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Leveraging Students' Prior Knowledge in Attaining Deep Structural Understanding of Domain  
General Models

By

Hillary Lucille Swanson

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Science and Mathematics Education

in the

Graduate Division

of the

University of California, Berkeley

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Spring 2015

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Hillary Lucille Swanson

## Abstract

### Leveraging Students' Prior Knowledge in Attaining Deep Structural Understanding of Domain General Models

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Doctor of Philosophy in Science and Mathematics Education

University of California, Berkeley

Professor Andrea A. diSessa, Chair

The Next Generation Science Standards charge U.S. teachers with the task of including *patterns*, as a crosscutting concept, in their science curricula. This study explores the learning processes and outcomes of a pattern-based curriculum that engages middle school students in the construction of models of particular patterns. These patterns are general behaviors or processes that can be found in a range of phenomena; examples of such patterns include threshold, equilibration, and oscillation. The study investigates 1) the development of students' pattern models in response to instruction, 2) the productivity of prior knowledge in students' construction of pattern models and 3) the features of instruction that support the process of pattern model construction. It addresses research questions through analysis of data collected during the implementation of a pattern-based curriculum. Findings show that students have a wealth of prior knowledge that can be leveraged by instruction toward their construction of models of threshold, equilibration, and oscillation patterns. These findings contribute to literatures concerned with deep structural and domain-general knowledge, the productive role of prior knowledge in conceptual change, and the design of constructivist instruction.

Dedicated to my mother, Sheila, and my father, Gerald.

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## Chapter 1: Introduction and Overview

The Next Generation Science Standards (NGSS) promote 7 crosscutting concepts as recurring themes throughout the K-12 science curriculum. The first of these crosscutting concepts is *patterns*. The Framework for K-12 Science Education argues that patterns are important for science and science learning because they are readily observed in nature across dimensions of structure, function, and behavior; noticing patterns is a first step to asking deeper questions about the mechanisms that drive their emergence; and patterns can be used as rules for categorizing similar phenomena within a unified explanatory framework (Schweingruber, Keller & Quinn, 2012). The present study makes a practical contribution to the field of science education through the design of curriculum that develops students' abilities to notice and articulate patterns of behavior that can be used to organize and explain phenomena across the sciences. Students engage in the study of these powerful scientific constructs through authentic practices such as modeling, explanation, and argumentation, also important foci of the NGSS.

### Patterns of Change and Control

The curriculum is focused on a particular category of patterns. Patterns of this category are general behaviors or processes that can be found in a range of phenomena. They are processes of change and control and are therefore more completely called *patterns of change and control*. Throughout this report they will be referred to simply as *patterns*. This study focuses on three patterns in particular: threshold, equilibration, and oscillation (Table 1). Each pattern can be found in a range of phenomena from both physical and psychosocial domains. Threshold, for example, can be seen in both the tipping point of a tower of blocks and the limit of a person's patience. Equilibration can be seen in both the warming to room temperature of a cold liquid and a strong emotional reaction that dissipates over time. Oscillation can be seen in both the swing of a pendulum and the vacillation of an indecisive mind.

Table 1. Pattern descriptions and examples

Pattern	Description	Example
Threshold	A parameter of a system is varied until it reaches some limiting value and the system transitions to a markedly different state.	A tower of blocks is tipped off balance and topples to the ground.  A person reacts when pushed beyond their limit of patience.
Equilibration	A system equilibrates quickly when it is far from equilibrium; its rate of equilibration decreases as the difference between the system and its equilibrium state decreases.	A cold liquid warms quickly at first and slows as it approaches room temperature.  An emotional reaction is strong at first and gradually dissipates over time.
Oscillation	When a system is displaced from equilibrium a restoring force drives it back towards equilibrium. It gains momentum and overshoots equilibrium. A restoring force	A pendulum bob is driven back and forth by gravity and momentum.

	again drives the system to equilibrium and it again gains momentum and overshoots. This cycle continues and the system is observed to fluctuate about equilibrium.	The indecisive mind vacillates between opposite choices.
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As general behaviors or processes, patterns are independent of surface features. This makes them powerful tools for scientists as abstract models for prediction and explanation that can be recruited to explain phenomena across social, biological, and physical sciences. It also makes them powerful tools for students, as foundations on which to construct conceptual and mathematical knowledge across the sciences. Finally, because they can be examined in a range of specific examples, patterns afford multiple points of entry to their rigorous academic study. This makes them powerful tools for helping teachers leverage students' individual ways of thinking to create equitable learning opportunities in heterogeneous classrooms.

### Research Objectives

My research investigates the productivity of students' prior knowledge in their development of models of threshold, equilibration, and oscillation patterns in the context of an instructional intervention called the Patterns Class. I outline my specific research questions below and address each in Chapter 5, through the separate analysis of data from each pattern unit.

#### Research Question 1: General Tendencies in Development

- 1a. What were general tendencies in the development of students' pattern models with respect to the *target model*?
- 1b. What were general tendencies in the development of students' pattern models as *domain-general models*?

#### Research Question 2: Productivity of Prior Knowledge

- 2a. What prior conceptions emerged as resources for students' construction of pattern models?
- 2b. How did resources contribute to individual students' construction of pattern models?

#### Research Question 3: Features of Instruction

- 3a. What aspects of the knowledge construction process emerged as important?
- 3b. What features of instruction supported important aspects of the knowledge construction process?

This work makes an intellectual contribution to the field of science education research by extending a particular line of work that lies at the intersection of research on the productive role of prior knowledge in conceptual change and the development of science instruction that builds on students' prior knowledge resources (Clark & Linn, 2013; Hammer, 2000; Hunt & Minstrell, 1994; Michaels, 2005; Smith, diSessa, & Roschelle, 1994; Tzou & Bell, 2010; Warren, Ballenger, Ogonowski, Rosebery, & Hudicourt-Barnes, 2001). It broadens the frontier by investigating prior knowledge that is productive in students' construction of domain-general patterns. In doing so it produces a microgenetic characterization of the construction of new knowledge on the basis of prior knowledge in an authentic classroom setting, an important

contribution as relatively few microgenetic characterizations of the learning process exist (diSessa, 2014).

### **Overview of the Dissertation**

I conclude my introduction by outlining the rest of my dissertation. In the next chapter I situate my work at the intersection of three strands of research and provide a high-level sketch of each, reviewing the literature that is most germane to my research questions. In Chapter 3, I describe my theoretical orientation, providing a rationale for the design and logic of my study, which I elaborate in Chapter 4. I present my analysis of each pattern unit separately, in subsections of Chapter 5. I conclude by discussing the theoretical, empirical, and practical contributions of my work and directions for future research in Chapter 6.

## Chapter 2: Empirical and Theoretical Foundations

In this chapter I situate my work at the intersection of three strands of research and provide a high-level sketch of each, reviewing the literature that is most germane to my research questions. The three strands relate to the *kind of knowledge* students are constructing, the *nature of naive knowledge* and its *role in conceptual change*, and *instruction designed to leverage naive knowledge* toward the construction of more sophisticated understanding.

The first area informs my development and operationalization of *pattern knowledge* as a theoretical construct and establishes a framework for understanding my findings in the context of previous empirical work. The second area characterizes views on the nature of naive knowledge and its role in conceptual change, thus informing my theoretical position with respect to my research questions and hypotheses, and my general orientation to the design of instruction. The third area presents instructional designs that have successfully leveraged students' naive knowledge toward learning and provides guidance with respect to general design decisions. For each area I illuminate points of contact between prior research and my own work, and consider the implications of findings of previous research for my own.

### Empirical and Theoretical Work on Pattern-Like Knowledge

A *pattern* is a theoretical construct that is a process or behavior that can be described as both *deep structural* and *domain-general*. Previous empirical work has separately investigated the nature of both deep structural and domain-general knowledge. I will report on selections from literature concerned with each type of knowledge, comparing previous constructs with patterns and considering the implications of past findings for the present study.

**Deep structural knowledge.** There is little agreement among researchers regarding deep structural knowledge, in part due to its definition and how it has been operationalized in inquiry. Findings from early research suggested that deep structural understanding was a distinguishing characteristic of expertise, while subsequent research provided evidence that such understanding was well within novice capacity.

Chi, Feltovich, and Glaser (1981) found that expert physicists categorized physics problems in terms of the underlying physics principles they would use to solve them (e.g., conservation of energy or Newton's second law). Novices, on the other hand, categorized the same set of problems based on similarities in surface features (e.g., springs or inclined planes). By defining the guiding physics principle as the *deep structure* underlying a problem, the researchers inferred from their findings that experts have knowledge of deep structure underlying physics phenomena, while novices do not.

The patterns of my study can be considered to exist at a level of deep structure as they are abstract structures that transcend surface features. However, as informal process models, patterns are quite different from the formal physics principles that Chi and colleagues defined as deep structure. Patterns are also more domain-general than the deep structure construct of Chi and colleagues, as they underlie not only physical phenomena, but psychological and social phenomena as well.

The findings of Chi, Feltovich, and Glaser suggest that, in the case of solving physics problems, experts attend to deeper structures while novices attend instead to surface features. The present study aims to add complexity to the picture of novice thinking by showing that novices can notice and articulate deeper structures that are patterns. In addition, my study aims to show that instruction can support students' construction of more sophisticated pattern models.

Chi, Feltovich, and Glaser did not investigate questions about the development of deep structure knowledge in response to instruction.

Work on analogical transfer produced evidence that suggested knowledge of deeper structures was well within the novice capacity. Holyoak and Koh (1987) found that similarity between *both* surface features and structural components influenced spontaneous analogical transfer. Later, Bassock and Holyoak (1989) found that their participants could recognize structural similarity between example problems that were superficially distinct (e.g., physics vs. algebra problems). These findings suggest that students can look beyond surface features and recognize structural similarities between superficially different examples and therefore provide grounds for a central hypothesis of my model: that students have resources for the construction of models of deep structural domain-general patterns.

**Domain-general knowledge.** The research discussed above focuses on knowledge of deeper structures within a given domain, however, the patterns of this study transcend domains. Research on knowledge of structures that are domain-general has shown that learners have such knowledge and that such knowledge facilitates transfer.

**Schema induction.** Gick and Holyoak (1983) found that research participants, when asked to describe the similarities between two analogs, often derived a *problem schema*: an abstract category that the individual analogs uniquely instantiated while preserving the schema's structure. They described the structure as consisting of *mapped identities*: the *core ideas* common to the two analogies. The researchers described the cognitive process of schema derivation as *abstraction by eliminative induction* whereby differences between two analogs are deleted while commonalities are preserved. Gick and Holyoak's schema is similar to a pattern in that it is an abstract structure common to different instantiations. Patterns differ from Gick and Holyoak's schemata in that patterns are articulated as process models as opposed to a set of solution pathways for a particular problem type.

The finding that students were able to derive an abstract core common to two analogies shows that students are able to articulate a deeper structure at the core of examples with different surface features from different domains. It suggests that it is reasonable to ask students to identify structural patterns that transcend both surface features and domains. It is important to note that while the *core ideas* identified by this study could be characterized as deeper structural knowledge, they are defined differently from the principle-based deep structure of Chi et al.

Gick and Holyoak also found a positive correlation between schema quality and transfer: students that articulated more complete schema structures were more successful in transferring the solution schema to analogous problems. This finding motivates teaching such structures in science classrooms because it suggests that constructing a generalized process model of a particular pattern might support students' understanding of the multiple specific instantiations to which the pattern applies. For example, building a general model of an oscillation pattern could be applied to learning about the mechanics of water, sound, and light waves. The finding that more completely articulated schemata support more successful transfer suggests the importance of helping students craft articulations, a key focus of Patterns Class instruction.

**Intermediate causal models.** White (1991) investigated the efficacy of a construct she called an *intermediate causal model* as a tool for classroom science instruction. She described intermediate causal models as partial abstractions that captured processes at a grain size between that of concrete mechanistic models and formal mathematical abstractions. The patterns of this study are articulated at a similar level of abstraction. One of White's intermediate causal models is in fact identical to the equilibration pattern that is one of three focal patterns of the present

study. Like the patterns of this work, intermediate causal models are generic and can therefore be used to describe phenomena across domains, ranging from the flow of electric charge, to the equilibration of both fluid matter and heat. As in the case of patterns, generalizability was an important characteristic of White's intermediate causal models.

White conjectured that the decontextualized quality of her models would facilitate transfer across domains. She found that intermediate causal models were effective tools for introducing students to formal mathematical representations of physics principles and that they also supported development of students' inquiry skills and their understanding of the form of scientific knowledge. White's findings motivate patterns as valuable tools for science instruction, suggesting that engagement in pattern modeling activities may support students' future science learning (of both structurally similar conceptual material and formal mathematical representations) and develop both skills for participation in and resources for understanding the scientific enterprise.

The research reported in Area 2 has investigated kinds of knowledge that share two defining characteristics of patterns knowledge, thus offering comparable theoretical constructs and implicating a variety of possible empirical outcomes. My study responds to this work and builds on prior research into the nature of naive patterns knowledge, presented below.

***Naive pattern knowledge.*** diSessa and Lewis (in preparation) conducted clinical interviews during which participants were shown simulations designed to illustrate patterns including threshold and oscillation. The simulation designed to illustrate threshold, for example, depicted a red circle to the left of a blue rectangle. The two shapes began at rest in the middle of the computer screen, separated by a small distance. The mouse could be used to drag the rectangle toward the circle. When it made contact with the circle, the rectangle could be used to push it leftward. If the mouse moved the rectangle back to the right, the circle would roll rightward, back to its original position. If the rectangle pushed the circle far enough to the left, the circle would fly leftward away from it.

Participants were asked to score a pre-existing list of examples that they were told other students had thought *worked like* the simulation. Items on the list ranged from examples that resembled the simulation in terms of surface features (e.g., a hockey stick and puck) to examples that connected at a level of deeper structure through a similarity in process (e.g., pushing someone's buttons until they reacted). Across simulations, participants consistently gave higher ratings to examples that were similar in terms of deeper structure than to those that showed surface feature resemblance. A significant finding of this study was that students of various ages and levels of education were able to recognize deeper structural connections between phenomena that were quite distinct in terms of surface features. In follow-up studies, Swanson (2012) and Fitzmaurice, Sayavedra, and Swanson (2013) found that these results extended to younger (8th and 9th grade) students representing a range of ethnic groups and socioeconomic classes.

Findings from work on deeper structural knowledge, domain-general knowledge, and naïve pattern knowledge suggest that students have rich resources for identifying connections between examples across domains at a deeper structural level. In particular they suggest that students have resources for constructing models of patterns like threshold, equilibration, and oscillation, setting the stage for the present investigation.

## **Empirical and Theoretical Work on Naive Knowledge and Its Role in Conceptual Change**

The present study addresses questions about the nature of naive knowledge and its role in learning. It is therefore informed by theoretical and empirical work grounded in the conceptual change literature. The study of conceptual change arose as educators and researchers began to pay attention to the pre-instructional ideas that students brought to their learning. Researchers found that many students held ideas that were very different from scientifically accepted models and that their ideas seemed to persist, despite instruction.

Naive knowledge and its role in conceptual change became a focus of empirical and theoretical work. Research questions have investigated the composition and structure of naive knowledge, the mechanics of conceptual change, and the role of naive knowledge in conceptual change. Over the last four decades, two main strands of work have emerged that take very different positions with respect to these questions. I will characterize each strand in terms of 1) its view on the nature of naive knowledge, 2) the mechanics of conceptual change, and 3) the role of naive knowledge in conceptual change, and illustrate my characterizations with examples from the literature.

### **Strand 1: The *Theory Theory***

**Nature of naive knowledge.** What is the nature of naive knowledge? What can be said of its components and structure? The first of the two main strands to emerge in the field of conceptual change research postulates that students' naive conceptions are bound together in an intuitive theory (McClosky, 1983; Clement, 1982; Carey, 1991; Wiser, 1995). The learner engages this theory consistently in sense-making across contexts. Because it views naive knowledge as a theory, this perspective has been called the *Theory Theory* perspective (diSessa, 2006).

Clement (1982) and McClosky (1983) published findings to support the claim that students had naive theories of motion. Through interviews with college undergraduates focused on qualitative questions about objects in motion, both researchers found that most of their participants explained motion as the result of an internal force, and the cessation of motion as the gradual dissipation of that force. Participants appeared to apply the same reasoning consistently to problems across a variety of contexts ranging from a swinging pendulum to a ball tossed into the air, to a marble shot out of a curved tube. McClosky termed the basic model shared by his participants the *naive impetus theory* while Clement summarized his subjects' commonsense theory as *motion implies a force*.

Carey (1991) claimed that the intuitive theories of children were incommensurable with those of adults, much in the way that central concepts of different research programs have been incommensurable throughout the history of science. Wiser (1995) explored middle school students' conceptions in the context of thermal physics and found that, unlike scientists, most students did not distinguish between heat and temperature. Wiser described her students' knowledge as bound in rigid structures, claiming: "... the beliefs of each student are interconnected, stable, consistent, and constrain each other, and a limited number of explanatory schemata are used to account for both familiar and novel phenomena in predictable ways" (pg. 29; Wiser, 1995). In addition to theories, researchers have characterized knowledge systems as unitary structures using constructs such as mental models (Vosniadou, 1992) and ontologies (Chi, 1992).

**Mechanics of conceptual change.** How does a novice develop expert knowledge? What is the process of conceptual change? Many advocates of the *Theory Theory* postulate conceptual change as a process whereby the naive theory is replaced by the expert theory in a kind of gestalt



switch (McClosky, 1983; Carey, 1991; Wiser, 1995). Carey (1991) characterized knowledge acquisition along a continuous spectrum from enrichment to conceptual change. Whereas the former occurs through the simple addition of new knowledge to existing knowledge structures, the latter occurs when one theory is replaced by another, incommensurable theory.

Vosniadou (1994) described conceptual change as both enrichment and revision. For Vosniadou, the existing cognitive structure was modified through the process of revision by changes in individual beliefs or changes in their relational structure. Wiser (1995) described conceptual change as a major restructuring, drawing analogy with the notion of incommensurability used in the history of science, where the meanings of concepts in the old system are incompatible with their meanings in the new system. Chi (1992) differentiated two types of change: conceptual change and radical conceptual change. Conceptual change referred to change *within* an ontological category (e.g., within the category of *matter*) while radical conceptual change referred to change *across* ontological categories (e.g., moving a conception from the category of *matter* to another category such as *events*). For Chi, the ontological shift necessary for radical conceptual change explained why students had robust misconceptions that interfered with their learning.

**Role of naive knowledge in conceptual change.** How does naive knowledge influence learning? A number of researchers aligned with the *Theory Theory* perspective have characterized naive knowledge as *misconceptions* that are strongly held and highly resistant to change, and therefore problematic to students' learning of normative scientific concepts (Clement, 1982; McClosky, 1983; Chi, 1992; Wiser, 1995). The view that naive ideas are misconceptions and learning consists of their replacement by scientific knowledge is not consistent with constructivist views that consider learning as a process in which the learner constructs new knowledge on the foundation of the old, by engaging and refining prior knowledge (Smith, diSessa & Roschelle, 1994). Implications of this perspective for instructional design focus on pointing out, to the student, that their naive understanding is incorrect, teaching them the scientifically accurate view, and inducing them to give up their previous view and replace it with the scientific one (McClosky, 1983; Wiser, 1995).

There is no denying that the misconceptions movement positively impacted the field of science education research because, for the first time, it focused educators' attention on student ideas and their role in learning (diSessa, 2006). Despite this positive contribution, the *Theory Theory* is problematic for a number of reasons along each of the three dimensions.

### **Problems with the *Theory Theory***

**Nature of naive knowledge.** The notion that learners employ a unitary theory across contexts to make sense of phenomena has been contested by research showing that knowledge is context-dependent (diSessa, 1993; diSessa, Gillespie, & Esterly, 2004; Hammer, Elby, Scherr, & Redish, 2005). It is also problematic that not much work has been done to build and test precise theoretical models of coherent knowledge structures (diSessa, 2006).

**Mechanics of conceptual change.** The notion that conceptual change occurs through a gestalt switch where one rigid knowledge system is replaced by another violates the basic principles of constructivism (Smith, diSessa, & Roschelle, 1994). This model of conceptual change is supported by the research aligned with the *Theory Theory* that limits sampling of knowledge to pre and post instruction. Their methodology is problematic as it is prone to confirmation and does not look for evidence of the alternative model - that change occurs in a piecemeal fashion over time. Microgenetic research is needed to examine the complexities of the process of conceptual change (diSessa, 2006).

**Role of naive knowledge in conceptual change.** Much of the conceptual change literature aligns with the misconceptions movement, documenting the problematic nature of prior knowledge (Carey, 1991; Chi, 1992; Clement, 1982; McClosky, 1983; Wiser, 1995). There is, however, a growing body of work that documents instances of prior knowledge playing productive roles in sense-making and learning, thus challenging the common characterization of preconceptions as misconceptions (diSessa, 2014; Hammer, Elby, Scherr, Reddish, 2005; Hunt & Minstrell, 1994; Rosebery, Ogonowski, DiSchino, & Warren, 2010; Smith, diSessa, & Roschelle, 1994).

In response to these and other issues with perspectives aligned with the *Theory Theory* and misconceptions views, a competing perspective emerged. Because of its explanatory power and constructivist orientation, I align my research with this strand of work and discuss it in greater detail below.

### **Strand 2: Knowledge-in-Pieces/Resources**

**Nature of naive knowledge.** In contrast with the *Theory Theory*, the second perspective models naive knowledge as a system of loosely connected elements. The elements are thought to be numerous and diverse and cued variously for sense-making depending on context. Because it views naive knowledge as a system comprised of pieces, this perspective is referred to as “Knowledge in Pieces” (KiP) (diSessa, 1993). diSessa modeled one class of intuitive knowledge elements that were responsible for making phenomena sensible to learners at the most basic explanatory level (diSessa, 1993). He named these knowledge elements phenomenological primitives (abbreviated as *p-prims*) and documented primitives such as *balancing*, *equilibration*, and *Ohm’s p-prim (more effort begets more result)*.

**Mechanics of conceptual change.** For proponents of Knowledge in Pieces, much of conceptual change consists of the reorganization and refinement of existing knowledge. Because it views learning as a process involving the construction of new knowledge on the basis of prior knowledge, KiP is a constructivist perspective (Smith, diSessa & Roschelle 1994). From a KiP perspective, capturing the process of conceptual change requires a microgenetic approach. Only close study of a learner’s thinking over time can reveal the minute shifts in their conceptualization of a phenomenon. Because of the close and extended nature of such research, few studies have been able to capture the process of conceptual change (diSessa, 2006).

**Role of naive knowledge in conceptual change.** Consistent with its reorganization and refinement vision of conceptual change, KiP views naive knowledge as a diverse conceptual ecology, rich with potential resources for the construction of new, more normative knowledge. KiP is therefore aligned with the *resources* perspective. Research within the resources perspective has documented productive intuitive and culturally learned knowledge (diSessa, 1993; Hunt & Minstrell, 1994; Rosebery, Ogonowski, DiSchino, & Warren, 2010), skills (diSessa, 2004; diSessa, Sherin, Hammer & Kolpakowski, 1991; Hudicourt-Barnes, 2003; Little, 2013; Michaels, 2005) and epistemologies that students bring to their classroom learning (Hammer & Elby, 2002).

**Conceptual resources.** diSessa advocates that intuitive knowledge such as *p-prims* can be leveraged by instruction to support students’ construction of more sophisticated physics knowledge. He demonstrated the productivity of previously documented *p-prims* in one student’s construction of Newton’s law of warming (diSessa, 2014). Hunt and Minstrell (1994) documented knowledge elements they called facets and explained how facets could be leveraged during high school physics instruction. Facets exist at a slightly larger grain-size than *p-prims*, they may be intuitive or a piece of previously-learned content knowledge. An example of a facet

is “Horizontal motion keeps things from falling as rapidly as they would if they were moving straight downwards” (pg. 52; Hunt & Minstrell, 1994). Rosebery, Ogonowski, DiSchino, & Warren (2010) documented the productive ideas students brought to their learning of heat transfer and the kinetic theory of matter. In particular they showed the productivity of ideas such as *the coat traps all your body heat* that were cued as a result of standing outside in the cold without a coat.

**Practice resources.** While the researchers mentioned above documented conceptual knowledge resources, others have documented productive sense-making and discourse practices that students bring to their science learning (diSessa, 2004; diSessa, Hammer, Sherin, & Kolpakowski, 1991; Hudicourt-Barnes, 2003; Little, 2013; Michaels, 2005). diSessa, Hammer, Sherin, & Kolpakowski (1991) and diSessa (2004) documented students’ native competencies regarding representations. They named this class of skill *metarepresentational competence* and identified skills relevant to the invention, critique, explanation of representations, as well as skills for learning new representations and understanding the purposes of representations. Little (2013) investigated students’ capacities for defining and found that students had native skills for crafting definitions.

Work within this area has concentrated on continuities between the everyday practices of students from non-dominant backgrounds and the practices of science. Michaels (2005) considered the affordances of working-class children’s storytelling for the development of argumentation expertise. Warren, Ballenger, Ogonowski, Rosebery, & Hudicourt-Barnes (2001) discussed the affordances of the everyday language and sense-making practices such as *imagining* that elementary school students brought to their science learning. Hudicourt-Barnes (2003) examined the affordances of *bay odeans* (a conversational practice common among Haitian-Creole speaking students) as a format for engaging in rich argumentation around science topics. She dissects the structure of the cultural discourse style to show how it positions speakers as *theoreticians* and challengers.

**Epistemic resources.** Research has been done on students’ resources for constructing more sophisticated epistemologies of science. Hammer and Elby (2002) documented a number of such resources for understanding the nature and sources of knowledge (e.g., knowledge as propagated stuff, knowledge as free creation, and knowledge as fabricated stuff), as well as epistemological activities (e.g., accumulation, formation, checking), forms (e.g., stories, rules), and stances (e.g., belief, disbelief, understanding, puzzlement).

### **Implications for Instruction**

With its view of prior knowledge as a productive foundation for learning, this perspective suggests my instructional design not only take students’ prior knowledge into consideration, but that it leverage their prior knowledge as a resource by engaging it in the construction of new and more sophisticated knowledge. This brings me to the next section that examines literature concerned with instruction designed to leverage student resources in knowledge construction.

## **Empirical and Theoretical Work on Instruction Designed to Leverage Student Resources**

This study addresses the question of how naive knowledge can be leveraged by instruction to play a productive role in learning. It is therefore informed by empirical work on the design of constructivist instruction. The fundamental assumption guiding the design of constructivist instruction is that students' prior knowledge plays an important role in their learning. Educators focused in particular on issues of equity talk about the importance of finding ways to help students engage their culturally learned knowledge and everyday sense-making skills in their classroom learning (Lee, 1993; 2001; 2006).

Consistent with the resources perspective presented earlier, design-based research has attempted to both investigate student resources in the context of instruction and refine the design of instruction to better leverage those resources. I will now review instructional approaches designed to leverage the conceptual knowledge resources and resources for scientific practices discussed in the previous section, pointing out aspects of design that inform the design of Patterns Class instruction.

### **Leveraging Students' Conceptual Knowledge Resources**

**Bridging analogies.** Clement, Brown and Zeitsman (1989) conjectured that situations demonstrating laws of physics that students found plausible could be used as *anchoring conceptions* for grounding conceptual change via analogical extension. Examples of anchoring conceptions for learning Newton's third law documented by the researchers included a situation in which a spring pushed back on someone when they pushed on it, and a situation in which one skater pushed another to the right, and moved, as a result, to the left. Bridging analogies can serve to map the anchoring conceptions to phenomena that are the targets of instruction (Brown & Clement, 1989). This work suggests that student-constructed pattern models might be used as anchoring conceptions for teaching specific phenomena that follow the pattern. This is not addressed by the research questions of the current study, however it is considered in the discussion of implications and future research at the end of Chapter 7.

**Benchmark Lessons.** Hunt & Minstrell (1994) describe benchmark lessons as a means for eliciting students' ideas and productively engaging them throughout physics instruction. They describe an exemplar benchmark lesson during which students share conjectures about the weight of an object measured by a scale in the air, as compared to its weight when measured by a scale in a vacuum. The teacher encourages students to articulate their reasoning and tries to get the range of different perspectives on the table for public consideration. Having tried to predict the behavior, the students are invested and the teacher runs the experiment. Observations are used to evaluate students' prior conjectures as problematic or potentially fruitful. The lesson provides a benchmark for comparison as later experiments and discussions are related back and students' developing ideas are compared with their initial thinking. In addition to the development of content knowledge, the experience of a benchmark lesson facilitates the development of scientific epistemologies and skills for scientific knowledge construction (diSessa & Minstrell, 1998). The design of Patterns Class instruction, with its focus of building on students' prior knowledge, draws substantially from the design of Benchmark Lessons, regularly eliciting student ideas in the context of whole class theory-building discussions.

**The Knowledge Integration framework.** Linn (2006) describes a general pattern to guide the design of instruction that she calls the Knowledge Integration (KI) framework. The KI framework consists of eliciting student ideas, adding scientific ideas through inquiry activities, and scaffolding students' reorganization and refinement of understanding. Linn and her collaborators have used the KI framework as the basis of a web-based inquiry environment that

can be used flexibly to design instruction for topics across middle and high school science curricula (Linn, Clark, & Slotta, 2003). The three steps of the KI framework are echoed in the general design of Patterns Class instruction and particular features related to each step are examined to illuminate their role in students' construction of new knowledge on the foundation of prior knowledge.

**Cultural funds of knowledge.** Educators working with diverse student populations have focused on ways to leverage the cultural resources of students from non-dominant communities. Moll et al. (1992) aimed to draw on the knowledge and skills of a school community's local households in their development of a rich and relevant curriculum. Based on ethnographic research of one community, they documented funds of knowledge pertaining to agriculture, mining, medicine, economics, and household management. One teacher researcher developed a unit on candy-making, tapping into students' interest in selling candy that they had brought back from a visit to Mexico and drawing on resources from students' homes, including a parent volunteer to teach the candy making process (Moll, Amanti, Neff & Gonzalez, 1992).

Tzou and Bell (2010) leveraged students' knowledge of their family's health-related practices (such as eating chicken soup or mangostein to treat a cold) in an instructional unit on consequential health decisions and scientific inquiry. Warren, Ogonowski, & Pothier (2004) analyzed accounts of classroom instruction on Newton's laws in which students used everyday experience to make sense of the relationship between force and motion. In another case the researchers showed how a classroom discussion leveraged students' commonsense ideas about heat to come to understand heat transfer in terms of scientific principles (Rosebery, Ogonowski, DiSchino, & Warren, 2010). The students participating in the Patterns Class have roots in Mexican, Central American, African American, and Eastern European cultures. The work on cultural funds of knowledge suggests that, in addition to more common elements of intuitive knowledge such as p-prims, there will also be particular culturally learned ideas and ways of knowing and doing that emerge as resources for engaging in the process of pattern construction.

### **Leveraging Students' Resources for Scientific Practices**

Some researchers of instructional design have focused on leveraging students' native and culturally learned competencies for engaging in the practices of science. While the Patterns Class is certainly designed to take advantage of students' native and culturally learned competencies, the productivity of these resources was not an explicit focus of the present study.

**Resources for representational practices.** In a classroom study that was part of a larger intervention, eight sixth grade students spent five days inventing and exploring ways to represent motion (diSessa, Hammer, Sherin, & Kolpakowski, 1991). The children demonstrated unexpected expertise in designing and critiquing representations. The researchers characterized these skills as *meta-representational competence* (MRC) and expanded the category of MRC to include more general abilities, such as understanding the purposes of representations, explaining representations, and learning new representations easily. Over the course of the five days the students successively refined their representations, arriving at a version of Cartesian graphing.

Lehrer and colleagues leveraged students' MRC and examined data from several classroom case studies in which students were provided more or less scaffolding to generate, critique, and revise representations of observations made during an inquiry activity (Lehrer & Schauble, 2003; 2004; Lehrer, Schauble, Carpenter & Penner, 2000). In one study, the researchers compared the outcomes of two versions of an elementary school lesson on the relationship between the steepness of a race track and a toy cart's speed. The researchers found that having students construct, evaluate, and revise their own representations was more powerful

for their learning than giving them representations to work with (Lehrer, Schauble, Carpenter & Penner, 2000).

