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Airline Delay Perturbation Problem Under Minor Disturbance

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Preface and Acknowledgments

This working paper presents the preliminary study of airline operational strategies under minor disturbance. In chapter 1, scopes and limitations of the minor delay problem are discussed. Chapter 2 then introduces previous related researches. Finally, in chapter 3, we built a mathematical model to formulate this delay problem and introduced possible ways to resolve this time-space network flow problem. Future studies are required to validate the model by adopting actual operation records.

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CHAPTER ONE INTRODUCTION

During the daily operations, the punctual performance of the flight schedule is subject to various factors. Schedule controllers/dispatchers might confront random flight delays due to rough weather, air space congestion, maintenance needs, prolonged ground holding …etc. Compared with such problems as aircraft shortage or temporary closure of airport, however, the consequence of the abovementioned delays is minor. Still, the operations could be severely disrupted without effective measures.

Although air carriers design "buffer time" to ease the schedule delays, for some bigger incidents mentioned above, the buffer time would not be able to absorb all the delays. Hence, schedule controllers/dispatchers are required to make on-the-spot decision to these incidents to maintain the punctuality of the operations.

Often, the phenomena of delay propagation won't result sever damage to international flights, since these flights contain more buffer time in both ground and airborne. The chance of absorbing these delays within the flight leg will be higher. This is not the case for domestic flights, becausethese flights tend to have less buffer time (both ground and airborne).

Comparing with international flights, the number of flight segments that each aircraft need to be served is higher for domestic ones. If we further consider the indirect cost to passengers generated from the delay propagation, the potential delay cost due to propagation from the previous flights will be enormous.

It entails much knowledge and experience to supervise air transportation. When schedule controllers anticipate a flight delay, as the circumstantial cases vary, they need to take measures including speeding up the on-going flights, putting off following ones to ensure connectivity, ferry spare aircraft to support operation, maneuver existing fleets to meet the timetable lag, and canceling flights…etc. Since it takes long to ease major perturbations like airport closure and aircraft shortage, accelerating fliers may prove unrealistic considering the limited engine-powered nature of aircraft.

Nevertheless, this strategy can play a significant role when minor delays occur. Since the scale is small, we can combine with delay flights as well as cancellations, if necessary, to allow better performance of recovery under the disruption of the system.

The aim of this research is to evaluate the optimal and sub-optimal solutions for minor delays that can not be absorbed by the built-in buffer time in domestic airline operation. Given the actual time-space network, we apply mathematical programming technique to build the models. The strategies mentioned above will be taken as side constraints inside the model.

Nature of airline perturbation under minor disturbance

Recurrent delays can be completely resolved by the built-in buffer time, but not so in the case of non-recurrent ones. For the low-fare, low-cost air carriers like Southwest, America West, and Delta Express, high utilization of aircraft is inevitable to make the bottom line. On average each flier under such short-haul airlines travels five to six legs per day in comparison to the long-distance counterpart flying over three to four (numbers of legs vary from different average stage length). Furthermore, the ground holding time for aircraft with multi-stop schedule may last for 20 to 30 minutes in each stop. Under such circumstance, the problem of delay propagation will only aggravate when the aircraft carries all these delays to subsequent flights.

Taking Southwest airlines flight 1775, for example, the flight departs from Oakland at 12:50PM and arrives at Baltimore at 11:25 PM via three stops-- Reno for 20 minutes, Salt Lake City for 25, and St. Louis 20 in sequence. The execution of multi-stop itinerary limited by such short ground time will be much disturbed if there is one or more non-recurrent delays, such as airspace congestion, air traffic control delay, airport closure, aircraft out of service ...etc.

Airline real-time network provides instantaneous aids to airline scheduling managers whenever there is an incident taking place in the system or the propagation of delays is massive enough to affect normal operation. These various perturbation causes include detrimental meteorological condition, aircraft malfunction, unexcused absence of crews, breakdown of airport ground facilities, and prolonged customs security inspection (as shown in the case of positive passenger bag match). The major goal of this real-time decision support framework is to maintain regular operations of the routing of aircraft, arrival and departure time of the flights despite the numerous factors that might decrease the possibility of keeping up the timetable. Although airline-scheduling research has been popular in recent years, very little work has been devoted to the real-time decision support framework under disruptions.

