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THE PRICING OF SOVEREIGN RISK: AN APPLICATION OF OPTION THEORY

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Abstract: Option theory is used here to determine the variables that should explain the price of bank loans to foreign governments. As usual, the key explanatory variable is the variance of the underlying state variable (in casu, government income). It is also shown that these bank loans can often be considered to be riskless in the quantity dimension, because repayment will be made with certainty. They are risky in the time dimension, however, in the sense that banks do not know with certainty the exact moment of repayment.

International bank loans have become increasingly important since the early 1970s. They have been contracted not only with firms, but also with governments. Indeed, especially governments of less developed countries have been borrowing large amounts of funds through foreign banks, while leaving other financial sources (like foreign bonds and Eurobonds) relatively untapped (contrary to what happened in the beginning of this century).

International bank loans have special features, which originate in the practices of the Eurocurrency market . They are medium to long term loans, contracted with a syndicate of banks. Several types of fees are paid to the members of the syndicate depending on whether they manage the loan or just underwrite it. Most international bank loans bear a floating interest rate. Every three or six months, the interest rate is adjusted to the ruling London Interbank Offer Rate (LIBOR) for the currency in which the loan is denominated (in the case of the US Dollar, the US prime rate is sometimes used in addition to the LIBOR). The debtor also pays a 'spread' above the LIBOR to compensate the banks for the risk they incur.

Actually, the risk of an international bank loan is a very complicated matter. One specific type of risk is called "sovereign risk" (or "country risk"). It is understood to be the risk that a country suddenly stops repaying its debt. Of course, this risk is present in all cross-border lending, but there is general agreement that it is the only risk in lending to foreign governments (at least for simple loans which do not bear special clauses like currency options etc.).

The notion of sovereign risk is ambiguous, as will be discussed later on. For the time being, it suffices to have an intuitive understanding of this type of risk by remembering an important case: Mexico's moratorium on foreign debt payments in August of 1982.

The purpose of this paper is to price sovereign risk. In Section I, existing pricing models will be discussed and it will be shown how the option pricing approach which will be used later on naturally evolves from them. Indeed, it is not surprising that sovereign risk can be viewed as an option the debtor country has to stop repaying its debt. In the next Section, attention will be paid to the ambiguity of the notion of sovereign risk and it will be questioned whether sovereign risk exists at all. In Section III, formulae will be derived from option pricing theory which determine (1) rescheduling terms and (2) the price of a new loan to a non-rescheduling country. The paper ends with a summary of the main points.

The theory behind the formulae, namely option pricing theory, is by now a standard approach in finance. It provides a conceptual framework to determine among other things which variables are relevant in pricing international bank loans. This is an improvement on existing pricing models (discussed in Section I) which often depend on ad hoc chosen variables. As they stand now, however, the pricing formulae of this paper cannot yet be used directly to price international bank loans, as for instance the Black-Scholes model can be employed to price European options on common stock. In this sense, the results of the present paper are preliminary.

I. EXISTING PRICING MODELS

In the early 1970s, the literature on sovereign risk was very much oriented towards assessing the magnitude of the risk of lending to a particular country. It dealt mainly with identifying the broad circumstances under which countries would experience debt servicing difficulties. It used ratios or "indicators" of debt servicing capacity. McDonald (1982) calls it the "indicator" approach. Most of the literature uses intuitive notions of creditworthiness, sometimes summarised in checklists. However, the predictive power of these sovereign risk assessment methods seemed to be very poor (and often worse than an "all countries are good debtors"-rule; see for instance Goodman (1977) and Blask (1978)). Because of this, the literature turned to more sophisticated methods, stimulated by Frank and Cline's 1971 article. Statistics was used to determine the exact weights to be given to the indicators of debt servicing capacity: principal components analysis (Dhonte (1975)), discriminant analysis (Frank and Cline (1971) and Sargen (1977)), logit analysis (Feder and Just (1977a); Feder, Just and Ross (1981) and Mayo and Barrett (1978)), and non-parametric methods (Fisk and Rimlinger (1979)).

More recently, the literature turned its attention to pricing problems. It was asked what the price of a cross-border loan should be in order for it to correctly reflect sovereign risk. Theoretical prices could then be compared with actual prices, so that a conclusion could be drawn with respect to the efficiency of the market for international bank loans.

The literature on statistical methods to determine sovereign risk naturally resulted in a first pricing model using logit analysis (see Feder and Just (1977b) and Edwards (1984)). The model is straightforward: it is assumed that debt servicing problems are generated according to a logit model, where the probability p depends on a set of indicators reflecting the circumstances under which countries tend to cancel payments:

$$p = \frac{\exp(b.x)}{1 + \exp(b.x)} \qquad (1)$$

where p = probability of debt servicing problems

x = vector of "indicators"

b = vector of weights.

Assuming risk neutrality, the risk premium (approximated by the spread s charged on bank loans) is expressed explicitly as a function of p:

$$s = f(p) \qquad (2)$$

Substituting (1) for p in (2):

$$s = g(b.x) \qquad (3)$$

Risk neutrality ensures that (3) is linear in x, hence the estimation of b does not pose problems (if it is assumed that there is no reverse causality from s on the indicators x).