The representations independently generated by the students were rich with meaning for them in contrast to the ones based on the teacher's suggestions, which lacked important features of the situation. Lehrer et al. argued for the importance of both generation and critique of representations to conceptual learning. They interpreted the classroom case study data to suggest that there was a connection between representational competence and conceptual development and argued that the progressive refinement of students' representations is mirrored by the progressive refinement of their own conceptual understanding (Lehrer & Schauble, 2003). Patterns Class activities encourage students' creation and critique of visual representations of patterns to help them think more carefully about their constructs on their own and with the help of their peers.

**Resources for discourse practices.** Ballenger (1997) presented *science talks*, whole class discussions that drew from students' experiences and ideas. Science talks could be employed at the beginning of a unit to develop questions to investigate, and later to interpret findings from investigations. Ballenger argued that through participation in whole class discussions in their own personal discourse styles, the English Language Learning students of a bilingual science classroom began to develop ways of arguing, making sense of evidence, and building theories. She noted that such a discussion format supported multiple points of entry and therefore increased participation. One cultural practice that has been leveraged in this format is *bay odeans*, discussed by Hudicourt-Barnes and reported in the previous section on student resources for learning the practices of science (Hudicourt-Barnes, 2003). One of the main findings of this work was that engagement increased when students were allowed to participate through a discourse format with which they were familiar. Informed by this finding, Patterns Class discussions were designed to support students' informal styles of discourse.

**Resources for inquiry practices.** Warren and colleagues demonstrated how inquiry-based instruction could leverage students' everyday sense-making practices. As evidence, they shared the case of a classroom inquiry into ant behavior that leveraged one student's ability to imagine himself in the place of an ant. Their ability for *embodied imagining* helped the student negotiate possible experimental designs to test whether ants preferred dark or light environments (Warren, Ballenger, Ogonowski, Rosebery, Hudicourt-Barnes, 2001). Inquiry is a main feature of Patterns Class instruction, as it is through investigation of exemplars that students are first introduced to each pattern.

In summary, all examples of instructional strategies share a commitment to engaging students' conceptual and practice relevant resources in their learning. Patterns Class instruction shares this commitment and strives to leverage both kinds of resources toward students' construction of more sophisticated patterns knowledge.

### Chapter 3: Theoretical Orientation

My research questions, hypotheses, design of instruction and analytic strategies are all grounded in a theoretical framework called *Knowledge in Pieces* (KiP), introduced in the literature review of Chapter 2 (diSessa, 1993). My research questions investigate the productivity of students' prior knowledge in their construction of pattern models; I have therefore looked for the emergence of prior knowledge resources, tracked their influence on the development of students' pattern models, and considered the features of instruction that were most important to the pattern model construction process. My hypotheses, design of instruction, and analytic approach are all grounded on the fundamental assumptions of KiP.

#### Foundations of Research Questions and Hypotheses: Fundamental Assumptions of KiP

**Knowledge as a complex system.** KiP views knowledge as a complex system of many diverse and loosely interconnected elements. Important distinctions between naive and expert knowledge systems include the elements of which they are composed, the relationships between those elements (organization and strength of connection), and the functions of those elements. A novice knowledge system is characterized by elements gained through previous experience and learning that are activated variously, depending on context. Misconceptions that arise are considered to be the result of an element cued in an unproductive context. An expert knowledge system contains additional elements gained through experience and learning. The main difference between the elemental compositions of novice and expert knowledge systems is that more experience and learning have added elements to the expert system. The relative absence of misconceptions is the result of elements being cued reliably in productive contexts. The present study examines prior knowledge elements that novices recruit in their construction of models of threshold, equilibration, and oscillation patterns.

**Learning as changes in composition, structure, and dynamics of a knowledge system.** A shift from novice to expert can be seen as a matter of adding new elements, organizing elements into new and more stable relationships, and creating and stabilizing connections between elements and productive contexts. The present study illustrates instances of all three shifts in real-time classroom learning, however, the instructional approach is focused primarily on the third: applying existing elements in contexts where they are productive. For each pattern, instruction is designed to accomplish this by eliciting prior knowledge and engaging productive knowledge elements in students' construction of more sophisticated pattern models.

Another feature that distinguishes the expert knowledge system from that of the novice is the consistency with which elements and sets of elements are appropriately applied to the explanation of phenomena across contexts. This consistency depends on both *cuing* and *reliability priorities*. Cuing priority is defined as the probability that a given context will activate a particular element; reliability priority is defined as how likely it is that an element, once cued, will remain active and therefore contribute to sense-making. An element is activated very easily in a given context when it has a high cuing priority. An element is more likely to be leveraged in reasoning when it has a high reliability priority. Both priorities are increased or decreased as a result of feedback regarding the productivity of the element in a given context. Tuning toward expertise is also a result, therefore, of an accumulation of experiences that increase priorities for elements that are productive in particular contexts and productive relationships between elements, and decrease priorities for those elements and relationships that are unproductive.

### **Prior knowledge as productive resources for the construction of new knowledge.**

In contrast to the misconceptions perspective, KiP considers elements of prior knowledge to be potentially productive in learners' construction of new knowledge. Misconceptions are viewed mainly as the effect of elements cued in contexts in which they are not productive. Prior knowledge is necessary for learning, which is made possible by processes of reorganization and refinement (more specifically processes that change the composition, structure and dynamics) of existing prior knowledge resources. For this dissertation I will track micro-changes in novice knowledge systems as students expand functions for existing knowledge elements by applying them to explain new phenomena. More specifically, I will look at elements of prior knowledge that are activated and then applied to the construction of threshold, equilibration, and oscillation pattern models. The importance of prior knowledge to learning avowed by KiP is central to my dissertation. It is therefore necessary to be specific and define *prior knowledge* and related epistemic constructs.

**Prior knowledge.** I define *prior knowledge* as the knowledge students bring to their learning and include knowledge acquired previously through school instruction and cultural experiences (*culturally learned knowledge*), as well as *naïve knowledge* or *intuitive knowledge*. Knowledge acquired through school instruction might include ideas that students learned in chemistry class about scientific concepts, for example *matter is made of molecules*, or general understanding of how to read a Cartesian graph that they might have been taught in math class. Knowledge acquired through cultural experiences might include ideas students have that shape their conceptions of words such as *alive*, (Bang, Warren, Rosebery, Medin, 2012) or how to participate in argumentative discourse (Hudicourt-Barnes, 2003). By naïve or intuitive knowledge I mean the tacit knowledge that students have acquired as a result of their physical experience of the world.

It is likely that the acquisition of intuitive knowledge is culturally mediated and I am not arguing against this by categorically separating intuitive knowledge from culturally learned knowledge. The main distinction I would like to make in naming intuitive knowledge as a separate category is to separate knowledge that is less articulable from knowledge that is more articulable. For my analysis I examine prior knowledge that is less articulable and therefore characterized as intuitive knowledge. I discuss the particular kind of intuitive knowledge that is germane to the present study in greater detail below.

**Intuitive Knowledge.** The particular elements of prior knowledge that I identify as resources and track through the development of students' pattern models are characterized as intuitive knowledge. More particularly, the elements belong to a category of intuitive knowledge characterized by KiP as *phenomenological primitives* or *p-prims*, for short (diSessa, 1993). P-prims are elements of the KiP model of the naïve knowledge system. They are basic explanations for why things behave the way they do. They are phenomenological in the sense that they are rooted in and explain our everyday phenomenal experience. They are primitive in that they are fundamental explanatory units. They can be linked into a larger explanatory structure, and unpacking a larger explanatory structure ultimately *bottoms out* at the p-prim level. An example of a well-documented p-prim (named *Ohm's p-prim* for its structural similarity with Ohm's law used in electric circuit analysis) is *greater effort begets greater result*. I identify this p-prim along with others that have been previously documented in my analysis of students' construction of threshold, equilibration, and oscillation models. The particular p-prims that are germane to my analyses are introduced and characterized in greater detail in Chapter 5.



**Resources.** *Resource* (introduced in Chapter 2) is a name that is used loosely in the constructivist literature to refer to prior knowledge that plays a productive role in the process of knowledge construction (Hammer, 2000). The present study is focused on identifying elements of intuitive knowledge that are resources for students' construction of threshold, equilibration, and oscillation models. My analyses track the contribution of each element to the development of students' pattern models with the intention of demonstrating its productivity.

### **Foundations of Instructional Design**

The fundamental assumption of KiP that guided my design of instruction is that naive knowledge can be leveraged toward students' construction of scientific knowledge. As I explained above, KiP views knowledge as a complex system of elements that can be cued independently or as parts of larger structures in response to the sense-making demands of a particular context. Learning requires, not the removal of one rigid network of ideas and replacement by another, but rather, finding existing knowledge elements that can be applied fruitfully in the construction of new understanding within the context at hand. Designing instruction, therefore, is about creating opportunities for the learner to activate and engage productive elements of prior knowledge - *resources* - in the construction of new knowledge (Hammer, 2000). This stands in contrast with instruction designed from the *misconceptions* perspective, which focuses on identifying incorrect knowledge and replacing it with correct knowledge (McClosky, 1983).

Patterns Class instruction was therefore designed to elicit and engage students' naive ideas in the construction of more sophisticated pattern models. Students created and refined pattern constructs by participating in activities that activated, elicited, and built on their prior knowledge. Students were given agency in the construction process and encouraged to produce models that represented their own thinking. Instruction was very student-centered, however the teacher played an important role and facilitated all of its phases. The design of Patterns Class curriculum and instruction is described next, in Chapter 4.

### **Foundations of Analytic Strategies**

In order to address my research questions, I drew on the set of qualitative strategies connected with the Knowledge in Pieces framework that are organized under the title Knowledge Analysis (KA). I organize and elaborate important characteristics of KA below.

**Focus on knowledge.** The KiP program is characteristically focused on knowledge and is situated within the cognitive subset of the learning sciences community. KiP views knowledge as something that exists within an individual's mind, rather than within their physical body, the artifacts they produce or use, or within their interactions with others. Knowledge Analysis, therefore, focuses on the *ideas* that learners communicate, rather than the means or modes through which they communicate their ideas. It does not ignore interaction, historical, social, or cultural aspects of learning, and recognizes these as playing fundamentally important roles in learning (diSessa, Levin, & Brown, in press). However, it does not foreground these elements as primary in the data, and it does not focus on building these elements into the models it produces. It is important to note that KA should not be seen as competing with analytic strategies that have grown out of situated cognition programs, but rather that the analytic perspectives are complementary, offering insights into different and important aspects of learning.

**Commitment to theory.** The KiP program grew out of work that was focused on building computational models of human thinking (diSessa, 2014). A high-level goal has been to

build a complex systems theory of knowledge that could be programmed into a computer and run to simulate human thinking and learning. The KiP program considers its work as a contribution to the initial stages of this larger effort. Modeling knowledge, its elements, their relational structures, and the processes in which they are involved and through which the system changes are key activities of the KiP enterprise. Knowledge Analysis, therefore, is committed to the generation, test, and refinement of models of knowledge toward the refinement and extension of the KiP theoretical base.

**Grounding in data.** Knowledge in Pieces is committed to the development, test, and refinement of general models of knowledge systems, however, it is firmly committed to the complexity and idiosyncratic nature of individual systems and therefore Knowledge Analysis attends to the data first. In this way KA is bottom-up. The researcher first considers the data and tries to characterize what is there and follows by applying a theoretical lens to model the elements and processes that emerge from the data. In practice it is difficult to say that one (an open look at the data) precedes the other (looking at data through the lens of theory), as observations tend to be theory-laden (Hanson, 1965). However, the important point is that Knowledge Analysis is a mixture of both bottom-up and top-down approaches. Data sources include written and verbal protocol, drawn representations, and physical indicators such as gesture and attention signaled through gaze.

**Microgenetic time scales.** The KiP program is concerned with modeling conceptual change at a fine enough grain size to illuminate the minute shifts in the individual elements of an individual's thinking over time. Knowledge Analysis therefore approaches the analysis of learning processes through microgenetic methods. *Genetic* in the word microgenetic is meant to communicate the idea of *genesis* (as in the origin of something vs. genes and hereditary traits), microgenesis meaning the genesis of something portrayed in tiny steps. A microgenetic view of learning therefore looks at data at finer time scales than pre and post assessment, attempting to characterize moment-by-moment shifts in student thinking.

Knowledge in Pieces provides the theoretical underpinnings of my research; my analytic approach is therefore grounded in Knowledge Analysis. I am focused explicitly on knowledge: in particular the role of naive knowledge in the construction of more sophisticated knowledge. I engage elements from KiP theory to model shifts in students' cognitive structures during this process. I begin each analysis grounded in the data with an open coding to illuminate general developmental trajectories within the class. I conduct careful analysis of students' knowledge construction at a finer time scale than pre and post assessment by sampling student thinking between 3 major snapshots in order to build a microgenetic picture of individual students' development of pattern models.

Having discussed the theoretical foundations of my research questions, hypotheses, design of instruction, and analytic strategies, I turn now to elaborating the details of the design and logic of my study.

## Chapter 4: Design and Logic

I approached my research questions by analyzing data collected during a single iteration of a design-based research cycle (Brown, 1992; Collins, Joseph & Bielaczyc, 2004). The design under test was a middle school science course called the Patterns Class. The Patterns Class was designed to activate and engage students' prior knowledge in their construction of models of patterns such as threshold, equilibration, and oscillation. The design of the class was refined over several iterations by a team of shifting composition. The core of the research and design team was comprised of the faculty PI and primary graduate student researcher (the author of this dissertation). Additional support was provided by a technology specialist, 2 graduate student research assistants, and several undergraduate research apprentices. Below, I describe my approach to investigating my research questions. I then describe the details of the participants, instructional design, and data collection procedures for the iteration that is the focus of the present report.

### Approach to Addressing Research Questions

My research questions, hypotheses and design of instruction are all intimately connected through an iterative process of design-based research. Elements of KiP theory and findings from previous research grounded my initial questions and hypotheses. By designing instruction according to my hypotheses and testing instruction in an authentic classroom context, I evaluated the theoretical assumptions on which the hypotheses were grounded. Analysis of student artifacts and videotape of classroom sessions led to findings, which in turn informed refinement of my evolving picture of novice pattern knowledge and learning and helped formulate the research questions, hypotheses, and instructional design of the present iteration. Below, I describe my research questions and hypotheses and their development with respect to previous iterations. I then describe my general design of instruction and approach to analysis

**Research questions and hypotheses.** In general, across iterations my research questions have investigated 1) students' development of models of threshold, equilibration, and oscillation patterns in the context of the Patterns Class, 2) the productivity of students' prior knowledge in their construction of pattern models, and 3) the aspects of instruction that support the pattern model construction process. Over the iterations my questions have narrowed to examine students' development of models with respect to both an instructional target and as domain-general models; the particular elements of prior knowledge that contribute to students' construction of models and how they are activated and engaged in the construction process; and how particular features of instruction support key aspects of the pattern model construction process.

I will now introduce my hypothesis for each research question and explain how it is backed by findings from previous iterations. For my first research question, I hypothesized that students could construct increasingly sophisticated pattern models (in terms of both the instructional target and as domain-general models) through participation in Patterns Class activities. This hypothesis was grounded on the foundational assumption that students could notice and articulate patterns instantiated by multiple examples. This assumption was backed by findings from previous iterations of pattern-based curriculum and research on novice pattern knowledge that showed students attended to deeper structural similarities in the behavior of examples that were different in terms of surface features, and that they were able to articulate those behaviors.

For the iteration directly preceding the focal iteration, I had used a coding scheme to characterize initial and final drafts of students' models of threshold and equilibration patterns. The coding schemes were predecessors of the coding schemes introduced in Part II of each subsection of Chapter 5. The coding schemes characterized students' models with respect to the target model by identifying the elements of the target model that students included in their own models. The proportion of student-generated models containing each element of the target model was compared across drafts and an increase in proportion for each element was noted. This finding backs the part of my hypothesis regarding the increase of sophistication of pattern models with respect to the target model. Crafting domain-general models had not been an instructional focus of previous iterations. However, it had been noted that some students articulated patterns using domain-general language (e.g., "adding more till you get a reaction"), while other students' descriptions of patterns were tied to particular examples (e.g., "adding more coins until the spaghetti breaks"). The observation that some students naturally described patterns using domain-general language inspired the addition of this as a goal of instruction, and provided backing for my hypothesis that students could refine their models to be articulated in more domain-general language.

For my second research question, I hypothesized that students had resources at the grain size of *p*-prims that could be leveraged in their construction of pattern models. For the iteration directly preceding this study I had treated resources as a broad category for collecting all ideas students had that were productive in their construction of pattern models. Resources were at the grain-size of facets (Hunt & Minstrell, 1994) for example, "the more something grows, the harder it becomes to resist" (for threshold) and "slowing down when they get to their destination" (for equilibration). The emergence and productivity of *Ohm's p-prim* during a whole class discussion during the equilibration unit led to the narrowing of the characterization of *resources* from ideas roughly the grain-size of facets to ideas the grain-size of *p*-prims. Findings from previous research had also shown the productivity of *Ohm's p-prim* in students' construction of equilibration, providing additional backing for my hypothesis in the context of equilibration (diSessa, 2014). The hypothesis was assumed to be generalizable to threshold and oscillation patterns.

For my third research question, I hypothesized aspects of instruction that supported students' construction of pattern models elicited students' prior knowledge and encouraged students to build on resources. Observations from the iteration directly preceding the present study suggested that students had a wealth of diverse ideas regarding both threshold and equilibration patterns. Among those ideas there were some that were very productive in students' construction of models of threshold and equilibration. It seemed like a good instructional approach was to elicit as many ideas as possible and then choose from among those the ideas that were most productive to nudge students toward the target model for each pattern.

From both the previous iteration of the Patterns Class as well as earlier tests of pattern-based curriculum it was known that particular activities supported eliciting and developing students' conceptions of patterns. Having students engage in the exploration of exemplars gave them a chance to notice the key characteristics of individual exemplars and what key characteristics overlapped. Whole class discussions in which students were asked to build theories to explain why patterns exhibited the behavior that they did moved student models from a description of behavior alone to one that included the processes that drove the behavior. Asking students to articulate the behavior or process common to two or more exemplars focused students on elements of pattern behavior and elicited their ideas for personal and public consideration.

Engaging students in the generation of examples further clarified students' own impression of a particular pattern and made the task of articulating the most important characteristics of the pattern more accessible, as students could carefully consider elements of the pattern in an example from their own regime of expertise.

Explanation and debate of examples prompted students to improve their articulation of patterns and identify what elements of the pattern were primary, versus secondary or superfluous. Consideration of examples that were boundary cases (i.e., examples that shared elements of the pattern but also differed in important ways) helped students increase the precision of their pattern model by drawing clear boundaries and articulating what elements were not included in the pattern. Revision of pattern models gave students second and third opportunities to adjust their models according to changes in thinking that were the result of participation in the activities described above.

**Design of instruction.** These hypotheses (in particular my hypothesis for research question 3) informed the design of the Patterns Class that is the focus of the present report. In general, each pattern unit (threshold, equilibration, and oscillation) was organized so that students followed a particular sequence of activities. Each unit opened with small group exploration of 2 or more examples of the pattern through hands-on investigations. Whole class discussions were used to elicit students' ideas about the processes driving the behavior observed during investigations. Following explorations, students worked individually to name and describe models of a deeper structural process or behavior exhibited by both examples. They were also asked to draw flow-charts of the pattern, to help them decide on and organize the key elements of the pattern. Next students explored an additional exemplar or boundary case. Following this they reviewed previous pattern models and then revised their models. Next they generated additional examples of the pattern and shared their pattern names, descriptions, and examples with the whole class. Problematic examples were selected from among those the students had generated and used to engage students in argumentation around the fit of examples and pattern models. Finally, prior pattern models were reviewed and students were given the opportunity to revise their pattern models. Several activities were dispersed throughout the curriculum to engage students in thinking about the nature of patterns (e.g., their domain-generalizability and their defining characteristics) and their usefulness to science.

**Evidence of students' pattern resources and model construction.** Artifacts produced by students' participation in the most recent iteration of the Patterns Class activities were used as data for addressing each of the three research questions. For this iteration, students wrote 3 drafts of descriptions of their pattern models. Students' individually written first, second, and third draft pattern descriptions were used to address all three research questions; posters produced by small groups, video footage of whole class discussions, teacher reflections, and researcher field notes were used to address the second and third research questions.

**Analytic strategies.** The first research question was addressed through a more macroscopic lens. The coding schemes for threshold and equilibration that were used for previous iterations of the Patterns Class and pattern-based curriculum were further refined based on artifacts produced by students during this iteration, and a coding scheme was produced to characterize a new pattern featured in this iteration: oscillation. These coding schemes were used to characterize students' first, second, and third pattern model drafts according to the elements of the target model that they contained. Proportions of student descriptions characterized as 1 of several increasingly sophisticated pattern structures were then compared across drafts to look for increase in sophistication. Additional coding schemes were created to characterize the domain-

generality of students' pattern models. Proportions of domain-general descriptions were compared across drafts to look for increase in domain-generality of pattern models over the course of each instructional unit. Statistical analysis of the development in students' pattern models was then used to check for the significance of the intervention in students' refinement of models.

The second research question was addressed through a more microscopic lens. Classroom activities and discussions were analyzed for the emergence of productive intuitive knowledge elements. Individual students' artifacts were then examined to learn about the productive role of intuitive knowledge in the development of their pattern model over the course of the unit. The third research question was addressed by looking for aspects of instruction that were important to the construction of pattern models and by describing the features of instruction that supported those aspects.

Having discussed my methodological approach and its development in response to the findings of previous research, I will now describe the participants, instructional intervention, and data collection methods of my study in greater detail.

## **Participants**

Twenty-one 8th grade students participated in the focal iteration of the Patterns Class. The majority of participating students were children of families that had emigrated from Mexico and Central America; however several identified as African American and European American. English was a second language for most, Spanish being the primary language spoken at home. The majority of students attending the school were designated as English Language Learners, and the majority qualified for free and reduced lunch (frequently used as a proxy for low-income socioeconomic status). English Language Learners from low-income homes are often mischaracterized by academic deficit narratives (Rosebery, Ogonowski, DiSchino, Warren, 2010). The students participating in the Patterns Class challenge a deficit characterization with a powerful counter-narrative: they are well supported by incredibly involved parents (as well as extended family and community) and they are committed to their own learning and bring a wealth of resources for constructing scientific knowledge and participating in science practices.

**Means of selection.** The group of students participating in the Patterns Class was selected based on accessibility: however particular characteristics were sought. A high-level goal of the research was documenting the resources of students from non-dominant communities and exploring how Patterns Class instruction could make challenging content engaging for these students by leveraging their intuitive and culturally learned resources. Previous studies had worked primarily with students from groups that were well represented in higher education and professional science (diSessa, 2014; diSessa & Lewis, in preparation). This research, therefore, sought to work with students from groups historically underrepresented in higher education and professional science, considered to be underserved by traditional classroom instruction. The Patterns Class therefore sought to partner with a school that primarily served students from low SES households and of ethnic and language minority.

The particular school was selected because the science teacher there was amenable to sharing her elective period students with our group for both fall and spring semesters. Her elective period had traditionally been used as a science enrichment period for students that had scored proficient or higher on tests of basic skills in English and math. The elective period did not begin meeting until the third week of school, in mid-September. Prior to this, the researcher met with the group of students who were eligible for the elective period to explain the goals and

design of the study and to invite the students to participate as both consumers and co-designers of the curriculum. The students were given assent, parental consent, and video release forms and given time to read the forms and ask questions. The researcher went over important points including the optional nature of participation in the class, data collection in the form of artifacts and video footage, the intended use of the data and procedures taken to protect participants' anonymity, and the dual role of the teacher as researcher. The 21 students who opted to participate and returned the necessary paperwork attended the Patterns Class.

### Instructional Intervention

**Patterns Class.** Patterns Class met for 40 minutes on Monday, Tuesday, and Thursday mornings. The class met both fall and spring semesters, accumulating approximately 60 hours of instructional time over the course of the school year. The researcher was the primary instructor and undergraduate research assistants doubled as teaching assistants, attending class about one morning a week with moderate consistency over the school year.

Patterns Class curriculum was designed to guide students through the systematic exploration of four patterns: threshold, equilibration, exponential growth and oscillation (though these names were never introduced to the class participants). Instruction was designed to support students in the modeling of each pattern by leveraging their prior knowledge in a general sequence of activities that alternated between exploring prototypical examples and generating and refining models of the patterns those examples followed. Research questions were addressed for 3 of the 4 pattern instructional units; details of the activities particular to each of the focal pattern units are presented in the threshold, equilibration, and oscillation sections of Chapter 5.

Pattern exploration activities such as investigations were done in groups of 4 students. Pattern modeling activities such as writing and revising pattern descriptions were done individually. Pattern example generation and critique activities were a mix of both individual and small group work, where individual work preceded group work and was used as a foundation for the construction of group artifacts (i.e., students worked alone to generate a list of examples that followed the pattern and then collaborated with their group to combine examples and produce a poster; students worked alone to decide how examples did or did not follow the pattern and then decided, as a team, which arguments to present during the whole class debate).

In addition to exploring particular patterns, students engaged in activities meant to help them develop meta-pattern knowledge by discussing the nature of pattern knowledge, the process through which it was created, and its general usefulness to science. These activities included an introductory unit that introduced students to the concept of a *pattern* and the general sequence of pattern construction activities in the context of a pattern underlying cookies, as well as reflection sessions between each of the pattern units. A general overview of Patterns Class curriculum is presented in Table 2, below.

Table 2. Patterns Class curriculum overview

Hours	Unit	Activities
6	Introduction to Patterns: The Cookie Pattern	1. Exemplar Inquiry: Chocolate Chip 2. Exemplar Inquiry: Oatmeal 3. Boundary Case Inquiry: Graham Crackers 4. Boundary Case Inquiry: Oreos
10	Pattern 1: Threshold	1. Exemplar Inquiry: Spaghetti Bridge 2. Exemplar Inquiry: Drops on a Coin

		<ol style="list-style-type: none"> <li>3. Describing the Pattern</li> <li>4. Boundary Case Inquiry: Egg in Salt Water</li> <li>5. Revising the Pattern Description</li> <li>6. Generating and Critiquing Examples</li> <li>7. Revising the Pattern Description</li> </ol>
20	Pattern 2: Equilibration	<ol style="list-style-type: none"> <li>1. Exemplar Inquiry: Cold Milk</li> <li>2. Exemplar Inquiry: Hot Tea</li> <li>3. Describing the Pattern</li> <li>4. Exemplar Inquiry: Beans in a Box</li> <li>5. Revising the Pattern Description</li> <li>6. Generating and Critiquing Examples</li> <li>7. Revising the Pattern Description</li> </ol>
1	Pattern Reflection	Describing the word "pattern"
8	Pattern 3: Exponential Growth	<ol style="list-style-type: none"> <li>1. Exemplar Inquiry: Spread of Disease</li> <li>2. Exemplar Inquiry: Growth of Population</li> <li>3. Describing the Pattern</li> <li>4. Generating and Critiquing Examples</li> <li>5. Revising the Pattern Description</li> </ol>
5	Pattern 4: Oscillation	<ol style="list-style-type: none"> <li>1. Exemplar Inquiry: Swinging Pendulum</li> <li>2. Exemplar Inquiry: Bouncing Magnets</li> <li>3. Exemplar Inquiry: Ping Pong Ball in an Airstream</li> <li>4. Exemplar Inquiry: Vibrating Rubber Band</li> <li>5. Describing the Pattern</li> <li>6. Exemplar Inquiry: Weighted Wheel</li> <li>7. Revising the Pattern Description</li> </ol>
2	Pattern Reflection	Revising descriptions of the word <i>pattern</i>

In addition to the design of activities, it is important to acknowledge the role of the teacher in facilitating the Patterns Class. The teacher's approach developed along with the curriculum in response to changes in design and experience with facilitating. By the focal iteration of the Patterns Class, a general orientation to teaching strategies had emerged for the teacher that is worth noting, as this orientation pervaded her facilitation of the particular activities of each pattern unit.

**General orientation to teacher strategies.** Instruction was very student-centered, however, the teacher played an important role and used particular strategies to support students' development of pattern models over the course of each unit. The teacher therefore assumed a general orientation throughout activities, of prioritizing student agency in the process of pattern model construction. Her strategies across activities addressed two halves of the construction process: 1) eliciting student ideas to generate pattern-relevant resources, and 2) engaging resources in students' construction of pattern models.

**Strategies for eliciting resources.** Because pattern models were constructed on the foundation of students' ideas, it was important to elicit ideas that were germane to particular elements of the target model of each pattern. Students could then consider those ideas carefully and construct their pattern model from the most sensible ideas. The teacher was therefore intentional in her facilitation of whole class discussions and encouraged broad student participation. To support this, she implemented a common teaching strategy called *think-pair-*



*share*. She regularly began whole class discussions by asking students to first respond to discussion prompts individually in writing. Students were then asked to share their ideas with the 3 other students of their small group. She would then open the whole class discussion by asking each group to present one idea that had come up at their table in response to the prompt, acknowledging their contribution by recording it on the board. Once all groups had shared their ideas she would ask for general responses to those ideas, or for students to volunteer ideas that had come up in small group discussion but had not yet been represented. The teacher varied her approach from here, sometimes asking for volunteers to share ideas, sometimes pulling names from a hat, and sometimes calling directly on students that had not yet participated in the discussion.

The intention was not merely to elicit a breadth of ideas, but to achieve depth as well. It was important for students to articulate their ideas more fully than they usually initially offered; exposing the depth of their reasoning yielded intuitive knowledge that included resources for the construction of pattern models. In order to expose the depth of their reasoning the teacher held students accountable to their ideas, pushing students (during class discussion, small group discussion, and individual conversations) to unpack their thinking by asking follow-up questions.

***Strategies for engaging resources in the construction of pattern models.*** With numerous ideas to consider, students had many options to choose from in constructing their pattern models. The teacher made it clear that students had agency in crafting their models, however, she played a role in slowly nudging their models in the direction of target models by selecting particular productive ideas for the focus of class discussions and actively steering students' attention back to those ideas when discussions moved in less productive directions. As well, she played a role in nudging models to be more domain-general by presenting students' patterns to the class and modeling and eliciting critique specifically focused on level of generality.

## **Data Collection**

Data were collected in four different forms: 1) student artifacts, 2) video footage, 3) field notes, and 4) teacher reflections.

**Student artifacts.** Students were asked to record their pattern-related ideas almost every class either individually (in response to worksheet prompts) or as part of a small group (in the form of a poster). Occasionally, artifacts were constructed during a whole class discussion (such as points shared by contributing students recorded by a student or teacher on the white board). Worksheets were collected and students' responses were typed into documents that organized their work for each unit chronologically. These documents were saved according to students' reference numbers on the lab computer and originals were transported to the locked research lab on campus. Written information on posters and white board notes were typed into documents that organized small group and whole class work for each unit chronologically. Student reference numbers were used to indicate which students contributed to small group artifacts recorded in these documents. The typed documents were stored on the lab computer and originals were transported to the locked research lab on campus. Student reference numbers were used for analysis and pseudonyms were given to students presented in case study analyses.

Students wrote descriptions of their pattern models 2 to 3 times over the course of each pattern unit. These drafts were analyzed to provide snapshots of students' initial and developing conceptions of particular patterns. The artifacts were produced following the exploration of examples of a particular pattern. To scaffold their description of the pattern model, students were asked to respond to the prompt: "you just documented the characteristics of two specific

behaviors that, while different on the surface, follow the same general pattern. Describe the pattern that the specific behaviors follow.” Drafts of descriptions were written by students and collected by the teacher/researcher pre, mid, and post pattern unit. Students responded to the prompt individually on their own worksheets, however, they were seated in groups of 3 or 4 around a common table and encouraged to work with their tablemates.