Airlines nowadays equipped with certain kind of decision support systems (DSS) to help schedulers/dispatchers with daily/weekly timetable scheduling. Since the duration of schedule planning stage often last for several weeks, solution optimization is a more concerned objective than the computational time required of the decision support system. Compared to the lengthy planning stage of decision support system, real-time decision support model has far limited computational back-up available. Rather than close optimal solution, real-time decision support systems often provide "good" feasible solutions. Hence, trade-off between computational time and optimization is the major difference between planning decision support systems and real-time ones.

For the research purposes, the real-time decision support framework is often divided into two parts, flight and crew. Flight real-time decision support framework deals with the impact of accidents inflicted on aircraft by adjusting the flights to bring all the aircraft back to normal schedule within shortest possible time. Crew real-time decision support framework tackles with the rerouting of crews after the flight adjustments are completed.

Often, after finishing re-scheduling flights, schedulers/dispatchers will pass the modified result to crew coordinators. Crew coordinators will then cull the possible re-route pairs to ensure that all available crew resources can be fitted into the updated schedule. If unable to do so, crew coordinators will return it to the schedule department pointing out certain constraints of such crew re-planning. Schedule department then has to revise the schedule once again according to these new references. This to-and-fro process will only repeat itself until both departments come to an agreement.

CHAPTER TWO LITERATURE REVIEW

Teodorovic (1988) summarized several possible decision-making strategies for the fleet perturbation problems. These strategies includes

A. Cancel a certain number of planned flights, with none of the remaining flights being delayed,

B. Introduce one or more reserved airplanes into operations, should there be any available,

C. Cancel a certain number of planned flights with a certain number of the remaining flights being delayed and

D. Design a new airline schedule so that none of the planned flights are canceled, but accepting a certain number of delayed flights.

These possible decisions formed the general strategies for the perturbation problems. The objective of Teodorovic's (1984) dispatching strategy for a disrupted airline network is to minimize the total number of passenger delayed because of breakdown of one or more aircraft in the beginning of the daily operations.

This problem is formulated as a network flow problem. In the network, nodes represent flights, while arcs stand for time loss on individual flights due to rescheduling departure time for the flights. One strategy in this model has been evaluated, which is delaying flights without canceling any flight. The model is solved by branch and bound technique. Although this model considers perturbation of the schedule, it restricted the incident time at very beginning of the day and no recovery of the aircraft is considered. This further limits the applicability of the model. Besides this, multiple fleet/capacity and passenger flow activities are not considered. Despite the fact that many assumptions are added to simplify the modeling difficulties, this paper opens up a wide researches and discussions of the perturbation problem.

Jarrah et al. (1993) considers a real-time perturbation problem under temporary shortage of aircraft. The paper starts from introducing a successive shortest path method (SSPM) and applies the method to the problem as a time-space network model. Two network models--delay and cancellation models-- are presented, the operation of which starts when the incident takes place and end at the recovery time of the aircraft. No combinations of the strategies were deliberated, though. The time-space network model used in the model composes aircraft and flight matching as well as the linkage between flights. With the setup, this model provides both aircraft scheduling and rotation solution in one integral framework. The problem was solved as a general minimum-cost network, which involves multiple sources and sinks. The algorithm only focuses on a station at a time, hence how efficient this greedy method is still unknown yet. Due to the complexity of the network formulation, and lack of large-scale evaluation in the paper, we don’t quite sure how efficient the CPU time will be for a large-scale problem.

Teodorovic and Stojkovic (1995) try to integrate crew, aircraft and maintenance scheduling together. With lexicographic optimization technique, the problem will be first optimized by the first-priority objective function, which is to maximize total number of flights flown. If there is a tie, then second priority objective function, which is to minimize total number of passenger delays, is used to break the tie, and so on.