In fact, what this model tests is whether the indicators x which Feder, Just and Edwards think should be reflected in the price of a loan can indeed explain part of the variation of s. What the model cannot test, of course, is whether s is a correct price. The model has, however, some disadvantages. First, the assumption that p can be described by a logit model means that debt servicing problems are independent drawings from a population, with probability of succes (or better: failure) equal to p (i.e., successive Bernoulli trials). This may be a very inaccurate description of the process generating debt servicing problems. Second, the assumption of risk neutrality may lead to an overestimation of p in a world of risk aversion. To illustrate these disadvantages, consider the following. Edwards (1984) calculates the probabilities p implicit in s and concludes that p for Spain in 1976 for instance was 0.078. Given that debt servicing problems are binomially distributed, as Edwards implicitly assumes, this means that the probability as perceived by the market that Spain would have had at least one debt servicing problem in ten years was equal to 0.556. This seems to be rather exaggerated.

The model Eaton and Gersovitz (1980,1981) construct is similar. But instead of having the spread depend on the probability p, they consider the total quantity of debt supplied to the country as a function of several "indicators" which would explain the probability of repudiation. Moreover, they introduce the possibility of a debt ceiling (as do Sachs and Cohn (1982) and Sachs (1982)). However, this ceiling depends (almost trivially) on their assumption that future income is limited. As in the former model, bankers are risk neutral.

Eaton and Gersovitz point out that a country will repudiate its debt whenever the cost of continuing debt service payments exceeds the benefit of repudiation. Costs and benefits depend on the debt service payments and the characteristics of the stochastic process income follows. In fact, their idea is that international bank loans include an option to repudiate whenever the present value of debt service payments exceeds the present value of future income flows the economy can generate. Of course, the latter value is the total wealth of the economy. It is this insight that will be the basis for the pricing model to be introduced in this paper.

The last pricing model discussed here is from Feder and Ross (1982). It is similar to Feder and Just (1977b) and Edwards (1984), but instead of describing p by a logit model, they take published subjective probabilities of default. These probabilities p are conditional probabilities of debt servicing problems (conditional on no default the year before). Feder and Ross assume that p is constant over time. They also consider bankers to be risk neutral.

II. DOES SOVEREIGN RISK EXIST ?

Let us now try to define sovereign risk and determine to what extent it exists.

A. A DEFINITION OF SOVEREIGN RISK

As pointed out in the introduction, there is some ambiguity around the notion of sovereign risk. It is best to start with a description of the materialisation of sovereign risk. What happens normally is that the government decides to stop debt service payments to foreign banks. At the same time, it blocks repayments by individuals and firms. After a while, refinancing of the official debt is agreed on (on new terms). In general, it takes much longer for a country to resume net debt repayments (it may never be able to resume them).

Contrary to the beginning of this century, country risk does not materialise in the form of outright debt repudiation, but rescheduling: refinancing of existing debt at new terms. According to Solomon (1981), repudiation occurred only twice after the second world war (Ghana and North Korea).

In fact, the government of a debtor country has an option to stop debt service payments and ask for a rescheduling of its debt. The problem is: why does a government want to reschedule its debt ? Is it because the government cannot pay anymore or because the economy as a whole cannot pay anymore ? It is here that the ambiguity enters: most people would be inclined to think the latter is true (hence the term "country" risk), whereas many elements point to the former, as explained below.

If the government cannot pay, it will be because the present value of debt service payments (D) exceeds its wealth (W), namely the present value of its receipts. If the country as a whole cannot pay, it will be because the present value of debt service payments (D) exceeds its wealth (W), namely the present value of the future income flows it can generate. Repayments will be made again when D < W.

The description of sovereign risk which was given above seems to point to the inability of the government to repay and not that of the country. Moreover, most often only official (as opposed to private) foreign debt is rescheduled. Indeed, many firms may still be creditworthy and willing to buy foreign exchange at a higher price than before the repayment stop, but they are not allowed to. Moreover, does it make sense to speak about a bankrupt economy (D > W) when there are private firms who could afford a high price to pay back their foreign debt ? Take an industrialized country: does the Belgian government pay a spread above the LIBOR because there is a probability that the country goes bankrupt or that the government goes bankrupt ?

If indeed a country may be unable to repay its debt, because D > W, a minor problem arises: should attention be restricted to the wealth (W) of the external sector ? This seems improbable, as resources can always be shifted from the domestic to the external sector. Taking the whole economy's wealth also seems inappropriate.

It should be emphasised that all the existing pricing models and a large part of the literature on assessing sovereign risk takes the view that an economy's potential inability to service the debt is the factor underlying sovereign risk. The government's ability to service its debt (the official debt) most often is not dealt with.