**Collection schedule.** The descriptions were written based on an accumulating list of examples that more or less followed the pattern. Table 3 (below) organizes, for each pattern, the timing of the written descriptions (labeled as draft 1, draft 2, etc.) as they were embedded in a sequence of example exploration activities. The examples were selected by the teacher as either exemplars of the pattern or as boundary cases (i.e., examples that had characteristics in common with the pattern but also important differences), and given to the students in the context of investigations and demonstrations. Toward the end of each unit, students generated their own examples and engaged in argumentation meant to help them decide the distinguishing features of the pattern.

Table 3. Collection schedule for pattern descriptions

Pattern	Explore	Articulate	Explore	Articulate	Explore	Articulate
Threshold	Example 1 Example 2	Draft 1	Example 3	Draft 2	Student generated examples	Draft 3
Equilibration	Example 1 Example 2	Draft 1	Example 3	Draft 2	Student generated examples	Draft 3
Oscillation	Example 1 Example 2 Example 3 Example 4	Draft 1	Example 5	Draft 2		

All students’ written descriptions were analyzed to address the first research question in the context of each pattern unit. Microgenetic analysis was applied to the work of particular students in order to address the second and third research questions. Students were selected for case study analysis because the developmental trajectory of their own pattern model matched the general developmental trajectory of the class.

**Video footage.** Two digital video cameras recorded the activities of each Patterns Class (see the map in Figure 1 below, for a bird's eye view of the camera positions within the classroom). One camera was positioned at the middle of one side of the room and pointed at an angle out across the classroom toward the front board. This camera was meant to capture the activity of the teacher and/or student(s) speaking at the front of the room and the artifacts recorded on the front board. A second camera was positioned at the front of the room just to the side of the front board. It was pointed at a slight angle out across the tables at which the students

sat. This camera was meant to capture the activity of the students (and assisting teachers) as they attended to the activity at the front of the room, or engaged in small group work.

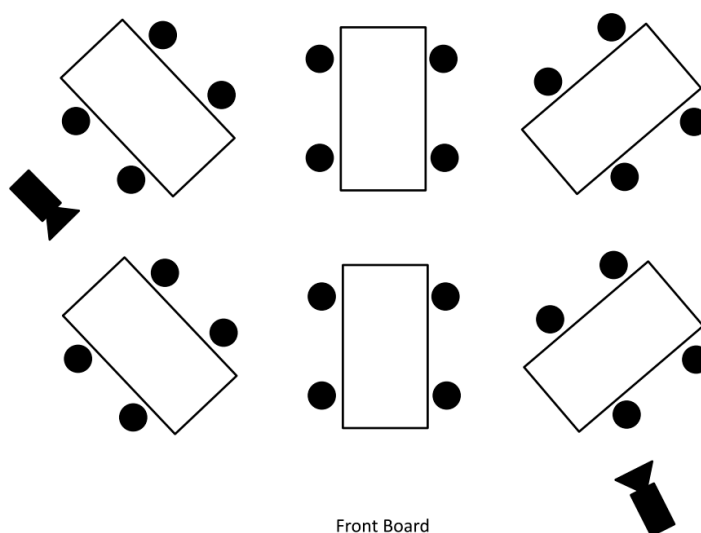


Figure 1. Bird's-eye view of classroom and video camera positions

During small-group work both cameras were turned to capture the activity of students at the nearest table. Digital files of the video recordings were downloaded, compressed, and stored on the lab computer. Video footage of whole-class and small-group interactions has been used to construct a more fine-grained picture of the emergence of students' prior knowledge and its role in their construction of pattern models.

**Transcription conventions.** I omit certain contributions that are interruptions or part of productive conversations that are not temporally linked to the contributions that are immediately relevant to my analysis. Omissions are summarized between brackets <...> and placed before the contributions they precede. I number each contribution for reference and use the symbols defined below to indicate the flow of speech:

// - a break in speech

/.../ - interruption or parallel speech

[...] - inaudible speech, text indicates my best guess.

(...) - pause

**Field notes.** In addition to the instructor, one of several undergraduate research assistants was present about one class each week. Along with managing video collection and interacting with the students as a teaching assistant, the research assistant was responsible for taking field notes. Three general categories of field note were recorded: notes on unexpected interesting phenomena related to learning or instructional design, notes on students' skills for articulating a pattern and notes on students' engagement in Patterns Class activities. Notes and timestamps for associated video footage were recorded in an electronic database according to category and stored on the lab computer. Field notes were used to identify video for more careful analysis.

**Teacher/researcher reflections.** Teacher/researcher reflections were written at the end of each class. Three categories of reflection were recorded about events in the class that connected to 1) student engagement, 2) instructional design and 3) cognition and learning.

Digital copies of these teacher reflections were stored on the lab computer. As with field notes, teacher notes were used to help identify particular dates and times for more careful analysis.

## Chapter 5: Analysis

I address my three research questions in each of the threshold, equilibration, and oscillation pattern units. I begin my analysis of each pattern unit by drawing a high-level sketch of general tendencies in the development of students' pattern models. I then turn to careful examination of the role of prior knowledge in students' construction of pattern models. I end by identifying important elements of the construction process and supporting features of instruction.

### Analytic Approach to Research Questions

**Research question 1: General tendencies in development.** What were general tendencies in the development of students' pattern models with respect to the *target model* and as domain-general models?

My first research question addresses general trends in the development of students' pattern models in response to the instructional unit. As this question focuses on general shifts in models, a coarse-grained analysis is appropriate and a macroscopic analytic strategy is employed. The data I analyzed were drafts students had written at various points along the instructional unit, to describe their pattern models. These drafts were meant to sample student thinking and provide snapshots of initial and later pattern conceptions. I developed coding schemes with which to characterize each description with respect to a target model (thereby addressing research question 1a), and as a domain-general model (thereby addressing research question 1b). I used these schemes to code initial and subsequent drafts of students' written pattern descriptions. I produced graphs to compare aggregate tendencies in students' pattern models with respect to target models and as domain-general models at each draft. Finally, I used a basic statistical hypothesis test to establish the likelihood of a causal link between instruction and development in students' pattern models.

**Research question 2: Productivity of prior knowledge.** What prior conceptions emerged as resources for students' construction of pattern models? How did resources contribute to individual students' construction of pattern models?

The second research question addresses the role of prior knowledge in individual students' construction of patterns. As the focus of this question is on the role of prior knowledge in the conceptual change of individual students, a more fine-grained analysis is appropriate and a microscopic analytic strategy is employed. Both sub-questions are approached through microgenetic case study analysis. A student was selected for each pattern whose shifts across drafts matched the general developmental trajectory of the class. The student's written descriptions of the pattern along with their other written classwork were examined and activities preceding major shifts in their thinking were identified as loci for closer analysis. Of these activities, those during which resources emerged were identified and episodes caught on video were transcribed. Written work, video transcriptions, teacher reflections, and researcher field notes were then coordinated to produce a map of the student's development over the unit, in relation to the emergence of resources. Finally, the details of the data and this map were used to track how prior knowledge contributed to the development of the student's construction of the pattern over the unit.

**Research question 3: Features of instruction.** What aspects of the process of pattern model construction emerged as particularly important? What features of instruction supported important aspects of the knowledge construction process?

The third research question addresses the features of instructional design that leveraged students' prior knowledge toward their construction of pattern models. This question requires a qualitative analysis at a fine grain size, though slightly coarser than that of question 2. I approached question 3 by examining case studies to identify aspects of the construction process that productively engaged prior knowledge and the features of instruction that supported these aspects.

## Section 5.1: Threshold Analysis

I begin this section by describing a model of the target threshold pattern and my design of the threshold instructional unit (Part I). As I describe in my introduction to Chapter 5, I address my first research question by presenting a high-level sketch of the general development of students' threshold models using the results of a macro analysis of students' written work (Part II). I then address my second research question and present a detailed picture of the role of prior knowledge in two students' construction of threshold models using the results of microgenetic analysis of written work, video transcripts, and field notes (Part III). I end by addressing my third research question, presenting the important elements of the threshold model construction process and the features of instruction that supported this process (Part IV).

### Part I. Introduction to the Threshold Pattern and Instructional Unit

**Threshold pattern.** A model of threshold that could be used as a benchmark for characterizing student models was crafted by our research team through a combined bottom-up (data driven) and top-down (scientific model driven) approach. We compared student models with the elements of scientific models of threshold, noting which elements were common to both, which elements were missing from students' models, and which elements students had included that were not included in scientific models, but seemed important to an intuitive sense of threshold. We created a coding scheme that consisted of scientific elements that students had included in their models (i.e., *pre-phase*, *post-phase*, and *transition*) and fine-tuned characterizations of those elements to be consistent with the intuitive sense conveyed by the students. The resulting model of the threshold pattern is a sequence of *pre-phase* (during which a system parameter is varied through a repeated action) that causes the system to reach a *limit* (which occurs when a system parameter reaches some maximum or minimum value) that results in a *post-phase* (which is characterized as either a *reaction* or a *terminal state*). It is this *pre-phase - limit - post-phase* characterization of threshold that is the target model for the threshold instructional unit.

**Threshold instructional unit.** The instructional unit was designed to support students' construction of threshold models through the exploration of exemplar phenomena and the generation and critique of familiar examples. The rationale for the general sequence of activities in each unit is explained in detail in Chapter 4. The threshold unit was comprised of 7 core activities that took place over approximately 10 instructional hours. The sequence of core activities is presented in Figure 2 and described below.

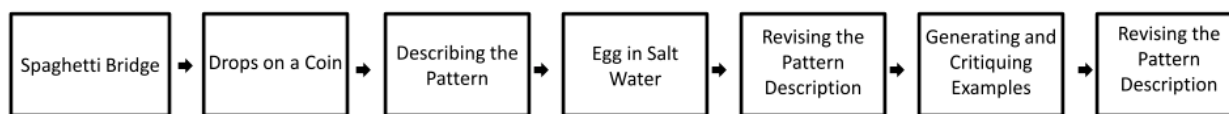


Figure 2. Sequence of core activities of the threshold instructional unit

**Core activity 1: Spaghetti bridge.** The unit opened with the investigation of an exemplar of the threshold pattern: a spaghetti bridge reaching its breaking point and snapping under the weight of a heavy load. A single stick of spaghetti was balanced across a 6-inch gap between two desks; students hung a small paper cup over the middle of the spaghetti and dropped in pennies, one at a time. The goal of the activity was to see what happened to the spaghetti as pennies were added to the cup. One class period (40 minutes) was allocated to this activity. This particular

example was selected because it clearly demonstrated the important elements of the threshold pattern. The pre-phase could be seen in the addition of pennies to the cup, the post-phase could be seen in the crash of pennies, cup, and spaghetti to the floor, and the reaching of a limit could be seen in the addition of the final penny that led to the destruction of the spaghetti bridge.

**Core activity 2: Drops on a coin.** Students next explored threshold in the surface tension of water. Each small group was given a penny, an eye-dropper, and a cup of water. Students took turns trying to see who could add the most water to the surface of the penny before the swelling bead burst and water flowed onto the table. One class period (40 minutes) was allocated to this activity. This particular example was selected because it demonstrated elements of the threshold pattern. The pre-phase is evident in the addition of water drops to the surface of the coin, the post-phase is marked by the water in a puddle on the table, and the reaching of a limit is evident in the addition of the drop that results in the overflow. The example was selected particularly as a partner activity to the spaghetti bridge because, in addition to sharing a deeper structural similarity, the two examples shared the inclusion of coins as activity materials. These parallel similarities would illuminate whether students naturally attended to similarities in deeper structure or surface features (responding to literature of Area 1 discussed in Chapter 2), and provide the teacher with a concrete example for helping students understand the key characteristics of a pattern (i.e., that it was a behavior at a deeper structural level, as opposed to a similarity in surface features).

**Core activity 3: Describing the pattern.** Following the exploration of the two exemplars, the teacher asked students to construct a model of the pattern illustrated by both activities. The purpose of this activity was to engage students in thinking about the deeper structural behavior common to both examples and articulating their initial impressions. Their initial ideas about the pattern would serve as the foundation for constructing more sophisticated pattern models over the course of the unit. At the beginning of the year the teacher had introduced *model* as a vocabulary word, distinguishing the sort of model students would use to describe patterns from the cell models they had constructed the year before in their life science class. She told students they would be making a kind of model called a *process model* to present the pattern behavior and how they thought it worked.

For each pattern students would write a description of their pattern model, represent it in a more visual way using a flowchart, and give it a name. She reminded students that the pattern was a behavior that was common to both examples. To scaffold the process of articulating the pattern the teacher told her students “one trick for doing this is to start by telling the story of both behaviors so that someone listening to your story would agree that you are talking about either one of the behaviors, but they would not know for sure which one you were talking about.” This was written at the top of the worksheet on which students wrote names and descriptions, and drew flowcharts of their pattern model. The prompt was developed during the previous iteration of the Patterns Class in response to students' initial confusion with how to approach the task of articulating the pattern and was observed to be fairly effective. The total instructional time allocated to initial pattern modeling activities was 1 hour 20 minutes.

**Core activity 4: Egg in salt water.** The next investigation supported students' exploration of a boundary case of the threshold phenomenon. For this investigation students stirred salt (one spoonful at a time) into a cup of water until a raw egg (that had been submerged at the start) floated to the surface of the water. This example was selected because it shared some of the important characteristics with the pattern but was also different in important ways. It was thought that exploration of a boundary case would increase the precision of students' pattern models by



prompting them to decide on key characteristics and non-characteristics of the pattern. The egg example was considered a boundary case because it illustrated a more continuous relationship between cause and effect, as opposed to the reaching of a limit resulting in an effect. An observant student might notice that a certain amount of salt had to be added to the water to make the egg initially buoyant, however with each spoonful of salt the egg would rise incrementally, finally floating to the surface. One hour and 20 minutes of instructional time were allocated to this activity.

**Core activity 5: *Revising the pattern description.*** Following the investigation, the teacher reviewed students' previous pattern descriptions and the results of the egg and salt water investigation. The students were then asked to revise their pattern names, descriptions, and flowcharts. It was thought that students' conceptualizations of the pattern might have been productively influenced by both the consideration of the boundary case, and by seeing the pattern names, descriptions, and flowcharts of their peers. Offering students a chance to revise their models gave them the opportunity to demonstrate their refinement in thinking and present their revised thinking in new models. The total instructional time allocated to these activities was one hour and 20 minutes.

**Core activity 6: *Generating and critiquing examples.*** Having generated and refined their own models of threshold, students worked in small groups to generate lists of examples from their own lives that followed the same general pattern. Students generated a number of examples, ranging from physical examples (e.g., "filling up a water balloon," "stretching gum until it snaps"), to psychophysical examples (e.g., "adding more sadness until you cry," "bothering someone until they burst"). They created posters to showcase their examples and reviewed the posters of other groups during a gallery walk. During the walk, they applied post-it notes to posters to communicate whether or not examples made sense to them (writing on post-its either "spot-on," "unique," or "please explain", and placing post-its beside particular examples).

The teacher used the examples on which students had not agreed as the basis of an argumentation activity. For this activity students wrote down points that either supported or challenged each of 11 problematic examples and then shared their points with the rest of the class for 3 particular examples – "bothering someone until they burst," "getting your hair cut," and "freezing water." Engaging students in the generation and critique of examples was meant to help them think carefully about the key characteristics of the pattern and to articulate their ideas as completely and as clearly as possible. Examples generated by students and the arguments they produced in favor of or against examples could also expose differences in conceptualizations of the pattern. Two hours of instructional time were allocated to these activities.

**Core activity 7: *Revising the pattern description.*** The final 2 hours and 40 minutes of instructional time were allocated to writing final draft descriptions of pattern models. The teacher introduced the activity with a review that surveyed previous pattern descriptions, names, and flowcharts. Students were asked to draw examples to illustrate their final draft descriptions. It was thought that students' conceptualizations of the pattern might have been productively influenced by the consideration of additional examples and by hearing their peers' arguments in favor of, and against, examples. Offering students a final chance to revise their models gave them the opportunity to demonstrate their further refinement in thinking.

## Part II. General Tendencies in Development

I turn now to addressing the first research question of my study in the context of the threshold instructional unit.

1a. What were general tendencies in the development of students' pattern models with respect to the *target model of the pattern*?

1b. What were general tendencies in the development of students' pattern models as *domain-general models*?

**Analytic approach.** I approached question 1 with a macro-analytic strategy. I developed coding schemes for characterizing students' written descriptions of their threshold models and used these to code students' initial and subsequent description drafts. I present coding outcomes graphically to show whole class tendencies in each pattern draft. I compare proportions across drafts and use basic statistics to identify significant shifts in whole class tendencies, which I present as general developmental trajectories.

**Research question 1a.** I begin by addressing the first sub-question of research question 1: What were general tendencies in the development of students' pattern models with respect to the *target model of the pattern*?

**Coding scheme.** I designed the coding scheme below (Table 4) to characterize students' descriptions in terms of the target model of threshold. The first part of the coding scheme describes the *elements* of threshold that students included in their written descriptions. The second half of the coding scheme (Table 5) lists sets of related elements that were the basic structures underlying students' patterns. These pattern structures contain one or more elements linked together by causal arrows. As shown in Table 5, pattern structures range from single elements to more complex chains of elements. The structures are ordered vertically according to level of sophistication and assigned rank scores. Beginning with a characterization of the pattern as one phase, phases are added as well as phase-specific details.

Table 4. Threshold elements coding scheme

Phase	Element	Description	Example
Pre-Phase	Action	An action is applied to the system.	"You need to add something on an object..."
	Repetition	An action is repeatedly applied to the system.	"I kept adding something..."
Limit	Implicit Limit	The word "until" implies that the repeated action results in a transition from pre- to post-phase.	"We added something till something happened." "We added something until it couldn't hold any more."
	Explicit Limit	The limit is explicitly specified.	"Trying to test the limit it can hold." "It reached its max capacity."

Post-Phase	Terminal State	The state of the system post-transition is characterized as a terminal state or state of saturation.	“...until it couldn't hold it anymore”
	Reaction	The system reacts to the (repeated) action.	“Adding something to something until it changes.”

Table 5. Threshold structures coding scheme

Pattern Structure	Description	Example	Score
Other	The description does not have one of the structures listed.	“They all involved household items.”	0
Pre-Phase	Some (repeated) action is applied to the system.	“Adding or taking away something.”	1
Limit	There is a limit for some value belonging to a system.	“Trying to test the limit it can hold.”	1
Reaction	A reaction occurs.	“They both ended up getting destroyed/broken.”	1
Pre → Reaction	Some (repeated) action causes a reaction to occur.	“We got our materials and then we added stuff to things. And then we repeated that. Then we waited for the original thing to change.”	2
Limit → Reaction	The system reaches a limit and a reaction occurs.	“Both were to see how much of something would it take before the object broke or spilled.”	2
Pre → Limit (Terminal State)	A repeated action causes the system to reach a limit that is its terminal state.	“Add more stuff until the object reaches its max capacity.”	3
Pre → Limit → Reaction	A repeated action causes the system to reach a limit and this causes a reaction.	“Repeating a process till something happens (explodes, pops, breaks, falls, bubbles, changes in color).”	3

**Coding.** These coding schemes were used together to characterize students' initial and subsequent written drafts of pattern model descriptions in terms of underlying structure. A team of 3 researchers (comprised of myself and two undergraduate assistants) coded each of the three drafts independently and then convened to compare scores, resolve discrepancies, and reach consensus.

**Results.** Coding outcomes are presented in the bar chart below to show aggregate tendencies in pattern conceptions for each draft. Comparison of proportions across drafts and a test of statistical significance provide evidence for general developmental trajectories.

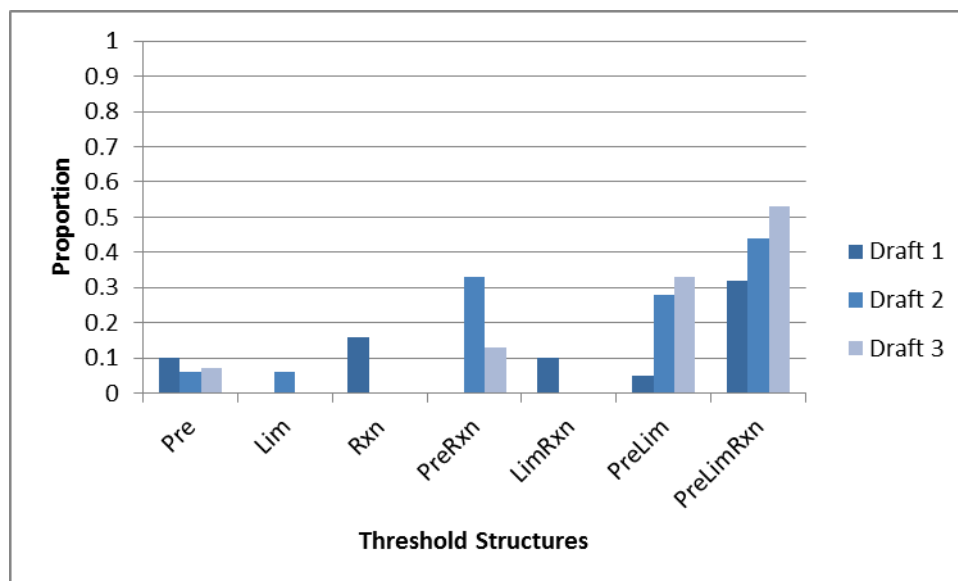


Figure 3. Characterization and comparison of threshold structures

*General tendencies in development.* For the first draft, about a third of the students characterized the pattern as a pre-phase that led to a limit and resulted in a reaction post-phase (e.g., “we had to keep on putting something until it broke”). A quarter of the descriptions were entirely about pre-phase (e.g., “we put pennies into a container”) or post-phase (e.g. “both were about something falling”). Fifteen percent characterized the pattern as a pre-phase leading to a limit that was a terminal state post-phase (e.g., “adding things to objects until they can't hold those objects anymore”) or a limit that resulted in a reaction (e.g., “only a certain amount weight had to be held before it interrupts the experiment”). The rest (about a quarter of the descriptions) were coded as *other*.

By the second draft, 44% of the descriptions were characterized as a pre-phase that led to a limit and resulted in a reaction post-phase, 33% were characterized as a pre-phase that led to a reaction post-phase (without mention of a limit), and 28% were characterized as a pre-phase that led to a limit that was a terminal state post-phase. By draft 3, 53% of the descriptions were characterized as a pre-phase that led to a limit and resulted in a reaction post-phase, 33% were characterized as a pre-phase that led to a limit that was a terminal state post-phase, and the remaining 14% are spread over a pre-phase that led to a reaction and only pre-phase. As both versions of the pattern (pre-phase that led to either a reaction or terminal state post-phase) are legitimate with respect to the target model of threshold, the proportions can be combined to show that 86% of the students crafted versions of the target model by the end of the unit.

Comparison of tendencies in pattern structures across drafts suggests movement away from a characterization of threshold that is *other* or incomplete toward one that is complete, and ending in the terminal state or reaction post-phase. Comparison of mean rank scores (Figure 4) for draft 1 and draft 3 pattern descriptions shows that as a class, students are writing descriptions of pattern models that increase in sophistication. The class mean increases from 1.73 (SD = 1.3) at draft 1 to 2.73 (SD = .57) at draft 2. This movement indicates that students, on the whole, are developing more sophisticated conceptions of threshold over the course of the unit.

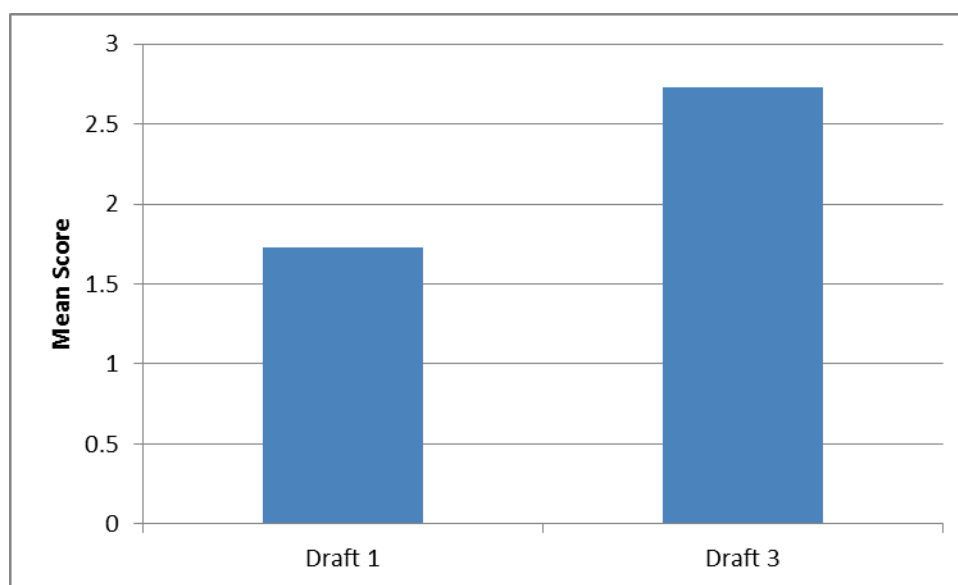


Figure 4. Mean scores for students' draft 1 and draft 3 threshold models

A basic statistical test was used to ascertain the probability that the threshold unit activities played a role in this development. In order to test the hypothesis that the unit activities did not have any effect on students' development of threshold models, their rank scores for first and final draft pattern model descriptions were compared using a paired difference test. Because rank scores (a discrete measure) were compared, and because the sample size was quite small and the data sets were not normally distributed, a non-parametric statistical hypothesis test was needed. The Wilcoxin signed-rank test is a hypothesis test that can be used for non-parametric data sets; it was used compare first and final draft scores. Results showed that students' gains in rank score were statistically significant ( $\alpha = .05$ ). This suggests that instruction played an important role in the development of students' models of threshold.

**Research question 1b.** I turn now to address the second sub-question of research question 1: What were general tendencies in the development of students' pattern models as *domain-general models*?

**Coding scheme.** I designed the coding scheme presented below (Table 6) to evaluate whether or not students' descriptions were domain-general. This coding scheme was used to classify descriptions as *general*, *specific*, or *other*.

Table 6. Domain-generality coding scheme

Characterization	Description	Example
General	The description is not embedded within one or multiple specific contexts.	“Adding something to something until it changes.”
Specific	Context or specific examples are given.	“We just kept adding water or pennies until the spaghetti broke or the water spilled.”
Other	The description does not contain any of the pattern elements.	“Both involved household items.”

**Coding.** This coding scheme was used to characterize students’ initial and subsequent written drafts of pattern descriptions. As in the case of the coding presented earlier, the team of 3 researchers coded every written description independently and then convened to compare scores, resolve discrepancies, and reach consensus.

**Results.** Coding outcomes are presented in the bar chart below (Figure 5) to show aggregate tendencies in the domain-generality of each draft. Comparison of frequencies across drafts show shifts in aggregate tendencies, which are presented as general developmental trajectories.

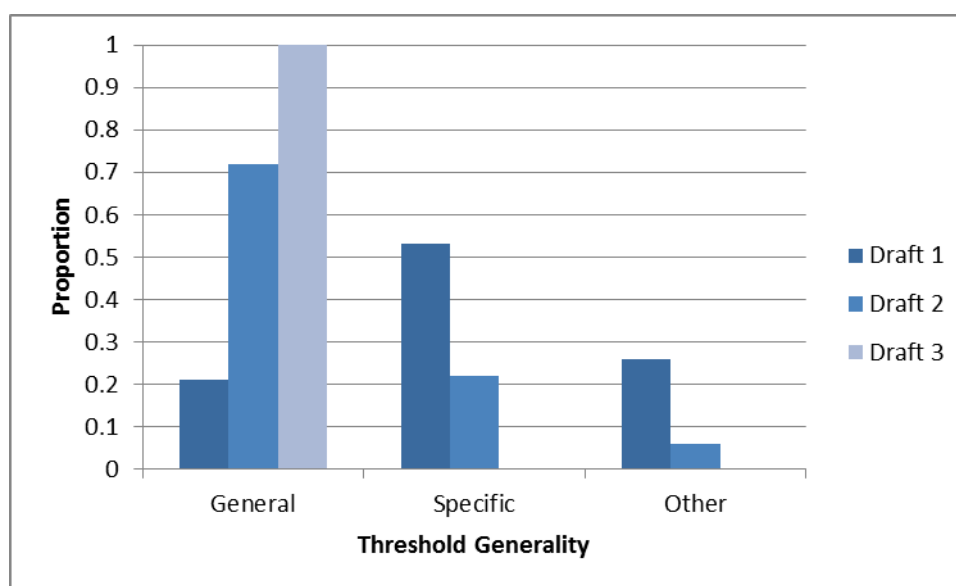


Figure 5. Proportion of descriptions coded as general, specific, or other

*General tendencies in development.* Comparison of proportions across drafts shows that students are crafting increasingly domain-general descriptions of threshold over the course of the unit. This change corresponds with revision activities for which students critiqued previous drafts of descriptions.

**Conclusion.** Instruction was designed to engage students’ prior knowledge in their construction of models of threshold. Instruction was primarily student-centered, and though it cannot be denied that the teacher played an important role, students’ models were constructed largely on the basis of their prior knowledge. The fact that over 85% of the class developed

descriptions of threshold that mapped directly to one of the two target models of threshold is a testament to the knowledge and skills this group of students brought to their learning. Closer analysis of the events during the unit suggests that students' conceptions of the pattern as consisting of either *terminal state* or *reaction* post-phase were influenced by the examples they felt best embodied the pattern. These results are considered more carefully through microgenetic examination of two students' construction of threshold models.

### Part III. Productivity of Prior Knowledge

I turn now to addressing the second research question of my study in the context of the threshold instructional unit.

2a. What prior conceptions emerged as resources for students' construction of pattern models?

2b. How did resources contribute to individual students' construction of pattern models?

#### Analytic approach.

I approached question 2 through microgenetic case study analysis. I selected two students whose shifts across drafts matched the two general developmental trajectories of the class illuminated in Part II. I examined their written classwork to identify major shifts in thinking and to look for connections between those shifts and particular episodes during which productive prior knowledge emerged. I used written work, video transcripts, teacher reflections, and researcher field notes to track how prior knowledge contributed to the development of each students' construction of threshold over the unit. The general trajectory of my presentation of this analysis will be to locate the emergence of key prior knowledge and then track how it is taken up and built upon productively by two particular students in their construction of models of threshold.

**Research question 2a.** I begin by addressing the first sub-question of research question 2: What elements of pre-instructional knowledge emerged as resources for students' construction of more sophisticated models of the pattern? I begin by introducing the two students whose developmental trajectories I follow more closely. I follow by introducing the key prior knowledge and its emergence in a focal lesson near the beginning of the unit.

**Case students.** Our case students are Araceli and Patricia. They are 2 of the 11 female students in the class of 21. Patricia is outgoing and vocal during small group and whole class work. Araceli is more introverted during whole class discussions but tends to play a central role during small group work. Both students were chosen for case study analysis because the development of their threshold models matched the two main aggregate tendencies of the group reported earlier in the macro analysis. Araceli's conception of threshold moved from being off-task at draft 1 (focused on materials and processes common to the first 2 examples), to consisting of pre-phase and post-phase as both *reaction* and *terminal state* at draft 2, to consisting of pre-phase, limit, and post-phase as *reaction* at draft 3. Patricia's conception of threshold moved from being off-task at draft 1 (focused as well on materials and processes), to consisting of pre-phase and post-phase as both *reaction* and *terminal state* at draft 2, to pre-phase and post-phase as *terminal state* at draft 3.

**Key prior knowledge.** The case studies presented here show how two elements of prior knowledge played key roles in the construction of two versions of the threshold pattern (shown in Figure 6). The knowledge elements have not been previously documented, however, they are candidates for phenomenological primitives: a class of intuitive knowledge elements within the KiP framework. The two particular knowledge elements are different versions of *limit*; I call them *abstract limit* and *dynamic limit*. *Abstract limit* is a limit that connects with a *terminal state*

post-phase: it is a point beyond which a parameter cannot be increased or decreased. A *dynamic limit* is a point at which a system transitions to a *reaction* post-phase. *Dynamic limit* is closer to our research team's original sense of threshold, presented in Part I. The students were encouraged to develop models of the pattern that they saw in all the examples. Both versions of the intuitive sense of limit were considered to be equally productive as each played an important role in one of the two threshold constructs that students' developed.