Unlike the conclusions of all related papers, this paper starts with regenerating new crew rotations and then aircraft rotations. They claimed this switch would substantially decrease the CPU time. In both rotations, two techniques, first in first out (FIFO) and a sequential approach based on dynamic programming, are developed to find the optimal solution. After obtaining the flow of network, network decomposition method is used to generate each crew and aircraft's rotation. However, since the problem is solved by heuristic methods, the effectiveness of the solution remains unknown. Also, aircraft rotation solutions, coming from the output of crew rotations, are likely to restrict aircraft assignment and thus unrealistic in terms of practice.

Cao and Kanafani (1995) improve the model from Jarrah et al. (1993) and combine two models, cancellation and delays, altogether. They formulate the problem as a quadratic 0-1 problem and consider ferry flight to give the model more flexibility. Since the order of stations in the network developed by Jarrah et al. (1993) will affect the solution result, Cao and Kanafani set higher priority for hub stations to generate better solutions. Furthermore, the cost function discussed in the paper is one of the most detailed in recent literatures.

Yan and Yang (1996) formulate an operational perturbation problem under one or more temporary aircraft breakdown as a dynamic network flow problem. Given the breakdown time and station as well as the aircraft recovery time and the termination of the cumulating delays, the objective function of this problem is to minimize the cost of the operations.

First, the model was built as a basic schedule perturbation model (BSPM), and then by adjusting objective functions, or adding some side constraints, this model can be modified to adopt such strategies as cancellations, delay and ferry flights, or a combinations of these kinds. Since the BSPM and cancellation strategies are both formulated as pure network flow problem, traditional network simplex method is capable of solving these two strategic problems. Delaying and ferrying flights-- these two strategies have side constraints, and the solution method for these strategies uses Lagrangian relaxation with subgradient method to approach the optimal solution. The formulation solves the problem in a systematic way, hence the efficiency can be predicted.

Yan and Lin (1997) carry out the research into the perturbation operations under temporary closure of the airports. The model uses time-space network similar to Yan and Yang's formulation. However, the model further provides flight delays, modification of multi-stop flights into non-stop flights, and ferry of idle aircraft. The basic model, cost minimization model and ferry of idle aircraft strategic model are formulated as pure network problems, and can be solved by network simplex method. Both delaying flights and modification of multi-stop flights strategic models are formulated as network flow problems with side constraints and can be solved by using Lagrangian relaxation with subgradient method.

CHAPTER THREE MODEL ESTABLISHMENT

This research will use time-space diagram as a graphic representation of the model. Basic concepts of the time-space diagram will be introduced and the basic model for minor delay perturbation problem will be presented.

Afterward, we will bring out some possible strategic models such as speed up, delay flights, cancellation, and swap operations. The combinations of these strategies are possible.

Introduction to time-space network

Time-space network models, also known as dynamic network models, have proved to be an effective modeling framework for a range of planning problems burgeoning in scheduling and routing. The use of time-space networks in an optimization framework was well established in the 1950's. Dantizig and Fulkerson (1954) formulate a tanker-scheduling problem using dynamic network. Time-space network provides a clear way of viewing the structure of the problem and it can be easily comprehended due to its logical setup.

Time-space network can be described as variations of two broad flavors. (Stochastic and dynamic networks and routing) In a fully dynamic network, every link moves forward in time from t to some time t+x, where x is larger than zero. A special case of a fully dynamic network occurs when x is equal to one for all links. We refer to this as a staged network. For a fully dynamic network problem, it is often convenient to transform networks with links that span more than one time period into networks where all links move forward exactly one time period. At another extreme are networks that are dynamic inventory networks, which are dynamic sequences of static problems.

The only dynamic arcs are inventory or holding arcs. Operational problems tend to look more like fully dynamic networks, while production planning problems are often modeled using a large time step, where all activities take place within a time period with the exception of inventory holding.

