions. Moreover, to justify  $p_a(REC||h) = 0$  for fully automated vehicles, one must also assume that just as the driver was able to find and execute a feasible crash avoiding trajectory, the automated vehicle controllers will also be able to do the same. This is one of the fundamental, and as yet partially solved problems of automated vehicle control design. For research on automated situation assessment required for safety refer [26, 27]. For research on automated emergency lane change maneuvers refer [24, 25].

It should be noted that an immediate consequence of the assumption  $p_a(REC||h) = 0$  for all  $h \in H_{of}^{nc}$  is that the expected number of crashes for the automated highway will not be greater than the number of crashes on the baseline highway. The rest of the section establishes an upper bound on the magnitude of improvement.

We begin with the statement

$$n_a = \sum_{h \in H_{of}} p_a(REC|h).$$

$p_a(REC|h) = 0$  for all  $h \in H_{of}^{nc}$  implies

$$n_a = \sum_{h \in H_{of}^c} p_a(REC|h).$$

Let  $l_k^h$  denote the  $k$ -th LVD associated with the hazard occurrence  $h$ . Then

$$n_a = \sum_{h \in H_{of}^c} \sum_{k=1}^{k=n_a^h} p_a(REC|l_k^h),$$

where  $n_a^h$  is the number of *LVD*'s caused by the hazard occurrence  $h$  and  $p_a(REC|l_k^h)$  is the probability that the first rear-end crash occurs in the  $k$ -th LVD, or between the  $(k+1)$ -th and  $k$ -th followers.

Recall that  $h$  on the baseline highway was associated with a sequence  $\langle v_{mk}^h, r_{mk}^h, \dot{r}_{mk}^h, d_{mk}^h, d_{mk}^{h,max}, \tau_{mk}^h \rangle_{k=1}^{k=n_a^h}$ . Likewise  $p_a(REC|l_k^h)$  depends on the values  $(v_{ak}^h, r_{ak}^h, \dot{r}_{ak}^h, d_{ak}^h, d_{ak}^{h,max}, \tau_{ak}^h)$  for  $k = 1 \dots n_a^h$ . Let  $v_0$  be the set speed,  $r_0$  be the space headway at the speed  $v_0$ ,  $\rho_{max}$  be the maximum velocity tracking error under normal traffic conditions, and  $\tau_r$  be the maximum reaction delay of the automated vehicle. Then

$$p_a(REC|l_{k-1}^h) = p_a(REC|v_{ak}^h, r_{ak}^h, \dot{r}_{ak}^h, d_{a(k-1)}^h, d_{ak}^{h,max}, \tau_{ak}^h) \leq p_a(REC|v_0, r_0, \rho_{max}, d_{a(k-1)}^h, d_{ak}^{h,max}, \tau_r),$$

since the automated vehicle will not exceed the set speed, come closer than the set headway, exceed the maximum range-rate error or the maximum delay. Since the crash probabilities increase monotonically with  $d_{a(k-1)}^h$ , we know that

$$p_a(REC|v_0, r_0, \rho_{max}, d_{a(k-1)}^h, d_{ak}^{h,max}, \tau_{max}) \leq p_a(REC|v_0, r_0, \rho_{max}, d_{a(k-1)}^h) = d_{a(k-1)}^{h,max}, d_{ak}^{h,max}, \tau_r),$$

which is the probability of a crash in the hard braking emergency given that the  $(k-1)$ -th vehicle has a deceleration capability  $d_{a(k-1)}^{h,max}$ . Therefore

$$n_a \leq \sum_{h \in H_{of}^c} \sum_{k=1}^{k=n_a^h} p_a(REC|v_0, r_0, \rho_{max}, d_{a(k-1)}^h) = d_{a(k-1)}^{h,max}, d_{ak}^{h,max}, \tau_r).$$