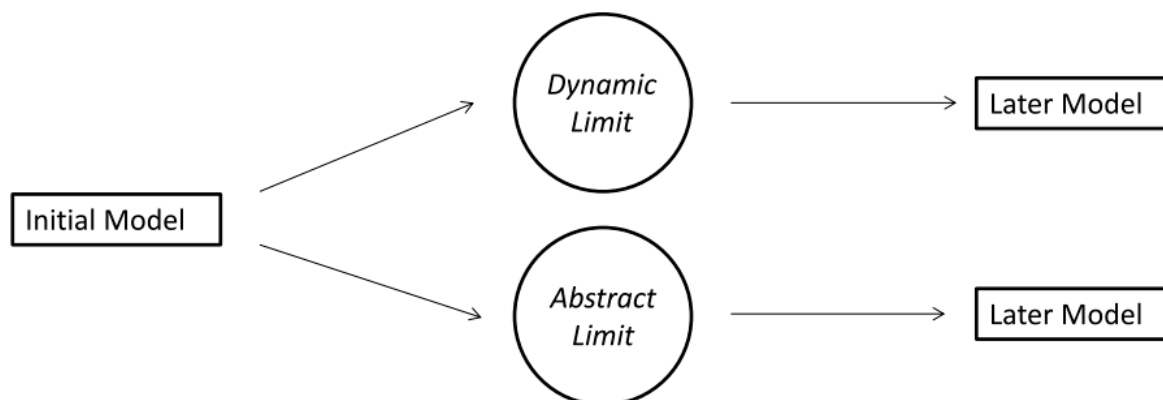


Figure 6. Basic trajectories showing the role of *abstract limit* and *dynamic limit* in students' development of threshold models<sup>1</sup>

***Lesson during which key prior knowledge emerged.*** The lesson during which both *abstract limit* and *dynamic limit* emerged for Araceli and Patricia was the investigation following the writing of the first pattern description. It appears that their different orientations to the investigation activated either *abstract limit* or *dynamic limit*. This influenced how they viewed later examples, as they appeared to attend to the features of examples that matched, and therefore reinforced, their own version of the pattern. The lesson was an investigation that illustrated two boundary examples of threshold. For the investigation students added teaspoons of salt to a glass of water and watched as an egg that had been submerged in the solution began to float. They recorded both the behavior of the egg (sinking vs. floating) and the appearance of the water (clear vs. foggy). The investigation worksheet oriented students' attention to the behavior of the egg in relation to the added salt and both Araceli and Patricia wrote their hypotheses about the egg, guessing that it would float as salt was added to the water. The observation section of the worksheet directed students' attention to both the behavior of the egg and the water by creating space for students to record their observations of both.

Araceli and Patricia's small group had finished collecting data and the teacher approached their table. The water was full of salt and the egg was bobbing at the surface of the water. The following transcript was taken from video of the small group. The transcript is divided into two

<sup>1</sup> It should be noted that the model in Figure 6 simply represents particular intuitive knowledge elements that can be used as resources for the construction of a model of threshold. It is not meant to represent the complexity of the model construction process, which likely involves other knowledge elements and might look more like fitting together pieces of a complicated puzzle.



segments (see Figure 7) to show how the first part of the conversation oriented students' attention to the relationship between salt and egg (activating *dynamic limit*) while the second part of the conversation oriented their attention to the relationship between salt and water (activating *abstract limit*).

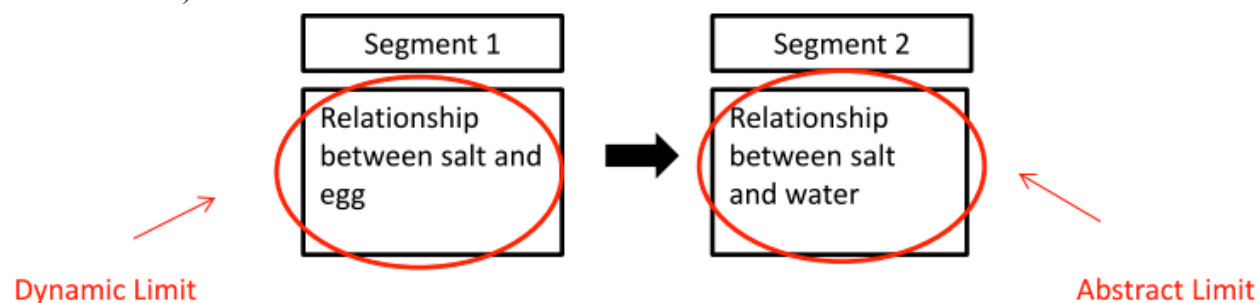


Figure 7. Prior knowledge cued during interaction between teacher and small group

*Segment 1: Relationship between salt and egg.*

1. Teacher: So you've gotten some result though// but did you actually measure exactly when that happened? <points to the floating egg>
2. Patricia: Yeah
3. Michelle: It happened at number 5

The teacher points to the floating egg and labels it as a *result*, asking if the group noticed exactly when it happened. She asks them if they can locate the moment the result occurred in order to expose the complexity of the boundary case. The floating didn't happen at one moment, in the way that the spaghetti bridge broke or the water overflowed the coin. Instead, it began to float gradually: rising higher and higher as more and more salt was added. This example was included in the unit to help students distinguish threshold from a direct relationship between cause and effect that results in gradual change. Patricia and Michelle seem to be orienting to the investigation in the same way as the teacher, as number 5 on Michelle's worksheet reads: *floats almost to the top*. Attention to the relationship between the addition of salt and the flotation of the egg makes the relationship between limit and a *reaction* post-phase salient, activating *dynamic limit*.

*Segment 2: Relationship between water and salt.*

5. Daniel: We need more salt
6. Teacher: I think// actually// well// I made more trials than we needed because I wasn't sure if people were going to measure salt in at the same rate// so don't worry about filling these out <points to empty slots in observation table> but do you see what's in the water at the bottom?
7. Michelle: Salt!
8. Daniel: Yeah it's water
9. Teacher: so do you think you can keep adding salt// do you think more salt can dissolve into that water?
10. Michelle: No
11. Teacher: Do you know what that's called what the water is right now?
12. Patricia: No
13. Teacher: It's called super-saturated and it means/
14. Michelle: / It can't take anymore
15. Teacher: Exactly// so it's saturated

Daniel is being playful when he suggests that the group needs more salt. The teacher explains that the students don't have to continue to add salt and fill out the blank slots in the observation table. Recognizing a chance to expose the second boundary example, the teacher turns the students' attention to the saturation of the water. One continues to add salt until the saturation point is reached, at which point no more salt will dissolve in the water. This example makes limit as a *terminal state* post-phase salient, activating *abstract limit*.

We will now examine how the two intuitive knowledge elements that emerged during this lesson influenced the development of Araceli and Patricia's conceptualizations of the threshold pattern in the context of the investigation. Following their conversation with the teacher, the students in the small group answered questions listed under the analysis section of the investigation worksheet. The first question asked students to "describe what you did and what you observed." Araceli appears to attend to the relationship between salt and egg, writing: "First we put water in a cup! Then we placed the egg in water. After we placed salt in water until we saw results. I saw that the egg was at the bottom then as we placed more salt it went up and up."

The first segment of her response is evidence that she is attending to pre-phase ("we placed salt in water"), followed by the reaching of a limit ("until") that triggers a *reaction* post-phase ("we saw results"). The second segment is evidence that she is attending to the relationship between the egg and added salt water. These segments taken together suggest that for Araceli, the relationship between the egg and added salt water has activated the prior knowledge element *dynamic limit*. She responds to questions about the similarity between this investigation and the spaghetti bridge and drops on a coin examples by noting similarities between the processes of doing the activities ("we repeated steps in both of them") and that both involved addition ("for both we had to add stuff like water in a coin and like salt in the egg"). This does not imply activation of either *dynamic limit* or *abstract limit*.

A look at Patricia's worksheet suggests that she has a different orientation to the activity, and instead has attended to the relationship between salt and water. To the first question Patricia responds: "I first dropped the egg in different densities of water according to 1-7 drops of salt. Then recorded results." A major difference between her description and Araceli's is that she does not indicate a causal connection between the addition of salt and the results, and she does not use the word *until* (which would be suggestive of *dynamic limit*). The fact that she notes "different densities of water" in her description suggests that she is at least attending to the state of the water, if not its degree of salt concentration. In her description of similarities between the investigation and earlier activities, she writes: "we had to see *how many drops an object would hold*." From this it appears that she is attending to limit as a terminal state post-phase, activating the prior knowledge element *abstract limit*.

Each girl attends to different characteristics of the investigation, thereby invoking different versions of intuitive knowledge about a limit. I will now trace how the activation of each version of limit moves each student's thinking toward a distinct version of the pattern by the end of the unit.

**Research question 2b.** I turn now to addressing the second sub-question of research question 2: How did resources contribute to individual students' construction of pattern models?

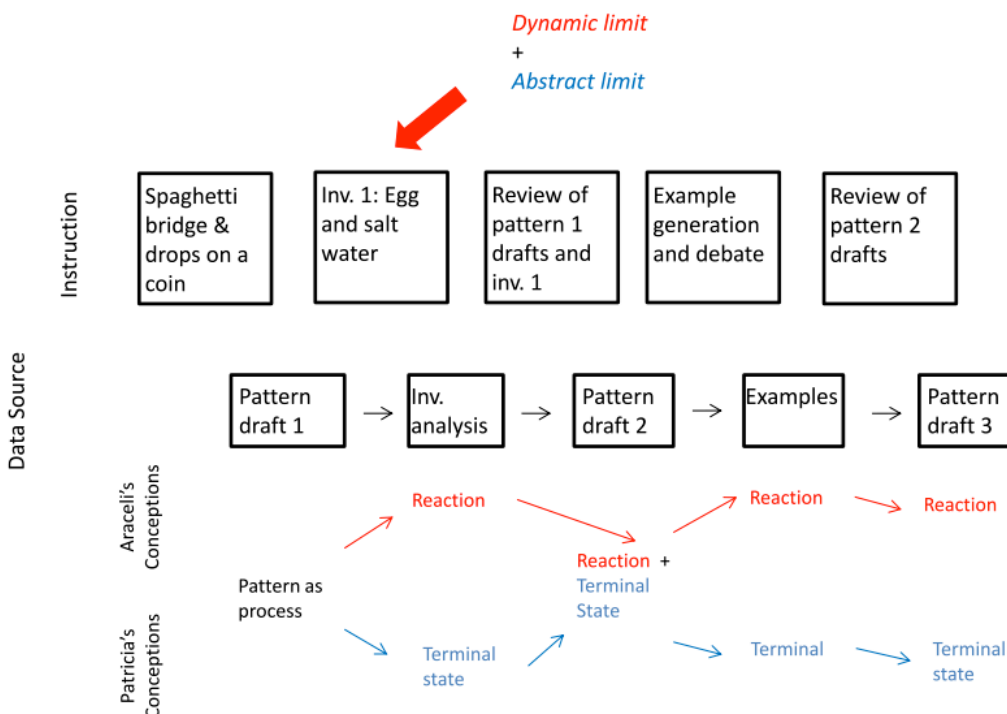


Figure 8. Instructional activities, data sources, and the development of Patricia and Araceli's models of threshold

***Tracing two students' construction of threshold over the unit.*** I present a map of Araceli and Patricia's development over the course of the unit (Figure 8); locating the emergence of the two intuitive versions of limit in relation to this map and tracking its influence on their construction of two different threshold models.

*Pattern draft 1.* The students wrote their first descriptions of the pattern following two activities that exemplified the threshold pattern. For the first activity the students added coins, one at a time, to a cup strung over the middle of the spaghetti bridge until the pasta snapped under the weight and the coins fell to the floor. For the second activity students added drops of water to the surface of a coin until the bead of water burst and spilled over the surface of the coin and onto the table. At the end of these two activities, the teacher asked the students to write their own descriptions of the pattern common to both examples. Araceli and Patricia worked in the same group for this activity. Students were encouraged to work with their group mates on the construction of the pattern description. Araceli and Patricia both named the pattern "getting more" and their descriptions were both focused on the material and social features of the activities, as opposed to the behavioral pattern underlying the phenomena they had explored. Both students listed common features. Araceli listed: "repeated a pattern, both involved household items, followed steps, needed techniques in order to do both of these experiments, both took patience." Patricia's list was much shorter: "both used pennies, had to have a special technique."

*Pattern draft 2.* From their second draft descriptions of the pattern model it is clear that both students are thinking of the pattern as having both pre and post-phases, however it is not clear whether they are thinking about the post-phase as characterized by a *reaction* or a *terminal state* (or both). The students wrote their second draft pattern descriptions following the

investigation with the salt water and egg. Araceli and Patricia were still working as a team and their descriptions are identical. As they had for their first draft, they named the pattern "adding more," this time describing the pattern as "they all involve adding something to get a *reaction* out of it. It mostly resulted in something *overflowing up* other objects." At this point it is clear that the girls' conceptions of the pattern include an action element from the pre-phase (i.e., "adding something") and a post-phase connected with the pre-phase (though not explicitly mediated by a limit). The post-phase element could be interpreted either as a reaction ("get a reaction out of it"), or a *terminal state* ("overflowing up other objects"), or both.

*Generating and debating examples.* Following these investigations, students worked individually and then in small groups to generate their own examples that followed the pattern. They listed their examples on posters that they shared with each other and then sorted 8 examples (chosen by the teacher) into two categories depending on whether or not they thought the examples followed the pattern. Students then participated in a whole class argumentation activity meant to help them decide whether or not 3 of those examples ("bothering someone until they burst," "getting a haircut," and "water freezing"), followed the pattern.

Araceli and Patricia were no longer working as a team for this activity and it is interesting to note differences in the way the two classified examples. Both students agreed that *getting a haircut* was an example, however their justifications imply their different conceptualization of post-phase at this point. Araceli wrote: "haircut does follow because you keep cutting until you have the haircut." This could be characterized as a repeated action pre-phase (*keep cutting*), limit (*until*), *reaction* post-phase (*you have the haircut*), if the outcome of a *haircut* is construed as a reaction. This suggests that Araceli is constructing her conception of the pattern on the basis of *dynamic limit*. Patricia, on the other hand, wrote: "getting a haircut (because the limit is going bald)." This suggests that Patricia sees the limit as connected with a *terminal state* post-phase and is constructing her conception of the pattern on the basis of *abstract limit*. This difference in the students' perspectives regarding the example of *getting a haircut* foreshadow the main difference between the girls' final descriptions of the pattern and is consistent with their different conceptualizations noted in my analysis of the students' comparisons of Investigation 1 with earlier activities.

Other differences between the girls' choice of examples included *low space on a device*. Araceli disagreed, Patricia agreed. Their differing response to the example makes sense if they are indeed conceptualizing the pattern in terms of *dynamic limit* and *abstract limit*, respectively. There is nothing that maps to a *reaction* post-phase in the example of *low space on a device*, so a student that conceptualizes the pattern on the basis of *dynamic limit* would not see it as an example of the pattern. The example does, however, have an apparent minimum limit (no space), making it a good example for a student conceptualizing the pattern on the basis of *abstract limit*. *Bothering someone until they burst* is an example to Araceli, but not to Patricia. If Araceli is building her conception on the basis of *dynamic limit*, this example is sensible, as *burst* is a clear *reaction* post-phase.

It is not necessarily obvious why this example does not fit Patricia's model of the pattern grounded on *abstract limit*, however her group mate's explanation during a whole class debate provides a clue. This student, publicly encouraged by Patricia during the debate to share his reasoning said: "Um// there's not really a maximum like you could keep annoying them// 'cause after they get burst you could still keep annoying them// it isn't// it won't// it's not going to get filled up on anything." This student appears to be arguing for a conceptualization of the pattern based on *abstract limit*: an example only represents his version of the pattern if it can reach a

*terminal state*. It is reasonable to assume that Patricia encouraged him to voice his reasoning because it represented her own position regarding the example.

There are a few examples for which the girls overlapped that are potentially puzzling. However, lacking explanation, they can be interpreted to support the frameworks that I have hypothesized the girls are operating within. They both agree with *stretching gum* and *engine overheating* and *when a bus fills up*. It is possible to imagine *gum* as having a *maximum* stretch length beyond which it can't be stretched, or as having a *reaction* when the limit of stretching is reached and it rips in two. An *engine overheating* could be interpreted as reaching a *terminal state*, beyond which it can't heat any more, or it could be imagined to *react* by bursting into flames, or quitting. A *bus filling up* could be a *terminal state* (beyond which another person cannot be fit into the bus), or as leading to a *reaction* post-phase (when you fill the bus beyond this point some reaction occurs such as a person falling out). This brings to mind both girls' draft 2 description "they all involve adding something to get a *reaction* out of it. It mostly resulted in something *overflowing up* other objects." For Araceli, it may have been that "resulted in something overflowing up other objects" may have been a *reaction* ("resulted" being the key linguistic indicator). Perhaps she even imagined that when you overfill something, it reacts by overflowing. For Patricia, it appears that "overflowing up other objects" may have been a terminal state: a point beyond which the system could not continue to be filled.

*Pattern draft 3*. For her final pattern description, Araceli wrote: "Maximum Capacity: You keep adding to an object until it breaks or burst." Her pattern structure clearly contains pre-phase (keep adding to an object") that leads to a limit ("until") that results in a *reaction* post-phase ("it breaks or burst"). Here it is clear that she is thinking of maximum capacity as the limit, that, when reached, causes the system to *react* and transition to a post-phase. This model of the pattern is consistent with evidence from her earlier work that suggests Araceli was constructing her model of the pattern on the basis of *dynamic limit*. The comic Araceli produced to illustrate her model of the pattern supports this interpretation of her model of threshold (Figure 9).

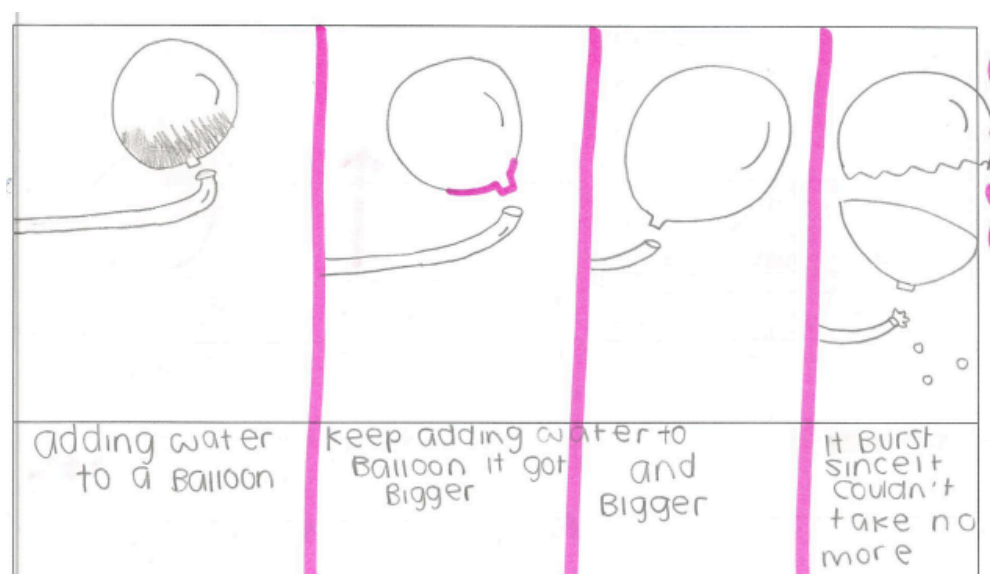


Figure 9. Araceli's comic of an example that follows her "Maximum Capacity" pattern

In contrast, Patricia appears to think of maximum capacity not as a limit that, when reached, results in a reaction, but rather, as a *terminal state*, beyond which the system cannot go. She writes "Maximum Capacity: Adding or taking something away till it reaches the maximum." "Till it reaches the maximum" implies that reaching the maximum is the final point of the process: the post-phase *is* the being at the maximum value. This model of the pattern is consistent with evidence from her earlier work that suggests Patricia was constructing her model of the pattern on the basis of *abstract limit*. The comic she produced to illustrate her conception of the pattern is further evidence of this interpretation. She illustrated her model of threshold with the example of a person getting a haircut (an example featured in the whole class debate). Her comic is presented below in Figure 10.

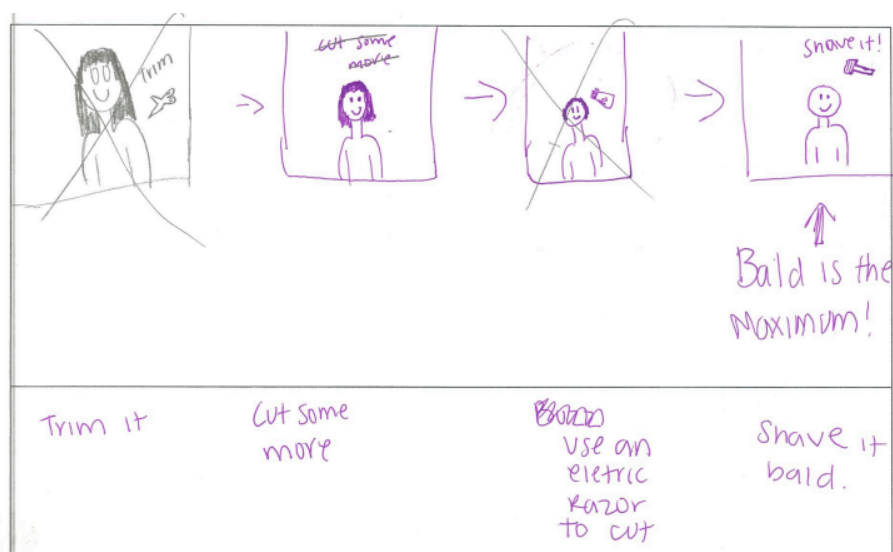


Figure 10. Patricia's comic of an example that follows her "Maximum Capacity" pattern

**Conclusion.** In summary, close examination of Araceli's and Patricia's developmental trajectories suggests that the intuitive knowledge elements *abstract limit* and *dynamic limit* played key roles in their construction of models of two different versions of the threshold pattern. Examination of other students' work indicates that these intuitive knowledge elements were not unique to the developmental trajectories of these two students.

#### Part IV. Features of Instruction

I turn now to addressing my third research question in the context of the threshold instructional unit.

3a. What aspects of the knowledge construction process emerged as important?

3b. What features of instruction supported important aspects of the knowledge construction process?

**Analytic approach.** I approached question 3 by examining case studies to identify aspects of the construction process that productively engaged prior knowledge and the features of instruction that supported these aspects.

In looking back at this iteration of the threshold unit through case study analyses, two general phases emerge as organizing aspects and activities that are important to the knowledge construction process. I refer to these phases as *activation* and *engagement*. During the first phase,

prior knowledge resources are activated. Activation depends on *context*, *attention*, and *orientation*. During the second phase, these resources are engaged in the construction of new knowledge through activities such as *articulating*, *combining*, *mapping*, *reinforcing*, *removing*, *generalizing*, *refining*, and *connecting*. In the threshold unit context, attention, and orientation played important roles during the activation phase, and articulating, mapping, reinforcing, removing and generalizing activities played important roles in the engagement phase. I will explain how each of these activities played an important role in the construction of threshold, and discuss the features of instruction that supported them, addressing both sub-questions of research question 3 together.

**Activation phase.** During this phase, resources are activated in the learner's mind. Which resources are activated depends on the context in which the learner is reasoning, what they attend to, and their orientation to the object of their attention. In the case of the threshold unit, the resources that are activated are *dynamic limit* and *abstract limit*.

**Context.** The two versions of limit appear to be activated for the two students, Araceli and Patricia, in the context of the same investigation. For the investigation the students observe what happens as they incrementally add salt to a cup of water in which a raw egg is submerged. The investigation exemplifies both dynamic and abstract limit and it is therefore possible to attend to one or the other, or both.

**Attention.** Within the context of the investigation, Araceli attends to the relationship between the egg and the added salt, while Patricia attends to the relationship between the water and the added salt. The worksheet is primarily structured to draw students' attention to the relationship between the egg and the added salt, and it explains the purpose of the investigation is "to see what happens to an egg placed in a cup of water as salt is dissolved in the water." It sends a bit of a mixed message, however, as it includes space for recording the appearance of the water in addition to the behavior of the egg, with each additional spoonful of salt. Through the observation stage of the experiment, most students in the class (except one group) have attended to the relationship between the egg and the added salt.

Between observation and analysis the teacher moves around the room pointing out the second example in the investigation, the saturation of the water with salt. It appears that while Araceli stayed focused on the relationship between the egg and the added salt, this move of the teacher encouraged Patricia to attend to the relationship between the water and the added salt. The different relationships to which they attend activate, for the two girls, different elements of prior knowledge. For Araceli, who attended to the relationship between the egg and the added salt ("we placed salt in water until we saw results. I saw that the egg was at the bottom then as we placed more salt it went up and up") *dynamic limit* is activated. For Patricia, who attended to the relationship between the water and the added salt ("we had to see how many drops an object would hold"), *abstract limit* is activated.

**Orientation.** Having each activated a different intuitive version of limit, each girl orients to subsequent examples through the lens of either *dynamic* or *abstract limit*, and consequently attends to the features of those examples that agree with their model of the pattern. A prime example of this is how both girls see *getting a haircut* as an example of their own particular version of the pattern. Araceli sees it as maximum capacity in which reaching a limit causes a *reaction* (*dynamic limit*). Patricia, on the other hand, sees it as maximum capacity in which a limit that is a *terminal state* is reached (*abstract limit*). Each girl's orientation with respect to future examples appears to be a result of their initial take on the pattern. It is probably generally

true that the first exemplars with which the students work leave a lasting impression of the pattern that influences their orientation to future examples.

Through this analysis, we see that a number of features of instruction supported the activation of both *dynamic* and *abstract limit*, including lesson content, worksheet design, and teacher moves. I will now discuss the activities of the engagement phase and the features of instruction that supported those activities.

**Engagement phase.** During this phase, the learner engages resources in the construction of patterns models. In the case of the threshold unit, *articulating*, *mapping*, *reinforcing*, *removing* and *generalizing* activities appear to have played important roles in the construction process.

**Articulating.** It is possible that the need for articulation, in fact, activates resources. At the end of the egg investigation, students were asked to compare and contrast the behavior they observed with that of the previous (spaghetti bridge and drops on a coin) activities. Araceli noted the “repeating steps,” and “adding stuff,” as similarities, neither of which appear to be related to the activation of *dynamic limit*. Patricia, on the other hand, noted that for all activities “we had to see how many drops an object would hold,” which does appear to be related to the activation of *abstract limit*.

**Mapping.** The process of articulating similarities between examples is a process of mapping. This is a part of the knowledge construction process that is particular to the construction of *pattern knowledge*. Patterns are general structures of behavior or process that can be found in many examples. To identify a pattern, students must identify the general structure of behavior or process that is common to two or more examples. Engaging students in mapping explicit connections between examples scaffolds their identification of the common behavior or process, helping them to generalize their understanding of the underlying concept. The worksheet specifically focused students’ attention on similarities between examples through prompts in the final questions of the analysis section. This scaffolded students’ explicit mapping between examples and facilitated their identification of the pattern.

**Reinforcing.** As mentioned in the discussion of *orientation*, the two case students appeared to see subsequent examples each through the lens of their own model of the pattern. Examples such as *getting a haircut*, therefore, tended to reinforce both models equally. Pattern modeling was framed as a student-driven activity, the goal of which was to create a model that represented the pattern as each student saw it. Students were therefore encouraged to follow their own view of the pattern and argue for or against examples. It didn’t matter which side they took as long as they supported their reasoning.

**Removing.** At the writing of their second draft, both Araceli and Patricia, who worked as partners, wrote identical descriptions of their pattern model, writing: “they all involve adding something to get a reaction out of it. It mostly resulted in something overflowing up other objects.” This appears to be a combination of both versions of threshold in sequence: the first part conveying a sense of threshold as reaching a limit and resulting in a *reaction*; the second part conveying a sense of threshold as reaching a limit that is a *terminal state*. It is possible that the two girls each contributed their own version of limit to this description and didn’t really resonate with the other half. They wrote their third draft of the pattern separately and each removed the part of draft 2 (that was possibly their partner’s contribution) that did not match their own conceptualization of the pattern. Providing multiple opportunities for revision supports *removing*. Because the students had time to think about the pattern and make revisions to their drafts, they were able to improve their drafts incrementally, as their conceptualizations of the pattern became clearer over time.



**Generalizing.** As shown in Part II, most of the students' pattern descriptions began as bound to specific contexts and then over the course of the 3 drafts, they became domain-general. This movement was probably influenced by the remark made by one student, during a review of prior drafts of pattern descriptions. The teacher was reading through previously written descriptions that she had projected onto the front board. One of the descriptions read "kept adding until it was destroyed." One student in the class responded to this, saying "destroyed is the wrong word." Asked by the teacher to explain what he meant the student clarified "they should have said change, because that would be more general." As the class had already talked about the meaning of general and the importance of generality to patterns, this remark provided a way for students to operationalize the value for generality through the adaptation of language.

The emergence of this remark was supported by two aspects of instruction. The first was that the teacher had explicitly taught the students about what it meant for a pattern to be general, introducing this to them through a lesson on what it meant to be a vampire in general, vs. what it meant to be a specific vampire. The second aspect of instruction that afforded this remark was that the teacher had created an opportunity for critique by framing the review as such, inviting students to comment on each other's descriptions (that were presented anonymously), and explicitly modeling the critique of descriptions for them.

Through this analysis, we see that a number of features of instruction facilitated the engagement of *dynamic limit* and *abstract limit* in students' construction of the threshold pattern. These include worksheet design, giving students agency, creating opportunities for revision, being explicit about values, creating opportunities for critique, and modeling productive engagement in a particular activity structure.

## Section 5.2: Equilibration Analysis

I begin this section by describing a model of the target equilibration pattern and my design of the equilibration instructional unit (Part I). As I describe in my introduction to Chapter 5, I address my first research question by presenting a high-level sketch of the general development of students' equilibration models using the results of a macro analysis of students' written work (Part II). I then address my second research question and present a detailed picture of the role of prior knowledge in one student's construction of a model of equilibration, using the results of microgenetic analysis of written work, video transcripts, and field notes (Part III). I end by addressing my third research question, presenting the important elements of the equilibration model construction process, and the features of instruction that supported this process (Part IV).

### Part I. Introduction to the Equilibration Pattern and Instructional Unit

**Equilibration pattern.** A model of equilibration that could be used as a benchmark for characterizing students' models was crafted by our research team through the same combined bottom-up (data driven) and top-down (scientific model driven) approach that we had used for the threshold pattern. We created a coding scheme out of the elements of a scientific model that were included in student models (*difference drives rate*) and fine-tuned those elements to be consistent with the intuitive sense conveyed in student models (e.g., using language like *far away* to mean a large difference). The resulting model characterizes equilibration as a process that occurs when two systems of different intensive quantities, such as density or temperature, are put in contact. The two systems equilibrate at a rate that is directly proportional to the difference in measure between the intensive quantities of the two equilibrating systems. When the difference between the two systems is very large, their rates of equilibration are also very large. As the systems equilibrate, their difference decreases and so do their rates of equilibration. When the systems have reached equilibrium, there is no difference and their rates of equilibration are equal to zero. The tendency for systems to equilibrate at a rate proportional to their difference can be summarized as *difference drives rate*. It is this *difference drives rate* characterization of equilibration that is the target model for the instructional unit.

**Equilibration instructional unit.** The instructional unit was designed to support students' construction of models of equilibration through the exploration, generation, and critique of examples. The same general sequence of activities of the threshold unit was used in the equilibration unit. The rationale for this general sequence of activities is explained in detail in Chapter 4. The unit was comprised of 7 core activities and ran for approximately 20 instructional hours. The sequence of core activities is presented in Figure 11 and described below.

