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Scarcity of Ideas and Options to Invest in R&D[1]

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Abstract

We consider a model of the innovative environment where there is a distinction between ideas for R&D investments and the investments themselves. We investigate the optimal reward policy and how it depends on whether ideas are scarce or obvious. By foregoing investment in a current idea, society as a whole preserves an option to invest in a better idea for the same market niche, but with delay. Because successive ideas may occur to different people, there is a conflict between private and social optimality. We argue that private incentives to create socially valuable options can be achieved by giving higher rewards where "ideas are scarce." We then explore how rewards should be structured when the value of an innovation comes from its applications, and ideas for the innovation may be more or less scarce than ideas for the applications.

JEL Classifications: O34, K00, L00

Keywords: Ideas; patents; intellectual property, innovation; options; nonobviousness

1 Introduction

Patent law doctrine distinguishes innovations that deserve a patent from those that do not according to the standards of novelty and nonobviousness. These doctrines feed a litigation industry since it is hard to know what they mean, even in light of considerable case law, and even harder to know what they *should* mean, when considered from the perspective of optimal incentives. In this paper we study a new model in which "nonobvious" has a clear meaning, linked to an exogenous parameter of the innovative environment. We illuminate optimal reward schemes as they depend on nonobviousness.

Most economic models of the R&D environment begin with some sort of production function for knowledge, which says how the investment of resources will accelerate innovation, increase the probability that innovation happens, or otherwise lead to the invention of new products or cost reductions. It is usually assumed that the production function is common knowledge.

The production-function model of innovation is hard to square with legal doctrine. In what sense is an innovation "nonobvious" if everyone knows how to achieve it? Legal doctrine has disparaged mere "sweat of the brow" (cost) as a standard for patentability, preferring some loftier ideal that involves creativity or imagination. It is hard to find creativity or imagination in the production function for knowledge.

The model in this paper tries to bridge the gap. As in O'Donoghue, Scotchmer and Thisse (1998), we distinguish between ideas and innovations. To innovate, the inventor must first have an idea, which we interpret as an act of imagination, and then have an incentive to invest in it. The notion is that "ideas" for innovations occur exogenously, perhaps influenced by the social institutions in which potential innovators interact, but that an idea is lost unless the recipient invests. The twist in this paper is that, if an idea is rejected, the market niche may nevertheless be filled by someone else who comes along later with a substitute idea that is even better.

We distinguish between obvious and nonobvious ideas on the basis of the frequency with which substitute ideas occur to the population of innovators as a whole. We say that ideas

are *scarce* if substitute ideas come along rarely. In contrast, an idea would be *common knowledge* if the same idea were available to everyone at the same time.

If all the substitute ideas were available at a given time to a given person, there would often be no conflict between the private incentive to invest and the socially efficient investment choice. It would be efficient from both points of view to invest in the best idea. However, because the ideas occur to random people at random times, the private decision problem is different from the social decision problem. The private investment decision is simply to invest if the idea at hand will generate positive expected profit. The socially efficient decision must account for the *option* created by not investing, namely, the option to invest later when someone thinks of an even better idea. If substitute ideas are likely to occur to different people, no individual will account for this option in his own investment decision. The problem is compounded by the fact that ideas arrive at random times.

Our main conclusion is that, to mediate the conflict between private and social incentives, patent rewards should reflect the scarcity of ideas. Specifically, patent rewards should be larger in environments where ideas are scarce than in environments where substitute ideas are likely to turn up. The legal doctrine of nonobviousness is a natural policy lever for distinguishing among such environments. In economic environments where better ideas for the same market niche come along rapidly, the option created by not investing is valuable. Since other ideas will come along quickly, the patent reward should not tempt investment in high-cost or inferior ways to fill the market niche. However, in economic environments where ideas are scarce, there may be a large cost of delay in waiting for a better idea. The reward system should therefore encourage investment in less good, or higher cost, ideas in order to avoid delay.

Because there is often no record of ideas that are rejected as too costly, or of better ideas that arrive after the market niche is filled, it is hard to track this phenomenon in practice. However, the history of computers is suggestive. The essential concept in computer hardware is a "switch" which can be set on or off. A switch codes bits (zeros or ones) and many switches together can implement logic. The first conception of a general purpose

computer (programming, logic, and memory) was by Babbage in the early 19th century. His idea for a switch was to use brass gears, which were expensive and had to be machined to very high precision. Although Babbage pursued his idea as a hobby, the cost could not be justified by the main computational challenge of the day, which was to produce astronomical tables for navigation. One hundred years later, urgent new computational problems had emerged – notably including tables for aiming artillery pieces – and Professor Howard Aiken of Harvard University had a cheaper way to make switches. Shortly afterward, there emerged a better idea for switches, namely to use electrical devices. First these were reed switches, then vacuum tubes, then transistors, and finally, integrated circuits. Today, switches are printed onto silicon using lithographic techniques and have become so cheap that the world produces more transistors than grains of rice.

In section 2, we present our main model and conclusions, first assuming that rewards can be linked to the rate at which ideas occur, and then assuming that the rate must be inferred from the delay in filling the market niche. The conclusions we draw about the optimal reward structure can be applied equally well to prizes or patents. Section 3 asks whether the possibility of keeping an innovation secret will subvert or reinforce our conclusions. In section 4, we pay closer attention to patents as the incentive instrument. Section 4 asks whether a common set of patent instruments can be used to set optimal incentives at two stages of innovation, where the main value of the first innovation is embodied in the applications it engenders. In line with our basic model, we assume that ideas for the innovation may be more or less scarce than ideas for the applications. In section 5, we discuss the implications of our findings for the various policy levers of patent law, including the nonobviousness standard, and how our findings relate to previous treatments of patentability in the literature.

2 Scarce ideas and optimal rewards

2.1 Model

There is market niche that may be filled with an innovation. There is an exogenous process by which the potential innovators receive ideas for filling this market niche. Each idea occurs at a random time, to a random recipient. Successive ideas for the given market niche are imperfect substitutes and may require different R&D costs. We use v for the flow value of having filled the market niche.

Each idea has associated to it an R&D cost that is drawn independently from a common distribution F with support in $[0, \infty)$ and density f. To create an innovation, the recipient of an idea must invest the cost. We assume that the ideas rain down on the population according to a Poisson process with parameter λ, and we take the parameter λ as a measure of scarcity. If the hit rate λ is low, ideas are scarce.

If the recipient of an idea discards it, it is not available to anyone else. If the recipient of an idea invests in it, the process stops because the market niche has been filled.

The optimal policy will therefore operate by getting the population of potential innovators to screen their ideas and discard those with costs that are too high. The social option created by not investing in a given idea is that another idea might entail a lower cost. There is thus a social trade-off between cost and delay. The policy objective is to manage this trade-off in a way that is socially optimal.

The option for lower cost that is preserved by discarding an idea is a social option, but not a private option for the recipient. We assume that each agent receives at most one idea. This is an intentionally extreme assumption that highlights the main premise of the paper. Ideas are scarce, not only for society as a whole, but especially from the perspective of any individual.

The social policy is described by a *threshold function* $c : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ such that the recipient of an idea at time t invests if the cost of the idea is less than $c(t)$. A threshold function is *stationary* if there exists $\bar{c}$ in $\mathbf{R}_+$ such that $c(t) = \bar{c}$ for all arrival times t. We say an idea at time t is *viable* if it has cost less than $c(t)$. The arrival rate of viable ideas at

time t is $\lambda F\left(c\left(t\right)\right)$. The investment process ends when the first viable idea arrives because the market niche is then filled.

The optimal threshold function will involve a trade-off between filling the market niche at time t or waiting for a cheaper idea. The expected cost of a viable idea at time t is

$$E_F\left(c\left(t\right)\right) = \int_0^{c(t)} \hat{c}\frac{f\left(\hat{c}\right)}{F\left(c\left(t\right)\right)}d\hat{c}. \tag{1}$$

As seen from time $t = 0$, the probability that there is no viable idea before time t is

$$e^{-\Lambda(t,c)} \quad \text{where} \quad \Lambda\left(t,c\right) = \int_0^t \lambda F\left(c\left(\tilde{t}\right)\right) d\tilde{t}$$

(Snyder and Miller, 1991, p. 51). As seen from time t, the probability that the first viable idea arrives at time $\hat{t} > t$ is the probability that no viable idea arrives between t and $\hat{t}$ times the probability that a viable idea arrives at time $\hat{t}$, namely,

$$\phi\left(\hat{t}|t,c,\lambda\right) = \lambda F\left(c\left(\hat{t}\right)\right) e^{-\left[\Lambda(\hat{t},c)-\Lambda(t,c)\right]} \tag{2}$$

2.2 Known arrival rate: Rewards increase with the scarcity of ideas

We first assume that the Poisson arrival rate λ is known, and characterize the optimal threshold function c, as well as optimal rewards. Our main result is that, given λ, the optimal threshold function is stationary, and further, that the optimal stationary value decreases with the arrival rate of ideas, λ.

Conditional on an arbitrary threshold function c, and assuming that no viable idea has occurred before t, social welfare measured from time t is V, defined by

$$V\left(t,c,\lambda\right) = \int_t^{\infty} e^{-r\left(\hat{t}-t\right)} \left(\frac{v}{r} - E_F\left(c\left(\hat{t}\right)\right)\right) \phi\left(\hat{t}|t,c,\lambda\right) d\hat{t}$$

The threshold function c is *optimal* if (3) holds at each t.

$$\left(\frac{v}{r} - c\left(t\right)\right) = V\left(t,c,\lambda\right) \tag{3}$$

The left hand side is the net social value of investing in the threshold idea at time t. The right hand side is the expected, discounted value of waiting for a better idea. If the left

hand side were greater than the right hand side, then social welfare could be improved by increasing the threshold cost. If the right hand side were greater than the left hand side, then social welfare could be increased by decreasing the threshold cost.

Taking the derivatives of (3), the optimal c satisfies the following at each t:

$$\frac{d}{dt}\left(\frac{v}{r} - c(t)\right) = \frac{d}{dt}V(t, c, \lambda) \tag{4}$$

Proposition 1 *Suppose that the arrival rate of ideas, λ, is fixed and known. Suppose that $c : \mathbf{R}_+ \to \mathbf{R}_+$ is the threshold function that maximizes $V(0, \cdot, \lambda)$. Then c is stationary.*

This is proved in the appendix. In particular, we show that the optimized value of $V(t, c, \lambda)$ is the same at all starting times t, from which it follows, using (4), that the optimal threshold function is stationary.

Welfare as a function of the stationary threshold $\bar{c}$ can be written as

$$\begin{aligned}
\bar{V}(t, \bar{c}, \lambda) &= \int_t^\infty e^{-r(\hat{t}-t)}\left(\frac{v}{r} - E_F(\bar{c})\right)\phi(\hat{t}|t, \bar{c}, \lambda)\,d\hat{t} \\
&= \left(\frac{v}{r} - E_F(\bar{c})\right)\int_t^\infty e^{-r(\hat{t}-t)}\lambda F(\bar{c})e^{-\lambda F(\bar{c})(\hat{t}-t)}\,d\hat{t} \\
&= \left(\frac{v}{r} - E_F(\bar{c})\right)\frac{\lambda F(\bar{c})}{\lambda F(\bar{c}) + r} \tag{5}
\end{aligned}$$

This expression shows the trade-off faced by the policy maker. If a higher stationary cost threshold $\bar{c}$ is tolerated, the innovation will arrive sooner since the hit rate of viable ideas, $\lambda F(\bar{c})$, is then higher, and the discounting expression, $\frac{\lambda F(\bar{c})}{(\lambda F(\bar{c})+r)}$, is larger.