We establish an upper bound on  $n_a^h$ . Recall that  $p_a(REC|l_k^h)$  is the probability that the first crash occurs between the  $(k+1)$ -th and  $k$ -th vehicles. We will find an  $n$  such that if the first crash has not occurred at or before the  $(n-1)$ -th and  $n$ -th vehicles, then the probability that it will occur after that is negligible. In an LVD vehicle pair, let  $d_l$  denote the deceleration of the lead vehicle and  $d_f$  the deceleration of the following vehicle required to come to a stop just behind the lead vehicle, if the lead vehicle brakes until it comes to rest at the rate  $d_l$ . Then if the time headway  $\tau_{head}$  is sufficiently large compared to the response time  $\tau_r$ , and neglecting the small range-rate errors,  $d_l$  and  $d_f$  may be related by the equation

$$\frac{v_0^2}{2d_l} + v_0\tau_{head} = \frac{v_0^2}{2d_f} + v_0\tau_r,$$

which implies

$$d_f = \frac{v_0 d_l}{v_0 + 2d_l(\tau_{head} - \tau_r)}.$$

We assume that in every LVD pair the follower vehicle brakes just as hard as necessary to avoid a crash. i.e., if the lead vehicle brakes at the rate  $d_l$  then the follower vehicle will brake at the rate  $d_f$  as determined by the above equation. Observe that the derivative of  $d_f$  w.r.t.  $d_l$  is positive on the interval  $[0, \infty)$ . Therefore increasing  $d_l$ , increases  $d_f$ . Consider the sequence of decelerations  $\langle d_{ak}^h \rangle_{k=1}^{n_a^h}$ . The positive derivative implies that maximizing  $d_{a1}^h$  maximizes  $d_{a2}^h$ , which maximizes  $d_{a3}^h$  and so on. Thus maximizing  $d_{a1}^h$  maximizes the disturbance propagation and the value of  $n_a^h$ . From the braking distribution described in section 3 we know that the deceleration capabilities of the vehicles lie in the range  $[0.4g, 1g]$ . Setting  $d_{a1}^h = 10m/s^2$  we obtain  $d_{a2}^h = 6.8m/s^2$ ,  $d_{a3}^h = 5.2m/s^2$ , and  $d_{a4}^h = 4.1m/s^2$ , with  $v_0 = 30m/s$ ,  $\tau_{head} = 1sec$ , and  $\tau_r = 0.3sec$ . Therefore it is assumed that if the first, second and third vehicles did not hit each other, then the probability that the fourth vehicle will hit the third vehicle is negligible, i.e.,  $p_a(REC||l_k^h) = 0$  for  $k > 3$ . Accordingly,

$$\begin{aligned} n_a &\leq \sum_{(h,k) \in H_{of}^c \times \{1,2,3\}} p_a(REC||v_0, r_0, \rho_{max}, d_{a(k-1)}^h = d_{a(k-1)}^{h,max}, d_{ak}^{h,max}, \tau_r) \\ &= \sum_{d_l^{max}, d_f^{max}} p_a(REC||v_0, r_0, \rho_{max}, d_l = d_l^{max}, d_f = d_f^{max}, \tau_r) n_{d_l^{max}, d_f^{max}}, \end{aligned}$$

where  $n_{d_l^{max}, d_f^{max}}$  is the number of vehicles with capability  $(d_l^{max}, d_f^{max})$  in the LVD set  $H_{of}^c \times \{1, 2, 3\}$ . But  $n_{d_l^{max}, d_f^{max}} = 3|H_{of}^c|p_{d_l^{max}, d_f^{max}}$ , where  $p_{d_l^{max}, d_f^{max}}$  is the probability that a vehicle pair on the highway has deceleration capability  $(d_l^{max}, d_f^{max})$ . Denoting  $p_a(REC||v_0, r_0, \rho_{max}, d_l = d_l^{max}, d_f = d_f^{max}, \tau_r)$  by  $p_a^{nom}(d_l^{max}, d_f^{max})$  we obtain

$$\begin{aligned} n_a &\leq \sum_{d^{max}} p_a^{nom}(d_l^{max}, d_f^{max}) 3|H_{of}^c| p_{d_l^{max}, d_f^{max}}, \\ &= 3|H_{of}^c| \sum_{d^{max}} p_a^{nom}(d_l^{max}, d_f^{max}) p_{d_l^{max}, d_f^{max}}, \\ &= 3|H_{of}^c| p_a^{nom, hb}, \\ &\leq 3p_a^{nom, hb} |H_c|, \end{aligned}$$

where  $p_a^{nom, hb}$  denotes the crash probability in the hard braking emergency for the nominal values  $v_0, r_0, \rho_{max}, \tau_r$ , and  $|H_c|$  is the number of rear-end crash incidents that occurred on the baseline highway as obtained from accident records.

Note that for  $v_0 = 30m/s, r_0 = 30m$  ( $\tau_{head} = 1sec$ , pipeline capacity 3600vphpl),  $p_a^{nom, hb}$  is 0.054. This says that no more than 16% of the crashes on the baseline highway would have occurred if the baseline highway had been replaced by an automated highway with a separation policy defined by 1 second headway.