The pricing model to be introduced in section III is completely general in this respect: W and D can equally well serve as government wealth and debt or total wealth and debt respectively. It is only in the empirical verification of the model that one has to make a choice, or one could test the model in both cases and compare the explanatory power. It may turn out that the distinction between "country" and "government" risk is of only minor importance since the fate of a country's government and a country's whole economy are intemately related (it is well known that the government's part of an economy remains very steady over time).

Before going on to the question whether sovereign risk is really a risk, the variables D and W should be specified more. The following assumptions will be used throughout:

A1 Let Y denote the income flow per unit time. lnY is assumed to follow an Ito process with drift σ and variance σ_y^2.

A2 $D(t) = D(0)e^{r_0 t}$ whenever $W(\tau) < D(\tau)$ $(\forall \tau \in [0,t]$, i.e., rescheduling does not occur from $\tau = 0$ on).

For notational convenience, small letters will denote logarithms of variables, i.e., w = lnW ; d = lnD ; y = lnY .

A1 is not very restrictive and indeed much more accurate than the probability laws used in the models discussed in Section I. A2 is restrictive, however: whenever rescheduling occurs, debt grows exponentially at a constant rate equal to the interest rate (r_o). Actually, r_o is not constant, but depends among other things on the LIBOR. But introducing a stochastic r_o creates considerable difficulties, as will be discussed in Section III.

Because W will be the relevant state variable in the pricing model of Section III (i.e., the variable that will determine when sovereign risk materialises - when the option a government has is exercised), it is necessary to determine the stochastic process it follows. Obviously:

$$W(t) = \int_t^{\infty} e^{-\rho(\tau-t)} E(Y(\tau)|Y(t))d\tau \qquad (4)$$

where ρ = risk adjusted discount rate for a security which pays Y each period. From A1:

$$W(t) = \int_t^{\infty} e^{-\rho(\tau-t)} Y(t) e^{(\upsilon+\frac{1}{2}\sigma_Y^2)(\tau-t)} d\tau$$

$$W(t) = \frac{Y(t)}{\rho-(\upsilon+\frac{1}{2}\sigma_Y^2)} \qquad (5)$$

From this, Proposition 1 will be obvious (the proof is given in the Appendix).

PROPOSITION 1

If A1 holds and the market is in equilibrium then w(t) will follow an Ito process with drift υ and variance σ_Y^2.

Hence, the stochastic process followed by W and Y are exactly the same. Only the initial value is different ($W(0) = Y(0)/(\rho-(\upsilon+\frac{1}{2}\sigma_Y^2))$). This property will be used in Section III.

One may object, of course, that a government will default on its international bank loans (i.e., exercise its option) long before the wealth level decreases to the debt level, especially since creditors cannot appropriate the government's belongings and realise them in order to get their money back as in the case of a levered firm. However, the subsequent analysis remains valid if the option is exercised when the ratio of wealth to debt takes on a certain value. In a more complete elaboration of the model, one should introduce specific costs and benefits of exercise, thus calculating the optimal exercise strategy. This will not be pursued here. If costs and benefits are constant across countries and over time, one can control for optimal exercise by including the ratio of wealth to debt in an empirical analysis on the pricing of sovereign risk. This merely reflects the fact that the closer this ratio is to one, the higher the probability of exercise is.

B. THE EXISTENCE OF SOVEREIGN RISK

This may seem a strange question. Indeed, very few people up to now challenged the idea that sovereign risk is really a "risk", i.e., an event the outcome of which is uncertain and which may eventually lead to a loss.

What is not known with certainty is at which t ($\in [0,\infty)$) a country stops repayment given at t = 0 (i.e., now), $W(0) > D(0)$. But it is known that when $W(t) < D(t)$ (i.e., the country stops repayment), debt will be rescheduled, or refinanced at new (market) terms, until W = D again. It is here that risk enters: it is not certain that there always exists a τ ($\in [t,\infty)$) such that $W(\tau) = D(\tau)$ given that the country stops repayment at t. At τ, $W(\tau) = D(\tau)$, that is, the country is creditworthy again and can repay its debt.

Proposition 2 states the conditions on which τ always exists (for the proof, see the Appendix). Then bank loans to the country's government will be paid back with probability one. To call them riskless is however not correct as will be explained later.

PROPOSITION 2

Let: A1, A2 hold and the market is in equilibrium

$W(0) < D(0)$

Then:
$$P[W(t)=D(t), \text{ for some } t \in [0,\infty)] = 1 \iff \upsilon - r_0 \geq 0$$

where: P[.] = Probability[.].

Hence, when $\upsilon - r_0 \geq 0$, repayment will occur with certainty. In such a case, we shall call the bank loan to be riskless "in the quantity dimension". A bank loan will be shown in general to not be riskless "in the time dimension". With this we mean that one does not know the exact time of repayment. The distinction between riskiness in the quantity and time dimension is subtle, but important, as will be clarified in the next Section.

One could argue that during the rescheduling period, banks are worse off because of non-performing loans on their balance sheet. This may not be true, as rescheduling is always done on new (market) terms, hence it would be possible for a bank to sell off their loans to others.