Since the optimal threshold function is stationary, we can conceive of the optimal policy as a value $c^*(\lambda) \in \mathbf{R}_+$, where $c(t) = c^*(\lambda)$ for each t. The first order condition for maximizing (5) can be written for each λ as

$$\left(\frac{v}{r} - c^*(\lambda)\right) - \frac{\lambda F(c^*(\lambda))}{(\lambda F(c^*(\lambda)) + r)}\left(\frac{v}{r} - E_F(c^*(\lambda))\right) = 0. \tag{6}$$

The (unique) solution $c^*(\lambda)$ has the property that investing in the marginal innovation today, and receiving net value $\left(\frac{v}{r} - c^*(\lambda)\right)$, is as valuable as waiting for the next viable idea, which will occur with some delay, but may have a lower cost. If the arrival rate of

6

ideas λ is larger, then the cost of waiting is reduced, and it is optimal to be more stringent in the ideas that are accepted for investment. One can see this by differentiating (6) implicitly, which leads to

Proposition 2 *If the arrival rate of ideas, λ, is fixed and known, the optimal cost threshold is stationary, and is decreasing with the arrival rate λ; that is $c^*(\cdot)$ is decreasing.*

To implement the optimal threshold function, let $\rho : \mathbf{R}_+ \to \mathbf{R}_+$ be a *reward function.* The reward function ρ *implements* the threshold function c if $\rho = c$. Then the recipient finds it profitable to invest rather than discard the idea if and only if investment is optimal according to the threshold function c. The reward function is *stationary* if $\rho(t) = \bar{\rho}$ for some $\bar{\rho} \in \mathbf{R}_+$, and the *optimal stationary reward* is $\rho^*(\lambda) = c^*(\lambda)$.

Proposition 3 *When the hit rate of ideas is known and fixed, the optimal reward function is stationary, and the optimal stationary value $\rho^*(\cdot)$ is decreasing with the arrival rate λ.*

We interpret this proposition as saying that rewards should be higher when ideas are scarce. This is true even though the distribution of costs associated with ideas remains the same.

2.3   Unknown arrival rate: Rewards increase with delay

Like all contracts, R&D incentives must depend on things that are verifiable. This is true whether the R&D incentive is provided as a prize or a patent. Although we have shown that the size of the reward should increase with the scarcity of ideas, the Poisson hit rate cannot be verified. At best, the prize or patent authority knows the date that the innovation materializes, but does not observe the distribution of arrival times or the arrival of ideas that were rejected.

In this section we investigate what the prize or patent authority should do when λ is unknown. We assume that the prize or patent authority takes the length of time without arrival as a signal of λ. A long delay with no arrival should make the observer more pessimistic about λ – it shifts the posterior distribution on λ toward lower values. However,

the posterior distribution on λ must also account for the fact that some ideas are rejected. Thus, the investment strategy by which recipients accept or reject ideas is an ingredient to forming a posterior belief on λ.

We showed above that, when λ is known, the minimum acceptable cost to fill the market niche is stationary. The stationary value is larger when ideas are scarce (λ is small) than when ideas are obvious. The stationarity is essentially because the optimized value function V is stationary, where V describes the social value of rejecting an idea and waiting for a lower-cost idea.

We now show that, when the posterior distribution on λ is changing as time passes, neither the optimized value function nor the optimal investment strategy is stationary. As time passes, the posterior distribution on λ shifts toward lower values. This implies that the (optimized) value of waiting for a better idea decreases with time. This in turn implies that society should optimally be less discriminating about which ideas are accepted. The socially optimal cost threshold is increasing instead of being stationary.

Let $\tilde{h}$ be the prior density function for the distribution of λ with support $[0, \infty)$. Then the posterior density, conditional on a threshold function c, and conditional on no viable hit having arrived by time t, is $h(\cdot|t, c)$ with cumulative distribution $H(\cdot|t, c)$, where $h(\cdot|t, c)$ satisfies

$$h(\lambda|t, c) = \frac{\tilde{h}(\lambda) e^{-\Lambda(t,c)}}{\int \tilde{h}(\lambda) e^{-\Lambda(t,c)} d\lambda} \text{ for each } \lambda \in (0, \infty)$$

The posterior depends on the threshold function c up to time t, through the value $\Lambda(t, c)$. Hence, the prior distribution $\tilde{h}$ can be written $h(\cdot|0, c)$ for any threshold function c, since at time 0 there is no prior history on which to condition a posterior, and $\Lambda(0, c) = 0$.

Let $E(\lambda|t, c)$ be the expected value of λ:

$$E(\lambda|t, c) = \int_0^\infty \lambda h(\lambda|t, c) d\lambda$$

The following lemma is proved in the appendix.

Lemma 1 *Given a threshold function c, if $t_1 < t_2$, then the distribution $h(\cdot|t_1, c)$ stochastically dominates $h(\cdot|t_2, c)$, and $E(\lambda|t_2, c) < E(\lambda|t_1, c)$.*

8

The probability that the first viable idea arrives at $\hat{t}$, conditional on λ and conditional on there being no viable idea before t, is given by (2). Then, $\phi\left(\hat{t}|t, c, \lambda\right) h\left(\lambda|t, c\right)$ is the joint density of λ and a first viable idea at time $\hat{t}$, conditional on there being no viable idea before t:

$$\phi\left(\hat{t}|t, c, \lambda\right) h\left(\lambda|t, c\right) = \lambda F\left(c\left(\hat{t}\right)\right) e^{-\left[\Lambda\left(\hat{t}, c\right) - \Lambda(t, c)\right]} h\left(\lambda|t, c\right)$$

Assuming that there has been no viable idea before t, the social value of continuing from time t is given by the function $\tilde{V}$ defined below. For generality and for use in the arguments below, we have allowed that the threshold function $\tilde{c}$ that determines the posterior distribution of λ can differ from the threshold function c that is relevant to the value $\tilde{V}$ going forward from time t. Since $\tilde{c}$ determines the posterior distribution on λ at time t, the only relevant values are those that occur before t.

$$\begin{aligned}
\tilde{V}\left(t, c, h\left(\cdot|t, \tilde{c}\right)\right) &= \int_0^\infty \int_t^\infty e^{-r\left(\hat{t}-t\right)} \left(\frac{v}{r} - E_F\left(c\left(\hat{t}\right)\right)\right) \phi\left(\hat{t}|t, c, \lambda\right) h\left(\lambda|t, \tilde{c}\right) \ d\hat{t}d\lambda \\
&= \int_0^\infty V\left(t, c, \lambda\right) h\left(\lambda|t, \tilde{c}\right) d\lambda
\end{aligned}$$

If the optimizing function c is followed at every t, the optimizing function determines both the posterior on λ at each time t and the value V going forward. By the principle of optimality, it will not be revised. Let $c : \mathbf{R}_+ \to \mathbf{R}_+$ be the threshold function that maximizes $\tilde{V}\left(0, \cdot, \tilde{h}\right)$. Then, analogously to (3) and (4) in the previous subsection, the optimal threshold function c satisfies the following at each t:

$$\frac{v}{r} - c\left(t\right) = \tilde{V}\left(t, c, h\left(\cdot|t, c\right)\right) \tag{7}$$

$$-c'\left(t\right) = \frac{d}{dt}\tilde{V}\left(t, c, h\left(\cdot|t, c\right)\right) \tag{8}$$

To show that $c'\left(t\right) > 0$ at t, it is enough to show that $\frac{d}{dt}\tilde{V}\left(t, c, h\left(\cdot|t, c\right)\right) < 0$, where c is the optimal threshold function. The intuitive reason that $\tilde{V}$ is decreasing is that the observer becomes more and more pessimistic about the arrival rate of ideas as time continues without a viable hit. Because of this pessimism, more delay is expected. To mitigate delay, it is optimal to tolerate higher cost. That is why the optimal c is increasing. In the appendix we prove the following result.

9

Proposition 4 *Suppose that the arrival rate of ideas, λ, has a prior distribution $\tilde{h}$ with support $[0, \infty)$, and suppose that the threshold function c maximizes $\tilde{V}\left(0, \cdot, \tilde{h}\right)$. Then c is increasing.*

The reward function that implements the optimal threshold function c is again $\rho = c$, from which it follows that:

Proposition 5 *Suppose that the optimal threshold function is increasing with the arrival time of the innovation. Then the optimal reward is also increasing with the arrival time.*

3   Optimal rewards, secrecy and scarce ideas

So far, we have assumed that the only compensation available to an innovator is the reward. In this section, we consider environments where it is possible to appropriate the value of the innovation, and ask whether the optimal reward must be modified if the innovator is tempted to keep the innovation secret while marketing it instead of claiming the reward. The key assumption is that the profit available from the secret innovation ends either when it leaks out or when another firm gets the reward for an innovation which also fills the given market niche. Hence, secrecy is more attractive precisely in those environments where ideas are scarce.

We assume that the innovator cannot both get the reward and keep the innovation secret. This is a reasonable assumption since the goal of most reward systems is to make innovations public. Hence, if the innovator gets the reward, the innovation is disclosed.

Since secrecy creates rewards through monopoly power, we must make an assumption about per-period profit. We will assume for simplicity that the monopolist collects the whole social value v during the period of market incumbency. This assumption allows us to focus on the aspect of efficiency discussed in this paper, which concerns the trade-off between delay and cost, without being sidetracked into a discussion of deadweight loss. We comment at the end on how these conclusions change if monopoly power entails deadweight loss.

10

Secrecy entails social costs, even in the absence of deadweight loss. In this setup, secrecy may cause the market niche to be filled twice – first by the innovator who keeps the innovation secret, and then possibly by a successor, who discloses his innovation.[1] Surprisingly, we show that under the optimal reward structure derived in section 2, the first innovator never has incentives to keep the innovation secret, even if he can earn the entire social value during the period of secrecy.

For simplicity, we revert to the model where λ is known and fixed. We assume that leakage occurs as a Poisson process with hit rate γ.[2] The hit rate for ideas that will displace the secret innovation is $\lambda F(\rho)$, where ρ is the reward.[3] Consequently, if the per-period reward is v, the expected revenue from keeping the innovation secret is

$$
\begin{aligned}
\rho^S(\rho, \lambda, \gamma) &= \int_0^\infty e^{-r\hat{t}} v \left(1 - e^{-r\hat{t}}\right) (\gamma + \lambda F(\rho)) \, e^{-(\gamma + \lambda F(\rho))\hat{t}} d\hat{t} \\
&= \frac{v}{\gamma + \lambda F(\rho) + r}.
\end{aligned}
\tag{9}
$$

The profit from secrecy depends on the reward ρ because the reward determines the delay before a disclosed, substitute product enters the market.