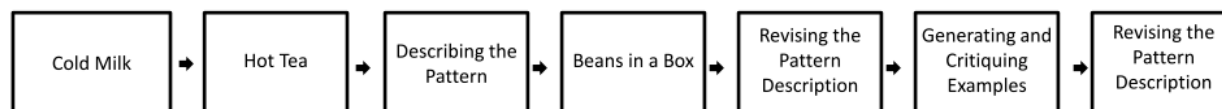


Figure 11. Sequence of core activities of the equilibration instructional unit

**Core activity 1: Cold milk.** The unit opened with the investigation of an exemplar of the difference drives rate pattern: the equilibration of a glass of cold milk with a warm room. Measuring and graphing the liquid's temperature over time revealed a pattern of thermal equilibration that could be explained by difference drives rate. At the start, the temperature of the liquid is furthest from room temperature and it is observed to warm at the greatest rate. As it

warms, the distance between its own temperature and the temperature of the room decreases and it is observed to warm at a progressively slower rate until it reaches room temperature. Five hours and 20 minutes of instructional time were devoted to students' generation and presentation of hypotheses, running the experiment, interpreting the data, and generating theories to explain the observed rate of temperature change. For each of these components of the investigation, students were asked to attend to the *rate* of temperature change over time. In addition to demonstrating the equilibration pattern, the activity was selected because it was known, from previous implementations of pattern-based curriculum, to work well in eliciting resources for the construction of the scientific model of thermal equilibration (Newton's law of warming) and to be highly engaging (see diSessa 2014 for a detailed treatment of one student's construction of Newton's law of warming in response to an inquiry into the thermal equilibration of cold milk).

**Core activity 2: Hot tea.** Following their investigation of the warming process, students explored the cooling process in the context of a glass of hot tea left to sit in the classroom. Measuring and graphing the liquid's temperature over time revealed, as for Investigation 1, a pattern of temperature change that matches the equilibration pattern. Two hours and 40 minutes of instructional time were devoted to the same sequence of activities as the first investigation: generating and presenting hypotheses, running the experiment, interpreting the data, and building theories to explain the changing rate of the liquid's thermal equilibration with the room. As in the case of the first investigation, students were asked to attend to the *rate* of temperature change over time. The two investigations were identical in terms of their general sequence; their difference in duration can be explained as a result of the students' increased familiarity with activity and participation structures. In addition to its clear demonstration of the equilibration pattern, this activity was predicted to be highly engaging, as during previous implementations of the thermal equilibration curriculum students had been passionately committed to opposite hypotheses about whether or not cooling processes worked in the same way as warming processes.

**Core activity 3: Describing the pattern.** Following the two thermal equilibration investigations the teacher led students in a whole class discussion to identify the aspects common to both warming and cooling processes. The teacher then connected students' ideas to Newton's law of warming (or cooling) that described the rate of temperature change as directly proportional to difference between equilibrating objects. The students were then asked to name and describe the pattern followed by both warming and cooling, and model it using a flowchart. These activities scaffolded students' identification of deeper structural similarities in behavior between the cold milk and hot tea examples. The opportunity to name, describe, and flowchart the pattern allowed them to articulate their initial conceptualizations and produce linguistic and visual artifacts of their thinking for further consideration and refinement. The total instructional time allocated to these activities was 1 hour and 20 minutes.

**Core activity 4: Beans in a box.** For the final investigation of the unit, students explored the equilibration process through another exemplar: the diffusion of beans across a semi-permeable boundary in a box. This particular activity was selected because the *difference* between the number of beans on either side of the box could be inferred to *drive the rate* of the redistribution of beans on either side of the box. The activity was also known, from previous implementations of pattern-based curriculum, to work well in eliciting resources for the construction of the target model of equilibration because it made both the balance of empty space and the density of beans particularly salient. For the activity students documented their observations and constructed a graph that could help them learn about the rate of bean diffusion

over time. A partitioned box was filled on one side with two tablespoons of dried beans. Students shook the box back and forth along the table, in the direction perpendicular to the partition in the box. As they shook the box, beans passed through a small gap in the middle of the partition. The students recorded the number of beans on the initially empty side of the box every 10 shakes and then graphed the total number of beans on that side over time. Their resulting graphs showed the same curve as the two previous investigations, as the bean diffusion process follows the equilibration pattern. Two hours and 40 minutes of instructional time were spread across generating hypotheses, running the experiment, interpreting the data, and building theories to explain the changing rate of bean diffusion over time.

**Core activity 5: Revising the pattern description.** Following the third investigation, the students revised their pattern names, descriptions, and models. Directly preceding this, the teacher presented the students with a review of their previous pattern descriptions. It was thought that students' exploration of the bean exemplar, along with their exposure to their peers' pattern models, would influence their conceptualizations of the pattern. The opportunity to revise would give them a chance to demonstrate the refinement in their thinking. The total instructional time allocated to these activities was 1 hour and 20 minutes.

**Core activity 6: Generating and critiquing examples.** Having refined their pattern models, students worked in small groups to generate lists of their own examples that followed the same general pattern as the warming and cooling of a liquid and the diffusion of beans. The students generated a number of examples that ranged from physical (e.g., “a car slowing to a stoplight;” “rainfall in a storm;” “lava exiting a volcano”), to psychophysical domains (e.g., “drinking more when you are thirsty and slowing down as your thirst is quenched;” “having strong emotions just after an event and becoming less emotional as time goes by;” “spending money quickly when you have it and slowing down as you run out”). The small groups created posters to share their examples with the rest of the class. They reviewed each other's posters, using post-its as they had in the previous unit to indicate whether or not the examples of their peers made sense to them.

The teacher used examples over which students disagreed as the basis of an activity that engaged students in argumentation. They provided points that either supported or challenged each of 10 problematic examples, and shared their points with the rest of the class for 2 examples: emotions that are strong at the beginning and fade away over time, and the movie *The Hunger Games* for which the number of children in a competition decreases quickly at the start and then more slowly as fewer children remain in the competition. As in the case of the threshold unit, engaging students in the generation and critique of examples was meant to help them think carefully about the key characteristics of the pattern and to articulate their ideas as completely and as clearly as possible. Examples generated by students and the arguments they produced in favor of or against examples could also expose differences in their conceptualizations of the pattern. Two hours and 40 minutes of instructional time were allocated to these activities.

**Core activity 7: Revising the pattern description.** The final hour and 20 minutes of instructional time were allocated to writing final drafts of the pattern. The teacher introduced the activity with a review that surveyed previous pattern descriptions, names, and flowcharts. Students were asked to illustrate an example of the pattern to go with their final drafts. As in the case of the threshold unit, it was thought that students' conceptualizations of the pattern might have been productively influenced by their consideration of additional examples and by hearing their peers' arguments in favor of and against examples. Offering students a final chance to

revise their models gave them the opportunity to demonstrate their further refinement in thinking.

## Part II. General Tendencies in Development

I turn now to addressing the first research question of my study in the context of the equilibration instructional unit.

1a. What were general tendencies in the development of students' pattern models with respect to the target *model of the pattern*?

1b. What were general tendencies in the development of students' pattern models as *domain-general models*?

**Analytic approach.** I approached question 1 with a macro-analytic strategy. I developed coding schemes for characterizing written descriptions of equilibration and used these to code students' initial and subsequent description drafts. I present coding outcomes graphically to show whole class tendencies in each pattern draft. I compare proportions across drafts and use basic statistics to locate significant shifts in whole class tendencies, which I present as general developmental trajectories.

**Research question 1a.** I begin by addressing the first sub-question of research question 1: What were general tendencies in the development of students' pattern models with respect to the *target model of the pattern*?

**Coding scheme.** I designed the coding scheme below to characterize students' descriptions in terms of the target model of equilibration. The first part of the coding scheme (Table 7) describes the *elements* of equilibration that students included in their written descriptions. The second half of the coding scheme (Table 8) lists sets of related elements that were the basic structures underlying students' patterns. These pattern structures contain one or more elements linked together by causal arrows. As shown in Table 8, pattern structures range from single elements to more complex chains of elements. The structures are ordered vertically according to level of sophistication and assigned rank scores. Beginning with a characterization of the behavior alone, the structures add explanations for the behavior that move from non-normative to both non-normative and normative, and finally to the normative explanation alone.

Table 7. Equilibration elements coding scheme

Element	Description	Example
Change in Rate of Change	Something is changing at a decreasing rate.	"The pattern must go fast and then it goes slow and stops like the hot tea and cold milk." "Slows down to reach equilibrium..."
Final State	There is a final state something is changing toward. The word "stop" counts as a final state.	"Slows down b/c reaching equilibrium." "It starts out fast then slows down and eventually stops."
Difference	There is a difference between two states (between initial and final states, or subsequent and final states).	"In the beginning it goes fast -- because there is a bigger difference and more space to cover..."

Table 8. Equilibration structures coding scheme

Pattern Structure	Description	Example	Score
Other	The description does not have one of the structures listed.	They go in the way of diffusion and try to reach equilibrium...	0
Change in Rate of Change	Something is changing at a decreasing rate.	"Fast → slow, starts off fast then it starts to slow down."	1
Final State → Rate of Change	The rate of change of state is driven by the final state.	"It was changing fast at first and then slows because it is reaching equilibrium."	2
(Final State + Difference) → Rate of Change	The rate of change of state is driven by both final state and difference between earlier and final states.	"Goes fast (fast b/c it has a lot to cover) → slows down (slows down b/c it's arriving to destination) → stops (can't go anymore)."	3
Difference → Rate of Change	The rate of change of state is driven by the difference between earlier and final states.	"There is a larger distance so it goes fast. There is less space so it goes slow/no more space so it stops."	4

**Coding.** These coding schemes were used together to characterize students' initial and later written drafts of students' model descriptions in terms of underlying structure. As in the case of the threshold analysis, a team of 3 researchers (comprised of myself and two undergraduate assistants) coded each of the three drafts independently and then convened to compare scores, resolve discrepancies, and reach consensus.

**Results.** Coding outcomes are presented in the bar chart below (Figure 12) to show aggregate tendencies in pattern models for each draft. Comparison of proportions across drafts and a test of statistical significance provide evidence for general developmental trajectories.

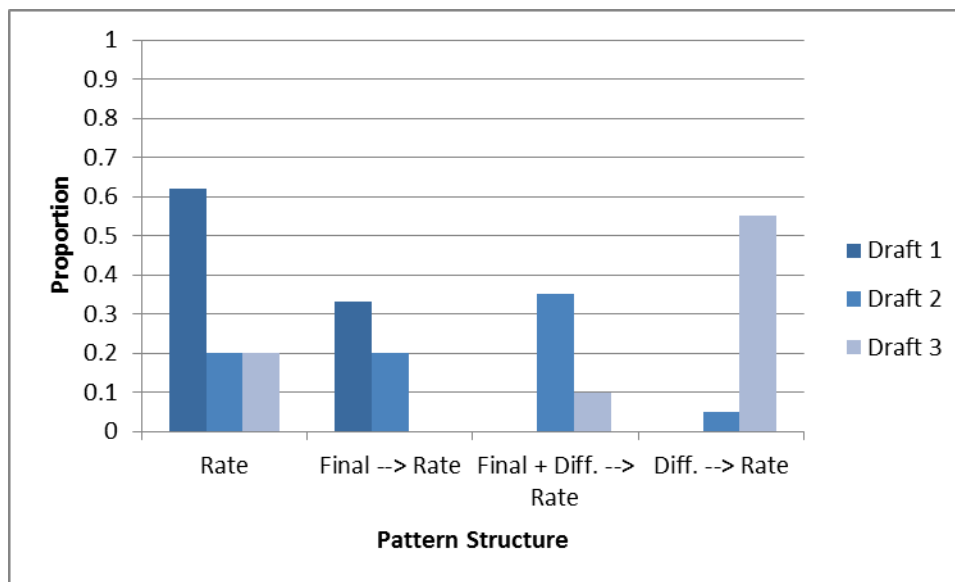


Figure 12. Characterization and comparison of equilibration structures

*General tendencies in development.* As evident in Figure 12, for the first draft, most students (62%) characterized the pattern in terms of the behavior alone (e.g., "the pattern goes fast and then slow"). About a third of the students included the non-normative explanation *final state* (e.g., "it goes slow to reach room temperature"). No students explain the behavior using the normative idea of *difference*. By the second draft, 55% of the descriptions have added the non-normative explanation *final state*, however, that 55% can be broken into 20% *final state* and 35% *final state* + *difference*. This means that 35% of the descriptions containing the non-normative *final state* explanation also include the normative *difference* explanation (e.g., "there is a long distance so it goes fast. Then it slows down to stop"). The proportion of descriptions including only the normative *difference* explanation is 5%. By draft 3, there are no descriptions that explain the fast to slow behavior *solely* in terms of the non-normative *final state* idea. Ten percent of the descriptions *do* include *final state* paired with *difference*. Fifty-five percent of the descriptions include *only* the normative explanation of *difference* (e.g., "the space is greater at first which makes it go fast, then it slows down as the amount of space decreases").

Comparison of tendencies in pattern structures across drafts suggests movement away from a characterization of equilibration that is solely behavioral and toward one that offers an explanation for the behavior. Within this, there is movement from solely non-normative, to both non-normative and normative, to solely normative explanation. Both the move to explain the behavior and the shift from non-normative to normative explanations are indicators that students, on the whole, are developing conceptions of equilibration that are closer to the target characterization of equilibration as *difference drives rate* over the course of the unit. Comparison of draft 1 and draft 3 class means for rank score show that the average level of sophistication for the class increased by 1.4 points from 1.3 (SD = .56) at draft 1 to 2.7 (SD = 1.61) at draft 3 (Figure 13).

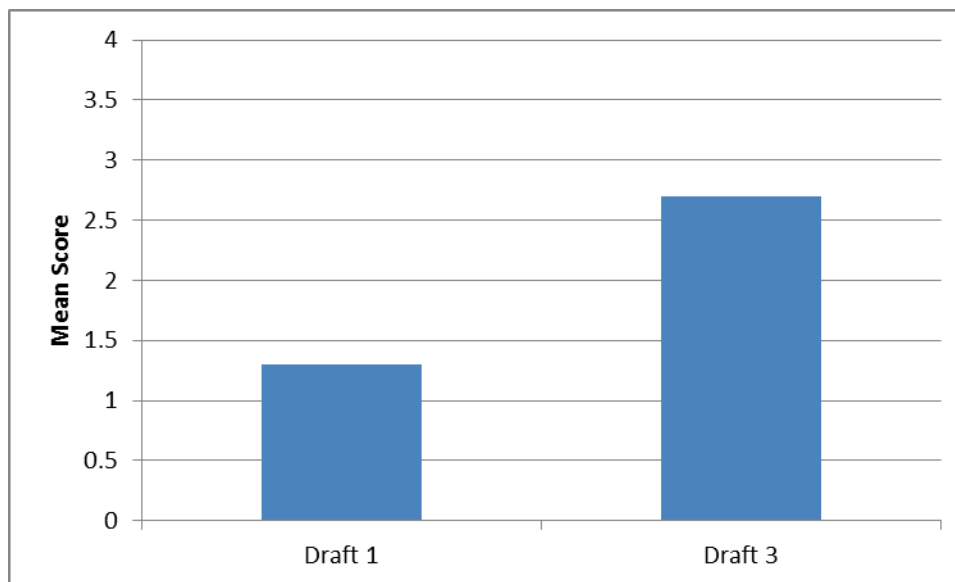


Figure 13. Mean scores for draft 1 and draft 3 student equilibration models

A basic statistical test was used to ascertain the probability that the equilibration unit activities played a role in this development. In order to test the hypothesis that the unit activities did not have any effect on students' development of equilibration models, their rank scores for first and final draft descriptions were compared using a paired difference test. Because rank scores (a discrete measure) were compared, and because the sample size was quite small and the data sets were not normally distributed, a non-parametric statistical hypothesis test was needed. The Wilcoxin signed-rank test is a hypothesis test that can be used for non-parametric data sets. Using this test to compare first and final draft scores showed that these gains were statistically significant ( $\alpha = .001$ ). This suggests that instruction played an important role in the development of students' models of equilibration.

**Research question 1b.** I turn now to addressing the second sub-question of research question 1: What were general tendencies in the development of students' pattern models as *domain-general models*?

**Coding scheme.** I designed the coding scheme presented below (Table 9) to evaluate whether or not students' descriptions were domain-general. This coding scheme is used to classify descriptions as *general*, *specific*, or *other*.

Table 9. Domain-general coding scheme

Characterization	Description	Example
General	The description is not embedded within one or multiple specific contexts.	"It starts out fast and then slows down."
Specific	Context or specific examples are given.	"...slows down as it reaches room temperature."
Other	The student is articulating a pattern made of elements that are not captured by the coding scheme.	"They go in the way of diffusion and try to reach equilibrium..."



**Coding.** This coding scheme was used to characterize students' initial and subsequent written drafts of pattern descriptions. As in the case of the coding presented earlier, the team of 3 researchers coded every written description independently and then convened to compare scores, resolve discrepancies, and reach consensus.

**Results.** Coding outcomes are presented in the bar chart below to show aggregate tendencies in the domain-generality of each draft. Comparison of proportions across drafts show shifts in aggregate tendencies, which are presented as general developmental trajectories.

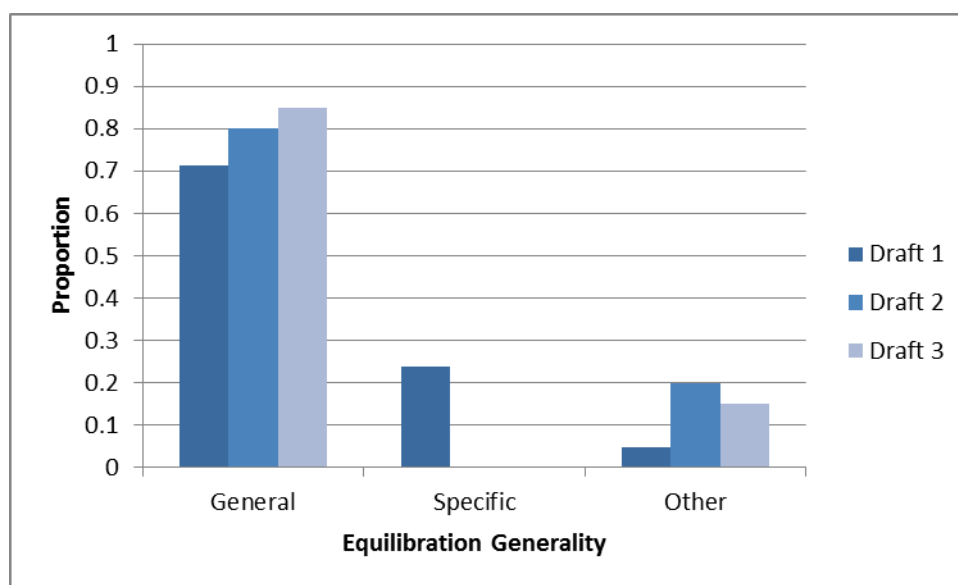


Figure 14. Proportion of descriptions coded as general, specific, or other

*General tendencies in development.* Comparison of proportions across drafts shows that students are crafting increasingly domain-general descriptions of equilibration over the course of the unit. A comparison of this graph (Figure 14) with the graph (Figure 5) of the previous unit on threshold, suggests that students are beginning to get a sense for *pattern* as a domain-general construct.

**Conclusion.** The unit was designed to elicit and engage students' prior knowledge in their construction of equilibration models. Instruction was primarily student-centered, and though it cannot be denied that the teacher played an important role, students were given a great deal of agency in crafting their own pattern descriptions. Over half the class developed descriptions of equilibration that mapped directly to the target characterization of equilibration as *difference drives rate*. This suggests that instruction designed to leverage prior knowledge can be successful, and supports the hypothesis that prior knowledge can be leveraged toward students' construction of scientifically powerful, domain-general knowledge. Closer analysis of the events within the instructional unit suggests that a particular element of intuitive knowledge played an important role in leading students to the difference drives rate characterization of equilibration. This analysis is presented next.

### Part III. Productivity of Prior Knowledge

I turn now to addressing the second research question of my study in the context of the equilibration instructional unit.

2a. What prior conceptions emerged as resources for students' construction of pattern models?

2b. How did resources contribute to individual students' construction of pattern models?

**Analytic approach.** I approached question 2 through microgenetic case study analysis. I selected a student whose shifts across drafts matched the general developmental trajectory of the class illuminated in Part II. I examined their written classwork to identify major shifts in thinking and to look for connections between those shifts and particular episodes during which productive prior knowledge emerged. I used written work, video transcripts, teacher reflections, and researcher field notes to track how prior knowledge contributed to the development of the case student's construction of equilibration over the unit. The general trajectory of my presentation of this analysis will be to locate the emergence of key prior knowledge and then track how it is taken up and built upon productively by one student in his construction of a model of equilibration.

**Research question 2a.** I begin by addressing the first sub-question of research question 2: What prior conceptions emerged as resources for students' construction of pattern models? I introduce the key prior knowledge and locate its emergence in a group discussion that occurred early in the unit. I then conduct a microgenetic analysis of the discussion and schematize the development of the key prior knowledge.

**Key prior knowledge.** Prior research on students' construction of equilibration in a thermal context showed that a well-documented intuitive knowledge element, *Ohm's p-prim* (*greater effort begets greater result*), played a productive role (see diSessa 2014 for a detailed treatment of this case). In this case, the focal student applied an agentive frame to a glass of cold milk to explain its equilibration curve and invoked *Ohm's p-prim*, leading to their construction of equilibration as difference drives rate. The student reasons that the cold milk is "freaked out" by the temperature difference and therefore works hard to warm to the room's temperature. This high level of effort causes the milk to warm up quickly. As it reaches room temperature, the milk becomes less "freaked out" and doesn't work as hard. This lower level of effort causes the milk to warm more slowly. The difference in temperature drives the level of effort of the milk, which in turn drives the rate of temperature change. The connection between level of effort and resulting rate is essentially *Ohm's p-prim*. The *causal scheme*, simplifies over time from *difference drives effort drives rate* to *difference drives rate*, as the anthropomorphic agentive language falls away.

The case presented below shows how the same p-prim played a key role in one student's construction of equilibration both as a thermal phenomenon and as a general pattern. The lesson during which *Ohm's p-prim* was initially invoked is in fact very similar in design to the one during which the "freaking out" episode (described above) occurred. A look across the data reveals that following this lesson, many student explanations began to use agentive language (suggestive of *Ohm's p-prim*) to describe equilibration as difference drives rate. Over the rest of the unit agentive language was gradually replaced by normative explanations for equilibration as versions of difference drives rate.

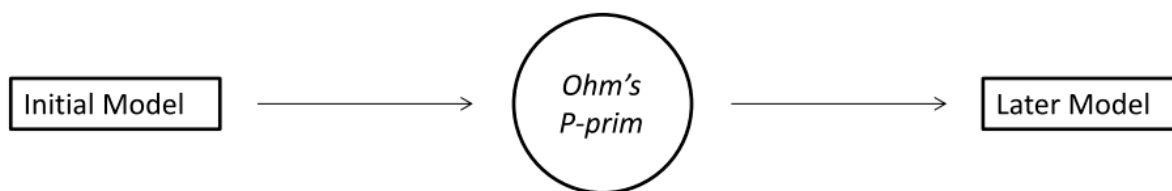


Figure 15. Basic trajectory showing the role of *Ohm's p-prim* in students' development of equilibration<sup>2</sup>

***Lesson during which key prior knowledge emerged.*** The lesson during which *Ohm's p-prim* first emerged was a whole class discussion following an investigation of the temperature change of a glass of cold milk as it warmed to room temperature. The discussion was facilitated by the teacher and focused on eliciting students' ideas about why the temperature of the cold milk followed a particular curve (Figure 16), warming fastest at the start and progressively slower over time. The teacher opened the discussion by reading aloud explanations that students had written at the end of the previous class. Two of the students wrote about the milk having to adjust quickly to the room temperature at the start - these explanations seem the most similar to the "freaking out" explanation. About half of the remaining students appealed to various mechanisms involving either molecules or the experimental apparatus. The other half had explained that the temperature change slowed because the milk was about to reach room temperature. Two of these explanations explicitly compared the milk's temperature change with running a race. The teacher asked the students to respond to one of these explanations that said: "because it was getting to room temperature at the end so it was slowing down. It's like a race, when you're getting to the destination you start to slow down." A lively discussion ensued, during which students' language became agentive, the milk's behavior became characterized as goal-directed, and *Ohm's p-prim* appeared to be invoked and used to explain the warming phenomenon.

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<sup>2</sup> It should be noted that the model in Figure 15 simply represents a particular p-prim that can be used as a resource for the construction of a model of equilibration. It is not meant to represent the complexity of the model construction process, which likely involves other elements of knowledge and might look more like fitting together pieces of a complicated puzzle.

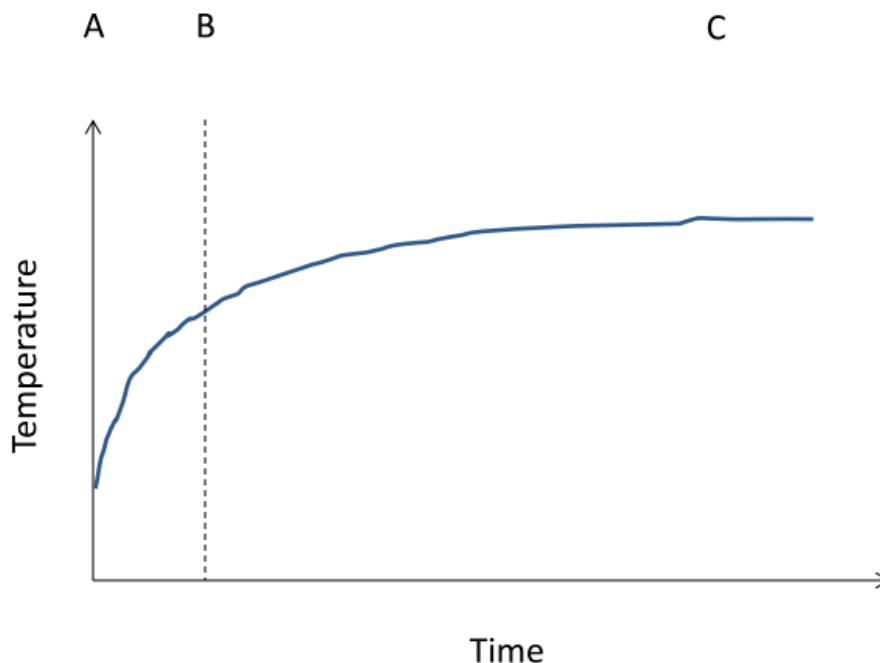


Figure 16. Approximate graph of temperature over time for cold milk equilibrating with a warm room

The discussion has been parsed into four segments and presented below. The four segments correspond to stages in the development of an explanation for the cause of the fast to slow behavior. In Segment 1, students focus on the *danger of crashing into a wall* as the cause of the slowing behavior. In Segment 2, the teacher asks students why the temperature changes quickly at the start and two main explanations arise – *space to run safely* and *energy*. In Segment 3, reaching the endpoint emerges as an explicit *goal* that drives the behavior at the start of the curve. Finally, the *relationship between the goal and the space between start and finish* is seen to drive the behavior. This is where *Ohm's p-prim* is first identified and seen to shift the explanation for equilibration in the direction of *difference drives rate*. The sequential development of thermal equilibration that occurred during the class discussion is represented by the flowchart below (Figure 17) and examined more carefully with reference to transcript from video.

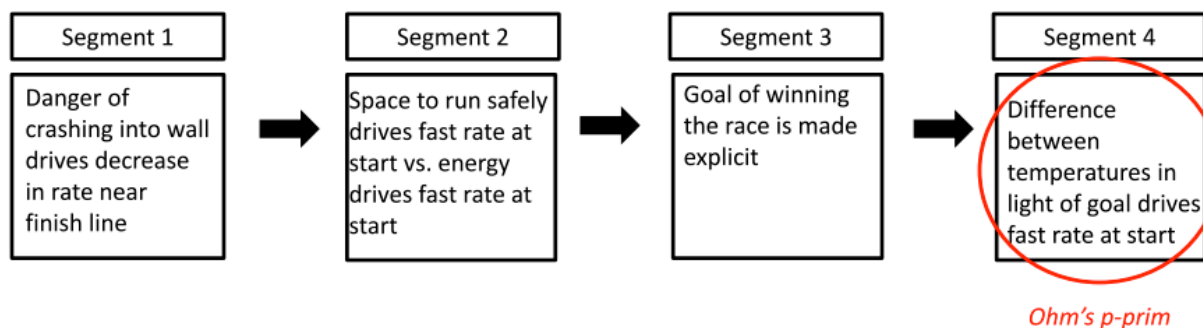


Figure 17. Sequential development of thermal equilibration conceptions over the course of the class discussion

*Segment 1: Danger of crashing into wall drives decrease in rate near finish line.*

Segment 1 opens as the teacher finishes reading the explanation that compares the milk's temperature change to running a race.

1. Leo: um//why would you slow down when you're about to finish a race? It doesn't make sense
2. Teacher: does anyone want to try and make a guess why?
3. Alvaro: say there's a wall// are you going to run straight into it Leo?
4. Leo: well I'm not gonna go slower though// because then I'll lose
5. Alvaro: like no no no no no like// say you're winning cause you're going as fast as you can// then when you're gonna reach the wall// don't you start to like <stomps feet on the ground> kinda/
6. Michelle: /is there a wall?
7. Alvaro: yes there's a wall
8. Michelle: there is no wall
9. Alvaro: there is a wall
10. Michelle: where?
11. Teacher: Alvaro, what would the wall be in the case of the milk warming up?
12. Alvaro: the room temperature// huh? huh Michelle what?  
<The teacher asks Alvaro to repeat what he has said for the whole class to hear>
13. Alvaro: this time when you're running// you're racing// you run as fast as you can// but then you're gonna go// you're going to hit a wall so you start to like slow down  
<Disruptions>
14. Alvaro: you have to slow down to stop

Leo's contribution (1) questions the race analogy on the grounds that it is not sensible to slow down at the end of a race. Indeed, competitive runners are trained to run as fast as they can *through* the finish line, in order to ensure their best possible time. Alvaro clarifies that the finish line in this case is a wall (3) and it is therefore prudent to slow down (5). Michelle accepts the idea of a race ending at a wall but questions its fit with the temperature scenario (6, 8), asking where the wall is in the case of the cold milk (10). Alvaro connects the wall to room temperature: the finish line of the cold milk's race (12).

It should be noted that all 3 students are making productive moves in the discussion. Both Leo and Michelle are being sensible in questioning the fit of the race analogy to the case of the milk's temperature change. In addition to the questions they raise, the analogy is a bit bizarre considering that nothing is actually moving forward in the case of increasing temperature. Despite shortcomings, the race analogy and the idea of slowing to a wall both play productive roles in the path to difference drives rate, as we will see.

Following this segment of transcript, the teacher asks Alvaro to repeat his explanation for the whole class and he does so using the walls of the school playground to create a familiar context for the race analogy. He emphasizes that you race as fast as you can but then you slow down before you hit the wall to avoid crashing into it. Between this and contributions 13 and 14 of Segment 1, Alvaro has offered an explicit explanation for the second half of the equilibration curve (segment BC, Figure 16): that temperature slows down to a stop in order to avoid an uncomfortable collision with the wall that is room temperature. At this point in the discussion it appears that the *danger of crashing into a wall drives the behavior* of the milk's temperature.<sup>3</sup>

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<sup>3</sup> It is worth pointing out that *slowing to a stop* is a strongly intuitive explanation, and though it has not been previously documented, it is a candidate for intuitive knowledge roughly the grain

Having elicited an explanation for the behavior shown in the second half of the equilibration curve, the teacher asks students to attend to the first half of the curve.

*Segment 2: Space to run safely drives fast rate at start vs. energy drives fast rate at start.*

15. Teacher: Let me ask you this: what if we need to do the other half of that - going really fast at the start? Why would the water start warming up really fast at the start?

