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**Heterogeneity in Organizational Form:  
Why Otherwise Identical Firms  
Choose Different Incentives for Their Managers**

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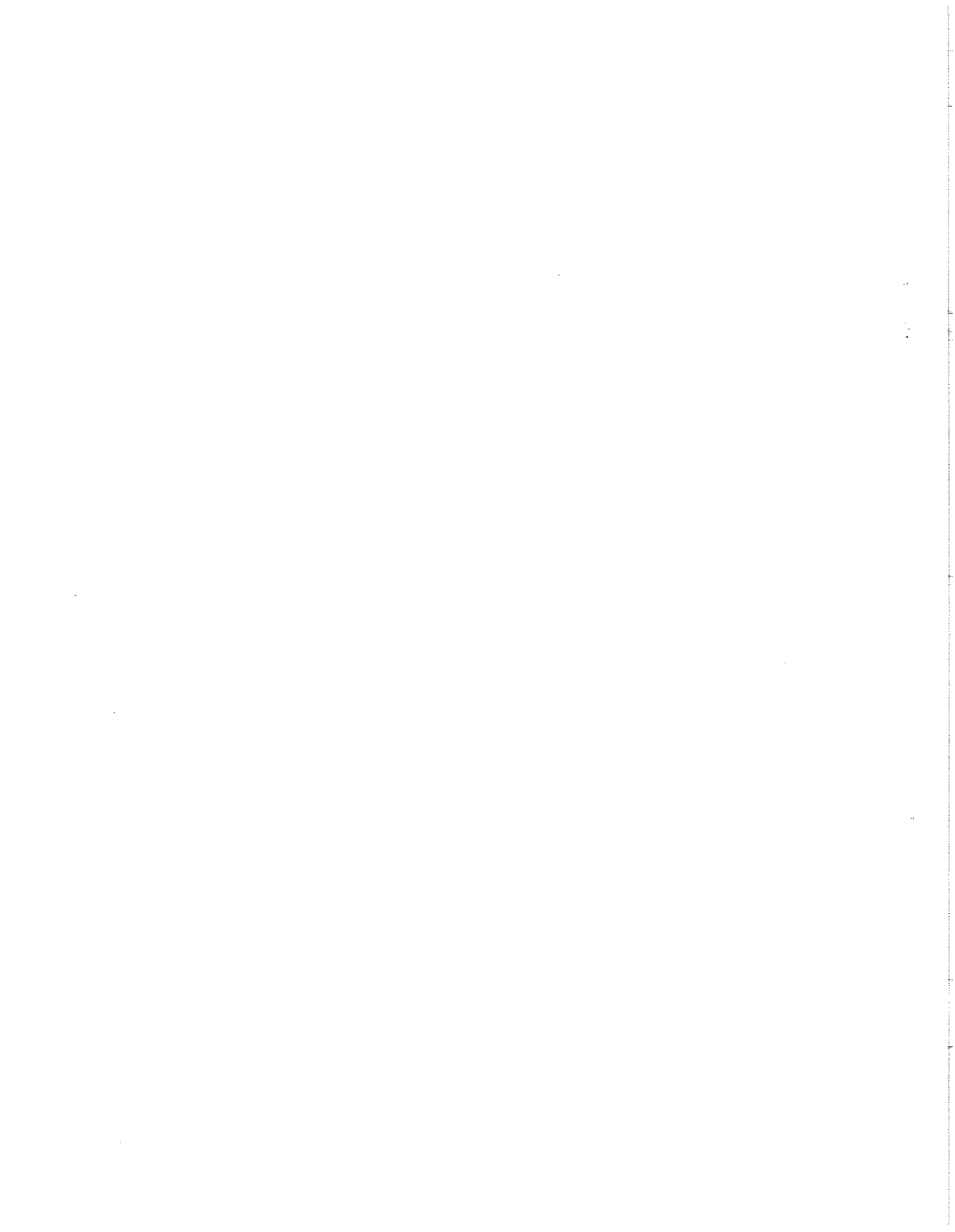
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Abstract

The benefits of giving incentives to managers depend on the nature of the product-market competition. In particular, the benefit from providing a manager with incentives to reduce costs is shrinking in the number of other firms that provide incentives. Indeed, the benefit can be negative, leading some firms not to provide incentives. Their not providing incentives increases the net benefit to firms that do provide incentives. Equilibria can, thus, emerge, in which only a fraction of firms provide incentives; particularly, if the set of actions the firms can make or want their managers to take is not convex. This set can be non-convex due to the production technology or the underlying agency problem. This paper also investigates whether increased competition increases or decreases the proportion of firms providing incentives in equilibrium.

JEL Classification: L22, D21, D23



# Heterogeneity in Organizational Form: Why Otherwise Identical Firms Choose Different Incentives for their Managers\*

## 1. Introduction

Even casual observation reveals variation in the ways firms in an industry are organized and their personnel provided with incentives. To consider just one example, there are mutual savings and loans, which have no shareholders (the depositors are the owners), and stock savings and loans, which have shareholders. Moreover, within a sample of savings and loans (S&Ls), there is wide variety in incentives: For instance, 57% have bonus plans for their CEOs, while 43% do not. Ninety percent have stock option plans for their CEOs, while 10% do not.<sup>1</sup> In addition, the CEO's shareholdings — typically the principle way compensation is tied to firm performance (Jensen and Murphy (1990)) — ranges from 0% to 61% (mean = 4% and standard deviation = 8%).<sup>2</sup> Why such heterogeneity in organizational design arises is largely an unanswered question. Here, I offer one possible answer: Specifically, I show how non-convexities introduced by the technology or the underlying agency problem, in combination with

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<sup>1</sup> These figures understate the amount of heterogeneity in incentives, as there is great variety in the structure of bonus plans and, to a lesser extent, in the structure of stock-option plans.

<sup>2</sup> Heterogeneity is also reflected in golden parachutes (65% have them) and CEO *base* salaries (range: \$60,000 to \$516,139; mean: \$168,701; standard deviation: \$104,544), among other variables. The data are from the annual (1986) proxy statements of 165 savings and loans, which were drawn essentially at random from the population of stock savings and loans. For details on the data see Hermalin and Wallace (1992).

product-market competition, can lead to heterogeneity among otherwise identical firms.<sup>3</sup>

Consider, for instance, a Cournot oligopoly of *ex ante* identical firms. Each firm's owner decides whether to provide her manager with incentives to invest effort in cost reduction. Providing incentives yields an owner smaller *net* benefits, the less output her firm will produce. Since competition is Cournot, her firm will produce less output when her rivals are expected to produce more output. They, in turn, can be expected to produce more output, the lower their costs are. Hence, an owner's net benefit from providing incentives decreases with the number of her rivals who also provide incentives to invest in cost reduction. At some point, the net benefit can, in fact, become negative, so the owner chooses not to provide incentives. By the same reasoning, if this owner chooses not to provide incentives, then she increases the other owners' net benefits from providing incentives. In this way, an equilibrium can arise in which only a fraction of the owners provide their managers with incentives. Moreover, under conditions discussed below, *all* equilibria exhibit heterogeneity, at least with positive probability.

The basic model is laid out in the following section and solved in Section 3. The basic model assumes that a manager can invest either zero or one unit of effort in cost reduction. A

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<sup>3</sup> This explanation, although independently derived, has some relation to Maksimovic and Zechner's (1991) analysis of heterogeneity in financial structure. Their model and emphasis are, however, very different from this paper's. In their paper, equity holders can choose between two technologies, *S* and *NS*, which have the same mean cost of production. Production costs are stochastic with technology *S*, but not with technology *NS*. If firms seek to maximize firm value, then some firms will choose *S* and some will choose *NS* in equilibrium. Technology *S*, however, has a riskier profit stream than technology *NS* in the sense of second-degree stochastic dominance. If their firm has enough debt, otherwise risk-neutral equity holders are risk loving and they strictly prefer *S*. All firms choosing *S* is not, however, an equilibrium: Firm value is increased by deviating from an all-*S* "equilibrium" (such a deviation is accomplished by decreasing the firm's leverage). Hence, there is a maximum number of firms that can be highly leveraged (or a homogenous maximum level of debt) that can exist in equilibrium. Given an exogenous motive for debt, such as taxes, heterogeneity can arise in the debt levels of the firms in the industry (absent an exogenous motive for debt, however, there also always exist homogenous equilibria in debt levels). There are other differences between this paper and Maksimovic and Zechner: Here, firms behave strategically in product-market competition; and, here, the owners of the firms are *not* limited to a restricted contract space (in Maksimovic and Zechner, the [initial] owners are limited to using just debt and equity).

discrete action space is not, however, necessary for heterogeneous equilibria to exist, as I show in Section 4. Moreover, a continuous action space alone is not sufficient for a homogeneous equilibrium to exist. What is important is whether the set of actions an owner can or wishes to implement is convex. This set can be non-convex due to the monitoring problem inherent in the agency relation, or due to non-convexities in the production technology.