In the following proposition, we show that the innovator never prefers secrecy.

Proposition 6 *Secrecy is never preferred to claiming the reward.*

Proof: It holds that $\rho^s(\rho^*(\lambda), \lambda, \gamma) < \rho^*(\lambda)$ if

$$
\frac{v}{r}\left(1 - \frac{\gamma + \lambda F(\rho^*(\lambda))}{\gamma + \lambda F(\rho^*(\lambda)) + r}\right) < \rho^*(\lambda)
$$

$$
\left(\frac{v}{r} - \rho^*(\lambda)\right) - \left(\frac{\gamma + \lambda F(\rho^*(\lambda))}{\gamma + \lambda F(\rho^*(\lambda)) + r}\right)\frac{v}{r} < 0
$$

---

[1] Implicitly we make the simplifying assumption that the second comer will not keep his innovation secret, even if the first innovator does. This is because the duopoly profit is not very lucrative relative to the reward.

[2] Hence, in section 2, we were assuming that $\gamma = \infty$ and leakage occurs immediately.

[3] Hence, we assume that if an innovation is kept secret, it does not prevent another innovator from qualifying for the reward. In the case that the reward is given as a patent, this means that secret innovations are not treated as prior art.

Using (6) and $\left(\frac{v}{r} - \rho^*(\lambda)\right) = \left(\frac{v}{r} - c^*(\lambda)\right)$, this implies

$$\left(\frac{\lambda F(\rho^*(\lambda))}{\lambda F(\rho^*(\lambda)) + r}\right)\left(\frac{v}{r} - E_F(\rho^*(\lambda))\right) - \left(\frac{\gamma + \lambda F(\rho^*(\lambda))}{\gamma + \lambda F(\rho^*(\lambda)) + r}\right)\frac{v}{r} < 0$$

$$\frac{v}{r}\left(\frac{\lambda F(\rho^*(\lambda))}{\lambda F(\rho^*(\lambda)) + r} - \frac{\gamma + \lambda F(\rho^*(\lambda))}{\gamma + \lambda F(\rho^*(\lambda)) + r}\right) - \frac{\lambda F(\rho^*(\lambda))}{\lambda F(\rho^*(\lambda)) + r}E_F(\rho^*(\lambda)) < 0$$

which always holds. $\square$

This proposition might be something of a surprise because the result does not depend on the magnitudes of λ and γ. From (9), we see that the direct effect of a decrease in λ or γ is to make secrecy more attractive at a given reward ρ. When ideas are scarce (λ is low) or secrets are easy to keep (γ is low), secrecy may give the innovator more profit than the reward, provided the reward is held fixed. However, as λ decreases, the policy maker tries to compensate for the decrease in the arrival rate of ideas by making the reward more attractive. Hence, when rewards are chosen optimally conditional on λ, secrecy is never preferred.[4]

An intuition is that, although the innovator earns the full consumption value of the innovation during the period of secrecy, he does not earn the full social value because he does not collect as profit the expected costs that are saved by investing at present rather than waiting. The optimal stationary reward $\rho^*(\lambda)$ grants the (larger) full social value.

The optimal stationary reward can be written as

$$\rho^*(\lambda) = \frac{v}{\lambda F(\rho^*(\lambda)) + r} + \frac{\lambda F(\rho^*(\lambda))}{\lambda F(\rho^*(\lambda)) + r}E_F(\rho^*(\lambda)) \tag{10}$$

The middle term is the additional consumption value of investing in the current idea rather than waiting, which is equal to the profit earned by the secret innovation when the leakage rate is zero ($\gamma = 0$). (This is the most favorable circumstance for the secret innovation.) The last term of (10) is the expected discounted cost of the next viable idea. By investing now, society gets the interim social value (the middle term) and avoids the costs of investing later. Thus, the optimal reward $\rho^*(\lambda)$ is the sum of both terms, and it is larger than the profit available from secrecy, $\rho^S(\rho^*(\lambda), \lambda, \gamma)$, which is only the middle term.

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[4] This result is in contrast with that of Erkal (2005) who shows that it would be optimal to have a lenient antitrust policy in industries where secrecy is an attractive option.

If market power imposes deadweight loss, the conclusions so far are strengthened. There is even less incentive to keep the innovation secret, and from a social point of view, secrecy is even less desirable. This does not depend on whether the reward ρ is given as a prize or a patent.

4 Balancing the rewards to basic innovations and applications

So far, we have argued in the context of a single innovation that the reward to innovation should increase with the scarcity of ideas when λ is known, and with delay when λ is unknown. We have assumed that the innovation has a fixed social value, v, per unit time and asked how to incite optimal screening of ideas in a way that balances delay against cost.

However, where innovation is cumulative, the social value of the innovation derives from the social welfare of applications. The value created by an innovation depends on future ideas to put the innovation to use. For example, the laser, which was patented in the 1950's and has no commercial value itself, has many applications with commercial value, such as laser surgery.

Ideas for applications may be scarce or obvious themselves. The innovation policy must then use a common set of policy levers to elicit investments at two stages. In the previous sections, there was no suggestion that the reward is given as a patent. In fact, the easiest interpretation is that the reward is a prize. In this section, we have in mind that the reward is given as a patent.

Since the patent treatments of the two innovations are intertwined, one of the questions is whether the first best can be achieved with the policy levers available. Suppose, in particular, that ideas for the first innovation are scarce (the arrival rate is low). Then, according to our conclusion in section 2, the reward to the first innovator should be high. However, since the reward is bounded by the value of the application, it might not be possible to give a high enough reward to the first innovator while also rewarding the application

13

developer.

The balancing of incentives between the two generations has features in common with other models of sequential innovation, but also differences. First, since both innovators share a common pot of money – the value of the applications – infringement is a key requirement for proper incentives. Without a claim on the profit generated by applications, the first innovator would not invest and the applications might never materialize (or might materialize with long delay) (Scotchmer, 1991). Second, there is a question of how to divide the profit generated by the application, as well as a question of how much total profit to create (Green and Scotchmer, 1995). Third, profit can be protected either because the application has a patent or because the application infringes a prior patent (Scotchmer, 1996; Denicolo, 2000). If both hold, the innovators must divide the profit. The difficulty added in our model is that ideas for applications occur at random times, leading to randomness in the periods of protection and infringement. Rewards for applications may therefore be random. This may cause inefficiencies.

We first characterize the inefficiencies that may arise in the case of a single application, and then investigate the extent to which they may be remedied when there are many potential applications.

4.1 Single application

The parameter λ is the Poisson hit rate of ideas for the underlying innovation and δ is the Poisson hit rate of ideas for the application. We assume that the substitute ideas for the application have costs drawn independently from a common distribution G with density g.

As before, we first consider the optimal investment strategy. We then ask whether the optimal investment strategy can be implemented in a patent regime. If the hit rates λ and δ are fixed and known, the optimal investment strategies are stationary, as derived in Proposition 1. There exist stationary threshold values, $c_A^* (\delta)$ and $c_B^* (\lambda, \delta)$, which maximize social welfare, where the subscripts A and B refer to the application and the basic innovation.

14

These are the values of c that maximize (11) and (12), respectively.

$$
\begin{aligned}
V_A\left(c;\delta\right) &= \int_0^\infty \left(\frac{v}{r} - E_G\left(c\right)\right) e^{-rt} \delta G\left(c\right)\ e^{-\delta G\left(c\right)t}\ dt \\
&= \frac{\delta G\left(c\right)}{\delta G\left(c\right) + r} \left(\frac{v}{r} - E_G\left(c\right)\right)
\end{aligned}
\tag{11}
$$

$$
V_B\left(c;\lambda,\delta\right) = \frac{\lambda F\left(c\right)}{\lambda F\left(c\right) + r} \left(V_A\left(c_A^*\left(\delta\right);\delta\right) - E_F\left(c\right)\right)
\tag{12}
$$

We will assume throughout that the per-period value v of the application is fixed.

Proposition 7 [Social Optimum] *Suppose that the innovation creates value only through an application with per-period social value v. Suppose that substitute ideas for the innovation occur at Poisson rate λ, with costs drawn independently from a distribution F, and that substitute ideas for the application occur at Poisson rate δ, with costs drawn independently from a distribution G. Then*

(a) The optimal stationary cost thresholds $\left(c_A^*\left(\delta\right), c_B^*\left(\lambda,\delta\right)\right)$ are the unique values that satisfy

$$
0 = \left(\frac{v}{r} - c_A^*\left(\delta\right)\right) - \left(\frac{\delta G\left(c_A^*\left(\delta\right)\right)}{\delta G\left(c_A^*\left(\delta\right)\right) + r}\right) \left(\frac{v}{r} - E_G\left(c_A^*\left(\delta\right)\right)\right)
\tag{13}
$$

$$
0 = \left(V_A\left(c_A^*\left(\delta\right);\delta\right) - c_B^*\left(\lambda,\delta\right)\right) - \left(\frac{\lambda F\left(c_B^*\left(\lambda,\delta\right)\right)}{\lambda F\left(c_B^*\left(\lambda,\delta\right)\right) + r}\right) \left(V_A\left(c_A^*\left(\delta\right);\delta\right) - E_F\left(c_B^*\left(\lambda,\delta\right)\right)\right)
\tag{14}
$$

(b) The optimal cost threshold $c_A^*\left(\delta\right)$ decreases with δ, and the optimal cost threshold $c_B^*\left(\lambda,\delta\right)$ decreases with λ.

(c) The optimal cost threshold $c_B^*\left(\lambda,\delta\right)$ increases with δ.

Proof: (a) The functions $V_A\left(c;\delta\right)$ and $V_B\left(c;\lambda,\delta\right)$ are strictly quasiconcave. These characterizations follow from the first order conditions $\frac{\partial}{\partial c} V_A\left(c;\delta\right) = 0$, evaluated at $c = c_A^*\left(\delta\right)$, and $\frac{\partial}{\partial c} V_B\left(c;\lambda,\delta\right) = 0$, evaluated at $c = c_B^*\left(\lambda,\delta\right)$.

(b) As in section 2, this follows from differentiating the first order conditions, (13) and (14).

15

(c) The optimizer $c_B^*(\lambda, \beta)$ of (12) increases with the value $V_A(c_A^*(\delta), \delta)$. It is easy to see from (11) that the optimized value $V_A(c_A^*(\delta), \delta)$ is increasing with δ even though the cost threshold $c_A^*(\delta)$ is decreasing with δ. $\square$

The only news in Proposition 7 is that, if the hit rate δ for the application is high, then the optimal cost threshold for the basic innovation is also high, even though the optimal cost threshold for the application is low.

The optimum can clearly be implemented by prizes that depend on the hit rates, namely $\rho_A(\delta) = c_A^*(\delta)$ and $\rho_B(\lambda, \delta) = c_B^*(\lambda, \delta)$. We now turn to whether the optimal investment strategies can be implemented with patent instruments, in particular, different patent lives for the two innovations, and how that should depend on the scarcity of ideas at both stages.