When $\upsilon - r_0 < 0$, sovereign risk does exist even in the quantity dimension, because there is a certain probability that a country (or the government) never becomes creditworthy again. That is,

$W(t) < D(t)$, $\forall\, t \in [0,\infty)$. Proposition 3 gives the complement of this probability, namely the probability that a country (or its government) becomes creditworthy again ($W = D$), given $W(0) < D(0)$ (For the proof, see Appendix).

PROPOSITION 3

Let: A1, A2 hold and the market is in equilibrium

$W(0) < D(0)$

$$P[W(t) = D(t) \text{ for some } t \in [0,\infty)] = \exp\left\{-\frac{2\,(d(0) - w(0))\,|\upsilon - r_0|}{\sigma_Y^2}\right\}$$

$$\Updownarrow$$

$$\upsilon - r_0 < 0$$

III. A MODEL FOR PRICING SOVEREIGN RISK

The two core Propositions of this paper will now be proved. As has already been discussed, the underlying theory for the following Propositions is the option pricing approach, a very familiar conceptual framework in finance. At this point, two remarks are necessary. First, option theory deals with the pricing of claims contingent on the value of a state variable. Hence, it requires in the first place the specification of a state variable, and, second, a continuously traded security perfectly correlated with it. It will by now be clear that the debtor government's wealth will be used here as the state variable. There is, however, no continuously traded security available directly to represent it. Therefore, we shall have to make the assumption that financial markets are complete (see A7 below) such that a perfectly correlated security can be constructed using existing financial assets. For instance, if the wealth of a small country's government can be considered to be perfectly correlated with the price of a major commodity in which the country trades, futures contracts for that commodity could be used as the mimicking security. Second, option theory requires the specification of boundary conditions, i.e., exact conditions on which an option is exercised. In order not to complicate matters, we shall assume that the boundary condition ($W(t) = D(t)$) evolves nonstochastically over time (see A3 below). This may not reflect actual exercise strategies, as explained in Section II.A.

More formally, the following assumptions will be made in addition to A1 and A2.

A3. $D(t) = D(0)e^{\delta t}$ whenever $W(\tau) \geqslant D(\tau), \forall \tau \in [0,t]$ (i.e., rescheduling does not occur in the interval $[0,t]$).

A4. The debt service of a loan consists of a set of fixed payments $K(t)$ at discrete moments in time $t=1,2,\ldots,n$, where n is the maturity of the loan; $K(t) = \overline{K}(t) + I(t)$, where $\overline{K}(t)$ = debt amortization at t, and $I(t)$ = interest payment at t.

A5. When rescheduling first occurs, wealth ($W(t)$) suddenly drops to $D(t)/\varepsilon$, for some $\varepsilon > 1$ (but close to 1).

A6. When rescheduling first occurs, all payments become due immediately. Repayments will be made when $W(t) \geqslant D(t)$ again.

A7. Financial markets are complete. This means that the stochastic process of W can be mimicked exactly by a linear combination of existing securities. Trade in such securities is assumed to be continuous over time.

A8. The market for international bank loans is efficient.

A9. The market is in equilibrium.

A3 is unrealistic. It could be relaxed but this would make the pricing formula much more complicated. One alternative would be to assume that d(t) follows an Ito process with drift δ and variance σ_D^2, independent of the process followed by w. (w-d) could be used as the state variable in the pricing formula. Indeed, (w-d) also follows an Ito process, as the convolution of normally distributed random variables (w and d) is distributed normally too.

In A4, K(t) is assumed to be fixed. In reality, K depends on r_0, which in turn depends on the LIBOR. As already mentioned, the introduction of a stochastic r_0 creates considerable difficulties.

A5 is mainly needed because of the mathematics of the problem. Indeed, when W = D initially, the density of the first passage time at W = D is a Dirac delta function. Actually, this assumption may not be irrealistic, since governments tend to impose many restrictions when rescheduling (price freezes, exchange and trade controls etc.), effectively immobilising a country's potential, i.e., lowering its wealth.

A6 constitutes what is known in international banking as the "cross-default clause": when the debtor country stops servicing (even part of) the debt, all payments become due immediately.

A7 is a crucial assumption. Because of it, W can be valued uniquely and hedging portfolios can be formed with W.

From 5:

$$W(t) = \frac{Y(t)}{\rho - (\upsilon + \frac{1}{2}\sigma_Y^2)}$$

Y(t) is known and υ and σ_Y^2 can be estimated from the (past) sample path of Y. And ρ can be computed using a suitable pricing model. According to Ross' Arbitrage Pricing Theory (see Ross (1976), and Solnik (1983) for an extension to international capital markets), ρ depends linearly on the senstivities of W to changes in pervasive (i.e., economy-wide) factors and the risk free rate. According to Proposition I, the characteristics of the stochastic processes of W and Y are equal. Hence, with a sample path of Y, the sensitivities of W to changes in the factors can be estimated. This is sufficient to compute W(t).

A8 says among other things that the market knows the probabilities of rescheduling, and once rescheduling occurs, repayment.

A9 implies that whenever two different securities have the same payoffs, they should have the same price.