Although this paper is primarily concerned with heterogeneity in organizational design, the model that I develop can also be used to study a conjecture — sometimes attributed to Hicks (1935) — that increased competition leads to managers working harder in equilibrium. Here, working harder means investing effort in cost reduction, and competition is measured by the number of firms in the oligopoly. In Sections 3 and 4, I show the "Hicks conjecture" to be false: An increase in the number of firms in the industry can decrease the number of firms that induce their managers to invest effort in a pure-strategy equilibrium (Proposition 2), reduces the probability that a firm will induce its manager to invest effort in a mixed-strategy equilibrium (Proposition 3), and reduces the equilibrium effort level in a continuous-effort model (Proposition 4). Section 5 presents some further discussion and conclusions.

## 2. The Basic Model

Suppose there are  $N \geq 2$  identical firms. In stage one, each firm's owner chooses an incentive contract for her firm's manager. In stage two, the firms' managers take actions that affect their firms' costs. In stage three, the firms engage in Cournot competition. Finally, payoffs are realized.

Momentarily passing to stage two, the  $N$  managers simultaneously choose whether to

invest effort in cost-reducing activities. Let  $e = 1$  denote investment and  $e = 0$  denote no investment. A firm's constant marginal cost is  $c^H$  if its manager invests no effort. Assume that investing effort can lower marginal cost to  $c^L$ , but only with probability  $q$ ,  $0 < q < 1$ . If investment fails to lower marginal cost, then it remains  $c^H$ . Cost reduction's stochastic nature reflects uncertainty in researching and engineering new production processes, as well as unforeseeable difficulties in adopting innovations. The success or failure of investments in cost reduction are not correlated across firms.

Let  $u(y) - de$ ,  $d > 0$ , denote the manager's utility from working for the firm when he is paid  $y$  and he invests effort  $e$ . Note that he finds investing effort personally costly.<sup>4</sup> Assume that  $u(y)$  is strictly increasing, strictly concave, and unbounded in  $y$ . The inverse function  $u^{-1}(\bullet)$  therefore exists and is defined for all  $u \in \mathbf{R}$ .

Assume that the firms' owners simultaneously choose whether to provide incentives to their managers. Assume, too, that the owners are risk-neutral, expected-profit maximizers.

After the marginal costs are determined, they are common knowledge. This has two implications: First, incentive contracts can be based on realized marginal cost; and, second, each firm knows its rivals' costs during the Cournot-competition stage (stage three). Relaxing this common-knowledge assumption would not substantially change the results.<sup>5</sup>

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<sup>4</sup> Investing in cost reduction may also have costs that are borne directly by the firm's owner. Rather than add to the notation, these costs are ignored. This is without loss of generality, since such costs could be added to  $C(1)$  (which is defined below). All the propositions given below would still be true as stated if  $C(1)$  includes these costs (the statement of Lemma 1 would, however, need to be slightly altered). I assume that if the owner bears direct costs, she cannot use them to monitor whether the manager has invested *effort* in cost reduction. The justification for this assumption is that a financial expenditure need not entail an effort expenditure; e.g., the manager could purchase a computerized inventory system, but not invest the effort sufficient to make it work.

<sup>5</sup> An alternative assumption would tie incentives to profits only. If incentives were tied to the firm's profit *and* the profits of the firm's rivals, then the analysis would be unchanged, since a firm's costs can be perfectly

I assume that the contract between an owner and her manager is unobservable to her rival owners or their managers. This is consistent with the observation that contracts are private documents between the parties to the contract.<sup>6</sup> This assumption has two consequences: First, an owner cannot make her manager's incentives contingent on whether the other owners provide incentives, since one firm's managerial contract cannot make reference to the other firms' managerial contracts. Second, unlike Fershtman and Judd (1987), an owner cannot credibly use her manager's compensation contract to become a Stackelberg leader in the product-market subgame. Instead, conditional on the belief that the other owners have instructed their managers to play their Cournot best responses, it is an owner's best response to instruct her manager to play his Cournot best response.<sup>7</sup>

The inverse market demand curve is  $P(X) = a - bX$ , where  $X$  is the firms' total output. Assume that no firm ever prefers shutting down to operating; specifically, assume  $a > (N+1)c^H - Nc^L$ . A well-known result under these assumptions is that a firm's profit is

$$\frac{(a + jc^L + (N - 1 - j)c^H - Nc)^2}{b(N + 1)^2} \equiv \pi(c | j).$$

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inferred from observing all the firms' profits. If a manager's incentives were tied to his firm's profit only, then the analysis would be somewhat changed — in particular, the optimal incentive contract would depend on how many other firms are expected to provide incentives. The basic results, however, would still hold, since they simply rely on it being more costly for the firm to provide incentives than not to provide incentives.

Assuming that a firm's costs were its private information would not change the results because what is at issue is whether the (expected) gain from lowering one's costs decreases in the number of firms which are also seeking to lower their costs. This property would still hold with privately known costs.

<sup>6</sup> For further justifications see Katz (1991).

<sup>7</sup> These instructions could be conveyed directly or through a forcing contract if the industry's cost structure is verifiable — a reasonable assumption given that it is common knowledge — or conveyed by paying the manager an  $\epsilon$ -percentage of the profits, where  $\epsilon$  is an arbitrarily small positive constant.



if its marginal cost is  $c$  and if  $j$  of its  $N - 1$  rivals have low marginal cost.

Whether the manager invests effort on cost reduction is unobservable to his firm's owner. Consequently, the owner must give him incentives that tie his compensation to realized marginal cost if he is to be willing to expend effort. Let his compensation be  $y_L$  if he achieves low marginal cost and  $y_H$  if he does not. These payments must satisfy

$$qu(y_L) + (1-q)u(y_H) - d \geq u(y_H)$$

if the manager is to prefer investing effort to not investing effort.

The incentive contract must yield the manager an expected level of utility sufficient to make him prefer working for the firm to working elsewhere. Let  $\underline{u}$  denote this level of utility. Hence, the payments must also satisfy

$$qe u(y_L) + (1-qe)u(y_H) - de \geq \underline{u}$$

if  $e$  is the equilibrium effort invested. If an owner chooses to induce her manager to expend effort, she will choose  $y_L$  and  $y_H$  to minimize her expected wage bill subject to these two constraints. Both constraints are readily shown to be binding, so  $y_L$  and  $y_H$  can be calculated by solving the constraints. Thus  $y_H = u^{-1}(\underline{u})$  and  $y_L = u^{-1}(\underline{u} + d/q)$ . If the owner decides not to induce the manager to expend effort, she will simply pay him  $u^{-1}(\underline{u}) = y_H$  — the minimum amount necessary to make him work for the firm. Define  $C(0) \equiv y_H$  and  $C(1) \equiv qy_L + (1-q)y_H$ . The function  $C(e)$  is the owner's expected cost of employing a manager whom she will induce to invest  $e$  units of effort.