We model the patent instruments in a flexible way as functions of the arrival date of the application. This more general modelling approach is convenient because we are not confined to constant patent lives. In fact, our first result is that if the patent lives for the two innovations are constant, it is not possible to implement the efficient cost thresholds. We therefore investigate whether it is possible to implement the efficient cost thresholds with the more flexible patent instruments. We show that even with such instruments, it may not be possible to implement the optimal cost thresholds. We comment in section 5 on how other levers of patent law might be used instead.

The flexible patent instruments are two patent-life functions (T_B, T_A), with values $T_B(t) \geq t$ and $T_A(t) \geq 0$, where t is the arrival date of the application, measured from the date of the basic innovation. A value $T_B(t) > t$ expresses the notion that the application infringes for the length of time $T_B(t) - t$ after the application arrives. The easiest interpretation is that the patent on the basic innovation ends at $T_B(t)$. Another interpretation is that the patent runs forever, and $T_B(t)$ is the date at which infringement ends. The value $T_B(t) = t$ expresses the notion that the patent has expired when the application arrives, so there is no period of infringement. $T_A(t)$ is the patent life of the application.

If an application developer has an idea at time t, the expected revenue from developing the application is given by Π^A, defined in (15) below, assuming that the profit from the
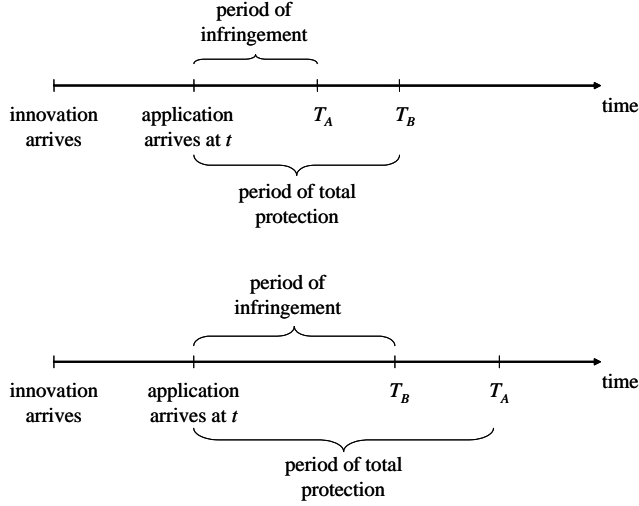
16

Figure 1: Patent-life functions

application is shared as $(\beta, 1 - \beta)$ during the period of infringement, where $\beta \in (0, 1)$. (This is an exogenous assumption about relative bargaining power in achieving a license.) The revenue earned by the application depends on the arrival time t because both the length of infringement $T_B(t) - t$ and the application patent life $T_A(t)$ may depend on t.

Profit is collected for a length of time that is the maximum of $T_B(t) - t$ and $T_A(t)$. Figure 1 represents the two possible cases. In the first case, $T_A(t) \leq T_B(t) - t$. The total period of protection is determined by the length of the patent on the basic innovation. The application developer pays a licensing fee to the first innovator until the application patent expires. This is reflected in the first line in the definition of Π^A, given in (15). The first innovator continues to earn the flow value v of the application until the patent on the first innovation expires even if the patent on the application has expired. In the second case, $T_A(t) > T_B(t) - t$. The total period of protection is determined by the length of the patent on the application while the period of infringement is determined by $T_B(t) - t$. As shown in the second line in the definition of Π^A, during this period, the application developer and first innovator share the returns from the application. After this period, the application developer keeps all the profits.

17

Given the patent-life functions (T_B, T_A), it is useful to partition arrival times into two sets, $\mathcal{T} \cup \bar{\mathcal{T}} = (0, \infty)$. At arrival times $t \in \mathcal{T}$, total profit is determined by the patent on the application. At arrival times $t \in \bar{\mathcal{T}}$, total profit is determined by the patent on the basic innovation.

$$T_A(t) \leq T_B(t) - t \text{ for each } t \in \bar{\mathcal{T}}$$
$$T_A(t) > T_B(t) - t \text{ for each } t \in \mathcal{T}.$$

$$\Pi^A(t; T_B, T_A) \tag{15}$$

$$= \begin{cases} (1 - \beta) \frac{v}{r} \left(1 - e^{-rT_A(t)}\right) & \text{if} & \begin{array}{l} T_A(t) \leq T_B(t) - t \\ (\text{or } t \in \bar{\mathcal{T}}) \end{array} \\ \\ \frac{v}{r} \left(1 - e^{-r(T_A(t))}\right) - \beta \frac{v}{r} \left(1 - e^{-r(T_B(t)-t)}\right) & \text{if} & \begin{array}{l} T_A(t) > T_B(t) - t \\ (\text{or } t \in \mathcal{T}) \end{array} \end{cases}$$

The application is developed by the first person with an idea that would make non-negative profit, namely, the first time t at which someone has an idea with cost less than $\Pi^A(t; T_B, T_A)$.

The profit available to the basic innovation is given by a function Π^B defined as

$$\Pi^B(T_B, T_A) = \int_{\bar{\mathcal{T}}} \left[\beta \frac{v}{r} \left(1 - e^{-rT_A}\right) + \frac{v}{r} \left(e^{-rT_A} - e^{-r(T_B(t)-t)}\right) \right] e^{-rt} \delta G(c(t)) e^{-\Delta(t,c)} dt$$
$$+ \int_{\mathcal{T}} \beta \frac{v}{r} \left(1 - e^{-r(T_B(t)-t)}\right) e^{-rt} \delta G(c(t)) e^{-\Delta(t,c)} dt \tag{16}$$

where

$$\Delta(t, c) = \int_0^t \delta G(c(\tilde{t})) d\tilde{t}$$

The basic innovation is developed by the first person whose idea has cost less than $\Pi^B(T_B, T_A)$.

Hence, given (T_B, T_A), the function $\Pi^A(\cdot; T_B, T_A)$ is a threshold cost function for investing in the application and $\Pi^B(T_B, T_A)$ is a stationary threshold cost for investing in the basic innovation. We say that the patent-life functions (T_B, T_A) *implement* (c_B, c_A) if $\Pi^B(T_B, T_A) = c_B$ and $\Pi^A(t; T_B, T_A) = c_A$ for each t. We say that the stationary cost

thresholds (c_B, c_A) are *implementable* if they are in the following set:

$$\mathcal{C} = \{(c_B, c_A) : \text{there exist patent life functions } (T_B, T_A) \text{ that implement } (c_B, c_A)\}$$

The most realistic restriction to put on the patent policy is that the patent lives are constant. We say the patent life functions (T_B, T_A) are *constant* if for some $\bar{T}_B \in \mathbf{R}_+$ and $\bar{T}_A \in \mathbf{R}_+$, the functions T_B, T_A satisfy $T_A(t) = \bar{T}_A$ for all t and $T_B(t) = \bar{T}_B$ for $t \leq \bar{T}_B$ and $T_B(t) = t$ for $t > \bar{T}_B$.

We point out in Proposition 8 that constant patent lives cannot implement a stationary cost threshold for the application. This is because the function $\Pi^A(\cdot, T_B, T_A)$ induces different cost thresholds at different arrival times t, even when the patent lives are constant. An application with a later arrival date faces a shorter period of infringement and gets more of the profit generated by its own patent.

Proposition 8 [Constant patent lives] *Suppose that the value of the innovation resides in an application with per-period value v. Assume that the application infringes the prior innovation's patent during its patent life. There are no constant patent life functions (T_B, T_A) that implement stationary cost thresholds.*

When the arrival rates are fixed and known, by Proposition 7 the efficient cost thresholds are stationary. Thus, constant patent lives cannot implement efficient outcomes.[5] Of course, we would not generally expect patent mechanisms to implement efficient outcomes due to deadweight loss. However, deadweight loss is not the source of the inefficiency here since we have assumed that patentholders earn the full value of the innovation during the patent life. The source of the inefficiency is that the incentive to invest in the application depends on when the idea occurs. The recipient of a great idea may invest if it occurs late, but

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[5] This proposition shows that the economic conclusions are different in our environment where ideas are scarce than in environments where ideas are common knowledge. Koo and Wright (2007) also study a model of applications with constant patent lives, and assume that the potential application developers know at the date of the first innovation how to achieve the application. Hence, inefficiencies may arise in their model, not because ideas for applications arrive at random times, but rather because market structure may result in delay or competitive rent dissipation.

may not invest earlier, due to the large licensing fees he would then owe to the prior patent holder.[6]

Even though constant patent lives will not implement stationary cost thresholds, it can be seen from (15) and (16) that other patent life functions (T_B, T_A) will do so. Lemma 2, proved in the appendix, says that all patent life functions (T_B, T_A) that implement given stationary thresholds (c_B, c_A) are equivalent to patent life functions for which the period $T_B(t) - t$ is constant and the patent life for the application is constant (but the patent life for the basic innovation is not constant). In Lemma 2, the constant k is the period of infringement.

Lemma 2 [Patent lives that implement stationary cost thresholds] *Suppose that the innovation gets value only through an application with per-period value v and that (c_B, c_A) are stationary cost thresholds implemented by $\left(\tilde{T}_B, \tilde{T}_A\right)$. Then there exist (T_A, T_B) that also implement (c_B, c_A) and satisfy the following for some $k \in \mathbf{R}_+, \bar{T}_A \in \mathbf{R}_+$ and every $t \in (0, \infty)$:*

$$
\begin{aligned}
T_B(t) - t &= k \qquad\qquad (17) \\
T_A(t) &= \bar{T}_A
\end{aligned}
$$

However, it is still not obvious whether there exist (T_B, T_A) which implement the *efficient* stationary thresholds. Proposition 9, proved in the appendix, characterizes the limits on profit sharing, even with the very flexible patent instruments we consider. It focuses on stationary thresholds (c_B, c_A) since stationarity is efficient.

Proposition 9 [Implementing the optimum with a single application] *Suppose that an innovation gets its value through an application with per-period value v. Suppose that substitute ideas for an application occur at Poisson rate δ, with costs drawn independently from a distribution G. Then the set $\mathcal{C}$ of implementable stationary thresholds is described by*

$$
\left\{ (c_B, c_A) \in \mathbf{R}_+ \times \mathbf{R}_+ \;\middle|\; \begin{array}{l} 0 \le c_A \le \frac{v}{r} \\[2mm] 0 \le c_B \le \frac{\delta G(c_A)}{\delta G(c_A) + r}\left(\frac{v}{r} - c_A\right) \end{array} \right\} \qquad (18)
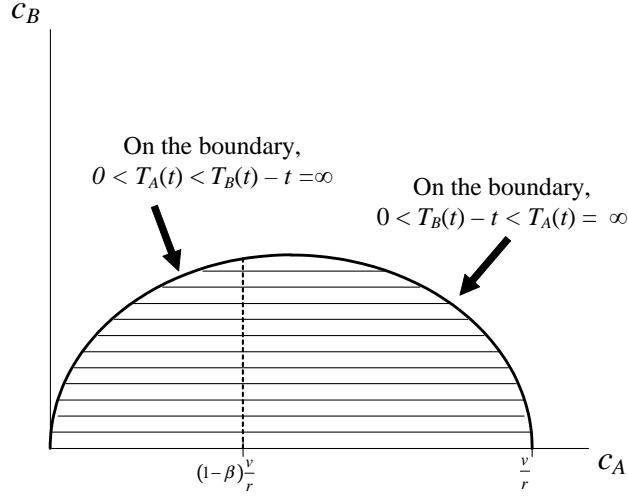$$

20

Figure 2: Implementable cost thresholds

Figure 2 shows the set (18) of implementable stationary thresholds. The boundary shows what is implementable when the two firms together collect the maximum possible profit, $\frac{v}{r}$.[7] An increase in c_A has two opposite effects on the values of c_B that can be implemented. First, since an increase in c_A brings forward the expected arrival time of the application, it increases the expected discounted profit of the first innovator. This is why the boundary on the left side of Figure 2 ($c_A < (1 - \beta) \frac{v}{r}$) is increasing. The points on the left side of the boundary are determined by setting $T_B(t) - t = \infty$ for each t, and letting the constant value $\bar{T}_A(t)$ range from ∞ to 0. When c_A is low, the maximum c_B which can be implemented is low because the innovator's reward, which comes from the application, is much delayed. The other effect arises because the firms are sharing a fixed total payoff, which is the private value of the application. When the payoff to one firm increases, the payoff to the other decreases. This is why the boundary on the right side of Figure 2 ($c_A > (1 - \beta) \frac{v}{r}$) is decreasing. The points on this part of the boundary are determined by setting $\bar{T}_A = \infty$ and letting the constant value $k = T_B(t) - t$ range from ∞ to 0.