The time-space network can be represented in several ways. First is the generic two-dimensional time-space network. Space (which is airport in airline scheduling problem) is represented in one dimension while time lies in the other dimension. Second is the complicate two-dimensional time-space network. In this network, we further divide space into finer segments, such as aircraft arrival time, aircraft ready for departure time and flight departure time in a specific airport. The major difference between these two representations is that the complicate network directly solves both fleet scheduling and aircraft rotation problems at once. The generic network considers the fleet scheduling problem first, and then solves the aircraft rotation problem by using network flow decomposition method. The third representation of the time space network is three-dimensional time-space network. This network often consists of a series of two-dimensional maps, stacked from bottom to top to represent time periods. Arc can move from a point on a map in one time period to a point in different map representing a later time period. However, not many researchers use this representation probably because of the high complexity of the graph.

This research will take generic two-dimensional time-space network as our graphical representation.

Basic concept of generic time-space network

The generic time-space network consists of two dimensions. One is space, which is airport in airline scheduling applications, and the other is time. Each node within the network represents an event taking place in a specific airport at a specific time. Each arc with in the network shows the linkage among different node events. Here we are going to define different types of nodes and links within our generic two-dimensional time-space network.

Classifications of Arc

In airline perturbation problem, the arcs can be represented as follows:

1. Flight arc

Flight arc connects aircraft departure node to aircraft arrival node, which means a possible flight. Generally, the amount of flow on this arc is one, but if we have two flights depart at the same time and go to the same location, the flow can be greater than one. The cost of flight arc is the operating cost.

2. Ground holding arc

This arc connects both arcs within the same airport but different time horizon. The amount of flow in the arc represents the number of aircraft kept during the ground holding time. There might be some costs of parking charges, facility charges, ...etc. Often, no revenue will be generated in a ground holding arc.

3. Overnight arc

Overnight arc is another type of the ground holding arc. The amount of flow means the number of aircraft staying overnight in the specific airport. The cost is most likely the same as that for ground holding arc, however, for some airports overnight charges may be applied.

4. Ferry arc

Ferry arc indicates the spare aircraft flying without carrying passengers to an airport to support the operation under perturbation. Ferry arc will start from a source node (supply node), which generates one more unit (or more) of flow whenever the schedule controllers consider ferrying a spare aircraft to support the operation when there is any available. We connect the source node of the spare aircraft to all the other airports and set the total flow of this bundle arcs less or equal to one. The cost for ferry arc is just the operating cost and some administrative costs. No revenue will be obtained.

5. Delay arc

Delay arc is the production of the delay strategy. We add up some parallel arcs to the scheduled flight arc with some discrete interval, such as five minutes. For example, Assuming that a flight departs from San Francisco International Airport at 9:00 PM and arrives at Los Angeles International Airport at 10:00 PM, we can add delay arcs departs at 9:05 PM, 9:10 PM, 9:15 PM ...etc., and arrive at 10:05 PM, 10:10 PM, 10:15 PM ...accordingly. The cost for the delay arc will be the delayed cost, as well as the operating cost due to the delay.

6. Speed up arc

Speed up arc will reduce the flight time even though the origin is the same with the aircraft departure node (or source node), and the arrival time with speed up arc will be earlier compared to the scheduled arrival time. Amount of time we can manage from speeding aircraft up will be determined by the distance between the departure and arrival airports. If the aircraft is on the air, it will depend on how far the aircraft is from the destination. The cost for speed up arc is just the fuel cost. No other penalty cost will be generated.

Classifications of Node

Several different types of node inside the time-space network will be introduced as follows:

1. Aircraft arrival node

Aircraft arrives at a specific time in a specific airport, and this node records the arrival time and airport. In the network, a flight link will emerge into the aircraft arrival node.

2. Aircraft departure node

Aircraft departs at a specific time from a specific airport. In the network, flight link exists from the aircraft departure node and connects to aircraft arrival node at the other station.