### 3. Equilibria of the Basic Model

Conditional on its marginal cost being  $c$  and  $i$  of its rivals' inducing their managers to invest in cost reduction, a firm's expected profit is

$$\sum_{j=0}^i \binom{i}{j} q^j (1-q)^{i-j} \pi(c|j) \equiv \Pi(c|i).$$

A pure-strategy equilibrium exists in which  $I$  ( $0 \leq I \leq N$ ) firms induce their managers to invest in cost reduction and  $N - I$  firms do not if and only if the following expressions hold:

$$q\Pi(c^L|I-1) + (1-q)\Pi(c^H|I-1) - C(1) \geq \Pi(c^H|I-1) - C(0) \quad (1)$$

and

$$\Pi(c^H|I) - C(0) \geq q\Pi(c^L|I) + (1-q)\Pi(c^H|I) - C(1). \quad (2)$$

Expression (1) states that a firm that induces investment cannot wish to deviate by not inducing investment, while expression (2) states that a firm that does not induce investment cannot wish to deviate by inducing investment. (If  $I = N$ , then only expression (1) is relevant, whereas if  $I = 0$ , then only (2) is relevant.) Expressions (1) and (2) can be summarized by

$$\Pi(c^L|I-1) - \Pi(c^H|I-1) \geq \frac{C(1) - C(0)}{q} \geq \Pi(c^L|I) - \Pi(c^H|I). \quad (3)$$

Below, I show that as  $d$ , the disutility-of-effort parameter, varies from 0 to  $+\infty$ ,  $[C(1) - C(0)]/q$  varies continuously from 0 to  $+\infty$ . Hence, if  $\Pi(c^L|i) - \Pi(c^H|i)$  is strictly decreasing in  $i$ , there will exist parameter values such that a pure-strategy equilibrium exists in

which  $I$  firms induce investment, while  $N - I$  firms do not, for any value of  $I \in \{0, \dots, N\}$ .

**Lemma 1:**<sup>8</sup>  $[C(1) - C(0)]/q$  varies continuously from 0 to  $+\infty$  as  $d$  varies from 0 to  $+\infty$ .

**Lemma 2:**  $\Pi(c^L|i) - \Pi(c^H|i)$  is decreasing in  $i$ .

The intuition behind Lemma 2 is as follows. The expected gain from having low marginal costs is smaller, the less output the firm expects to produce. The firm, in turn, can expect to produce less output, the lower its rivals' marginal costs are on average. Its rivals will have lower costs, on average, when more of them have provided their managers incentives to reduce costs. Hence, the expected gain from having low marginal costs decreases as more firms provide their managers with incentives.

Putting Lemmas 1 and 2 together with the preceding discussion proves

**Proposition 1:** Fix an  $I$  in  $\{0, \dots, N\}$ , where  $N$  is the number of firms in the industry. Then there are parameter values such that a pure-strategy equilibrium exists in which  $I$  firms induce their managers to invest in cost reduction and  $N - I$  firms do not induce their managers to invest in cost reduction. Moreover, for these parameter values, there are no other pure-strategy equilibria.

When the parameters are such that  $0 < I < N$ , Proposition 1 states there is *no symmetric* pure-strategy equilibrium; that is, all equilibria entail heterogeneity in incentives (at least with positive probability — although not yet discussed, there also exist mixed-strategy equilibria).

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<sup>8</sup> The proofs of all lemmas may be found in the appendix.

This conclusion can be extended to any model of product-market competition in which cost reduction is a strategic substitute (these include some models of Bertrand competition with differentiated products, as well as other Cournot models).<sup>9</sup> Specifically, let  $c \in \{c^L, c^H\}$  denote a cost-function parameter and let  $\tilde{\pi}(c_n | c_{-n})$  denote the  $n$ th firm's profit when its cost parameter is  $c_n$  and its rivals' cost parameters are  $c_{-n}$ , then if

$$\frac{\partial^2 \tilde{\pi}(c_n | c_{-n})}{\partial c_m \partial c_n} < 0, \forall m \neq n, \quad (\text{C - P})$$

Lemma 2 continues to hold.<sup>10</sup> Formally,

**Lemma 2':** *Suppose condition (C - P) holds, then  $\Pi(c^L | i) - \Pi(c^H | i)$  is decreasing in  $i$ .*

From above, this is sufficient for Proposition 1 to hold (it is also sufficient for the first half of Proposition 3 below to hold).

In addition to asking is there heterogeneity with  $N$  firms, one can also ask how this

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<sup>9</sup> Whether investments in cost reduction are strategic substitutes is an important issue in the strategic trade literature (see, e.g., Bagwell and Staiger (1992), especially Section III, for a survey of this issue and a summary of some models in which investments in cost reduction are strategic substitutes). One model of Bertrand competition in which they are substitutes is when firm  $n$ 's demand is  $\alpha - \beta p_n + \gamma \sum_{k \neq n} p_k$ , where  $p$  is price, firms have constant marginal cost,  $c \in \{c^L, c^H\}$ , and  $(N-1)\gamma^2 + (N-2)\beta < 2\beta^2$  (note for the case of duopoly, this last condition matches the condition given by Bagwell and Staiger, footnote 16).

<sup>10</sup> The condition (C - P) also plays an important role in Dana (1991). He considers, among other questions, a Cournot duopoly in which the firms choose one of two available production technologies,  $A$  or  $B$ . If they choose the same technology, then their marginal costs are the same. If they choose different technologies, then their marginal costs differ (almost surely). The marginal distribution of costs given either technology is the same, so there is no inherent advantage to  $A$  or  $B$ . Provided, however, that (C - P) holds, the duopolists will choose different technologies in a pure-strategy equilibrium. The intuition is similar to here: One's relative gain from achieving low costs decreases if one's rivals also achieve low costs. In part, Lemma 2' can be seen as showing that this intuition is so powerful that the prediction of heterogeneity extends to the case where one technology is inherently superior to the other technology.

heterogeneity changes as  $N$  changes. Returning to the basic model and continuing to focus on pure-strategy equilibria, one finds:

**Lemma 3:**<sup>11</sup>  $\Pi(c^L|I) - \Pi(c^H|I)$  is decreasing in  $N$  if

$$(1-N)(a - c^H) + [N + qI(N-1)](c^H - c^L) < 0 \quad (4)$$

Moreover, a sufficient condition for this to hold is that  $I \leq N - 2$ .

The intuition, for the most part, is similar to before: More firms means more rivals, which means greater aggregate production by rivals. This, in turn, means a firm optimally wishes to produce less, which shrinks the relative value of cost reduction,  $\Pi(c^L|I) - \Pi(c^H|I)$ . There is, however, a second, strategic effect: By successfully lowering its costs, a firm induces its rivals to produce less. This effect is greater, the more rivals there are that can be induced to produce less (i.e., the greater is  $N$ ).<sup>12</sup> For a wide range of parameter values, this strategic effect is dominated by the first effect, so the relative benefit of cost reduction is still decreasing in the number of firms in the industry.

The quantity  $[C(1) - C(0)]/q$  is independent of the number of firms. So, (3) and Lemma 3 imply that, as the number of firms increases, the number offering incentive contracts to their managers in a pure-strategy equilibrium is non-increasing if  $0 \leq I \leq N - 2$ . Clearly, this number increases *at most* one-for-one with  $N$  if  $I = N$  (it may also decrease or stay constant). The

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<sup>11</sup> I am grateful to Marcus Schäfer for pointing out an algebra error in the proof of an earlier version of this lemma.

<sup>12</sup> A firm's profit is  $(a - b\sum x_n - c)x$ , so  $d\pi/d(-c) = [1 + (N-1)/(N+1)]x$ , using the envelope theorem and the fact that  $dx_n/dc_m = (b[N+1])^{-1}$ .  $(N-1)/(N+1)$  — the strategic effect — is increasing in  $N$ .

difficult case is  $I = N - 1$ ; however, here, too, the number of firms offering incentive contracts increases at most one-for-one with  $N$  (see the proof of Proposition 2 in the appendix).

**Proposition 2:** *The number of firms that induce their managers to invest in cost reduction in a pure-strategy equilibrium is non-increasing in the number of firms if expression (4) above holds — for instance, if  $N - 2$  or fewer firms initially induced their managers to invest. Moreover, in no case does the number of firms that induce their managers to invest increase by more than the increase in the number of firms.*

Consider this proposition's empirical implications. If one looked at the number of firms providing incentives to their managers, one could easily find that number decreasing as the level of competition, measured by the number of firms, increased. A tempting explanation for this phenomenon is that product-market competition "substitutes" for direct incentives; i.e., competition "naturally spurs" managers to work harder, so their compensation schemes need to provide them fewer incentives. Although tempting, this explanation is incorrect: Greater competition does not mean that fewer direct incentives are *needed*, rather it means that fewer direct incentives are *wanted*. A more competitive environment can mean owners value effort investment by their managers less, which will reduce their desire to provide incentives.