---

[7] Although the upper limit on c_B is drawn in Figure 2 as a single-peaked function, this does not necessarily hold.

A first look at Figure 2 suggests that there will be problems in implementing the efficient cost thresholds when innovation ideas are scarce but application ideas are frequent (λ is low but δ is high). The optimal $c_B^*(\lambda, \delta)$ is then high and the optimal $c_A^*(\delta)$ is low. Problems also seem likely in environments where both innovation and application ideas are scarce (λ and δ are low), since the optimal $c_B^*(\lambda, \delta)$ and $c_A^*(\delta)$ are both high. However, the boundary in Figure 2 shifts as δ changes. The following proposition, proved in the appendix, states that, for any fixed λ, the efficient thresholds $(c_B^*(\lambda, \delta), c_A^*(\delta))$ lie within the set in Figure 2 as $\delta \to \infty$ and $\delta \to 0$. As $\delta \to \infty$, the optimal reward to the application converges to zero, so all the social value can be given to the first innovator. As $\delta \to 0$, the value of the application becomes zero, so it is not optimal to support the basic innovation.

Nevertheless, efficiency may not always be achievable. We show this by considering what happens as $\lambda \to 0$, with δ fixed. For low λ, efficient incentives require a high reward for the first innovator. This may be inconsistent with giving adequate reward to the application developer.

Proposition 10 *Let $(c_B^*(\lambda, \delta), c_A^*(\delta))$ stand for the efficient stationary cost thresholds. (i) For given $\lambda \in (0, \infty)$, as $\delta \to \infty$ and as $\delta \to 0$, $(c_B^*(\lambda, \delta), c_A^*(\delta))$ are implementable. (ii) For given $\delta \in (0, \infty)$, as $\lambda \to 0$, $(c_B^*(\lambda, \delta), c_A^*(\delta))$ are not implementable.*

4.2 Multiple applications

We now ask whether the problem of inefficiency stated in Proposition 10 is mitigated when there are n applications rather than one. On one hand, if there are n applications, each application developer only pays $1/n$ of the innovator's total reward. Thus, even if ideas for each application are frequent, so they optimally receive low rewards, the first innovator might be highly rewarded because he collects licensing fees from many applications. On the other hand, the social value of the innovation increases proportionately with the number of applications. Delay in achieving the first innovation becomes more costly, which calls for a higher reward. This may restore the conflict.

We investigate which of these intuitions dominates. Proposition 11 shows a sense in

which, with many applications, the constraint described in Proposition 10(ii) will not bind very severely as n becomes large, regardless of how low the reward to applications is.

If there are n applications, each with the same arrival rate of ideas δ, the same per-period social value v, and the same distribution of costs, the optimized value of the applications is $nV_A\left(c_A^*\left(\delta\right);\delta\right)$ instead of $V_A\left(c_A^*\left(\delta\right);\delta\right)$. Analogously to (14), the optimal investment threshold for the basic innovation, $c_{B,n}^*\left(\lambda,\delta\right)$, satisfies

$$
\begin{aligned}
0 \; = \; & \left(nV_A\left(c_A^*\left(\delta\right);\delta\right) - c_{B,n}^*\left(\lambda,\delta\right)\right) \\
& - \left(\frac{\lambda F\left(c_{B,n}^*\left(\lambda,\delta\right)\right)}{\lambda F\left(c_{B,n}^*\left(\lambda,\delta\right)\right) + r}\right)\left(nV_A\left(c_A^*\left(\delta\right);\delta\right) - E_F\left(c_{B,n}^*\left(\lambda,\delta\right)\right)\right)
\end{aligned}
\tag{19}
$$

Since the value of the innovation is n times its value with one application, there is more social cost to delaying the innovation, and the innovation should be more highly rewarded. It is easy to see from (19) that $c_{B,n}^*\left(\lambda,\delta\right)$ is increasing in n and also that $F\left(c_{B,n}^*\left(\lambda,\delta\right)\right) \to 1$ as $n \to \infty$. That is, with high probability, the first idea for the innovation should be accepted.

We now ask whether the optimal stationary cost thresholds $\left(c_{B,n}^*\left(\lambda,\delta\right), c_A^*\left(\delta\right)\right)$ can be implemented. Given $\left(T_A, T_B\right)$, the first innovator's profit scales with the number of applications. Namely, it is $n\Pi^B\left(T_B, T_A\right)$. Hence, the equilibrium cost threshold for the underlying innovation also scales with the number of applications. With one application, the cost threshold is $\Pi^B\left(T_A, T_B\right)$. With n applications, the cost threshold is $n\Pi^B\left(T_A, T_B\right)$.

Proposition 11 shows that with many applications, the optimal stationary cost thresholds can be implemented if the support of F is bounded. Even if the applications receive a low reward, the first innovation's reward will be high enough so that every cost from the bounded support is accepted. The difficulty in sharing profit vanishes.

This is not true if the support of F is unbounded. The first idea that arrives may have such high cost that it exceeds the profit available to the first innovator from licensing n applications. However, as n grows, the probability that this happens becomes lower and lower. The tails of the distribution F have only small probability if the mean of F is finite. Therefore, the probability of discarding the first idea goes to zero. In expectation, the delay in achieving the first innovation also goes to zero.

Proposition 11 [Implementing the optimum with many applications] *Suppose that an innovation gets its value through n applications, each with per-period value v. Suppose that substitute ideas for the innovation occur at Poisson rate λ, with costs drawn independently from a distribution F, and that substitute ideas for each application occur at Poisson rate δ, with costs drawn independently from a distribution G.*

(a) If F has bounded support $[\underline{c}, \bar{c}]$, then there exists $\bar{n}$ such that if $n > \bar{n}$, the optimal stationary cost thresholds $\left(c^*_{B,n}(\lambda, \delta), c^*_A(\delta) \right)$ are implementable.

(b) If the support of F is $[0, \infty)$, there exist a sequence of implementable stationary cost thresholds $(\bar{c}_{B,n}, c_A)$ such that $\left(\frac{\lambda F(\bar{c}_{B,n})}{\lambda F(\bar{c}_{B,n}) + r} \right) \to \left(\frac{\lambda F(c^*_{B,n}(\lambda, \delta))}{\lambda F(c^*_{B,n}(\lambda, \delta)) + r} \right)$ as $n \to \infty$.

Proof: With n applications, the following describes the thresholds $(c_{B,n}, c_A)$ that can be implemented, analogously to Proposition 9:

$$
\left\{ (c_{B,n}, c_A) \in \mathbf{R}_+ \times \mathbf{R}_+ \;\middle|\; \begin{array}{l} 0 \leq c_A \leq \frac{v}{r} \\[2mm] 0 \leq \frac{c_{B,n}}{n} \leq \frac{\delta G(c_A)}{\delta G(c_A) + r} \left(\frac{v}{r} - c_A \right) \end{array} \right\} \tag{20}
$$

(a) Since $c^*_{B,n}(\lambda, \delta) \leq \bar{c}$, it holds that $\frac{c^*_{B,n}(\lambda, \delta)}{n} \to 0$ as $n \to \infty$. Hence, since $c_A < \frac{v}{r}$, it follows directly from (20) that $\left(c^*_{B,n}(\lambda, \delta), c^*_A(\delta) \right)$ is implementable for sufficiently large n.

(b) The inequalities in (20) imply that $\left(c^*_{B,n}(\lambda, \delta), c^*_A(\delta) \right)$ are implementable if $c^*_{B,n}(\lambda, \delta) \leq \bar{c}_{B,n}$, where $\bar{c}_{B,n} = n \left(\frac{\delta G(c^*_A(\delta))}{\delta G(c^*_A(\delta)) + r} \right) \left[\frac{v}{r} - c^*_A(\delta) \right]$. Then if $c^*_{B,n}(\lambda, \delta) \leq \bar{c}_{B,n}$, $\left(c^*_{B,n}(\lambda, \delta), c^*_A(\delta) \right)$ are implementable. Suppose instead that $c^*_{B,n}(\lambda, \delta) > \bar{c}_{B,n}$ for some large n. It follows from (19) and the fact that $n V_A\left(c^*_A(\delta); \delta \right) \to \infty$ as $n \to \infty$ that $c^*_{B,n}(\lambda, \delta) \to \infty$ as $n \to \infty$. It also holds that $\bar{c}_{B,n} \to \infty$. Therefore, $\left(\frac{\lambda F(c^*_{B,n}(\lambda, \delta))}{\lambda F(c^*_{B,n}(\lambda, \delta)) + r} \right) \to \left(\frac{\lambda}{\lambda + r} \right)$ and $\left(\frac{\lambda F(\bar{c}_{B,n})}{\lambda F(\bar{c}_{B,n}) + r} \right) \to \left(\frac{\lambda}{\lambda + r} \right)$, hence $\left(\frac{\lambda F(c^*_{B,n}(\lambda, \delta))}{\lambda F(c^*_{B,n}(\lambda, \delta)) + r} \right) \to \left(\frac{\lambda F(\bar{c}_{B,n})}{\lambda F(\bar{c}_{B,n}) + r} \right)$. $\square$

Proposition 11 can be interpreted to mean that any conflict in rewarding the two generations of innovators almost vanishes when there are many applications. Even if each application generates low revenue, the revenue earned by the first innovation can be high since the innovator receives licensing fees from each application.

5 Economic Concepts and Legal Concepts

There are two sets of arguments in this paper. One set of arguments is about the optimal size of rewards. The other is about how the rewards should be structured when two generations of innovations are at stake.

We have argued that rewards (whether prizes or patents) should be higher in environments where ideas are scarce. If ideas are scarce, higher cost should be tolerated in order to reduce delay. The same principle applies in the context of basic and applied research. If rewards can be given separately for the two generations of innovations, then in each case, the optimal reward depends on the scarcity of ideas for achieving it.