We shall now state the two core Propositions. The first one gives a formula to determine rescheduling terms. It will clarify the distinction we have been making between risk in the quantity dimension and risk in the time dimension. The second formula prices a fresh loan to a non-rescheduling country. It depends only on a limited set of specific explanatory variables, notably, the debtor's income and income variance, and the debtor's current wealth. This contrasts with existing pricing models, which often include ad hoc variables (see Section I).

Let B be the face value of a bank loan to a foreign government. Denote by K the total amount of payments which become due because of a rescheduling (cf. A6). The amount to be repaid when W = D again, is $Ke^{r_r t}$, where r_r is the interest rate of the rescheduling agreement. Let r_F be the risk free rate. The problem is to determine r_r, given B and K.

PROPOSITION 4

Let: A1, A2, A5, A6, A7, A8, A9 hold

rescheduling occurs at t=0 $(W(0) = D(0)/\varepsilon)$

Then: r_r is the solution to $L(r_F - r_r) = 1$

where L is the Laplace transform of g, the first passage density of W at D, with σ replaced by $(r_F + \frac{1}{2}\sigma_Y^2 - Y(t)/W(t))$

The proof is given in the Appendix. It is based on the option pricing model, in the sense that a riskless hedge is constructed enabling to determine the rescheduling terms as a function of the underlying state variable (W(t)). Notice that neither σ nor ρ appear in the pricing formula.

From Proposition 2, L(0) = 1 when $r_F + \frac{1}{2}\sigma^2 - Y/W \geq r_o$. Hence, in that case, $r_r = r_F$. Even when $\sigma \geq r_o$ (i.e., the loan is riskless in the sense that repayment is guaranteed, as shown in Proposition 2; we denoted this by "riskless in the quantity dimension"), r_r may be strictly larger than r_F. An intuitive explanation for this is the following. Assume continuous-time two-fund separation holds. If Y (and hence, W) is highly correlated with the market protfolio, its β will be high. Repayment will most likely occur in those states of nature in which aggregate wealth is high, and vice-versa. In such a case, investors are willing to pay only a lower price for this investment opportunity. In other words, they demand a higher return. They know they will be repaid some time in the future ($\upsilon \geqslant r_o$), but they also know they will most probably be repaid at a moment aggregate wealth is high, hence they ask a premium above the risk free rate ($r_r > r_F$). The link is given by ρ which is higher than r_F because β is high, so that $r_F + \frac{1}{2}\sigma_Y^2 - Y(t)/W(t)$ may be smaller than r_o even when $\upsilon > r_o$. Of course, it can also be the case that $r_F + \frac{1}{2}\sigma_Y^2 - Y(t)/W(t) \geqslant r_o$ (hence $r_r = r_F$) even when $\upsilon < r_o$, for instance when Y is negatively correlated with the market portfolio. It is the time dimension that makes international bank loans risky, in a subtle way, even when at first they seem to be riskless (by Proposition 2).

Proposition 5 gives the price of a loan to a government given no rescheduling (i.e., W(t) > D(t)). Its proof is given in the Appendix.

PROPOSITION 5

Let: A1 through A9 hold

W(0) > D(0)

n = 2

Then:

$$B(W(t),t) = e^{-r_F(2-t)} \int_a^\infty K(2) \frac{1}{\sigma_Y \sqrt{2\pi(t-2)}} \left[\exp\left\{ - \frac{(x-(\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)(2-t))^2}{2\sigma_Y^2 (2-t)} \right\} \right.$$

$$- \exp\left\{\frac{2(\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)a}{\sigma_Y^2} - \frac{(x - 2a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)(2-t))^2}{2\sigma_Y^2(2-t)}\right\}\Bigg] dx$$

$$+ e^{-r_F(1-t)} \int_a^{\infty} K(1) \frac{1}{\sigma_Y\sqrt{2\pi(1-t)}} \Bigg[\exp\left\{-\frac{(x - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)(1-t))^2}{2\sigma_Y^2(1-t)}\right\}$$

$$- \exp\left\{\frac{2(\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)a}{\sigma_Y^2} - \frac{(x - 2a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)(1-t))^2}{2\sigma_Y^2(1-t)}\right\}\Bigg] dx$$

$$+ \int_t^1 e^{-r_F\tau} \left(\sum_{i=1}^{2} \bar{K}(i) + I(1)\tau\right) \frac{a}{\sigma_Y\sqrt{2\pi\tau^3}} \exp\left\{-\frac{(a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)\tau)^2}{2\sigma_Y^2\tau}\right\} d\tau$$

$$+ \int_1^2 e^{-r_F\tau} \left(\bar{K}(2) + I(2)(\tau - 1)\right) \frac{a}{\sigma_Y\sqrt{2\pi\tau^3}} \exp\left\{-\frac{(a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)\tau)^2}{2\sigma_Y^2\tau}\right\} d\tau$$

where: $t \in [0,1)$

a = absorbing barrier = $w(t) - d(t) + \ln\varepsilon$

Note once again that the pricing formula depends only on a limited set of variables.