Proposition 2 also sheds light on the Hicks conjecture that more competition leads to the managers' working harder. From Proposition 2, this conjecture is essentially false in this model: An increase in competition (as measured by more firms in the industry) can easily lead to a smaller number of firms providing incentives to their managers — and hence to a smaller number of managers "working hard" (investing in cost reduction). Moreover, even if the

number of managers who work hard increases, it increases at most by the number of new firms — in a loose sense, the *original* firms' managers need not working harder due to increased competition. As such, Proposition 2 complements Scharfstein (1988) and Hermalin (1992), which also present models in which competition leads to less "hard work."<sup>13</sup> This model and Scharfstein's differ, in part, because he exogenously divides firm into "managerial" firms and "entrepreneurial" firms. Hermalin considers the various ways competition can affect incentive contracts — including the "reduced-value-of-cost-reduction" effect identified in Lemma 3 — but he does not consider the question of heterogeneity.

Turning to mixed-strategy equilibria: If no *symmetric* pure-strategy equilibrium exists, then a symmetric mixed-strategy equilibrium exists. Moreover, in this equilibrium, the probability any given firm provides its manager with incentives decreases as the number of firms increases; that is, the Hicks conjecture is false for symmetric mixed-strategy equilibria.

**Proposition 3:** *Suppose no symmetric pure-strategy equilibrium exists; that is,*

$$\Pi(c^L|0) - \Pi(c^H|0) > \frac{C(1) - C(0)}{q} > \Pi(c^L|N-1) - \Pi(c^H|N-1).$$

*Then there exists a symmetric mixed-strategy equilibrium in which each owner mixes between providing her manager with an incentive contract and not providing her manager with an incentive contract. The equilibrium probability that an owner provides her manager with an*

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<sup>13</sup> Proposition 2 is also reminiscent of the literature on patent races (see Reinganum (1989) for a survey), which shows in some cases that more firms leads to each firm investing less in R&D. The intuition is similar to here: More firms reduce the benefits of investing. This literature does not, however, focus on asymmetric equilibria.

*incentive contract is decreasing in the number of firms in the industry.*

Proposition 3 is proved in the Appendix. Although the owners play the same strategy *ex ante* in a symmetric mixed-strategy equilibrium, *ex post*, with positive probability, there will be heterogeneity: Some managers will have incentive contracts, while others will not.

#### 4. Continuous Investment Levels

The reader may, at this point, wonder to what extent heterogeneity in organizational design is due solely to the discrete investment choices available to the managers. That is, can heterogeneity arise in situations other than those in which the managers are limited to discrete investment choices?

The answer is yes: What matters are non-convexities, not the discrete action space. As the following analyses show, non-convexities can arise in many ways. Common to all these analyses, suppose that  $e \in [0,1]$ , that the probability of successful cost reduction is  $qe$  ( $0 < q < 1$ ), and that the disutility-of-effort function is  $d(e)$ . Assume further that  $d(e)$  is strictly increasing, twice differentiable, and  $d(0) = 0$ . Define  $C(e)$  as the owner's cost of inducing her manager to choose effort level  $e$  (if  $e$  can be induced). Let  $\tilde{\Pi}(c | \mathbf{e}_{-n})$  denote the  $n$ th firm's expected profit conditional on its marginal cost being  $c$  and its rivals' inducing the vector of investment levels  $\mathbf{e}_{-n} \equiv (e_1, \dots, e_{n-1}, e_{n+1}, \dots, e_N)$ :



$$\tilde{\Pi}(c | e_{-n}) \equiv \sum_{j=0}^{N-1} \pi(c | j) \sum_{k=1}^{\binom{N-1}{j}} \prod_{n \in N(k,j)} q e_n \prod_{m \in N(k,j)} (1 - q e_m),$$

where  $N(k,j)$  is the set of firms in the  $k$ th combination of  $j$  firms. Note that  $\tilde{\Pi}(c | e_{-n})$  is differentiable in  $e_{-n}$ . Conditional on its rivals inducing  $e_{-n}$ , an owner's expected profit from inducing an investment level of  $e$  is

$$q e \tilde{\Pi}(c^L | e_{-n}) + (1 - q e) \tilde{\Pi}(c^H | e_{-n}) - C(e). \quad (5)$$

Maximizing this last expression yields the owner's best-response correspondence,  $e(e_{-n})$ . Although, as shown below, asymmetric equilibria can exist even if  $e(e_{-n})$  is continuous, it is "easier" to generate asymmetric equilibria if  $e(e_{-n})$  is not continuous. Moreover, symmetric equilibria cannot be ruled out unless  $e(e_{-n})$  is not continuous. To see this last point, note that, since the first two terms of (5) are linear in  $e$ ,  $e(e_{-n})$  is continuous if  $C(e)$  is convex:

**Lemma 4:** *Assume that  $C(e)$  is convex. Then a unique symmetric pure-strategy equilibrium exists. Moreover, the symmetric equilibrium investment level is decreasing in the number of firms,  $N$ .*

In contrast, if  $C(e)$  is non-convex, then the only pure-strategy equilibria could be asymmetric. For instance, if  $C(e)$  is concave or if it is defined for  $e = 0$  and  $e = 1$  only, then the analysis would be the same as in the previous section — only  $e = 0$  and  $e = 1$  would ever be implemented. Since the focus of this paper is on heterogeneity, the next two sub-sections will focus on how such non-convex  $C(e)$  functions can arise.

A. *Non-Convexities Due to a Hidden-Action Problem*

An agency problem is created by two distortions: One, the owner and manager's interests diverge; and, two, the owner cannot monitor the manager's actions. These distortions can be a source of non-convexities if, because of them, the owner is unable to induce her manager to choose certain investment levels. To see this, suppose there are increasing returns (possibly slight) to effort invested: Specifically, the disutility-of-effort function,  $d(e)$ , is concave. From Hermalin and Katz's (1991) Proposition 2, the only implementable investment levels are  $e = 0$  and  $e = 1$ . Intuitively,  $\hat{e} \in (0,1)$  is not implementable because, at the very least, the manager would do better to mix between  $e = 1$  and  $e = 0$  with probabilities  $\hat{e}$  and  $1-\hat{e}$  respectively — the distribution over his monetary compensation would be the same as if he had chosen  $\hat{e}$ , but his expected disutility of effort would be less. So, despite a continuum of possible investment levels, exactly the same analysis would apply here as in Sections 2 and 3.

Increasing returns to effort, alone, need not yield this non-convexity. The hidden-action problem — the owner's inability to monitor her manager's actions — can also be necessary. To see this, momentarily suppose no hidden-action problem existed. Since  $u(\cdot)$  is unbounded, the owner could, then, induce any action she wished using a forcing contract. In equilibrium, her cost of implementing action  $e$  would be  $u^{-1}[\underline{u} + d(e)]$ . This cost can be convex in  $e$  even if  $d(e)$  is concave in  $e$  (consider, e.g.,  $u(y) = \log(y)$  and  $d(e) = 2 \cdot \log(e+1)$ ). So, without a hidden-action problem there would be a symmetric equilibrium (Lemma 4), but with a hidden-action problem there could be no symmetric equilibrium (Proposition 1).

Even with non-increasing returns to effort, the owners' best-response correspondences need not be continuous. This is because the convexity of  $d(e)$  is not sufficient for  $C(e)$  to be

convex.<sup>14</sup> To show this, the following result is needed:

**Lemma 5:** *Assume that the disutility-of-effort function,  $d(e)$ , is convex. Then all investment levels,  $e$ , in the interval  $[0,1]$  are implementable; moreover, the expected cost of implementing a given investment level  $e$ ,  $C(e)$ , is given by the following expression.*

$$C(e) = qe u^{-1}\left(\underline{u} + d(e) + (1 - qe)\frac{d'(e)}{q}\right) + (1 - qe)u^{-1}\left(\underline{u} + d(e) - qe\frac{d'(e)}{q}\right).$$

Consider the following case:  $\underline{u} = 0$ ,  $u(y) = \log(y)$ ,  $d(e) = e^{3/2} + 2e$ , and  $q = .95$ .