In the context of basic and applied research, there are two reasons that first-best investment incentives might not be achievable with intellectual property.[8] The first is that there is a natural problem of budget balance. Both generations of innovators must be rewarded from the same pot of money, which is the value of the intellectual property right on the applications. The second is that there is a problem of how to divide profit. Even if applications provide enough revenue to reward both innovators optimally, the intellectual property regime restricts how it can be divided (see Figure 2).

This paper has illuminated a new subtlety in the problem of dividing profit. Because ideas for applications arrive at random times, different applications may face different periods of infringement, and thus receive different rewards. For this reason, a patent system with constant patent lives cannot generally create optimal incentives. What matters for incentives are the period of infringement and the patent life on the applications, but not otherwise the length of the patent on the basic innovation.

We now ask whether patent law has levers that can achieve our economic prescriptions. The main requirements for obtaining a patent are novelty, nonobviousness, utility and enablement. Nonobviousness governs the breadth of claims that are granted. When the statutory patent life is the same for all patentable innovations, breadth is the main lever

---

[8] These also show up in other models of basic and applied research, e.g., Green and Scotchmer (1995), Scotchmer (1996) and Denicolo (2000), and other papers referenced in chapter 5 of Scotchmer (2004).

to differentiate rewards. Our main prescription is that, when ideas are scarce, the nonobviousness requirement should be interpreted to grant generous claims, or broad patents.

In our model, a broad patent on a basic innovation is interpreted as a long period of collecting license fees from applications developers. There is probably no policy lever to fine-tune the period of infringement, but the same effect can be achieved by creating uncertainty as to whether applications infringe, and providing for a higher probability of infringement when ideas for the basic innovation are scarce. However it is achieved, a broad patent on the basic innovation impinges on the profit of applications developers. This demonstrates the basic conflict that arises when both innovators must be rewarded from the same pot of money. The profit of the applications developers might be restored by lengthening or broadening their own patents, but there is no provision in patent law that allows courts or the PTO to adjust patents according to duties owed to previous patent holders.

Despite these limitations in patent law, there is one redeeming feature that we have not yet mentioned. In our discussion of applications, we made the simplifying assumption that ideas for every application arrive at the same rate δ, and therefore all applications require the same reward. Suppose, however, that ideas for different applications arrive at different rates and the time of arrival is a signal of the arrival rate. Then, as in section 2.3, patent rewards should increase with the time of arrival. Rewards will automatically have this feature if the patent lives of both the basic innovation and applications are constant. Applications that arrive later will receive a higher reward because they face a shorter period of infringement and smaller licensing fees.

We close this section by comparing our interpretation of nonobviousness to others proposed by economists. Previous models which try to link economic concepts to novelty and nonobviousness focus either on the amount of progress that is required for a patent (Scotchmer and Green, 1990; O'Donoghue, 1998; Hunt, 2004), or the amount of progress that is required to escape infringement (O'Donoghue, Scotchmer and Thisse, 1998). Our own notion of nonobviousness is not defined by increments to progress, but rather by scarcity of

ideas for investment in the first instance.[9]

Our new interpretation is rooted in a different model of the R&D process. We interpret ideas, and the fact that ideas are private, as a model of imagination or creativity. Ideas have economic value because they are scarce. Because ideas are not common knowledge, innovators make positive profit in expectation. This conclusion contrasts with most "racing" models, where opportunities to invest are common knowledge and profit is dissipated by entry. In those models, resources are scarce, but ideas are not.

Since our model of the ideas process is closely related to that of O'Donoghue, Scotchmer and Thisse (1998), we clarify the differences. The policy variable in the earlier paper is the quality difference between an earlier product and a later product such that the later product does not infringe. They call this parameter "leading breadth." Leading breadth governs the length of time that an innovator remains the market incumbent, and also governs the per-period profit during the period of incumbency. In the environment they consider, ideas come in different "sizes" (increments to quality), and leading breadth establishes the minimum size idea that will become an innovation.

In the model of this paper, ideas are distinguished by cost rather than quality. However, that is not the essential difference. The essential difference is that we allow the patent treatment to depend on the arrival rate of ideas, rather than on aspects of the realized technology, such as the increment to progress, or the cost of achieving it. Our argument thus lends support for the patent authority's use of "long-felt need" as a standard for patentability and a reason to be more generous with patents.[10]

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[9] Like us, legal commentators have advocated that courts consider the innovative environment in addition to the technology under consideration. Merges (1993) advocates that the court considers the uncertainty of success, as well as rival technologies. Barton (2003) and Duffy (2007) argue, in a spirit similar to this paper, that the court should be less generous in environments with rapid turnover.

[10] Patent offices instruct their examiners to consider long-felt need in deciding what merits a patent. See European Patent Office (2007), Australian Patent Office (2007), and U.S. Patent and Trademark Office (2007).

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6   Appendix

6.1   Proof of Proposition 1

We first show that the optimized value of V is stationary.

Claim 1 *Given $t_1 < t_2$, let $c_1 : (t_1, \infty) \rightarrow \mathbf{R}_+$ be the function that maximizes $V(t_1, c, \lambda)$, and let $c_2 : (t_2, \infty) \rightarrow \mathbf{R}_+$ be the function that maximizes $V(t_2, c, \lambda)$. Then $V(t_1, c_1, \lambda) = V(t_2, c_2, \lambda)$.*

Proof: Define a function $\tilde{c}_1 : (t_1, \infty) \rightarrow \mathbf{R}_+$ by

$$\tilde{c}_1\left(\hat{t}\right) = c_2\left(\hat{t} + t_2 - t_1\right) \tag{21}$$

The function $\tilde{c}_1$ is the same function as c_2, except shifted to begin at t_1 instead of t_2. Then by definition, $V(t_1, c_1, \lambda) \geq V(t_1, \tilde{c}_1, \lambda)$, and by construction, $V(t_1, \tilde{c}_1, \lambda) = V(t_2, c_2, \lambda)$. Hence, $V(t_1, c_1, \lambda) \geq V(t_2, c_2, \lambda)$.

Now reverse the roles and define a threshold function $\tilde{c}_2 : (t_2, \infty) \rightarrow \mathbf{R}_+$ by

$$\tilde{c}_2\left(\hat{t}\right) = c_1\left(\hat{t} - t_2 + t_1\right)$$

Then by definition, $V(t_2, c_2, \lambda) \geq V(t_2, \tilde{c}_2, \lambda)$, and by construction, $V(t_2, \tilde{c}_2, \lambda) = V(t_1, c_1, \lambda)$. Hence, $V(t_2, c_2, \lambda) \geq V(t_1, c_1, \lambda)$. Together with $V(t_1, c_1, \lambda) \geq V(t_2, c_2, \lambda)$, this proves the result. $\square$

Claim 1 implies that $\frac{d}{dt} V(t, c, \lambda) = 0$. Using (4), this implies that $c'(t) = 0$ at each t.

6.2   Proof of Lemma 1

We need to show that $H(\lambda | t_1, c) < H(\lambda | t_2, c)$ for each λ in the interior of the support $[0, \infty)$ and for each $t_2 > t_1 \geq 0$, holding c fixed. It is enough to show that $(d/dt) H(\lambda | t, c) > 0$. We use

$$\frac{d}{dt} h(\lambda | t, c) = F(c(t)) h(\lambda | t, c) [E(\lambda | t, c) - \lambda]$$

Define a function g by $g\left(\hat{\lambda}\right) = \left[E(\lambda | t, c) - \hat{\lambda}\right]$. Because g is decreasing, if λ is such that $g(\lambda) > 0$, it holds that

$$\frac{d}{dt} H(\lambda | t, c) = \int_0^\lambda \frac{d}{dt} h\left(\hat{\lambda} | t, c\right) d\hat{\lambda} = F(c(t)) \int_0^\lambda h\left(\hat{\lambda} | t, c\right) g\left(\hat{\lambda}\right) d\hat{\lambda} > 0$$

Now consider λ such that $g(\lambda) \leq 0$. Then since

$$0 = \frac{d}{dt} \int_0^\infty h\left(\hat{\lambda}|t,c\right) d\hat{\lambda} = \frac{d}{dt} \int_0^\lambda h\left(\hat{\lambda}|t,c\right) d\hat{\lambda} + \frac{d}{dt} \int_\lambda^\infty h\left(\hat{\lambda}|t,c\right) d\hat{\lambda}$$

and

$$\frac{d}{dt} \int_\lambda^\infty h\left(\hat{\lambda}|t,c\right) d\hat{\lambda} = F\left(c\left(t\right)\right) \int_\lambda^\infty h\left(\hat{\lambda}|t,c\right) g\left(\hat{\lambda}\right) d\hat{\lambda} < 0$$

it holds that

$$\frac{d}{dt} H\left(\lambda|t,c\right) = \frac{d}{dt} \int_0^\lambda h\left(\hat{\lambda}|t,c\right) d\hat{\lambda} = F\left(c\left(t\right)\right) \int_0^\lambda h\left(\hat{\lambda}|t,c\right) g\left(\hat{\lambda}\right) d\hat{\lambda} > 0$$

Therefore, $h\left(\cdot|t_1,c\right)$ stochastically dominates $h\left(\cdot|t_2,c\right)$ and $E\left(\lambda|t_2,c\right) < E\left(\lambda|t_1,c\right)$.

6.3 Proof of Proposition 4

The conclusion that c is increasing follows from (7) and (8), since we can show that $\tilde{V}$ is decreasing. For the derivative of $\tilde{V}$, we need the derivative of the conditional density function at $\hat{t} > t$,

$$\frac{d}{dt} \phi\left(\hat{t}|t,c,\lambda\right) h\left(\lambda|t,c\right) = F\left(c\left(t\right)\right) E\left(\lambda|t,c\right) \phi\left(\hat{t}|t,c,\lambda\right) h\left(\lambda|t,c\right)$$

Differentiating $\tilde{V}$ with respect to t gives

$$\frac{d}{dt} \tilde{V}\left(t,c,h\left(\cdot|t,c\right)\right)$$
$$= -\left(\frac{v}{r} - E_F\left(c\left(t\right)\right)\right) F\left(c\left(t\right)\right) E\left(\lambda|t,c\right) + r\tilde{V}\left(t,c,h\left(\cdot|t,c\right)\right)$$
$$+ \int_t^\infty \int_0^\infty e^{-r\left(\hat{t}-t\right)} \left(\frac{v}{r} - E_F\left(c\left(\hat{t}\right)\right)\right) \frac{d}{dt}\left[\phi\left(\hat{t}|t,c,\lambda\right) h\left(\lambda|t,c\right)\right] d\lambda \, d\hat{t}$$
$$= -\left(\frac{v}{r} - E_F\left(c\left(t\right)\right)\right) F\left(c\left(t\right)\right) E\left(\lambda|t,c\right) + r\tilde{V}\left(t,c,h\right)$$
$$+ F\left(c\left(t\right)\right) E\left(\lambda|t,c\right) \int_t^\infty \int_0^\infty e^{-r\left(\hat{t}-t\right)} \left(\frac{v}{r} - E_F\left(c\left(\hat{t}\right)\right)\right) \phi\left(\hat{t}|t,c,\lambda\right) h\left(\lambda|t,c\right) d\lambda \, d\hat{t}$$
$$= -\left(\frac{v}{r} - E_F\left(c\left(t\right)\right)\right) F\left(c\left(t\right)\right) E\left(\lambda|t,c\right) + \left(r + F\left(c\left(t\right)\right) E\left(\lambda|t,c\right)\right) \tilde{V}\left(t,c,h\left(\cdot|t,c\right)\right)$$

$$= \left(r + F\left(c\left(t\right)\right) E\left(\lambda|t,c\right)\right) \left[-\left(\frac{v}{r} - E_F\left(c\left(t\right)\right)\right) \frac{F\left(c\left(t\right)\right) E\left(\lambda|t,c\right)}{r + F\left(c\left(t\right)\right) E\left(\lambda|t,c\right)} + \tilde{V}\left(t,c,h\left(\cdot|t,c\right)\right)\right]$$
$$= \left(r + F\left(c\left(t\right)\right) E\left(\lambda|t,c\right)\right) \left[-\left(\frac{v}{r} - E_F\left(c\left(t\right)\right)\right) \frac{F\left(c\left(t\right)\right) E\left(\lambda|t,c\right)}{\left(r + F\left(c\left(t\right)\right) E\left(\lambda|t,c\right)\right)} + \left(\frac{v}{r} - c\left(t\right)\right)\right] \quad (22)$$

31

where the last line follows from (7).