Propositions 4 and 5 are empirically verifiable. They basically list the variables that are and those that are not important in explaining international bank loan prices. Using a linear or quadratic approximation (the latter will take into account the usual convexity of option prices), one could regress bank loan prices (spreads) crossectionally and over time on explanatory variables. The model is then verified by testing for instance whether the expected return on the state variable has significant explanatory power (ideally, it should not have so).

IV. CONCLUSION

Option theory was used here to determine which variables are important for pricing bank loans to foreign governments. This approach naturally evolves from existing pricing models. As usual in option theory, the variance of the state variable (in this case, government wealth) is a key explanatory variable. It was also shown that sovereign risk may not exist 'in the quantity dimension' because banks know they will be paid back some time in the future and because loans are rescheduled at market terms. But even then, the loans are risky 'in the time dimension', in the sense that banks do not know when they will be paid back - they may be paid back exactly at the moment the marginal utility of an extra dollar as perceived by the market is lowest.

The results in the paper are preliminary: the formulae do not lend thenselves readily to application as the Black-Scholes model can easily be applied to pricing European options on continuously traded stock. The model does have enough structure, however, to start empirical verification. Indeed, it determines the variables that are relevant in pricing international bank loans, and the ones that are not. This improves upon existing pricing models.

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APPENDIX

Proof of Proposition 1

From A1:

$$f(Y(t) \mid Y(0)) = \frac{1}{Y(t)\sigma_y\sqrt{2\pi t}} \exp\left\{-\frac{(y(t)-y(0)-\upsilon t)^2}{2\sigma_y^2 t}\right\}$$

Carry out the change of variables:

$W(t) = Y(t) /(\rho - (\upsilon + \frac{1}{2}\sigma_y^2)$

$W(0) = Y(0) /(\rho - (\upsilon + \frac{1}{2}\sigma_y^2)$

which gives:

$$f(Y(t) \mid Y(0)) = \frac{1}{(\rho+(\upsilon+\frac{1}{2}\sigma_y^2)W(t)\sigma_y^2\sqrt{2\pi t}} \exp\left\{-\frac{(w(t)+(\rho-\upsilon)-w(0)-(\rho-\upsilon)-\upsilon t)^2}{2\sigma_y^2 t}\right\} \left|\rho-(\upsilon+\tfrac{1}{2}\sigma_y^2)\right|$$

Market equilibrium will ensure that $\rho \geq \upsilon + \frac{1}{2}\sigma_y^2$. Hence:

$$f(W(t) \mid W(0)) = \frac{1}{W(t)\,\sigma_y\sqrt{2\pi t}} \exp\left\{-\frac{(w(t)-w(0)-\upsilon t)^2}{2\sigma_y^2 t}\right\}$$

This means that w(t) follows an Ito process with drift υ and variance σ_y^2.

Proof of Proposition 2

P[W(t) = D(t) for some $t \in [0,\infty)$] is the probability, given W(0) < D(0), of ever reaching the boundary D(t).

From A1 and Proposition 1:

$$f(w(t) \mid w(0)) = \frac{1}{\sigma_y\sqrt{2\pi t}} \exp\left\{-\frac{(w(t)-w(0)-\upsilon t)^2}{2\sigma_y^2 t}\right\}$$

Carry out the change of variables:

x(t) = w(t) - d(t) - (w(0) - d(0))

From A2:

$d(t) = d(0) + r_o t$

$x(t) = w(t) - w(0) - r_o t$

Hence:

$$f(x(t)\,|\,x(0)=0) = \frac{1}{\sigma_y\sqrt{2\pi t}} \; exp\left\{-\frac{(x(t)-(\upsilon-r_0)t)^2}{2\sigma_y^2 t}\right\}.|1|$$

x(t) follows an Ito process with drift $\upsilon - r_0$ and variance σ_y^2.

P[W(t) = D(t) for some $t \in [0,\infty)$] is then the probability, given x(0) = 0, of ever reaching the boundary a = d(0) - w(0) (>0).

$$P[.] = \int_0^\infty \frac{a}{\sigma_y\sqrt{2\pi t^3}} \; exp\left\{-\frac{(a-(\upsilon-r_0)t)^2}{2\sigma_y^2 t}\right\} dt \quad = 1 \text{ if and only if}$$

$\upsilon - r_0 \geq 0$ (see Cox and Miller, 1965, p. 212).

Proof of Proposition 3

Carry out the same changes of variables as in the proof of Proposition 2. Here:

$$P[.] = exp\left\{-\frac{2a|\upsilon-r_0|}{\sigma_y^2}\right\}$$

if and only if $\upsilon - r_0 < 0$ (see Cox and Miller, 1965, p.212). Note that in this case the probability distribution of first passage at a defines an improper probability measure because its limit for t going to infinity is strictly less than unity.