Using Lemma 5 and rearranging,  $C(e)$  becomes

$$C(e) = \exp\left[-\frac{1}{2}e^{\frac{3}{2}}\right]\left[.95e \times \exp\left[\frac{3e^{\frac{1}{2}} + 4}{1.9}\right] + 1 - .95e\right).$$

It is straightforward — albeit tedious — to show that  $C(e)$  has an inflection point at  $e \approx .683$ :  $C''(e) < 0$  for  $e > .683$ . Hence, it is never a best response to induce an  $e \in (.683,1)$ . Consequently, it is possible to find parameters such that no symmetric equilibrium exists: Some owners will induce  $e = 1$ , while others will induce an  $e \in [0, .683]$ .

As before, heterogeneity arises from the hidden-action problem: Were it possible to monitor the manager's actions, the cost of implementing  $e$  would again be  $u^{-1}[\underline{u} + d(e)]$ , which

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<sup>14</sup> A result that is well known for principal-agent models in which compensation is contingent on a continuous variable (e.g., a model with a continuum of possible cost realizations). In fact, the so-called "problem" with the first-order approach to principal-agent models (see Grossman and Hart, 1983, or Jewitt, 1988 for a discussion) has to do with the fact that  $C(e)$  need not be convex. Moreover, sufficient conditions for  $C(e)$  to be convex in these models are typically stringent (see, e.g., Jewitt, 1988). Consequently, non-convexities — and, hence, heterogeneity — are robust to enriching the model by allowing for more than two possible cost realizations.

is a convex function in  $e$  for all  $e$  and convex  $d(e)$ . Hence, from Lemma 4, a symmetric equilibrium would exist.

The function  $C(e)$  is convex for all levels of effort under the following conditions:

**Lemma 6:** *If  $q \leq 1/2$  and  $d'''(e) \geq 0$ , then  $C(e)$  is convex.*

Lemmas 4-6 imply

**Proposition 4:** *Assume that the probability of successful cost reduction is  $qe$ , where  $e \in [0,1]$ .*

*Let the disutility-of-effort function,  $d(e)$ , be strictly increasing and thrice differentiable. If*

- $d(e)$  is convex,
- $d'''(e) \geq 0$ ,

*and*

- $q \leq 1/2$ ,

*then a unique symmetric pure-strategy equilibrium exists. Moreover, under these same assumptions, the symmetric equilibrium level of investment is decreasing in the number of firms.*

Note that Proposition 4 further refutes the Hicks conjecture: An increase in competition, as measured by the number of firms, leads to managers' investing less effort in equilibrium.

Proposition 4 only states conditions for a symmetric equilibrium to exist. Asymmetric equilibria are not precluded from also existing. To see why, consider a duopoly. If

$$\bar{\Pi}(c^L|0) - \bar{\Pi}(c^H|0) > C'(e)/q > \bar{\Pi}(c^L|1) - \bar{\Pi}(c^H|1) \quad (6)$$

for all  $e \in [0,1]$ , then there exist pure-strategy equilibria in which one firm induces  $e = 1$  and

the other firm induces  $e = 0$ . Since  $\tilde{\Pi}(c^L|0) - \tilde{\Pi}(c^H|0) > \tilde{\Pi}(c^L|1) - \tilde{\Pi}(c^H|1)$  by Lemma 2, it is possible that (6) holds. For example, suppose  $\underline{u} = 0$ ,  $u(y) = 100 \times \log(y)$ ,  $q = 1/2$ ,  $a = 3$ ,  $b = 3.25$ ,  $c^H = 1$ ,  $c^L = 0$ , and  $d(e) = 15e + e^2$ ; then it is straightforward to show that

$$\frac{C'(e)}{q} = \left[ 1 + \frac{4e - 2e^2}{100} \right] \left[ \exp\left(\frac{4e - e^2 + 30}{100}\right) - \exp\left(\frac{-e^2}{100}\right) \right];$$

$$\tilde{\Pi}(c^L|0) - \tilde{\Pi}(c^H|0) \approx .4103 ;$$

and

$$\tilde{\Pi}(c^L|1) - \tilde{\Pi}(c^H|1) \approx .3419 .$$

Since  $.4089 \approx C'(1)/q > C'(e)/q > C'(0)/q \approx .3499$ , it follows that (6) holds. That is, under these assumptions, an *asymmetric* pure-strategy equilibrium exists.

These asymmetric equilibria disappear if the hidden-action problem disappears. With no hidden-action problem, the marginal cost of implementing  $e$  is

$$(.15 + .02e) \times \exp[(15e + e^2)/100],$$

and it readily follows that both owners induce  $e \approx .5679$  in the only pure-strategy equilibrium.

To summarize, this section has shown that a hidden-action problem can result in asymmetric equilibria (different owners offer their managers different incentive contracts) for a number of reasons. First, a hidden-action problem can make it impossible to implement certain effort levels, so the owners are necessarily limited to a non-convex set of *implementable* actions. Even if all effort levels are implementable, this is not sufficient to ensure that the *owners'* costs of implementing these effort levels exhibit decreasing returns to scale, which can mean the set of effort levels they *wish* to implement is non-convex. Finally, decreasing returns

to scale in effort implementation is not even sufficient to ensure that the only pure-strategy equilibrium is symmetric — asymmetric equilibria can still exist. Moreover, all of this has been illustrated with the simplest of agency models; if one extended the model to allow for multiple tasks or more-than-two cost realizations, then, as the literature makes clear, issues of implementability and the global concavity of the owners' maximization problems could become even more pressing.<sup>15</sup>

*B. Non-Convexities Due to an Increasing Returns to Scale Technology*

Discontinuities in the owners' best-response correspondences over effort levels can also arise because the underlying production technology exhibits increasing returns to scale over some range. The following example illustrates this.<sup>16</sup>

Assume a duopoly, in which the assumptions of Lemmas 5 and 6 hold. In addition, assume that  $C'(0) = 0$  (this would be the case if  $d(e) = e^2$ ). Finally, assume that the cost-of-output function exhibits increasing returns to scale; specifically, this function,  $K(x,c)$ , is

$$K(x,c) = \begin{cases} 9x, & x \leq 10 \\ cx + 90 - 10c, & x > 10 \end{cases}$$

where  $c \in \{c^L = 2, c^H = 3\}$ . Assume the inverse market demand curve is  $P(X) = 36 - X$ . If one duopolist's output is  $x$  and its rival's output is  $z$ , then denote the first duopolist's marginal

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<sup>15</sup> See, e.g., Hermalin and Katz (1991) and Holmstrom and Milgrom (1991) for analyses of the implementability issue. The problems with more than two cost realizations were discussed in footnote 14.

<sup>16</sup> What follows can be generalized without changing the basic message. For the sake of brevity, however, only a specific example will be considered.

revenue schedule by  $MR(x|z)$ . For certain values of  $z$ ,  $MR(x|z)$  crosses the firm's marginal cost ( $MC$ ) curve twice, both times from above. This means that there are two local maxima. This is illustrated in Figure 1: The two local maxima are reached at  $\underline{x}$  and  $\bar{x}$ . Which is the global maximum depends on whether  $MR(10|z)$  is greater or less than  $\frac{1}{2}(9 + c)$ : By expanding from  $\underline{x}$  to  $\bar{x}$ , the firm loses the area of triangle A in profits but gains the area of triangle B in profits. If  $MR(10|z) > \frac{1}{2}(9 + c)$ , then the global maximum occurs to the right of  $x = 10$ , but if  $MR(10|z) < \frac{1}{2}(9 + c)$ , then the global maximum occurs to the left of  $x = 10$ . As pictured in Figure 1,  $MR(10|z) = \frac{1}{2}(9 + c)$ , so the duopolist is indifferent between  $\underline{x}$  and  $\bar{x}$ . Define  $z^*(c)$  as the value of  $z$  that gives a firm two best responses (i.e.,  $MR[10|z^*(c)] = \frac{1}{2}(9 + c)$ ). Calculations reveal that  $z^*(2) = 10.5$  and  $z^*(3) = 10$ . Since  $MR(x|z)$  shifts to the left as  $z$  increases, the unique best response is less than  $\underline{x}$  if  $z > z^*(c)$  and the unique best response is greater than  $\bar{x}$  if  $z < z^*(c)$ . Using these results, reaction curves are illustrated in Figure 2. The solid lines are the reaction curves assuming no cost reduction, while the dotted lines are the reaction curves assuming cost reduction (the two kinds of reaction curves coincide for a firm producing less than  $8\frac{1}{4}$  units of output).