We mentioned earlier that an automated highway may experience crashes due to vehicle and highway failures that do not occur on the baseline highway. This analysis tells us that as long as the fault tolerance of the AHS is such that the expected number of the new fault induced crashes does not exceed  $0.84|H_c|$ , the safety of the AHS, as measured by the expected number of rear-end crashes, will surpass that of the baseline highway.

## 6 Summary and Future Work

We have provided a safety comparison of manual and automated driving. The two modes are compared in the same scenario, i.e., the hard braking emergency. Our measure of safety is the frequency and severity of rear-end crashes.

Frequency is quantified by the total collision probability given the occurrence of hard braking, and severity by the mean square collision velocity given the occurrence of a collision. Note that we restrict our attention to the frequency and severity of the first rear-end crash. Safety is related to driver, vehicle and highway operating characteristics. There may be large variations in these characteristics. Such variations are modeled by probability distributions. The equations of motion are used to map the deterministic and stochastic vehicle and highway parameters to collision velocity distributions. The collision frequency and severity metrics are computed by post-processing these distributions.

The results show that by both measures, automated driving is safer than the most alert manual drivers, at similar speeds and capacities. Note that no lane changing response is considered for either mode of driving. If the lane changing capabilities of the two modes are substantially different it could alter relative safety.

The safety of automated driving is derived primarily from constant headway and speed control. This suggests that the judicious selection of separation policy is very important for safe automated driving.

We have also undertaken a more detailed investigation of four AHS concepts, representing four kinds of automated driving, i.e., autonomous individual vehicles, low cooperative individual vehicles, high cooperative individual vehicles, and platoons. The analyses establish that safety and capacity are inversely related through separation policy, although the decrease in safety at high capacity is much less for platooned AHS as compared to individual vehicle based AHS. We believe that such safety-capacity frontiers are useful for AHS policy decisions. The study has also quantified the impact of inter-vehicle cooperation and operating speed on AHS safety. It also compares and contrasts the safety characteristics of an individual vehicle and platooned AHS.

The principal findings may be summarized as follows. At speeds of  $67\text{ mph}$  all four AHS concepts compare favorably with manual driving. For a given set of automated vehicle capabilities and AHS capacity, higher speeds reduce safety. There is a significant improvement in safety as one goes from no inter-vehicle co-operation (i.e., autonomous) to low inter-vehicle co-operation. The safety improvements from low to high co-operation are not quite as dramatic. Two vehicle platoons have comparable safety characteristics to low cooperative individual vehicles at capacities of approximately  $3000\text{ vplph}$ . However, platoons do better in terms of the severity metric whereas the low cooperative Individual Vehicles do better in terms of the frequency metric.

In all cases we have restricted our attention to rear-end crash safety in the hard braking emergency scenario, i.e., the vehicle in front of the subject vehicle brakes hard until it comes to rest. This scenario serves well to establish an upper bound on the likelihood of rear-end crashes caused by LVD occurrences. National crash statistics indicate that this is a significant class of crashes. We have used the hard braking safety metrics to estimate AHS rear-end crash mitigation benefits. The benefits promise to be substantial if AHS can be designed to react well to hazardous situations where drivers react well, or if such hazardous situations can be prevented from occurring altogether. It may be noted that the hard braking emergency is a severe disturbance that serves well to elicit differences between the different kinds of AHS. It is arguably not the most dangerous hazard for which AHS

safety systems have to be designed.

It should be emphasized that this analysis considered only the first rear-end crash between two vehicles on a highway. The safety impacts of secondary crashes in a string of vehicles is not analyzed. The safety analyses are conducted in the hard braking scenario. We believe that most LVD disturbances on current highways are not so severe. More data on such disturbances is gradually becoming available [30]. We hope this will lead to more accurate safety estimates in the future. There is also scope for improvement in the capacity estimation. Pipeline capacity is clearly an upper bound on the AHS lane flow. Capacity estimates that account for losses due to transient effects caused by entry, exit, etc. are desirable. The AHS research community has also considered concepts beyond the four considered here. We would like to be able to extend these analyses to the more recent AHS concepts.