Proof of Proposition 4

B is the value of an option which promises to pay $Ke^{r_r t}$ whenever W crosses the boundary D for the first time, given that at t=0, W = D/ε (cf. A5). Consider the portfolio worth P dollars by buying u units of the portfolio perfectly mimicking W and borrowing uW - P dollars. From A1 and A9,

$$dP = udW - (uW - P)r_F dt + uYdt$$

$$= [u(\upsilon + \tfrac{1}{2}\sigma_y^2 - r_F)W + uY + r_F P]dt + uW\sigma_y dz$$

Let H be a twice differentiable function of W and t. By Ito's lemma:

$$dH = H_t dt + H_W dW + \frac{1}{2} H_{WW}(dW)^2$$
$$= [(\upsilon + \frac{1}{2}\sigma_y^2)H_W W + \frac{1}{2}\sigma_y^2 H_{WW} W^2 + H_t]dt + \sigma_y W H_W dz$$

Let $\partial H(W,t)/\partial W = u$ and define H to be the solution to:

$$0 = \frac{1}{2}\sigma_y^2 W^2 H_{WW} + [r_F W - Y]H_W + H_t - r_F H \qquad (1')$$

subject to $H(D,t) = Ke^{r_r t}$ and $H(0,t) = 0$. Consider the quantity $dP - dH$:

$$dP - dH = [-(r_F W - Y)H_W + r_F P - \frac{1}{2}\sigma_y^2 H_{WW} W^2 - H_t]dt$$
$$= r_F(P - H)dt$$

If we choose $P(0) = H(W,0)$, then $P(t) = H(W(t),t)$ ($\forall t > 0$), hence the portfolio P has at maturity a value equal to the value of the option we are interested in. Consequently, H must be the price of the option (from A7, A8 and A9). Now rewrite (1') (remember from (5) that $Y = W \,.\, (\rho - (\upsilon + \frac{1}{2}\sigma_y^2))$):

$$0 = \frac{1}{2}\sigma_y^2 W^2 H_{WW} + [r_F - (\rho - (\upsilon + \frac{1}{2}\sigma_y^2))]WH_W + H_t - r_F H$$

The solution of this equation, subject to the boundary conditions, is:

$$B(W,t) = H(W,t) = \int_t^\infty e^{-r_F(\tau-t)} K e^{r_r(\tau-t)} g(\tau-t)\,d\tau$$

($W < D$ by assumption). $g(\tau - t)$ is the first passage density of W at D, with υ replaced by $(r_F + \frac{1}{2}\sigma_y^2 - Y(t)/W(t))$. From A2 and (5):

$$g(\tau - t) = \frac{a}{\sigma_y\sqrt{2\pi(\tau-t)^3}} \exp\left\{-\frac{(a - (r_F + \frac{1}{2}\sigma_y^2 - Y(t)/W(t) - r_o)(\tau - t))^2}{2\sigma_y^2(\tau-t)}\right\}$$

where $a = d(t) - w(t)$ (see also the proof of Proposition 2).

We know that $B(W(0),0) = K$. Indeed, the purpose of a rescheduling agreement is to refinance the amount due at $t = 0$, namely K, and to ask repayment of this amount, plus interest (i.e., $Ke^{r_r t}$), at τ, when $W = D$ again.

$$\frac{B(W(0),0)}{K} = 1 = \int_0^\infty e^{-(r_F - r_r)\tau} g(\tau)\, d\tau$$

But the integral on the right hand side is the Laplace Transform of $g(\tau)$. Hence, r_r is given by the solution to:

$$L(r_F - r_r) = 1$$

Proof of Proposition 5

B is the value of a contingent claim paying a dividend Z(W,t) with the following payoff characteristics:

$$\hat{Z}(W(1),1) = K(1) = \bar{K}(1) + I(1)$$

$$\hat{Z}(W(t),t) = 0 \quad (\text{for } t \neq 1)$$

$$B(W(t),t) \leq W(t)$$

$$B(W(2),2) = K(2) = \bar{K}(2) + I(2) \tag{2'}$$

$$B(D(t)/\varepsilon,t) = \sum_{i=1}^{2} \bar{K}(i) + I(1)t \quad (\text{for } t \in [0,1])$$

$$B(D(t)/\varepsilon,t) = \bar{K}(2) + I(2)(t-1) \quad (\text{for } t \in [1,2])$$

(cf. A2, A3, A5, A6, and remember that W(0) > D(0) and n = 2). The last two payoffs are absorbing barriers, i.e., whenever W reaches the limit D/ε, the indicated amount is paid out and the option expires. Consider the portfolio worth P dollars by buying u units of the portfolio perfectly mimicking W and borrowing uW - P dollars. From A1 and A9:

$$dP = u\,dW - (uW - P)r_F dt + (uY dt - \hat{Z})dt$$

$$= [u(v + \tfrac{1}{2}\sigma^2_y - r_F)W + uY - \hat{Z} + r_F P]dt + uW\sigma_y dz$$

where it is assumed that only the net dividends from the portfolio mimicking W over those paid on the contingent claim are reinvested. Let H be a twice differentiable function of W and t. By Ito's lemma,

$$dH = H_t dt + H_W dW + \tfrac{1}{2}H_{WW}(dW)^2$$

$$= [(\upsilon + \tfrac{1}{2}\sigma_y^2)H_W W + \tfrac{1}{2}H_{WW}W^2 + H_t]dt + \sigma_y W H_W dz$$