From Figure 2, the reaction curves do not intersect on the  $45^\circ$  line, so there is no symmetric pure-strategy equilibrium in the Cournot subgame regardless of the realized cost structure — one firm will produce more than the other. Suppose the firms anticipate that Firm 2 will produce more than Firm 1. Since Firm 1 will never produce more than 10 units, there is no value to Firm 1's owner from providing her manager with incentives to reduce costs.<sup>17</sup>

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<sup>17</sup> Asymmetric equilibria in the provision of incentives would still emerge even if cost reduction were possible for fewer than 10 units. To keep this example as short as possible, I limit cost reduction to units produced beyond the 10th unit.

The story, however, is different for Firm 2's owner: If her firm fails to lower cost, her profit is 109, but if her firm succeeds in lowering cost, her profit is  $1051/9 \approx 116.78$ . She will induce the level of investment that solves the first-order condition:

$$q(116.78 - 109) = C'(e) .$$

Since  $C'(0) = 0$ , it follows that she will provide some incentives to reduce cost.

## 5. Discussion and Conclusions

This paper has shown how, in equilibrium, managers of otherwise identical firms can work under different incentive contracts. Although the model used to derive these results is admittedly simple, it is rich enough to generate results that are suggestive about how non-convexities can arise from the technology or the underlying agency (hidden-action) problem, and how these non-convexities can explain heterogeneity in contractual and organizational forms. In addition, it illustrates how product-market competition affects the choice of contractual and organizational form within a firm, and, more importantly, variation in them across firms. Moreover, these results are fairly general: As noted above they are robust to changes in the model of product-market competition and changes in the agency model.

The comparative statics results — the rejections of the Hicks conjecture — are somewhat less robust. As shown in Hermalin (1992), they are sensitive to the assumption — common to most principal-agent models — that increased competition has *no* effect on a manager's well-being: His expected utility continues to equal his reservation utility,  $\underline{u}$ . To give the Hicks conjecture a "fair chance," competition should be allowed to affect the manager's expected



utility, since the Hicks conjecture, in part, postulates that managers work harder to reduce the adverse effects of competition *to them*.

One way to do this is to follow Hermalin (1992) and change the bargaining game between owner and manager to allow the manager's bargaining position to be affected by increased competition.<sup>18</sup> This introduces income effects because the manager's expected income given a *fixed* investment level falls as competition increases. *Not* investing is a normal good for a manager with preferences such as those assumed here. Consequently, the manager's propensity not to invest *decreases* as competition increases. Moreover, this income effect can — but need not — outweigh such effects as the reduced-value-of-cost-reduction effect found in Lemma 3, so increased competition can make managers work harder (invest more).

Although explaining heterogeneity in incentives is important in itself, there is a second reason to desire a model that predicts heterogeneity. A theory must predict variation for it to be testable: If *all* firms, for instance, found it profit-maximizing to provide strong incentives for their managers, then it would be impossible to test the effectiveness of incentives on firm performance. Clearly, incentive theory is testable here, since some owners will provide incentives, while others will not. Moreover, because it explains the source and nature of the

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<sup>18</sup> There are other ways to give the Hicks conjecture a fair chance. One would be to allow correlation across firms in the success or failure of investments in cost reduction. Because of this correlation, competition benefits an owner by allowing her to better distinguish industry-wide shocks from lack of effort by her manager. Hart (1983) and Nalebuff and Stiglitz (1983) develop models in which this increased ability to distinguish industry-wide shocks lowers the cost of providing incentives to the point that the owners implement harder actions. However, something like the reduced-value-of-cost-reduction effect still exists and, as Scharfstein (1988) and Hermalin (1992) demonstrate, the Hicks conjecture can still fail because of it.

As David Levine pointed out to me, a second way to give the Hicks conjecture a fair chance is to assume that there is a *qualitative* change in competition. For example, suppose, with a small number of firms, the industry could sustain a cartel in which output is severely restricted, but, with more firms, the cartel breaks down. The failure of the cartel could lead all firms to expand their output, which would supply their owners with a motive to induce *more* cost-reducing investments.

variation, this model can be used to inform empirical tests, such as cross-firm regressions. For instance, since incentives are endogenously chosen, this model suggests a weak correlation (at best) between incentives and profits; indeed, if mixed-strategy equilibria (Proposition 3) are played, profits for firms with incentives and those without incentives will, *on average*, be the same. On the other hand, firms that provide incentives will, on average, have lower direct-production costs (i.e., costs excluding managerial compensation). So a regression of direct-production costs on incentives should reveal a negative relation. Moreover, since firms with lower costs will produce more, incentives should strongly predict sales.

A reasonable surmise is that heterogeneity within an industry is *initially* due many factors: Firms enter at different times, they are endowed with different proprietary technologies and different locations, and bounded rationality leads them to experiment. What this paper addresses is whether this initial heterogeneity can be sustained: After there is equal access to technologies and locations and after the outcomes of the experiments have been learnt can some degree of heterogeneity remain in equilibrium? This paper suggests that the answer is yes. Indeed, to use a biological metaphor, just as competition among organisms has led not to a single species but a variety of species — despite a potential common ancestor — so too might product-market competition not only sustain but increase the variety of organizational forms.

Admittedly this paper has only shown that heterogeneity can be sustained as an equilibrium. The dynamics and evolutionary processes that lead to this heterogeneity remain topics for future research.<sup>19</sup>

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<sup>19</sup> One evolutionary model is Friedman and Fung (1991), which looks for evolutionary equilibria in a dynamic process in which firms choose between two available organizational modes.

## Appendix

**Proof of Lemma 1:**  $[C(1) - C(0)]/q = u^{-1}(\underline{u} + d/q) - u^{-1}(\underline{u})$ , which is clearly a continuous and increasing function of  $d$ . Moreover,  $[C(1) - C(0)]/q \rightarrow 0$  as  $d \rightarrow 0$  and  $[C(1) - C(0)]/q \rightarrow +\infty$  as  $d \rightarrow +\infty$  (since  $u^{-1}(\bullet)$  is convex). *Q.E.D.*

**Proof of Lemma 2:** The proof of this and other lemmas and propositions is facilitated by defining the function  $\Delta\pi(j)$ :

$$\Delta\pi(j) \equiv \pi(c^L | j) - \pi(c^H | j) .$$

The function  $\Delta\pi(j)$  is strictly decreasing in  $j$ :

$$\frac{d\Delta\pi(j)}{dj} = -\frac{2N(c^H - c^L)^2}{b(N+1)^2} < 0 .$$

The key to this proof is a comparison between two expected values:

$$\begin{aligned} \Pi(c^L | i) - \Pi(c^H | i) &= \sum_{j=0}^i \binom{i}{j} q^j (1-q)^{i-j} (\pi(c^L | j) - \pi(c^H | j)) \\ &= \sum_{j=0}^i \binom{i}{j} q^j (1-q)^{i-j} \Delta\pi(j) \end{aligned} \tag{A.1}$$

and

$$\Pi(c^L | i - 1) - \Pi(c^H | i - 1) = \sum_{j=0}^{i-1} \binom{i-1}{j} q^j (1-q)^{i-j} \Delta \pi(j). \quad (\text{A.2})$$

The lemma will be proved if it can be shown that the binomial distribution over  $\Delta \pi(j)$  resulting from  $i-1$  firms inducing their managers to invest first-degree stochastically dominates the binomial distribution over  $\Delta \pi(j)$  resulting from  $i$  firms inducing their managers to invest; that is, if it can be shown that  $\text{Prob}\{j \geq k | i\} \geq \text{Prob}\{j \geq k | i-1\}$  for all  $k \in \{0, \dots, i\}$ . Clearly this holds for  $k = i$ , so consider  $k < i$ . Define

$$\Psi(k) = \sum_{j=k}^i \binom{i}{j} q^j (1-q)^{i-j} - \sum_{j=k}^{i-1} \binom{i-1}{j} q^j (1-q)^{i-j-1} = q^i + \sum_{j=k}^{i-1} \binom{i}{j} q^j (1-q)^{i-j-1} \left[ \frac{j}{i} - q \right].$$

$\text{Prob}\{j \geq k | i\} \geq \text{Prob}\{j \geq k | i-1\}$  if  $\Psi(k) \geq 0$ . Clearly  $\Psi(k) > 0$  for  $k \geq iq$ . For  $k < iq$ , the following result applies.