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## A Composite Safety-Capacity Relationship at Other Speeds

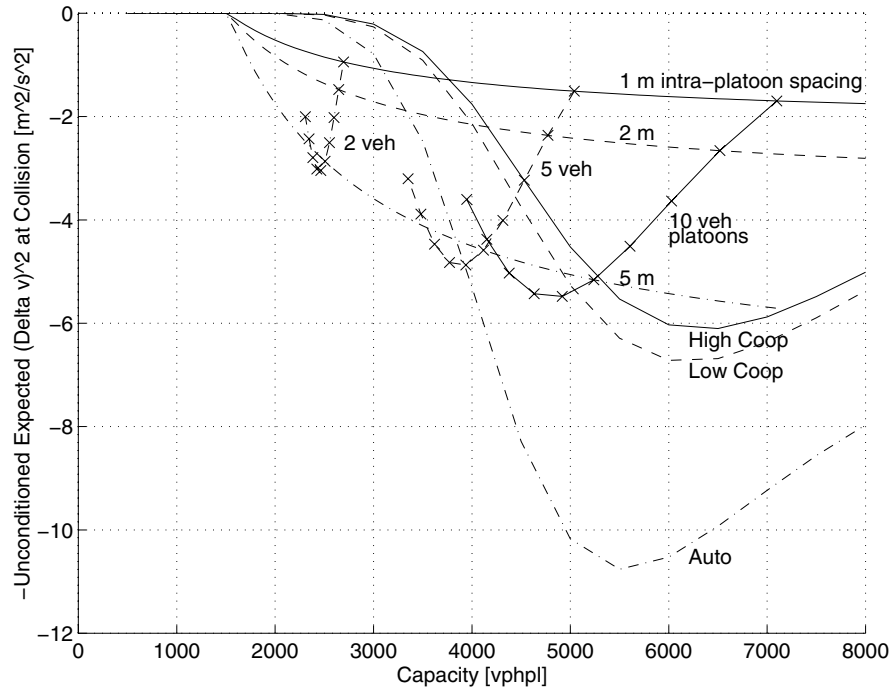


Figure 14: The Safety/Capacity Relationship for all Separation Policies at 20 m/s: Composite Metric

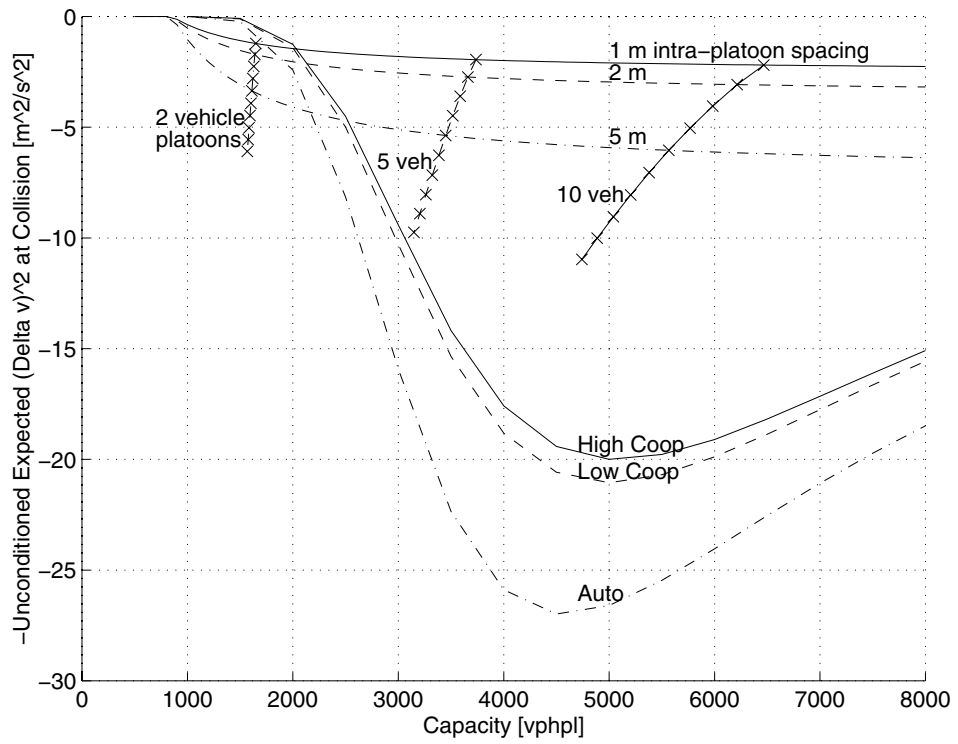


Figure 15: The Safety/Capacity Relationship for all Separation Policies at 40 m/s: Composite Metric