3. Source (Supply) node

Source node represents number of aircraft available by request in a specific airport. The source node can be either in the beginning of the day, when the initial supply comes into use, or later at some specific moments of the day, at which time the recovery of the aircraft is assured or the spare aircraft becomes available.

4. Sink (Demand) node

The demand node can be taken as the incident node, if it appears in the middle of the day. Incident demand node indicates a shortage of aircraft at the time moment in a specific airport. If it appears at the end of the day, this is just the demand node. This means that after all the operations we have to have as many aircraft located at the airport at the end of the operating period as that in the morning.

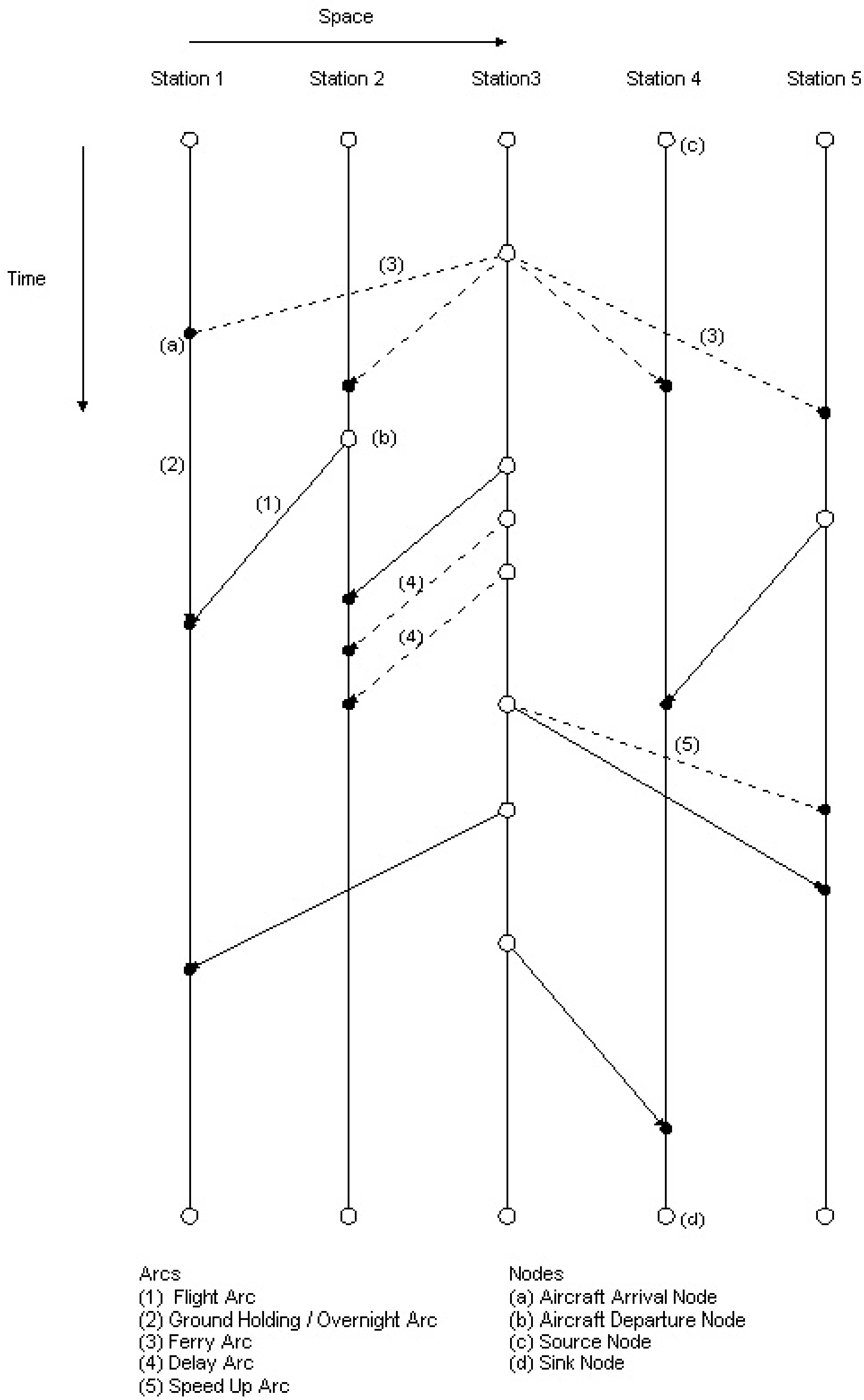

Figure 1 Time-space network

Basic model for minor delay perturbation problem

The basic model for the minor delay perturbation problem is to model the consequence of delay whenever there is an incident. The network will be segmented into several perturbation time zones, with a starting time point, a recovery time point and an ending time point. Starting time point shows the time schedule controllers anticipate and are prepared to make some control decisions. At this moment, the controllers will enumerate all the flow at the starting time point, both ground holding arcs and flight arcs. We will then set up all the initial supply nodes for each station by adding up the flow on the ground at the starting point.

Recovery time is the duration needed to supply the want of aircraft. However, the delay propagation will still cumulate at this time moment. Ending time will finish all the perturbation and return the schedule back to normal operations.

The basic model for the minor delay perturbation problem can be formulated as follows:

Basic Model

$$\text{Minimize} \quad Z = \sum_{i,j \in links} C_{ij} X_{ij} \tag{3.1}$$

$$\text{Subject to} \quad \sum_{j \in O(m)} X_{mj} - \sum_{k \in D(m)} X_{km} = b_m - \alpha_m \quad \text{for all } m \in \text{incident point sets} \tag{3.2}$$

$$\sum_{j \in O(r)} X_{rj} - \sum_{k \in D(r)} X_{kr} = b_r + \alpha_r \quad \text{for all } r \in \text{recovery point sets} \tag{3.3}$$

$$\sum_{j \in O(i)} X_{ij} - \sum_{k \in D(i)} X_{ki} = b_i \quad \text{for all } i \in \text{Node} - \{m, r\} \tag{3.4}$$

$$L_{ij} \le X_{ij} \le U_{ij} \quad \text{for all } i, j \tag{3.5}$$

$$X_{ij} \in \text{integer} \quad \text{for all } i, j \tag{3.6}$$