Claim 1:  $\Psi(k) \geq 0$  for  $\forall k < iq$ .

Proof: Since distributions sum to one,  $\Psi(0) = 0$ . In addition,

$$\Psi(k) - \Psi(k-1) = -\binom{i}{k-1} q^{k-1} (1-q)^{i-k} \left[ \frac{k-1}{i} - q \right] > 0, \quad \forall k < iq.$$

Hence,  $0 = \Psi(0) < \Psi(1) < \dots < \Psi(\bar{k})$ , where  $\bar{k} = \max\{k \in \mathbb{N} | k < iq\}$ . ■

It follows that (A.2) is strictly greater than (A.1).

*Q.E.D.*

**Proof of Lemma 2':** Let the  $(N-1)$ -element vector  $c^j$  denote the parameters when  $j$  of a firm's rivals have parameter  $c^L$  and  $N-1-j$  of its rivals have parameter  $c^H$ . Analogously to what was done in the proof of Lemma 2, define  $\Delta\pi(j) = \bar{\pi}(c^L | c^j) - \bar{\pi}(c^H | c^j)$ . From the proof of Lemma 2, Lemma 2' is proved if  $\Delta\pi(j)$  is decreasing in  $j$ :

$$\Delta\pi(j+1) - \Delta\pi(j) = \int_{c^H}^{c^L} \left[ \frac{\partial \bar{\pi}(c | c^{j+1})}{\partial c} - \frac{\partial \bar{\pi}(c | c^j)}{\partial c} \right] dc < 0;$$

where the inequality follows because the expression in brackets is positive from (C - P) and the direction of integration is reversed ( $c^H > c^L$ ). *Q.E.D.*

**Proof of Lemma 3:** From (A.1),

$$\frac{d(\Pi(c^L | I) - \Pi(c^H | I))}{dN} = \sum_{j=0}^I \binom{I}{j} q^j (1-q)^{I-j} \frac{d\Delta\pi(j)}{dN}. \quad (\text{A.3})$$

To derive  $d\Delta\pi(j)/dN$ , define

$$\gamma = a + jc^L + (N-1-j)c^H$$

and write  $\Delta\pi(j)$  as

$$\Delta\pi(j) = \frac{(\gamma - Nc^L)^2 - (\gamma - Nc^H)^2}{b(N+1)^2}.$$

Differentiating this last expression with respect to  $N$  (noting  $d\gamma/dN = c^H$ ) yields

$$\frac{d\Delta\pi(j)}{dN} = \frac{2b(N+1)^2[(\gamma - Nc^L)(c^H - c^L)] - 2b(N+1)[-2\gamma Nc^L + (Nc^L)^2 + 2\gamma Nc^H - (Nc^H)^2]}{b^2(N+1)^4}$$

Simplifying,

$$\frac{d\Delta\pi(j)}{dN} = \frac{2}{b(N+1)^3}(c^H - c^L)[(1-N)\gamma + N^2c^H - Nc^L].$$

Substituting back for  $\gamma$ ,

$$\begin{aligned} \frac{d\Delta\pi(j)}{dN} &= \frac{2}{b(N+1)^3}(c^H - c^L)[(1-N)(a - c^H + jc^L + (N-j)c^H) + N^2c^H - Nc^L] \\ &= \frac{2}{b(N+1)^3}(c^H - c^L)[(1-N)(a - c^H) + [N + (N-1)j](c^H - c^L)]. \end{aligned}$$

Substituting this back into (A.3) and using the fact that distributions sum to one yields

$$\begin{aligned} \frac{d(\Pi(c^L|I) - \Pi(c^H|I))}{dN} &= \frac{2}{b(N+1)^3}(c^H - c^L)[(1-N)(a - c^H) + N(c^H - c^L)] \\ &\quad + \frac{2}{b(N+1)^3}(N-1)(c^H - c^L)^2 \sum_{j=0}^I \binom{I}{j} q^j (1-q)^{I-j}. \end{aligned}$$

Using the well-known formula for the mean of a binomial distribution, this last expression can be rewritten as

$$\frac{d(\Pi(c^L|I) - \Pi(c^H|I))}{dN} = \frac{2}{b(N+1)^3} (c^H - c^L) [(1-N)(a - c^H) + N(c^H - c^L) + qI(N-1)(c^H - c^L)].$$

The first part of the lemma follows. Now, since no firm shuts down,  $a - c^H > N(c^H - c^L)$ .

Hence, because  $1 - N < 0$ ,

$$\frac{d(\Pi(c^L|I) - \Pi(c^H|I))}{dN} < \frac{2}{b(N+1)^3} (c^H - c^L)^2 [2N - N^2 + qIN - qI].$$

The expression in brackets is negative if  $I \leq N - 2$ .

*Q.E.D.*

**Proof of Proposition 2:** The discussion preceding the statement of the proposition proved everything, except that the number of firms that induce their managers to invest increases at most one-for-one with an increase in the number of firms when  $I = N - 1$ . To do this, it is necessary to keep track of industry size; hence, I will write  $\Delta\pi(j,N)$  instead of  $\Delta\pi(j)$  and  $\Pi(c|I,N)$  instead of  $\Pi(c|I)$ . Define  $\Delta\Pi(I,N) = \Pi(c^L|I,N) - \Pi(c^H|I,N)$ . The following claim is crucial:

Claim 2:  $\Delta\Pi(N-1,N) > \Delta\Pi(N,N+1)$ .

Proof: From the proof of Lemma 2,

$$\Delta \pi(j, \hat{N}) - \Delta \pi(j+1, \hat{N}) = \frac{2\hat{N}(c^H - c^L)^2}{b(\hat{N} + 1)^2}.$$

Using (A.1) and the fact that  $\binom{i}{j} = \binom{i-1}{j-1} + \binom{i-1}{j}$ ,

$$\begin{aligned} \Delta \Pi(I+1, \hat{N}) &= \Delta \pi(0, \hat{N})(1-q)^{I+1} + \Delta \pi(I+1, \hat{N})q^{I+1} \\ &\quad + \sum_{j=1}^I \left[ \binom{I}{j-1} + \binom{I}{j} \right] q^j (1-q)^{I+1-j} \Delta \pi(j, \hat{N}) \\ &= q \sum_{j=0}^I \binom{I}{j} q^j (1-q)^{I-j} \Delta \pi(j+1, \hat{N}) + (1-q) \sum_{j=0}^I \binom{I}{j} q^j (1-q)^{I-j} \Delta \pi(j, \hat{N}). \end{aligned}$$

Hence,

$$\Delta \Pi(N, N+1) = \Delta \Pi(N-1, N+1) - \frac{2q(N+1)(c^H - c^L)^2}{b(N+2)^2}.$$

Straightforward calculations reveal that  $\Delta \Pi(I, \hat{N})$  is convex in  $\hat{N}$ ; hence

$$\Delta \Pi(N-1, N+1) < \Delta \Pi(N-1, N) + \left. \frac{d\Delta \Pi(N-1, \hat{N})}{d\hat{N}} \right|_{\hat{N}=N+1}.$$

Combining these last two results, and substituting for  $d\Delta \Pi/d\hat{N}$  from Lemma 3:



$$\Delta\Pi(N, N+1) < \Delta\Pi(N-1, N)$$

$$+ \frac{2}{b(N+2)^3} (c^H - c^L) [-N(a - c^H) + (N+1)(c^H - c^L) + q(N-1)N(c^H - c^L)]$$

$$- \frac{2q(N+1)(c^H - c^L)^2}{b(N+2)^2}.$$

The claim is, therefore, proved if

$$[-N(a - c^H) + (N+1)(c^H - c^L) + q(N-1)N(c^H - c^L)] - q(N+2)(N+1)(c^H - c^L) < 0.$$