Let $\partial H(W,t)/\partial W = u$ and define H to be the solution of:

$$0 = \tfrac{1}{2}\sigma_y^2 W^2 H_{WW} + [r_F W - Y]H_W + H_t - r_F H + \hat{Z} \qquad (3')$$

subject to (2') (where H is substituted for B). Consider the quantity dP - dH:

$$dP - dH = [-(r_F W - Y)H_W + r_F P - \tfrac{1}{2}\sigma_y^2 H_{WW}W^2 - H_t - \hat{Z}]dt$$

$$= r_F(P - H)dt$$

If we choose P(0) = H(W,0), then P(t) = H(W(t),t) (for all t > 0), hence the portfolio P has at maturity a value equal to the value of the option we are interested in and it pays the same dividends. Consequently, H must be the price of the option (from A7, A8 and A9). Rewrite (3') using (5):

$$0 = \tfrac{1}{2}\sigma_y^2 W^2 H_{WW} + [r_F - (Y(t)/W(t) - \tfrac{1}{2}\sigma_y^2)]WH_W + H_t - r_F H + \hat{Z}$$

The solution of this partial differential equation, subject to the boundary conditions, is:

$$\begin{aligned} B(W(t),t) = {} & e^{-r_F(2-t)} \int K(2)\, f(W(2),2)\, dW(2) \\ & + e^{-r_F(1-t)} \int \hat{Z}(W(1),1)\, f(W(1),1)\, dW(1) \\ & + \int_t^1 e^{-r_F(\tau-t)} \left(\sum_{i=1}^{2} \bar{K}(i) + I(1)\tau\right) g(\tau)d\tau \\ & + \int_1^2 e^{-r_F(\tau-t)} \left(\bar{K}(2) + I(2)(\tau-1)\right) g(\tau)d\tau \end{aligned}$$

(for $t \in [0,1]$)

where: f = (defective) density of W, given W(t) and an absorbing barrier at D/ε

g = density of the first passage of W at D/ε

with υ replaced by $r_F + \tfrac{1}{2}\sigma_y^2 - Y/W$.

Let: $x(\tau) = d(\tau) - w(\tau) - [(d(t) - w(t)] = \delta(\tau - t) - w(\tau) + w(t)$ (hence $x(t) = 0$)

a = absorbing barrier = $w(t) - d(t) + \ln \varepsilon$

x will follow an Ito process with drift $\delta - \upsilon$. Consequently,

$$B(W(t),t) = e^{-r_F(2-t)} \int_a^\infty K(2) f(x(2),2)\, dx(2) + e^{-r_F(1-t)} \int_a^\infty K(1) f(x(1),1)\, dx(1) + \int_t^1 e^{-r_F(\tau-t)} \left(\sum_{i=1}^{2} \bar{K}(i) + I(1)(\tau-1) \right) g(\tau)\, d\tau + \int_1^2 e^{-r_F(\tau-t)} \left(\bar{K}(2) + I(2)(\tau-1) \right) g(\tau)\, d\tau$$

$$B(W(t),t) = e^{-r_F(2-t)} \int_a^\infty k(2) \frac{1}{\sigma_y \sqrt{2\pi(t-2)}} \Big[\exp\Big\{ - \frac{(x-(\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_Y^2)(2-t))^2}{2\sigma_Y^2 (2-t)} \Big\} - \exp\Big\{ \frac{2(\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_y^2) a}{\sigma_y^2} - \frac{(x - 2a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_y^2)(2-t))^2}{2\sigma_y^2(2-t)} \Big\} \Big] dx$$
$$+ e^{-r_F(1-t)} \int_a^\infty k(1) \frac{1}{\sigma_y \sqrt{2\pi(1-t)}} \Big[\exp\Big\{ - \frac{(x-(\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_y^2)(1-t))^2}{2\sigma_y^2 (1-t)} \Big\} - \exp\Big\{ \frac{2(\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_y^2) a}{\sigma_y^2} - \frac{(x - 2a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_y^2)(1-t))^2}{2\sigma_y^2(1-t)} \Big\} \Big] dx$$
$$+ \int_t^1 e^{-r_F\tau} \Big(\sum_{i=1}^{2} \bar{K}(i) + I(1)\tau \frac{a}{\sigma_y \sqrt{2\pi\tau^3}} \exp\Big\{ - \frac{(a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_y^2)\tau)^2}{2\sigma_y^2 \tau} \Big\} \Big] d\tau$$
$$+ \int_1^2 e^{-r_F\tau} \Big(\bar{K}(2) + I(2)(\tau-1) \Big) \frac{a}{\sigma_y \sqrt{2\pi\tau^3}} \exp\Big\{ - \frac{(a - (\delta - r_F + Y(t)/W(t) - \frac{1}{2}\sigma_y^2)\tau)^2}{2\sigma_y^2 \tau} \Big] d\tau$$

which is the formula given in Proposition 5.