Decision Variables are:

Cij = Cost for arc (i, j). If the arc is a flight arc, the cost will be the operating cost minus revenue for passenger on board. If the arc is a ground holding arc, the cost will be the ground holding cost.

Xij = Flow in arc (i, j). Flow will be bounded by its upper bound Uij as well as lower bound Lij. If Xij belongs to flight arc, it’s mostly likely that the upper bound is one. However, having two or more flow in the flight arc is possible, but not practically useful. If Xij belongs to the ground holding/overnight arc, the upper bound will be the capacity of the parking and holding space the carriers allow to use in the airport.

Bi = net flow in node i. If node i belongs to supply node, Bi will be positive. If node i belongs to sink node, Bi will be negative. Otherwise, Bi is zero for all transient nodes.

_i = number of new aircraft available or unavailable/out of service in node i. _i means the number of aircraft needed when node i belongs to the incident node, and the number of extra aircraft when node i belongs to the recovery node.

O(m) = the set of nodes that emanates from node m.

D(m) = the set of nodes that goes to node m.

The basic model for minor delay perturbation problem considers no strategy. Objective function (3.1) is to minimize the operating cost. Of course, the objective function can be changed because of different company policies. The constraints consist of several flow conservation equations and some upper and lower bounds of the decision variables. Equation (3.2) indicates the flow conservation in the nodes where incidents occur. We define the incident as the time that the aircraft need to arrive at the node but are unable to do so. Therefore, at incident points, all the flow going from the incident node i to all the downstream nodes have to be deducted by the number of aircraft that can't arrive at the node on time. Equation (3.3) indicates another flow conservation equation when recovery occurred. Aircraft that couldn’t catch the scheduled arrival time will then arrive at the recovery node. At the recovery nodes, the flow going on these nodes to downstream nodes equals to the flow going these recovery nodes plus the number of recovery aircraft. Equation (3.4) shows the flow conservation in all the other nodes. Equation (3.5) indicates the upper and lower bound for the flow in the arcs. Finally, equation (3.6) denotes integer requirement for the flow in every single arc.