16. Leo: because at the start you have a lot of energy to run so you run

17. Teacher: Oh cool// so based on the running analogy// because at the start you have a lot of/

18. Alvaro: /no

19. Teacher: /energy/

20. Alvaro: /I disagree/

21. Teacher: /to run// and Alvaro do you want to add a different reason?

22. Alvaro: yeah

23. Teacher: Ok// why's that?

24. Alvaro: Because you have more space <gestures spreading hands apart>// you're not going to crash into a wall <moves whole body forward> so that's why you run faster/

Leo responds to the question "why does the temperature change faster at the start?" by moving back into the context of the race and offering a suggestion that is perfectly sensible within the realm of the analogy: at the start of the race *you have more energy* (16). Alvaro is quick to disagree with this (18, 20) and offer a competing reason: that at the start of the race *you have more space* (24). His notion of space is directly related to the way he has been thinking about the race as ending at a wall - "because you have more space// you're not going to crash into a wall so that's why you run faster." It seems that Alvaro's conception of *space* is invoked as a result of the way he is thinking about the race with respect to the wall: with both the start of the race and the wall in mind *the space between the starting line and wall is made salient*. At the start, there is a great deal of space between the two and it is therefore *safe to run fast*, near the finish there is less space and the runner is in *danger of crashing into the wall* and must therefore slow to a stop. In Alvaro's case, it appears that the *space to run safely drives the fast rate at the start*. Leo, on the other hand, is not attending to the wall and therefore differently orients to the start of the race, cuing instead ideas about having more energy at the start. In his case, it appears that *energy drives the fast rate at the start*.

*Segment 3: Goal of winning the race is made explicit.*

25. Leo: /that's not really true/

26. Alvaro: /you try to win// Shhh!/  
 27. Leo: /that's adding on/

size of a p-prim. It is close to the p-prim *slowing equilibration*: the notion that things slow as they approach equilibrium, however I would argue that it is different in three important ways. The first difference is the level of urgency: while slowing equilibration is described by a natural and gradual easing to a stop, the slowing described by Alvaro is marked by an urgent need to stop. The second difference is in the location of the *impetus* for slowing: in the case of slowing equilibration the object slows because that is its natural internal tendency, whereas in the case of Alvaro's slowing, the object is pressured to stop by an external entity. The third difference is related to the final destination: in the case of slowing equilibration the object is returning to its natural state or state of balance, whereas in the case of Alvaro's slowing, the object is moving to a new destination.

28. Alvaro: /Shhh! So you can try to like// win// that's why you're running faster// but then// when you're like approaching the wall// you're gonna start like slow down// so you don't want to like crash into it

The major contribution of this segment is that the *goal* of the behavior is made explicit - *you are running fast because you are trying to win*. This is the first time an agentive frame has been *explicitly* applied to the race analogy. It is true that the goal of winning is implicit within the race analogy: however, I would argue that the explicit articulation of the goal (along with the agentive framing of the scenario) plays an important part in cuing *Ohm's p-prim* for the students who are asked to respond to both Leo's and Alvaro's explanations. The power of agentive framing in activating *Ohm's p-prim* has been previously documented (diSessa, 2014).

*Segment 4: Difference between temperatures in light of goal drives fast rate at start.* The teacher asks several students in sequence to comment on any of the explanations they have just heard. The teacher begins by asking Sofia to explain the ideas that have made sense to her.

29. Sofia: They all make sense// because the last one// because the temperature is super different // I guess it

<The teacher asks Sofia why it goes fast at the start and then slows down>

30. Sofia: It wants to get like warmer// to reach room temperature

<The teacher asks Mateo to explain the ideas that have made sense to him during the discussion>

31. Mateo: There's a big difference in the temperature

<Disruptions>

32. Mateo: um like it has a lot to cover so it wants to do it fast

<The teacher asks Mateo to explain the second half of the warming curve>

33. Mateo: Like JJ said, it slows down 'cause there's a wall

<The teacher asks Martin to explain the ideas that have made sense to him>

34. Martin: um// I agree? That it has lots to cover so it just does it fast and then it starts slowing down because it's almost reaching its max

The teacher asks the students in the classroom to respond to the two proposed explanations for *faster at the start*. Alvaro's explanation is the main one taken up, however *two interesting and important adaptations* are made: 1) the context is shifted from the analogy of the race back to temperature (29, 30, 31), 2) instead of interpreting what Alvaro has said as more space so *you can safely* run faster (28), the students appear to interpret his idea as more space *so you want to* run faster (32, 34). It is possible that this is what Alvaro originally meant in expressing his idea. It seems more likely, however, that the introduction of agency has cued *Ohm's p-prim* for these students and caused them to orient to his analogy in a way that was different from his own orientation to it. For whatever reason, the three students that respond to his idea in the last five minutes of class reframe Alvaro's conception of *faster because more space at the start* in a way that appears to activate *Ohm's p-prim* and move the explanation in the direction of conceptualizing thermal equilibration as *difference drives rate*.

The language used here implicates agency in equilibration: the temperature "wants to get warmer" (30), "it has a lot to cover so it wants to do it fast" (32). By the last response the agency has dropped out and only the difference drives rate structure remains (34). Mateo's description holds on to Alvaro's non-normative cause for slowing down at the end to avoid hitting the wall (33), but by Martin's description, the wall has given way to an implicit version of difference drives rate (34).

In their novel interpretation of Alvaro's relationship between speed and distance the students appear to invoke *Ohm's p-prim*: intuitive knowledge that has been previously

documented as helpful to understanding the equilibration pattern as difference drives rate.<sup>4</sup> *Ohm's p-prim* is invoked when a goal and an agentive frame are applied to the temperature. If we interpret "has a lot to cover" as implicating a large result to produce and "wants to do it fast" as implicating a desire to put forth effort, Contribution 32: "it has a lot to cover so it wants to do it fast" can be interpreted as "greater result necessitates greater effort." This is the key relationship between effort and result that defines *Ohm's p-prim*. It is important to note that line 32 is only meant to explain the beginning of the equilibration curve and the student invokes Alvaro's notion of *slowing to a wall* to explain the rate at the end of the equilibration process.

In summary, there is a sequential development of ideas during the whole class discussion. During Segment 1, contributing students orient their attention to the second half of the equilibration curve and explain the changing rate as driven by the endpoint. In Segment 2 the teacher asks the class to orient to the first half of the curve and this elicits two causes for the fast rate of change at the start: *space* and *energy*. In Segment 3 the goal of winning the race is explicitly named as a factor driving the rate. Finally, the *difference in temperatures in light of the goal* appears to activate *Ohm's p-prim* and lead to a difference drives rate explanation for the first half of the curve, possibly the second half.

It appears that *Ohm's p-prim* was first activated during the class discussion and its influence on the development of students' equilibration models can be observed throughout the unit. I will now trace the development of one student's construction of equilibration over the course of the unit, attending to the influence of *Ohm's p-prim*.

**Research question 2b.** I turn now to addressing the second sub-question of research question 2: How did resources contribute to individual students' construction of pattern models? I introduce the case student and present a map of his conceptions over the course of the unit. I locate the public emergence of *Ohm's p-prim* in relation to this map and track its influence on the development of his model of equilibration.

**Tracing one student's construction of equilibration over the unit.** Our case student is called Emre. In a class of 21 8th graders, he is one of 10 male students. He engages well in individual work and is often centrally involved in small group work. He is willing to share during whole class discussions but tends to be shy in this context and is therefore less likely to make contributions without encouragement. Emre was chosen for case study analysis because the development of his conception of equilibration matched the aggregate tendencies of the group reported earlier. His initial conception was focused on the *fast to slow* behavior, his conception partway through the unit explained the behavior using both normative (*fast when far*) and non-normative (*slows down as approaching the final destination*) explanations, and his final conception mapped nicely to the scientific model of equilibration as *difference drives rate*. For this analysis I will look at data collected across the unit that sampled Emre's thinking before and

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<sup>4</sup> This is not to say that Alvaro's idea of *going fast when there is space to run fast safely and slowing down as you approach a wall* could not lead to a difference drives rate model of equilibration. *Greater distance allows one to go faster* does map to difference drives rate, if the entity does what it is allowed to do. *Greater distance allows one to go faster* is a different intuition, however, from *greater distance makes one want to go faster*. It is the latter that connects with *Ohm's p-prim* and is taken up by participating students at the end of the class discussion, and by the case student.



after major instructional activities. A map coordinating the sequence of instructional activities, data sources, and Emre's conceptions are presented in Figure 18, below.

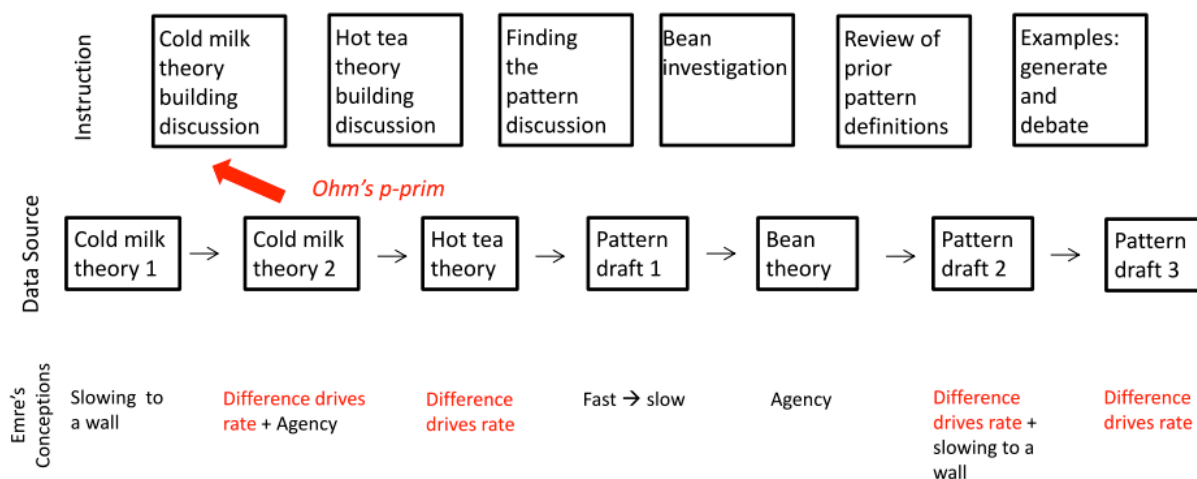


Figure 18. Instructional activities, data sources, and the development of Emre's model of equilibration

*Cold milk theory 1.* Following the cold milk investigation, students were asked to write explanations for why the temperature of the milk warms up quickly at the start and then slows down. Emre and Alvaro were partners, both writing explanations that related slowing to room temperature with slowing to a wall at the end of a race. The teacher opens the whole class discussion by reading these explanations and asks students to respond to the one Alvaro has written. Emre's is quite similar, reading: "The temp changes fast and then slow like if me and Alvaro where racing we would start fast and slow down towards the wall."

*Cold milk theory 2.* During the class period following the whole class discussion, the teacher asked the students to again write explanations for why the milk warmed up quickly at the start and then slowed down. Emre seems to have been influenced by the discussion, writing: "A reason why the cold water might warm up fast is because it *wants* to get to a room temperatuer as fast as it can so that it doesn't feel like left out and so *the bigger the distance is from the temepartures that faster it will heat up and then when they are about the same it will slow down.*"

There is a clear goal in his scenario and the language is agentive, even explicitly anthropomorphic. His explanation is strongly suggestive of *Ohm's p-prim*. When he writes: "it *wants* to get to a room temperatuer as fast as it can so that it doesn't feel like left out" he attributes the milk with a desire to be at room temperature and explains the reason for that desire. He follows with "and so the bigger the distance is from the temepartures that faster it will heat up." In light of the *desire* revealed in his previous statement, it is possible that Emre is thinking that the further the milk is from room temperature, the stronger its desire is to reach room temperature. This greater desire motivates a greater effort to reach room temperature and the greater effort results in a greater speed. *Greater effort results in greater speed* maps directly to *Ohm's p-prim*, the element of intuitive knowledge that had appeared to be invoked at the end of the class discussion preceding this writing assignment.

Emre's explanation appears to be a version of difference drives rate, suggesting that the discussion from the previous day (in particular the final minutes during which *Ohm's p-prim* was

publicly invoked) shifted his conceptualization of the phenomenon from *slowing to the wall* to one aligned with difference drives rate. He doesn't mention the wall here, however it is not clear whether his explanation for the second half of the curve connects to the wall or difference drives rate. His phrase "when they are about the same" seems to be more of a comparison with "bigger the distance," suggesting difference drives rate as opposed to the wall, however, there is not enough data to say for certain.

*Hot tea theory.* The students conduct a second investigation to explore the cooling of hot tea. At the end of a theory building discussion the teacher asks Emre to share why he thinks the hot tea cools fast and then slow and he says, in front of the class: "the *bigger the difference in temperature the faster* its gonna start to like cool down but then once it reaches room temperature it will just slow down." The first half of the curve can be characterized as difference drives rate. The second half of the curve is not easy to characterize; there is no mention of the wall as a cause of slowing and it is possible that he is thinking about the rate as driven by difference. His explanation only weakly suggests this; he has not explicitly stated it.

*Pattern draft 1.* Following the hot tea investigation, the teacher facilitates a discussion through which the students recap the behaviors and explanations for both cold milk and hot tea investigations. She then asks students to write descriptions of the pattern common to both phenomena. Ideas are voiced about *going fast at the start because there is more space* and *slowing down to avoid crashing into a wall*, and the teacher explicitly connects these ideas to the formal law (Newton's law of warming/cooling) that explains the rate of temperature change as driven by difference in temperatures. Most students, however, include only observed behavior in their pattern descriptions. The few that do include explanations for behavior focus on slowing to a final destination. Emre's description is somewhat anomalous, as he seems to be describing a different equilibration curve. He writes: "It speeds up until it can't go anymore then it starts to slow down until it reaches equilibrium."

This seems to imply an S-curve where something moves slowly at the start, increasing in speed and then decreasing in speed until it reaches equilibrium. It is possible that he has construed an increase in amount as an increase in rate of change for the first half of the real equilibration curve and has then switched back to an increase in amount for the second half of the curve. It is also possible that Emre is not considering the equilibration curve at all in this construction of the pattern and is instead describing a process of going fast and then slow, that begins, for him, with a gradual warming up to speed. Emre had initially proposed the race analogy to explain the change in rate and it is possible that the analogy has pervaded his thinking from the start. A gradual warming up to speed would be quite sensible if this were the case.

*Bean theory.* The students engage in a third investigation that illustrates the equilibration pattern. As part of the investigation the teacher asked the students to write an explanation for why more beans move at the beginning than the end. It is clear from the agency in their language that many students are orienting to the phenomenon as a goal-directed behavior. Emre exemplified this, writing: "Because it is trying to reach equilibrium and it wanted to go fast then slow." It is possible that this agentic framing is an artifact of the cold milk discussion. It is also possible that this is the way the bean diffusion naturally makes sense to him. It is also possible that he is being playful.

*Pattern draft 2.* At the end of the investigation the teacher gives a short PowerPoint presentation to remind the students of their previous pattern descriptions. Following this she asks them to write a description of the pattern followed by the cold milk, hot tea, and beans. Emre

writes: "Fast then slow. Fast, faster, fast, slow, slower, slower, stop. slows down bc reaching equilibrium *goes fast in the beginning because it has more room to cover.*"

It is clear that ideas from the first whole class discussion have been integrated into Emre's explanation. The first two lines describe the behavior, detailing the change in rate of change. It is interesting to note a hint of the S-curve that had surfaced in his first description of the rate of change, suggested by the words "fast, faster, fast." The third line is his original explanation for the changing rate, and the one Alvaro voiced during the discussion. The fourth line connects to students' interpretations of Alvaro's "more space so faster" explanation and is nicely aligned with difference drives rate. Agentive language has fallen away.

*Pattern draft 3.* Students worked in teams to generate lists of other examples that followed the pattern, and after sharing examples they engaged in argumentation to sort out which examples truly followed the pattern and why. Following these activities, the teacher showed students a final PowerPoint presentation to remind them of their previous pattern descriptions. Students were then asked to write final draft descriptions of the pattern. Emre writes: "In the beginning it goes *fast because there is a bigger difference and more space to cover* and it slows down because *every time less space is available so it slows down.*" This description is precisely focused on the difference drives rate phenomenon. The agentive language has completely fallen away and the "slowing to a wall" explanation has disappeared, in its place a difference drives rate explanation for the second half of the equilibration curve.

**Conclusion.** In summary, close examination of Emre's developmental trajectory shows that, similar to diSessa's (2014) finding, *Ohm's p-prim*, a well-documented element of intuitive knowledge, played a key role in his construction of the normative version of equilibration. Emre's construction differs from the "freaking out" scenario because it was based on a spatial orientation to the context, as opposed to a sensory orientation.

#### Part IV. Key Features of Instruction

I turn now to addressing the third research question of my study in the context of the equilibration instructional unit.

3a. What aspects of the knowledge construction process emerged as important?

3b. What features of instruction supported important aspects of the knowledge construction process?

**Analytic approach.** I approached question 3 by examining case studies to identify aspects of the construction process that productively engaged prior knowledge and the features of instruction that supported these aspects. In reviewing case studies, the two phases of the knowledge construction process that emerged in the previous unit emerged as important for equilibration: *activation* and *engagement*. I will explain how each of these parts played an important role in the construction of equilibration, discussing the features of instruction that supported them and addressing both sub-questions of research question 3 together.

**Activation phase.** During this phase, resources are activated in the learner's mind. Which resources are activated depend on the context in which the learner is reasoning, what they attend to, and their orientation to the object of their attention. In the case of the equilibration unit, the resource that is activated is *Ohm's p-prim*.

**Context.** Although I have argued in the case analysis of Part III that *Ohm's p-prim* is first publicly invoked during a whole class discussion in the context of *temperature* by the student Mateo, I would argue that his reasoning was an interpretation of an earlier idea that was proposed by Alvaro, who worked his idea out in the context of a *race*. The race was therefore a

key context in the activation of *Ohm's p-prim*. The emergence of the race context is likely connected with a data interpretation activity that took place directly before students wrote their first explanations for why the milk warmed fastest at the start and then slowed down. For this activity, students were invited to the front of the classroom to demonstrate the changing rate of temperature change by walking along a horizontal thermometer (drawn on the board) to their classmates' counting. Alvaro was the student that volunteered to walk out the temperature change and it is possible that his experience led him to think about temperature change as analogous with a race. This suggests the influential role of the lesson content, on students' thinking and learning.

**Attention.** Working on the problem of a decreasing rate of change in the context of the analogy of a race to a wall, Alvaro shifts his attention among the different characteristics of the analogy and articulates his reasoning for different parts of the curve. Some important shifts in his attention can be explained as the result of the teacher's moves. Alvaro naturally attends to the slowing to the finish line and explains the change in rate as a result of avoiding crashing into the finish line, which is a wall. The teacher shifts his attention to the high speed at the start, and with the wall fresh in his awareness he attends to the fact that there is a great amount of space between the start and the wall so it is safe to go fast. This suggests the importance of the teacher's role in directing student attention to productive phenomena, or aspects of phenomena.

**Orientation.** Along the way he applies an *agentive frame* to the race analogy and identifies the goal as getting to the finish line. This agentive frame is applied by two other students, Sofia and Mateo, to the context of the changing temperature. The student Sofia shifts reasoning back to the temperature context, mapping the notion of *space between start and finish* of Alvaro's explanation to the *difference between the temperatures*, and applying the agentive frame to the temperature increase, making the goal reaching room temperature. Attending to the difference between temperatures while orienting to the goal activates *Ohm's p-prim* for the student Mateo. Alvaro applied the agentive frame to the race; this led to the goal-directed orientation that other students mapped back to the temperature context. He worked the agentive frame out naturally, as a result of clarifying his explanation to his classmates and teacher. His classmates worked out their reasoning and as a result of clarifying their interpretations of Alvaro's explanation, *Ohm's p-prim* was publicly invoked. This suggests the importance of creating time for students to think out loud, both for their own sake in coming to understand how they are thinking about something, and for the sake of their peers, who benefit from hearing the ideas they share.

Through this analysis, we see that a number of features of instruction supported the activation of *Ohm's p-prim*, including lesson content, the role of the teacher, and lesson structure. I will now discuss the activities of the engagement phase and the features of instruction that supported those activities.

**Engagement phase.** During this phase, the learner engages resources in the construction of patterns models. In the case of the equilibration unit, *articulating*, *reinforcing*, *removing*, *refining*, and *connecting* activities appear to have played important roles in the construction process.

**Articulating.** Once activated, resources need to be articulated so that they can be productively engaged in the construction of new knowledge. In this case, the activation and articulation of *Ohm's p-prim* are described as occurring in the same episode. It is likely that knowledge is often tacitly cued, and then engaged, or not, in the construction process. In the case outlined in the equilibration unit, the articulation of *Ohm's p-prim* appeared to play an important

role in many students' construction of equilibration. Had Mateo not first articulated it publicly, it is possible that other students would have thought it and engaged it in their own construction of equilibration but it is probable that it would not have had such a far reaching influence and affected so many students' thinking, and it may not even have crystallized as a conscious thought in Mateo's own awareness if he had not articulated it.

The aspect of instruction that supported articulation of *Ohm's p-prim* was first and foremost, the whole class theory-building discussion. Important features of this discussion were *asking students to share their own ideas, framing their ideas as productive* (i.e., no right or wrong ideas, just individual ways of thinking about the phenomenon) and making sure *students treat each other's ideas with respect, pressing them to articulate their ideas* (i.e., holding them accountable to unpacking explanations), and encouraging students to *revoice, question, and argue* with other students' ideas. It is worth noting that at the same time that the teacher is holding students accountable to their ideas, she is also creating a space for discussion that allows students to re-think and re-state their ideas without the pressure to be correct or even persuasive. It may be that this sort of gentle approach to argumentation is better for eliciting resources than more strict versions of classroom argumentation, as intuitive knowledge is difficult to articulate and perhaps best drawn out in the context of thinking out loud in a safe and supportive space.

**Reinforcing.** It is interesting to note the delay between the introduction of *Ohm's p-prim* and difference drives rate and the uptake of those ideas by students in their pattern descriptions. Emre, for example, clearly invoked *Ohm's p-prim* in his explanation for the warming of the cold milk directly following the whole class theory-building discussion. For his explanation of the cooling of hot tea, the agentive language is absent from his explanation and he is left with a difference drives rate explanation. He does not include difference drives rate in his first model of the pattern, however. His second draft includes difference drives rate for the first half of the curve but invokes *slowing to the wall*, for the first time since his first explanation, to explain the second half of the curve. It is not until his third draft that Emre refines his description of the pattern to be entirely difference drives rate. Perhaps Emre initially leaves difference drives rate out of his model because he doesn't see it as salient to the pattern. It is also possible that he has an aesthetic for parsimony and only adds details to his pattern as he sees that this is preferred by his teacher and the norm among his classmates. It is also possible, however, that the *repetition* of ideas related to difference drives rate shared by his classmates reinforced Emre's sense that it was a productive explanation and stabilized its place in his model of the pattern. Whole class discussions such as the one described above create venues for students to share their thinking and reinforce productive ideas.

**Removing.** In many students' developmental trajectories, we observe the inclusion of the *slowing to the wall* idea in their second, but not third drafts of the pattern descriptions. In the second draft, some students include both the wall and the difference drives rate ideas, together. The difference drives rate explanation is sufficient for both halves of the equilibration curve and it therefore makes the wall explanation obsolete. It is likely that students remove the wall example when this becomes apparent to students, leaving only the normative difference drives rate explanation.

**Refining.** We observe in the development of Emre's equilibration model what we observe in the development of many students' models: *the refinement of language over time*. Beginning with his second explanation for the cold milk which uses highly anthropomorphic language, Emre's explanations gradually replace anthropomorphic language with scientific language. Providing students with multiple opportunities for revision supports both *removing* and

*refining*. Giving students the chance to revise their drafts allows them to improve their drafts, incrementally reflecting a change in both conceptualization of the pattern, and their ability to articulate it.

**Connecting.** At the end of a class discussion meant to help students find the pattern followed by both the cold milk and the hot tea, the teacher drew a connection between students' ideas and Newton's law of warming/cooling. In this case, *Ohm's p-prim* makes the proper scientific explanation intuitively sensible to students who include it in their conception of temperature change. Drawing explicit mappings between student ideas and scientific theories helps students see scientific ideas as intuitively sensible.

Through this analysis, we see that a number of features of instruction supported students' engagement of *Ohm's p-prim* in their construction of equilibration pattern models. These include accountability to ideas, sharing ideas, opportunities for revision, and drawing explicit connections.

### Section 5.3: Oscillation Analysis

I begin this section by describing a model of the target oscillation pattern and my design of the instructional unit (Part I). As I describe in my introduction to Chapter 5, I address my first research question by presenting a high-level sketch of the general development of students' oscillation models using the results of a macro analysis of students' written work (Part II). I then address my second research question and present a detailed picture of the role of prior knowledge in one student's construction of a model of oscillation using the results of microgenetic analysis of written work, video transcripts, and field notes (Part III). I end by addressing my third research question, presenting the important elements of the oscillation model construction process and the features of instruction that supported this process (Part IV).

#### Part I. Introduction to the Oscillation Pattern and Instructional Unit

**Oscillation pattern.** A model of oscillation that could be used as a benchmark for characterizing students' models was crafted by our research team through the same combined bottom-up (data driven) and top-down (scientific model driven) approach that we had used for both threshold and equilibration patterns. We created a coding scheme out of the elements of a scientific model that were included in student models (e.g., *displacing force*, *restoring force*, *momentum*) and fine-tuned elements to be consistent with the intuitive sense conveyed in student models (e.g., *pushing the weight back*, *gravity pulls it back*, *momentum makes it shoot out*). The resulting model characterizes oscillation as a movement back and forth about a fixed point. For the mechanical systems explored by the students, this movement begins when the system is displaced from its equilibrium state. A restoring force returns the system to equilibrium, however, the displaced element gains momentum and overshoots the equilibrium position. A restoring force counters the new displacement and the process continues indefinitely. The movement back and forth about the equilibrium state is called oscillation. The back and forth movement along with equilibrium, forces (displacing and restoring), and momentum comprise the target model of the oscillation pattern.<sup>5</sup>

**Oscillation instructional unit.** The instructional unit was designed to support students' construction of the oscillation pattern through the exploration of exemplar phenomena. This is an abbreviated version of the same sequence of activities used in both threshold and equilibration units. Due to time constraints, the oscillation unit was cut short. It therefore does not feature the generation or debate of examples, or the final revision activity. Additionally, some activities were compressed into more teacher-centered formats (e.g., class discussions were converted to lectures). The unit was comprised of 4 core activities that took place over approximately 5 instructional hours. The sequence of core activities is presented in Figure 19 and described below.

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<sup>5</sup> Some physicists may choose to model mechanical oscillation differently (for example using conservation of energy). Our research team chose to use a model comprised of forces and momentum as we felt the particular example of the weighted wheel elicited intuitive knowledge that could be refined and combined toward this model.

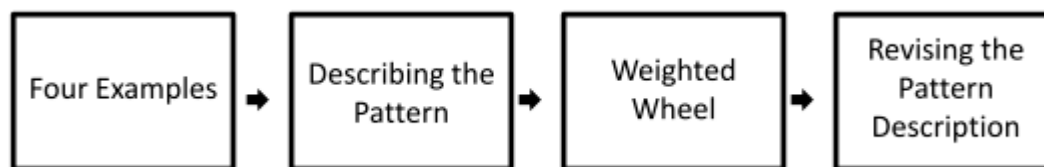


Figure 19. Sequence of core activities of the oscillation instructional unit

**Core activity 1: Four examples.** The unit opened with a demonstration of 4 exemplars of the oscillation pattern: 1) the back and forth swinging of a pendulum, 2) the up and down bouncing of magnets constrained to move along a rod, 3) the up and down motion of a ping pong ball in a vertical stream of air, and 4) the quick back and forth vibration of a rubber band. The teacher demonstrated each of the 4 examples at the front of the classroom and then walked around to each small group so that students could have a chance to view the examples more closely and experiment with them. These particular examples were selected because they demonstrated the behavior of the oscillation pattern and could be parsed into segments that made understanding the mechanisms that drove the behavior more accessible.

For example, the pendulum begins hanging at rest in an equilibrium position. A displacing force is applied to move the bob away from equilibrium and when released, a restoring force (gravity) returns it to equilibrium. It gains momentum during the return and overshoots the equilibrium position. In the second example, magnets are constrained to move along a rod and positioned so that their north (or south) poles are facing each other. They repel each other and one magnet appears to float above the other in a state of rest (this is the equilibrium position). Applying a force and pinching the magnets together displaces the system from its equilibrium position, and when released, a restoring force (from the magnet's magnetic fields) returns the system to equilibrium, moving the top magnet away from the bottom magnet (whose position is fixed at the bottom of the rod). In the return, the top magnet gains momentum and overshoots the equilibrium position. In this case, a different restoring force (gravity) acts to bring the system back to equilibrium. The top magnet again gains momentum in the return and overshoots equilibrium, and the system continues to oscillate. The third and fourth examples of the ball in the air stream and the rubber band also demonstrate oscillatory behaviors and they were included to make the back and forth behavior salient. One class period (40 minutes) was allocated for this activity.

**Core activity 2: Describing the pattern.** Following the demonstration, the teacher asked students to name, describe, and flowchart the pattern followed by the 4 examples. As in the case of both threshold and equilibration units, the purpose of this activity was to engage students in 1) thinking about the deeper structural behavior common to the examples and 2) articulating their initial impressions of the pattern. These ideas would serve as foundations for constructing more sophisticated pattern models over the course of the unit.

**Core activity 3: Weighted wheel investigation.** The next activity supported students' exploration of the mechanism that drove the back and forth behavior of the oscillation pattern. For this investigation students observed the workings of a wheel that had been outfitted with weights positioned away from its center. Because of its asymmetric distribution of mass, the wheel tended to roll back to a position where the weights were at the bottom of the wheel, closest with the table or the floor. Because of this tendency, the wheel was a nice example of a simple harmonic oscillator. Students explored different segments of the wheel's dynamics in the context of a guided inquiry and then shared their findings with the rest of the group during a whole class



discussion. Following the investigation and debrief, the teacher gave a lecture that built on students' ideas and explained the oscillation of the wheel explicitly in terms of physics principles. These activities were given three class periods, totaling 2 hours of instructional time. The example of the weighted wheel was selected because it could be parsed into segments that made the exploration of the mechanism behind its back and forth behavior accessible. It was also predicted to be generative of productive prior knowledge as, for example, the resistance of the weight could be physically felt and used to invoke intuitive knowledge that could be used as a foundation for understanding restoring force. Finally, the wheel was an especially fruitful example because it demonstrated not only oscillation, but threshold and equilibration as well, thereby creating an opportunity to review the two previous patterns.

**Core activity 4: Revising the pattern description.** Following the activities around the wheel investigation, the teacher led a discussion to connect the mechanisms driving the back and forth behavior of the pendulum and magnet examples to the mechanism behind the wheel's behavior. The next class period students revised their pattern names, descriptions, and flowcharts. As in the case of both threshold and equilibration units, it was thought that students' conceptualizations of the pattern might have been productively influenced by the consideration of an additional example. Offering students a final chance to revise their models gave them the opportunity to demonstrate their further refinement in thinking. The total instructional time allocated to the two activities was one hour and 20 minutes.

## Part II. General Tendencies in Development

I turn now to addressing the first research question of my study in the context of the oscillation instructional unit.

1a. What were general tendencies in the development of students' pattern models with respect to the *target model of the pattern*?

1b. What were general tendencies in the development of students' pattern models as *domain-general models*?

**Analytic approach.** I approached question 1 with a macro-analytic strategy. I developed coding schemes for characterizing written descriptions of oscillation and used these to code students' initial and subsequent description drafts. I present coding outcomes graphically to show whole class tendencies in each pattern draft. I compare proportions across drafts and use statistical tests to locate significant shifts in whole class tendencies, which I present as general developmental trajectories.