Since no firm shutdowns, the lefthand side is less than

$$(c^H - c^L)[-N^2 + (N+1) + q(N-1)N - q(N+2)(N+1)] < 0. \quad \blacksquare$$

To finish the proof, suppose that although only  $N-1$  owners induce their managers to invest when the industry size is  $N$ ,  $N+1$  owners induce their managers to invest when the industry size is  $N+1$ . From expressions (1) and (2), this entails

$$\Delta\Pi(N, N+1) \geq [C(1) - C(0)]/q \geq \Delta\Pi(N-1, N);$$

but this contradicts Claim 2. Hence, if only  $N-1$  owners induce their managers to invest when the industry size is  $N$ , at most  $N$  owners can induce their managers to invest when the industry size is  $N+1$ . *Q.E.D.*

**Proof of Proposition 3:** Define  $\hat{\Pi}(c|\beta)$  as a firm's expected profit if its cost is  $c$  and each of its opponents engages in cost reduction with probability  $\beta$ ; that is,

$$\hat{\Pi}(c|\beta) = \sum_{j=0}^{N-1} \binom{N-1}{j} (\beta q)^j (1 - \beta q)^{N-j-1} \pi(c|j).$$

A firm's owner is willing to mix over incentive contracts if

$$q\hat{\Pi}(c^L|\beta) + (1-q)\hat{\Pi}(c^H|\beta) - C(1) = \hat{\Pi}(c^H|\beta) - C(0).$$

Or, equivalently if

$$\hat{\Pi}(c^L|\beta) - \hat{\Pi}(c^H|\beta) = [C(1) - C(0)]/q. \quad (\text{A.4})$$

From the definition of  $\hat{\Pi}(c|\beta)$  and the assumption that no symmetric pure-strategy equilibrium exists, it follows that there exists a  $\beta \in (0,1)$  that satisfies (A.4). Since the firms are identical,  $\beta$  is common to all firms in equilibrium.

Since the righthand side of (A.4) is independent of  $N$ , the total change in the lefthand side of (A.4) with respect to a change in  $N$  must also be zero as one compares symmetric mixed-strategy equilibria. Momentarily fixing  $\beta$ , one can treat  $\beta q$  as equivalent to  $q$  and, hence,  $\hat{\Pi}(c|\beta)$  as equivalent to  $\Pi(c|N-1)$ . Thus, for fixed  $\beta$ , the lefthand side of (A.4) is decreasing in  $N$  from Claim 2, which was used in the proof of Proposition 2. So if (A.4) is to remain an equality,  $\beta$  must adjust to balance the decrease in the lefthand side of (A.4) due to an increase in  $N$ . As the following claim proves, this means that  $\beta$  must fall.

**Claim 3:**  $\hat{\Pi}(c^L|\beta) - \hat{\Pi}(c^H|\beta)$  is a decreasing function of  $\beta$ .

**Proof:** The derivative of  $\hat{\Pi}(c^L|\beta) - \hat{\Pi}(c^H|\beta)$  with respect to  $\beta$  is

$$\sum_{j=0}^{N-1} \binom{N-1}{j} [j(\beta q)^{j-1}(1-\beta q)^{N-1-j} - (N-1-j)(\beta q)^j(1-\beta q)^{N-2-j}] q \Delta \pi(j).$$

Simplifying, this derivative can be rewritten as

$$q(N-1) \sum_{j=0}^{N-2} \binom{N-2}{j} (\beta q)^j (1-\beta q)^{N-2-j} [\Delta \pi(j+1) - \Delta \pi(j)] < 0,$$

which is negative since  $\Delta \pi(j)$  is a decreasing function of  $j$ . ■

*Q.E.D.*

**Proof of Lemma 4:** Suppose all other firms induce an investment level of  $\bar{e}$ , then a given firm's expected profit conditional on its marginal cost's being  $c$  is

$$\tilde{\Pi}(c|\bar{e}) \equiv \sum_{j=0}^{N-1} \binom{N-1}{j} (q\bar{e})^j (1-q\bar{e})^{N-1-j} \pi(c|j).$$

An owner seeks to maximize  $qe\tilde{\Pi}(c^L|\bar{e}) + (1-qe)\tilde{\Pi}(c^H|\bar{e}) - C(e)$ . Since,  $C(e)$  is convex,  $e(\bar{e})$ , the best-response correspondence, is a function; moreover, by the theorem of the maximum (see, e.g., Varian (1992, p. 506), it is continuous in  $\bar{e}$ . By the usual fixed-point arguments, there exists an  $\hat{e} \in [0,1]$  such that  $\hat{e} = e(\hat{e})$ . From Claim 3, which was used in the proof of Proposition 3,  $\tilde{\Pi}(c^L|\bar{e}) - \tilde{\Pi}(c^H|\bar{e})$  is decreasing in  $\bar{e}$  ( $\bar{e}$  plays the same role that  $\beta$  did in Claim 3). That  $e(\bar{e})$  is, therefore, decreasing follows from the convexity of  $C(e)$ . Hence, there is a unique symmetric equilibrium.

Fixing  $\tilde{e}$ ,  $\tilde{e}q$  plays the same role that  $q$  did in Section 3. Hence,  $\tilde{\Pi}(c^L|\tilde{e}) - \tilde{\Pi}(c^H|\tilde{e})$  is equivalent to  $\Pi(c^L|N-1) - \Pi(c^H|N-1)$ . Hence, from Claim 2, which was used in the proof of Proposition 2,  $\tilde{\Pi}(c^L|\tilde{e}) - \tilde{\Pi}(c^H|\tilde{e})$  is decreasing in  $N$  for any  $\tilde{e}$ . Hence,  $e(\tilde{e})$  is also decreasing in  $N$ . Thus  $d\hat{e}/dN < 0$ . *Q.E.D.*

**Proof of Lemma 5:** As before, let the manager's compensation be  $y_L$  if he achieves low marginal costs and  $y_H$  if he does not. Define  $\Delta u = u(y_L) - u(y_H)$ . A contract  $(y_L, y_H)$  feasibly implements an action  $\hat{e}$  if

$$\hat{e} \in \text{Argmax} \{qeu(y_L) + (1-qe)u(y_H) - d(e)\} \quad (\text{A.5})$$

and

$$q\hat{e}u(y_L) + (1-q\hat{e})u(y_H) - d(\hat{e}) \geq \underline{u}. \quad (\text{A.6})$$

Constraint (A.6) is binding (see Proposition 2 of Grossman and Hart (1983)). Since the program in (A.5) is globally concave in  $e$  by assumption, it follows that  $\hat{e}$  is implementable if there exists a  $\Delta u$  solving

$$q\Delta u = d'(\hat{e}). \quad (\text{A.7})$$

Since  $u(\bullet)$  has an unbounded range, such a  $\Delta u$  must exist. Use (A.7) to define  $\Delta u(e)$ . From (A.7),  $\Delta u(e)$  is continuous and strictly increasing in  $e$ . From (A.6),

$$C(e) = qeu^{-1}[\underline{u} + d(e) + (1-qe)\Delta u(e)] + (1-qe)u^{-1}[\underline{u} + d(e) - qe\Delta u(e)].$$

*Q.E.D.*

**Proof of Lemma 6:** Clearly,  $C(e)$  is continuous and differentiable in  $e$ . Differentiating  $C(e)$

yields

$$C'(e) = q(u^{-1}[u_L] - u^{-1}[u_H]) + (qe - (qe)^2)(\partial u^{-1}[u_L] - \partial u^{-1}[u_H])\Delta u'(e) ,$$

where  $u_L = u(y)$  and  $\partial u^{-1}$  is the first derivative of  $u^{-1}(\bullet)$ . Clearly, this is a differentiable function of  $e$ . Now  $du_L/de > 0$  and  $du_H/de < 0$ , so if  $q < 1/2$  ( $qe - (qe)^2$  is increasing in  $e$ ) and  $d'''(e) > 0$  ( $\Delta u'(e)$  is increasing in  $e$ ), then  $C'(e)$  is increasing in  $e$ ; that is, it is convex.

*Q.E.D.*

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