This basic model is a pure network flow network. According to all these researches before, the problem can be solved by using network simplex method, one of the most popular techniques for network models.

Strategies models for minor delay perturbation problem

Five Strategies were considered in the research. They include speeding up, canceling, delaying flights, swapping operations and finally ferrying spare aircraft.

<u>Speed up strategic model</u>

In the speed up strategic model, we build up extra arcs that link to the origin node but the travel time between two points is shorter than the original one. The figure below shows the way we arrange the speed up bundle.

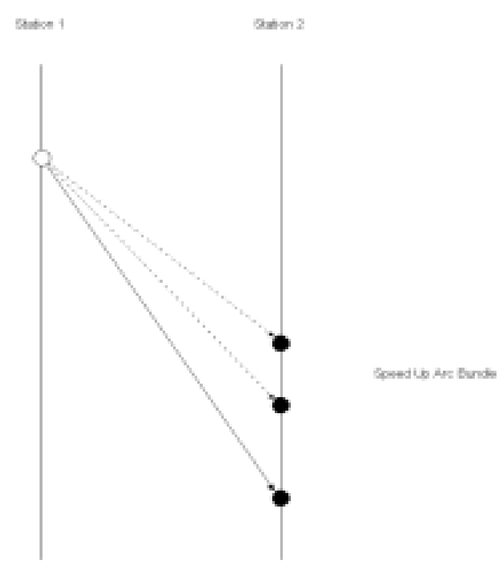

Figure 2 Speed-up flight arc bundle

$$\sum_{(i,j)\in B(K)} X_{ij} \le 1 \qquad \text{for all } i,j \text{ belongs to Bundle } K \qquad (3.7)$$

Where

B(K) = the bundle set K contains all possible speed up arcs.

The objective function is the same as the basic model, as well as all the constraints. We have to add one more speed up arc bundle constraint. Also, we make the constraints less or equal to one to allow cancellation for this speed up flight.

Cancellation Strategic Model

In cancellation strategic model, according to Yan and Yang, this strategic model can be achieved by simply changing the cost function to allow cancellation. The objective function can be changed as follows:

$$\text{Min } Z = \sum_{(i,j)\in \text{ground holding arcs}} C_{ij} X_{ij} + \sum_{(i,j)\in \text{flight arcs}} C_{dij} X_{ij} + \sum_{(i,j)\in \text{flight arcs}} C_{cij} \qquad (3.8)$$

Where we have

C_{dij} = $C_{ij} - C_{cij}$

C_{cij} = the cancellation cost of flight (i,j)

Swap Operations Strategic Model

If there is an aircraft that can not catch up with the next flight scheduled to serve, we will then find another aircraft which is already or will soon be available before the departure time of the flight and swap the delayed aircraft assignment with the available one. This strategic model requires a pose-processor program to accomplish the task. Since the research uses generic time-space network, it is required to separate flight scheduling problem and fleet assignment problem. Hence, in the post-processor, flow decomposition method will be used to carry out the strategy.

Delay flight strategic model

Delay flight strategic model has the same objective function. The only Difference between the basic and delay flight model is that the delay flight model has one more constraint, which is the delay bundle arc constraint. The formulation of this constraint is similar to the speed up one.

$$\sum_{i,j \in B(K)} X_{ij} \leq 1 \qquad \text{for all } i,j \text{ belongs to Bundle } K \qquad (3.9)$$

The network representation is shown as follows:

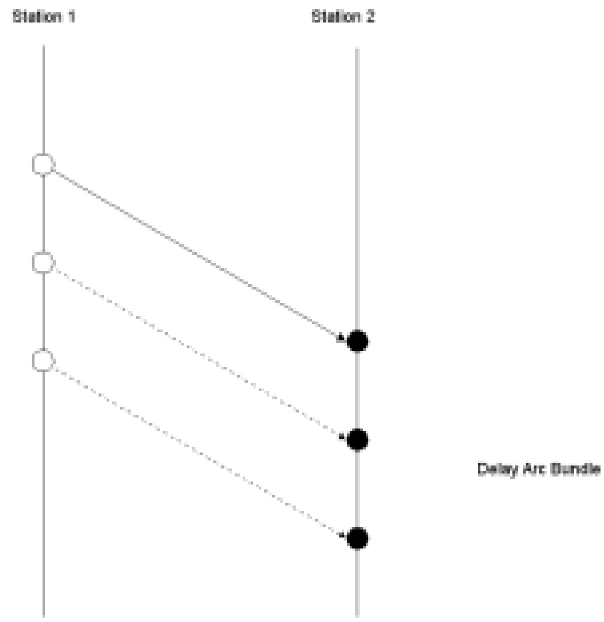

Figure 3 Delay flight arc bundle

As we can see, we generate some parallel arcs shift downward from original flight arc and the mathematical programming formulation constraints the upper bound flow of the bundle arc to be one.

Ferry flights

In case of incidents, airline companies always reserve some spare aircraft in specific stations. Once incidents happen, airlines could ferry these spare aircraft to some stations to support the operations. We place some position arcs, and set the upper bound of these bundle arcs to be the number of available aircraft. The formulation can be formulated as follows:

$$\underset{i,j\ B(K)}{X_{ij}} \leq 1 \qquad \text{for all } i,j \text{ belongs to Bundle } K \qquad (3.10)$$

In addition, the network representation can be shown as follows:

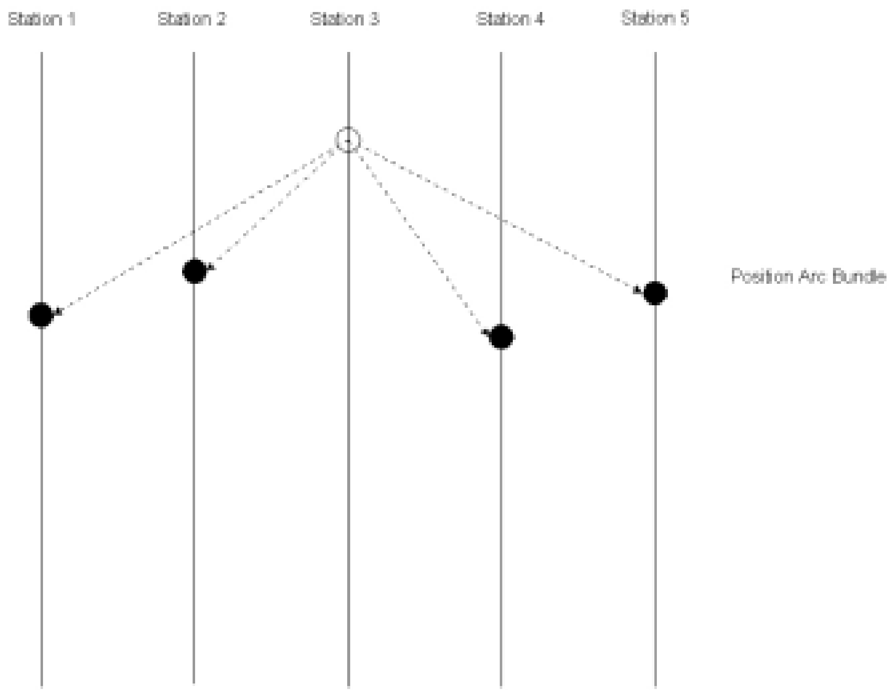

Figure 4 Ferry flight position arc bundle

Conclusion

Among these strategies, it is possible that we just combine some of the strategies, and theoretically speaking, this will lead to better solutions.

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