**Research question 1a.** I begin by addressing the first sub-question of research question 1: 1a. What were general tendencies in the development of students' pattern models with respect to the *target model of the pattern*?

**Coding scheme.** I designed the coding scheme below to characterize students' descriptions in terms of the target model of oscillation. The first part of the coding scheme (Table 10) describes the *elements* of oscillation that students included in their written descriptions. The second half of the coding scheme (Table 11) lists sets of related elements that were the basic structures underlying students' patterns. These pattern structures contain elements of behavior and elements of mechanism. As shown in Table 11, pattern structures range from single behavior elements to elements of behavior explained by mechanisms. The structures are ordered vertically according to level of sophistication and assigned rank scores. Beginning with a characterization as purely movement, details of mechanisms (e.g., restoring force; momentum) are added.

Table 10. Oscillation elements coding scheme

Element	Description	Example
Movement	There is movement in the system.	"all involve movement"
Movement Between Opposite Values	The system is moving from one value to its opposite.	"The objects move back and forth from left to right or up and down."
Repetition	The behavior repeats itself.	"going to and going back, constant motions"
Damping	The behavior repeats but lessens over time and eventually stops.	"It goes back and forth and then less and less and it eventually stops."
Initial State	The system begins in an initial state.	"It starts with the weight at the bottom"
Equilibrium	The initial or natural state of the object is one of equilibrium.	"It goes back to equilibrium"
Initiation	The system is displaced from its initial state.	"push it to side it momentu it shoots out but gravity pushes it down."
External Force Causes Displacement	An external force causes the displacement of the system from its initial state.	"Force is applied to the object to start movement to one side."
Return	The system returns to its initial state.	"push it to side it momentu it shoots out but gravity pushes it down."
Restoring Force Causes Return	A restoring force within the system acts to counter the displacement and return it to its initial state.	"push it to side it momentu it shoots out but gravity pushes it down."
Overshoot	The system overshoots its initial state.	"The Gravity causes the object to overshoot with speed"
Momentum Causes Overshooting	Momentum causes the system to overshoot the initial state.	"push it to side it momentu it shoots out but gravity pushes it down."

Table 11. Oscillation structures coding scheme

Structure	Description	Example	Score
Other	The description cannot be characterized as one of the structures listed.	"You could hear the rubber band"	0

Movement	The description focuses on movement in general.	"all involve movement"	1
Movement Between Opposite Values	The description of the movement specifies movement between opposite values.	"The objects move back and forth from left to right or up and down."	2
Repeated Movement Between Opposite Values	There is repetition of the movement between opposite values.	"going to and going back, constant motions"	3
Damped Movement Between Opposite Values	The movement between opposite values repeats, diminishing over time and eventually stopping.	"It goes back and forth and then less and less and it eventually stops."	3
Repetition with Mechanism	The description focuses on repetition in general and provides some causal mechanism.	"They repeat something over and over until it ends. Because of Gravity and momentum."	4
Movement with Mechanism	The description focuses on movement in general and provides some causal mechanism.	"Force is applied to make some sort of movement"	4
Movement Between Opposite Values with Mechanism	The description of the movement specifies movement between opposite values and provides some causal mechanism.	"It moves back and forth movement and gravity cause it."	4
Repeated Movement Between Opposite Values with Mechanism	The description of the movement specifies repeated movement between opposite values and provides some causal mechanism.	"It goes in a specific direction again + again. push it to side it momentu it shoots out but gravity pushes it down."	4
Damped Movement	The description of the movement specifies diminishing movement between opposite values and provides some causal	"Force is applied to the object to start movement to one side. Then gravity or momentum make it go to the other side and	4

	mechanism.	keep going back and forth. It slows down and then comes to a stop."	
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**Coding.** These coding schemes were used together to characterize students' initial and later written drafts of pattern descriptions in terms of underlying structure. As in the analyses of both the threshold and equilibration units, a team of 3 researchers (comprised of myself and two undergraduate assistants) coded each of the three drafts independently and then convened to compare scores, resolve discrepancies, and reach consensus.

**Results.** Coding outcomes are presented in the bar chart below (Figure 20) to show aggregate tendencies in pattern models for each draft. Comparison of proportions across drafts and a test of statistical significance provide evidence for general developmental trajectories.

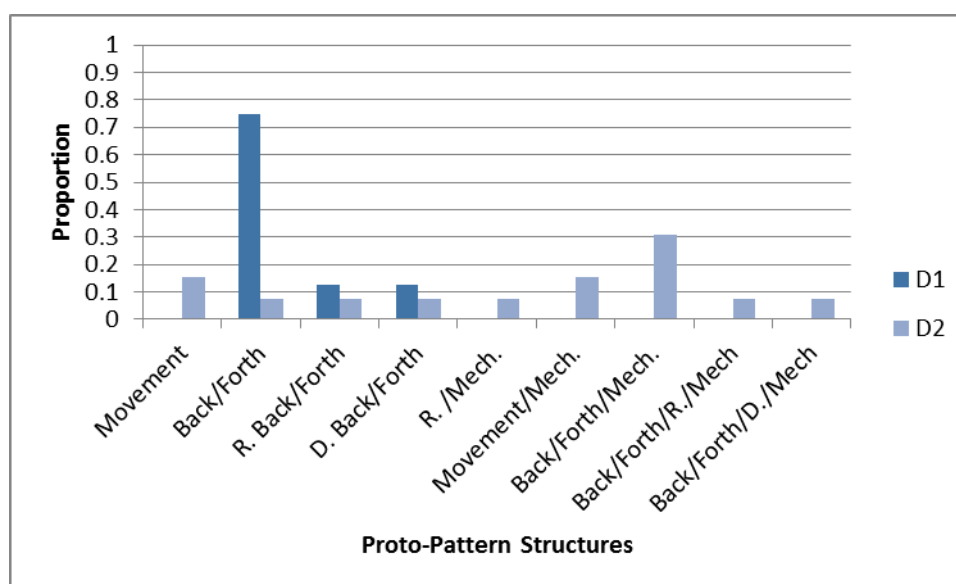


Figure 20. Characterization and comparison of oscillation structures

*General tendencies in development.* At the writing of the first draft, all students characterized the pattern in terms of behavior alone. Seventy-five percent described it as some version of back and forth (e.g. "the objects move from left to right or up and down"), 12.5% described it repeated back and forth (e.g., "it goes back and forth and it keeps on going"), and 12.5% described it as damped back and forth movement (e.g., "it goes back and forth and then less and less and it eventually stops"). At the writing of the second draft, 38% of descriptions were spread across these categories and the additional category: *movement*. The remaining descriptions were distributed across the same variations of behavior, however they include one or more of the mechanism elements listed in the oscillation elements coding scheme (Table 10) (e.g., "it moves back and forth movement and gravity cause it").

Comparison of tendencies in pattern structures across drafts suggests movement away from a solely behavioral characterization of oscillation toward one that includes elements of mechanism. This move to include elements of mechanism is a step in the direction of crafting more sophisticated models of oscillation. Comparison of draft 1 and draft 2 mean rank scores

(Figure 21) shows that the average level of sophistication for the class increased by 1.5 points from 2.2 (SD = .4) at draft 1 to 3.7 (SD = .64) at draft 2.

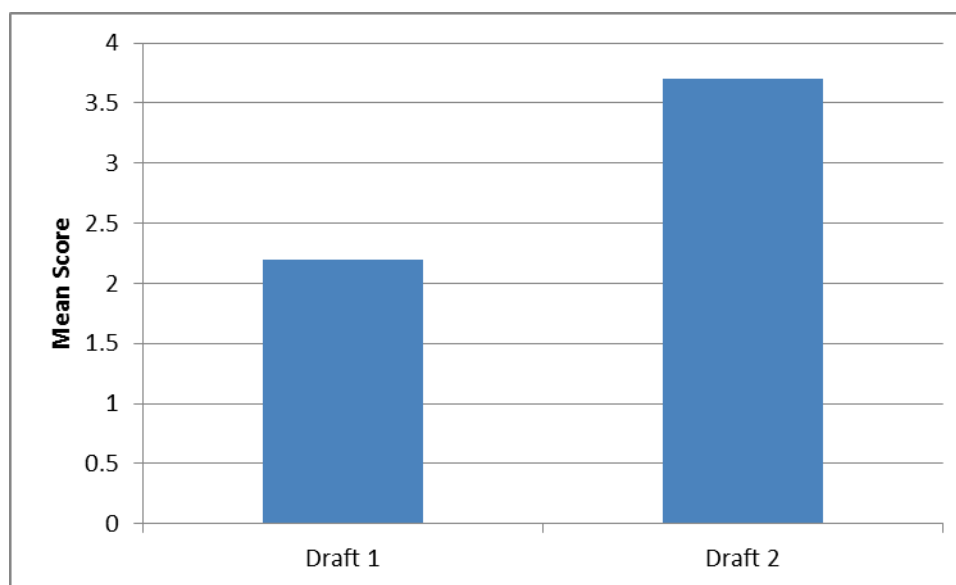


Figure 21. Mean scores for draft 1 and draft 3 student oscillation models

A basic statistical test was used to ascertain the probability that the oscillation unit activities played a role in this development. In order to test the hypothesis that the unit activities did not have any effect on students' learning, their rank scores for first and final draft descriptions were compared using a paired difference test. Because rank scores (a discrete measure) were compared (as opposed to a continuous measure), and because the sample size was quite small and the data sets were not normally distributed, a non-parametric statistical hypothesis test was needed. The Wilcoxin signed-rank test is a hypothesis test that can be used for non-parametric data sets. Using this test to compare first and final draft scores showed that gains in rank score were statistically significant ( $\alpha = .05$ ). This suggests that instruction played an important role in the development of students' models of oscillation.

**Research question 1b.** I turn now to addressing the second sub-question of research question 1: What were general tendencies in the development of students' pattern models as *domain-general models*?

**Coding scheme.** I designed the coding scheme presented below (Table 12) to evaluate whether or not students' descriptions were domain-general. This coding scheme is used to classify descriptions as *general*, *specific*, or *other*.

Table 12. Domain-generality coding scheme

Characterization	Description	Example
General	The description is not embedded within one or multiple specific contexts.	"The movement is in opposite directions."
Specific	Context or specific examples are given. (Equilibrium does not count as context.)	"The objects go up and down or left and right."

Other	The student is articulating a pattern made of elements that are mostly not present in the coding scheme.	“All involve household objects.”
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**Coding.** This coding scheme was used to characterize students’ first and second written drafts of pattern descriptions. As in the case of the coding presented earlier, the team of 3 researchers coded every written description independently and then convened to compare scores, resolve discrepancies, and reach consensus.

**Results.** Coding outcomes are presented in the bar chart below to show aggregate tendencies in the domain-generality of each draft. Comparison of proportions across drafts is used to infer general developmental trajectories.

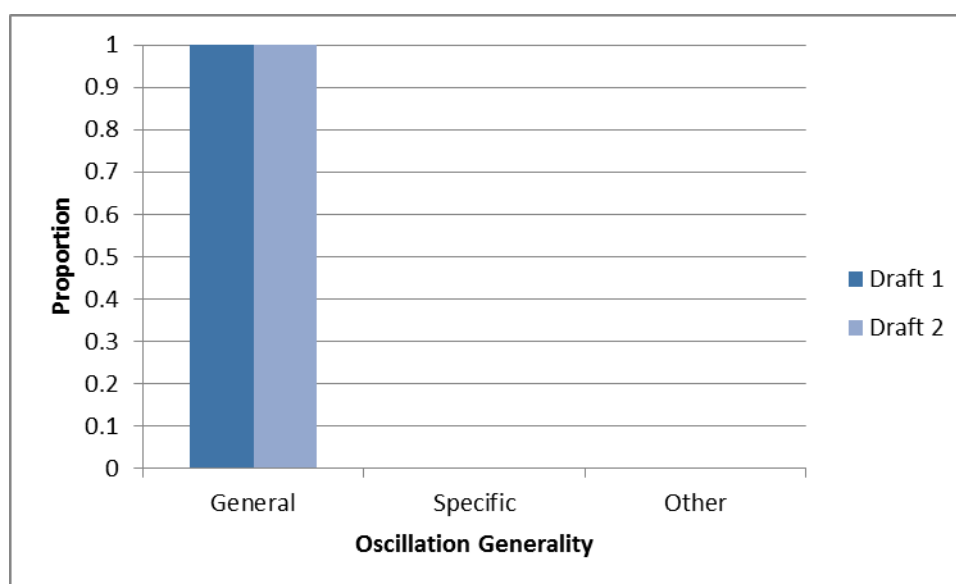


Figure 22. Proportion of descriptions coded as general, specific, or other

*General tendencies in development.* Comparison of proportions across drafts indicates that students are crafting domain-general descriptions of oscillation throughout the unit. This suggests that students understand the domain-general nature of a pattern model and that they have a firm grasp of how to leverage language to articulate the pattern so that it is domain-general.

**Conclusion.** Instruction was designed to engage students' prior knowledge in the task of constructing models of oscillation. Instruction was primarily student-centered, and though it cannot be denied that the teacher played an important role, students were given agency in crafting their own pattern descriptions. The fact that over half of the class developed descriptions of oscillation that included elements of mechanism is a testament to the power of the knowledge and skills this group of students brought to their learning and suggests success of the instructional design, on the whole. Closer analysis of the events within the instructional unit suggests that particular elements of prior knowledge played important roles in introducing elements of mechanism to students' understanding of oscillation, and that the first investigation

of the unit played a particularly important role in facilitating that development. This analysis is presented next.

### Part III. Productivity of Prior Knowledge

I turn now to addressing the second research question of my study in the context of the oscillation instructional unit.

2a. What prior conceptions emerged as resources for students' construction of pattern models?

2b. How did resources contribute to individual students' construction of pattern models?

**Analytic approach.** I approached question 2 through microgenetic case study analysis. I selected a student whose shifts across drafts matched the general developmental trajectory of the class illuminated in Part II. I examined their written classwork to identify major shifts in thinking and to look for connections between those shifts and particular episodes during which productive prior knowledge emerged. I used written work, video transcripts, teacher reflections, and researcher field notes to track how prior knowledge contributed to the development of the student's construction of oscillation over the unit. The general trajectory of my presentation of this analysis will be to locate the emergence of key prior knowledge and then track how several elements are taken up and built upon productively by one student in his construction of a model of oscillation.

**Research question 2a.** I begin by addressing the first sub-question of research question 2: What prior conceptions emerged as resources for students' construction of pattern models? I begin by introducing the key prior knowledge and then locate its emergence within a focal lesson near the beginning of the unit.

**Key prior knowledge.** The model of oscillation described at the beginning of this chapter has a number of components and may therefore be assembled out of a number of knowledge elements. Figure 23 below uses the example of the weighted wheel to describe components of the target model of mechanical oscillation.

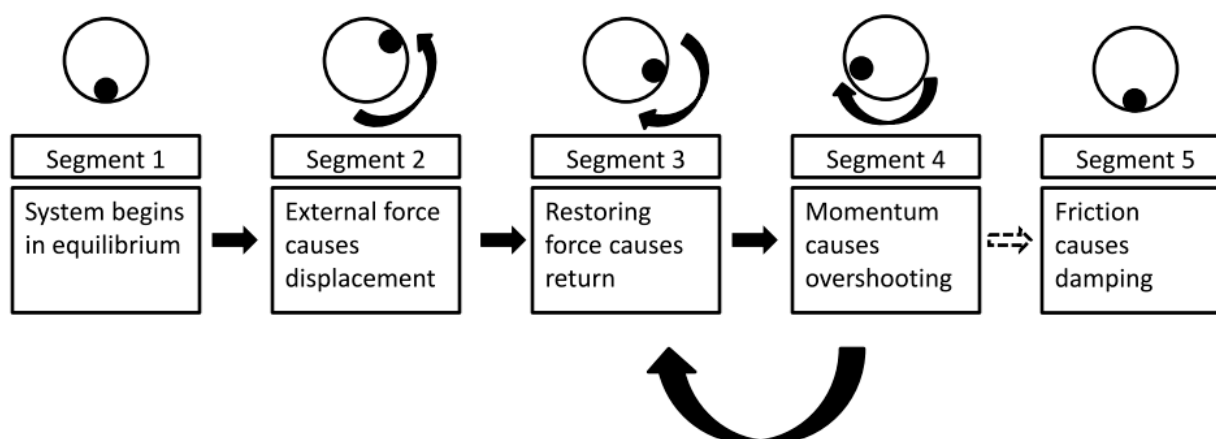


Figure 23. Model of oscillation in the case of a weighted wheel

The wheel begins in a state of rest with the weight at the equilibrium position (Segment 1). A perturbation in the form of an external force causes the weight to be displaced from the equilibrium position (Segment 2). A restoring force (gravity in the case of the wheel) pulls the weight back toward the equilibrium position (Segment 3). The restoring force is proportional to the displacement: the greater the horizontal displacement from equilibrium, the greater the

restoring force. As the wheel rolls the weight back to the equilibrium position, it gains momentum. This momentum causes the weight to overshoot the equilibrium position (Segment 4). A restoring force acts on the displaced weight, slowing it down and pulling it back toward the equilibrium position, but again, it gains momentum in the process and overshoots. The cycle in which the weight is displaced on alternate sides of the equilibrium position is called oscillation. It would continue forever in a frictionless environment, but in the observable case, due to friction, the amplitude of the displacement decreases with each cycle, the motion of the wheel gradually diminishes and the system ends in a state of static equilibrium (Segment 5).

Ten previously documented intuitive knowledge elements emerged in the context of the students' investigation of the weighted wheel. These elements belong to the class of intuitive knowledge called phenomenological primitives. Six of these p-prims were determined to be productive resources (when applied to the appropriate segment) for the construction of oscillation. These are *equilibration*, *abstract balance*, *generalized springiness*, *Ohm's p-prim*, *force as a mover*, and *overcoming*.

*Equilibration* explains the return of a system to equilibrium. In the case of the wheel it provides a foundation for understanding why the displaced weight returns to its lowest position.

*Abstract balance* explains, on the basis of symmetry, why a system is found in a state of balance. In the case of the wheel it can be refined to explain the mechanics behind the equilibrium position, and help students understand the meaning of equilibrium in the context of the wheel.

*Generalized springiness* explains the innate tendency of an object to resist being deformed, and that a resulting deformation is proportional to an applied force. In the case of the wheel it can be refined to explain the initial state of the wheel, its displacement from equilibrium, and the resulting return. If students see the weighted wheel as a springy system, the wheel's "preferred" state of equilibrium is explainable as the state in which it is not deformed. It remains in this state, unless a deforming force is applied to it, in which case it resists the deformation. The greater the applied deformation force the greater the displacement (of the weight) from the equilibrium position. The restoring force is understood in terms of the springy system's tendency to resist deformation. It is also a way of understanding the return to equilibrium that can be generalized to other examples (as opposed to the force of gravity which is specific to the weighted wheel). While it can play an important role in the construction of oscillation, *generalized springiness* cannot explain the phenomenon without ideas that explain the overshoot of the equilibrium position.

*Ohm's p-prim* can be invoked to make intuitively sensible the fact that the restoring force becomes stronger as the deformation (or displacement of the weight from equilibrium) increases.

*Force as a mover* explains motion of an object as the result of an applied force. In the case of the wheel, this p-prim provides a foundation for understanding both displacing (a push) and restoring (gravity) forces that explain the behavior of the wheel as a specific example of a springy system.<sup>6</sup>

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<sup>6</sup> For physicists, the displacement of the weight from the equilibrium position that initiates oscillation is not salient. Physicists instead use a general (mathematical) description of the system's "initial conditions." The displacement that initiates the oscillation is salient for the students, however, so we include it in our design of instruction.



*Overcoming* explains motion despite resistance in a case where one force is perceived to "win" over an opposing force. In the case of the wheel, *overcoming* provides an intuitive foundation for understanding momentum as the cause of overshooting, because the momentum of the weight overcomes the tendency of the weight to be in its lowest position.

It is my hypothesis that the refinement and combination of these elements will ultimately, with care, lead students to construct the target model of oscillation. Figure 24 (below) illustrates a hypothetical developmental trajectory for oscillation.

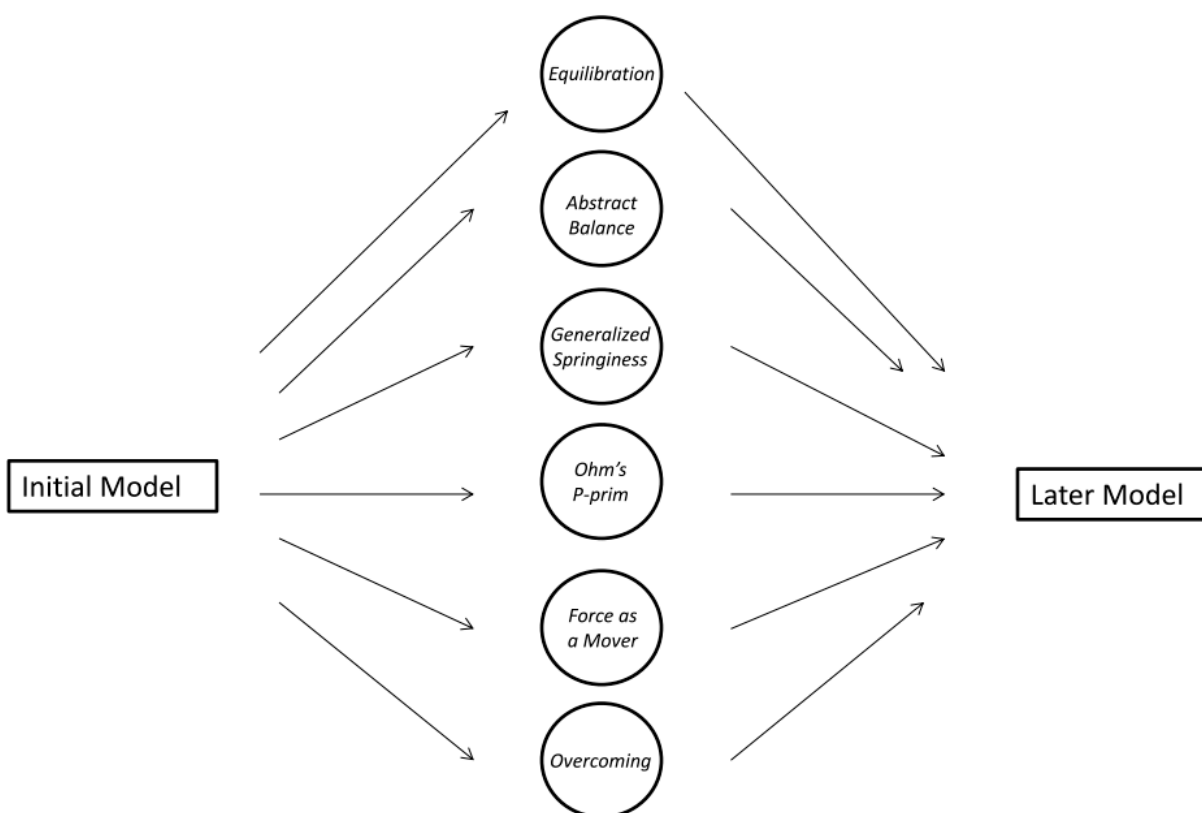


Figure 24. Basic trajectory showing the role of key pre-instructional ideas in students' development of oscillation<sup>7</sup>

Because the unit was cut short by end of year activities, students only wrote 2 drafts of pattern descriptions and I do not have evidence that any one student traversed a complete version of this path over the unit. However, all intuitive knowledge elements were elicited during the weighted wheel investigation (distributed across the students in the class) and leveraged by the teacher during a whole class discussion as a basis for an introduction to the mechanics of the wheel and other examples. Bits of the mechanism that explained the wheel's oscillation were

<sup>7</sup> It should be noted that the model in Figure 24 simply represents the particular p-prims that can be used as resources for the construction of a model of oscillation. It is not meant to represent the complexity of the model construction process, which might look more like fitting together pieces of a complicated puzzle.

integrated into many students' second draft pattern descriptions (as reported in the macro analysis at the beginning of the chapter).

I will now turn to describing the intuitive knowledge that was elicited in the context of the weighted wheel investigation, discussing first the productive ideas (outlined above) followed by other ideas that were elicited. I will then trace one student's construction of oscillation across their first and second descriptions, noting where resources may have had productive influence on the student's thinking.

***Lesson during which key prior knowledge emerged.*** Following their observation of four oscillating systems (swinging pendulum, bouncing magnets, ball bouncing in airstream, vibrating rubber band), the students investigated the behavior of a weighted wheel (pictured below). Each group had their own weighted wheel, made of two 10-inch diameter rubber tires connected by a metal shaft and weighted with metal washers. The students were given a worksheet that scaffolded their investigation by focusing their attention on the five segments shown in the model above (Figure 25).



Figure 25. The weighted wheel

I will now go through each of the segments of the oscillation pattern and discuss the pre-instructional ideas that emerged in response to the weighted wheel investigation. Responses to guiding questions (reported below) were either written on individual student's worksheets or given as verbal contributions to the whole class discussion following the investigation.

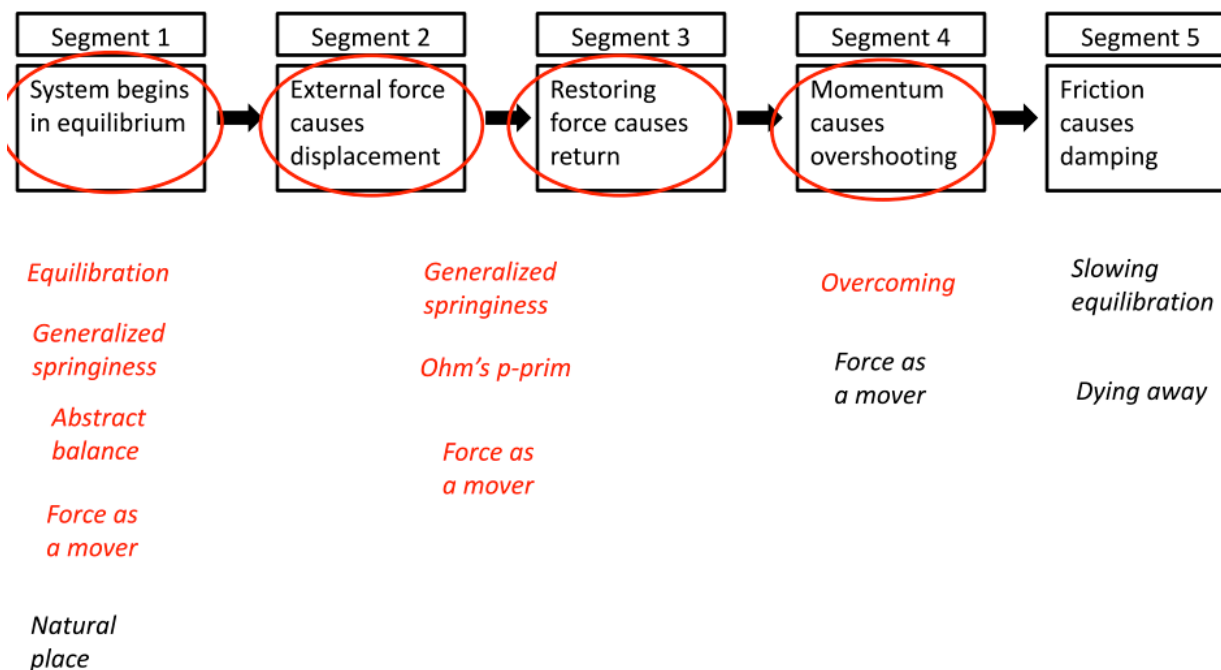


Figure 26. Prior knowledge invoked during weighted wheel investigation<sup>8</sup>

**Segment 1.** Prior knowledge corresponding to Segment 1 was shared by students was in response to the investigation question: "What position does the wheel stop in? Why do you think it stops in this position?"

*Equilibration.* One of the students simply wrote "equilibrium" as an explanation for why the weight stopped at the bottom. This is indicative of the p-prim *equilibration* that explains the return of a system to an equilibrium state as a natural tendency. This idea can be powerful for making intuitively sensible the fact that a perturbed system is always driven back toward an equilibrium state. At a high level description of behavior, this is accurate. The behavior though, left as a natural tendency, is incompletely modeled and the teacher can direct students' attention to the need for a restoring force, pushing students' explanations to the level of mechanism. The replacement of a top-level sketch of a pattern by a more elaborate process that results in the same behavior was observed in the test of earlier pattern-based curriculum and defined as causal interpolation (diSessa, 2014).

*Abstract balance.* Two of the students produced potentially productive explanations for why the weight remains in the equilibrium position. One student, on her worksheet wrote: "Because it's equal weight at all sides." Another student, during the class discussion elaborated a similar idea in response to their classmate's question about why the weight could also be in a state of balance at the top of the wheel.

1. Teacher: So why can it be balanced at the top? Tomas?
2. Tomas: If you split it in half then there'll be the same amount of weight on each side
3. Teacher: So why does that keep it from rolling one way or the other?

<sup>8</sup> Elements in red ink were identified as resources for the construction of the target model of oscillation.

4. Tomas: because it's not like one side has more weight than the other// and if it had more weight then it rolls// but since it's equal then it won't begin to roll

It appears that this student is invoking the p-prim *abstract balance*. In this case, *abstract balance* explains the absence of motion as the result of the symmetric distribution of weight about a balance point. This p-prim can be refined to explain the state of equilibrium as one in which the weight feels equal tension in both of the directions that it is free to move.

*Force as a mover*. Four of the students wrote something along the lines of "the force is pushing it down." This suggests that the p-prim *force as a mover* has been activated. A force (identified as gravity by two of the students) is responsible for having moved the weight to its resting position. It is true that the force of gravity is pulling the weight down, and this primitive can be tapped to play a productive role in students' exploration of restoring force. In addition to eliciting these three ideas considered to be potential resources, the question elicited an intuitive explanation that is hypothesized to be less productive in the construction of oscillation.

*Natural place*. Half of the students that responded to the question of why the wheel stops with the weight at the bottom wrote something along the lines of "because it's heavy." Though not a previously documented p-prim it seems like it would be a good candidate for this category of knowledge, as it seems that the students take it to be explanatorily primitive. The idea itself seems almost an instantiation of the Aristotelean concept of Natural Place: heavy things belong lowest to the ground.

**Segments 2 and 3**. Prior knowledge corresponding to Segment 2 was shared by students in response to the investigation question: "What do you have to do to start the wheel rocking back and forth?"

Following the episode in the class discussion focused on the balance of the weight at the top of the wheel, the teacher, still holding the weight at the top of the wheel, asked the question "what do you have to do to start the wheel rocking back and forth?" One student responded to this question with "you have to tip it over." Her response was refined later, during more teacher-centered treatment of oscillation to introduce the idea that an external force is needed to cause displacement from equilibrium. It is likely that allowing students more time to respond to this question would elicit the productive intuition *force as a mover*.

Prior knowledge corresponding to both Segments 2 and 3 was shared by students in response to the investigation question: "Rest your fingers on the top of the wheel and give it a gentle push so that it rolls forward a few inches. What do you notice about the way the wheel pushes back when you do this? Does it push back more or less?"

*Generalized springiness*. All of the students noticed that the wheel pushes back more as the weight moves from the bottom out to the 90 degree position. This is a sensory experience but it can invoke the p-prim *generalized springiness*. *Generalized springiness* explains the natural tendency of an inanimate object to resist deformation and that any displacement from an equilibrium position is proportional to the force applied. It is therefore helpful for making the mechanics of Segment 1, 2, and 3 intuitively sensible. One student seems to communicate this idea when he says "it pushes back more as you move it up. The wheel will move back stronger if you apply more force." It appears that he is equating "pushes back more" with "move back stronger," and "move it [the weight] up" with "apply more force." It is therefore likely that his reasoning is something along the lines of: "if you apply more force, it will move the weight up and the wheel will push back more and (if released) move back stronger." His statement: "as you move it up" describes an increasing displacement, and "apply more force" describes a corresponding increasing force. Because he is connecting displacement with restoring force, it

seems that the p-prim *generalized springiness* is invoked. *Generalized springiness* can be refined and combined with the *force as a mover* p-prim to describe the fact that (for a simple harmonic oscillator such as the weighted wheel) displacement is proportional to both the applied force and the resulting restoring force.