First, the optimizing function c cannot be "U-shaped" on any domain. If the function c is "U-shaped" on some domain, there exist t_1 and t_2 such that $t_1 < t_2$, $c(t_1) = c(t_2)$, and $c'(t_1) < 0 < c'(t_2)$. However, this generates a contradiction. It holds that $\left(\frac{v}{r} - E_F(c(t_1))\right) = \left(\frac{v}{r} - E_F(c(t_2))\right)$, $\left(\frac{v}{r} - c(t_1)\right) = \left(\frac{v}{r} - c(t_1)\right)$, $F(c(t_1)) = F(c(t_2))$, and (using Lemma 1) $E(\lambda|t_1, c) > E(\lambda|t_2, c)$. Hence, using (22), $\frac{d}{dt}\tilde{V}(t_1, c, h(\cdot|t_1, c)) < \frac{d}{dt}\tilde{V}(t_2, c, h(\cdot|t_2, c))$. Together with $c'(t_1) < 0 < c'(t_2)$, this contradicts (8).

Proposition 4 then follows from Claim 2 and Claim 3 below. By Claim 3, if c is the optimal threshold function, $\tilde{V}(t, c, h(\cdot|t, c))$ is decreasing with t on a domain $(\bar{t}, \infty)$. Therefore, using (8), it also holds that c is increasing on that domain. But it then follows that the entire function c is nondecreasing, since c cannot be U-shaped on any domain. And, in fact, c is increasing because the derivative (22) is not constant on any interval.

Claim 2 Let c be the threshold function that maximizes $V\left(0, \cdot, \tilde{h}\right)$. Then there exists $\bar{t}$ such that the function $t \to e^{-rt}\left(\frac{v}{r} - E_F(c(t))\right)$ is decreasing on the domain $(\bar{t}, \infty)$.

Proof of Claim 2: Because the optimal c cannot be U-shaped, it is either nonincreasing or nondecreasing for sufficiently large t. Further, because c is bounded above and below, it holds that $c'(t) \to 0$, $c(t) \to c^*$, $E_F(c(t)) \to E_F(c^*)$ for some $c^* \in \left[0, \frac{v}{r}\right]$. The result follows because

$$
\frac{d}{dt}e^{-rt}\left(\frac{v}{r} - E_F(c(t))\right) = e^{-rt}\left[-r\left(\frac{v}{r} - E_F(c(t))\right) - \frac{dE_F(c(t))}{dc(t)}c'(t)\right]
$$
$$
\to -re^{-rt}\left(\frac{v}{r} - E_F(c^*)\right) \qquad \qquad \square
$$

To show that c is increasing for sufficiently large t, we show that $\tilde{V}$ is decreasing for sufficiently large t, when evaluated at the optimal c that satisfies (8).

Claim 3 Let c be the threshold function that maximizes $V\left(0, \cdot, \tilde{h}\right)$. Then there exists a domain $(\bar{t}, \infty)$ for which

$$
\tilde{V}(t_1, c, h(\cdot|t_1, c)) > \tilde{V}(t_2, c, h(\cdot|t_2, c)) \qquad if \qquad \bar{t} \le t_1 < t_2
$$

Proof of Claim 3: We will take the domain $(\bar{t}, \infty)$ as the domain on which $e^{-rt} \left(\frac{v}{r} - E_F(c(t)) \right)$ is decreasing, by Claim 2. We will show that

$$\tilde{V}(t_1, c, h(\cdot|t_1, c)) \geq \tilde{V}(t_1, \tilde{c}, h(\cdot|t_1, c)) > \tilde{V}(t_2, c, h(\cdot|t_2, c)) \quad \text{if } \bar{t} \leq t_1 < t_2 \qquad (23)$$

where $\tilde{c}$ is defined by $\tilde{c}(t) = c(t)$ for $t < t_1$ and $\tilde{c}(t) = c(t + t_2 - t_1)$ for $t \geq t_1$.

The first inequality in (23) is true by the principle of optimality. Beginning from time t_1, the optimizing function is still c, as it was when optimized from the beginning. If $\bar{c}$ satisfies $\tilde{V}(t_1, \bar{c}, h(\cdot|t_1, c)) \geq \tilde{V}(t_1, \hat{c}, h(\cdot|t_1, c))$ for all threshold functions $\hat{c}$, then $\bar{c}(t) = c(t)$ for every $t \geq t_1$.

It is the second inequality in (23) that we must show. The function $\tilde{c}$ in $\tilde{V}(t_1, \tilde{c}, h(\cdot|t_1, c))$ is defined by the function c restricted to (t_2, ∞) and shifted back in time to t_1. For a fixed λ, it would therefore hold that $V(t_1, \tilde{c}, \lambda) = V(t_2, c, \lambda)$. However, $\tilde{V}(t_1, \tilde{c}, h(\cdot|t_1, c)) \neq \tilde{V}(t_2, c, h(\cdot|t_2, c))$ because λ is unknown, and the posterior distribution on λ is different at t_1 than at t_2. In particular, using Lemma 1, $h(\cdot|t_1, c)$ puts relatively high weight on high values of λ, where the value $V(t_1, \tilde{c}, \lambda)$ is relatively high (under the hypothesis that the integrand $e^{-rt} \left(\frac{v}{r} - E_F(c(t)) \right)$ is decreasing), and $h(\cdot|t_2, c)$ puts high weight on relatively low values of λ, where $V(t_2, c, \lambda)$ is lower (under the hypothesis that the integrand $e^{-rt} \left(\frac{v}{r} - E_F(c(t)) \right)$ is decreasing). Thus, $\tilde{V}(t_1, \tilde{c}, h(\cdot|t_1, c)) > \tilde{V}(t_2, c, h(\cdot|t_2, c))$.

We now show this formally.

$$
\begin{aligned}
&\tilde{V}(t_1, \tilde{c}, h(\cdot|t_1, c)) \\
&= \int_0^\infty \int_{t_1}^\infty e^{-r(\hat{t} - t_1)} \left(\frac{v}{r} - E_F(\tilde{c}(\hat{t})) \right) \phi(\hat{t}|t_1, \tilde{c}, \lambda) h(\lambda|t_1, c) \, d\hat{t} d\lambda \\
&= \int_0^\infty \int_{t_2}^\infty e^{-r(\tilde{t} - t_2)} \left(\frac{v}{r} - E_F(\tilde{c}(\tilde{t} - t_2 + t_1)) \right) \phi(\tilde{t} - t_2 + t_1|t_1, \tilde{c}, \lambda) h(\lambda|t_1, c) \, d\tilde{t} d\lambda \\
&= \int_0^\infty \int_{t_2}^\infty e^{-r(\tilde{t} - t_2)} \left(\frac{v}{r} - E_F(c(\tilde{t})) \right) \phi(\tilde{t}|t_2, c, \lambda) h(\lambda|t_1, c) \, d\tilde{t} d\lambda \\
&= \int_0^\infty h(\lambda|t_1, c) V(t_2, c, \lambda) \, d\lambda
\end{aligned}
$$

Thus, to show (23), it is enough to show that

$$\int_0^\infty h(\lambda|t_1, c) V(t_2, c, \lambda) \, d\lambda > \int_0^\infty h(\lambda|t_2, c) V(t_2, c, \lambda) \, d\lambda = \tilde{V}(t_2, c, h) \qquad (24)$$

33

Since $e^{-r(t-t_2)}\left(\frac{v}{r} - E_F\left(c\left(t\right)\right)\right)$ is decreasing with t for $t \in \left(\bar{t}, \infty\right)$, $V\left(t_2, c, \cdot\right)$ increases with λ. Then (24) follows because the distribution $h\left(\cdot | t_1, c\right)$ stochastically dominates $h\left(\cdot | t_2, c\right)$. This means that $h\left(\cdot | t_2, c\right)$ puts relatively more weight on low values of λ, where the value of $V\left(t_2, c, \lambda\right)$ is low, and $h\left(\cdot | t_1, c\right)$ puts relatively more weight on high values of λ, where the value of $V\left(t_2, c, \lambda\right)$ is high. $\square$

6.4 Proof of Lemma 2

Let c be the threshold function with stationary value c_A. Let $\hat{T}_A \in \mathbf{R}_+$ be defined by

$$c_A = (1-\beta)\frac{v}{r}\left(1 - e^{-r\hat{T}_A}\right)$$

Since $\left(\tilde{T}_B, \tilde{T}_A\right)$ implement the stationary thresholds (c_B, c_A), arrival times can be partitioned into $\mathcal{T} \cup \bar{\mathcal{T}} = (0, \infty)$ such that

$$\tilde{T}_A\left(t\right) \leq \tilde{T}_B\left(t\right) - t \quad \text{and} \quad \tilde{T}_A\left(t\right) = \hat{T}_A \quad \text{for each} \quad t \in \bar{\mathcal{T}}$$
$$\tilde{T}_A\left(t\right) > \tilde{T}_B\left(t\right) - t \quad \text{for each} \quad t \in \mathcal{T}$$

Further, it holds that

$$
\begin{aligned}
c_B &= \int_{\bar{\mathcal{T}}}\left[\beta\frac{v}{r}\left(1 - e^{-r\hat{T}_A}\right) + \frac{v}{r}\left(e^{-r\hat{T}_A} - e^{-r\left(\tilde{T}_B(t)-t\right)}\right)\right]\delta G\left(c_A\right)e^{-(\delta G(c_A)+r)t}dt \\
&\quad + \int_{\mathcal{T}}\beta\frac{v}{r}\left(1 - e^{-r\left(\tilde{T}_B(t)-t\right)}\right)\delta G\left(c_A\right)e^{-(\delta G(c_A)+r)t}dt
\end{aligned}
$$

There are two cases.