*Ohm's p-prim.* One student responded by drawing a picture where the weight was shown at about 90 degrees and writing "when it's like that because it wants to go to its normal position." It is possible that *generalized springiness* (as the tendency of an object to resist deformation) is behind this student's logic. However, *generalized springiness* is a p-prim that has been documented primarily in novices' explanations for inanimate objects. Here, this student has applied an agentive frame to the wheel when they ascribe to it a desire to attain a goal. It is possible that *Ohm's p-prim* has been invoked: as the weight has been displaced and is further from its "normal position" (where it prefers to be) it has to work harder and therefore pushes back more. Invoking this p-prim can likely make the proportional relationship between displacement and displacing/restoring force more intuitively sensible, however it is important to treat it carefully in the context of the wheel, helping students clarify that the 90 degree position is the furthest displacement from either of the two equilibrium positions: either with the weight on the bottom (stable equilibrium) or the weight balanced on the top (unstable equilibrium). If considering distance or amount rolled away from the bottom balance point, *Ohm's p-prim* is inconsistent with experience because the wheel pushes back less and less from 90 degrees to 180 degrees.

*Force as a mover.* One student notes that: "the wheel pushes back," suggesting that the wheel supplies the push back to the position from which the weight has been displaced. This idea can be productive in the sense that it suggests a resistance to the displacement and a return to the equilibrium position. Another student explains the motion in terms of gravity as the external force. Gravity is indeed pulling the weight down, and this can be identified as the specific instantiation of the restoring force in this particular oscillating system.

**Segment 4.** Prior knowledge corresponding to Segment 4 was shared by students in response to the investigation question: "Notice that, when released, the wheel initially goes through the "settled" position, rather than just stopping directly there. Why does it do this? Recall that the wheel only pushed back on your finger (tried to move) when it was away from the "settled" position."

*Overcoming.* Several students' responses indicate that a different p-prim, *overcoming*, has been activated. *Overcoming* is the idea that in the case of unbalanced opposing agents, one "wins" over the other. These students wrote things like "can't reach equilibrium right away because of speed...because speed beats equilibrium," and "can't stop to equilibrium...there's more pressure with the weights." Words like "beats" and comparative language like "more" suggest that one entity is *overcoming* another. *Overcoming* is a powerful intuitive building block in this case as it can be refined and connected to the momentum that causes the wheel to overshoot its equilibrium position.

During the class discussion around this question one student suggested momentum as a cause for the overshoot. It is interesting to note that on his worksheet he had written: "the force pushes it past," but that for whatever reason, in the group discussion format, he used the idea of momentum instead. Transcript from the discussion is presented below.

1. Teacher: So this is really interesting// I pull the weight away and it naturally rolls back// but// instead of just going straight back to earth it overshoots and it goes back up// can anyone raise their hand and tell me why they think the weight overshoots this point and rolls back up?

2. Tomas: because there's too much momentum
3. Teacher: momentum is an excellent word// Tomas said because there's too much momentum// can you say more about what you mean by that Tomas?
4. Tomas: There's probably too much momentum and that's like that// that means that there's too much like I guess bulk// I mean like weight force// or like the speeds too high for it to come to a complete stop

Tomas's explanation needs only a small amount of language refinement, moving the definition of momentum from bulk, weight force, or speed, into mass with speed to match the scientific model.

In addition to eliciting these productive ideas related to the *overcoming* p-prim, the question elicits *force as a mover*. This p-prim, while hypothesized to be productive during the previous 3 segments, is hypothesized to be unproductive in this context. *Force as a mover* is unproductive in explaining the wheel's overshoot because it is not a force that pushes the weight past equilibrium. Additionally it is problematic because it obviates the need for momentum as an explanation. *Force as a mover* is an intuitive primitive that explains an object's motion as the result of a force. In this case, a number of students attribute the motion of the weight through the equilibrium position to a force acting from the side: "because of the forces on the side," "the force pushes it past," "the force pushes the ball around."

**Segment 5.** Prior knowledge corresponding to Segment 5 was shared by students in response to the investigation question: "Start the wheel again and let it go. What do you observe?"

Almost every student predicted that the wheel would eventually slow to a stop. It appears that two p-prims are invoked separately to explain the phenomenon. One is *slowing equilibration*, the other is *dying away*. Neither *slowing equilibration* nor *dying away* are considered to be particularly productive in the construction of a model of oscillation, as both p-prims obviate the need for a deeper explanation.

*Slowing equilibration.* *Slowing equilibration* explains why an object, displaced from equilibrium, returns to equilibrium. One version of *slowing equilibration* is a return that gradually slows down to a stop, like a train pulling into a station. Another version of *slowing equilibration* invoked to explain oscillatory systems is *damped bobbing*. This explains a buoy bobbing up and down at ever decreasing amplitudes until it comes to a stop. It is not clear based on the language that students are seeing *damped bobbing*, though this description fits the decreasing rise and fall of the weight on the wheel. In any case, some version of *slowing equilibration* seems present in this prediction, written by one student but echoed by others: "I think the wheel will keep rolling, then eventually slow down and stop."

*Dying away.* One student predicts that the wheel "eventually stops" "because the force runs out." This student had explained the overshooting "because the force pushes the ball around" so it is likely that they see the eventual stopping as the result of this force (that is causing the overshoot) running out. Force running out over time is explained by the p-prim *dying away*, the tendency of all earthly motion to come to rest.

**Research question 2b.** I turn now to addressing the second sub-question of research question 2: How did resources contribute to individual students' construction of pattern models? I introduce the student and present a map of their development over the course of the unit. I locate the emergence of key intuitive knowledge elements in relation to this map and track its influence on the student's construction of equilibration.

**Tracing one student's construction of oscillation over the unit.** Our case student is called Tomas. In a class of 21 8th graders, he is one of 10 male students. Tomas was chosen for case study analysis of this pattern because his trajectory is representative of the general tendency in the class to move from a pattern structure that is focused on basic back and forth behavior to one that includes mechanisms for the behavior. Tomas is thoughtful and quiet, but comfortable speaking in front of the whole class. He excels at independent work and is thoughtful in his response to questions. His ideas are usually taken up by his small group. Though he is quiet, he is a leader in the class and his ideas are well respected by his classmates. We will now look at how the ideas elicited during the wheel investigation played productive roles in Tomas's construction of oscillation.

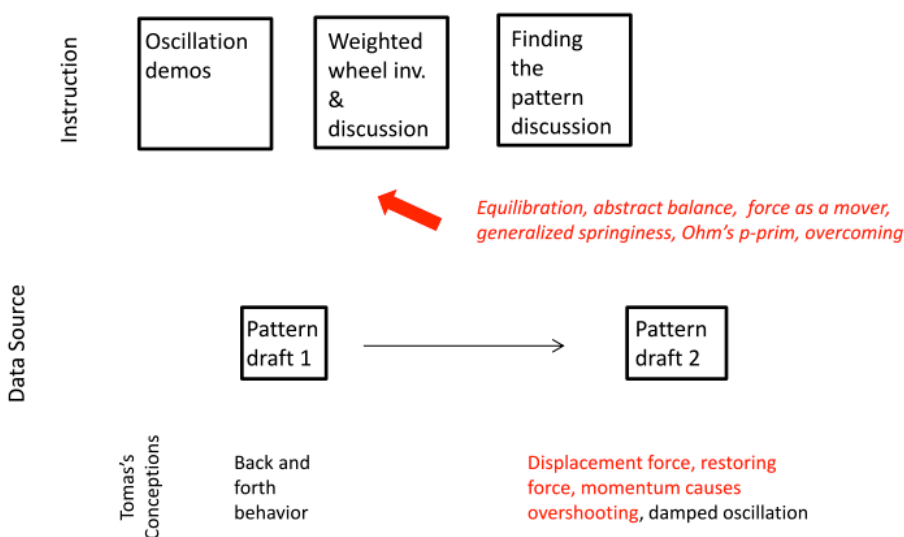


Figure 27. Instructional activities, data sources, and the development of Tomas's model of oscillation

Figure 27 (above) shows the development of Tomas's model of oscillation over the course of the short, 5-hour intervention. Prior to the wheel investigation, students' descriptions of the pattern were focused on the *observable behavior* of the examples. Tomas's description was typical of this: "The objects move back and forth from left to right or up and down." At the writing of this description, Tomas and his classmates had observed four oscillating systems: a swinging pendulum, magnets bouncing along a wooden rod, a ping pong ball bobbing in an airstream, and a vibrating rubber band.

Following the wheel investigation, over half of the students' descriptions (62%) included *elements of mechanism* that, although particular to the wheel scenario, are part of the target model of mechanical oscillation. Tomas's description illustrates this tendency: "Force is applied to the object to start movement to one side. Then gravity or momentum make it go to the other side and keep going back and forth. It slows down and then comes to a stop." He has integrated the *external force* that initiates the cycle by displacing the weight and then, although he has not articulated the details, he weaves in gravity and momentum, which "make it go to the other side and keep it going back and forth," implicating both *restoring force* and *overshooting*. He notes that the behavior will diminish and terminate, though he does not offer a reason why.

**Conclusion.** The elements of mechanism that Tomas has added to his model of oscillation and the elements present in his contributions to the class discussion suggest that *force as a mover* and *overcoming* have productively influenced his conceptualization of oscillation. These elements appear to have been introduced to his thinking in response to the weighted wheel investigation, suggesting that the activity helped improve his understanding of oscillation.

#### Part IV. Key Features of Instruction

I turn now to addressing the third research question of my study in the context of the oscillation instructional unit.

3a. What aspects of the knowledge construction process emerged as important?

3b. What features of instruction supported important aspects of the knowledge construction process?

**Analytic approach.** I approached question 3 by examining case studies to identify aspects of the construction process that productively engaged prior knowledge and the features of instruction that supported these aspects. In reviewing case studies, the two phases of the knowledge construction process that emerged as important for both previous units emerged as important for oscillation. These are *activation* and *engagement*. I will explain how the elements of each phase played an important role in students' construction of oscillation models, and discuss the features of instruction that supported each phase, addressing both sub-questions of research question 3 together. It is important to note that, due to time constraints, this unit was more teacher-centered than the threshold or equilibration units. The role of the teacher is apparent in many of the aspects of both activation and engagement phases and is addressed more fully in the concluding remarks of the chapter.

**Activation phase.** During this phase, resources are activated in the learner's mind. Which resources are activated depend on the context in which the learner is reasoning, what they attend to, and their orientation to the object of their attention. In the case of the oscillation unit, the activated resources were *equilibration*, *abstract balance*, *generalized springiness*, *Ohm's p-prim*, *force as a mover*, and *overcoming*.

**Context.** Students are reasoning within the context of a weighted wheel investigation when the resources are activated. More precisely, the *equilibration* and *force as a mover* p-prims are activated in the context of the wheel's return to its settled position; *abstract balance* is activated in the context of the weight balanced at the top of the wheel; *generalized springiness* and *Ohm's p-prim* are activated in the context of the displacement of the weight from equilibrium; and *overcoming* is activated in the context of the displaced weight's overshoot of the equilibrium position. The wheel is a particularly productive context because it affords the separation of the oscillation pattern into intuitively accessible segments, and because interaction with the wheel is good at activating intuitive knowledge because its behavior is closely observable and can be felt (e.g., the strength with which the wheel pushes back on your finger when raised to different heights).

**Attention.** Each of the knowledge elements is activated when students are attending to different segments of the wheel's oscillation. When they attend to the segment where the weight is at the bottom of the wheel (or at the top) before it begins oscillating or once it has come back to a stop, the resources *equilibration*, *abstract balance* and *force as a mover* are activated. When they attend to the force of the wheel pushing back on their finger when they displace the weight, *generalized springiness*, *Ohm's p-prim*, and *force as a mover*, are activated. When they attend to the overshooting of the weight beyond the equilibrium position, *overcoming* is activated. The



investigation was specifically designed to focus students' attention on the features of each segment that instantiated productive p-prims, with the hope of activating them for recruitment in the construction of a model of oscillation. During a whole class review of students' investigation findings, the teacher *explicitly directed* students' attention to the important features of each segment, if it appeared they had not already attended to those.

Through this analysis, we see that a number of features of instruction supported the activation of a number of elements of intuitive knowledge, including lesson content, teacher moves, and lesson structure. I will now discuss the activities of the engagement phase and the features of instruction that supported those activities.

**Engagement phase.** During this phase, the learner engages resources in the construction of patterns models. In the case of the oscillation unit, articulating, combining, refining, and connecting activities appear to have played important roles in the construction process.

**Articulating.** While experiencing separate steps in the wheel's oscillation may have activated particular elements of intuitive knowledge, it was necessary for students to vocalize these intuitive explanations, in particular because the unit was shorter and the teacher chose from among the intuitions to build a complete model of oscillation for the students. The worksheet that guided the investigation created a space for and scaffolded students' articulation, as it asked them to explain particular segments of the wheel's behavior on the basis of their prior knowledge. The teacher also played an important role in pushing students to fully articulate their ideas during the whole class discussion that reviewed the wheel investigation.

**Combining.** The teacher separated the wheel's oscillation into segments and elicited resources that could be used as foundations for scientific explanations of each segment. Once she had elicited resources for each segment, she connected them to scientific ideas and combined them to explain oscillation. The wheel example afforded separation into visually observable segments. Other examples, such as the vibrating rubber band, did not afford segmentation that was visually accessible to students.

**Refining.** The teacher refined the productive intuitions that students articulated in the context of the whole class discussion. She refined phrases like "speed beats equilibrium" to "momentum causes the wheel to overshoot its equilibrium position," by explaining that a mass with speed was what scientists referred to as momentum. The teacher used student language as the basis for defining scientific language. She revoiced students' ideas and drew connections between their ideas and scientific words.

**Connecting.** Once the teacher had built a model of oscillation (by combining the ideas students had articulated and refining those into scientific language) she drew connections between the example of the wheel and 2 of the examples the students had observed during the demonstration on the first day (i.e., swinging pendulum and vibrating rubber band), pointing out where, in each of the examples, each of the stages of the oscillation pattern was illustrated. The key feature of drawing connections between students' ideas and scientific ones is to draw explicit mappings between examples and the model, or examples and other examples.

Through this analysis, we see that a number of features of instruction supported the engagement of a number of prior knowledge elements in the construction of the oscillation pattern. These include worksheet design, using examples that can be deconstructed into constituent parts, connecting student language with scientific language, and drawing explicit connections. For this unit in particular we see the importance of the teacher. The centrality of the teacher during this unit was not the norm of the Patterns Class and as a result, the construction of the oscillation model was achieved through the collaboration of students and teacher. The teacher

focused students' attention on particular aspects of the wheel's behavior in order to elicit particular p-prims. She used these as foundations for introducing scientific concepts that she linked together in a model of oscillation. The students engaged with the model productively and showed evidence of understanding parts of it. The success of this unit is evidence that even a more teacher-centered model of constructivist instruction can help students build understanding of oscillation as a pattern and suggests that pattern-based curriculum can be implemented with varying degrees of student agency.

## Chapter 6: Discussion

I will now address each research question, presenting my general findings and placing those findings in conversation with those of previous research in order to delineate the theoretical, empirical, and practical contributions of my dissertation. I will end with implications of my research and questions for future investigation.

### Theoretical and Empirical Contributions

**Research question 1.** What were general tendencies in the development of students' pattern models with respect to *target models of the pattern* and as *domain-general models*?

The results of the macro analysis for all three patterns suggest that in general, students' pattern models increased in sophistication over the course of each instructional unit. Shifts in general tendencies of the model indicate that, despite occasional backsliding, over time more students included in their model more elements of the target model and gradually came to use domain-general language in its description. For all 3 units, students' models showed statistically significant increases in level of sophistication, indicating the importance of instruction in the development of their models.

**Deep structural knowledge.** These findings suggest that the 8th grade Patterns Class students were able to identify and articulate deeper structures underlying superficially diverse examples. They suggest that Chi and colleagues' characterization of novices as attending solely to surface features is incomplete, and that at least in the case of patterns such as threshold, equilibration, and oscillation, novices have the ability to see beyond surface features and identify deeper structures that underlie distinct phenomena.

**Domain-general knowledge.** These findings are consistent with those of Gick and Holyoak (1983) in which participants were able to derive a solution schema representing the abstract structure common to superficially different story problems. However, while the work of Gick and Holyoak similarly focused on domain-general knowledge, the kind of knowledge participants derived was a solution pathway for a particular type of problem. My findings therefore broaden the category of domain-general knowledge to include process models, in particular models that approximate threshold, equilibration, and oscillation patterns.

Students engaged effectively with patterns and developed more sophisticated models of each pattern over the course of the instructional unit. This is consistent with White's (1991) findings that students engaged well with intermediate causal models. White introduced intermediate causal models to students in order to facilitate their learning of both microscopic and formal mathematical models of phenomena. My research shows that students were able to create and refine their own pattern models, and therefore their own versions of powerful intermediate causal models, with the support of instruction. This suggests that it might be possible to broaden student agency within White's instructional approach by involving students in the construction of focal intermediate causal models.

My findings are consistent with the argument that students have a wealth of pre-instructional ideas related to patterns of change and control (diSessa & Lewis, in preparation; Swanson, 2012; Fitzmaurice, Sayavedra & Swanson, 2013). My findings extend this work by investigating how specific intuitive knowledge elements can be leveraged by classroom instruction in the construction of pattern models.

**Research question 2.** What prior conceptions emerged as resources for students' construction of pattern models? How did resources contribute to individual students' construction of pattern models?

In the case of each pattern (threshold, equilibration, and oscillation) prior knowledge was shown to play a productive role in students' construction of the pattern model. This finding and others described below challenge the *Theory Theory*/misconceptions view of naïve knowledge and provide evidence in support of the Knowledge in Pieces/resources perspective.

**Nature of naïve knowledge.** The microgenetic case analyses illuminated particular elements of naïve knowledge as resources for students' construction of models of threshold, equilibration, or oscillation. Most of the resources identified appeared to be previously documented phenomenological primitives. These findings provide support for basic theoretical elements of Knowledge in Pieces. My work extends the space of possible knowledge elements by proposing three new candidate p-prims. Two of these, which I called *abstract limit* and *dynamic limit*, were the pre-instructional ideas that were shown to play important roles in students' construction of the threshold pattern. I called the third idea *natural place* based on its connection to the Aristotelean notion. This idea arose in the context of the wheel investigation and did not appear to be productive in helping students construct a model of oscillation.

**Mechanics of conceptual change.** While the analysis of development in pattern models across drafts provides only snapshots of student thinking at different points in time, case-study analyses trace the development of students' pattern conceptions between drafts, and reveal shifts in thinking at a finer grain size. This microgenetic style of analysis is the gold standard of Knowledge in Pieces research, which places importance on capturing the details of conceptual change in order to document the complexity of the learning process and debunk theories that portray learning as a *gestalt switch* in which the learner goes from one way of thinking to an entirely new and even incommensurable way of thinking. My case analyses reveal complex processes that do not correspond with a simple gestalt switch. Rather, the processes appear to involve many micro-shifts in student thinking that depend on learner, content, and instruction.

**Role of naïve knowledge in conceptual change.** My microgenetic analyses identify productive contributions of prior knowledge to students' construction of models of threshold, equilibration, and oscillation. These findings clearly support the resources perspective of naïve knowledge. This study makes a particular contribution by crafting a detailed picture of how prior knowledge is leveraged in the construction of scientific knowledge in the complexity of real-time classroom learning. This is an important contribution, as even among empirical work done within the KiP perspective there exist few microgenetic studies that track the construction of normative knowledge on the foundation of naïve knowledge in the classroom context (diSessa, 2014). This work also contributes to the particular line of work within the resources perspective that challenges deficit narratives of students from non-dominant communities. The students participating in the Patterns Class were from non-dominant communities; their success in constructing sophisticated models of patterns is evidence of the power of their prior knowledge and everyday sense-making practices and their commitment to their own learning.

**Research question 3.** What aspects of the process of pattern model construction emerged as particularly important? What features of instruction supported important aspects of the knowledge construction process?

Instruction was designed to leverage students' prior knowledge toward their construction of pattern models. It was refined in a bottom-up fashion over cycles of design, test, and refinement. The initial Patterns Class consisted of basic activities that elicited students' prior

knowledge and engaged it in the process of model construction. Students explored pattern exemplars, described the behavior common to examples, generated additional examples of the pattern, and refined pattern descriptions. In the pattern models and examples that emerged, our research team looked for resources for the construction of more sophisticated pattern models. Over iterations we fine-tuned the structure and content of each pattern unit, however the basic structure of exploring examples and generating and refining pattern models remained the same.

In looking back at case-study data, certain aspects of the process of pattern model construction emerged as particularly important (see Figure 28, below). These aspects can be organized within two general phases of the knowledge construction process. I call these phases: *activation* and *engagement*. During the first phase, prior knowledge, including ideas that are resources, are activated. Activation depends on context, attention, and orientation. During the second phase, particular ideas among those activated are engaged in the construction process through activities such as articulating, combining, mapping, reinforcing, removing, generalizing, refining, and connecting. Findings from the threshold, equilibration, and oscillation units suggest a general model for the process of pattern model construction, as mediated by instruction.

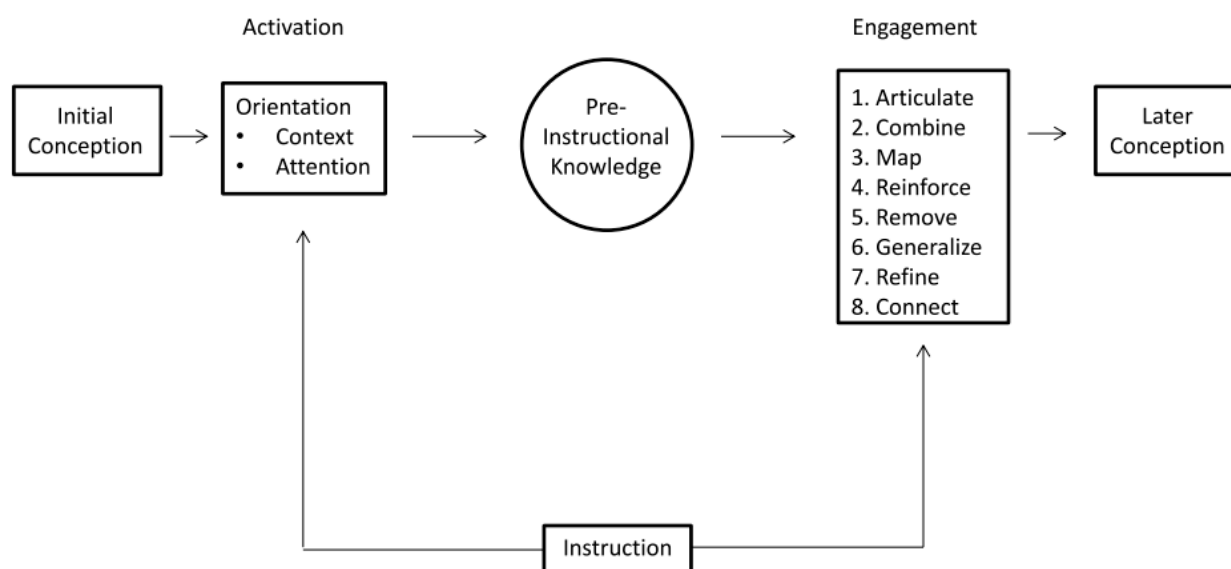


Figure 28. A model of the process of pattern construction as mediated by Patterns Class instruction

***The Knowledge Integration framework.*** This model of instructionally-mediated knowledge construction connects with the Knowledge Integration (KI) framework put forth by Linn et al. (Linn, Clark, & Slotta, 2003; Linn, 2006). Linn's framework for instructional design consists of *eliciting student ideas*, *adding new ideas* through inquiry activities, and scaffolding reorganization and refinement of students' thinking through *distinguishing ideas* and *reflecting* and *sorting out ideas*. *Eliciting student ideas* is a general category of activities that correspond to the activation and articulation of prior knowledge included in my model. *Adding new ideas* is related to the 8th activity listed in the construction phase, which refers to building connections between scientific ideas and resources. *Adding* and *distinguishing ideas* correspond with construction activities such as combining, reinforcing, removing, and refining. *Reflecting* and

*sorting out ideas* are steps of the KI framework under which mapping and generalizing activities can be organized.

The general model that I offer suggests designing instruction that facilitates or supports activation and engagement phases. A number of features of Patterns Class instruction support activation and engagement phases, one of the most prominent is the whole class theory-building discussion. The whole class discussions used frequently in the Patterns Class are comparable to *Benchmark Lessons* developed by a high school physics teacher, Jim Minstrell (Hunt & Minstrell, 1994; diSessa & Minstrell, 1998).

**Benchmark lessons.** Benchmark Lessons were designed to take place at the beginning of a new unit and functioned to elicit students' prior knowledge around the new topic. They served as a benchmark for comparison with new ideas that emerged over the course of the unit and helped students actively modify their thinking and bear witness to the transformation process. The whole class theory-building discussions of the Patterns Class were not limited to the beginning of a unit. They served the function of eliciting students' prior knowledge around a particular topic. In addition to supporting the activation and articulation of ideas, these discussions created space for students to participate in other aspects of the knowledge construction phase outlined in the model above. The students in the cold milk theory-building discussion engaged in *reinforcing* when they revoiced each other's ideas, *removing* when they removed the wall from the explanation, and *refining* as the anthropomorphic language fell away from their descriptions of the pattern. The model of instructional design proposed by this research therefore extends the functionality of the Benchmark Lesson beyond an introductory activity focused on eliciting prior knowledge, to one that supports knowledge construction, as well.

### Practical Contribution

In addition to the theoretical and empirical contributions discussed in the context of each research question, I make a practical contribution to the field of science education by objectively validating the design of instruction that demonstrably develops students' abilities to notice and articulate patterns of behavior that can be used to organize and explain phenomena across the sciences. Students engage in the study of these powerful scientific constructs through authentic practices such as modeling, explanation, and argumentation. Both the knowledge and practices of the Patterns Class are important foci of the Next Generation Science Standards and the curriculum is among the first to focus on the development of students' understanding of patterns.

**A novel version of science.** In addition to the knowledge and skills outlined in the NGSS, the Patterns Class presents students with a novel picture of the scientific enterprise. Much science curriculum that has been designed to emulate the practices of professional science has focused on empirical activities, such as inquiry-based learning. While certain versions of inquiry have focused on the theoretical side (White, Frederiksen, Collins, 2009; Lehrer, Schauble, Carpenter & Penner, 2000; Lehrer & Schauble, 2003; 2004), much inquiry curriculum is focused on engaging students in practices of observation, data collection, and representation. The Patterns Class includes empirical investigations but the theoretical half of the inquiry process is more heavily weighted. Moreover the theoretical half is not focused on engaging students in activities around *existing theories*, rather, it is focused on scaffolding students' construction of *their own theories*. Much instructional time is devoted to activities such as theory building (constructing theories that explain the behavior of pattern examples), and pattern modeling and refinement (constructing models of the general processes that underlie multiple examples).

Participation in the theoretical activities of the Patterns Class is also beneficial because it develops deep structural understanding and engages students in epistemic practices that are not traditionally the foci of authentic science-based classroom instruction such as *pattern identification* and the *characterization* and *classification* of phenomena according to *deep structure similarity*. Engagement in such practices not only develops skills for playing the game of science, but a broadened sense for the nature of that game.

**Equitable science instruction.** The Patterns Class features a strengths-based curriculum that values and leverages the prior knowledge that individual students bring to their learning. The responsive nature of the instructional design does not privilege *formal knowledge* over *everyday knowledge*. It supports students in the task of knowledge construction by building on their strengths. Though instruction has been designed with target scientific models in mind, it is meant to support students' construction of their *own* pattern models and strives to treat students as autonomous agents of knowledge construction.

The general nature of patterns makes them accessible to students from diverse backgrounds. Threshold, for example, can be explored in contexts ranging from the stability of physical objects to the relationships and emotions of people. Students can think carefully about the aspects of an example with which they resonate and construct their model of threshold on the basis of those. Because they afford a multitude of diverse entry-points to their exploration, patterns are an excellent object of thought around which to engage students from all backgrounds and levels of academic preparation in abstract thinking and authentic practices of science.

**Engagement.** Students were highly engaged in Patterns Class activities; their engagement in theoretical activities was particularly striking. Challenging cognitive activities such as theory building are made accessible because students can approach them within the context of familiar examples, and because pattern models can be constructed on the basis of intuitive knowledge. Challenging activities within one's *regime of competence* are a recipe for engagement (diSessa, 2001).

**From managing hyperrichness to leveraging hyperrichness.** Despite the power of constructivist instruction, many teachers avoid it due to a practical problem with its implementation. It is easy enough to dedicate time to eliciting student ideas, but what does the teacher do with all of the ideas their students share? Students have so many ideas, some of which are productive with respect to a topic of instruction, and some of which are not. The wealth and diversity of student ideas has been termed *hyperrichness* within the KiP paradigm (diSessa, 2001). How does a teacher respond to this *Pandora's Box* once they have opened it and populated the learning space with so many diverse ideas?

Perhaps the answer lies in transforming the question from one of *managing* hyperrichness to one of *leveraging* hyperrichness. In this case a teacher's goal becomes selecting from a plethora of ideas those that will be most productive in the construction of powerful knowledge, as opposed to responding to every idea that their students share. Knowing which ideas will be productive follows from experience. Interestingly, when given the opportunity to spend enough time exploring examples, thinking about the pattern on their own, and talking with their classmates, many students in the Patterns Class seemed to autonomously select and build on the most productive of the publicly shared ideas.

### **Implications and Questions for Future Research**

My findings imply that students have a wealth of prior knowledge that can be leveraged by instruction toward their development of models of threshold, equilibration, and oscillation patterns. Questions for future research are motivated both by implications of prior research reported in the literature review, and by findings of the present study.

Findings from literature on patterns-like knowledge suggest that knowledge of domain-general structures like patterns can be powerful for transfer (Gick & Holyoak, 1983). It would therefore be fruitful to test instruction that teaches scientific phenomena by first introducing the corresponding model of the pattern. Once students had constructed a general model for the pattern by considering familiar examples, they could apply that model to make sense of other phenomena that follow the same pattern. For example, students might learn about the periodic pulsation of Cepheid variable stars or the oscillation of the Briggs-Rauscher chemical reaction after first constructing a model of oscillation based on careful investigation of the weighted wheel. The question of transfer could be investigated in the context of instruction through design-based research. Such instructional design is closely related to the use of bridging analogies and anchoring conceptions suggested by Clement, Brown, and Zeitsman (1989).

White's (1991) findings about the success of intermediate causal models for teaching mathematical formalisms suggest that patterns could be leveraged as an on-ramp to the study of particular functions and even basic differential equations. For example, the development of the equilibration model in the context of temperature change could be used as the foundation for understanding the differential equation that describes Newton's law of warming and cooling. The ability of patterns to act as intermediate causal models and provide a conceptual bridge to mathematical formalisms could be investigated, again, in the context of instruction through design-based research.

Finally, both conceptual change and constructivist instruction literatures suggest that students have a wealth of resources, not only for constructing conceptual understanding, but also for participating in the practices of science, and for understanding the nature of the scientific enterprise. This suggests that, in addition to the conceptual resources identified by this study, students have resources for engaging in pattern-related practices and for constructing meta-pattern knowledge (regarding the distinguishing features of pattern knowledge and what makes it useful, as well as how one can go about developing it). Resources for engagement in practices of pattern construction and the construction of meta-pattern knowledge are subjects of future research. Existing video and student work from this iteration of the Patterns Class will be used as sources of data for analysis that will inform the design of subsequent iterations of Patterns Class instruction focused on leveraging students' resources for practices and meta-pattern knowledge toward their development of those capacities.



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