Case (a). Suppose that

$$c_B < \int_{\bar{\mathcal{T}}}\beta\frac{v}{r}\left(1 - e^{-r\hat{T}_A}\right)\delta G\left(c_A\right)e^{-(\delta G(c_A)+r)t}dt$$

Choose $k < \hat{T}_A$ and $\bar{T}_A \in \left(k, \hat{T}_A\right)$ such that

$$
\begin{aligned}
c_B &= \int_0^\infty \beta\frac{v}{r}\left(1 - e^{-rk}\right)\delta G\left(c_A\right)e^{-(\delta G(c_A)+r)t}dt \\
c_A &= \frac{v}{r}\left(1 - e^{-r\bar{T}_A}\right) - \beta\frac{v}{r}\left(1 - e^{-rk}\right)
\end{aligned}
$$

34

Case (b). Suppose that

$$c_B \geq \int_{\bar{T}} \beta \frac{v}{r} \left(1 - e^{-r\hat{T}_A}\right) \delta G\left(c_A\right) e^{-(\delta G(c_A)+r)t} dt$$

Choose $\bar{T}_A = \hat{T}_A$ and $k \geq \bar{T}_A$ such that

$$c_B = \int_0^\infty \left[\beta \frac{v}{r} \left(1 - e^{-r\bar{T}_A}\right) + \frac{v}{r} \left(e^{-r\bar{T}_A} - e^{-rk}\right)\right] \delta G\left(c_A\right) e^{-(\delta G(c_A)+r)t} dt$$

In each case, define the function T_A by $T_A\left(t\right) = \bar{T}_A$, and define the function T_B by $T_B\left(t\right) = k + t$. Then the pair (T_B, T_A) satisfies (17) and implements (c_B, c_A).

6.5   Proof of Proposition 9

We must show that (c_B, c_A) is in the set (18) if and only if $(c_B, c_A) \in \mathcal{C}$. We first assume that (c_B, c_A) is in the set (18) and show that $(c_B, c_A) \in \mathcal{C}$. Let (c_B, c_A) be an arbitrary element of (18). Then, using Lemma 2, $(c_B, c_A) \in \mathcal{C}$ if it can be implemented by (T_B, T_A) defined as $T_B\left(t\right) = k + t$ for some $k \in (0, \infty)$ and $T_A\left(t\right) = \bar{T}_A$ for some $\bar{T}_A \geq 0$. Thus, using $c_A = \Pi^A\left(t; T_B, T_A\right)$ for each t and $c_B = \Pi^B\left(T_B, T_A\right)$, the pair (c_B, c_A) can be implemented if it holds for some $k \geq 0, \bar{T}_A \geq 0$ that

$$
\begin{align}
c_A &= (1 - \beta) \frac{v}{r} \left(1 - e^{-r\bar{T}_A}\right) & \text{if } k \geq \bar{T}_A \tag{25} \\
c_B &= \frac{\delta G\left(c_A\right)}{\delta G\left(c_A\right) + r} \left[\frac{\beta}{1 - \beta} c_A + \frac{v}{r} \left(e^{-r\bar{T}_A} - e^{-rk}\right)\right] & \text{if } k \geq \bar{T}_A \tag{26}
\end{align}
$$

or

$$
\begin{align}
c_A &= \frac{v}{r} \left(1 - e^{-r\bar{T}_A}\right) - \beta \frac{v}{r} \left(1 - e^{-rk}\right) & \text{if } k \leq \bar{T}_A \tag{27} \\
c_B &= \frac{\delta G\left(c_A\right)}{\delta G\left(c_A\right) + r} \beta \frac{v}{r} \left(1 - e^{-rk}\right) & \text{if } k \leq \bar{T}_A \tag{28}
\end{align}
$$

which also imply that

$$c_B = \begin{cases} \frac{\delta G(c_A)}{\delta G(c_A)+r} \left[\frac{v}{r} \left(1 - e^{-rk}\right) - c_A\right] & \text{if } \quad k \geq \bar{T}_A \\[2mm] \frac{\delta G(c_A)}{\delta G(c_A)+r} \left[\frac{v}{r} \left(1 - e^{-r\bar{T}_A}\right) - c_A\right] & \text{if } \quad k \leq \bar{T}_A \end{cases} \tag{29}$$

35

We partition the set (18) into three subsets:

$$\mathcal{C}_1 = \left\{ (c_B, c_A) : c_A \in \left[0, (1-\beta)\frac{v}{r}\right], c_B \in \left(\frac{\delta G(c_A)}{\delta G(c_A) + r}\left(\frac{\beta}{1-\beta}c_A\right), \frac{\delta G(c_A)}{\delta G(c_A) + r}\left(\frac{v}{r} - c_A\right)\right] \right\}$$

$$\mathcal{C}_2 = \left\{ (c_B, c_A) : c_A \in \left[0, (1-\beta)\frac{v}{r}\right], c_B \in \left[0, \frac{\delta G(c_A)}{\delta G(c_A) + r}\left(\frac{\beta}{1-\beta}c_A\right)\right] \right\}$$

$$\mathcal{C}_3 = \left\{ (c_B, c_A) : c_A \in ((1-\beta)\frac{v}{r}, \frac{v}{r}), c_B \in \left[0, \frac{\delta G(c_A)}{\delta G(c_A) + r}\left(\frac{v}{r} - c_A\right)\right] \right\}$$

If $(c_B, c_A) \in \mathcal{C}_1$, the pair (c_B, c_A) can be implemented by choosing $\bar{T}_A$ such that (25) holds, and $k \geq \bar{T}_A$ such that (26) holds. As k ranges between $\bar{T}_A$ and ∞, c_B ranges between $\frac{\delta G(c_A)}{\delta G(c_A)+r}\left(\frac{\beta}{1-\beta}c_A\right)$ and $\frac{\delta G(c_A)}{\delta G(c_A)+r}\left(\frac{v}{r} - c_A\right)$. Thus, every pair in $\mathcal{C}_1$ can be implemented.

If $(c_B, c_A) \in \mathcal{C}_2 \cup \mathcal{C}_3$, the pair (c_B, c_A) can be implemented by k such that (28) holds, and $\bar{T}_A \geq k$ such that (27) holds. As $\bar{T}_A$ ranges between k and ∞, c_A ranges between $\frac{\delta G(c_A)+r}{\delta G(c_A)}\left(\frac{1-\beta}{\beta}c_B\right)$ and $\frac{v}{r} - \frac{\delta G(c_A)+r}{\delta G(c_A)}c_B$. Thus, every pair in $\mathcal{C}_2$ and $\mathcal{C}_3$ can be implemented.

We now show that if $(c_B, c_A) \in \mathcal{C}$, then (c_B, c_A) is in the set defined by (18). Using Lemma 2, we show the converse instead: If (c_B, c_A) is not in the set (18), then (c_B, c_A) cannot be implemented by any (T_B, T_A) defined by $k \in \mathbf{R}_+$ and $\bar{T}_A \in \mathbf{R}_+$. Given $c_A > \frac{v}{r}$ and any c_B, it follows from (25) that the pair (c_B, c_A) cannot be implemented. Given $c_A \in \left[0, \frac{v}{r}\right]$ and $c_B > \frac{\delta G(c_A)}{\delta G(c_A)+r}\left(\frac{v}{r} - c_A\right)$, it follows from (29) that the pair (c_B, c_A) cannot be implemented.

6.6 Proof of Proposition 10

Since $V_A(c; \delta)$ and $V_B(c; \lambda, \delta)$ are strictly quasiconcave functions of c, the solutions that satisfy the first order conditions (13) and (14) are unique, namely, $(c_B^*(\lambda, \delta), c_A^*(\delta))$.

(i) First consider $\delta \to \infty$. The first order condition (13) implies that $\left(\frac{\delta G(c_A^*(\delta))}{\delta G(c_A^*(\delta))+r}\right) \to 1$, since it holds that $c_A^*(\delta) \to 0$ and $E_G(c_A^*(\delta)) \to 0$ as $\delta \to \infty$.

From Proposition 9, the set $\mathcal{C}$ of implementable stationary thresholds is defined by (18). For each c_A, let $\bar{c}_B(c_A)$ stand for the upper bound on the stationary c_B values that can be implemented:

$$\bar{c}_B(c_A) = \frac{\delta G(c_A)}{\delta G(c_A) + r}\left(\frac{v}{r} - c_A\right)$$

As $\delta \to \infty$, $\bar{c}_B (c_A) \to \frac{v}{r}$ for each c_A. (The boundary of Figure 2 converges to a line at height $\frac{v}{r}$.) Since the efficient $c_B^* (\lambda, \delta)$ cannot be larger than the gross social value created, $\frac{v}{r}$, the efficient cost thresholds are implementable.

Now consider $\delta \to 0$, which implies $V_A (c_A^* (\delta) ; \delta) \to 0$. Since the first derivative of $V_B (c; \lambda, \delta)$ with respect to c would be negative for any positive value of c, $c_B^* (\lambda, \delta) \to 0$.

(ii) Taking the first derivative of $V_B (c; \lambda, \delta)$ with respect to c gives

$$\frac{\partial}{\partial c} V_B (c; \lambda, \delta) = \frac{\lambda f (c)}{\lambda F (c) + r} \left(V_A (c_A^* (\delta) ; \delta) - c - \frac{\lambda F (c)}{\lambda F (c) + r} (V_A (c_A^* (\delta) ; \delta) - E_F (c)) \right).$$

Evaluating at $\bar{c}_B (c_A^* (\delta))$ yields

$$\frac{\partial}{\partial c} V_B (c; \lambda, \delta) \Bigg|_{c = \bar{c}_B (c_A^* (\delta))} \tag{30}$$

$$= \frac{\lambda f (\bar{c}_B)}{\lambda F (\bar{c}_B) + r} \left[V_A (c_A^* (\delta) ; \delta) - \frac{\delta G (c_A^* (\delta))}{\delta G (c_A^* (\delta)) + r} \left(\frac{v}{r} - c_A^* (\delta) \right) - \frac{\lambda F (\bar{c}_B)}{\lambda F (\bar{c}_B) + r} (V_A (c_A^* (\delta) ; \delta) - E_F (\bar{c}_B)) \right].$$

Substituting from (13),

$$\frac{\partial}{\partial c} V_B (c; \lambda, \delta) \Bigg|_{c = \bar{c}_B (c_A^* (\delta))}$$

$$= \frac{\lambda f (\bar{c}_B)}{\lambda F (\bar{c}_B) + r} \left[V_A (c_A^* (\delta) ; \delta) \left(1 - \frac{\delta G (c_A^* (\delta))}{\delta G (c_A^* (\delta)) + r} \right) - \frac{\lambda F (\bar{c}_B)}{\lambda F (\bar{c}_B) + r} (V_A (c_A^* (\delta) ; \delta) - E_F (\bar{c}_B)) \right].$$

The bracketed term is positive as $\lambda \to 0$, and therefore (using quasiconcavity of $V_B (c; \lambda, \delta)$) positive for all $c \leq \bar{c}_B (c_A^* (\delta))$. Hence, the efficient $c_B^* (\lambda, \delta)$ will be higher than $\bar{c}_B (c_A^* (\delta))$ as $\lambda \to 0$.